

Partial compositeness: From anarchy to symmetry

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Within the composite Higgs paradigm, partial compositeness has emerged as an elegant mechanism for generating large flavor hierarchies such as are observed in the quark and lepton masses and mixings. This mechanism exploits the strong renormalization group effects of the compositeness dynamics when these are *not* flavor symmetric. Despite its remarkable properties, at this point it is stringently constrained by the body of flavor- and *CP*-violation tests, so that the compositeness scale must be at least $O(100)$ TeV, beyond the direct reach of proposed colliders. On the other hand, composite Higgs theories with flavor-symmetric strong dynamics, but with realistic flavor-violating hierarchies introduced in an *ad hoc* manner, can extend the Glashow–Iliopoulos–Maiani mechanism of the standard model and thereby be far less constrained, at the edge of LHC reach and well within reach of future colliders. We show how the best features of both of these types of dynamics can be combined if flavor symmetries of the strong composite dynamics are emergent in the IR near the compositeness scale but absent in the far UV. In this case, flavor hierarchies can be generated by the renormalization group flow in the UV, followed by an IR stage in which the dynamics flows towards accidental flavor and *CP* symmetries. We point out how the collider and low-energy phenomenology is significantly impacted by the IR stage. Our analysis includes a discussion of the distinctive features of the small neutrino masses and their large mixings.

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I. INTRODUCTION

The Standard Model’s (SM) Higgs mechanism and Higgs boson remain among its most enigmatic features and are the focus of intense experimental efforts to probe. Its couplings are central to the hierarchical structure of particle physics that we see, both in the sense of the hierarchy problem related to the scale of ElectroWeak (EW) symmetry breaking, $v \ll m_{\text{Planck}}$, as well as the pattern of hierarchical quark and lepton masses and mixings. The composite Higgs beyond-SM (BSM) paradigm offers one of the boldest proposals for addressing these mysteries (see, e.g., [1] for a review). The defining hypothesis is that the Higgs doublet is a composite of some as-yet unknown strongly coupled constituents assembled at a scale $\sim m_*$. The Higgs is significantly lighter than m_* because it is a pseudo-Nambu-Goldstone boson (pNGB) of the strong dynamics. If m_* is not too far above v , then this compositeness solves the bulk of the hierarchy problem. The new strong dynamics creating the Higgs then also offers the possibility of understanding how its hierarchical Yukawa

couplings emerge. Experimentally, the generic manifestation of the new physics would be the appearance of other composite excitations, also carrying SM gauge charges, with masses $\sim O(m_*)$, which may well be within collider reach. Even for collider energies $E < m_*$, off-shell composites will mediate new nonrenormalizably weak interactions which can be probed at colliders or, if combined with distinctive breaking of approximate SM symmetries, within low-energy precision tests.

Partial compositeness [2] is a generic and important possibility: the observed particle mass eigenstates can be superpositions of elementary particles and composite particles, where the superposition is quantitatively determined by the strong dynamics renormalization group (RG) flow from the far UV. Phenomenologically, these superpositions determine the degree to which different mass eigenstates participate in the new interactions.

Beyond these general considerations, such strongly coupled BSM theories are hard to study or even formulate microscopically. However, we can take advantage of two powerful expansion parameters to develop provisional theories. One is the $1/N \ll 1$ expansion familiar from QCD [3], where N is the number of colors, which can readily generalize to, and simplify, BSM strong dynamics by making the composites couple with typical strength $g_* \sim 4\pi/\sqrt{N}$, intermediate between SM couplings and maximal strong coupling 4π . Furthermore, the simplest

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robust form of strong dynamics is one which is approximately conformally (scale) invariant above m_* , governed in large part by the nontrivial scaling dimensions $\Delta_{\mathcal{O}}$ of its primary composite operators $\mathcal{O}$.

If only a finite subset of operators $\mathcal{O}$ have $\Delta_{\mathcal{O}} \leq \Delta_{\text{gap}}$ for some $\Delta_{\text{gap}} \gg 1$, then we can also expand in $1/\Delta_{\text{gap}} \ll 1$. Remarkably, via AdS/CFT duality, this expansion is given precisely by AdS₅ effective field theory (EFT) keeping only the finite set of 5D fields dual to this subset of operators (see, e.g., the discussion in [4]). When the strong dynamics is coupled to external elementary particles/fields, the dual description is given by warped 5D EFTs of Randall-Sundrum I (RS1) type [5] (see, e.g., [6,7] for reviews). The fifth dimension is now an interval which roughly geometrizes and reflects the physics of the strong RG flow between a far UV scale Λ_{UV} and m_* [8,9]. Partial compositeness is reflected in the extra-dimensional profiles of the lightest 4D modes of the 5D fields [10–12], and new strong interactions are modulated by the extra-dimensional overlap of the participating 4D particles [13,14]. Currently, the most explicit realistic modeling is formulated at the level of such nonrenormalizable 5D RS1 EFTs.

Anarchic partial compositeness (APC) is the simplest theoretically compelling realization of this dynamics, where “anarchy” refers to the minimal use of (approximate) symmetries. Here, SM fermion and gauge fields, each couple to different $\mathcal{O}$ of the strong sector, whose scaling dimensions then determine the partial compositeness of the observed lightest states through the RG flow. For a “generic” spectrum of scaling dimensions the EFT $< m_*$ has hierarchical structure *qualitatively* similar to that of the SM EFT we observe, and quantitatively can be fit to it. In this framework, the lightest mass eigenstates are correlated with having the smallest superposition with strong composites, so that they have the weakest new interactions in general. This greatly reduces sensitivity to the new physics within our most powerful experimental precision low-energy tests precisely because these apply to the lightest particles [14–17].¹ Consequently, most of the precision constraints are compatible with $m_* \sim 10\text{--}50$ TeV, at the boundary of plausible collider reach (see, e.g., [18] for updated bounds).

Dramatic improvements of electron EDM measurements in the past decades provide one of the most powerful constraints on BSM CP violation in general, and APC in particular, for which $m_* > \text{PeV}$, far above any contemplated collider reach. However, just mild use of leptonic symmetries of the strong dynamics relaxes the leptonic constraints back to $m_* \sim 10\text{--}50$ TeV [19].

¹The standard strong CP problem associated with neutron electric dipole moment constraints is not mitigated in this way and requires a separate solution, such as the QCD axion mechanism with which partial compositeness is fully compatible.

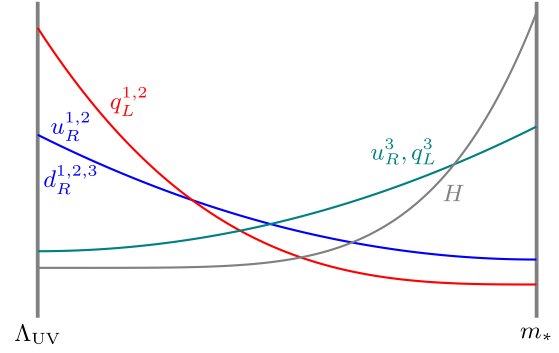


FIG. 1. Holographic realization of APC. Chiral-zero mode profile overlaps with the Higgs are the key determinant of the SM Yukawa couplings.

Figure 1 sketches the form of the dual 5D warped EFT structure, in particular the distinctive profiles of the different species of light 4D modes. These are determined by the 5D masses, related to the $\Delta_{\mathcal{O}}$ via the standard AdS/CFT dictionary.

Given the plausibility of accidental/emergent symmetries of strongly coupled QFTs (e.g., isospin or baryon-number within QCD, or even CP symmetry within massless QCD) and their power to relax flavor/ CP /EW precision constraints, it is important to assess how low m_* can be if the strong dynamics at this scale is governed by more symmetry. Remarkably, maximal flavor symmetry of the strong dynamics, with explicit breaking by spurions proportional to SM Yukawa matrices [20–23] [minimal flavor violation (MFV) [24]], can relax precision constraints to $m_* \gtrsim 10$ TeV [18], now well within proposed future collider reach [25–28]. A more nuanced symmetry implementation [18,29,30] can even remove precision low-energy constraints as the dominant bound on m_* , leaving $m_* \gtrsim 5$ TeV bounded only by direct resonance searches at the LHC.

Such highly symmetric composite Higgs frameworks with the lowest allowed m_* considerably mitigate residual EW fine-tuning and strongly motivates ongoing resonance searches at the LHC and future colliders. References [18,21–23,29,30] leave important questions unanswered: the emergence of the requisite flavor symmetries and the hierarchical Yukawa matrices that explicitly break them.² They also do not address the consistency of hierarchical charged lepton masses with the small but nonhierarchical structure of neutrino masses and mixings. In this paper, we will address how all of these features can emerge from a strong RG flow from the UV in the context of partial compositeness.

An overview of our proposal can be seen from the dual 5D perspective, sketched in Fig. 2(b). A central feature is

²See also [31–34] for an incomplete list of UV realizations of composite Higgs models that also address an explanation of flavor hierarchies or suppression of flavor violation [35–38].

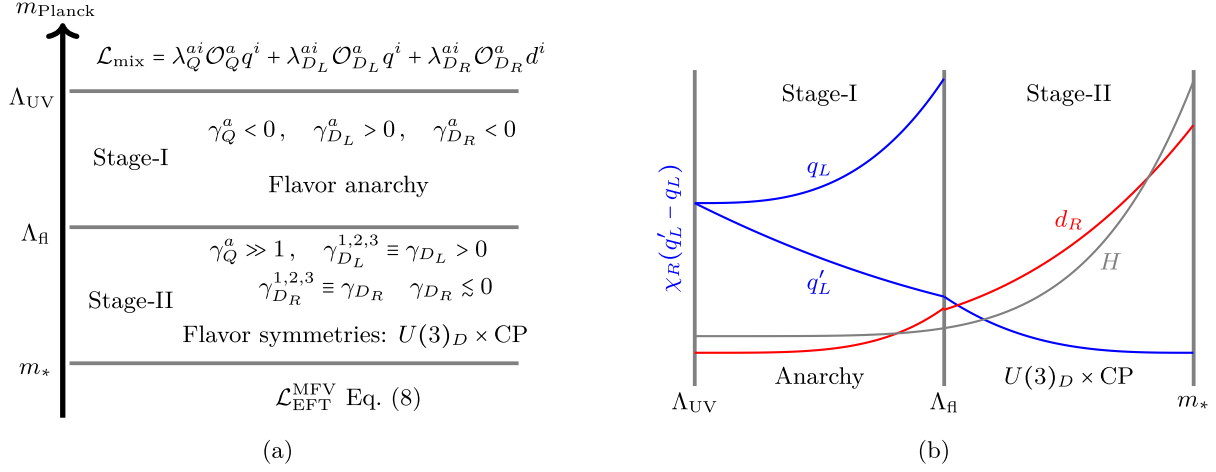


FIG. 2. Schematic description of our symmetric model from the Ads-CFT dual perspective. Only the down-quark sector is shown. The model can be extended to account for the up sector and leptons, as explained in the text. (a) RG flow of the symmetric model from the compositeness perspective. (b) Holographic realization of the same model within a warped 5D description.

the presence of an intermediate defect/“brane” dividing the 5D “bulk” into two sections. These reflect two stages of RG flow, each of which is approximately conformal (close to a RG fixed point). The UV stage depicted on the left has no flavor symmetries, but the flavor-violating Yukawa hierarchies are generated here, while the IR stage depicted on the right has “protective” flavor symmetries (dual to 5D gauge fields) and CP symmetry. We therefore denote the juncture of these two stages as the flavor scale Λ_{fl} , dual to the midbrane, reflecting a rapid flow or “tumbling” (for related idea see [39,40]) between the two RG fixed points governing the two stages [see Fig. 2(a)]. From the 5D EFT viewpoint, it is straightforward to introduce and stabilize the midbrane via generalizing [41,42] the Goldberger-Wise mechanism [43]. 5D masses can change significantly across this brane, and effectively this allows it to be semi-permeable. As can be seen, this primarily affects the EW doublet (“left-handed”) fermionic light modes, which have a peculiar shape. One can think of this as the composition of simpler profiles of two light modes from two 5D fields, q_L and q_L' , the first confined to propagate in the UV section and the second propagating in the entire bulk. They are tied together by UV mass mixing on the UV boundary to a boundary-localized field χ , which below Λ_{UV} effectively becomes a Lagrange multiplier identifying the q_L and q_L' light modes.³ Furthermore, notice that all of the right-handed fields u_R are peaked towards m_* in contrast to APC (see Fig. 1).

³Equivalently, we can avoid having the brane-localized fermion by choosing different boundary conditions (BCs) for q_L and q_L' . In the language of $Z_2 \times Z_2'$ orbifold compactifications (see, e.g., [44]), q and q' have, respectively, $(++)$ and $(-+)$ BCs, where the first/second sign is for UV/IR boundary. Such q and q' can have a mass term on the UV boundary, resulting in a single chiral zero mode housed partly in q_L and partly in q_L' as depicted in Fig. 2(b).

For concreteness, in the rest of the paper we will derive the hierarchical and symmetry(-breaking) structure of our model purely from the 4D RG perspective, AdS/CFT dual to the RS 5D EFT of Fig. 2. We start in Sec. II where we introduce the general setup and review APC in more detail. Then, in Sec. III, we describe our symmetric model, including both quark and lepton hierarchies. We conclude in Sec. IV with comparison between the symmetry-based and APC frameworks and in particular the significant phenomenological distinctions. Appendix A provides more detail on the RG evolution of our model and in particular nuances associated to the large top Yukawa coupling. Appendix B expands the discussion to less-symmetric, yet not anarchic, models and Appendix C summarizes the current experimental constraints on the various scenarios.

II. ANARCHIC PARTIAL COMPOSITENESS

We start from a bottom-up effective Lagrangian for just the SM fields,

$$\mathcal{L}_{\text{EFT}} = \frac{m_*^4}{g_*^2} \hat{\mathcal{L}}_{\text{EFT}} \left(\frac{g_* H}{m_*}, \frac{D_\mu}{m_*}, \frac{\lambda_\psi^{ai} \psi^i}{m_*^{3/2}}, \dots \right), \quad (1)$$

where $\psi = q_L, u_R, d_R, e_R, l_L$ are the usual chiral SM multiplets, with $i = 1, 2, 3$ denoting generation.⁴ This is assumed to arise after integrating out composite resonances with typical masses $\sim m_*$ and interactions $\sim g_* > g_{\text{SM}}$ [45].

⁴The ellipsis in (1) may include multilinear terms involving the light fields, such as bilinear interactions like $\bar{q}^i H d^j$, which we will discuss later, or four-fermion interactions. These terms are assumed to be subleading compared to those obtained directly by expanding the terms explicitly in (1). In addition, the ellipsis encodes the formal loop dependence, i.e., terms suppressed by factors such as $g_{\text{SM}}^2/16\pi^2$, $g_*^2/16\pi^2$, or $\lambda_\psi^{ai} [\lambda_\psi^{\beta i}]^* / 16\pi^2$.

The elementary SM fermions are assumed to linearly mix with the composite states at m_* via (multiple) couplings $\lambda_{\psi}^{\alpha i} \lesssim g_*$ $\alpha = 1, 2, 3, \dots$. In the cases of emergent flavor symmetries, we will take these to act on the α index whose nature and multiplicity we will describe below. The Higgs H represents the exceptionally light composite pNGB that remains in the EFT. The Higgs has an approximate shift symmetry which is respected by g_* interactions, but broken weakly by the elementary fields via gauge couplings and the λ s. The Wilson coefficients arising from expanding (1) in a Taylor series, subject to any flavor or approximate shift symmetry, are $O(1)$.⁵

Equation (1) at $O(m_*^0)$ reproduces the renormalizable SM and, in particular, the Yukawa couplings. For quarks,

$$Y_u^{ij} \simeq \frac{(\lambda_q^{\alpha i})^* c_u^{\alpha\beta} \lambda_u^{b\beta}}{g_*}, \quad Y_d^{ij} \simeq \frac{(\lambda_q^{\alpha i})^* c_d^{\alpha\beta} \lambda_d^{b\beta}}{g_*}, \quad (2)$$

where $c_{u/d}^{\alpha\beta} \sim O(1)$ but subject to any flavor symmetry constraint. Together with these symmetries the $\lambda_{\psi}^{\alpha i}$ will determine the precision phenomenological impact of the nonrenormalizable terms in (1).

The minimal and most appealing option is “anarchy,” where there are no symmetries acting on $\alpha \equiv 1, 2, 3$. For example, the $c_{u,d}^{\alpha\beta}$ in (2) are anarchic matrices with $O(1)$ entries. By contrast, the couplings to the elementary fermions at m_* must be determined by RG evolution from the far UV,

$$\mathcal{L}_{\text{mix}} = \sum_{\psi} \lambda_{\psi}^{\alpha i} \mathcal{O}_{\psi}^{\alpha} \psi^i. \quad (3)$$

We will normalize the fermionic operators $\mathcal{O}_{\psi}^{\alpha}$ to have engineering dimension 5/2, so that the λ s are dimensionless. The mixings are understood as functions of the RG scale, $\lambda_{\psi}^{\alpha i} \equiv \lambda_{\psi}^{\alpha i}(\mu)$. In the UV, the couplings $\lambda_{\psi}^{\alpha i}(\Lambda_{\text{UV}})$ are assumed to be $O(1)$ and anarchic, but they must be run down to the scale m_* , where (1) is defined. This running produces hierarchies via

$$\frac{d\lambda_{\psi}^{\alpha i}(\mu)}{d \log \mu} = \gamma_{\psi}^{\alpha} \lambda_{\psi}^{\alpha i}(\mu) + \dots, \quad (4)$$

where $\Delta_{\mathcal{O}_{\psi}^{\alpha}} \equiv \frac{5}{2} + \gamma_{\psi}^{\alpha}$, with Δ_{ψ} the scaling dimension of $\mathcal{O}_{\psi}^{\alpha}$ and the ellipsis denotes higher order terms in λ . In the simplest possibility, where all $\gamma_{\psi} > 0$, this gives

⁵The Higgs potential requires shift-symmetry breaking but it is not directly connected to the gauge couplings and the λ s, so that it arises from these sources of shift-symmetry breaking only radiatively, naturally making the Higgs lighter than m_* . A further tuning is required to obtain $v < m_*/g_*$ which is the composite Higgs incarnation of the “little hierarchy problem,” which we do not address here (see, again [1]).

$$\lambda_{\psi}^{\alpha i}(m_*) \simeq \eta_{\psi}^{\alpha} \lambda_{\psi}^{\alpha i}(\Lambda_{\text{UV}}), \quad \eta_{\psi}^{\alpha} = \left(\frac{m_*}{\Lambda_{\text{UV}}} \right)^{\gamma_{\psi}^{\alpha}}. \quad (5)$$

Remarkably for $\Lambda_{\text{UV}} \lesssim m_{\text{Planck}}$ and $\gamma_{\psi}^{\alpha} \sim O(0.1)$ and anarchic, (2) yields SM-like flavor hierarchies, and can readily be fit to the actual SM values.

A subtlety arises for the top, since (5) fails to generate a sizable top Yukawa. Instead, the heavy top must involve relevant deformations. In this case we have to be careful about higher order terms. For instance, just focusing on $\gamma_u^3 < 0$,

$$\frac{d\lambda_u^{33}}{d \log \mu} = \lambda_u^{33} \gamma_u^3 + c \frac{(\lambda_u^{33})^3}{g_*^2} \left(1 + O\left(\frac{\lambda^2, g_*^2}{16\pi^2} \right) \right), \quad (6)$$

where we neglected subleading contributions. Notice that the $c \sim O(1)$ is necessarily positive [46], thus λ_u^{33} approaches the IR fixed point,

$$\lambda_u^{33}|_{\text{fp}} = g_* \sqrt{-\gamma_u^3}, \quad (7)$$

setting $c = 1$. A parallel statement applies to λ_q^{33} with $\gamma_q^3 < 0$. With these relevant couplings we can achieve fully realistic SM Yukawa couplings.⁶ At the other extreme, small Dirac neutrino masses with anarchic mixings can be consistently included by considering irrelevant couplings to the strong dynamics which are bilinear in ψ [47].

However, the m_* -suppressed effects in (1) generate a peculiar pattern of signatures, including sizable flavor and CP -violating effects, which are strongly constrained (see, e.g., [18] for a recent analysis). In particular, CP violation in the quark sector requires $m_* \gtrsim O(10 \text{ TeV})$ due to constraints from the neutron EDM, CP violation in $K - \bar{K}$ transitions, and D -meson decays. The situation is even more severe in the lepton sector, where bounds from $\mu \rightarrow e\gamma$ and the electron EDM imply $m_* \gtrsim O(10^2 - 10^3 \text{ TeV})$.

III. SYMMETRIC MODELS

An economical option to relax the experimental constraints and lower m_* is to construct flavor-symmetric models. The simplest is the “minimal flavor violation”

⁶Given sizable $\lambda_{q,u}^{33}$ (i.e., coupling of elementary top/LH (left handed) bottom quark to strong sector), it is clear that the SM top (and LH bottom) quark has a significant admixture of composite fermion interpolated by $\mathcal{O} \dots$. Alternatively, we could assume that the top/LH bottom quark is (purely) a massless composite of the strong sector, i.e., there is no elementary fermion at all. The two alternatives are not distinct, but are rather “dual” in the sense of [11]. For simplicity/uniformity with treatment of other SM fermions (which are predominantly elementary), we will continue to use the original viewpoint with the elementary top/LH bottom quark.

(MFV), defined from the bottom up as in (1) by [21,22],⁷

$$\mathcal{L}_{\text{EFT}}^{\text{MFV}} = \frac{m_*^4}{g_*^2} \hat{\mathcal{L}}_{\text{EFT}} \left(\frac{\lambda_{U_L}^{ai} q^i}{m_*^{3/2}}, \frac{\lambda_{D_L}^{ai} q^i}{m_*^{3/2}}, \frac{\rho_{U_R}^a u^a}{m_*^{3/2}}, \frac{\rho_{D_R}^a d^a}{m_*^{3/2}}, \dots \right), \quad (8)$$

where $a = 1, 2, 3$. We arrive at this structure from (1) starting with $\alpha = 1, \dots, 6$, where the $\lambda_{q,u,d}$, 6×3 matrices (boldface in matrix notation), are given by

$$\lambda_q = \begin{bmatrix} \lambda_{U_L} \\ \lambda_{D_L} \end{bmatrix}, \quad \lambda_u = \rho_{U_R} \begin{bmatrix} \mathbf{I}_3 \\ \mathbf{0}_3 \end{bmatrix}, \quad \lambda_d = \rho_{D_R} \begin{bmatrix} \mathbf{0}_3 \\ \mathbf{I}_3 \end{bmatrix}, \quad (9)$$

where λ_{U_L, D_L} are 3×3 matrices with elements $\lambda_{U_L}^{ai}, \lambda_{D_L}^{ai}$, $\mathbf{I}_3$ and $\mathbf{0}_3$ are the identity and zero 3×3 matrices, and ρ_{U_R, D_R} are complex numbers. This notation helps clarify a $U(3)_U \times U(3)_D$ broken global symmetry with spurions λ_{U_L, D_L} under which

$$\lambda_{U_L}^{ai} q^i \in \bar{\mathbf{3}}_{U(3)_U}, \quad u^a \in \mathbf{3}_{U(3)_U}, \quad (10)$$

and similarly for the down sector.⁸ Respecting the symmetry being broken by just these spurions in (2) yields

$$Y_u^{ia} = c_u \frac{(\lambda_{U_L}^{ai})^* \rho_{U_R}}{g_*}, \quad Y_d^{ia} = c_d \frac{(\lambda_{D_L}^{ai})^* \rho_{D_R}}{g_*}, \quad (11)$$

where $c_{u,d}$ are two $O(1)$ numbers. Charged leptons can be treated in complete analogy to down-type quarks.

Note that since λ_{U_L, D_L} are proportional to the Yukawa matrices, then those are the only flavor spurion appearing in (8) [via (11)], thereby generalizing the SM GIM (Glashow–Iliopoulos–Maiani) mechanism. This results in a significant relaxation of experimental constraints to $m_* \gtrsim 10$ TeV, well within reach of future colliders [18,21,22]. This is reviewed in Appendix C (see in particular Fig. 3) for completeness.

The main goal of this work is to explain the peculiar ansatz in (8) and (9) within the top-down framework, depicted in Fig. 2. Specifically, we now show that this ansatz can be achieved in analogy to the partial compositeness construction of (3), but with a more nuanced RG flow which is AdS/CFT dual to the nonminimal features of Fig. 2(b).

The two extra-dimensional segments are dual to two stages of RG flow [see Fig. 2(a)]:

- (i) *Stage I* $\Lambda_{\text{UV}} > \mu > \Lambda_{\text{IR}}$: *Anarchy*. Hierarchical mixings λ_{U_L, D_L} between the elementary and composite

left-handed fields are generated by RG flow, leading to the observed SM Yukawa hierarchies by (11).

- (ii) *Stage II* $\Lambda_{\text{IR}} > \mu > m_*$: *Symmetries*. The strong dynamics possesses emergent *CP* and flavor symmetries. The *right-handed* elementary fields align, via RG evolution, with their composite partners so as to produce the $U(3)_U \times U(3)_D$ invariant structure $\lambda_{u,d}$ in (9).

For simplicity, we focus for now only on the quark sector and neglect subtleties associated to the large top Yukawa. These are discussed in Appendix A. Later, we discuss the generalization to leptons and particularly how to include neutrino masses.

In the far UV (stage I) the composite sector mixes to the elementary sector as

$$\mathcal{L}_{\text{mix}} = \sum_{i,a=1,2,3} \left(\lambda_Q^{ai} \mathcal{O}_Q^a q^i + \lambda_{U_L}^{ai} \mathcal{O}_{U_L}^a q^i + \lambda_{U_R}^{ai} \mathcal{O}_{U_R}^a u^i + \lambda_{D_L}^{ai} \mathcal{O}_{D_L}^a q^i + \lambda_{D_R}^{ai} \mathcal{O}_{D_R}^a d^i \right). \quad (12)$$

Note that the SM doublets q mix with $\mathcal{O}_Q^a, \mathcal{O}_{U_L}^a, \mathcal{O}_{D_L}^a$ in this stage but λ_Q does not appear in the m_* -scale EFT in (8). We will see later on that λ_Q participates in the RG for λ_{U_L, D_L} . At Λ_{UV} no flavor and *CP* symmetries are present ($\Lambda_{\text{UV}} \lesssim m_{\text{Planck}}$) and the mixings $\lambda(\Lambda_{\text{UV}})$ are complex and anarchic $O(1)$. Furthermore, the stage I strong dynamics has no flavor symmetries.⁹

In the following, we will just focus on the operators relevant for down-type Yukawa couplings. The scaling dimensions of the composite operators, $\Delta_{\mathcal{O}_\psi^a} \equiv \frac{5}{2} + \gamma_\psi^a$, are taken in the ranges

$$\text{Stage I: } \begin{cases} -1 < \gamma_Q^1 < \gamma_Q^2 < \gamma_Q^3 < 0 \\ \gamma_{D_L}^1 > \gamma_{D_L}^2 > \gamma_{D_L}^3 > 0 \\ -1 < \gamma_{D_R}^1 < \gamma_{D_R}^2 < \gamma_{D_R}^3 < 0. \end{cases} \quad (13)$$

To describe the RG, we now consider, for simplicity, a single family, say $a = i = 1$. We report in Appendix A the straightforward generalization. The RG equations read

$$\begin{aligned} \frac{d\lambda_{D_L}^{11}}{d \log \mu} &= \gamma_{D_L}^1 \lambda_{D_L}^{11} + \frac{\lambda_{D_L}^{11} |\lambda_Q^{11}|^2}{g_*^2} \\ \frac{d\lambda_Q^{11}}{d \log \mu} &= \gamma_Q^1 \lambda_Q^{11} + \frac{\lambda_Q^{11} |\lambda_Q^{11}|^2}{g_*^2}, \end{aligned} \quad (14)$$

where we neglected higher-order contributions in $(\lambda_{D_L}^{11})^3$ since these are suppressed in the IR. Indeed the coupling λ_Q^{11} in the second equation of (14) approaches the fixed point in the IR,

⁹We only assume baryon number to suppress proton decay mediating operators.

⁷There are other realizations of MFV in composite Higgs models (see, e.g., [18]). For concreteness we only consider (8) but it is straightforward to generalize the main construction of the paper to other realizations.

⁸With some abuse of notation, the representations $\mathbf{3}_{U(3)}$ and $\bar{\mathbf{3}}_{U(3)}$ denote fields in the fundamental and antifundamental of $SU(3)$, with arbitrary but opposite charges under the Abelian $U(1)$ factor of $U(3)$.

$$|\lambda_Q^{11}|_{\text{fp}}^2 = -g_*^2 \gamma_Q^1. \quad (15)$$

Inserting this in (14) gives

$$\begin{aligned} \frac{d\lambda_{D_L}^{11}}{d\log\mu} &= \gamma_{D_L}^1 \lambda_{D_L}^{11} - \lambda_{D_L}^{11} \gamma_Q^1 \\ \Rightarrow \lambda_{D_L}^{11}(\mu) &= \lambda_{D_L}^{11}(\Lambda_{\text{UV}}) \left(\frac{\mu}{\Lambda_{\text{UV}}} \right)^{\gamma_{D_L}^1 - \gamma_Q^1}, \end{aligned} \quad (16)$$

where, for simplicity and robustly, we assumed λ_Q^{11} to have reached the fixed point very quickly.¹⁰ Generalizing to all three generations we find

$$\lambda_{D_L}^{ai}(\Lambda_{\text{fl}} \lesssim \mu \ll \Lambda_{\text{UV}}) = \eta_Q^a(\mu) \lambda_{D_L}^{ai}(\Lambda_{\text{UV}}) \eta_{D_L}^i(\mu), \quad (17)$$

with

$$\eta_Q^a(\mu) = \left(\frac{\mu}{\Lambda_{\text{UV}}} \right)^{-\gamma_Q^a}, \quad \eta_{D_L}^i(\mu) = \left(\frac{\mu}{\Lambda_{\text{UV}}} \right)^{\gamma_{D_L}^i}. \quad (18)$$

The RH mixings are driven by a relevant deformation and they flow toward to a fixed point, similarly to (7). In particular, at this stage $\lambda_d^{ai}(\mu) \lesssim g_*$.

At the “flavor scale” $\Lambda_{\text{fl}} (\Lambda_{\text{UV}} \gg \Lambda_{\text{fl}} \gg m_*)$, the dynamics is assumed to undergo a sudden and significant transition from flavor anarchy of the strong dynamics toward emergent flavor symmetries. Above and below this transition the dynamics is taken to be governed by distinct RG fixed points, close to distinct CFTs. In particular, the spectrum of scaling dimensions of low-lying operators is different. We will see that precision flavor tests constrain $\Lambda_{\text{fl}} \gtrsim (10^4 - 10^5) g_* \text{TeV}$. In 5D this threshold can be robustly modeled as an intermediate brane, separating the two “slices” of approximately AdS_5 geometry. The matching coefficients of the RG at this threshold are represented by brane-localized couplings.

Below the transition the dynamics are assumed to be near a new IR CFT. This is assumed to possess an accidental $U(3)_U \times U(3)_D$ global symmetry, with nearly marginal operators $\mathcal{O}_{U_{L,R}, D_{L,R}}$, but no longer including $\mathcal{O}_Q$:

$$\begin{aligned} \text{Stage II: } \mathcal{O}_{U_{L,R}}^a &\in \bar{\mathbf{3}}_{U(3)_U}, & \mathcal{O}_{D_{L,R}}^a &\in \bar{\mathbf{3}}_{U(3)_D}, \\ \gamma_{D_L}^{1,2,3} &\equiv \gamma_{D_L} > 0, & -1 < \gamma_{D_R}^{1,2,3} &\equiv \gamma_{D_R} < 0. \end{aligned} \quad (19)$$

Additionally, the strong dynamics below Λ_{fl} is assumed to be accidentally CP invariant to protect from large effects in neutron electric dipole moment [18]. The strong dynamics at or above Λ_{fl} might also be CP invariant or not; see IV for implications.

¹⁰Indeed, as we explain in Appendix A, we can estimate $\lambda_q^{11}(\mu) \sim \lambda_q^{11}|_{\text{fp}} (1 + O(\delta_\mu^{-2\gamma_q^1}))$ with $\delta_\mu = \mu/\Lambda_{\text{UV}}$.

The RG flow in stage II is

$$\begin{aligned} \frac{d\lambda_{D_L}^{ai}}{d\log\mu} &= \gamma_{D_L} \lambda_{D_L}^{ai} \\ \frac{d\lambda_{D_R}^{ai}}{d\log\mu} &= \gamma_{D_R} \lambda_{D_R}^{ai} + \frac{\lambda_{D_R}^{aj} [\lambda_{D_R}^\dagger \lambda_{D_R}]^{ji}}{g_*}. \end{aligned} \quad (20)$$

The RH mixings $\lambda_{D_R}^{ai}$, similarly to (6), approach the diagonal and universal fixed point

$$\lambda_{D_R}(m_*) \simeq \lambda_{D_R}|_{\text{fp}} = g_* \sqrt{-\gamma_{D_R}} \mathbf{I}_3. \quad (21)$$

The previous equation matches with (9): we can identify $\rho_{D_R} = g_* \sqrt{-\gamma_{D_R}}$.

The linear RG for λ_{D_L} in (20) results in a universal rescaling, namely

$$\lambda_{D_L}^{ai}(m_*) = \left(\frac{m_*}{\Lambda_{\text{fl}}} \right)^{\gamma_{D_L}} c_{D_L}^{ab} \eta_Q^b(\Lambda_{\text{fl}}) \lambda_{D_L}^{bi}(\Lambda_{\text{UV}}) \eta_{D_L}^i(\Lambda_{\text{fl}}). \quad (22)$$

Here, $c_{D_L}^{ab}$ is an anarchic $O(1)$ matrix arising from flavor nonsymmetric matching of the couplings between the I and II stage at Λ_{fl} (again, dual to couplings localized on the intermediate brane).

The previous expression can be written as

$$\lambda_{D_L}^{ai} = \left(\frac{m_*}{\Lambda_{\text{fl}}} \right)^{\gamma_{D_L}} U_D^{ab} \eta_Q^b(\Lambda_{\text{fl}}) \lambda_{D_L}^{bi}(\Lambda_{\text{UV}}) \eta_{D_L}^i(\Lambda_{\text{fl}}), \quad (23)$$

where U_D^{ab} is unitary and we have reabsorbed $c_{D_L}^{ab}$ in the anarchic matrix λ'_{D_L} .¹¹ With the rotation $\mathcal{O}_{D_{L,R}}^a \rightarrow \mathcal{O}_{D_{L,R}}^b (U_D^{ab})^\dagger$ and $d^i \rightarrow U_D^{ij} d_j$, we can remove U_{D_L} from the previous expression still keeping (21). In this new basis, we have

$$\lambda_{D_L}^{ai}(m_*) = \left(\frac{m_*}{\Lambda_{\text{fl}}} \right)^{\gamma_{D_L}} \eta_Q^b(\Lambda_{\text{fl}}) \lambda_{D_L}^{bi}(\Lambda_{\text{UV}}) \eta_{D_L}^i(\Lambda_{\text{fl}}). \quad (24)$$

The previous equation shows that plugging λ_{D_L} in (11) yields the same attractive pattern of hierarchical Yukawa couplings found in APC by plugging (5) in (2). In the up sector, the presence of a relevant deformation associated to the large top Yukawa generates a different structure than APC with suppressed mixing angles. Yet, the down sector will dominate the contributions to the physical Cabibbo–Kobayashi–Maskawa (CKM) angles, as we discuss in Appendix A.

Notice that, as we pointed out before, in stage I the theory is essentially anarchic, allowing for generic flavor-violating effects. Given that $\lambda_{Q,U_R,D_R}^{ai}(\Lambda_{\text{fl}}) \lesssim g_*$ are not suppressed at this stage, we expect $\Delta F = 2$ four-fermion

¹¹In particular, via rotation U_D we can put $c_{D_L}^{ab}$ into a lower triangular form $\hat{c}_{D_L}^{ab}$. Then we use $\hat{c}_{D_L}^{ab} \eta_Q^b(\Lambda_{\text{fl}}) \lambda_{D_L}^{bi} \simeq \eta_Q^b(\Lambda_{\text{fl}}) \lambda_{D_L}^{bi}(\Lambda_{\text{fl}})$ with $\lambda_{D_L}^{bi}(\Lambda_{\text{fl}})$ another anarchic matrix.

operators suppressed by $g_*^2/\Lambda_{\text{fl}}^2$. The coefficients of some of these operators, namely the one involving right-handed fields, will be further suppressed by the RG flow in stage II in line with (20) and (21). Yet, the coefficients of four-fermion operators made purely out of q^i have no strong RG suppression since the $\mathcal{O}_Q^a$ are not relevantly coupled to the strong dynamics in stage II. Bounds on $\Delta S = 2$ transitions [48] of this form then imply $\Lambda_{\text{fl}} \gtrsim 10^5\text{--}10^6 g_* \text{ TeV}$.

A similar procedure can be applied to the up sector, though again a subtlety arises in connection with the top quark. In order to generate a sizable Yukawa, $\gamma_{U_L}^3$, in stage I and/or γ_{U_L} in stage II must be sufficiently negative. This generalization of (14) is straightforward and we report the details in Appendix A.

The benchmark values $m_* = 10 \text{ TeV}$ and $g_* = 3$, at the boundary of the allowed parameter space (see Appendix C for details), together with $\Lambda_{\text{UV}} = m_{\text{Planck}} = 10^{16} \text{ TeV}$, $\Lambda_{\text{fl}} = 10^6 \text{ TeV}$, and

$$\begin{aligned} \text{Stage I: } & \gamma_{U_L}^a \sim (0.2, 0.05, -0.05), \quad -\gamma_Q^a \sim (0.4, 0.3, 0.05), \\ & \gamma_{D_L}^a \sim (0.1, 0.05, 0.05), \quad -\gamma_{U_R, D_R}^a \sim O(0.1) \\ \text{Stage II: } & \gamma_{U_L} \sim -0.15, \quad \gamma_{D_L} \sim 0.05, \\ & \gamma_{U_R} \sim -0.5, \quad \gamma_{D_R} \sim -0.6 \end{aligned} \quad (25)$$

can readily be fit to Yukawa matrices, matching the real world quark masses and mixings for anarchic $O(1)$ $\lambda(\Lambda_{\text{UV}})$ and c_{U_L, D_L}^{ab} .¹² As can be seen, these γ are consistent with themselves being anarchic and $\sim O(0.1)$.

Leptons. The previous construction can be generalized to include leptons. In particular, we extend (8) to include leptonic arguments,

$$\mathcal{L}_{\text{EFT}}^{\text{MFV}} \supset \frac{m_*^4}{g_*^2} \hat{\mathcal{L}}_{\text{EFT}} \left(\frac{\tilde{\lambda}_{D_L}^{ai} l^i}{m_*^{3/2}}, \frac{\tilde{\rho}_{D_R} e^a}{m_*^{3/2}}, \frac{g_*^2 \tilde{\mathcal{Y}}_{\nu}^{ij} \bar{l}^i H^C \nu^j}{m_*^4}, \dots \right), \quad (26)$$

with $H^C \equiv i\sigma^2 H^*$, the usual conjugate of the H doublet. Here, the first two terms describe linear mixings responsible for generating hierarchical charged lepton masses, in complete analogy with the down-quark sector. We have also included a gauge invariant term bilinear in leptons to generate a Dirac mass for neutrinos, where ν^i denote the three right-handed neutrinos. Throughout the rest of the text, we include a tilde (e.g., $\tilde{\lambda}$) to distinguish lepton-sector quantities from their quark-sector counterparts. In (26) we could have included analogous bilinear terms unrelated to neutrinos. We will see that all such $\tilde{\mathcal{Y}}$ can flow to be very

small but anarchic, so that they will only be important when other Yukawa contributions (from linear mixings) are highly suppressed, as is however the case only for neutrinos, in the spirit of [47]. We thus arrive at

$$Y_e^{ia} \simeq c_e \frac{(\tilde{\lambda}_{D_L}^{ai})^* \tilde{\rho}_{D_R}}{g_*}, \quad Y_{\nu}^{ij} \simeq \tilde{\mathcal{Y}}_{\nu}^{ij}, \quad (27)$$

so that the charged leptons can be hierarchical consistently with small but anarchic neutrino masses, as observed in the data.

The origin of (26) begins in the UV with

$$\begin{aligned} \mathcal{L}_{\text{mix}}(\Lambda_{\text{UV}}) \supset \sum_{i,a=1,2,3} & \left(\tilde{\lambda}_L^{ai} \tilde{\mathcal{O}}_L^a l^i + \tilde{\lambda}_{U_L}^{ai} \tilde{\mathcal{O}}_{U_L}^a l^i + \tilde{\lambda}_{U_R}^{ai} \tilde{\mathcal{O}}_{U_R}^a \nu^i \right. \\ & \left. + \tilde{\lambda}_{D_L}^{ai} \tilde{\mathcal{O}}_{D_L}^a l^i + \tilde{\lambda}_{D_R}^{ai} \tilde{\mathcal{O}}_{D_R}^a e^i \right), \end{aligned} \quad (28)$$

with $\tilde{\mathcal{O}}_L^a, \tilde{\mathcal{O}}_{U_L, D_L}^a, \tilde{\mathcal{O}}_{U_R, D_R}^a$ the composite operators.¹³ All the $\tilde{\lambda}$ are anarchic and $O(1)$. In stage I we assume $-1 < \tilde{\gamma}_{L, U_R, D_R}^{1,2,3} < 0$, $\tilde{\gamma}_{U_L, D_L}^{1,2,3} > 0$ in analogy to the quark sector. Furthermore, we assume lepton number conservation which suppresses a Majorana mass term for the right-handed neutrinos.

At the transition at Λ_{fl} we match onto stage II as we did for quarks. However, we will now include irrelevant effects bilinear in the elementary fields:

$$\mathcal{L}_{\text{mix}}(\Lambda_{\text{fl}}) \supset \tilde{\mathcal{Y}}_{\nu}^{ij} \bar{l}^i \mathcal{O}_H^C \nu^j + \tilde{\mathcal{Y}}_e^{ij} \bar{l}^i \mathcal{O}_H e^j, \quad (29)$$

where $\mathcal{O}_H^{(C)}$ denotes the (conjugate) operator interpolating the composite Higgs, normalized $\mathcal{O}_H$ to have engineering dimension 1. Assuming for simplicity that $\tilde{\lambda}_L^{ai}(\Lambda_{\text{fl}}) \sim g_*$, we estimate $\tilde{\mathcal{Y}}_{\nu}^{ij}(\Lambda_{\text{fl}})$ as an anarchic matrix with entries of order $O(g_*)$.

In stage II, the theory develops emergent global $U(3)_N \times U(3)_E$ symmetry, so that

$$\begin{aligned} \tilde{\mathcal{O}}_{U_L, U_R}^a & \in \bar{\mathbf{3}}_{U(3)_N}, \quad \tilde{\mathcal{O}}_{D_L, D_R}^a \in \bar{\mathbf{3}}_{U(3)_E}, \\ \tilde{\gamma}_{U_L, D_L}^{a=1,2,3} & \equiv \tilde{\gamma}_{U_L, D_L} > 0, \quad \tilde{\gamma}_{U_R, D_R}^{a=1,2,3} \equiv \tilde{\gamma}_{U_R, D_R} < 0. \end{aligned} \quad (30)$$

Paralleling the quark sector, in stage II we have

$$\tilde{\lambda}_{U_L}^{ai}(m_*) \sim \left(\frac{m_*}{\Lambda_{\text{fl}}} \right)^{\tilde{\gamma}_{U_L}} \left(\frac{\Lambda_{\text{fl}}}{\Lambda_{\text{UV}}} \right)^{\tilde{\gamma}_{U_L} - \tilde{\gamma}_L^a} \tilde{\lambda}_{U_L}^{ai}(\Lambda_{\text{UV}}), \quad (31)$$

¹²We have numerically checked, by solving the RG equations at $O(\lambda^3)$, that these values generate Yukawa couplings and mixing angles in the ballpark of the observed SM values, starting from $O(1)$ mixings $\lambda(\Lambda_{\text{UV}})$ and $O(1)$ matching matrices c^{ab} at Λ_{fl} . These values are consistent with the analytic expressions in Appendix A.

¹³Bilinear interactions of the form $\bar{l} \mathcal{O}_H^C \nu$, with $\mathcal{O}_H^C$ the conjugate of the operator interpolating the Higgs, are also generated at Λ_{UV} and can be included in (28). However, these turn out to be suppressed by the RG.

$$\tilde{\lambda}_{D_L}^{ai}(m_*) \sim \left(\frac{m_*}{\Lambda_{\text{fl}}}\right)^{\tilde{\gamma}_{D_L}} \left(\frac{\Lambda_{\text{fl}}}{\Lambda_{\text{UV}}}\right)^{\tilde{\gamma}_{D_L} - \tilde{\gamma}_L^a} \tilde{\lambda}_{D_L}^{ai}(\Lambda_{\text{UV}}). \quad (32)$$

We now focus on the bilinear terms. For instance $\tilde{\mathcal{Y}}_\nu^{ij}$ runs as

$$\begin{aligned} \frac{d\tilde{\mathcal{Y}}_\nu^{ij}}{d\log\mu} &= \gamma_H \tilde{\mathcal{Y}}_\nu^{ij} + \tilde{\mathcal{Y}}_\nu^{ik} \frac{[\tilde{\lambda}_{U_R}^\dagger \tilde{\lambda}_{U_R}]^{kj}}{g_*^2} \\ \frac{d\tilde{\lambda}_{U_R}^{ai}}{d\log\mu} &= \tilde{\gamma}_{U_R} \tilde{\lambda}_{U_R}^{ai} + \frac{\tilde{\lambda}_{U_R}^{aj} [\tilde{\lambda}_{U_R}^\dagger \tilde{\lambda}_{U_R}]^{ji}}{g_*^2}, \end{aligned} \quad (33)$$

where $\Delta_H = 1 + \gamma_H$ is the scaling dimension of $\mathcal{O}_H$. We consider again the solution where the right-handed mixings in the second equation of (33) reach the fixed point $\tilde{\lambda}_{U_R}^{ai}|_{\text{fp}} \simeq g_* \sqrt{-\tilde{\gamma}_{U_R}} \mathbf{I}_3$. Substituting this in the first equation, we get

$$\tilde{\mathcal{Y}}_\nu^{ij}(m_*) \sim \left(\frac{m_*}{\Lambda_{\text{fl}}}\right)^{\gamma_H - \tilde{\gamma}_{U_R}} \tilde{\mathcal{Y}}_\nu^{ij}(\Lambda_{\text{fl}}), \quad (34)$$

and similarly

$$\tilde{\mathcal{Y}}_e^{ij}(m_*) \sim \left(\frac{m_*}{\Lambda_{\text{fl}}}\right)^{\gamma_H - \tilde{\gamma}_{D_R}} \tilde{\mathcal{Y}}_e^{ij}(\Lambda_{\text{fl}}). \quad (35)$$

The final Yukawa couplings are the sum of the bilinear contributions (34) and (35) and the one arising from linear terms $(\tilde{\lambda}_{U_R}^{ai})^* \tilde{\rho}_{U_R}/g_*$ and $(\tilde{\lambda}_{D_R}^{ai})^* \tilde{\rho}_{D_R}/g_*$.

Note that taking $\Lambda_{\text{fl}} \gtrsim 10^6$ TeV and $m_* \sim \mathcal{O}(10)$ TeV, for $\gamma_H \gtrsim 1^{14}$ and $-\tilde{\gamma}_{D_L} \gtrsim \mathcal{O}(0.1)$ the bilinear interaction $\tilde{\mathcal{Y}}_e^{ij}$ is at most comparable to the electron Yukawa. Thus, bilinear terms are expected to be subdominant for charged lepton, and analogously quark, hierarchies.

Turning to neutrinos, the data does not support a hierarchical structure so we need the anarchic bilinear contribution of (34) to dominate over $\tilde{\lambda}_{U_R}^{ai} \tilde{\rho}_{U_R}/g_*$. This robust possibility holds for $\tilde{\gamma}_{U_L} - \tilde{\gamma}_L^a + \tilde{\gamma}_{U_L}^i \gtrsim \gamma_H - \tilde{\gamma}_{U_R}$ for the choice of an intermediate scale $\Lambda_{\text{fl}} \sim \sqrt{\Lambda_{\text{UV}} m_*}$. Furthermore, for (34) to match the data on neutrino masses we need $\gamma_H - \tilde{\gamma}_{U_R} \sim 2.5$, assuming $m_*/\Lambda_{\text{fl}} \sim 10^{-5}$. Consistently this requires, for instance, $\tilde{\gamma}_{U_L}, \tilde{\gamma}_{U_L}^a \gtrsim 1$ to suppress the bilinear contribution.

IV. OUTLOOK

We have developed a framework of minimal flavor violation (MFV) for the composite Higgs scenario which emerges in two stages of RG evolution from the far UV. In the first stage at the highest energies the dynamics is flavor

anarchic, but evolves to generate realistic flavor hierarchies for the effective SM Yukawa couplings, while in the second stage the dynamics flows towards accidental flavor (generational) $U(3)$ symmetries, broken only by the Yukawa matrices. By the Higgs compositeness scale m_* , this realizes the proposal of [18,21–23,29] in which the SM GIM mechanism is extended to the host of compositeness effects, so that these can robustly evade low-energy precision CP violation and flavor-violating tests for $m_* \sim 10$ TeV.

Obviously, for phenomenological purposes it is important to ask just how low m_* can be, given this type of framework. We have discussed briefly in Appendix B how to generalize to the less symmetric variant (for the up-type right-handed quarks) in our two-stage evolution, in which $m_* \sim 5$ TeV is possible, at the edge of LHC sensitivity.

We also described the generation of hierarchies among the charged leptons, while obtaining (Dirac) neutrinos with much smaller but anarchic mass matrices. The neutrino mixing can be either CP conserving or significantly CP violating depending on whether the intermediate “flavor” scale separating the two stages of strong dynamics preserves CP or not, either of which is allowed.

There is also a sharp contrast in the phenomenology of composite Higgs between the anarchic and flavor-symmetric variants. The anarchic composite Higgs framework possesses an imperfect realization of the GIM mechanism, and therefore is strongly constrained by low energy CP and flavor tests, generally bounding $m_* > 10$ –50 TeV but with the electron EDM constraint giving $m_* > \text{PeV}$ [19]. If CP violation is restricted to the quark sector in the UV, then the electron EDM constraint is eliminated, and plausible future colliders could probe the composite physics. Because the heaviest SM particles are the most (partially) composite, any composites produced at colliders decay primarily into these heaviest particles: tops, Higgs, W, Z. Because the valence quarks and leptons at any collider have negligible partial compositeness, production of composites is mostly mediated by SM gauge bosons. On the other hand, flavor-symmetric composite Higgs allows significantly lower $m_* \sim 5$ –10 TeV, but now *all* generations of (right-handed) fermions have significant partial compositeness. Therefore valence quarks and leptons at colliders can directly couple to and create the composites, and these will decay into all generations. Indeed, the tightest bounds on m_* currently arise from quark compositeness bounds from the LHC. At future muon or electron colliders, the leptonic compositeness opens up powerful and exciting new channels, with significantly higher sensitivity to m_* than anarchic models. Moreover, the enlarged flavor symmetry group is tied to the presence of peculiar KK gauge bosons at m_* , unlike in the anarchic case, which may provide additional tests. We leave a detailed investigation of this for future work.

¹⁴This ensures that the gauge invariant scalar $\mathcal{O}_H^C \mathcal{O}_H$ is not a relevant operator (in a large- N approximation for the strong dynamics in which its dimension is approximately twice that of $\mathcal{O}_H$) that would reintroduce a hierarchy problem.

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX A: FROM ANARCHY TO MFV

In this appendix we discuss in more detail the RG evolution that leads to the phenomenological Lagrangian in (8) and (9).

The starting point in the UV is (12), which involves the three elementary quark doublets q^i coupled a total of nine doublet operators. These operators (and their corresponding mixings) are conveniently grouped into three sets: $\mathcal{O}_Q^a$, $\mathcal{O}_{U_L}^a$, and $\mathcal{O}_{D_L}^a$, with $a = 1, \dots, 3$. Focusing on the LH fields, the full RG at cubic order in λ can be written as

$$\begin{aligned} \frac{d\lambda_Q^{ai}}{d\log\mu} &= \gamma_Q^a \lambda_Q^{ai} + \lambda_Q^{aj} \frac{[\sum_{x=Q,U_L,D_L} \lambda_x^\dagger \lambda_x]^{ji}}{g_*^3} \\ \frac{d\lambda_{U_L}^{ai}}{d\log\mu} &= \gamma_{U_L}^a \lambda_{U_L}^{ai} + \lambda_{U_L}^{aj} \frac{[\sum_{x=Q,U_L,D_L} \lambda_x^\dagger \lambda_x]^{ji}}{g_*^3} \\ \frac{d\lambda_{D_L}^{ai}}{d\log\mu} &= \gamma_{D_L}^a \lambda_{D_L}^{ai} + \lambda_{D_L}^{aj} \frac{[\sum_{x=Q,U_L,D_L} \lambda_x^\dagger \lambda_x]^{ji}}{g_*^3}, \end{aligned} \quad (\text{A1})$$

where we have reabsorbed $O(1)$ coefficients in the cubic terms [see (7)] in the definitions of the λ s.

We assume the following scaling dimensions:

$$\text{Stage I: } \begin{cases} -1 < \gamma_Q^1 < \gamma_Q^2 < \gamma_{U_L}^3 < \gamma_Q^3 < 0 \\ \gamma_{D_L}^1 > \gamma_{D_L}^2 > \gamma_{D_L}^3 > 0 \\ \gamma_{U_L}^1 > \gamma_{U_L}^2 > 0, \end{cases} \quad (\text{A2})$$

where we have used the freedom to rearrange the indices a to achieve this ordering. Note the requirement $\gamma_{U_L}^3 < \gamma_Q^3$, needed to generate a sizable top Yukawa.

To begin with, we solve analytically (A1), making suitable approximations as follows. We first focus on the most relevant interaction, associated with $\mathcal{O}_Q^1$, namely λ_Q^{1i} , and neglect all other couplings $\lambda_Q^{2i}, \lambda_Q^{3i}, \lambda_{U_L,D_L}^{ai}$ for the purpose of solving the RG for λ_Q^{1i} . As we will show later, the subset of these couplings that could potentially contribute to the running of λ_Q^{1i} is in fact suppressed by their own RG equations. Within this approximation, the RG for λ_Q^{1i} simply reads

$$\frac{d\lambda_Q^{1i}}{d\log\mu} = \gamma_Q^1 \lambda_Q^{1i} + \lambda_Q^{1j} (\lambda_Q^{j1})^* \lambda_Q^{1i}, \quad (\text{A3})$$

with solution

$$\lambda_Q^{1i}(\mu) = \frac{\lambda_Q^{1i}(\Lambda_{UV})}{\sqrt{\delta_\mu^{-2\gamma_Q^1} - \frac{\|\lambda_Q^1(\Lambda_{UV})\|^2}{g_*^2 \gamma_Q^1} (1 - \delta_\mu^{-2\gamma_Q^1})}}, \quad (\text{A4})$$

where $\delta_\mu \equiv \mu/\Lambda_{UV}$ and

$$\|\lambda_Q^1(\mu)\| = \sqrt{\sum_{i=1,2,3} (\lambda_Q^{1i}(\mu))^2}. \quad (\text{A5})$$

In the IR, (A4) reaches the fixed point

$$\lambda_Q^{1i}|_{\text{fp}} = g_* \sqrt{-\gamma_Q^1} \frac{\lambda_Q^{1i}(\Lambda_{UV})}{\|\lambda_Q^1(\Lambda_{UV})\|}, \quad (\text{A6})$$

and by a rotation of the elementary quarks q^i we can set

$$\lambda_Q^{11}|_{\text{fp}} = g_* \sqrt{-\gamma_Q^1}, \quad \lambda_Q^{12}|_{\text{fp}} = \lambda_Q^{13}|_{\text{fp}} = 0. \quad (\text{A7})$$

Note that for $\delta_\mu \ll 1$, i.e., for an RG scale μ far below Λ_{UV} , we can expand as (A4)

$$\lambda_Q^{1i}(\mu) \simeq \lambda_Q^{1i}|_{\text{fp}} \left(1 + O(\delta_\mu^{-2\gamma_Q^1})\right), \quad (\text{A8})$$

i.e., $\delta_\mu^{-2\gamma_Q^1}$ controls corrections from the fixed point value in the IR.

Assuming λ_Q^{1i} reaches the fixed point soon after Λ_{UV} , we can replace (A7) into (A1) and iterate for λ_Q^{2i} , once again neglecting λ_{U_L,D_L}^{ai} entering the RG. We get

$$\begin{aligned} \frac{d\lambda_Q^{21}}{d\log\mu} &= (\gamma_Q^2 - \gamma_Q^1) \lambda_Q^{21} \\ \frac{d\lambda_Q^{2i}}{d\log\mu} &= \gamma_Q^2 \lambda_Q^{2i} + \frac{\lambda_Q^{2i} \|\lambda_Q^{2i}\|^2}{g_*^2} \quad i = 2, 3. \end{aligned} \quad (\text{A9})$$

The off-diagonal mixing λ_Q^{21} is exponentially suppressed, since $\gamma_Q^2 - \gamma_Q^1 > 0$, while $\|\lambda_Q^{2i=2,3}\|$ approaches itself the fixed point (we assume this also happens close to Λ_{UV} itself),

$$\lambda_Q^{22}|_{\text{fp}} = g_* \sqrt{-\gamma_Q^2}, \quad \lambda_Q^{23}|_{\text{fp}} = 0, \quad (\text{A10})$$

where we used the residual symmetries of the elementary quarks to set $\lambda_Q^{23}|_{\text{fp}}$ to zero.

Moving on to λ_Q^{3i} , it is straightforward to see from the associated RGs that $\lambda_Q^{3i=1,2} \rightarrow 0$ in the IR, since

$\gamma_Q^3 - \gamma_Q^{1,2} > 0$, similarly to the fate of λ_Q^{21} discussed above. On the other hand, the RG for λ_Q^{33} is more involved, as it requires including the third most relevant interaction, corresponding to $\mathcal{O}_{U_L}^3$. We can verify that $\lambda_{U_L}^{3i=1,2} \rightarrow 0$ under RG flow, due to $\gamma_{U_L}^3 - \gamma_Q^{1,2} > 0$, thereby justifying our earlier neglect of these terms in the RGs for $\lambda_Q^{3i=1,2}$. As for $\lambda_{U_L}^{33}$, it reaches the fixed point

$$\lambda_{U_L}^{33}|_{\text{fp}} = g_* \sqrt{-\gamma_{U_L}^3}, \quad (\text{A11})$$

which in turn drives $\lambda_Q^{33} \rightarrow 0$.

All of the remaining entries of λ_{U_L, D_L} , aside from $\lambda_{U_L}^{33}$, are suppressed by RG evolution. Approximating $\lambda_Q^{1i=1,2,3}$, $\lambda_Q^{2i=2,3}$, and $\lambda_{U_L}^{33}$ to be already close to their fixed points given in (A7), (A10), and (A11) near Λ_{UV} , by a straightforward but tedious evaluation of the RG, we find

$$\lambda_{U_L}(\mu) \simeq \begin{bmatrix} \delta_\mu^{\gamma_{U_L}^1 - \gamma_Q^1} & \delta_\mu^{\gamma_{U_L}^1 - \gamma_Q^2} & \delta_\mu^{\gamma_{U_L}^1 - \gamma_{U_L}^3} \\ \delta_\mu^{\gamma_{U_L}^2 - \gamma_Q^1} & \delta_\mu^{\gamma_{U_L}^2 - \gamma_Q^2} & \delta_\mu^{\gamma_{U_L}^2 - \gamma_{U_L}^3} \\ \delta_\mu^{-\gamma_Q^1} & \delta_\mu^{-\gamma_Q^2} & g_* \sqrt{-\gamma_{U_L}^3} \end{bmatrix},$$

$$\lambda_{D_L}(\mu) \simeq \begin{bmatrix} \delta_\mu^{\gamma_{D_L}^1 - \gamma_Q^1} & \delta_\mu^{\gamma_{D_L}^1 - \gamma_Q^2} & \delta_\mu^{\gamma_{D_L}^1 - \gamma_{U_L}^3} \\ \delta_\mu^{\gamma_{D_L}^2 - \gamma_Q^1} & \delta_\mu^{\gamma_{D_L}^2 - \gamma_Q^2} & \delta_\mu^{\gamma_{D_L}^2 - \gamma_{U_L}^3} \\ \delta_\mu^{\gamma_{D_L}^3 - \gamma_Q^1} & \delta_\mu^{\gamma_{D_L}^3 - \gamma_Q^2} & \delta_\mu^{\gamma_{D_L}^3 - \gamma_{U_L}^3} \end{bmatrix}, \quad (\text{A12})$$

where we drop $O(1)$ factors deriving from the specific $\lambda_{Q, U_L, D_L}(\Lambda_{UV}) \sim O(1)$. We emphasize, again, that ultimately only the entries λ_Q^{11} , λ_Q^{22} , and $\lambda_{U_L}^{33}$ are sizable in the IR while the others flow to zero, suppressed by the RG. This feature indeed justifies our initial (approximate) approach of solving independently for λ_Q^{11} , λ_Q^{22} , and $\lambda_{U_L}^{33}$ while basically neglecting other terms.

As explained in the main text, the RG evolution of the RH mixings is not significant in stage I since we always have $1 \lesssim \lambda_{U_R, D_R} \lesssim g_*$.

We now turn to stage II. Flavor symmetries constrain the scaling dimensions the various operators:

$$\text{Stage II: } \begin{cases} \gamma_{U_L}^{a=1,2,3} \equiv \gamma_{U_L} < 0 & \gamma_{D_L}^{a=1,2,3} \equiv \gamma_{D_L} > 0 \\ \gamma_{U_R}^{a=1,2,3} \equiv \gamma_{U_R} < 0 & \gamma_{D_R}^{a=1,2,3} \equiv \gamma_{D_R} < 0. \end{cases} \quad (\text{A13})$$

Let us start with the RH mixings. By a rotation of the elementary u^i , d^i and composite $\mathcal{O}_{U_R, D_R}^a$, we can bring $\lambda_{U_R, D_R}(\Lambda_{\text{fl}})$ into diagonal form, $\lambda_{U_R, D_R}^{a \neq i}(\Lambda_{\text{fl}}) \equiv 0$. The RG evolution is then simply

$$\frac{d\lambda_{U_R}^{ii}}{d \log \mu} = \gamma_{U_R} \lambda_{U_R}^{ii} + \frac{(\lambda_{U_R}^{ii})^3}{g_*^2}, \quad (\text{A14})$$

for $i = 1, 2, 3$. Again, λ_{U_R} reach the universal fixed point,

$$\lambda_{U_R}|_{\text{fp}} = g_* \sqrt{-\gamma_{U_R}} \mathbf{I}_3 \equiv \rho_{U_R} \mathbf{I}_3, \quad (\text{A15})$$

and analogously for the down sector.

The starting point of the RG for λ_{U_L, D_L} in stage II is

$$\lambda_{U_L}^{ai}(\Lambda_{\text{fl}}) = c_u^{ab} \lambda_{U_L}^{bi}(\Lambda_{\text{fl}}^+), \quad (\text{A16})$$

for the up sector, where $\lambda_{U_L}^{bi}(\Lambda_{\text{fl}}^+)$ is (A12) evaluated at $\mu = \Lambda_{\text{fl}}$ and c_u^{ab} accounts for the rearrangement of the degrees of freedom at the transition at Λ_{fl} (see the main text). It is convenient to perform a rotation of the composite operators $\mathcal{O}_{U_L}^a$ in order to put c_u^{ab} in lower triangular form. Then, given the hierarchical structure of $\lambda_{U_L}(\Lambda_{\text{fl}}^+)$, $\lambda_{U_L}(\Lambda_{\text{fl}})$ in (A16) takes the same form of (A12) up to irrelevant $O(1)$ factors. Note that the same rotation can be performed on $\mathcal{O}_{U_R}^a$, following the $U(3)_U$ symmetry, and on u^i , in order to maintain (A15). A similar consideration applies to the down sector. Furthermore, all couplings in (A12) are assumed to be rather small, with the only exception given by $\lambda_{U_L}^{33}$. We thus approximate the RG in stage II as

$$\begin{aligned} \frac{d\lambda_{U_L}^{ai}}{d \log \mu} &= \gamma_{U_L} \lambda_{U_L}^{ai} + \lambda_{U_L}^{ai} \frac{(\lambda_{U_L}^{33})^2}{g_*^2} \delta^{i3} & a = 1, 2 \\ \frac{d\lambda_{U_L}^{ai}}{d \log \mu} &= \gamma_{U_L} \lambda_{U_L}^{ai} + \lambda_{U_L}^{ai} \frac{(\lambda_{U_L}^{33})^2}{g_*^2} & a = 3 \\ \frac{d\lambda_{D_L}^{ai}}{d \log \mu} &= \gamma_{D_L} \lambda_{D_L}^{ai} + \lambda_{D_L}^{ai} \frac{(\lambda_{U_L}^{33})^2}{g_*^2} \delta^{i3}. \end{aligned} \quad (\text{A17})$$

As usual, $\lambda_{U_L}^{33}$ is assumed to approach its fixed point $\lambda_{U_L}^{33} \sim g_* \sqrt{-\gamma_{U_L}}$ rapidly, while the remaining couplings are either rescaled by (noninteger) powers of $\delta'_\mu = \mu/\Lambda_{\text{fl}}$ or stay approximately constant. At m_* we estimate

$$\lambda_{U_L}(m_*) \sim (\delta'_{m_*})^{\gamma_{U_L}} \begin{bmatrix} \delta_{\Lambda_{\text{fl}}}^{\gamma_{U_L}^1 - \gamma_Q^1} & \delta_{\Lambda_{\text{fl}}}^{\gamma_{U_L}^1 - \gamma_Q^2} & \frac{\delta_{\Lambda_{\text{fl}}}^{\gamma_{U_L}^1 - \gamma_{U_L}^3}}{(\delta'_{m_*})^{\gamma_{U_L}}} \\ \delta_{\Lambda_{\text{fl}}}^{\gamma_{U_L}^2 - \gamma_Q^1} & \delta_{\Lambda_{\text{fl}}}^{\gamma_{U_L}^2 - \gamma_Q^2} & \frac{\delta_{\Lambda_{\text{fl}}}^{\gamma_{U_L}^2 - \gamma_{U_L}^3}}{(\delta'_{m_*})^{\gamma_{U_L}}} \\ \frac{\delta_{\Lambda_{\text{fl}}}^{-\gamma_Q^1}}{(\delta'_{m_*})^{\gamma_{U_L}}} & \frac{\delta_{\Lambda_{\text{fl}}}^{-\gamma_Q^2}}{(\delta'_{m_*})^{\gamma_{U_L}}} & \frac{g_* \sqrt{-\gamma_{U_L}}}{(\delta'_{m_*})^{\gamma_{U_L}}} \end{bmatrix}, \quad (\text{A18})$$

where we neglected $O(1)$ numbers in the matrix in brackets. A completely analogous expression holds for the down sector, namely

$$\lambda_{D_L}(m_*) \sim (\delta'_{m_*})^{\gamma_{D_L}} \begin{bmatrix} \delta_{\Lambda_\Pi}^{\gamma_{D_L}^1 - \gamma_Q^1} & \delta_{\Lambda_\Pi}^{\gamma_{D_L}^1 - \gamma_Q^2} & \frac{\delta_{\Lambda_\Pi}^{\gamma_{D_L}^1 - \gamma_{U_L}^3}}{(\delta'_{m_*})^{\gamma_{U_L}^3}} \\ \delta_{\Lambda_\Pi}^{\gamma_{D_L}^2 - \gamma_Q^1} & \delta_{\Lambda_\Pi}^{\gamma_{D_L}^2 - \gamma_Q^2} & \frac{\delta_{\Lambda_\Pi}^{\gamma_{D_L}^2 - \gamma_{D_L}^3}}{(\delta'_{m_*})^{\gamma_{D_L}^3}} \\ \delta_{\Lambda_\Pi}^{-\gamma_{D_L}^3 - \gamma_Q^1} & \delta_{\Lambda_\Pi}^{-\gamma_{D_L}^3 - \gamma_Q^2} & \frac{\delta_{\Lambda_\Pi}^{-\gamma_{D_L}^3 - \gamma_{U_L}^2}}{(\delta'_{m_*})^{\gamma_{U_L}^2}} \end{bmatrix}, \quad (\text{A19})$$

Finally, as dictated by (11), (A18) and (A19) exhibit a hierarchical structure, similar to that of the Yukawa couplings in APC. However, in the up sector, both the left- and right-handed mixing angles are suppressed. In contrast, the left-handed mixing angles in the down sector are analogous to those in APC. Therefore, the V_{CKM} mixing angles are dominated by the down-sector contribution in (A19), and are estimated to be of the order

$$\theta_{12} \sim \delta_{\Lambda_\Pi}^{-\gamma_Q^1 + \gamma_Q^2}, \quad \theta_{23} \sim \frac{\delta_{\Lambda_\Pi}^{-\gamma_Q^2 + \gamma_{U_L}^3}}{(\delta'_{m_*})^{-\gamma_{U_L}^3}}, \quad \theta_{13} \sim \frac{\delta_{\Lambda_\Pi}^{-\gamma_Q^1 + \gamma_{U_L}^3}}{(\delta'_{m_*})^{-\gamma_{U_L}^3}}. \quad (\text{A20})$$

We have checked that the above approximate analytic solution of the RGs for the various λ s are in full agreement with a numerical evaluation of the full RG at $O(\lambda^3)$. Indeed, our benchmark points presented in (25) are based on such a complete analysis.

APPENDIX B: A LESS SYMMETRIC MODEL

In the main text, we discussed in detail an UV realization of a maximally symmetric scenario (MFV). Less symmetric scenarios, yet not completely anarchic, are compelling options still compatible with a compositeness scale m_* at the boundary of the (HL)LHC reach $\sim O(5 \text{ TeV})$ [18] (see Fig. 3).

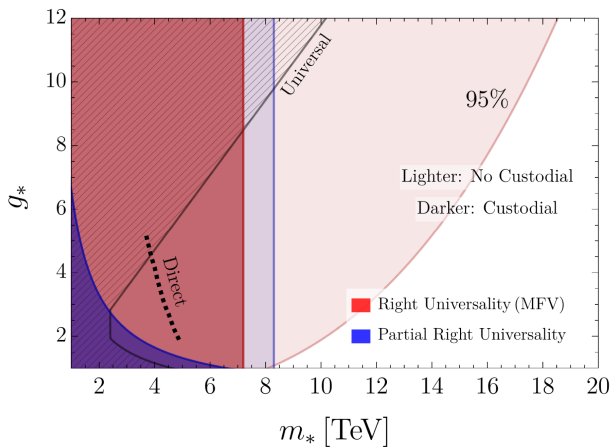


FIG. 3. Main sensitivity bounds for the flavor symmetric models presented in the text.

We here briefly discuss how to generalize our construction to this class of models. For concreteness, we focus on the quark sector of the model with partial up-type universality, which constitutes a minimal departure from (8). Namely, from the bottom up we consider the following EFT:

$$\mathcal{L}_{\text{EFT}} = \frac{m_*^4}{g_*^2} \hat{\mathcal{L}}_{\text{EFT}} \left(\frac{\lambda_{U_L}^{Ai} q^i}{m_*^{3/2}}, \frac{\lambda_{T_L}^i q^i}{m_*^{3/2}}, \frac{\lambda_{D_L}^{ai} q^i}{m_*^{3/2}}, \frac{\rho_{U_R} u^A}{m_*^{3/2}}, \frac{\rho_{T_R} t}{m_*^{3/2}}, \frac{\rho_{D_R} d^a}{m_*^{3/2}}, \dots \right), \quad (\text{B1})$$

where $A = 1, 2$ and $a = 1, 2, 3$. In the notation of (1), the previous mixings are

$$\lambda_q = \begin{bmatrix} \hat{\lambda}_{U_L} \\ (\vec{\lambda}_{T_L})^T \\ \lambda_{D_L} \end{bmatrix}, \quad \lambda_u = \begin{bmatrix} \rho_{U_R} \mathbf{I}_2 & \mathbf{0}_{2 \times 1} \\ \mathbf{0}_{1 \times 2} & \rho_{T_R} \\ & \mathbf{I}_3 \end{bmatrix}, \quad \lambda_d = \rho_{D_R} \begin{bmatrix} \mathbf{0}_3 \\ \mathbf{I}_3 \end{bmatrix}, \quad (\text{B2})$$

with λ_{D_L} a 3×3 matrix, $\hat{\lambda}_{U_L}$ a 2×3 matrix, and $\vec{\lambda}_{T_L}$ a three-vector. The global symmetry group is now $U(2)_U \times U(1)_T \times U(3)_D$, with $\hat{\lambda}_{U_L}$, $\vec{\lambda}_{T_L}$, and λ_{D_L} such that

$$\lambda_{U_L}^{Ai} q^i \in \bar{\mathbf{2}}_{U(2)_U}, \quad u^A \in \mathbf{2}_{U(2)_U}, \quad \lambda_{D_L}^{ai} q^i \in \bar{\mathbf{3}}_{U(3)_D}, \quad d^a \in \mathbf{3}_{U(3)_D}, \quad (\text{B3})$$

and t and $\vec{\lambda}_{T_L} \cdot q$ transforming under $U(1)_T$ with opposite charges.

The down sector reproduces the Yukawa as in (11), while the up sector has

$$Y_u^{ia} \simeq \left[c_u \frac{(\lambda_{U_L}^{Ai})^\dagger \rho_{U_R}}{g_*} c_t \frac{\lambda_{T_L}^{i*} \rho_{D_R}}{g_*} \right], \quad (\text{B4})$$

with $c_{u,t}$ $O(1)$ Wilson coefficients.

We want to generate the previous ansatz dynamically as a result of RG evolution. For that it is straightforward to generalize the two-stages RG presented in the main text. Indeed, the starting point is exactly (12) with the additional constraint that stage I is not completely anarchic but possesses a $U(1)_T$ symmetry. Under that the composites operators $\mathcal{O}_{T_L} \equiv \mathcal{O}_{U_L}^{a=3}$ and $\mathcal{O}_{T_R} \equiv \mathcal{O}_{U_R}^{a=3}$ have the same charge. Aside from this, the RG evolution in stage I is completely analogous to what was described in the main text and detailed in Appendix A. Indeed, the additional $U(1)_T$ symmetry is completely negligible in this first stage. The LH mixings evolve as (A18) and (A19), while the RH mixings are always $\lesssim g_*$.

In stage II, the flavor symmetries $U(2)_U \times U(3)_D$ are emergent. The $U(1)_T$ symmetry is assumed to be the same in both stages. The RG flow closely resembles that of MFV, with the key difference

$$\text{Stage II: } \begin{cases} \gamma_{U_L}^{1,2} \equiv \gamma_{U_L} & \gamma_{U_L}^3 \equiv \gamma_{T_L} \\ \gamma_{U_R}^{1,2} \equiv \gamma_{U_R} & \gamma_{U_R}^3 \equiv \gamma_{T_R} \end{cases} \quad (\text{B5})$$

as mandated by the $U(2)_U \times U(1)_T$ symmetry. We assume $\gamma_{U_R}, \gamma_{T_R}, \gamma_{T_L} \lesssim 0$ while $\gamma_{U_L} \gtrsim 0$, differently from the fully symmetric model. The $\lambda_{U_R}^{ai}$ approaches a nonuniversal fixed point,

$$\begin{aligned} \lambda_{U_R}^{ai} \big|_{\text{fp}} &= \begin{pmatrix} \rho_U & 0 & 0 \\ 0 & \rho_U & 0 \\ 0 & 0 & \rho_T \end{pmatrix} \\ &= g_* \begin{pmatrix} \sqrt{-\gamma_U} & 0 & 0 \\ 0 & \sqrt{-\gamma_U} & 0 \\ 0 & 0 & \sqrt{-\gamma_T} \end{pmatrix}, \end{aligned} \quad (\text{B6})$$

matching (B2).

Regarding the LH mixings, the starting point of stage II is (A16). The $U(1)_T$ symmetry constrains the structure of the anarchic matrix c_u^{ab} entering in (A16), enforcing the entries $c_u^{13} = c_u^{23} = c_u^{31} = c_u^{32} = 0$. Similarly to the MFV case, we can put the 2×2 block of c_u^{ab} ($a, b = 1, 2$) in upper triangular form performing a $U(2)_U$ rotation on $\mathcal{O}_{U_L}^a$. This can be reabsorbed in the $O(1)$ factor of the hierarchical matrix in (A18). Note that the same rotation must be applied to $\mathcal{O}_{U_R}^a$ following the $U(2)_U$ symmetry and to u^a to maintain the form of (B6). The resulting RG flow is very similar to (A18). In particular, we get

$$\lambda_{U_L}^{ai}(m_*) \sim (\delta'_{m_*})^{\gamma_{U_L}} \begin{bmatrix} \delta_{\Lambda_{\Pi}}^{\gamma_{U_L}^1 - \gamma_Q^1} & \delta_{\Lambda_{\Pi}}^{\gamma_{U_L}^1 - \gamma_Q^2} & \frac{\delta_{\Lambda_{\Pi}}^{\gamma_{U_L}^1 - \gamma_{T_L}^3}}{(\delta'_{m_*})^{\gamma_{T_L}}} \\ \delta_{\Lambda_{\Pi}}^{\gamma_{U_L}^2 - \gamma_Q^1} & \delta_{\Lambda_{\Pi}}^{\gamma_{U_L}^2 - \gamma_Q^2} & \frac{\delta_{\Lambda_{\Pi}}^{\gamma_{U_L}^2 - \gamma_{T_L}^3}}{(\delta'_{m_*})^{\gamma_{T_L}}} \\ \frac{\delta_{\Lambda_{\Pi}}^{-\gamma_Q^1}}{(\delta'_{m_*})^{\gamma_{U_L}}} & \frac{\delta_{\Lambda_{\Pi}}^{-\gamma_Q^2}}{(\delta'_{m_*})^{\gamma_{U_L}}} & \frac{g_* \sqrt{-\gamma_{T_L}}}{(\delta'_{m_*})^{\gamma_{U_L}}} \end{bmatrix}, \quad (\text{B7})$$

and we can identify $\lambda_{U_L}(m_*) = [\hat{\lambda}_{U_L}^T(m_*), \vec{\lambda}_{T_L}^T(m_*)]^T$. The down sector is completely equivalent to MFV and we get (A19) with the replacement $\gamma_{U_L} \rightarrow \gamma_{T_L}$.

The benchmark values $m_* = 5$ TeV, $g_* = 3$, not excluded by present constraints, with $\Lambda_{\Pi} = 10^6$ TeV, $\Lambda_{UV} = m_{\text{Planck}}$, and

$$\begin{aligned} \text{Stage I: } & \gamma_{U_L}^a \sim (0.1, 0.05, -0.05), \\ & -\gamma_Q^a \sim (0.35, 0.25, 0.05), \\ & \gamma_{D_L}^a \sim (0.1, 0.05, 0.05), \\ & -\gamma_{U_R, D_R}^a \sim O(0.1) \end{aligned}$$

$$\begin{aligned} \text{Stage II: } & \gamma_{U_L} \sim 0.05, & \gamma_{U_R} \sim -0.15 \\ & \gamma_{D_L} \sim 0.05, & \gamma_{D_R} \sim -0.5 \\ & \gamma_{T_L} \sim -0.07, & \gamma_{T_R} \sim -1 \end{aligned} \quad (\text{B8})$$

give quark masses and mixings in the ballpark of the SM values. Furthermore, notice that $\gamma_{T_R} = -1$ corresponds to the limit where $\mathcal{O}_{T_R}$ becomes free. It can be shown [10] that this is dual to having the top t_R fully composite.

APPENDIX C: EXPERIMENTAL CONSTRAINTS

Here we discuss the main low-energy experimental constraints on the models presented in the main text and Appendix B. A comprehensive analysis is provided in [18], from which all of the results quoted below are taken. We organize the discussion into two parts: quark-sector and lepton-sector constraints. All of the quoted bound refer to exclusion of new physics at 95% CL. A summary of the resulting constraints on the (m_*, g_*) plane are shown in Fig. 3.

Quarks. For convenience, we present the relevant constraints on the Lagrangian in (B1). These can be readily translated to (8) by the substitution $\rho_T \rightarrow \rho_U$. It is further convenient to normalize $\rho_{U,T,D}$ in units of g_* :

$$\rho_U \equiv \varepsilon_u g_*, \quad \rho_T \equiv \varepsilon_t g_*, \quad \rho_D \equiv \varepsilon_d g_*, \quad (\text{C1})$$

with $\varepsilon_u, \varepsilon_t \leq 1$.

The models feature sizably composite light right-handed quarks, controlled by ε_u and ε_d (ρ_U and ρ_D). These are tested by measurements of four-fermion interactions in dijet events at the LHC, which require (at 95% CL)

$$\frac{m_*}{g_* \varepsilon_u^2} \gtrsim 4.8 - 7.8(8.2 - 13) \text{ TeV}, \quad (\text{C2})$$

$$\frac{m_*}{g_* \varepsilon_d^2} \gtrsim 3.0 - 3.6(5.1 - 5.8) \text{ TeV}, \quad (\text{C3})$$

where the numbers in parentheses refer to the projected sensitivity at the end of run 3 (300 fb⁻¹). In each case, the two values correspond to destructive and constructive interference with the Standard Model.¹⁵

Among flavor transitions, the most significant effects are $\Delta F = 1$ in $b \rightarrow s$ transitions mediated by a Z exchange. These are tightly constrained by the decay $B_s \rightarrow \mu^+ \mu^-$,

¹⁵Note, however, that integrating out an s -channel heavy spin-1 resonance results only in destructive interference.

which leads to the bound

$$m_* \gtrsim \frac{6.5 - 8.3}{\varepsilon_t} \text{TeV}, \quad (\text{C4})$$

at 95% CL.

In the model with partial up-type universality prefers $\varepsilon_t \sim 1$. In contrast, in the model with complete up-type universality (MFV) $\varepsilon_t = \varepsilon_u$, and thus the lowest allowed m_* for any g_* is obtained combining (C4) with (C2) (see Fig. 3).

The effect in (C4) can be suppressed by imposing custodial protection of the Z couplings, as proposed in [49]. In such scenarios, $\Delta F = 1$ effects become effectively negligible, and the dominant flavor transitions are associated with B_d oscillations, leading to the bound

$$m_* \gtrsim \frac{6.6}{g_* \varepsilon_t^2} \text{TeV}, \quad (\text{C5})$$

at 95% CL. Again, for models with full RH universality $\varepsilon_t = \varepsilon_u$ so that (C5) must be combined with (C2). On the contrary, the model of (B1) prefers $\varepsilon_t \sim 1$.

In Fig. 3 we show the sensitivity reaches for models with full and partial universality, with and without custodial protection of the Z couplings, optimized on $\varepsilon_{u,t}$ to allow the lowest m_* . For any bound quoted above, it is conservatively considered the stronger side of the interval. The figure also shows universal constraint (gray region), arising from the measurement of bosonic operators and assuming the strong sector is protected by large effects in $\hat{T}$ (see [50]). The black, dashed line shows the direct LHC reach on spin-1 resonances kinetically mixed with the SM gauge bosons.

In the main text, in (25), we consider the following benchmark point:

$$m_* = 10 \text{ TeV}, \quad g_* = 3, \quad \varepsilon_u = 0.7, \quad \varepsilon_d = 0.8, \quad (\text{C6})$$

which is at the edge of present constraints on MFV without custodial protection and well within that in the opposite case.

As a more optimistic option, in (B8) we also consider the benchmark point,

$$m_* \sim 5 \text{ TeV}, \quad g_* \sim 3, \quad \varepsilon_t \sim 1, \\ \varepsilon_u \sim 0.4, \quad \varepsilon_d \sim 0.7, \quad (\text{C7})$$

which is at the boundary of the current LHC direct reach yet well within the indirect flavor bounds in partially symmetric models with custodial protection.

Because the RH flavor universality in (9) arises dynamically from the RG, there will be deviations at m_* , which will result in flavor violation at m_* . Consider, for instance, the down sector, where corrections to (A15) at m_* are at most of the order

$$\lambda_{D_R}^{ai}(m_*) = \rho_{D_R} \mathbf{I}_3 \left(1 + O\left(\frac{m_*}{\Lambda_{\text{fl}}}\right)^{-2\gamma_{D_R}} \right), \quad (\text{C8})$$

as can be seen from (A8). Bounds on C_1 for $\Delta F = 2$ transitions in the sd system require [48]

$$\frac{g_*^2 \varepsilon_d^4}{m_*^2} \left(\frac{m_*}{\Lambda_{\text{fl}}} \right)^{-4\varepsilon_d^2} \lesssim (10^4 - 10^5 \text{ TeV})^{-2}, \quad (\text{C9})$$

where recall $\varepsilon_d^2 = -\gamma_{D_R}$. Assuming $\Lambda_{\text{fl}} = 10^6$, the previous constraint is compatible with both (C6) and (C7). A similar expression holds for the up-sector current bounds from the uc system $\Delta F = 2$ [48] are well compatible with (C6) and marginally compatible with (C7). Furthermore, notice that these effects can be simply suppressed by pushing Λ_{fl} further into the UV.

Leptons. In the lepton sector, (26), via (27), leads to no dangerous flavor transition in the approximation of neglecting the very small $\tilde{Y}_\nu$. Yet, the compositeness of right-handed leptons, i.e., the parameter $\varepsilon_d \equiv \tilde{\rho}_D/g_*$ in (26), is bounded by large electron positron data. Assuming a rough 10^{-3} precision on Z couplings, we can estimate

$$\frac{g_*^2}{m_*^2} v^2 \varepsilon_e^2 \lesssim 10^{-3} \quad \Rightarrow \quad m_* \gtrsim 8g_* \varepsilon_e \text{TeV}. \quad (\text{C10})$$

Simultaneous quark and lepton compositeness can also lead to effects in high-energy di-lepton Drell–Yan measurements at the LHC. In particular, the projected sensitivity of the LHC with 300 fb^{-1} for the dimension-6 operator $(\bar{u}\gamma^\mu u)(\bar{e}\gamma_\mu e)$ [51], at 95% CL implies

$$\frac{g_*^2 \varepsilon_u^2 \varepsilon_e^2}{m_*^2} \lesssim 4 \times 10^{-3} \text{ TeV}^{-2}. \quad (\text{C11})$$

The benchmark point in (C6) is compatible with both (C10) and (C11), provided that $\varepsilon_e \lesssim 0.4$.

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