

J/ψ polarization and p_T distribution in $c\bar{c}$ associated hadroproduction at $\mathcal{O}(\alpha_s^5)$

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The quarkonium production in association with a heavy-quark pair of the same flavor was found to play an important role in various production schemes and provide a distinctive signature that could be studied in experiment. Within the nonrelativistic QCD framework, we perform the first complete calculation of the hadroproduction process $gg \rightarrow J/\psi + c\bar{c}$ at the next-to-leading order (NLO) in the expansion of strong coupling constant α_s . The result tells us that the NLO corrections substantially enhance the color-singlet yield and alter the predicted polarization patterns and, hence, lead to a better agreement with experimental data across the p_T spectrum. In contrast to charm-quark fragmentation, the fixed-order J/ψ and $c\bar{c}$ associated production channel displays a markedly different kinematic behavior. While the inclusion of this process greatly reduces the discrepancy between color-singlet theoretical prediction and experimental measurement, a residual gap of approximately 1 order of magnitude still remains.

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Introduction. Heavy quarkonium production serves as a unique probe into both perturbative and nonperturbative aspects of QCD, including the mechanisms of hadronization and the nature of color confinement. Despite decades of progress, the quarkonium production mechanism remains an unsettled issue. The nonrelativistic QCD (NRQCD) factorization framework [1] provides a systematic approach to separate the perturbative calculable short-distance sector from the long-distance hadronization part via a double expansion in the strong coupling constant and heavy-quark velocity. However, studies on the higher-order velocity expansion color-octet (CO) contributions show somewhat incompatible results [2].

Within NRQCD, the production cross section is expressed as a sum over intermediate quark-antiquark states, each weighted by a short-distance coefficient (SDC), calculable in perturbation theory, and a long-distance matrix element (LDME) that encodes nonperturbative transition probabilities. While SDCs are process dependent, LDMEs are assumed to be universal, allowing global fit across multiple production channels.

Extensive investigations of J/ψ transverse momentum (p_T) distributions at colliders have established the essential

role of CO mechanisms [3–19]. However, significant discrepancies persist between theoretical predictions and experimental measurements, especially on charmonium polarization and high- p_T region. The discrepancies can be mainly traced back to the fitting schemes in extracting CO LDMEs. Although the feed-down effects from higher charmonium states, such as $\psi(2S)$ and χ_{cJ} , may help reconcile some of the inconsistencies in the total prompt J/ψ yield [18], the determination of a universal and consistent set of CO LDMEs remains an open challenge.

From an experimental aspect, the CMS Collaboration reported the measurements of J/ψ polarization at the LHC Run I with colliding energy $\sqrt{s} = 7$ TeV [20] and Run II with $\sqrt{s} = 13$ TeV [21]. The observed polar anisotropy parameter λ_θ shows only mild variation with p_T and tends to a value around 0.3 in the high- p_T region. This behavior suggests a suppression or cancellation between transverse and longitudinal components, implying the presence of unaccounted production channels with nearly unpolarized characteristics.

In this context, the associated production process $pp \rightarrow J/\psi + c\bar{c}$ emerges as a promising candidate. Earlier studies [7,8,22–25] examined $J/\psi + c\bar{c}$ associated production across various production mechanisms. These works reveal sizable contributions relative to inclusive J/ψ production, particularly in pp collision. Research also indicates that the next-to-leading-order (NLO) corrections to the charmonium production tend to be substantial [8,23–25]. In particular, it was found that the gluon-gluon fusion process may yield nearly unpolarized J/ψ mesons [7,8], in agreement with experimental data. Despite the potential importance of the

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$gg \rightarrow J/\psi + c\bar{c}$ process in the color-singlet (CS) channel, nevertheless, the full NLO QCD corrections to it have never been computed, mainly due to the complexity of the multi-body final-state phase space and the large number of contributing Feynman diagrams.

In this Letter, we report the first complete calculation of the $gg \rightarrow J/\psi + c\bar{c}$ process at NLO QCD [$\mathcal{O}(\alpha_s^5)$], retaining leading-order accuracy in the NRQCD velocity expansion. We present results for both the J/ψ transverse momentum distribution and polarization and demonstrate that the NLO corrections significantly enhance the CS contribution and yield polarization behavior in improved agreement with experimental observations.

Calculations. The spin density matrix $d\sigma_{s_z s'_z}$ characterizes both the production rate and polarization of the produced J/ψ . At leading order in the NRQCD velocity expansion, it factorizes as

$$\begin{aligned} d\sigma_{s_z s'_z}(pp \rightarrow J/\psi + c\bar{c}) \\ = \sum_{i,j} \int d\eta_i d\eta_j f_{i/p}(\eta_i, \mu^2) f_{j/p}(\eta_j, \mu^2) \\ \times d\hat{\sigma}_{s_z s'_z}(ij \rightarrow c\bar{c}[^3S_1] + c\bar{c}) \langle \mathcal{O}^{J/\psi}(^3S_1) \rangle, \end{aligned} \quad (1)$$

where $\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle$ denotes the CS LDME. The partonic spin density matrix is built from helicity amplitudes via

$$d\hat{\sigma}_{s_z s'_z} \propto \sum_{s_i, s_j, s_c, s_{\bar{c}}} \mathcal{A}^{s_i s_j s_c s_{\bar{c}}} (\mathcal{A}^{s_i s_j s'_c s'_{\bar{c}}})^*. \quad (2)$$

Here, $\mathcal{A}^{s_i s_j s_c s_{\bar{c}}}$ signifies the helicity amplitude with spin indices $(s_i, s_j, s_c, s_{\bar{c}})$ corresponding to the initial partons i, j , the final J/ψ , and the final charm quark and antiquark, respectively. The polarization of the J/ψ is determined by its dilepton decay angular distribution in the J/ψ rest frame, parametrized as [26,27]

$$\frac{d\sigma(J/\psi \rightarrow \ell^+ \ell^-)}{d\cos\theta} \propto 1 + \lambda_\theta \cos^2\theta, \quad (3)$$

where λ_θ characterizes the reconstructed J/ψ polarization, and $\lambda_\theta \rightarrow \pm 1$ correspond to transverse polarization (+1) and longitudinal polarization (−1) of J/ψ , respectively. It can be expressed in terms of spin density matrix as

$$\lambda_\theta = \frac{d\sigma_{11} - d\sigma_{00}}{d\sigma_{11} + d\sigma_{00}}. \quad (4)$$

The $g(k_1) + g(k_2) \rightarrow J/\psi(p_0) + c(p_1) + \bar{c}(p_2)$ associated process involves 42 leading-order (LO) and over 2000 NLO Feynman diagrams. Typical NLO Feynman diagrams are shown in Fig. 1. Our calculation largely follows the strategy outlined in [25]. Scattering amplitudes are first analytically generated and projected onto the CS $c\bar{c}[^3S_1]$ state. We adopt the spinor helicity formalism to numerically evaluate amplitudes, except for loop diagrams containing Coulomb divergences. For the latter, we apply the conventional squared-amplitude method to compute both the total and the longitudinal J/ψ cross sections. The longitudinal polarization cross section $d\sigma_L = d\sigma_{00}$ is evaluated by employing the longitudinal polarization projection tensor

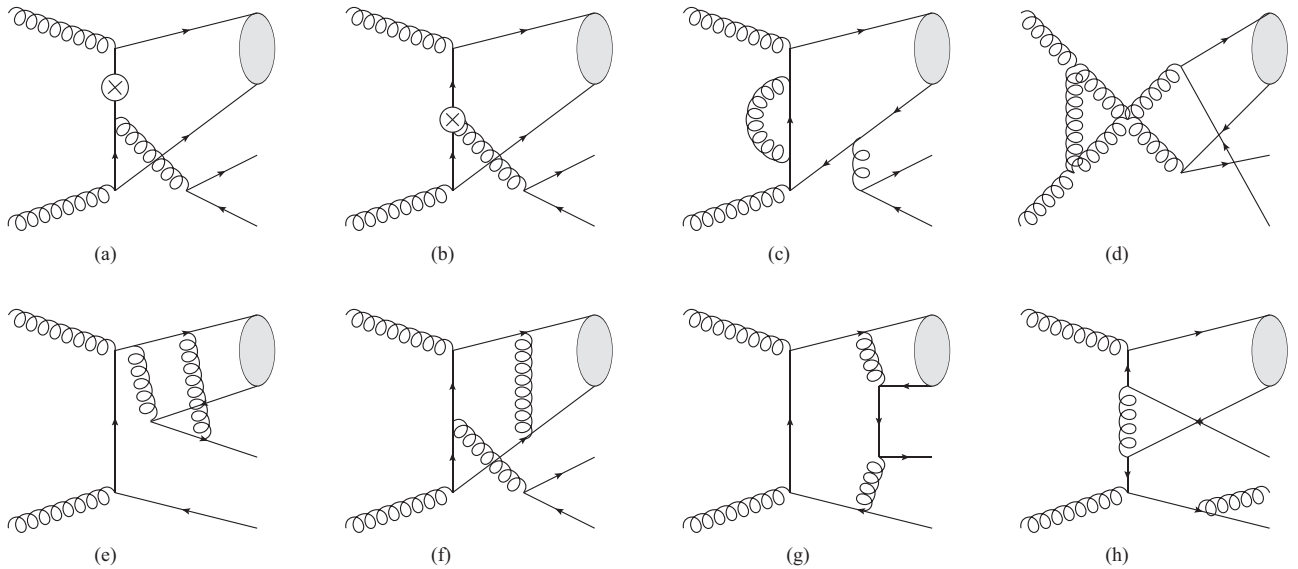


FIG. 1. Typical NLO diagrams of $gg \rightarrow J/\psi + c\bar{c}$ process. (a),(b) Counterterm diagrams. (c)–(g) Two- to six-point one-loop diagrams, incorporating UV, soft, collinear, Coulomb divergences, respectively. (h) Represents real correction.

$$\Pi_L^{\mu\nu} = \varepsilon_L^\mu(p_0)\varepsilon_L^{*\nu}(p_0) = \frac{1}{M^2} \left(p_0^\mu - \frac{M^2}{k_1 \cdot p_0} k_1^\mu \right) \times \left(p_0^\nu - \frac{M^2}{k_1 \cdot p_0} k_1^\nu \right), \quad (5)$$

where M is the mass of the J/ψ . The transverse cross section is obtained via $d\sigma_T = d\sigma_{11} + d\sigma_{-1,-1} = d\sigma - d\sigma_L$, where $d\sigma$ is the total cross section.

Loop integrals without Coulomb singularities are evaluated by using integrand reduction via the Laurent-expansion method [28,29], while those terms containing Coulomb poles are handled via integration-by-parts reduction using NeatIBP [30]. For loop diagrams, the calculation may be greatly simplified through subtracting those contributions related by symmetric permutations of the initial gluons and the final (anti)charm quarks. Ultraviolet (UV) divergences are canceled via renormalization. The renormalization constant Z_g is defined in the modified minimal subtraction ($\overline{\text{MS}}$) scheme, while Z_2 , Z_m , and Z_3 are defined in the on-shell scheme. Infrared (IR) divergences arising in the real corrections are extracted by means of the two-cutoff phase space slicing method [31]. Thus, the total cross section at NLO accuracy can be expressed as

$$\sigma_{\text{tot}} = \int_{3\text{-body}} \left(d\sigma^{\text{LO}} + d\sigma^{\text{Virtual}} + d\sigma_{\text{S}}^{\text{Real}} + d\sigma_{\text{HC}}^{\text{Real}} \right) + \int_{4\text{-body}} d\sigma_{\text{HC}}^{\text{Real}}. \quad (6)$$

Here, $d\sigma^{\text{LO}}$, $d\sigma^{\text{Virtual}}$, and $d\sigma^{\text{Real}}$ denote the LO, virtual, and real contributions, respectively. The virtual term comprises three separately computed contributions: counterterms from renormalization, loop diagrams with Coulomb singularities, and those without. The real contributions are partitioned into soft (S), hard collinear (HC), and hard noncollinear ($\text{HC}\bar{C}$) regions.

As an analytic check, the IR divergences of the loop amplitudes are extracted following the method of [32]. The cancellation between virtual and real corrections leads to

$$2\mathcal{A}_{\text{div}}^{\text{Virtual}}(\mathcal{A}^{\text{LO}})^* = -|\mathcal{A}_{\text{div}}^{\text{Real}}|^2 = \sum_{i,j=1}^3 \hat{\mathcal{A}}_i^{\text{LO}} \Delta^{ij} (\hat{\mathcal{A}}_j^{\text{LO}})^*. \quad (7)$$

Here, $\hat{\mathcal{A}}_i^{\text{LO}}$ represent the color-decomposed Born-level amplitudes,

$$\mathcal{A}^{\text{LO}} = \delta^{g_1 g_2} \delta_{c\bar{c}} \hat{\mathcal{A}}_1^{\text{LO}} + (T^{g_1} T^{g_2})_{c\bar{c}} \hat{\mathcal{A}}_2^{\text{LO}} + (T^{g_2} T^{g_1})_{c\bar{c}} \hat{\mathcal{A}}_3^{\text{LO}}, \quad (8)$$

with $g_{1,2}$ and $c\bar{c}$ denoting the color indices of the initial gluons and final-state charm quarks, respectively. The IR-divergent kernel Δ reads

$$\Delta = \begin{pmatrix} -\frac{144}{\epsilon^2} + \frac{8(4C_0-35)}{\epsilon} & -\frac{24}{\epsilon^2} + \frac{2(8C_0+9(C_1-C_2-C_3+C_4)-70)}{3\epsilon} & -\frac{24}{\epsilon^2} + \frac{2(8C_0-9(C_1-C_2-C_3+C_4)-70)}{3\epsilon} \\ -\frac{24}{\epsilon^2} + \frac{2(8C_0+9(C_1-C_2-C_3+C_4)-70)}{3\epsilon} & -\frac{32}{\epsilon^2} + \frac{C_0+72C_1-9C_2-9C_3+72C_4-560}{9\epsilon} & \frac{4}{\epsilon^2} + \frac{10C_0-9(C_1+C_2+C_3+C_4)+70}{9\epsilon} \\ -\frac{24}{\epsilon^2} + \frac{2(8C_0-9(C_1-C_2-C_3+C_4)-70)}{3\epsilon} & \frac{4}{\epsilon^2} + \frac{10C_0-9(C_1+C_2+C_3+C_4)+70}{9\epsilon} & -\frac{32}{\epsilon^2} + \frac{C_0-9C_1+72C_2+72C_3-9C_4-560}{9\epsilon} \end{pmatrix}. \quad (9)$$

Here,

$$C_0 = \frac{4(s_{45} - 2m_c^2)}{\sqrt{s_{45}(s_{45} - 4m_c^2)}} \ln \left(\frac{\sqrt{s_{45}} + \sqrt{s_{45} - 4m_c^2}}{2m_c} \right), \quad C_1 = \ln \left(\frac{(m_c^2 - t_{14})^2}{m_c^2 s_{12}} \right), \\ C_2 = \ln \left(\frac{(m_c^2 - t_{15})^2}{m_c^2 s_{12}} \right), \quad C_3 = \ln \left(\frac{(m_c^2 - t_{24})^2}{m_c^2 s_{12}} \right), \quad C_4 = \ln \left(\frac{(m_c^2 - t_{25})^2}{m_c^2 s_{12}} \right), \quad (10)$$

with s_{ij} and t_{ij} being the corresponding Mandelstam invariants.

Numerical results. In numerical calculation, the charm-quark mass is taken as half of the J/ψ mass, i.e., $m_c = 1.5$ GeV. The renormalization and factorization scales are set as $\mu_r = \mu_f = m_T \equiv \sqrt{p_T^2 + 4m_c^2}$, where m_T is the transverse mass of the J/ψ . The running coupling

constant is evaluated at one-loop (two-loop) accuracy for LO (NLO) calculation, and the proton parton distribution functions CTEQ6L1 (CTEQ6M) [33] are employed at LO (NLO). The CS LDME satisfies $\langle \mathcal{O}^{J/\psi}({}^3S_1) \rangle = \frac{9}{2\pi} |R(0)|^2$, where the squared radial wave function at the origin,

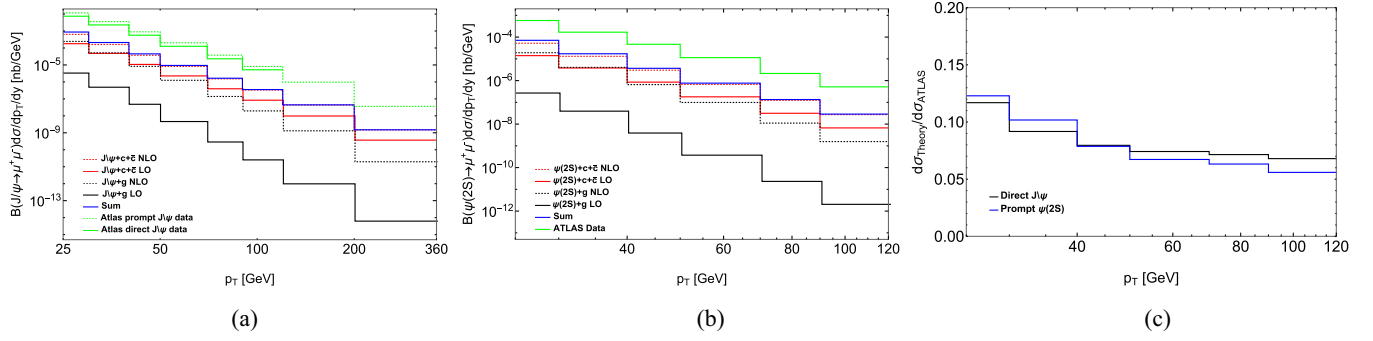


FIG. 2. At $\sqrt{s} = 13$ TeV LHC, differential cross section distributions in p_T for (a) J/ψ and (b) $\psi(2S)$ production at LO and NLO in the CS contribution. (c) The ratios of the direct J/ψ and prompt $\psi(2S)$ cross sections over the ATLAS data.

$|R(0)|^2$, is extracted from the leptonic decay width $\Gamma(J/\psi \rightarrow e^+e^-) = 5.55$ keV [34].

To isolate direct J/ψ production from prompt experimental data, feed-down contributions from $\psi(2S)$ and χ_{cJ} states are included. The $\psi(2S)$ feed-down is fitted up to 120 GeV using ATLAS data [35], with branching ratios $B(J/\psi \rightarrow \mu^+\mu^-) = 0.0593$, $B(\psi(2S) \rightarrow J/\psi) = 0.595$, and $B(\psi(2S) \rightarrow \mu^+\mu^-) = 0.0077$. The transverse momentum of the feed-down J/ψ is approximated by $p_T^{J/\psi} = \frac{m_{J/\psi}}{m_{\psi(2S)}} p_T^{\psi(2S)}$. The χ_{cJ} feed-down contribution, taken from [36], amounts to approximately 26%.

Figure 2 presents the p_T differential cross sections at $\sqrt{s} = 13$ TeV LHC experiment. Figure 2(a) shows the J/ψ production results. For comparison, we also show the LO and NLO CS contributions of the $J/\psi + g$ production process, which dominates inclusive $J/\psi + X$ production. To facilitate comparison, ATLAS prompt J/ψ data [35] are rebinned to match our binning, and direct J/ψ production is obtained by subtracting feed-down contributions. Our results indicate that, despite being higher order in α_s , the CS contribution from associated $J/\psi + c\bar{c}$ production dominates over $J/\psi + g$, exhibiting a milder falloff with the increase of p_T . The shape of the CS $J/\psi + c\bar{c}$ process

matches the direct J/ψ data well, though a normalization gap remains.

Given the negligible feed-down to $\psi(2S)$, its production provides a clean probe of short-distance dynamics. Figure 2(b) shows the p_T distribution of $\psi(2S)$, where the difference from J/ψ in the color-singlet model arises solely from the LDME; we use $|R_{\psi(2S)}|^2 = 0.639$ GeV³ from [37]. Figure 2(c) displays the ratios of the direct J/ψ and prompt $\psi(2S)$ cross sections to the ATLAS data. The similar p_T dependence of these ratios suggests analogous behavior in the CS channel.

The J/ψ polarization parameters are shown in Fig. 3. Unlike the longitudinal polarization observed in $J/\psi + g$ production, the $J/\psi + c\bar{c}$ channel yields nearly unpolarized J/ψ at both LO and NLO, improving agreement with experimental measurements. The polarization tends toward unpolarized as p_T increases.

The fragmentation process is expected to contribute comparably to $J/\psi + c\bar{c}$ production. Since charm-quark fragmentation dominates among CS fragmentation contributions, we estimate the fragmentation contribution from the dominant $gg \rightarrow c\bar{c}$ process in NLO accuracy,

$$d\sigma_{\text{frag}}^{\text{NLO}}(p_0) = 2 \left(d\hat{\sigma}_{gg \rightarrow c\bar{c}}^{\text{NLO}} \left(\frac{p_0}{z}, \mu_f \right) \otimes D_{c \rightarrow J/\psi}^{\text{LO}}(z, \mu_f) + d\hat{\sigma}_{gg \rightarrow c\bar{c}}^{\text{LO}} \left(\frac{p_0}{z}, \mu_f \right) \otimes D_{c \rightarrow J/\psi}^{\text{NLO}}(z, \mu_f) \right). \quad (11)$$

Here, $\otimes$ denotes convolution product over z . $D_{c \rightarrow J/\psi}^{\text{LO,NLO}}(z, \mu_f)$ represents the charm to J/ψ fragmentation function at LO and NLO, respectively. The overall factor of 2 accounts for identical c and $\bar{c}$ contributions. The fragmentation functions are taken from [38,39], and evolved via the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi equation from the initial scale $\mu_0 = 3m_c$,

$$\mu_f^2 \frac{d}{d\mu_f^2} D_{c \rightarrow J/\psi}^{\text{LO,NLO}}(z, \mu_f) = \frac{\alpha_s(\mu_f)}{2\pi} P_{cc}(z) \otimes D_{c \rightarrow J/\psi}^{\text{LO,NLO}}(z, \mu_f), \quad (12)$$

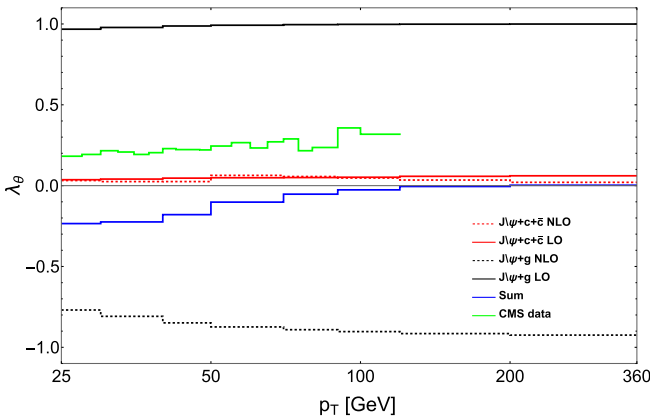


FIG. 3. J/ψ polarization parameter λ_θ as a function of p_T in the helicity frame at $\sqrt{s} = 13$ TeV LHC.

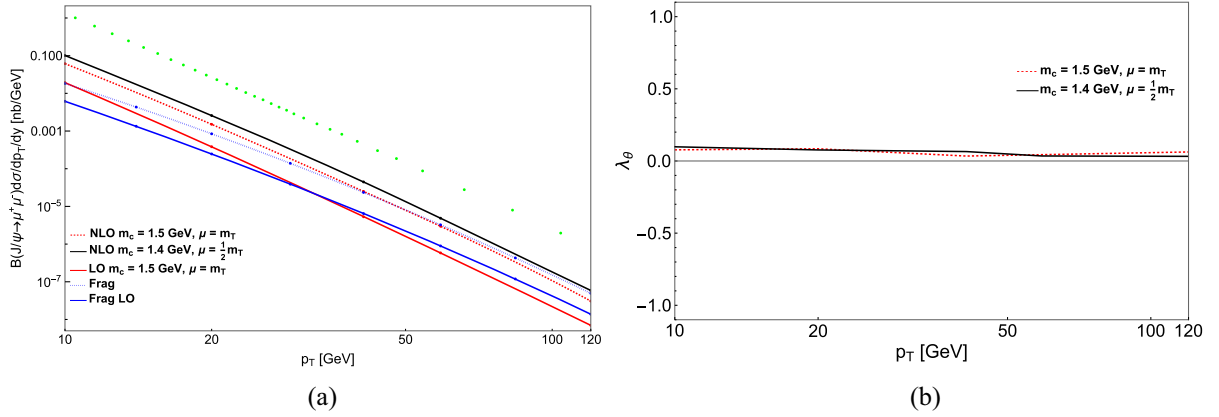


FIG. 4. At $\sqrt{s} = 7$ TeV LHC, (a) p_T differential cross sections for J/ψ production. The green dots are CMS data. (b) J/ψ polarization in $J/\psi + c\bar{c}$ production with varied charm mass and scale.

where $P_{cc}(z)$ is the regularized splitting function. Figure 4(a) compares the $J/\psi + c\bar{c}$ hadroproduction, charm-quark fragmentation, and CMS data at $\sqrt{s} = 7$ TeV LHC. At low p_T , the associated production dominates over fragmentation at both LO and NLO, in agreement with Ref. [7]. At high p_T , the fragmentation contribution becomes larger than the associated one for $m_c = 1.5$ GeV and $\mu = m_T$, consistent with [40] depicted. As p_T increases, the NLO fragmentation contribution approaches the result with $m_c = 1.4$ GeV, $\mu = \frac{1}{2}m_T$, exhibiting a clear upward trend that suggests it may overtake the associated process at even higher p_T . This indicates that the fragmentation contribution cannot adequately describe the associated process at fixed order in QCD, and the discrepancy is expected to be mitigated by resummation of higher-order logarithmic terms. Considering mass and scale uncertainties, the cross section increases roughly twofold, yet a gap with experimental data persists. Figure 4(b) shows the polarization parameter with varied charm mass and scale, indicating little sensitivity.

Conclusions. The quarkonium production in association with a heavy-quark pair of the same flavor turn out to be the dominant processes among various color-singlet production schemes, which bear evident signatures that could be studied in experiment. We present in this Letter the first calculation of J/ψ hadroproduction associated with a $c\bar{c}$ pair at the NLO QCD in the framework of NRQCD. Results show that the NLO QCD corrections are substantial and may enhance the LO contribution by several times. This

confirms the persistence of sizable color-singlet contributions beyond the LO.

According to the calculation, the predicted p_T spectrum of the associated production process exhibits a similar shape to the experimental data, though an obvious discrepancy in magnitude remains. The polarization of J/ψ in the associated production behaves approximately unpolarized in huge transverse momentum scope, consistent with the recent CMS measurement. The inclusion of the NLO corrections of the concerned process may greatly reduce the uncertainty in the global fitting of color-octet LDMEs.

Last, it is worth noting that we have compared our calculation with the charm-quark fragmentation approximation and find that the full results are distinct from the latter in both p_T behavior and overall magnitude.

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Data availability. The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

- [1] G. T. Bodwin, E. Braaten, and G. P. Lepage, *Phys. Rev. D* **51**, 1125 (1995); **55**, 5853(E) (1997).
- [2] G. T. Bodwin, *arXiv:1208.5506*.
- [3] C. H. Chang, R. Li, and J. X. Wang, *Phys. Rev. D* **80**, 034020 (2009).
- [4] P. Artoisenet, J. M. Campbell, F. Maltoni, and F. Tramontano, *Phys. Rev. Lett.* **102**, 142001 (2009).
- [5] J. M. Campbell, F. Maltoni, and F. Tramontano, *Phys. Rev. Lett.* **98**, 252002 (2007).
- [6] J. P. Lansberg, *Phys. Lett. B* **695**, 149 (2011).
- [7] P. Artoisenet, J. P. Lansberg, and F. Maltoni, *Phys. Lett. B* **653**, 60 (2007).
- [8] B. Gong and J. X. Wang, *Phys. Rev. Lett.* **100**, 232001 (2008).
- [9] M. Kramer, J. Zunft, J. Steegborn, and P. M. Zerwas, *Phys. Lett. B* **348**, 657 (1995).
- [10] M. Krämer, *Nucl. Phys.* **B459**, 3 (1996).
- [11] M. Butenschoen and B. A. Kniehl, *Phys. Rev. Lett.* **104**, 072001 (2010).
- [12] M. Butenschoen and B. A. Kniehl, *Phys. Rev. Lett.* **108**, 172002 (2012).
- [13] M. Butenschoen and B. A. Kniehl, *Phys. Rev. Lett.* **107**, 232001 (2011).
- [14] K. T. Chao, Y. Q. Ma, H. S. Shao, K. Wang, and Y. J. Zhang, *Phys. Rev. Lett.* **108**, 242004 (2012).
- [15] Y. Q. Ma, K. Wang, and K. T. Chao, *Phys. Rev. D* **84**, 114001 (2011).
- [16] Y. Q. Ma, K. Wang, and K. T. Chao, *Phys. Rev. Lett.* **106**, 042002 (2011).
- [17] Y. J. Zhang, Y. Q. Ma, K. Wang, and K. T. Chao, *Phys. Rev. D* **81**, 034015 (2010).
- [18] B. Gong, L. P. Wan, J. X. Wang, and H. F. Zhang, *Phys. Rev. Lett.* **110**, 042002 (2013).
- [19] N. Brambilla, M. Butenschoen, and X. P. Wang, *Phys. Rev. D* **112**, L011902 (2025).
- [20] S. Chatrchyan *et al.* (CMS Collaboration), *Phys. Lett. B* **727**, 381 (2013).
- [21] A. Hayrapetyan *et al.* (CMS Collaboration), *Phys. Lett. B* **858**, 139044 (2024).
- [22] C. F. Qiao and J. X. Wang, *Phys. Rev. D* **69**, 014015 (2004).
- [23] Y. J. Zhang and K. T. Chao, *Phys. Rev. Lett.* **98**, 092003 (2007).
- [24] Z. Q. Chen, L. B. Chen, and C. F. Qiao, *Phys. Rev. D* **95**, 036001 (2017).
- [25] Q. M. Feng and C. F. Qiao, *Phys. Rev. D* **110**, 094047 (2024).
- [26] E. Braaten and J. Russ, *Annu. Rev. Nucl. Part. Sci.* **64**, 221 (2014).
- [27] P. Faccioli, C. Lourenco, J. Seixas, and H. K. Wohri, *Eur. Phys. J. C* **69**, 657 (2010).
- [28] P. Mastrolia, E. Mirabella, and T. Peraro, *J. High Energy Phys.* **06** (2012) 095; **11** (2012) 128(E).
- [29] T. Peraro, *Comput. Phys. Commun.* **185**, 2771 (2014).
- [30] Z. Wu, J. Boehm, R. Ma, H. Xu, and Y. Zhang, *Comput. Phys. Commun.* **295**, 108999 (2024).
- [31] B. W. Harris and J. F. Owens, *Phys. Rev. D* **65**, 094032 (2002).
- [32] S. Dittmaier, *Nucl. Phys.* **B675**, 447 (2003).
- [33] J. Pumplin, D. R. Stump, J. Huston, H. L. Lai, P. M. Nadolsky, and W. K. Tung, *J. High Energy Phys.* **07** (2002) 012.
- [34] P. A. Zyla *et al.* (Particle Data Group), *Prog. Theor. Exp. Phys.* **2020**, 083C01 (2020).
- [35] G. Aad *et al.* (ATLAS Collaboration), *Eur. Phys. J. C* **84**, 169 (2024).
- [36] G. Aad *et al.* (ATLAS Collaboration), *J. High Energy Phys.* **07** (2014) 154.
- [37] Y. J. Zhang, Y. Q. Ma, and K. T. Chao, *Phys. Rev. D* **78**, 054006 (2008).
- [38] E. Braaten, K. m. Cheung, and T. C. Yuan, *Phys. Rev. D* **48**, 4230 (1993).
- [39] X. C. Zheng, C. H. Chang, and X. G. Wu, *Phys. Rev. D* **100**, 014005 (2019).
- [40] B. Gong, R. Li, and J. X. Wang, *arXiv:1102.0118*.