

Searching for gluon saturation effects in the momentum transfer dependence of coherent charmonium electroproduction off nuclei

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We study for the first time the transverse momentum transfer distributions $d\sigma/dt$ in coherent production of charmonia in nuclear ultraperipheral and electron-ion collisions within the QCD color dipole approach based on a rigorous Green function formalism. This allows us to treat properly the color transparency effects, as well as the higher and leading-twist shadowing corrections associated with the $|Q\bar{Q}\rangle$ and $|Q\bar{Q}nG\rangle$ Fock components of the photon. While the multigluon photon fluctuations represent the dominant source of nuclear shadowing at kinematic regions related to the recent LHC and its future upgrade to LHeC, the upcoming electron-ion collider at RHIC will additionally require the proper incorporation of reduced quark shadowing. The latter effect leads to a significant decrease in the differential cross sections $d\sigma/dt$ compared to standard calculations based on the eikonal form for the dipole-nucleus amplitude. The leading-twist shadowing corrections, corresponding to a nonlinear QCD evolution of a partial dipole-nucleus amplitude, reduce substantially charmonium t -distributions in the LHeC energy range. We predict a nonmonotonic energy dependence of $d\sigma/dt$ suggesting so possible gluon saturation effects with increased onset at larger t -values. In addition to shadowing corrections, we study how the color transparency effects affect the shape of t -dependent nuclear modification factor.

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I. INTRODUCTION

Recent experiments with ultraperipheral collisions (UPC) at Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) (see e.g., [1]) offer new possibilities for investigation of exclusive photoproduction of vector mesons on protons and nuclei. [2]. Whereas the LHC kinematic region allows to study the space-time pattern of the diffraction mechanism [3] in coherent photoproduction of heavy quarkonia at high photon energies, the experiments on the planned Large Hadron-Electron Collider (LHeC) from the High Luminosity-LHC [4] will extend such studies also to an electroproduction process with a nonzero photon virtuality Q^2 . Here the theoretical description is simplified due to the long coherence length (CL) l_c for the $Q\bar{Q}$ photon fluctuation, much longer than the nuclear radius R_A ,

$$l_c = \frac{W^2 + Q^2 - m_N^2}{m_N(M_V^2 + Q^2)} \gg R_A, \quad (1.1)$$

where W is the c.m. energy of the photon-nucleon system; m_N and M_V are the masses of the nucleon and vector meson, respectively. The inequality (1.1) is related to the strongest onset of the initial state quark shadowing. Then the corresponding expressions for the nuclear cross sections lie on the eikonal form for the $Q\bar{Q}$ dipole-nucleus cross section and/or for the impact-parameter-dependent partial amplitude [5–7]. The eikonal form is frequently adopted in the literature also in kinematic regions where the condition (1.1) is not completely fulfilled (see e.g., Ref. [8]).

Future experiments at the Electron-Ion Collider (EIC) [9–11] using the present RHIC facility will provide an opportunity to look inside a diffraction mechanism also in the kinematic region where $l_c \lesssim R_A$. Here, one cannot use the eikonal approximation anymore and more sophisticated formalism is required to describe quantum coherence (QC) effects. In heavy quarkonium electroproduction, a short lifetime of the $Q\bar{Q}$ photon fluctuation, $l_c \ll R_A$, is correlated with a much longer formation time (length) (FL) [12], $l_f \sim R_A$, controlling the evolution of the $Q\bar{Q}$ wave packet during propagation through the medium. The magnitude of l_f can be obtained in the rest frame of the nucleus from the uncertainty principle,

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$$l_f = \frac{W^2 + Q^2 - m_N^2}{m_N(M_{V'}^2 - M_V^2)} \approx l_c \cdot \frac{M_{V'}^2 + Q^2}{M_{V'}^2 - M_V^2} = l_c \cdot \left[\frac{M_{V'}^2 + Q^2}{M_{V'}^2 - M_V^2} - 1 \right], \quad (1.2)$$

where $M_{V'}$ is the mass of radially excited quarkonium. The propagation of the $Q\bar{Q}$ pair in the medium is related to the color transparency (CT) effect (see e.g., Refs. [13–16]), which represents the final state absorption of the produced quarkonia. Here the medium becomes more transparent for $Q\bar{Q}$ dipole configurations with smaller transverse sizes. Note that in the electroproduction of heavy quarkonia there is a strong inequality, $l_c \ll l_f$, as follows from Eq. (1.2).

The description of the diffraction mechanism in terms of color dipoles has a long-standing history starting from Ref. [13] and has been applied to exclusive photo and electroproduction of heavy quarkonia on protons and nuclei (see e.g., Refs. [14,17–20]). In our recent work [21] we have studied the coherent process $\gamma^*A \rightarrow V A$ [$V = J/\Psi(1S), \psi'(2S)$] including properly the above mentioned effects of QC and CT within a rigorous quantum mechanical approach based on the Green function formalism. We have performed corresponding predictions for t -integrated¹ production cross sections that can be verified by future experiments at the EIC.

In Ref. [6], the t -dependent differential cross sections $d\sigma^{\gamma^*A \rightarrow VA}/dt$ have been studied in the LHC kinematic region of recent experiments with UPC, where the eikonal approximation related to the condition (1.1) can be safely adopted. In the present work, we improve such predictions extending them also to the kinematic region beyond the validity of Eq. (1.1). This requires to incorporate the reduced effects of QC by applying the path integral technique. Simultaneously we aim to minimize all known theoretical uncertainties of the QCD dipole formalism, related to quarkonium S-wave functions together with elimination of D-wave component, dipole orientation, as well as the magnitude of the leading-twist shadowing corrections. Consequently, this may lead to increased efficiency in searching for a conclusive signal of gluon saturation effects [22].

We include the correlation between the dipole transverse orientation $\vec{r}$ and the impact parameter of the collision $\vec{b}$ [23]. This effect causes that the $Q\bar{Q}$ dipole-nucleon interaction vanishes when $\vec{r} \perp \vec{b}$ but reaches maximal strength when $\vec{r} \parallel \vec{b}$. However, it is missed in many calculations. The $\vec{r}$ - $\vec{b}$ correlation has been treated in Ref. [24], but not properly incorporated, since the $\vec{b}$ -dependent dipole-nucleon amplitude $\mathcal{A}_{Q\bar{Q}}^N(\vec{r}, \vec{b})$ after integration over $\vec{b}$ exhibits a nonmonotonic behavior as a function of the

dipole size r . Besides, the ratio of dipole-nucleon amplitudes, $R_{\perp/\parallel}(b) = \mathcal{A}_{Q\bar{Q}}^N(\vec{r}, \vec{b}, \Theta = \pi/2) / \mathcal{A}_{Q\bar{Q}}^N(\vec{r}, \vec{b}, \Theta = 0)$ leads to incorrect limiting values that differ from expected magnitudes 1 and 0 when $b \rightarrow 0$ and $b \sim R_A$.

The impact of the $\vec{r}$ - $\vec{b}$ correlation on the magnitude of differential cross sections in exclusive photoproduction of heavy quarkonia on protons has been studied in Ref. [20]. The nodal structure of the wave functions for radially excited heavy quarkonium states enhances the onset of the correlation effect and thus provides additional constraints on the models for the b -dependent dipole amplitude. The importance of the color dipole orientation in other processes has been discussed in Ref. [20] (see also references therein). The $\vec{r}$ - $\vec{b}$ correlation has been also omitted in Ref. [25] analyzing the impact-parameter dependence in the initial condition of the Balitsky-Kovchegov (BK) equation [26,27].

However, in comparison with nucleon target, the effect of $\vec{r}$ - $\vec{b}$ correlation is diluted in processes on nuclear targets [6]. A rather weak impact of the dipole orientation on the azimuthal asymmetry of photons and pions has been found and analyzed in Refs. [23,28–30]. Nevertheless, in the present paper, we implement this effect in calculations of $d\sigma^{\gamma^*A \rightarrow VA}/dt$, thus minimizing the theoretical uncertainties.

The LHC and LHeC kinematic region, related to the condition (1.1), gives rise to a maximal strength of the higher-twist quark shadowing. However, such a shadowing correction is rather small for the photo and electroproduction of heavy quarkonia due to the large heavy quark mass. Then the main nuclear effect comes from the leading-twist gluon shadowing, which is related to the higher Fock components of the projectile photon containing gluons. According to the analysis in Ref. [6], the dominant contribution to nuclear shadowing comes from the $|Q\bar{Q}G\rangle$ Fock component. We implement the gluon shadowing (GS) corrections in our calculations of nuclear cross sections from Refs. [6,18,31]. Inclusion of higher multi-gluon Fock components $|Q\bar{Q}nG\rangle$ is still a challenge. However, their effect is essentially taken into account by the eikonalization of the GS correction factor.

The LHeC kinematic region allows to analyze how a modification of the gluon distribution in nuclei by the GS corrections affects the t -dependent differential cross sections. In the infinite momentum frame, the phenomenon of GS looks similar to gluon-gluon fusion corresponding to a nonlinear term in evolution equations [26,27,32,33]. We expect a suppression of small- x gluons and a precocious onset of saturation effects, especially for heavy nuclei. This may lead to a nonmonotonic energy dependence of $d\sigma^{\gamma^*A \rightarrow VA}/dt$ at fixed t -values. Such an expectation is explored in the present work and represents one of the main goals of our study. Note that a nonmonotonic energy dependence of $d\sigma/dt$ has been presented at large $t \sim 1 \text{ GeV}^2$ also in Ref. [34] studying the incoherent production

¹ $t = -q^2$, where $\vec{q}$ is the transverse component of the momentum transfer.

of J/ψ on the lead target within the hot-spot model. However, the important effect of gluon shadowing has been ignored. This may have a large impact on the reliability of the predicted onset of saturation effects.

The present paper is organized as follows. In Sec. II we briefly present the dipole formalism for electroproduction on a proton target, $\gamma^* p \rightarrow V p$, in terms of the $\vec{b}$ -dependent dipole amplitude. Section III is devoted to the coherent heavy quarkonium production on nuclear targets, $\gamma^* A \rightarrow V A$. Here we present expressions for the t -dependent differential cross sections $d\sigma^{\gamma^* A \rightarrow V A}/dt$ within a rigorous Green function formalism. Consequently, in Sec. IV A we first compare our model calculations of $d\sigma^{\gamma^* p \rightarrow V p}/dt$ ($V = J/\psi(1S)$ and $\psi'(2S)$) and $\sigma^{\gamma p \rightarrow \psi'(2S)p}$ with available data from experiments at the Hadron-Electron Ring Accelerator (HERA) and the LHC. Then in Sec. IV B we propose how manifestations of CT effects may be recognized by the future EIC measurements. Section IV C is devoted to predictions for $d\sigma/dt$ in the coherent photo and electroproduction of 1S and 2S charmonium states off nuclei in kinematic regions accessible by upcoming EIC experiments at RHIC and the LHeC. Here we analyze the impact of reduced quantum coherence effects for $Q\bar{Q}$ photon fluctuations on magnitudes of the t -dependent nuclear differential cross sections at small photon energies W , when $l_c \lesssim R_A$, for fixed values of Q^2 and t . Moreover, for the LHeC kinematic region, we predict in Sec. IV D the possible onset of saturation effects, manifested via a nonmonotonic energy behavior of $d\sigma^{\gamma^* A \rightarrow V A}/dt$. Section V contains summary and discussion of our results.

II. ELECTROPRODUCTION OF HEAVY QUARKONIA ON PROTONS

Within the light-front (LF) color dipole formalism, the amplitude for the electroproduction of heavy quarkonia with the transverse component of momentum transfer $\vec{q}$ takes the following factorized form [14],

$$\begin{aligned} \mathcal{A}^{\gamma^* p \rightarrow V p}(x, Q^2, \vec{q}) &= \langle V | \tilde{\mathcal{A}} | \gamma^* \rangle \\ &= \int d^2 r \int_0^1 d\alpha \Psi_V^*(\vec{r}, \alpha) \mathcal{A}_{Q\bar{Q}}(\vec{r}, x, \alpha, \vec{q}) \\ &\quad \times \Psi_{\gamma^*}(\vec{r}, \alpha, Q^2). \end{aligned} \quad (2.1)$$

Here $\mathcal{A}_{Q\bar{Q}}(\vec{r}, x, \alpha, \vec{q})$ is the amplitude for the elastic scattering of the color dipole on a nucleon target.

It is convenient to study differential cross sections $d\sigma/dq^2$ treating the partial dipole scattering amplitude in the impact parameter representation $\mathcal{A}_{Q\bar{Q}}(\vec{r}, x, \alpha, \vec{b})$ related to the $\vec{q}$ -dependent amplitude by Fourier transform,

$$\mathcal{A}_{Q\bar{Q}}(\vec{r}, x, \alpha, \vec{q}) = \int d^2 b e^{i\vec{b} \cdot \vec{q}} \mathcal{A}_{Q\bar{Q}}(\vec{r}, x, \alpha, \vec{b}), \quad (2.2)$$

which correctly reproduces the dipole cross section at $\vec{q} = 0$,

$$\begin{aligned} \sigma_{Q\bar{Q}}(r, x) &= \text{Im} \mathcal{A}_{Q\bar{Q}}(\vec{r}, x, \alpha, \vec{q} = 0) \\ &= 2 \int d^2 b \text{Im} \mathcal{A}_{Q\bar{Q}}^N(\vec{r}, x, \alpha, \vec{b}). \end{aligned} \quad (2.3)$$

In Eq. (2.1), the variable $\Psi_V(r, \alpha)$ is the LF wave function for heavy quarkonium and $\Psi_{\gamma^*}(r, \alpha, Q^2)$ is the LF distribution of the $Q\bar{Q}$ Fock component of the real ($Q^2 = 0$) or virtual ($Q^2 > 0$) photon, with transverse separation $\vec{r}$. The variable α is the fractional LF momentum carried by a heavy quark or antiquark from a $Q\bar{Q}$ Fock component of the photon.

Combining Eqs. (2.1)–(2.3) the electroproduction amplitude $\mathcal{A}^{\gamma^* p \rightarrow V p}(x, Q^2, \vec{q})$ has the following form:

$$\begin{aligned} \mathcal{A}^{\gamma^* p \rightarrow V p}(x, Q^2, \vec{q}) &= 2 \int d^2 b e^{i\vec{b} \cdot \vec{q}} \int d^2 r \int_0^1 d\alpha \Psi_V^*(\vec{r}, \alpha) \\ &\quad \times \text{Im} \mathcal{A}_{Q\bar{Q}}^N(\vec{r}, x, \alpha, \vec{b}) \Psi_{\gamma^*}(\vec{r}, \alpha, Q^2), \end{aligned} \quad (2.4)$$

where the impact parameter $\vec{b}$ of the dipole is the transverse distance from the target to the dipole center of gravity, which varies with the fractional LF momenta of Q or $\bar{Q}$.

The amplitude (2.4) depends on Bjorken x evaluated in [35] in the leading $\log(1/x)$ approximation,

$$x = \frac{M_V^2 + Q^2}{s} = \frac{M_V^2 + Q^2}{W^2 + Q^2 - m_N^2}. \quad (2.5)$$

The essential feature of the dipole-proton partial amplitude $\mathcal{A}_{Q\bar{Q}}^N(\vec{r}, x, \alpha, \vec{b})$ in Eq. (2.4) is the $\vec{r}$ - $\vec{b}$ correlation. Its explicit form was proposed in Refs. [20,23,28–30] and is as follows:

$$\begin{aligned} \text{Im} \mathcal{A}_{Q\bar{Q}}^N(\vec{r}, x, \alpha, \vec{b}) &= \frac{\sigma_0}{8\pi \mathcal{B}(x)} \left\{ \exp \left[-\frac{[\vec{b} + \vec{r}(1-\alpha)]^2}{2\mathcal{B}(x)} \right] \right. \\ &\quad + \exp \left[-\frac{(\vec{b} - \vec{r}\alpha)^2}{2\mathcal{B}(x)} \right] \\ &\quad \left. - 2 \exp \left[-\frac{r^2}{R_0^2(x)} - \frac{[\vec{b} + (1/2 - \alpha)\vec{r}]^2}{2\mathcal{B}(x)} \right] \right\}, \end{aligned} \quad (2.6)$$

where the function $\mathcal{B}(x)$ was defined in Ref. [30] as

$$\mathcal{B}(x) = B_{el}^{\bar{q}q}(x, r \rightarrow 0) - \frac{1}{8} R_0^2(x). \quad (2.7)$$

Here $R_0^2(x)$ controls the x dependence of the saturation cross section, introduced in [36,37], $\sigma_{Q\bar{Q}}(r, x) = \sigma_0(1 - \exp[-r^2/R_0^2(x)])$. We adopt parameters from the recent work [38], $\sigma_0 = 25.21$ mb, $R_0(x) = 0.4 \text{ fm} \times (x/x_0)^{0.1405}$ with $x_0 = 0.80 \times 10^{-4}$, referred to as the GBS-0 model.

The dipole-proton slope in the limit of vanishingly small dipoles $B_{el}^{qq}(x, r \rightarrow 0)$ can be measured in the electroproduction of vector mesons with highly virtual photons $Q^2 \gg 1 \text{ GeV}^2$. The measured slope $B_{\gamma^* p \rightarrow \rho p}(x, Q^2 \gg 1 \text{ GeV}^2) \approx 5 \text{ GeV}^{-2}$ [39] is, as expected [40], defined by the proton charge radius.

The GBS-0 model mentioned above lacks the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution. However, Ref. [38] also contains the same model with such an evolution. Here the saturation scale is related to the gluon density, which is subject to the DGLAP evolution, $R_0^2(x, \mu^2) = 4/Q_s^2(x, \mu^2) = \sigma_0 N_c / (\pi^2 \alpha_s(\mu^2) xg(x, \mu^2))$, where $\mu^2 = C/r^2 + \mu_0^2$, Q_s^2 is the saturation scale, and the gluon distribution function $xg(x, \mu^2)$ is obtained as a solution of the DGLAP evolution equation with the form at the initial scale $Q_0^2 = 1 \text{ GeV}^2$, $xg(x, Q_0^2) = A_g x^{-\lambda_g} (1-x)^{5.6}$. Here $A_g = 1.07$, $\lambda_g = 0.11$, $\mu_0^2 = 1.74 \text{ GeV}^2$, $C = 0.27$, and $\sigma_0 = 22.93$ mb. In what follows, we will refer to the GBS dipole model as to the GBS-0 model containing the DGLAP evolution and the color dipole orientation (2.6).

Treating the electroproduction of heavy quarkonia and assuming the s -channel helicity conservation, the t -dependent differential cross section can be expressed as a sum of T and L contributions for transversely and longitudinally polarized photons and vector mesons, respectively,

$$\begin{aligned} & \frac{d\sigma^{\gamma^* p \rightarrow Vp}(x, Q^2, t = -q^2)}{dt} \\ &= \frac{d\sigma_T^{\gamma^* p \rightarrow Vp}(x, Q^2, t)}{dt} + \tilde{\epsilon} \frac{d\sigma_L^{\gamma^* p \rightarrow Vp}(x, Q^2, t)}{dt} \\ &= \frac{1}{16\pi} (|\mathcal{A}_T^{\gamma^* p \rightarrow Vp}(x, Q^2, \vec{q})|^2 + \tilde{\epsilon} |\mathcal{A}_L^{\gamma^* p \rightarrow Vp}(x, Q^2, \vec{q})|^2), \end{aligned} \quad (2.8)$$

where we have taken the photon polarization $\tilde{\epsilon} = 0.99$.

We also include a small real part [41–43] of the $\gamma^* p \rightarrow Vp$ amplitude performing the following replacement in Eq. (2.4):

$$\begin{aligned} \text{Im } \mathcal{A}_{Q\bar{Q}}^N(\vec{r}, x, \alpha, \vec{b}) &\Rightarrow \text{Im } \mathcal{A}_{Q\bar{Q}}^N(\vec{r}, x, \alpha, \vec{b}) \cdot \left(1 - i \frac{\pi\Lambda}{2}\right), \\ \Lambda &= \frac{\partial \ln(\text{Im } \mathcal{A}_{Q\bar{Q}}^N(\vec{r}, x, \alpha, \vec{b}))}{\partial \ln(1/x)}. \end{aligned} \quad (2.9)$$

In order to include the skewness correction [44] we perform the following modification,

$$\text{Im } \mathcal{A}_{Q\bar{Q}}^N(\vec{r}, x, \alpha, \vec{b}) \Rightarrow \text{Im } \mathcal{A}_{Q\bar{Q}}^N(\vec{r}, x, \alpha, \vec{b}) \cdot R_S(\Lambda), \quad (2.10)$$

where the skewness factor $R_S(\Lambda) = (2^{2\Lambda+3}/\sqrt{\pi}) \cdot \Gamma(\Lambda + 5/2)/\Gamma(\Lambda + 4)$.

In our previous works [5,19–21,45–47] (see also Sec. II.A in Ref. [6]) we ruled out the possibility of a photonlike vertex for the transition of a heavy quarkonium to a $Q\bar{Q}$ pair due to the unusually large weight of a D -wave component in the rest frame wave function, in contrast to solutions of the Schrödinger equation. Instead, we rely on the quarkonium LF wave functions obtained from a solution of the Lorentz boosted Schrödinger equation [48] with realistic potentials. Although several such potentials can be found in the literature, in the present paper we choose only the powerlike potential (POW) [49]. This potential in combination with the GBS model [38] for the dipole cross section provides the best description of the UPC data, as shown in Ref. [21]. Since the quark transverse momenta are not parallel to the boost axis, a significant correction to the boosting prescription [50] from the rest frame to the LF frame is taken into account, known as the Melosh spin rotation [51] (see also [19,52]). This leads to the following specific form of the electroproduction amplitudes given by Eq. (2.4) for T and L polarizations:

$$\mathcal{A}_T^{\gamma^* p \rightarrow Vp}(x, Q^2, \vec{q}) = N_p \int d^2r \int_0^1 da \int d^2b e^{i\vec{b}\cdot\vec{q}} \text{Im } \mathcal{A}_{Q\bar{Q}}^N(\vec{r}, x, \alpha, \vec{b}) [\Sigma_T^{(1)}(r, \alpha, Q^2) + \Sigma_T^{(2)}(r, \alpha, Q^2)],$$

with

$$\begin{aligned} \Sigma_T^{(1)}(r, \alpha, Q^2) &= K_0(\eta r) \int_0^\infty dp_T p_T J_0(p_T r) \Psi_V(\alpha, p_T) \left[\frac{2m_Q^2(m_L + m_T) + m_L p_T^2}{m_T(m_L + m_T)} \right], \\ \Sigma_T^{(2)}(r, \alpha, Q^2) &= K_1(\eta r) \int_0^\infty dp_T p_T^2 J_1(p_T r) \Psi_V(\alpha, p_T) \left[\eta \frac{m_Q^2(m_L + 2m_T) - m_T m_L^2}{m_Q^2 m_T(m_L + m_T)} \right], \end{aligned} \quad (2.11)$$

and

$$\mathcal{A}_L^{\gamma^* p \rightarrow V p}(x, Q^2, \vec{q}) = N_p \int d^2 r \int_0^1 d\alpha \int d^2 b e^{i\vec{b} \cdot \vec{q}} \text{Im} \mathcal{A}_{Q\bar{Q}}^N(\vec{r}, x, \alpha, \vec{b}) \Sigma_L(r, \alpha, Q^2),$$

with

$$\Sigma_L(r, \alpha, Q^2) = K_0(\eta r) \int_0^\infty dp_T p_T J_0(p_T r) \Psi_V(\alpha, p_T) \left[4Q\alpha(1-\alpha) \frac{m_Q^2 + m_L m_T}{m_Q(m_L + m_T)} \right], \quad (2.12)$$

where $N_p = Z_Q \sqrt{2N_c \alpha_{em}} / 2\pi$, $\eta^2 = m_Q^2 + \alpha(1-\alpha)Q^2$, m_Q is the heavy quark mass, $\alpha_{em} = 1/137$ is the fine-structure constant, the factor $N_c = 3$ represents the number of colors in QCD, $Z_Q = 2/3$ is the charge isospin factor for charmonium production, variables $m_T = \sqrt{m_Q^2 + p_T^2}$ and $m_L = 2m_Q \sqrt{\alpha(1-\alpha)}$, and $J_{0,1}$ and $K_{0,1}$ are the Bessel functions of the first kind and the modified Bessel functions of the second kind, respectively.

III. COHERENT ELECTROPRODUCTION OFF NUCLEI: MOMENTUM TRANSFER DEPENDENCE IN THE GREEN FUNCTION FORMALISM

In this section we focus on the lowest $|Q\bar{Q}\rangle$ Fock component of the projectile photon. Since the $Q\bar{Q}$ transverse size is small, $\propto 1/\eta$, the shadowing corrections are as small as $1/\eta^2$, so they should be treated as a higher-twist effect. Within the Green function formalism, the amplitude for quarkonium electroproduction on a nuclear target, $\gamma^* A \rightarrow VA$, is given by the following expression:

$$\mathcal{A}^{\gamma^* A \rightarrow VA}(x, Q^2, \vec{q}) = 2 \int d^2 b \int d^2 b_A \exp[i(\vec{b} + \vec{b}_A) \cdot \vec{q}] \int_{-\infty}^\infty dz \rho_A(\vec{b} + \vec{b}_A, z) F_1(x, \vec{b}, \vec{b}_A, z), \quad (3.1)$$

where $\rho_A(b, z)$ is the nuclear density distribution, for which we employ the realistic Wood-Saxon form with parameters taken from Ref. [53]; $\vec{b}_A$ is the nuclear impact parameter and the function $F_1(x, \vec{b}, \vec{b}_A, z)$ can be expressed as

$$F_1(x, \vec{b}, \vec{b}_A, z) = \int_0^1 d\alpha \int d^2 r_1 d^2 r_2 \Psi_V^*(\vec{r}_2, \alpha) G_{Q\bar{Q}}(z' \rightarrow \infty, \vec{r}_2; z, \vec{r}_1 | \vec{b}_A) \mathcal{A}_{Q\bar{Q}}^N(\vec{r}_1, x, \alpha, \vec{b}) \Psi_\gamma(\vec{r}_1, \alpha, Q^2). \quad (3.2)$$

Here the function $F_1(x, \vec{b}, \vec{b}_A, z)$ describes the coherent (elastic) production of the colorless $Q\bar{Q}$ pair of initial separation $\vec{r}_1$ at point z with its subsequent evolution during propagation through the nucleus which is completed by the formation of the heavy quarkonium wave function at $z' \rightarrow \infty$ with final separation $\vec{r}_2$. Such an evolution of an interacting $Q\bar{Q}$ pair is described by the corresponding Green function $G_{Q\bar{Q}}(z', \vec{r}_2; z, \vec{r}_1 | \vec{b}_A)$, which satisfies the two-dimensional Schrödinger equation [12,31],

$$i \frac{d}{dz_2} G_{Q\bar{Q}}(z_2, \vec{r}_2; z_1, \vec{r}_1 | \vec{b}_A) = \left[\frac{\eta^2 - \Delta_{r_2}}{2p\alpha(1-\alpha)} + V_{Q\bar{Q}}(z_2, \vec{r}_2, \alpha, \vec{b}_A) \right] G_{Q\bar{Q}}(z_2, \vec{r}_2; z_1, \vec{r}_1 | \vec{b}_A), \quad (3.3)$$

where p is the photon energy in the target rest frame and the Laplacian Δ_{r_2} acts on the coordinate r_2 .

The real part of the LF potential $V_{Q\bar{Q}}(z_2, \vec{r}_2, \alpha)$ in Eq. (3.3) describes the interaction between the Q and $\bar{Q}$. In Ref. [21] we have proposed a procedure how to obtain $\text{Re} V_{Q\bar{Q}}(z_2, \vec{r}_2, \alpha)$ in the LF frame from various realistic $Q\bar{Q}$ interaction models defined in the rest frame. Such potentials lead to a correct shape of quarkonium wave functions in the LF frame. As in the case of the nucleon target, for the coherent process $\gamma^* A \rightarrow VA$ we also adopt the POW model [49] for the realistic interaction potential in the $Q\bar{Q}$ rest frame.

The imaginary part of the LF potential $V_{Q\bar{Q}}(z_2, \vec{r}_2, \alpha)$ in Eq. (3.3) controls the attenuation of the $|Q\bar{Q}\rangle$ Fock state of the photon in the medium. It has the following form:

$$\begin{aligned} \text{Im} V_{Q\bar{Q}}(z_2, \vec{r}, \alpha, \vec{b}_A) \\ = - \int d^2 b' \mathcal{A}_{Q\bar{Q}}^N(\vec{r}, x, \alpha, \vec{b}') \rho_A(\vec{b}' + \vec{b}_A, z_2). \end{aligned} \quad (3.4)$$

The final form of the nuclear t -dependent production amplitudes for both the T and L polarizations, including the spin rotation effects, has a structure that is more complicated compared to the proton target, Eqs. (2.11) and (2.12), and reads,

$$\begin{aligned} \mathcal{A}_T^{\gamma^*A \rightarrow VA}(x, Q^2, \vec{q}) = & 2N_p \int d^2b \int d^2b_A \exp[i(\vec{b} + \vec{b}_A) \cdot \vec{q}] \int_{-\infty}^{\infty} dz \rho_A(\vec{b} + \vec{b}_A, z) \\ & \times \int d^2r_1 \int d^2r_2 \int_0^1 d\alpha \mathcal{A}_{Q\bar{Q}}^N(\vec{r}_1, x, \alpha, \vec{b}) G_{Q\bar{Q}}(z' \rightarrow \infty, \vec{r}_2; z, \vec{r}_1 | \vec{b}_A) [\Gamma_T^{(1)}(r_1, r_2, \alpha, Q^2) + \Gamma_T^{(2)}(r_1, r_2, \alpha, Q^2)], \end{aligned}$$

with

$$\begin{aligned} \Gamma_T^{(1)}(r_1, r_2, \alpha, Q^2) &= K_0(\eta r_1) \int_0^\infty dp_T p_T J_0(p_T r_2) \Psi_V(\alpha, p_T) \left[\frac{2m_Q^2(m_L + m_T) + m_L p_T^2}{m_T(m_L + m_T)} \right], \\ \Gamma_T^{(2)}(r_1, r_2, \alpha, Q^2) &= K_1(\eta r_1) \int_0^\infty dp_T p_T^2 J_1(p_T r_2) \Psi_V(\alpha, p_T) \left[\eta \frac{m_Q^2(m_L + 2m_T) - m_T m_L^2}{m_Q^2 m_T(m_L + m_T)} \right], \end{aligned} \quad (3.5)$$

and

$$\begin{aligned} \mathcal{A}_L^{\gamma^*A \rightarrow VA}(x, Q^2, \vec{q}) &= 2N_p \int d^2b \int d^2b_A \exp[i(\vec{b} + \vec{b}_A) \cdot \vec{q}] \int_{-\infty}^{\infty} dz \rho_A(\vec{b} + \vec{b}_A, z) \\ &\times \int d^2r_1 \int d^2r_2 \int_0^1 d\alpha \mathcal{A}_{Q\bar{Q}}^N(\vec{r}_1, x, \alpha, \vec{b}) G_{Q\bar{Q}}(z' \rightarrow \infty, \vec{r}_2; z, \vec{r}_1 | \vec{b}_A) \Gamma_L(r_1, r_2, \alpha, Q^2), \end{aligned}$$

with

$$\Gamma_L(r_1, r_2, \alpha, Q^2) = K_0(\eta r_1) \int_0^\infty dp_T p_T J_0(p_T r_2) \Psi_V(\alpha, p_T) \left[4Q\alpha(1 - \alpha) \frac{m_Q^2 + m_L m_T}{m_Q(m_L + m_T)} \right]. \quad (3.6)$$

The expression for the t -dependent differential cross section reads

$$\frac{d\sigma^{\gamma^*A \rightarrow VA}(x, Q^2, t = -q^2)}{dt} = \frac{1}{16\pi} (|\mathcal{A}_T^{\gamma^*A \rightarrow VA}(x, Q^2, \vec{q})|^2 + \tilde{\epsilon} |\mathcal{A}_L^{\gamma^*A \rightarrow VA}(x, Q^2, \vec{q})|^2). \quad (3.7)$$

Note that the Green function formalism can be applied to coherent processes without any restrictions for the coherence length l_c , for arbitrary realistic $Q - \bar{Q}$ interaction potential, as well as for any phenomenological model for the impact parameter-dependent partial dipole amplitude.

At high photon energies, corresponding to $x \lesssim 0.01$, the condition (1.1) is valid and the so-called *long coherence length* (LCL) regime is at work. Then the eikonal approximation can be applied as a limiting case of the Green function formalism, when the Green function acquires a simple form,

$$G_{Q\bar{Q}}(z_2, \vec{r}_2; z_1, \vec{r}_1) \Rightarrow \delta^{(2)}(\vec{r}_1 - \vec{r}_2) \exp \left[- \int d^2b \mathcal{A}_{Q\bar{Q}}^N(\vec{r}_1, x, \alpha, \vec{b}) \int_{z_1}^{z_2} dz \rho_A(\vec{b} + \vec{b}_A, z) \right], \quad (3.8)$$

and the Lorentz time dilation freezes the transverse sizes of such long-lived $Q\bar{Q}$ photon fluctuations during propagation through the medium. Such an LCL effect is also known as the “frozen” approximation. Then the t -dependent amplitude of quarkonium electroproduction on a nuclear target, $\gamma^*A \rightarrow VA$ is given by the expression (2.4), but replacing the dipole-nucleon by dipole-nucleus amplitude $\mathcal{A}_{Q\bar{Q}}^N(\vec{r}, x, \alpha, \vec{b}) \Rightarrow \mathcal{A}_{Q\bar{Q}}^A(\vec{r}, x, \alpha, \vec{b}_A)$ and $\vec{b} \Rightarrow \vec{b}_A$,

$$\mathcal{A}^{\gamma^*A \rightarrow VA}(x, Q^2, \vec{q}) = 2 \int d^2b_A e^{i\vec{q} \cdot \vec{b}_A} \int d^2r \int_0^1 d\alpha \Psi_V^*(\vec{r}, \alpha) \mathcal{A}_{Q\bar{Q}}^A(\vec{r}, x, \alpha, \vec{b}_A) \Psi_V(\vec{r}, \alpha, Q^2), \quad (3.9)$$

where one can rely on the eikonal form for the dipole-nucleus partial amplitude at nuclear impact parameter $\vec{b}_A$,

$$\begin{aligned} \text{Im } \mathcal{A}_{Q\bar{Q}}^A(\vec{r}, x, \alpha, \vec{b}_A)|_{l_c \gg R_A} &= 1 - \left[1 - \frac{1}{A} \int d^2b \text{Im } \mathcal{A}_{Q\bar{Q}}^N(\vec{r}, x, \alpha, \vec{b}) T_A(\vec{b}_A + \vec{b}) \right]^A \\ &\approx 1 - \exp \left[- \int d^2b \text{Im } \mathcal{A}_{Q\bar{Q}}^N(\vec{r}, x, \alpha, \vec{b}) T_A(\vec{b}_A + \vec{b}) \right]. \end{aligned} \quad (3.10)$$

Here $T_A(\vec{b}_A) = \int_{-\infty}^{\infty} dz \rho_A(\vec{b}_A, z)$ is the nuclear thickness function normalized as $\int d^2b_A T_A(\vec{b}_A) = A$.

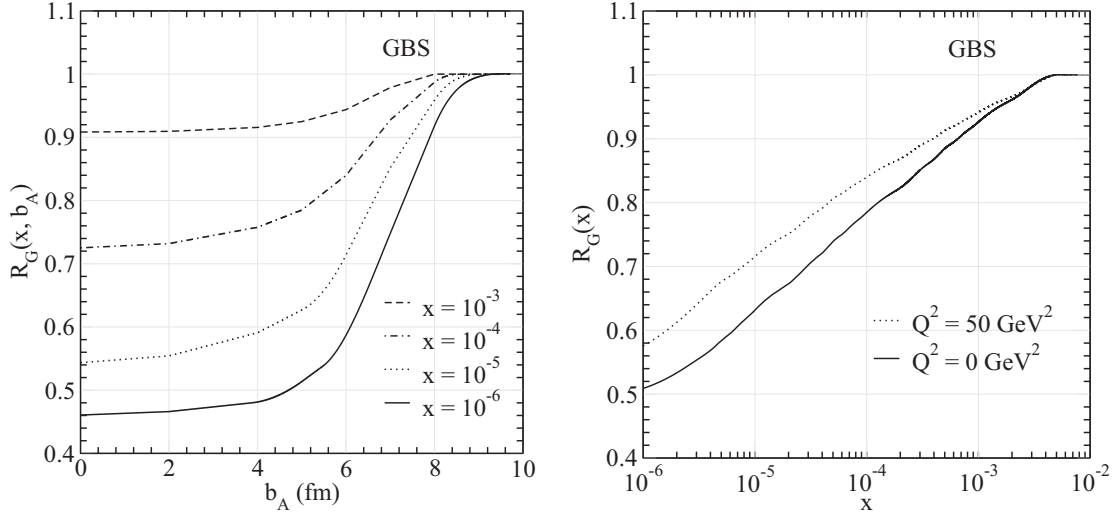


FIG. 1. Gluon shadowing factor R_G for photoproduction of J/ψ on the gold target as function of impact parameter b_A at several fixed values of x (left), and as function of x at fixed values of the photon virtuality $Q^2 = 0$ and 50 GeV^2 (right).

Similarly as for the proton target, the real part of the t -dependent nuclear $\gamma^*A \rightarrow VA$ amplitude and the skewness correction are incorporated via substitutions (2.9) and (2.10) into Eqs. (3.4)–(3.6).

Besides the effect of the reduced coherence length when $l_c \lesssim (1/2) \cdot R_A$, the photoproduction of heavy quarkonia is also affected by the gluon shadowing as shown in Refs. [5,6,21,54]. The leading-twist gluon shadowing was introduced in Ref. [31] within the dipole representation as a shadowing correction corresponding to higher Fock components of the photon containing gluons, i.e., $|Q\bar{Q}G\rangle$, $|Q\bar{Q}2G\rangle$, ... $|Q\bar{Q}nG\rangle$. However, in agreement with the analysis and discussion in Ref. [6], the high-energy part of the EIC kinematic region at RHIC and UPC kinematic region at the LHC generate a dominant contribution to the nuclear shadowing coming only from one-gluon Fock state $|Q\bar{Q}G\rangle$ of the photon. The corresponding coherence (radiation) length has the form

$$l_c^G = \frac{(W^2 + Q^2 - m_N^2) \cdot \alpha_g(1 - \alpha_g)}{m_N[k_T^2 + (1 - \alpha_g)m_g^2 + \alpha_g M_{\bar{Q}Q}^2 + \alpha_g(1 - \alpha_g)Q^2]}, \quad (3.11)$$

where α_g is the LF fraction of the photon momentum carried by the gluon, $M_{\bar{Q}Q}$ is the effective mass of the $\bar{Q}Q$ pair and the effective gluon mass $m_g \approx 0.7 \text{ GeV}$ is fixed by data on gluon radiation [31,55]. The condition for the onset of GS is a sufficiently long $l_c^G \gg d$, where $d \approx 2 \text{ fm}$ is the mean separation between bound nucleons. One can also see that $l_c^G \ll l_c$ due to small $\alpha_g \ll 1$, in agreement with calculations in Ref. [56]. We have incorporated the GS correction in our calculations as a reduction of the partial b -dependent dipole amplitude in nuclear reactions with respect to processes on nucleon [6,57],

$$\text{Im } \mathcal{A}_{\bar{Q}Q}^N(\vec{r}, x, \alpha, \vec{b}) \Rightarrow \text{Im } \mathcal{A}_{\bar{Q}Q}^N(\vec{r}, x, \alpha, \vec{b}) \cdot R_G(x, |\vec{b}_A + \vec{b}|), \quad (3.12)$$

where the correction factor $R_G(x, b)$, related to the $Q\bar{Q}G$ component of the photon, was calculated within the Green function formalism [12,15,18,21,31,57–59].

In the left panel of Fig. 1 we present the b_A dependence of the R_G factor at several fixed values of x covering the LHC and LHeC kinematical regions. The right panel of Fig. 1 shows the b_A -integrated gluon shadowing factor at fixed values of the photon virtuality $Q^2 = 0$ and 50 GeV^2 .

The real part of the nuclear $\gamma^*A \rightarrow J/\psi A$ amplitude and the corresponding skewness correction has been included in consistency with Eqs. (2.9) and (2.10). Here we checked that final results for t -dependent differential cross section of the coherent process $\gamma^*Au \rightarrow J/\psi Au$ are insensitive to the order in which such corrections are implemented, whether after the substitution (3.12) or first without it with subsequent multiplication by the R_G factor.

Note that the calculations of higher multi-gluon Fock components are very complex within the Green function formalism and represent thus a challenge. However, using renormalization (3.12), the corresponding shadowing corrections are included essentially via eikonalization of the factor $R_G(x, b)$ (see discussion in Ref. [57]).

IV. PREDICTIONS WITHIN THE GREEN FUNCTION FORMALISM

A. Model calculations vs data

In the present paper we apply the recent GBS model with DGLAP evolution [38] for the dipole-proton partial amplitude including correlation between the dipole transverse orientation $\vec{r}$ and the impact parameter of the collision $\vec{b}$,

Eq. (2.6). Our calculations rely on the quarkonium wave functions generated by the POW $Q - \bar{Q}$ interaction potential [49], since in combination with the GBS model provides the best description of the UPC data on rapidity distributions $d\sigma/dy$ (see Ref. [21]). We also include the leading-twist gluon shadowing representing the main nuclear effect in the kinematic region when $x \lesssim x_g$, where x_g has the following form [18]

$$x_g \approx \frac{\sqrt{3}}{f_g m_N R_A^{ch}}. \quad (4.1)$$

Here, the factor $f_g = \langle l_c \rangle / \langle l_c^G \rangle \sim 10$ [56] and R_A^{ch} is the mean value of the nuclear charge radius.

As a first step, we verified in Fig. 2 that the model calculations of t -distributions $d\sigma^{\gamma^* p \rightarrow J/\psi p}(t, W, Q^2 \approx 0)/dt$, $d\sigma^{\gamma^* p \rightarrow J/\psi p}(t, W, Q^2 = 8.9 \text{ GeV}^2)/dt$, as well as t -integrated $\sigma^{\gamma^* p \rightarrow \psi' p}(W, Q^2 \approx 0)$ are in good agreement with the available data on the proton target from the H1 [60] and LHCb [61] Collaborations. Then we checked that our calculations describe well the first ALICE data [62] on the t dependence of coherent J/ψ photonuclear production, as demonstrated in the lower right panel of Fig. 2. In the next sections, we focus only on the heavy gold target due to

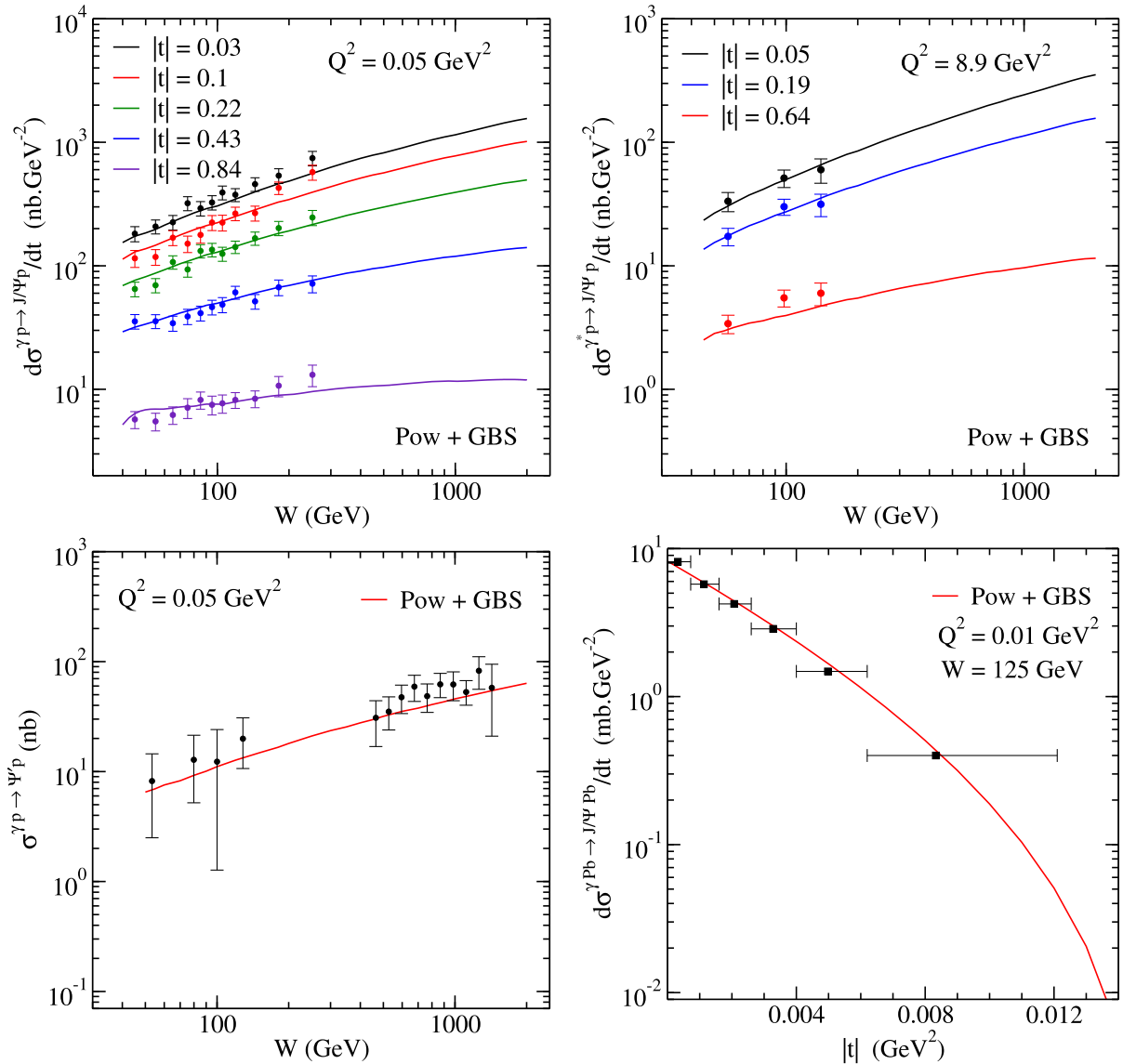


FIG. 2. Model calculations of $d\sigma^{\gamma(\gamma^*) p \rightarrow J/\psi p}(t)/dt$ at $Q^2 = 0.05 \text{ GeV}^2$ (upper left panel) and at $Q^2 = 8.9 \text{ GeV}^2$ (upper right panel) as a function of photon energy W at several fixed values of t vs data from the H1 collaboration [60]. Results for the t -integrated differential cross section in the photoproduction of $\psi'(2S)$ are shown in the lower left panel and compared with data from the LHCb [61] collaboration. The lower right panel shows the model calculations of $d\sigma^{\gamma Pb \rightarrow J/\psi Pb}(t)/dt$ in comparison with data from the ALICE experiment [62].

the expected maximum strength of all relevant nuclear phenomena, such as the CT and (reduced) CL effects, as well as the gluon shadowing.

B. Manifestations of CT effects

The effect of CT leads to a modification of the diffractive pattern in $d\sigma^{\gamma^*A \rightarrow VA}/dt$ which results in a shifted positions of the diffractive minima to larger values of t , as shown in Fig. 3. Here we present model calculations in the LCL regime ($l_c \gg R_A$) of the t -dependent nuclear transparency, defined as

$$Tr_A^{\text{coh}}(t) = \frac{d\sigma^{\gamma^*A \rightarrow VA}/dt}{A^2 d\sigma^{\gamma^*N \rightarrow VN}/dt|_{t=0}} = \frac{|\mathcal{A}^{\gamma^*A \rightarrow VA}(x, Q^2, t)|^2}{A^2 |\mathcal{A}^{\gamma^*p \rightarrow VP}(x, Q^2, t=0)|^2}, \quad (4.2)$$

at two fixed values of $Q^2 = 0$ and 50 GeV^2 . In the LCL regime given by Eqs. (3.9) and (3.10), the amplitude in the numerator of Eq. (4.2) can be expressed as

$$\mathcal{A}^{\gamma^*A \rightarrow VA}(x, Q^2, t = -q^2) = 2 \int d^2 b_A e^{i\vec{q} \cdot \vec{b}_A} \mathcal{M}_A(\vec{b}_A). \quad (4.3)$$

To better understand the manifestation of CT effects, we can use the quadratic approximation of the dipole cross section in Eq. (3.10), $\int d^2 b \text{Im} \mathcal{A}_{Q\bar{Q}}^N(\vec{r}, x, \alpha, \vec{b}) \approx C(x)r^2/2$, as well as the Gaussian approximation of the product of the photon and vector meson wave functions, $\Psi_V^*(\vec{r}, \alpha) \Psi_{\gamma^*}(\vec{r}, \alpha, Q^2) \propto \exp[-r^2/\langle r^2 \rangle]$. Then the b_A -dependent amplitude in Eq. (4.3) reads,

$$\mathcal{M}_A(\vec{b}_A) = \pi \langle r^2 \rangle \cdot \frac{C(x) T_A(\vec{b}_A) \langle r^2 \rangle}{2 + C(x) T_A(\vec{b}_A) \langle r^2 \rangle} \quad \text{and} \quad \mathcal{A}^{\gamma^*p \rightarrow VP}(x, Q^2, t=0) = \pi C(x) \langle r^2 \rangle^2, \quad (4.4)$$

which gives the following simplified form for the t -dependent nuclear transparency,

$$Tr_A^{\text{coh}}(t) = \frac{4}{A^2} \left| \int d^2 b_A e^{i\vec{q} \cdot \vec{b}_A} \frac{T_A(\vec{b}_A)}{2 + C(x) T_A(\vec{b}_A) \langle r^2 \rangle} \right|^2. \quad (4.5)$$

It can be seen from Eqs. (4.4) and (4.5) that the partial amplitude $\mathcal{M}_A(\vec{b}_A)$ is suppressed with Q^2 due to diminishing of the mean $Q\bar{Q}$ -dipole size $\langle r^2 \rangle$ and that the suppression is larger at smaller b_A -values. Consequently, the slope $B_{\gamma^*A} = \langle b^2 \rangle / 2$ of the differential cross section $d\sigma^{\gamma^*A \rightarrow VA}/dt$ should be reduced with Q^2 , shifting the diffractive minima to larger values of t , as indeed shown in Fig. 3. However, observing experimentally such a CT signal by Q^2 modification of the diffractive pattern is questionable due to a rather weak effect. Here, the coherent production of light vector mesons gives a better chance to investigate the CT effects since their manifestations are more visible compared to the heavy quarkonium production, as analyzed in Refs. [12,15]. On the other hand, the CT effects could be recognized and measured by EIC experiments at RHIC and the LHeC as an increase in t -dependent nuclear transparency $Tr_A^{\text{coh}}(t)$ with Q^2 at small fixed values of t , as predicted in the left panel of Fig. 3 for electroproduction of $J/\psi(1S)$. The coherent electroproduction of $\psi'(2S)$ gives almost negligible chance

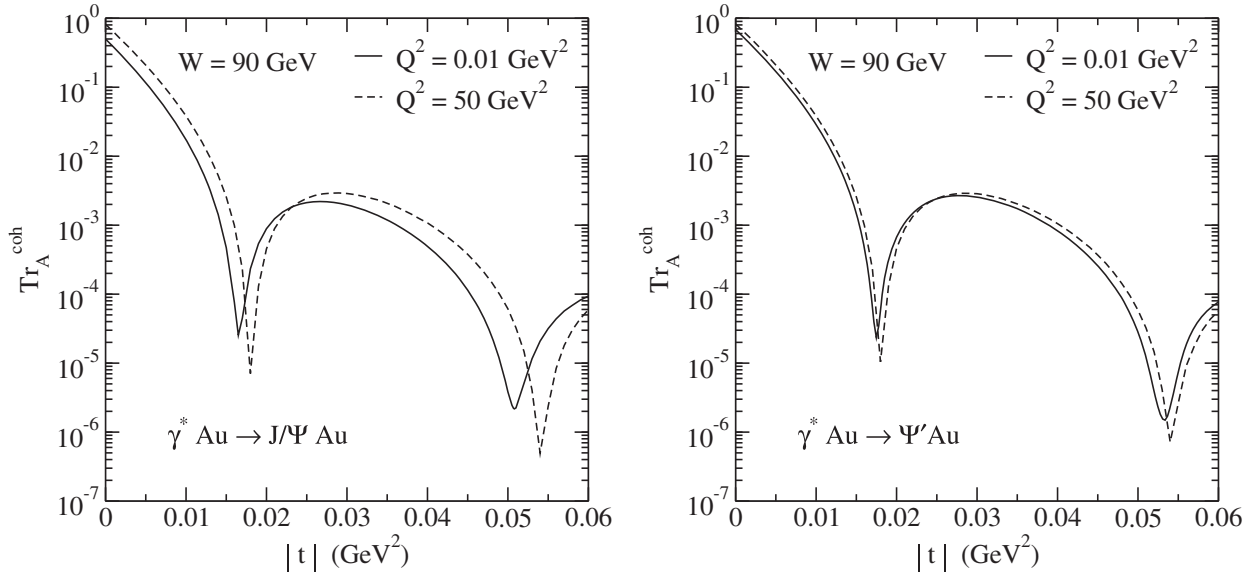


FIG. 3. The t -dependent nuclear transparency, Eq. (4.2), for the coherent electroproduction of $J/\psi(1S)$ (left panel) and $\psi'(2S)$ (right panel) in the limit $l_c \gg R_A$ and at two fixed values of $Q^2 = 0$ and 50 GeV^2 .

for investigation of such effects (see the right panel of Fig. 3).

C. Reduced higher-twist corrections at EIC

In the EIC kinematic region at RHIC corresponding to Bjorken $x \gtrsim x_c \approx 0.01$, where x is given by Eq. (2.5), one should also include reduced quantum coherence effects for the lowest $|Q\bar{Q}\rangle$ Fock state of the photon, i.e., going beyond the “frozen” eikonal approximation, Eq. (3.10). Instead of a standard approximate incorporation of such effects via the nuclear form factor depending on the longitudinal momentum transfer $q_L = 1/l_c$ [63], we use a rigorous Green function formalism as described in the previous Sec. III. Such a formalism contains all multiple scattering terms and treats the nuclear form factor correctly. The reduced quark shadowing significantly modifies the t -dependent charmonium yields compared to the eikonal approximation at small W values, as can be seen in Fig. 4. Here we present $d\sigma^{\gamma^* Au \rightarrow V Au}/dt$ for the coherent photoproduction of 1S and 2S charmonium states, $V = J/\psi(1S)$ and $\psi'(2S)$, on the gold target and their 2S-to-1S ratio as a function of the photon energy W for several fixed values of

t (upper panels). The lower panels of Fig. 4 show the analogous model predictions as the upper panels but for the coherent electroproduction process at $Q^2 = 50 \text{ GeV}^2$.

Figure 4 demonstrates that in the EIC kinematic region, the onset of reduced l_c -effects at the same photon energy is stronger in charmonium electroproduction compared to photoproduction process, in consistency with Eq. (1.1). Whereas in the photoproduction limit ($Q^2 \approx 0$) such effects can be experimentally recognized only at rather small $W \lesssim 20 \text{ GeV}$, i.e., at the lower limit of the kinematic region of future EIC experiments (see the upper left and middle panels of Fig. 4), charmonium electroproduction at $Q^2 = 50 \text{ GeV}^2$ provides a better chance for experimental investigation of reduced quark shadowing, which occurs over a wider energy range, $W \lesssim 50 \text{ GeV}$ (see differences between solid and dashed lines in the lower left and lower middle panels of Fig. 4).

D. Searching for the gluon saturation at LHeC

The LHeC kinematic region provides a good basis for searching for possible manifestations of gluon saturation effects. Here the charmonium photoproduction is more

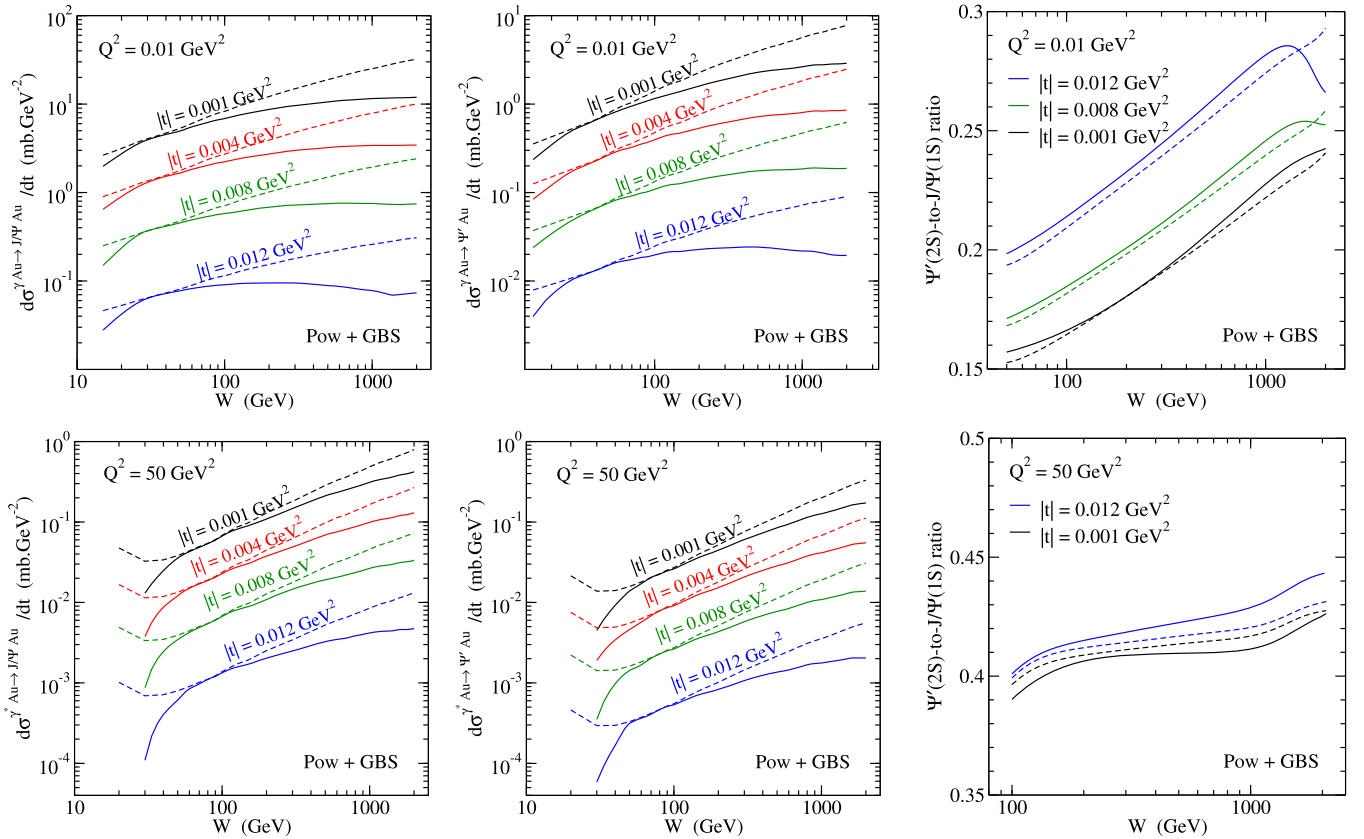


FIG. 4. Model predictions for the energy behavior of the t -dependent differential cross section $d\sigma/dt$ for coherent photoproduction of $J/\psi(1S)$ (upper left panel), $\psi'(2S)$ (upper middle panel) and for the $\psi'(2S)$ -to- $J/\psi(1S)$ ratio of $d\sigma/dt$ (upper right panel) at several fixed values of $t = 0.001, 0.004, 0.008$ and 0.012 GeV^2 . The lower panels show the electroproduction process at $Q^2 = 50 \text{ GeV}^2$. In all panels the Green function formalism with gluon shadowing corrections (solid lines) is compared with the standard eikonal approximation without gluon shadowing (dashed lines).

suitable compared to electroproduction process due to a stronger onset of the leading-twist shadowing corrections. The corresponding factor R_G in Eq. (3.12) causes a sizable reduction of the gluon distribution function in a free nucleon at small dipole separations $r \lesssim R_0(x, \mu^2)$ and only very slowly (logarithmically) is approaching unity as $Q^2 \rightarrow \infty$. The interpretation of the gluon shadowing phenomenon is reference-frame dependent. It can be treated as glue-gluon fusion in the infinite momentum frame of the nucleus. The corresponding Lorentz contraction of the nucleus keeps still a sufficiently large separations of the bound nucleons, except the gluon clouds of the nucleons that are contracted less due to their smaller momentum fraction x of the nucleon. Then gluons originating from different nucleons can overlap and interact with each other with their subsequent fusion leading to a reduction in gluon density. This corresponds to a nonlinear QCD evolution incorporated into evolution equations [26,27,32,33].

The leading-twist shadowing corrections in coherent charmonium photoproduction off nuclei, studied in the present paper, may represent an effective tool in searching for gluon saturation effects. Figure 4 shows how the gluon shadowing phenomenon affects the energy behavior of the t -dependent differential cross sections (see the differences between solid and dashed lines at large photon energies) in the production of 1S and 2S charmonium states and their 2S-to-1S ratios at different fixed values of t and at $Q^2 \approx 0$ (upper panels) and $Q^2 = 50 \text{ GeV}^2$ (lower panels). One can see that, within the LHeC energy range of $W \lesssim 2000 \text{ GeV}$, a monotonic rise of $d\sigma/dt$ with the photon energy at $t = 0$ is gradually changed for a nonmonotonic shape with a maximum that is more visible at larger t values and simultaneously shifted towards smaller values of W . This result is consistent with an increase of the gluon shadowing correction with t , since the $R_G(x, \vec{b}_A)$ factor is scanned at smaller impact-parameter values b_A (see Fig. 1 in Ref. [6]).

The upper panels of Fig. 4 demonstrate that in the photoproduction of charmonia off gold target the t -dependent positions of maxima in $d\sigma/dt(W)$ at $W = W_{\max}$ cover a broad photon energy range, namely $W_{\max} \sim 1500 \text{ GeV}$ at $t = 0.004 \text{ GeV}^2$, $W_{\max} \sim 800 \text{ GeV}$ at $t = 0.008 \text{ GeV}^2$ and $W_{\max} \sim 200 \text{ GeV}$ at $t = 0.012 \text{ GeV}^2$. This gives an opportunity for a recognition of gluon saturation effects by future experiments at the LHeC. However, finding such effects may require larger t values and, consequently, the data with higher statistics.

Here we would like to note that the b_A dependence of gluon shadowing, calculated in the present paper, is crucial for evaluation of the t -dependent differential cross sections $d\sigma/dt$. This is in contrast to the global data analyses providing only the b_A -integrated gluon shadowing. Note that various models for the dipole-nucleon scattering amplitude can also slightly modify the kinematic region for the onset of saturation effects when a nonmonotonic

scenario for the energy dependence of $d\sigma/dt$ can be shifted to smaller or larger values of t . The coherent charmonium electroproduction off nuclei reduces substantially the chance to investigate saturation effects compared to photoproduction reaction due to weaker leading-twist corrections corresponding to a weaker onset of the nonlinear QCD evolution, as seen in lower panels of Fig. 4.

The potential existence and position of the maxima in the W dependence of $d\sigma/dt$, as a manifestation of possible gluon saturation effects, depend on the magnitude of gluon shadowing. Its determination is related to the distance r_g of the gluon propagation from the $Q\bar{Q}$ pair, i.e., to the size of the GG dipole. The value of r_g limits the validity of the approximation $Q\bar{Q}G \sim GG$ used for calculation of gluon shadowing. The corresponding condition reads $\eta^2 \gg 1/r_g^2$, which is safely satisfied in coherent charmonium photo and electroproduction studied in this paper. In the opposite case the $Q\bar{Q}$ pair cannot be treated as a pointlike object compared to the size of the entire $Q\bar{Q}G$ Fock state of the photon. The single diffractive data in hadronic collisions allow to extract a magnitude of r_g as was done in Ref. [31]. Here the corresponding diffraction cross section $\propto r^4$ exhibits a larger sensitivity to the dipole separation r compared to total cross section $\propto r^2$. This led to the determination of the mean GG dipole size of the order of $r_g \sim 0.3 \text{ fm}$ [31]. Such a small gluon propagation radius represents so far the only way how to resolve the long-standing problem related to the small size of the triple-Pomeron coupling. The above value of r_g is incorporated into the LF color dipole formalism via a nonperturbative G - G interaction using the path integral technique.

Note that the small value of r_g is consistent with the results of other approaches, based on the instanton liquid model [64], on the QCD sum rule analysis of the gluonic formfactor of the proton [65], as well as on the lattice calculations [66]. This indicates rather small leading-twist shadowing effects in accordance with the next-to-leading-order analysis of available DIS data on nuclei [67].

We predict a stronger energy dependence for $d\sigma^{\gamma Au \rightarrow \psi'(2S)Au}/dt$ than for the coherent photoproduction of $J/\psi(1S)$ due to the nodal structure of the radial wave function for the 2S charmonium state (compare the upper middle and upper left panel of Fig. 4). This also leads to an increase of the $\psi'(2S)$ -to- $J/\psi(1S)$ ratio $R_{\psi'/J/\psi}(t)$ of t -dependent differential cross sections with the photon energy, as demonstrated in the upper right panel of Fig. 4. The node effect for the 2S charmonium state gradually fades with energy and we predict a similar scenario for a nonmonotonic energy behavior of $d\sigma/dt$ as for the 1S charmonium coherent photoproduction (compare upper left and upper middle panels of Fig. 4 at large W values). The corresponding maxima of $d\sigma/dt$ are shifted towards larger values of W , $W_{\max} \sim 1300 \text{ GeV}$ at $t = 0.008 \text{ GeV}^2$ and $W_{\max} \sim 500 \text{ GeV}$ at $t = 0.012 \text{ GeV}^2$.

Since the leading-twist shadowing corrections are very similar for both the 1S and 2S charmonium states, the rise of the ratio $R_{\psi'/J/\psi}(t)$ with energy does not differ much from that without such nonlinear QCD effects, as demonstrated in right panels of Fig. 4. The nonmonotonic W -dependence of $d\sigma/dt$ for both the 1S and 2S charmonium states with different positions of maxima may generate nonmonotonic energy behavior also for the $R_{\psi'/J/\psi}(t)$, especially at larger t values, as demonstrated in the upper right panel of Fig. 4. However, it is questionable whether such a manifestation of gluon saturation effects can be recognized experimentally at the LHeC.

At large $Q^2 = 50 \text{ GeV}^2$ the node effect for the $\psi'(2S)$ state is very weak and we predict very similar scenarios for the W -dependent $d\sigma/dt$ in coherent electronuclear production of the both charmonium states, $J/\psi(1S)$ and $\psi'(2S)$ (see the lower left and lower middle panel of Fig. 4). This also leads to a very weak energy dependence of the ratio $R_{\psi'/J/\psi}(t)$ (see the lower right panel of Fig. 4). Here we present the results only for fixed $t = 0.001$ and $t = 0.012 \text{ GeV}^2$ because the curves at all values of t lie very close to each other.

Note that in all calculations of differential cross sections on nuclear targets, we have verified that in comparison with the proton target the effect of the correlation between the dipole orientation and impact parameter of a collision is minimal, causing a very small reduction of $d\sigma^{\gamma^*Au \rightarrow J/\psi(1S)Au}/dt$ and $d\sigma^{\gamma^*Au \rightarrow \psi'(2S)Au}/dt$ by only a few percents.

In the present work, we tried to minimize the theoretical uncertainties related to the elimination of D-wave admixture in charmonium wave functions, to the choice of realistic $Q - \bar{Q}$ potential together with the Melosh spin effects, to the inclusion of dipole orientation with respect to the impact parameter of the collision, as well as to the proper calculations of the leading-twist shadowing corrections consistent with other models and available diffractive data. We believe that our predictions for manifestation of gluon saturation effects are realistic and can be verified by the future LHeC experiments.

V. SUMMARY

The transverse momentum transfer dependence of differential cross sections $d\sigma/dt$ for the coherent electroproduction of heavy quarkonia on nuclei was studied in the framework of the dipole description based on a rigorous Green function formalism. We have analyzed a significance of the reduced effects of quantum coherence in the EIC kinematic region at RHIC, as well as leading-twist corrections at LHeC energies. The latter effect can also be treated as a manifestation of nonlinear QCD effects related to gluon saturation.

In order to minimize theoretical uncertainties in our predictions, we consider the following:

- (i) We include the correlation between impact parameter of a collision $\vec{b}$ and the dipole orientation $\vec{r}$;
- (ii) We use the structure for the $V \rightarrow Q\bar{Q}$ transition which eliminates the exaggerated weight of the D-wave component in the rest frame quarkonium wave function, in consistency with the solutions of the Schrödinger equation;
- (iii) We rely on LF wave functions of heavy quarkonia generated from a solution of the Lorentz boosted Schrödinger equation with realistic potentials together with Melosh spin rotation effects.

Each Fock component of the incoming photon, i.e., $|\bar{Q}Q\rangle$, $|\bar{Q}QG\rangle$, $|\bar{Q}Q2G\rangle$, ..., contributes independently to the heavy quarkonium electroproduction on nuclear targets in accordance with the corresponding coherence lengths. The UPC energy range at the LHC allows the eikonalization of the partial $\vec{b}$ -dependent dipole $Q\bar{Q}$ -proton amplitude [see Eq. (3.10)], since the coherence length for the lowest $Q\bar{Q}$ Fock state considerably exceeds the nuclear size, $l_c \gg R_A$. The corresponding quark shadowing gradually diminishes with the heavy quarkonium mass and thus represents a higher-twist effect.

However, a part of the kinematic region of the prepared EIC experiments at RHIC will cover quite large $x \gtrsim x_c \approx 0.01$, and one eventually has to deal with reduced coherence effects, when $l_c \lesssim R_A$. Therefore, we apply a rigorous path integral technique for calculations of t -dependent differential cross sections and derive the corresponding formulas [see Eqs. (3.5) and (3.6)]. The incorporation of the reduced l_c effects represents the main advancement of our present work. The reduced l_c effects lead to a significant decrease in the yields of t -dependent charmonium production. The corresponding predictions for $d\sigma^{\gamma^*A \rightarrow VA}/dt$ ($V = J/\psi(1S), \psi'(2S)$) can be tested by future EIC experiments at RHIC.

The high-energy part of the EIC kinematic region, when $x \lesssim x_g$, requires also to include shadowing corrections from the higher Fock components of the photon, $|\bar{Q}QG\rangle$, $|\bar{Q}Q2G\rangle$, etc. Such corrections are known as the gluon shadowing, which is the leading-twist effect because the transverse size of the $Q\bar{Q}$ - G dipoles is large and nearly scale independent. They are much stronger than the higher-twist shadowing and their calculations have to rely on the path integral technique, because the dipoles $Q\bar{Q} - G, \dots$, cannot be treated as “frozen” even at very high energies due to a divergent $d\alpha_g/\alpha_g$ behavior.

Besides effects of quantum coherence, the Green function formalism also allows us to study the manifestations of color transparency effects as a modification of the shape of $d\sigma/dt$ and/or $Tr_A^{\text{coh}}(t)$ with Q^2 . We have found that such effects lead to a shift of the diffraction minima towards larger values of t (see Fig. 3), caused by a Q^2 -reduction of the slope $B_{\gamma^*A}(Q^2)$ of the t -distributions. However, it will be difficult to detect it experimentally.

The only way to detect CT effects by EIC experiments at RHIC and the LHeC is to measure an increase of t -dependent nuclear transparency with Q^2 at fixed values of t , as seen in Fig. 3 for the electroproduction of $J/\psi(1S)$.

Coherent photoproduction of charmonia off nuclei at large LHeC energies, where the coherence length $l_c^G \gg 2$ fm, favors to analyze the dominant role of gluon shadowing corrections allowing a subsequent study of a significance of gluon saturation effects. We predict a nonmonotonic energy dependence of $d\sigma/dt$ (see Fig. 4) that is more visible at larger t values with corresponding maxima shifted towards smaller photon energies. Such an energy behavior of $d\sigma/dt$ can be verified by future EIC experiments at planned LHeC. This can shed more light on the magnitude of the leading twist gluon shadowing and can help us to rule out various phenomenological models devoted to study of nuclear shadowing. Specifically, this will also contribute to resolving the issue of the

applicability of the BK evolution equation in its current form in combination with the high-energy eikonal approximation for the partial dipole-nucleus amplitude for description of processes on nuclear targets, frequently used in the literature.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [60–62].

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