

Robustness of the derivative expansion in asymptotic safety

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We analyze the renormalization group flow of quantum gravity at sixth order in the derivative expansion within the background field approximation. Nonlinear field redefinitions are used to ensure that only essential couplings flow. Working within the universality class of general relativity, with a vanishing cosmological constant, redundant couplings are fixed to their values at the Gaussian fixed point. This reduces the theory space to two dynamical essential couplings given by Newton's and the Goroff-Sagnotti coupling. Furthermore, it implements the condition that no extra degrees of freedom are present beyond those of general relativity, in contrast to higher derivative theories and derivative expansions in a conventional renormalization scheme. We find a unique ultraviolet fixed point with a single relevant direction and analyze the phase diagram of the theory. Our results suggest resilience of the gravitational Reuter fixed point under the inclusion of higher order curvature invariants and show several signs of near perturbativity. The regulator dependence of our results is investigated in detail and shows that qualitative and quantitative features are robust to a large extent.

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I. INTRODUCTION

Finding a quantum theory of gravity is one of the most fascinating open problems in theoretical physics. Such a theory is supposed to shine light on fundamental questions about the origin of the Universe and the microstructure of black holes. However, pure Einstein gravity is well known to be perturbatively nonrenormalizable due to Newton's coupling having negative mass dimension. As a result, perturbative renormalizability fails from two loops onward as signaled by the presence of the Goroff-Sagnotti counterterm [1–3]. This indicates that perturbative Einstein quantum gravity has to include an infinite number of parameters, rendering the theory nonpredictive. Below the Planck scale, predictivity can be regained by treating gravity as an effective field theory [4]. In this regime, higher order

couplings are suppressed by powers of the Planck mass and can be neglected. The remaining finite number of couplings can be fixed by observations, at least in principle [4]. However, this approach breaks down at the Planck scale and requires an ultraviolet (UV) completion.

The asymptotic safety scenario has emerged as one of the key contenders for a viable UV completion of quantum gravity [5–7]. It is based entirely on standard quantum field theory, and suggests that quantum gravity becomes scale-invariant at short distances. This can be realized by an interacting fixed point of the renormalization group (RG) flow. Evidence for the existence of such a fixed point has accumulated, both in pure gravity, where it is known as the Reuter fixed point [8], and when coupled to matter, see, e.g., the collection of book chapters [9–16] for an up-to-date overview of the field. For some recent work in the field see, e.g., [17–29].

Since asymptotic safety is a nonperturbative phenomenon, small coupling expansions cannot be relied on and a mathematical proof for the existence of a fixed point seems out of reach for the moment. Nevertheless, there are strong indications that asymptotic safety is realized in a near-perturbative regime: it is as nonperturbative as it needs to be, but as perturbative as it can be [30–33]. This gives some evidence that central aspects of the theory are already well-captured by current approximations.

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In this work, we provide more evidence for this observation. Specifically, we investigate asymptotic safety in a derivative expansion up to sixth order on a general background, for the first time considering a complete basis of operators at this order. This goes significantly beyond previous work, and breaks a barrier in terms of technical feasibility. Previous computations were either limited to fourth order in derivatives [34–36], or to selecting very specific backgrounds. Consequently, this lead to either an identification of operators with an equal number of derivatives, or to a straight neglect of certain operators [31,37–42].

In our endeavor, we rely on the freedom to perform field redefinitions of the metric to fix all inessential couplings at all scales. This follows recent technical progress in the field [43–46]. Apart from technical simplifications, the resulting clear distinction between essential and inessential couplings is beneficial when counting the number of free parameters in the theory. Moreover, it imposes that no extra degrees of freedom are present apart from those included in general relativity (GR). This is contrary to higher derivative theories such as quadratic gravity or derivative expansions in the standard renormalization scheme, in which higher order curvatures lead to the presence of (fiducial or actual) ghosts [47–49].

This work is structured as follows. In Sec. II, preliminaries including the usage of field redefinitions at sixth order in derivatives for pure gravity are explained. The results of our computation are presented in Sec. III. We discuss properties of the UV fixed point (Sec. III A), the phase diagram

(Sec. III B) and regulator dependence (Secs. III C and III D). In Sec. IV we summarize the main results and give future perspectives. In Appendix A, we refer to an earlier computation including the Goroff-Sagnotti term in the standard scheme, while Appendix B contains some remarks about the topological Gauss-Bonnet term.

II. ESSENTIAL RENORMALIZATION GROUP FOR GRAVITY

In this section, we discuss some basics of implementing field redefinitions in non-perturbative RG flows as well as our setup and technical choices.

A. Generalized flow equations

In order to study the nonperturbative renormalizability of quantum gravity, we employ the functional renormalization group (FRG) [50–52]. It realizes Wilson’s formulation of the RG whereby fluctuations are integrated out with respect to a fiducial momentum scale k . For every scale k , this gives rise to an effective average action (EAA) $\Gamma_k[\Phi]$ that describes physics at the scale k , where modes with momentum $p^2 > k^2$ have been integrated out. In the generalized formulation [43,44,53], we allow for the freedom to change the description of physics at different scales k by means of local field redefinitions. Thus, the dependence on k originates from the Wilsonian infrared (IR) cutoff and from the k -dependent choice of field variables. Explicitly, the EAA can be defined by the integro-differential equation

$$e^{-\Gamma_k[\Phi]} = \int [D\chi] e^{-S[\chi] + (\hat{\Phi}_k[\chi] - \Phi)^A \frac{\delta}{\delta \Phi^A} \Gamma[\Phi] - \frac{1}{2} (\hat{\Phi}_k[\chi] - \Phi)^A [\mathcal{R}]_{AB} (\hat{\Phi}_k[\chi] - \Phi)^B}. \quad (1)$$

We use DeWitt notation with a repeated index implying both a sum over field indices and an integration over spacetime. The argument of the EAA is the expectation value of the microscopic field $\Phi = \langle \hat{\Phi}_k \rangle_{k,\Phi}$. Exploiting the freedom to choose the microscopic field variables $\hat{\Phi}_k[\chi]$ as a function of k allows for general nonlinear local field redefinitions. These enter the flow equation for the EAA via the RG kernel Ψ_k , which is defined as the expectation value of the k -derivative of the microscopic field,

$$\Psi_k = \langle \partial_t \hat{\Phi}_k \rangle_{k,\Phi}, \quad (2)$$

where $t = \log(k/k_0)$ is the RG time and k_0 an arbitrary reference scale.

The EAA obeys the exact functional flow equation [43]

$$\begin{aligned} \left(\partial_t + \Psi^A \frac{\delta}{\delta \Phi^A} \right) \Gamma_k &= \frac{1}{2} (-1)^{[A]} \left[\left(\Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} \right]^{AB} \\ &\times \left(\partial_t \delta_B^C + 2 \frac{\delta \Psi^C}{\delta \Phi^B} \right) [\mathcal{R}_k]_{CA}, \end{aligned} \quad (3)$$

where $[A]$ takes values 1 or -1 depending on whether the field corresponding to the index A is commuting or anti-commuting. The flow equation depends on the IR cutoff $\mathcal{R}_k$, which implements a gradual integration of fluctuations in the path integral, and the RG kernel Ψ_k , which parametrizes field redefinitions.

In the case of pure gravity, the fields Φ^A include both the metric tensor $g_{\mu\nu}$ and the Faddeev-Popov ghosts c_μ and $\bar{c}_\mu$. Furthermore, the EAA depends on a background metric $\bar{g}_{\mu\nu}$ introduced both to fix the gauge and to employ a covariant regularization. The total metric is split into the background plus a fluctuation field $h_{\mu\nu}$,

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}. \quad (4)$$

In this paper, we use the background field approximation, where $\Gamma_k[\Phi]$ is approximated by a diffeomorphism invariant functional $\bar{\Gamma}_k[g]$ in addition to gauge fixing and ghost terms,

$$\Gamma_k = \bar{\Gamma}_k[g] + S_{\text{gf}}[g; \bar{g}] + S_{\text{gh}}[g, c, \bar{c}; \bar{g}]. \quad (5)$$

To close the background field approximation, we identify the full metric $g_{\mu\nu}$ with the background metric $\bar{g}_{\mu\nu}$ by setting $h_{\mu\nu} = 0$ in (3). One then obtains the β functions for the couplings in $\bar{\Gamma}_k[g]$, which multiply diffeomorphism invariant operators. Importantly, as we indicate below, the gauge fixing and ghost terms in the action depend on k only through Newton's coupling $G_N = G_N(k)$. This reflects that we study the universality class with the same degrees of freedom as GR.

B. Continuum limit

In the absence of a UV fixed point, the functional integral (1) needs UV regularization to be well defined. This means that $S[\chi]$ requires a UV cutoff such that modes with momentum $p^2 \gg \Lambda^2$ are suppressed. Then, the EAA $\Gamma_k[\Phi]$ is replaced by $\Gamma_{k,\Lambda}[\Phi]$ with

$$\Gamma_k[\Phi] = \lim_{\Lambda/k \rightarrow \infty} \Gamma_{k,\Lambda}[\Phi]. \quad (6)$$

For $k \ll \Lambda$ the effects of the cutoff are negligible. On the other hand, if k approaches Λ , $\Gamma_{k,\Lambda}[\phi]$ necessarily involves the UV cutoff since $\Gamma_{\Lambda,\Lambda}[\phi]$ tends to the UV regulated bare action. For more details on the EAA with a UV cutoff, and its relation $\Gamma_k[\phi]$, see [54].

If there is a UV fixed point, one can first send $\Lambda/k \rightarrow \infty$. Then, the EAA tends to the unregulated microscopic action as $k \rightarrow \infty$. To understand this, let us observe that, from a physical point of view, a fixed point of the RG is associated with a scale invariant theory. In statistical physics, this corresponds to a continuous phase transition. Such transitions realize universality and, thus, are indifferent to microscopic details. Indeed, we can understand the continuum limit, i.e., asymptotic safety, as a limiting case where we start from a UV regulated theory, for example by some discrete model of spacetime, and tune the parameters of this theory so we pass close to a scale invariant theory before shooting away [55]. Let us call k_T the scale where we pass closest to the fixed point and $\delta\ell$ the corresponding minimum distance to the fixed point in coupling space. Then, if we pass arbitrarily close to the fixed point so $\delta\ell$ is small, Λ/k_T gets arbitrarily large, since the flow slows down as we approach the fixed point. Therefore, by (6), the portion for the flow for $k < k_T$ is arbitrarily well approximated by $\Gamma_k[\Phi]$. Universality is realized to a better and better approximation as $\delta\ell \rightarrow 0$, meaning that the theory forgets about its discrete nature. As $\delta\ell \rightarrow 0$ the half of the trajectory corresponding to $k > k_T$ approaches the phase transition where the fixed point is realized in the IR. On the other hand, the other half of the trajectory with $k < k_T$ gets closer to the asymptotically safe trajectory, i.e., the one that originates at the fixed point. As such this trajectory is described by completely Γ_k .

Since we are using field redefinitions, one might be concerned about the reparametrization invariance of the UV

limit $k \rightarrow \infty$. In particular, this involves the potential necessity of including the effects of a Jacobian that could modify the flow equation. In this regard, let us note that the field transformations in (1) are made by changing the operator that couples to the source and the regulator and so no Jacobian in the measure is required. On the other hand, in the limit $k \rightarrow \infty$ the relation between $\Gamma_\infty[\Phi]$ and $S[\chi]$ does involve a Jacobian [44]. To see this, note that (apart from a constant factor)

$$e^{-\Gamma_\infty[\Phi]} = \det \frac{\delta \chi_\infty[\Phi]}{\delta \Phi} e^{-S[\chi_\infty[\Phi]]} \quad (7)$$

where $\chi_k[\Phi]$ is the inverse of the transformation $\hat{\Phi}_k[\chi]$. However, $\Gamma_\infty[\Phi]$ and $S[\chi]$ are related by redundant operators [53] and thus physically equivalent. Indeed, (7) implies that $\int [D\chi] e^{-S[\chi]} = \int [D\hat{\Phi}] e^{-\Gamma_\infty[\hat{\Phi}]}$. Thus, we conclude that UV fixed points of the flow equation automatically involve the Jacobian required for reparametrization invariance.

C. Essential and redundant operators

In the next section, we will discuss the type of scheme used in this paper whereby only the essential couplings flow. Here we recall the technical details [44,45] relating to essential and redundant operators, that are the basis of the scheme.

Physics is independent of which field variables $\Phi_k[\hat{\chi}]$ we couple to the source and the regulator in the EAA (1). If we send¹ $\hat{\Phi}[\hat{\chi}] \rightarrow \hat{\Phi}[\hat{\chi}] + \epsilon \hat{\Xi}[\hat{\chi}]$ where ϵ is infinitesimal then

$$\Gamma \rightarrow \Gamma + \epsilon \mathcal{T}[\Gamma, \Xi], \quad (8)$$

where $\mathcal{T}[\Gamma, \Xi]$ is by definition a *redundant operator* and is given by

$$\mathcal{T}[\Gamma, \Xi] = \Xi^A \frac{\delta}{\delta \Phi^A} \Gamma - (-1)^{|A|} G^{AB} \frac{\delta \Xi^C}{\delta \Phi^B} R_{CA}, \quad (9)$$

with

$$\Xi = \langle \hat{\Xi} \rangle, \quad (10)$$

the expectation value of $\hat{\Xi}$, and

$$G^{AB} = [(\Gamma_k^{(2)} + \mathcal{R}_k)^{-1}]^{AB} \quad (11)$$

denoting the propagator. Essential operators are those operators that are linearly independent of all redundant operators. Importantly, a given operator will be essential only in a particular region of theory space, i.e., for certain values of the coupling constants. Thus, fixing a certain set

¹We drop the subscript k in this subsection.

of essential operators confines the analysis to a specific region in theory space.

Let us now take a complete basis of local terms that enter the action $\{O_i\}$ and a complete basis of local terms $\{\Xi_\alpha\}$ that enter the redefinition Ξ such that the most general action is given by (where we use the Einstein summation convention)

$$\Gamma = g^i O_i[\Phi], \quad (12)$$

and the most general RG kernel

$$\Psi = \gamma^\alpha(g) \Xi_\alpha[\Phi], \quad (13)$$

with g^i denoting the coupling constants of the theory. Furthermore, we choose the basis $\{O_i\}$ so it splits into the operators, which are a complete set of essential operators $\{O_a\}$, within a region of theory space $\mathcal{E}$ to be defined shortly, and a set of operators $\{O_\alpha\}$, which completes the set (thus Latin indices and Greek indices from the beginning of the alphabets take values in two disjoint sets, while i runs over the union of the two index sets). As such the most general action can be written as

$$\Gamma = g^a O_a[\Phi] + g^\alpha O_\alpha[\Phi]. \quad (14)$$

Let us now determine the region of the theory space $\mathcal{E}$. To this end, we note that we can expand each redundant operator $\mathcal{T}_\alpha \equiv \mathcal{T}[\Gamma, \Xi_\alpha]$, which is generated by Ξ_α , as

$$\mathcal{T}_\alpha = \Upsilon_\alpha^\beta(g) O_\beta[\Phi] + y_\alpha^b(g) O_b[\Phi]. \quad (15)$$

Within $\mathcal{E}$, we must be able to express every operator O_i as the sum of the essential operators $\{O_a\}$ and the redundant operators $\mathcal{T}_\alpha$. This means that we must be able to write

$$O_\alpha \stackrel{!}{=} x_\alpha^\beta(g) \mathcal{T}_\beta + x_\alpha^b(g) O_b. \quad (16)$$

We can achieve this if and only if the matrix Υ with components Υ_α^β is invertible since it follows from the consistency of (15) and (16) that $x_\alpha^\beta = (\Upsilon^{-1})_\alpha^\beta$. It also follows that when Υ is invertible, the operators O_a are linearly independent of the redundant operators $\mathcal{T}_\alpha$. Therefore, as long as Υ is invertible, $\{O_a\}$ is a complete basis of essential operators. Thus we conclude that the region $\mathcal{E}$ is defined by

$$g^i \in \mathcal{E} \quad \text{iff} \quad \det \Upsilon(g) \neq 0. \quad (17)$$

Within $\mathcal{E}$, the couplings g^a are essential.

Turning to the flow equation (3), it is convenient to work with dimensionless variables. Let us denote by $D\Gamma$ the term generated in the flow equation we get when changing from dimensionful to dimensionless variables, i.e.,

$$\frac{d}{dt} \Gamma = \partial_t \Gamma + D\Gamma, \quad (18)$$

where ∂_t is the derivative taken at constant dimensionless field. The flow equation (3) can then be written in dimensionless form as

$$\partial_t \Gamma = \frac{1}{2} (-1)^{[A]} G^{AB} \partial_t \mathcal{R}_{BA} - D\Gamma - \mathcal{T}[\Gamma, \Psi]. \quad (19)$$

Then let us define coefficients $\bar{\beta}^a$ and $\bar{\beta}^\alpha$ as

$$\frac{1}{2} (-1)^{[A]} G^{AB} \partial_t \mathcal{R}_{BA} - D\Gamma = \bar{\beta}^a O_a + \bar{\beta}^\alpha O_\alpha. \quad (20)$$

In other words, $\bar{\beta}^a$ and $\bar{\beta}^\alpha$ would be the beta functions in a standard scheme where we put $\Psi = 0$.

In an essential scheme, the dimensionless couplings g^a are fixed so they do not run, thus $\partial_t g^a = 0$. Therefore, expanding the flow equation, from terms proportional to O_a we obtain

$$\partial_t g^a = -\gamma^\alpha y_\alpha^a + \bar{\beta}^a, \quad (21)$$

while from the terms proportional to O_α

$$\gamma^\alpha \Upsilon_\alpha^\beta = \bar{\beta}^\beta. \quad (22)$$

Then we note that we can solve for γ^α if Υ_α^β is invertible and thus if O_a are essential. Furthermore, if the flow approaches a point where Υ_α^β has a zero eigenvalue, the gamma functions

$$\gamma^\alpha = \bar{\beta}^\beta (\Upsilon^{-1})_\beta^\alpha \quad (23)$$

will generically diverge.

Thus, we can conclude that in an *essential scheme* where we fix the values of the couplings g^a , we will be able to solve the flow equation in the region of theory space $\mathcal{E}$ where the couplings g^a are essential. Furthermore, when we reach the boundary of this region $\partial\mathcal{E}$, this will be indicated by the divergence of the γ functions γ^α . In this way, we understand that essential schemes are self-consistent: we will be able to solve the generalized flow equation in an essential scheme when we are in the region of theory space (17). On the other hand, it is clear that fixed points outside the region (17) will be inaccessible to the essential scheme.

As such, there are different regions of theory space where different sets of operators will be essential. Adopting an essential scheme, therefore, limits the RG flows that can be studied to those within the defined region $\mathcal{E}$. These regions can be associated with different universality classes, assuming we restrict the field redefinitions to a local set that cannot change the number of dynamical degrees of freedom. In the Appendix C we discuss how these regions

can be characterized by the Gaussian fixed points they contain and can, in turn, be related to a particular essential scheme.

D. Minimal essential scheme at sixth order

In this paper, we implement the minimal essential scheme (MES) for gravity [44,45]. According to this approach, we fix all inessential couplings to their values at the Gaussian fixed point associated with GR, characterized by a vanishing Newton coupling, $G_N \rightarrow 0$. The EAA therefore has a restricted ansatz in any approximation. According to the analysis of the previous section, there is a region of theory space $\mathcal{E}$, containing the Gaussian fixed point, for which the essential couplings identified at the Gaussian fixed point remain essential. Although no β functions for the inessential couplings appear in the flow equation, one can solve for γ functions that appear as coefficients of operators in the RG kernel [see (26) below]. As long as this is possible, the restricted ansatz contains all essential couplings. At the Gaussian fixed point we can easily identify the essential terms as those that do not vanish when we apply the equations of motion. This follows since the terms in the redundant operator proportional to the Einstein equations are proportional to $1/G_N$ and therefore dominate at the Gaussian fixed point.

Accordingly, in practice, the MES for gravity consists of putting to zero all the couplings for all operators that are proportional to the vacuum Einstein equations, $R_{\mu\nu} = 0$, apart from the Einstein-Hilbert term. It is minimal in the sense that the inessential couplings are simply set to zero hence minimizing the complexity of calculations. This can be achieved within a consistent approximation where the flow equation is expanded in derivatives with an appropriate choice of the RG kernel Ψ_k . In particular, if we include terms with up to n derivatives of the metric in the action, we include terms with $(n - 2)$ derivatives in the RG kernel. Previously, this scheme has been carried out to order $n = 4$ for both pure gravity [45] and gravity coupled to a shift-symmetric scalar field [46]. We extend this to the sixth order ($n = 6$), whence the EAA reads

$$\bar{\Gamma}_k = \int d^4x \sqrt{g} \left[\frac{\rho}{8\pi} - \frac{1}{16\pi G_N} R + \sigma_{\mathfrak{G}} \mathfrak{G} + G_{C^3} C^{\rho\sigma}{}_{\mu\nu} C^{\mu\nu}{}_{\alpha\beta} C^{\alpha\beta}{}_{\rho\sigma} \right]. \quad (24)$$

Here, $\rho = \Lambda/G_N$ gives the vacuum energy of the field, $\mathfrak{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\rho\sigma\mu\nu}R^{\rho\sigma\mu\nu}$ is the topological Gauss-Bonnet term, and $C_{\rho\sigma\mu\nu}$ is the Weyl tensor. In addition to Newton's coupling G_N and the topological Gauss-Bonnet coupling $\sigma_{\mathfrak{G}}$, the Goroff-Sagnotti coupling G_{C^3} [1,2] enters as the second dynamical essential coupling. All other couplings to operators with up to six derivatives are set to zero.

An important additional element of the MES for gravity is the treatment of the cosmological constant [45]. The scheme specifies that ρ is fixed to be proportional to the fourth power of the cutoff scale k . The constant of proportionality is fixed at the Gaussian fixed point and depends on the regulator. This imposes that the physical cosmological constant vanishes, i.e., when k goes to zero. Moreover, we note that this condition also dictates that the dimensionless product $G_N\Lambda$ is given by $G_N\Lambda = \tilde{\rho}k^4G_N^2$, where the proportionality constant $\tilde{\rho}$ depends on the regularization. The vacuum energy has also been fixed in an alternative fashion in [26], which involves rescaling the metric and modifying the regulator. We also note that using certain gauges and parametrization of the metric the cosmological constant can be removed from the trace part flow equation [56–58]. Furthermore in unimodular gravity the Newton's constant is essential and the cosmological constant appears to play no role [59]. For a more general discussion on the redundant nature of the cosmological constant see [60].

As we have mentioned, the MES for gravity fixes the inessential couplings to zero as they are identified at the Gaussian fixed point, defined by taking each coupling G_N , ρ and G_{C^3} to zero. In the case of gravity, the redundant operators (9) involved are symmetric covariant tensors composed of the metric and its derivatives $\Xi^A = \Xi_{\mu\nu}(x)$.² Maintaining the form of the action (24) along the flow, with $\tilde{\rho}$ also fixed, we will be able to solve the flow equation for the beta functions of the essential couplings G_N , G_{C^3} , and $\sigma_{\mathfrak{G}}$, and for the γ functions provided we are in the region $\mathcal{E}$ corresponding to the action (24).

At sixth order in the derivative expansion, the redundant operator basis is determined by the following linearly independent set of $\Xi_{\mu\nu}$ inserted in (9),

$$\{g_{\mu\nu}, Rg_{\mu\nu}, S_{\mu\nu}, R^2g_{\mu\nu}, S_{\rho\sigma}S^{\rho\sigma}g_{\mu\nu}, C_{\rho\sigma\alpha\beta}C^{\rho\sigma\alpha\beta}g_{\mu\nu}, S_{\mu\rho}S^{\rho}{}_{\nu}, C_{\mu\rho\nu\sigma}S^{\rho\sigma}, g_{\mu\nu}\Delta R, \Delta S_{\mu\nu}, R S_{\mu\nu}, \nabla_{\mu}\nabla_{\nu}R\}. \quad (25)$$

Here, $\Delta = -\nabla^2$ and $S_{\mu\nu} = R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R$ is the trace-free Ricci tensor. A general ansatz for the RG Kernel is set by the choice in (25) and is given by

$$\begin{aligned} \Psi_{\mu\nu}^g = & \gamma_g g_{\mu\nu} + \gamma_R Rg_{\mu\nu} + \gamma_S S_{\mu\nu} + \gamma_{R^2} R^2 g_{\mu\nu} \\ & + \gamma_{S^2} (S_{\rho\sigma}S^{\rho\sigma}g_{\mu\nu} + (R - 4\Lambda)S_{\mu\nu}) \\ & + \gamma_{C^2} (C_{\rho\sigma\alpha\beta}C^{\rho\sigma\alpha\beta}g_{\mu\nu} - 8\Lambda S_{\mu\nu}) \\ & + \gamma_{SSTL} \left(S_{\mu\rho}S^{\rho}{}_{\nu} - \frac{1}{4}g_{\mu\nu}S^{\rho\sigma}S_{\rho\sigma} \right) + \gamma_{CS} C_{\mu\rho\nu\sigma}S^{\rho\sigma} \\ & + \gamma_{\Delta R} g_{\mu\nu}\Delta R + \gamma_{\Delta S} \Delta S_{\mu\nu} + \gamma_{RS} R S_{\mu\nu} \\ & + \gamma_{DDR} \nabla_{\mu}\nabla_{\nu}R + \mathcal{O}(\partial^6). \end{aligned} \quad (26)$$

²We do not allow for redefinitions of the ghosts since we do not compute the flow of their couplings in our approximation.

The explicit dependence on the cosmological constant $\Lambda = G_N \rho$ is simply introduced for convenience—it brings the classical contribution of the RG kernel to the flow

equation [the contribution on the left-hand side of (3)] into an almost diagonal form for our chosen basis of curvature invariants,

$$\begin{aligned} \Psi_{\mu\nu}^g \frac{\delta \bar{\Gamma}_k}{\delta g_{\mu\nu}} = & \int d^4x \sqrt{g} \frac{1}{16\pi G_N} \left[4\Lambda\gamma_g + (4\Lambda\gamma_R - \gamma_g)R + \left(-\gamma_R + 4\Lambda\gamma_{R^2} - \frac{2}{3}\Lambda\gamma_{C^2} \right) R^2 \right. \\ & + \gamma_S S_{\mu\nu} S^{\mu\nu} + 4\Lambda\gamma_{C^2} \mathfrak{E} - \gamma_{R^2} R^3 + \gamma_{RS} R S_{\mu\nu} S^{\mu\nu} - \gamma_{C^2} R C_{\rho\sigma\mu\nu} C^{\rho\sigma\mu\nu} + \gamma_{SSTL} S_{\mu\nu}^{\mu} S_{\rho}^{\nu} S^{\rho}_{\mu} \\ & \left. + \gamma_{CS} S_{\mu\nu} S_{\rho\sigma} C^{\mu\rho\nu\sigma} - 16\pi G_N \gamma_g G_{C^3} C^{\rho\sigma}_{\mu\nu} C^{\mu\nu}_{\alpha\beta} C^{\alpha\beta}_{\rho\sigma} - \gamma_{\Delta R} R \Delta R + \gamma_{\Delta S} S_{\mu\nu} \Delta S^{\mu\nu} \right]. \end{aligned} \quad (27)$$

From this expression, we can see explicitly that we can choose the γ functions in such a way that we do not generate inessential couplings along the flow. Note that the terms proportional to γ_{DDR} and γ_{S^2} of the RG kernel (26) do not contribute to (27). This fact is explained in the following subsection.

Inspecting (27), we can also see the justification for our statement that we can absorb the running of all couplings multiplying terms that are proportional to at least one Ricci tensor (or Ricci scalar, but not pure Weyl invariants). All these terms are multiplied by one of the γ functions, and, thus, can be eliminated order by order in the derivative expansion. By contrast, essential terms like the C^3 term cannot be absorbed since the γ function γ_g has been used to fix the running of the cosmological constant. Importantly, this does *not* rely on imposing the (GR or other) equations of motion in one way or another—this is only a rule of thumb to identify terms that can be systematically removed by a field redefinition as above. The only qualifier to this discussion is that one has to choose the universality class at the beginning [27]—this means that the kinetic term(s) that define the spectrum *cannot* be removed by a field redefinition. In this way, if one were to investigate higher derivative gravity in the framework of essential RG flows, one could *not* remove R^2 and $S^{\mu\nu} S_{\mu\nu}$, since this would change the degrees of freedom. Put differently, at the four derivative Gaussian fixed point R^2 and $S^{\mu\nu} S_{\mu\nu}$ are essential operators and thus we cannot fix their couplings. In this paper, we only consider the universality class that has the spectrum of GR based on the Gaussian fixed point where $G_N \rightarrow 0$ and higher derivative couplings vanish.

E. Symmetries of the RG kernel

The RG kernel as defined in (26) includes terms that represent symmetries of the underlying action. Due to this fact, they do not lead to contributions to the left-hand side of the essential RG equation (3), as seen in (27). One example for such terms is $\gamma_{DDR} \nabla_\mu \nabla_\nu R$. Since

$$\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = \nabla_\mu \nabla_\nu R, \quad (28)$$

with $\xi_\mu = \nabla_\mu R/2$, this term is a diffeomorphism and does not contribute to the left-hand side of (3). However, it does

lead to a nonvanishing contribution on the right-hand side. Moreover, its contribution to the left-hand side of (3) only vanishes within the background field approximation. If nondiffeomorphism invariant terms are included in the action, there is also a nontrivial contribution to the left-hand side. We conclude that the corresponding tree-level redundant operator is outside of our approximation. Therefore, we set the term $\gamma_{DDR} \nabla_\mu \nabla_\nu R$ to zero in our ansatz for Ψ_k . The same reasoning applies to other diffeomorphism invariant contributions to the RG kernel that would be encountered at higher orders of the derivative expansion.

Another example for a term of the RG kernel that does not contribute to the left-hand side of (3) is

$$\gamma_{S^2} (S_{\rho\sigma} S^{\rho\sigma} g_{\mu\nu} + (R - 4\Lambda) S_{\mu\nu}). \quad (29)$$

In contrast to $\gamma_{DDR} \nabla_\mu \nabla_\nu R$, it is not a diffeomorphism term, but rather an example of another class of symmetries [61]. Any contribution to the RG kernel of the form

$$\Psi_{\mu\nu}^{\text{sym}}(x) = \int d^4y \sqrt{g} T_{\mu\nu,\rho\sigma}(x, y) \frac{\delta \Gamma}{\delta g_{\rho\sigma}(y)} \quad (30)$$

is a symmetry, provided that the two-point function $T_{\mu\nu,\rho\sigma}(x, y)$ satisfies $T_{\mu\nu,\rho\sigma}(x, y) = -T_{\rho\sigma,\mu\nu}(y, x)$. Note that such contributions are not specific to gravity, but also arise in other theories, e.g., in scalar field theories. In the case of (29), we have

$$T_{\mu\nu,\rho\sigma}(x, y) = \delta(x - y) [g_{\mu\nu}(x) S_{\rho\sigma}(y) - S_{\mu\nu}(x) g_{\rho\sigma}(y)], \quad (31)$$

up to proportionality factors. Similarly to diffeomorphisms, the contribution of (29) vanishes on the left-hand side of (3) within the background field approximation, while it leads to a nonvanishing contribution on the right-hand side. However, such contributions on the right-hand side are proportional to the equations of motion of the EAA for any term of the form (30). This can be shown in general by noting that the contribution is proportional to the field derivative of the RG kernel. To see this, we write the general form of (30) in DeWitt notation as

$$\Psi^{\text{sym}C} = T^{CD} \Gamma_D^{(1)}, \quad T^{CD} = -T^{DC}. \quad (32)$$

Using that T^{CD} is an antisymmetric two-point function while the regulator and propagator are symmetric, it can be shown that

$$\mathcal{G}^{AB} \frac{\delta \Psi^{\text{sym}C}}{\delta \Phi^B} [\mathcal{R}_k]_{CA} = \mathcal{G}^{AB} \frac{\delta T^{CD}}{\delta \Phi^B} [\mathcal{R}_k]_{CA} \Gamma_D^{(1)}. \quad (33)$$

In addition to this observation, we note that in the absence of a regulator, redundant operators of the effective action only have the tree-level term, and hence the redundant operator associated with (30) vanishes, but we expect the expectation values of local operators to be nonlocal. As a result, nonlocal terms in Ψ can arise from local field redefinitions.

F. Gauge fixing and regularization

For practical computations utilizing the FRG in gravity, we require to fix the gauge and implement a regulator. In the MES, we can find convenient choices for both by only considering the Einstein-Hilbert part of the action. This is due to the fact that, within the MES, the propagator on a conformally flat spacetime coincides with that derived from GR. Hence, we implement a standard background field harmonic gauge of the form

$$S_{\text{gf}}[g; \bar{g}] = \frac{1}{2} \int d^4x \sqrt{\bar{g}} \chi_\mu [h, \bar{g}] \chi^\mu [h, \bar{g}], \quad (34)$$

with

$$\chi_\mu [h; \bar{g}] = -\frac{1}{\sqrt{16\pi G_N}} [\bar{\nabla}_\nu h_\mu^\nu - \frac{1}{2} \bar{\nabla}_\mu h]. \quad (35)$$

Bars denote that respective quantities are constructed from the background metric $\bar{g}_{\mu\nu}$. The resulting Hessian including the Einstein-Hilbert term together with the gauge fixing is given by

$$\delta^2(S_{\text{EH}} + S_{\text{gf}}) = \frac{1}{32\pi G_N} \int d^4x \sqrt{\bar{g}} h_{\mu\nu} \bar{\Delta}_{\text{EH}}^{\mu\nu}{}_{\rho\sigma} h^{\rho\sigma}. \quad (36)$$

The gauge fixing (34) together with (35) leads to a particularly simple form for the Einstein-Hilbert Laplacian $\bar{\Delta}_{\text{EH}}$,

$$\bar{\Delta}_{\text{EH}}^{\mu\nu}{}_{\rho\sigma} = [-\bar{\nabla}^2 - 2\Lambda] \bar{K}^{\mu\nu}{}_{\rho\sigma} + \left[-2\bar{C}^{\mu}{}_{\rho}{}^{\nu}{}_{\sigma} + \frac{2}{3} \bar{R} \bar{\Pi}^{\text{TL}\mu\nu}{}_{\rho\sigma} \right], \quad (37)$$

where the matrix $\bar{K}^{\mu\nu}{}_{\rho\sigma}$ is given by

$$\bar{K}^{\mu\nu}{}_{\rho\sigma} = \bar{\Pi}^{\text{TL}\mu\nu}{}_{\rho\sigma} - \bar{\Pi}^{\text{Tr}\mu\nu}{}_{\rho\sigma}, \quad (38)$$

and $\bar{\Pi}^{\text{TL}}$ and $\bar{\Pi}^{\text{Tr}}$ denote the projectors on the traceless and trace modes of the graviton, respectively,

$$\begin{aligned} \bar{\Pi}^{\text{TL}\mu\nu}{}_{\rho\sigma} &= \delta^\mu{}_{(\rho} \delta^\nu{}_{\sigma)} - \frac{1}{4} \bar{g}^{\mu\nu} \bar{g}_{\rho\sigma}, \\ \bar{\Pi}^{\text{Tr}\mu\nu}{}_{\rho\sigma} &= \frac{1}{4} \bar{g}^{\mu\nu} \bar{g}_{\rho\sigma}. \end{aligned} \quad (39)$$

Other choices for the gauge fixing term typically lead to nonminimal, or higher order differential operators in (37). Both are avoided in this gauge and in the MES.

The gauge fixing (34) is accompanied by the usual Faddeev-Popov ghosts in the path integral. Their contribution reads

$$S_{\text{gh}} = \int d^4x \sqrt{\bar{g}} \bar{c}_\mu \mathcal{M}^{\mu\nu} c_\nu, \quad (40)$$

where c_μ and $\bar{c}_\mu$ are complex Grassmann fields, and the operator $\mathcal{M}^{\mu\nu}$ is given by

$$\mathcal{M}^{\mu\nu} = \frac{\delta \chi^\mu[\bar{g}, h[\xi]]}{\delta \xi_\nu}. \quad (41)$$

Within the background field approximation, it suffices to set $h_{\mu\nu} = 0$ in $\mathcal{M}_{\mu\nu}$. This gives rise to

$$\bar{\mathcal{M}}^{\mu\nu} = \frac{1}{\sqrt{16\pi G_N}} (-\bar{g}^{\mu\nu} \bar{\nabla}^2 - \bar{R}^{\mu\nu}). \quad (42)$$

Next, we choose a regulator. Motivated by the form of (37), for the graviton we pick

$$\begin{aligned} \mathcal{R}_k^{g\mu\nu}{}_{\rho\sigma} &= R_k^g(\bar{\Delta}) \bar{K}^{\mu\nu}{}_{\rho\sigma} \\ &= R_k^g(\bar{\Delta}) \bar{\Pi}^{\text{TL}\mu\nu}{}_{\rho\sigma} - R_k^g(-\bar{\nabla}^2) \bar{\Pi}^{\text{Tr}\mu\nu}{}_{\rho\sigma}. \end{aligned} \quad (43)$$

The Laplacian $\bar{\Delta}$ contains a nontrivial endomorphism,

$$\bar{\Delta}^{\mu\nu}{}_{\rho\sigma} = -\bar{\Pi}^{\text{TL}\mu\nu}{}_{\rho\sigma} \bar{\nabla}^2 + \beta \left[-2\bar{C}^{\mu}{}_{\rho}{}^{\nu}{}_{\sigma} + \frac{2}{3} \bar{R} \bar{\Pi}^{\text{TL}\mu\nu}{}_{\rho\sigma} \right], \quad (44)$$

that only contributes to the traceless, but not the trace mode of the graviton. Similarly, for the ghosts, the regulator is of the form $\mathcal{R}_k^{gh}{}_\nu = R_k^{gh}(-\bar{\nabla}^2 \delta^\mu{}_\nu - \beta \bar{R}^\mu{}_\nu)$. The parameter β controls the magnitude of the endomorphism. For $\beta = 0$ it vanishes, and we recover a type I cutoff. For $\beta = 1$, the Laplacian $\bar{\Delta}$ becomes equal to the Laplacian of the Einstein-Hilbert action, see (36). We also refer to this choice as the type II cutoff. Most of our analysis below is focused on $\beta = 1$, however, we also consider the dependence of our results on β below.

We derive the flow equation for general shape functions $R_k^g(z)$ and arbitrary endomorphism parameters β . To solve the threshold functions in the resulting β functions explicitly, we mostly use the Litim cutoff [62] with a prefactor α ,

$$R_k^g(z) = \frac{\alpha k^2}{32\pi G_N} \left(1 - \frac{z}{k^2} \right) \theta \left(1 - \frac{z}{k^2} \right). \quad (45)$$

While analytical expressions for the threshold functions are obtained for general α , they take a particularly simple form for $\alpha = 1$.

A second shape function that still allows obtaining analytical results for all threshold functions is the “square” of the Litim cutoff,

$$R_k^g(z) = \frac{\alpha k^2}{32\pi G_N} \left(1 - \frac{z}{k^2}\right)^2 \theta\left(1 - \frac{z}{k^2}\right), \quad (46)$$

which we also refer to as the Litim-2 cutoff. This additional shape function allows us to study some of the dependence of our results on the shape function.

The shape function for the ghosts R_k^{gh} is chosen to be the same as the one for the graviton R_k^g , with the factor $1/(32\pi G_N)$ replaced by $1/\sqrt{16\pi G_N}$.

III. RESULTS

In this section, we present and discuss our findings for quantum gravity at sixth order in the derivative expansion of the MES. The flow equation underlying our results has been obtained by an implementation of the setup outlined above in a computer algebra system. To ensure its correctness, two completely independent technical setups have been created based on (i) the `xAct` package for *Mathematica* [63–66] as implemented previously in the context of asymptotic safety [35,46,67], and (ii) self-written *Mathematica* code together with `Form` [68,69]. This allows for important cross-checks at this order of the derivative expansion, where previously only very limited partial results have been available.³ We provide the final result for all β and γ functions for arbitrary shape functions in a *Mathematica* notebook [70].

First, we report our results for fixed technical parameters α and β and discuss the resulting UV fixed point and its RG flow toward the IR. Following that, we also discuss the dependence of our results on α and β and analyze the stability of the fixed point.

A. UV fixed point

In this subsection, we choose fixed values for the technical parameters of our setup and search for nontrivial UV fixed points. We fix a type II cutoff with the natural endomorphism ($\beta = 1$), while the shape function of the regulator is set to the Litim cutoff with $\alpha = 1$. To search for nontrivial fixed points, we define dimensionless couplings g_N and g_{C^3} via

$$g_N = k^2 G_N, \quad g_{C^3} = k^2 G_{C^3}, \quad (47)$$

as well as dimensionless γ functions

$$\tilde{\gamma}_X = k^{-d_X} \gamma_X, \quad (48)$$

where d_X is the mass dimension of γ_X . To find the value for the vacuum energy ρ , which we fix along the RG flow, we set all γ functions to zero and send $g_N \rightarrow 0$. At this point we look for a solution to the flow of $\rho = \tilde{\rho} k^4$. For $\alpha = 1$ and the Litim cutoff, we find for the vacuum energy⁴

$$\rho = \frac{k^4}{8\pi}. \quad (49)$$

In the following, fixed point values are indicated by asterisks.

To investigate convergence properties of our system, we evaluate the flow order by order in the derivative expansion. At orders $\mathcal{O}(\partial^2)$ and $\mathcal{O}(\partial^4)$, Newton’s coupling is the only dynamical coupling in the MES. The fixed point results with a type I cutoff ($\beta = 0$) were first obtained in [44]. With the type II cutoff, these results are qualitatively unchanged, see also [46]. At leading order $\mathcal{O}(\partial^2)$ and with the type II cutoff, the fixed point for Newton’s coupling and its corresponding critical exponent read

$$g_N^* = 0.368, \quad \theta = 2.239. \quad (50)$$

At this order, the RG kernel is parametrized by a single γ function that takes the value

$$\tilde{\gamma}_g^* = -0.987. \quad (51)$$

Due to the Gauss-Bonnet term being topological, the fixed point at order $\mathcal{O}(\partial^4)$ is still described by a single dynamical coupling while the RG kernel contains three γ functions,

$$g_N^* = 0.364, \quad \theta = 2.228, \quad (52)$$

$$\tilde{\gamma}_g^* = -0.998, \quad \tilde{\gamma}_R^* = 0.014, \quad \tilde{\gamma}_S^* = 0.057. \quad (53)$$

For a discussion of the topological term, we refer to Appendix B.

A second essential and dynamical coupling is encountered at order $\mathcal{O}(\partial^6)$. This is the Goroff-Sagnotti coupling g_{C^3} . However, its inclusion does not lead to any qualitative changes in the fixed point. There is still a unique nontrivial fixed point at this order connected to the Gaussian fixed point. Its coupling coordinates are given by

$$g_N^* = 0.364, \quad g_{C^3}^* = 4.490 \times 10^{-7}. \quad (54)$$

Note the numerically small value of the Goroff-Sagnotti coupling. The critical exponents of the fixed point are

$$\theta_1 = 2.225, \quad \theta_2 = -3.850. \quad (55)$$

³Where a comparison was possible, we completely agree with [27].

⁴Note that the vacuum energy depends on α as well as the shape function of the cutoff, but not on β .

TABLE I. Comparison of critical exponents at different orders of the derivative expansion, using $\alpha = \beta = 1$ and the Litim shape function. The last line gives PMS values obtained in (63) for the critical exponents at $\mathcal{O}(\partial^6)$.

Order	θ_1	θ_2
$\mathcal{O}(\partial^2)$	2.239	Not applicable
$\mathcal{O}(\partial^4)$	2.228	Not applicable
$\mathcal{O}(\partial^6)$	2.225	-3.850
PMS at $\mathcal{O}(\partial^6)$	2.145	-2.697

The nontrivial fixed point only admits one relevant direction—the classically irrelevant Goroff-Sagnotti term stays irrelevant, confirming the findings in the standard scheme [39]. This means that, after fixing the RG scale, no free parameters are left in the theory.

Compared to the lower order results, (50) and (52), the fixed point value of Newton’s coupling only receives minor corrections at order $\mathcal{O}(\partial^6)$. This is also true for the critical exponents that we compare in Table I. The first two significant figures of θ_1 do not change between the leading order and $\mathcal{O}(\partial^6)$. The relative change between $\mathcal{O}(\partial^4)$ and $\mathcal{O}(\partial^6)$ is as small as 0.1%. This behavior of the derivative expansion suggests convergence toward higher orders. Moreover, we note that the value of the critical exponent θ_1 is close to its canonical value $\theta_1^{\text{cl}} = 2$, achieved in a one-loop approximation. This means that the induced quantum correction for θ_1 is small.

The stability of the fixed point in the MES can be quantitatively understood by the RG kernel (26). At the fixed point, we find the following values for the γ functions,

$$\begin{aligned}
 \tilde{\gamma}_g^* &= -0.996, & \tilde{\gamma}_R^* &= 0.011, \\
 \tilde{\gamma}_S^* &= 0.056, & \tilde{\gamma}_{R^2}^* &= 0.008, \\
 \tilde{\gamma}_{C^2}^* &= -0.005, & \tilde{\gamma}_{RS}^* &= -0.012, \\
 \tilde{\gamma}_{SSTL}^* &= -0.024, & \tilde{\gamma}_{CS}^* &= 0.025, \\
 \tilde{\gamma}_{\Delta R}^* &= -0.011, & \tilde{\gamma}_{\Delta S}^* &= 0.007.
 \end{aligned} \tag{56}$$

Except for $\tilde{\gamma}_g$, which is already included in the Einstein-Hilbert approximation, the absolute values of all of the γ functions are much smaller than one. Together with the small value of the Goroff-Sagnotti coupling at the fixed point, this quantitatively explains why higher order effects on the fixed point are small.

To close this subsection, let us note that the nontrivial fixed point shows several signs of a near-perturbative behavior. This includes the fixed point value of the Goroff-Sagnotti coupling, quantum corrections to the critical exponents, and the γ functions that parametrize the RG kernel. All of these take very small values within the MES. The smallness of the RG kernel also implies that the structure of the redundant operators is a small deformation

of the Gaussian fixed point. Together with earlier indications of a near-perturbative behavior of quantum gravity [30–33], our results strengthen the evidence that the nontrivial fixed point is in a near-perturbative region. As a matter of fact, if we were to neglect all γ functions except γ_g in the above computation, the results change only insignificantly. In this reduced setup, we find

$$g_N^* = 0.368, \quad g_{C^3}^* = 7.704 \times 10^{-7}, \quad \tilde{\gamma}_g^* = -0.987, \tag{57}$$

and

$$\theta_1 = 2.239, \quad \theta_2 = -3.881. \tag{58}$$

This simplified approximation compares very favorably with the full computation, and opens up a path forward to high order computations: keep all essential couplings and γ_g , while neglecting all other γ functions. Curiously, the same procedure in the standard scheme yields a highly noncanonical critical exponent [39], see also Appendix A.

B. RG flow and IR behavior

To visualize the flow of asymptotically safe trajectories and their IR properties, Fig. 1 shows the phase diagram within the MES for $\alpha = \beta = 1$ and Litim cutoff. Since we require Newton’s coupling to be small and positive at low energies, any physical RG trajectory must reach the vicinity of the Gaussian fixed point with a positive Newton coupling in the IR. How couplings arrive at the Gaussian fixed point from the UV can be read off from Fig. 1. Among these trajectories that have an IR limit corresponding to perturbative GR, there are two possibilities for their UV origin.

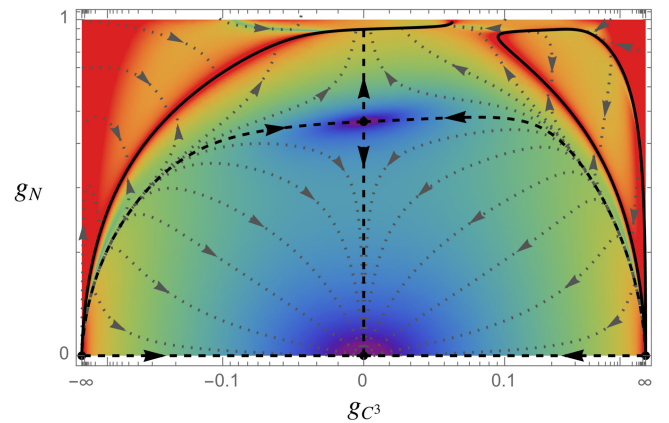


FIG. 1. Phase diagram with the Litim shape function and technical parameters $\alpha = \beta = 1$. Arrows point from the UV to the IR, and the color gradient indicates the flow velocity (increasing velocity from blue to red). Solid black lines indicate divergences of β functions, dashed black lines represent separatrices, whereas dotted grey lines are sample trajectories.

The first and generic option consists of trajectories for which the Goroff-Sagnotti coupling diverges at high energies. These trajectories begin in the UV at $g_N = 0$ and $g_{C^3} = \pm\infty$ and therefore do not correspond to asymptotically safe theories. Based on scaling arguments [6], one expects that observables such as scattering amplitudes have a divergent high energy behavior.

Finite values for the couplings are only reached on the separatrix between the regions where the Goroff-Sagnotti coupling grows toward $+\infty$ and $-\infty$. This is the asymptotically safe trajectory that originates from the fixed point (54) in the UV, and it is the only trajectory that has a well-defined behavior at all energy scales and connects to the region of perturbative quantum GR at low energy scales. There are no free parameters that need to be fixed and no second nontrivial fixed point is available as a UV completion for trajectories that are connected to the Gaussian fixed point.

The UV safe separatrix arises from the relevant deformation of the UV fixed point. The irrelevant deformation spans separatrices that split the phase diagram into two parts: all trajectories with smaller Newton coupling than these separatrices reach the Gaussian fixed point in the IR, while trajectories with larger Newton coupling do not. The latter eventually end up in a divergence of the RG flow. Such divergences arise when denominators of the β functions develop a zero. The zeros of those denominators form hypersurfaces in coupling space at which the β functions diverge. In Fig. 1 we see that this happens whenever Newton's coupling becomes too large, and we attribute this to a breakdown of our approximation.

The asymptotically safe trajectory that connects the nontrivial fixed point in the UV with the Gaussian fixed point in the IR leads to a nontrivial prediction at low energies. For small k , we can solve the k dependence of the Goroff-Sagnotti coupling in terms of Newton's coupling,

$$g_{C^3} = [G_N k^2] \left(A - \frac{43}{645120\pi^3} \ln G_N k^2 \right). \quad (59)$$

The prefactor of the logarithm is identical for all trajectories reaching the Gaussian fixed point. By contrast, the Wilson coefficient A depends on the trajectory. For the asymptotically safe trajectory, the value of A is predicted by the fixed point. Numerically, we find

$$A = -3.988 \times 10^{-6}, \quad (60)$$

once again for the choice $\alpha = \beta = 1$ and the Litim shape function.

In principle, the RG running in the IR can be compared to perturbative β functions. On the perturbative side, additional attention must be paid to the fact that the Gauss-Bonnet term acts as an evanescent operator that leads to a nontrivial relation between the perturbative two-

loop divergence and its RG running [71,72]. Taking this into account, the perturbative β function of the Goroff-Sagnotti coupling is given by

$$\partial_t g_{C^3} = 2g_{C^3} + \frac{1}{7680\pi^3} g_N + \dots \quad (61)$$

Our result using the type II cutoff is

$$\partial_t g_{C^3} = 2g_{C^3} - \frac{43}{322560\pi^3} g_N + \dots \quad (62)$$

Although the two results for the quantum corrections do not agree, it should be noted that beyond the one-loop order, a comparison with perturbative results requires approximation schemes other than a derivative expansion in the FRG. Indeed, in [73,74] it is shown that two-loop results for the quartic coupling of an $O(N)$ symmetric scalar field can be obtained with an infinite number of terms in the derivative expansion. Thus, it is reasonable to think that also the case of gravity requires an infinite expansion in derivatives to reproduce the perturbative result (61).

C. Regulator dependence with type II cutoff

To test the reliability of our results, we analyze the regulator dependence. For this purpose, the parameters α and β are introduced in the regulator. In this subsection, we focus on the dependence on one of these parameters while keeping the other fixed: we continue using the type II cutoff with $\beta = 1$, while changing the strength of the regulator by varying α .

In Fig. 2, we show the dependence of the fixed point on α . There is a large area in which the fixed point is stable. This reaches from $\alpha \approx 0.05$ to $\alpha \approx 50$. Going beyond this regime, the couplings and critical exponents grow large. Furthermore, the fixed point becomes unphysical due to a sign change of Newton's coupling at $\alpha \approx 57$. For $\alpha > 57$, the nontrivial fixed point is located at negative Newton coupling. Due to zeros and divergences of the β functions at $g_N = 0$ and at large g_N , such a fixed point cannot admit an RG trajectory connecting it to a low energy regime with small positive Newton's coupling. Moreover, the sign change of Newton's coupling at $\alpha \approx 57$ is accompanied by a divergence of the Goroff-Sagnotti coupling. Thus, we expect that the physical region in which our approximation scheme is well defined does not reach beyond $\alpha = 57$.

Similar observations hold for the small α regime. Although we do not observe a divergence in the couplings, the critical exponents diverge for $\alpha \rightarrow 0$. This is indicated in Fig. 2, which shows how the relevant critical exponent becomes large for $\alpha < 0.1$. The limit $\alpha \rightarrow 0$ is typically characterized by log tails in α for even dimensions: in [75], this has been discussed for a scalar field in two and four dimensions.

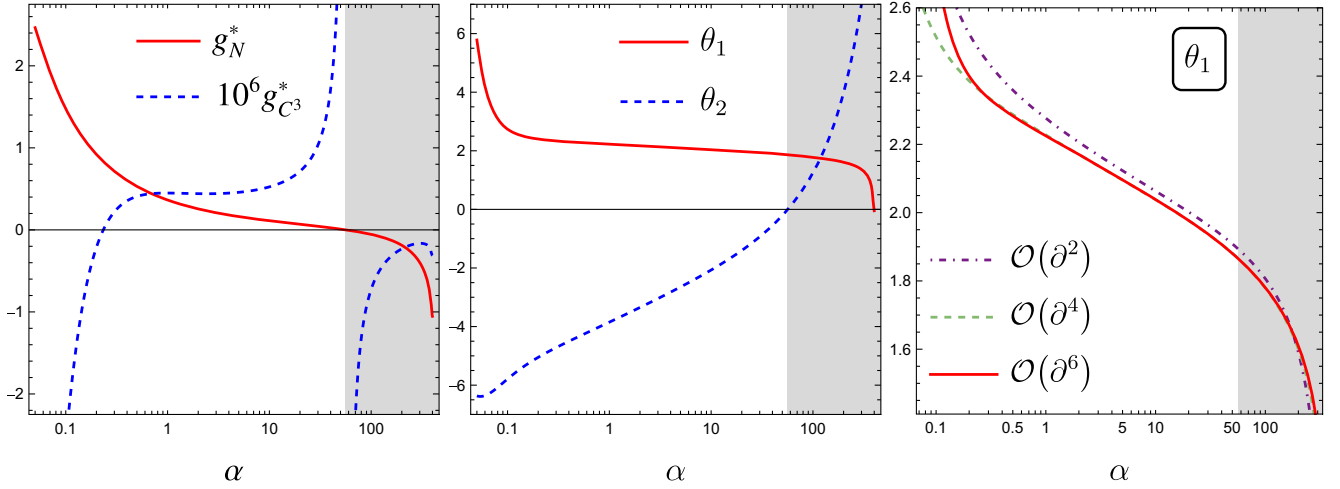


FIG. 2. Fixed point dependence on α with the type II cutoff ($\beta = 1$) and the Litim cutoff. The left plot shows the couplings of the fixed point at order $\mathcal{O}(\partial^6)$ in the derivative expansion. The central plot shows the critical exponents at order $\mathcal{O}(\partial^6)$. The right panel compares the α dependence of the relevant critical exponent obtained at orders $\mathcal{O}(\partial^2)$, $\mathcal{O}(\partial^4)$, and $\mathcal{O}(\partial^6)$. In all plots, the grey-shaded area indicates regions of negative Newton's coupling.

In the intermediate range, $0.1 < \alpha < 57$, we observe a large area of stability for the fixed point. Most notably, this includes a plateau for the relevant critical exponent. From $\alpha \approx 0.2$ up to $\alpha \approx 50$, we find a mild dependence of θ_1 on α , taking values from 2.4 to 1.9. This is not only true at order $\mathcal{O}(\partial^6)$, but also at lower orders. In fact, the relevant critical exponent only receives minor numerical modifications between different orders from $\mathcal{O}(\partial^2)$ up to $\mathcal{O}(\partial^6)$ throughout the whole region $0.1 < \alpha < 57$. Starting from $\alpha = 0.3$ and up until $\alpha = 57$, the difference between $\mathcal{O}(\partial^4)$ and $\mathcal{O}(\partial^6)$ is of the order of 0.1%.

This observed convergence between different orders of the derivative expansion can, again, be explained by the smallness of the Goroff-Sagnotti coupling and the RG kernel. In Fig. 3, we show the γ functions of the RG kernel at the fixed point in dependence of α . Within the previously

identified region, all γ functions except for γ_g are essentially vanishing. Thus, different orders in the derivative expansion are almost disentangled from each other. Higher orders of the derivative expansion only have minimal effects on results obtained at lower orders. Therefore, we expect our results to be robust against effects from higher order curvature contributions.

Following this discussion, it is worth noting that the α dependence of the relevant critical exponent is virtually unaffected by higher order operators beyond four powers of derivatives, see right panel of Fig. 2. This behavior is different compared to other models that have been studied in the derivative expansion such as the Ising model [76]. There, the dependence on α usually becomes stronger with increasing orders. This signals that the derivative expansion only converges for specific values of α . In contrast, Fig. 2

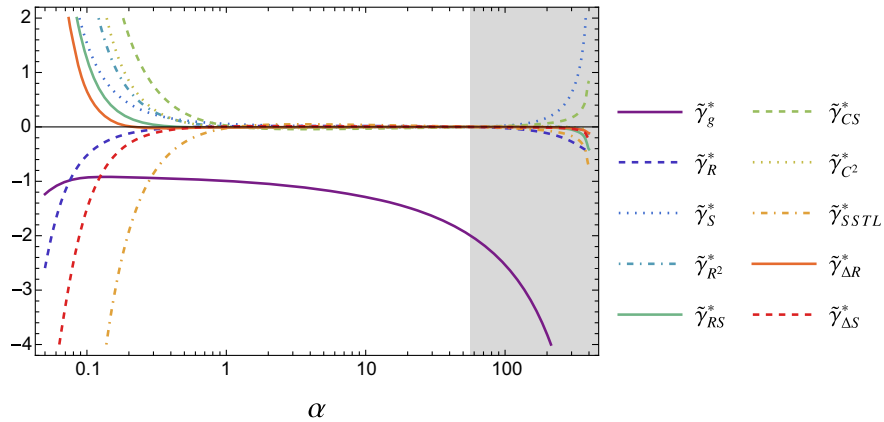


FIG. 3. γ functions are shown in dependence of α for the type II cutoff ($\beta = 1$) and the Litim cutoff. In the grey-shaded area, Newton's coupling takes negative values.

suggests a large range for α in which the α dependence does not change notably beyond $\mathcal{O}(\partial^4)$. This could point toward a finite α range in which the derivative expansion converges. However, the α dependence does not become flat in this regime. Most likely, this can be explained by truncation artifacts of the background field approximation. The α dependence would then be caused by neglecting nondiffeomorphism invariant terms in the action. The inclusion of those would potentially level out the α dependence within the domain of convergence. To test this hypothesis and control this effect, it is necessary to go beyond the background field approximation.

Compared to the relevant critical exponent θ_1 , the critical exponent of the Goroff-Sagnotti coupling θ_2 shows a stronger dependence on α . This is not surprising since the Goroff-Sagnotti term is included for the first time in the derivative expansion at order $\mathcal{O}(\partial^6)$. Most likely, its critical exponent is stabilized by other operators at higher orders. From $\alpha = 0.2$ to $\alpha = 50$, the critical exponent θ_2 takes values from $\theta_2 = -5$ to $\theta_2 = -0.2$. It is decreasing in magnitude with increasing α . At $\alpha \approx 57$ it changes sign and becomes relevant. However, this is the point where Newton's coupling becomes negative and falls out of the physical regime. Thus, within our approximation the Goroff-Sagnotti term stays irrelevant within the physical region.

Let us note that an analysis of the α dependence of the fixed point shows that previously observed indications of near perturbativity are not an artifact of a specific value for α . Instead, the smallness of the Goroff-Sagnotti coupling, the γ functions, and corrections to the critical exponents are

independent of α within the region in which we expect our approximation to be trustworthy. Hence, we tend to conclude that the nontrivial fixed point is near perturbative, independently of α .

D. Regulator dependence beyond type II cutoff

We now analyze the full regulator dependence of our results, varying both α and β . Thus, in addition to the regulator strength encoded in α , we test the dependence of our results on the endomorphism parameter β . For $\beta = 1$, the regulator corresponds to a type II cutoff, which has been studied in detail above. For $\beta = 0$, we recover the type I cutoff. We have computed the fixed point for all values of $0.5 < \alpha < 4$, and $0 < \beta < 7$, using the two cutoff shapes given in (45) and (46).

Apart from parameter choices for which the fixed point becomes unstable, our qualitative results are independent of α and β . Within the scanned region, instabilities only occur for small α and large β . In this area, the fixed point moves close to a hypersurface on which the RG flow develops a singularity. This leads to rapidly changing and large values for the fixed point couplings, its critical exponents, and the RG kernel. As a consequence, we cannot find the fixed point reliably anymore, and we expect that our approximation scheme cannot be trusted in this regime. Accordingly, this regime is excluded from all plots.

One of the main results of the scan in the (α, β) plane is the observed special role of $\beta = 1$. As can be seen in Fig. 4, the Goroff-Sagnotti coupling is zero at the fixed point almost exactly along the line of $\beta = 1$, while taking comparatively large positive values for other values of β .

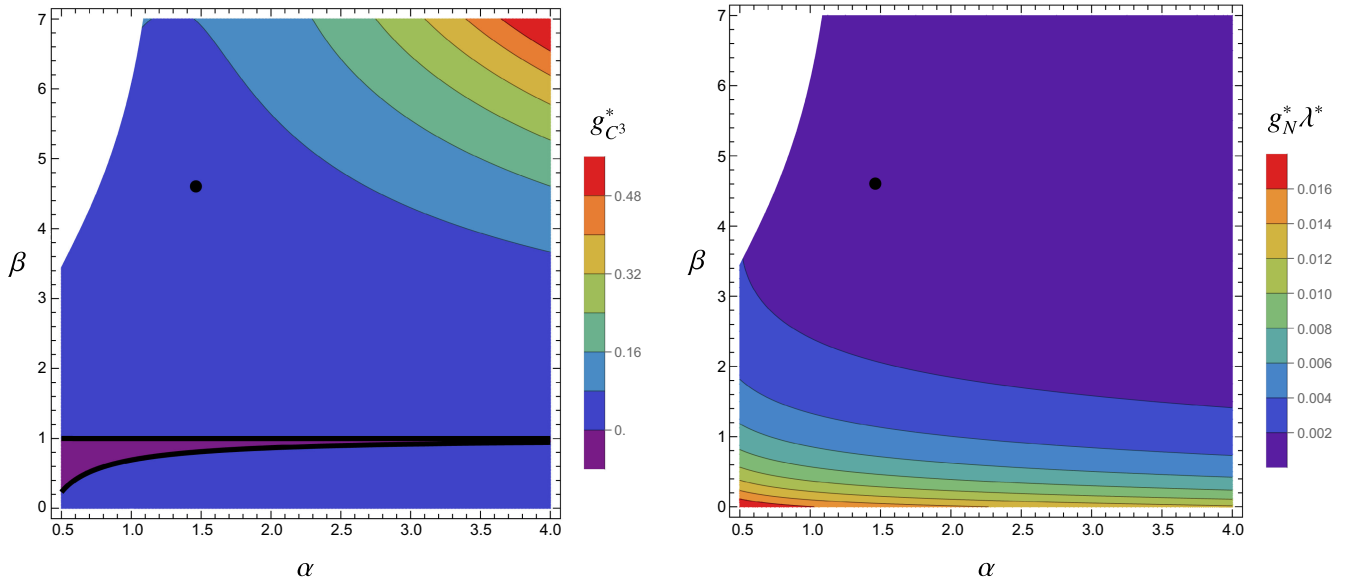


FIG. 4. The Goroff-Sagnotti coupling (left panel) and the product of Newton's coupling and the cosmological constant (right panel) in dependence of α and β with the Litim cutoff. Thick black lines indicate zeros of the Goroff-Sagnotti coupling. Note the pronounced zero along $\beta = 1$ corresponding to the type II cutoff. Similar zeros are also seen for the γ functions that parametrize the RG kernel, see Fig. 5. Similar results are found for the Litim-2 cutoff.

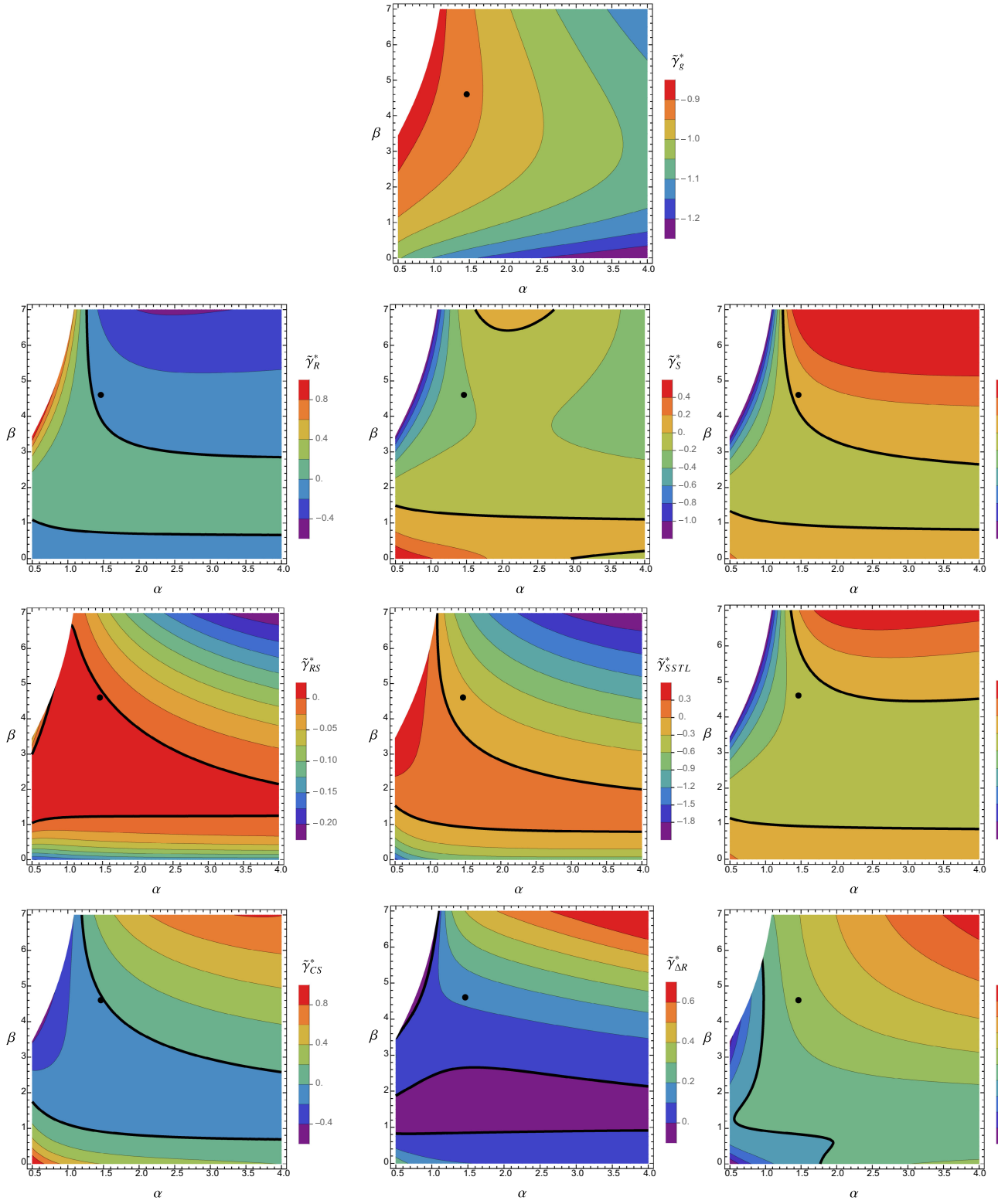


FIG. 5. Values for the γ function at the fixed point in dependence of α and β with the Litim cutoff. Zeros are indicated by thick black lines.

Thus, the smallness of the Goroff-Sagnotti coupling is tied to $\beta = 1$. A similar effect is observed for the RG kernel in Fig. 5. Except for γ_g , all γ functions develop zeros close to $\beta = 1$. These observations motivate our usage of the type II

cutoff ($\beta = 1$) above: it is this technical choice that makes the near perturbativity of the nontrivial fixed point explicit, and leads to a small Goroff-Sagnotti coupling and RG kernel, with the exception of γ_g . We might expect our

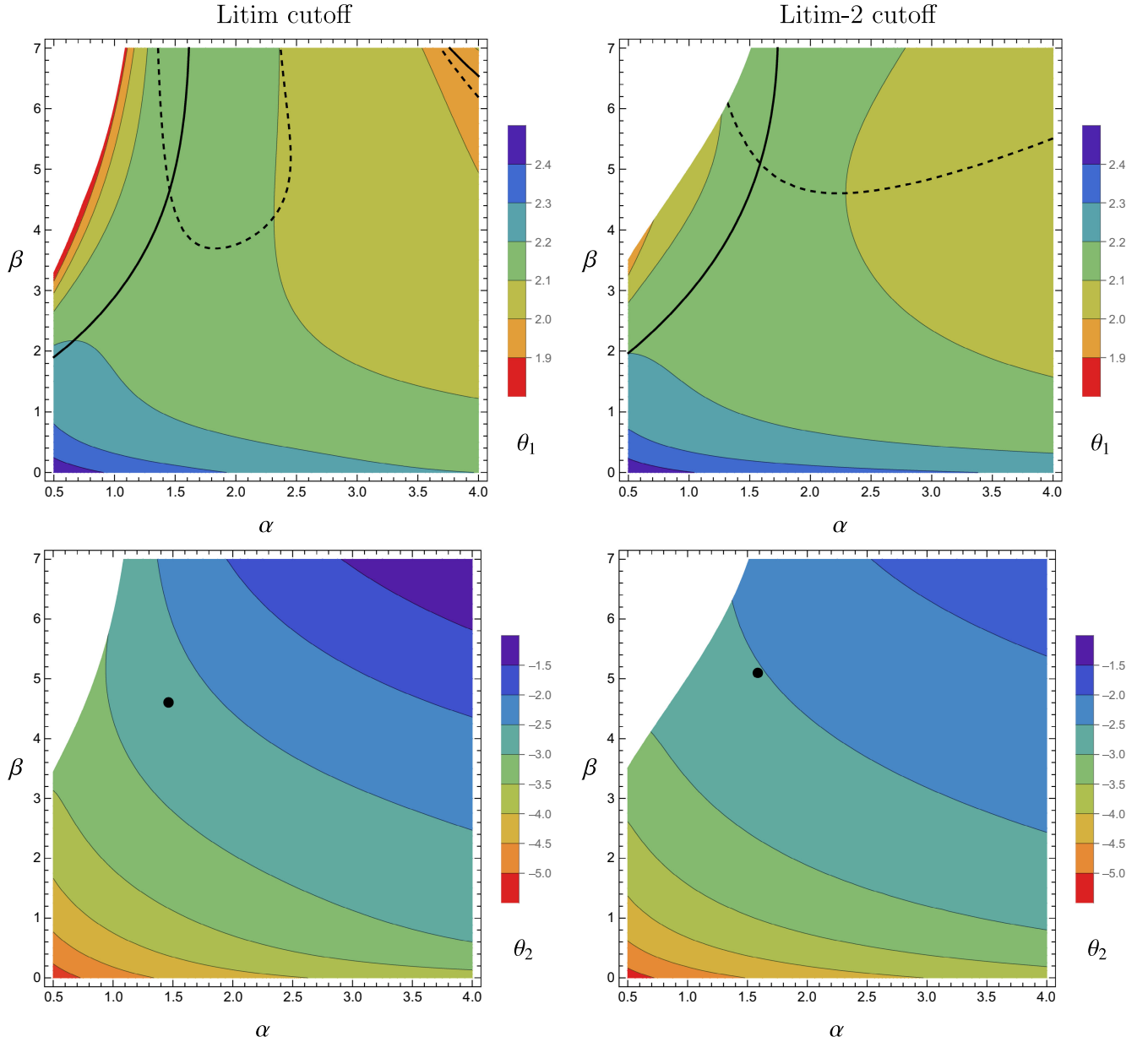


FIG. 6. Regulator dependence of the critical exponents on α and β with Litim (left column) and Litim-2 (right column) cutoff. Thick solid (dashed) black curves show where the dependence of the relevant exponent on α (β) vanishes. A black dot denotes the PMS value. For small α and large β , we find that the γ functions become large, and critical exponents fluctuate strongly. This unstable area has been cut out from the plots.

approximation to converge most quickly in this regime as higher order curvatures are as disentangled as possible from lower order curvatures.

In Fig. 6, we show the dependence of the relevant and irrelevant critical exponent on α and β , using the Litim as well as the Litim-2 cutoff. With both cutoffs, the relevant critical exponent takes values between 2.0 and 2.2 in almost the entire scanned area. This includes most of the $\beta = 1$ line, and shows that the relevant critical exponent is also stable away from $\beta = 1$. Toward smaller values of α , the fluctuations of the relevant critical exponent become stronger. This

is in line with our results for the type II cutoff in Sec. III C and similar studies in scalar field theories [75]. Compared to the relevant critical exponent, the irrelevant critical exponent shows stronger variations. Within the scanned region, these are almost uniform and take values -1.5 and -5 . Thus, the irrelevant critical exponent is strictly irrelevant.

We can also apply the principle of minimal sensitivity (PMS) to determine preferred values for α and β for which we expect the derivative expansion to rapidly converge. This has proven successful in determining the critical exponents of the Ising model [76]. The PMS is based on

the fact that physical observables will not depend on the regulator at the exact level. In approximations, this will not be the case but we can impose it as an extra restriction in addition to the fixed point condition and therefore determine PMS values for α, β , the fixed point and the critical exponents. We therefore consider the PMS, and determine technical parameters for which the dependence of physical observables on α and β vanishes. This is possible if an observable obtains an extremum as a function of technical parameters. Applying this concept to the critical exponents, we do not find a PMS point for α at $\beta = 1$. This is clear from Fig. 2, which shows that the critical exponents do not take an extremum anywhere within the physical regime for α . However, a PMS point does exist if we allow for $\beta \neq 1$ and require the relevant critical exponent θ_1 to have an extremum in both α and β . In Fig. 6, solid black lines show where the dependence of the relevant critical exponent on α vanishes, while dashed black lines indicate where the dependence on β vanishes. Within the scanned region, we find a unique intersection of both—this holds for the Litim and the Litim-2 cutoff. For the Litim cutoff, the resulting PMS value is located at

$$\begin{aligned}
 \alpha &= 1.454, & \beta &= 4.655, \\
 \theta_1 &= 2.145, & \theta_2 &= -2.697, \\
 g_N^* &= 0.108, & g_{C^3}^* &= 0.034, \\
 \tilde{\gamma}_g^* &= -0.933, & \tilde{\gamma}_R^* &= -0.036, \\
 \tilde{\gamma}_S^* &= -0.226, & \tilde{\gamma}_{R^2}^* &= 0.095, \\
 \tilde{\gamma}_{RS}^* &= 0.004, & \tilde{\gamma}_{CS}^* &= -0.011, \\
 \tilde{\gamma}_{C^2}^* &= -0.430, & \tilde{\gamma}_{SSTL}^* &= -0.189, \\
 \tilde{\gamma}_{\Delta R}^* &= 0.125, & \tilde{\gamma}_{\Delta S}^* &= 0.280,
 \end{aligned} \tag{63}$$

while we find for the Litim-2 cutoff,

$$\begin{aligned}
 \alpha &= 1.586, & \beta &= 5.098, \\
 \theta_1 &= 2.117, & \theta_2 &= -2.521, \\
 g_N^* &= 0.119, & g_{C^3}^* &= 0.101, \\
 \tilde{\gamma}_g^* &= -0.894, & \tilde{\gamma}_R^* &= 0.061, \\
 \tilde{\gamma}_S^* &= -0.787, & \tilde{\gamma}_{R^2}^* &= -1.861, \\
 \tilde{\gamma}_{RS}^* &= 0.735, & \tilde{\gamma}_{CS}^* &= -0.865, \\
 \tilde{\gamma}_{C^2}^* &= -5.071, & \tilde{\gamma}_{SSTL}^* &= 1.626, \\
 \tilde{\gamma}_{\Delta R}^* &= -0.107, & \tilde{\gamma}_{\Delta S}^* &= 0.740.
 \end{aligned} \tag{64}$$

Comparing both results, the relevant critical exponents differ by just 1%, while the irrelevant exponents agree within 7%. Similar small differences arise when comparing with the results with the type II cutoff, i.e., Table I. The results for the relevant critical exponent agree within 5%.

Stronger variations arise for the results of the irrelevant critical exponent, which shows larger dependencies on technical parameters, once again indicating that higher order operators are likely necessary to stabilize its value.

Similar to the type II cutoff, the PMS point is an anchor point for zeros of most of the γ functions, see Fig. 5. Except for $\gamma_g, \gamma_S, \gamma_{\Delta R}$, and $\gamma_{\Delta S}$ all γ functions develop a zero close to the PMS point. The major difference compared with the type II cutoff at $\beta = 1$ is the value of the Goroff-Sagnotti coupling. While it vanishes close to $\beta = 1$, it does not have a zero in the vicinity of the PMS point and takes a comparatively large value.

In summary, we have found that our qualitative—and to a large extent quantitative—results are independent of the technical parameters α and β . In particular, the critical exponent of the Goroff-Sagnotti coupling is always irrelevant. Moreover, the relevant critical exponent is remarkably stable, and takes values between 2.0–2.3 within the trustworthy region of our setup. On the other hand, the smallness of the RG kernel and the Goroff-Sagnotti coupling are to some extent related to special choices for technical parameters, such as the type II cutoff ($\beta = 1$) or the PMS point, see (63) and (64). In both cases, the RG kernel becomes small, while the former also leads to an extremely small Goroff-Sagnotti coupling. We expect that this near perturbativity of couplings and RG kernel is particularly useful for the convergence of our expansion and we expect the best convergence in this region.

IV. DISCUSSION

In this work, we have provided further evidence for the existence of a nontrivial fixed point in quantum gravity. Employing the FRG, we have performed a consistent derivative expansion to sixth order on general backgrounds. This is the first complete work at this order in four dimensions,⁵ and marks a considerable step forward compared to previous approaches that were limited to fourth order of the derivative expansion, or the use of simpler background geometries. Moreover, we have used recent insights on the use of field redefinitions in the FRG by implementing the MES. In this scheme, only essential couplings are retained in the action, while redundant couplings are fixed at the expense of nontrivial field redefinitions. In this way, it is possible to eliminate the inessential couplings from the flow equations, which then depend only on essential couplings. Two main advantages of this approach are (i) a simpler structure for the propagator, which makes computations to this order of the derivative expansion feasible, and (ii) focusing on the essential couplings of the theory when searching for nontrivial fixed points and evaluating their critical exponents. The latter point is important for the asymptotic safety scenario since redundant couplings do not affect physical

⁵For a recent discussion of one-loop β functions of cubic gravity in its critical dimension, $d = 6$, see [67].

observables, and consequently they should not be taken into account when counting the number of free parameters of the theory. Furthermore, the assumptions of the MES, which identifies the inessential couplings at the free fixed point, restrict the theory space to the subspace where those couplings remain inessential. Namely, we focus on the universality classes connected to GR, and characterized by the same redundant operator structure. The simple form of the propagator then suggests that fixed points found in the MES have the same propagating degrees of freedom as GR, as also indicated by recent results using momentum-dependent field redefinitions [27]. More technically, on any conformally flat background, the propagator has the same pole structure as GR.

Although a nontrivial gravitational fixed point has previously been identified at second and quartic orders in derivatives within the MES [45] at sixth order a second dynamical essential coupling enters that could have strongly altered results obtained at lower orders. Nevertheless, as seen in Fig. 1, the results of our analysis suggest that the fixed point survives the inclusion of terms at sixth order in derivatives, and moreover, that it is remarkably stable. Newton's coupling and its critical exponent are essentially unaffected by the extension, even though a new essential coupling enters the system. Between fourth and sixth order of the derivative expansion, the relevant critical exponent receives corrections of only $\approx 0.1\%$, taking a value of $\theta_1 = 2.225$ at sixth order, see Table I. Furthermore, the Goroff-Sagnotti term stays irrelevant at the fixed point, which confirms previous observations in the standard scheme [39]. Therefore, its Wilson coefficient is a prediction of the theory, see (59) and (60). The fact that the sixth order still gives rise to a unique nontrivial fixed point with just one relevant direction points toward an asymptotic safety scenario of pure quantum gravity that features no free parameters. Compared to the standard scheme, which usually gives rise to three to four relevant directions [31,34,36–38,40,41,77], we see that identifying redundant couplings is key to understanding the predictivity of the fixed point within asymptotic safety.

We have also observed several features of near perturbativity at the nontrivial fixed point. The clearest indication is the very small value of the Goroff-Sagnotti coupling at the fixed point, $g_{C^3} = 4.5 \times 10^{-7}$, see (54). Additionally, we have shown in (56) that the RG kernel that parametrizes the field redefinitions takes small absolute values as well, with the sole exception of γ_g taking values of the order of one. Lastly, critical exponents receive minor corrections when compared to their canonical counterparts. All these indications line up with previous observations [30–33], and suggest a near-perturbative behavior of quantum gravity. It is as nonperturbative as necessary, but as perturbative as it gets.

This observation can lead to possible simplifications for future investigations: a possible way to compute higher

order corrections is to keep all essential couplings in the action to a given order, while only retaining γ_g in the RG kernel. Remarkably, implementing this seemingly crude approximation, the results at sixth order in the derivative expansion are virtually unchanged, see (58). This is to be contrasted with the standard scheme, where such an approximation leads to a strongly noncanonical irrelevant critical exponent, see [39] and Appendix A.

Finally, we have checked the dependence of our results on technical parameters. This serves as an important tool to estimate truncation errors that are present in applications of the FRG. Our analysis shows that our results are largely independent of such technical choices. We have tested this for two different shape functions and with two different technical parameters: α , which enters as a prefactor in front of the shape function of the regulator (45), and β , which parametrizes the endomorphism of the Laplacian (44). A sophisticated numerical analysis shows that qualitative properties are independent of technical parameters. As seen in Fig. 6, a large area is found in which critical exponents plateau within which we find a point of minimum sensitivity with respect to both parameters, see (63) and (64). Outside of this area, stronger fluctuations are present. We expect that in this regime, our approximation scheme breaks down. Most importantly, the critical exponent of the Goroff-Sagnotti coupling is always irrelevant, and the relevant critical exponent takes mostly values of $\theta_1 \approx 2.1 \pm 0.2$. These results also hold for different shape functions. In this work, we compared the Litim cutoff and its “square,” and found qualitative agreement. This analysis also showed the special role played by the type II cutoff ($\beta = 1$). For this choice, we find the strongest indications of near perturbativity. Except for γ_g , all γ functions that parametrize the RG kernel develop a zero close to $\beta = 1$, see Fig. 5. Figure 4 shows that this also applies to the fixed point value of the Goroff-Sagnotti coupling, which develops a zero close to $\beta = 1$ as well.

It will be of great interest to apply our findings to future endeavors, including even higher orders in the derivative expansion. This might be particularly feasible due to the near perturbativity of the fixed point, which potentially allows for more simplified yet accurate approximations. Similarly, the application of this scheme to gravity-matter systems is intriguing. First steps have been made in [46]. Observing the fixed point to persist the inclusion of matter would be a strong indication for a unitary and UV-complete theory of quantum gravity. By studying gravity-matter systems it would also be interesting to compute the scaling dimension of relational observables [78] at the higher orders of the derivative expansion. Last but not least, a critical open question is the extension of the MES to setups beyond the background field approximation. This can be explored within the fluctuation approach [16], within perturbation theory [12] or within a manifestly background independent approach such as [79].

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DATA AVAILABILITY

We provide a *Mathematica* notebook with all β and γ functions for arbitrary shape functions and technical parameters Supplemental Material [70]. No other data were created or analyzed in this study.

APPENDIX A: THE TWO-LOOP COUNTERTERM IN THE STANDARD SCHEME

With our setup, it is straightforward to try to reproduce the original FRG computation that included just the Einstein-Hilbert action amended by the Goroff-Sagnotti counterterm [39]. We found some inaccuracies in the β function for g_{C^3} (called σ in the reference).⁶ Borrowing a similar notation, we parametrize the β function as

$$\partial_t g_{C^3} = c_0 + (2 + c_1)g_{C^3} + c_2 g_{C^3}^2 + c_3 g_{C^3}^3. \quad (A1)$$

With the standard notation $\eta_N = \frac{\partial_t g - 2g}{g}$, the correct coefficients c_i read

$$\begin{aligned} c_0 &= \frac{1}{64\pi^2(1-2\lambda)} \left(\frac{2-\eta_N}{2(1-2\lambda)} - \frac{3+\eta_N}{(1-2\lambda)^3} - \frac{\eta_N}{756} \right), \\ c_1 &= \frac{g}{48\pi(1-2\lambda)^2} \left(45(6-\eta_N) + 59 \frac{8-\eta_N}{1-2\lambda} + \frac{63}{10} \frac{10+\eta_N}{(1-2\lambda)^2} \right), \\ c_2 &= \frac{g^2}{2(1-2\lambda)^3} \left(\frac{43}{5}(12-\eta_N) - \frac{9}{7} \frac{112-\eta_N}{1-2\lambda} \right), \\ c_3 &= 6\pi g^3 \frac{18-\eta_N}{(1-2\lambda)^4}. \end{aligned} \quad (A2)$$

Despite these corrections, the overall picture is extremely similar to the one reported in [39]. On the qualitative level, this is obvious due to the general arguments given in [39]

⁶There is also a typo in the expression for B_1 (the last term in the brackets should be -7 instead of -5), but the numerics were done with the correct expression for B_1 .

about the cubic structure of the above β function. Quantitatively, we again only find two real fixed points. The shifted Gaussian fixed point is at

$$g_N^* = \lambda^* = 0, \quad g_{C^3}^* = \frac{1}{64\pi^2}, \quad (A3)$$

with canonical critical exponents

$$\theta^{sGFP} \in \{2, -2, -2\}. \quad (A4)$$

The nontrivial, interacting fixed point now sits at

$$g_N^* = 0.707, \quad \lambda^* = 0.193, \quad g_{C^3}^* = 1.368 \times 10^{-4}, \quad (A5)$$

with critical exponents

$$\theta_{1,2} = 1.475 \pm 3.043i, \quad \theta_3 = -20.097. \quad (A6)$$

The irrelevant critical exponent is still rather large compared to the one that we find in the MES, independent of whether we do or do not take into account all operators up to six powers in derivatives.

APPENDIX B: THE TOPOLOGICAL TERM

We have already mentioned in the main text that the fourth order of the derivative expansion features the topological Gauss-Bonnet term,

$$\mathfrak{G} = R_{\rho\sigma\mu\nu}R^{\rho\sigma\mu\nu} - 4R_{\mu\nu}R^{\mu\nu} + R^2, \quad (B1)$$

whose spacetime integral is related to the Euler characteristic,

$$\chi = \frac{1}{32\pi^2} \int d^d x \sqrt{g} \mathfrak{G}. \quad (B2)$$

Due to its topological nature, $\mathfrak{G}$ is locally a total derivative. Hence, the Gauss-Bonnet coupling does not appear in any β functions of the theory. A consequence of this is that the β function of the Gauss-Bonnet ($\beta_{\mathfrak{G}}$) is independent of itself and receives only contributions from other couplings in the theory. Even though these other couplings can have a nontrivial fixed point, the Gauss-Bonnet coupling only stops running if its β function vanishes due to nontrivial cancelations between the contributions of other couplings. However, we are not aware of any arguments why such a cancelation would take place.

If $\beta_{\mathfrak{G}}$ does not vanish at the nontrivial fixed point, the Gauss-Bonnet coupling either runs to $+\infty$ or $-\infty$. In [35], it was argued that this leads to a suppression/enhancement of certain topologies in the path integral. Depending on the sign of $\beta_{\mathfrak{G}}$, the contribution of topologies with a given sign of the Euler characteristic is enhanced over the other.

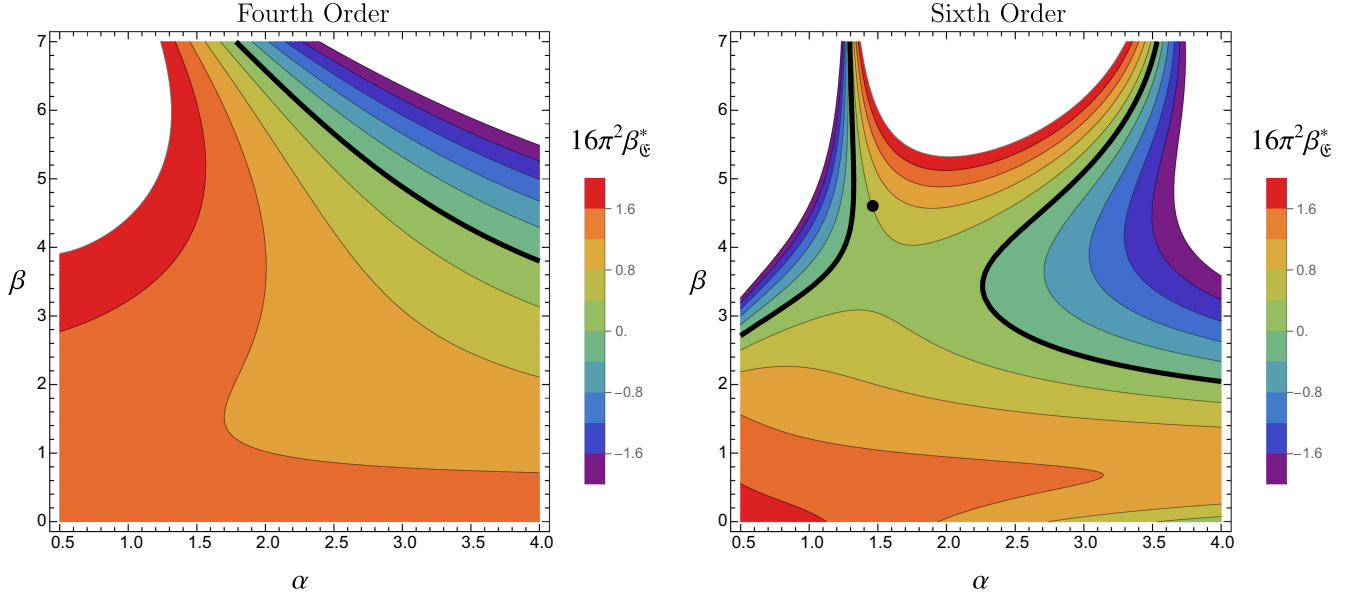


FIG. 7. Value for the β function of the topological invariant at the Reuter fixed point in dependence of α and β with the Litim cutoff. The left panel shows the fourth order and the right panel the sixth order in derivatives. White areas denote either instabilities of our approximation, or areas where $\beta_{\mathbb{G}}$ takes values outside the plotted domain.

We present our findings for $\beta_{\mathbb{G}}$ at the nontrivial fixed point in Fig. 7 in dependence on α and β . To understand the convergence of the result for $\beta_{\mathbb{G}}^*$, we show the results at fourth and sixth order in the derivative expansion. Comparing both orders, we conclude that the biggest stability for $\beta_{\mathbb{G}}^*$ is found for the type II cutoff, i.e., $\beta = 1$. This is in line with our findings in the main text, see, e.g., Fig. 5. In this regime, we find a positive value for $\beta_{\mathbb{G}}^*$. For $\alpha = \beta = 1$ we have

$$16\pi^2\beta_{\mathbb{G}}^* \approx 1.273. \quad (\text{B3})$$

This indicates that the Gauss-Bonnet coupling runs toward $+\infty$ in the UV. In the Euclidean path integral, this means that the weight of topological configurations with a positive Euler characteristic is suppressed, while topologies with negative Euler characteristics are enhanced. We also find parameter regions with vanishing or negative $\beta_{\mathbb{G}}^*$, however, comparing fourth and sixth orders, we find that our approximation is less stable in these ranges. Thus, our results suggest a positive $\beta_{\mathbb{G}}$ at the nontrivial fixed point, in line with previous findings [27,35,46].

These findings can also be compared to the value of $\beta_{\mathbb{G}}$ at the Gaussian fixed point. There, we rediscover the known result

$$16\pi^2\beta_{\mathbb{G}}^{\text{Gauss}} = \frac{53}{45} \approx 1.178. \quad (\text{B4})$$

Within the uncertainties of our computation, this is identical to (B3) at the nontrivial fixed point. The topological β function is virtually unchanged at the nontrivial fixed point, similar to the results at fourth order in derivatives [45].

APPENDIX C: GAUSSIAN FIXED POINTS AND ESSENTIAL SCHEMES

In Sec. II C we have explained how different universality classes can be characterized by the set of essential operators. Identifying the essential operators in any universality class is an involved task and generally requires the computation of quantum effects included on the right-hand side of (3). However, this task is significantly reduced at a Gaussian fixed point. Then, quantum effects can be neglected and the set of essential operators can be determined using the classical field equations at the Gaussian fixed point. This allows determining the set of essential operators in any universality class that contains a Gaussian fixed point.

In Sec. II D, we have derived the set of essential operators within the universality class of GR, i.e., at the Gaussian fixed point $G_N \rightarrow 0$. Up to cubic order in curvature, all higher order operators are inessential in this class, with the exception of the topological Gauss-Bonnet term, and the Goroff-Sagnotti term. To study fixed points in this universality class, we then used field redefinitions to fix all inessential operators to zero, such that only essential operators have to be taken into account in the ansatz for the effective average action. This is what we refer to as the MES.

Note that the set of essential operators can differ in other universality classes. In any universality class that contains essential operators in addition to those identified in the GR universality class, the MES cannot be utilized. This is because the MES assumes that certain operators can be fixed to zero by field redefinitions. If there are additional essential operators, this is no longer possible. This subtlety

is of particular importance for the application to higher derivative gravity.

As is well known, gravity not only features the Gaussian fixed point at $G_N \rightarrow 0$, but also contains higher derivative Gaussian fixed points. This is not a special feature of gravity, but can also be observed in simpler field theories, e.g., scalar field theories [80]. Crucially, in general such higher derivative Gaussian fixed points belong to different universality classes. To see this explicitly for the case of gravity, let us consider Gaussian fixed points of the form

$$\bar{\Gamma}_k = \int d^4x \sqrt{g} \frac{1}{\lambda} [\omega R \nabla^{2n} R + S_{\mu\nu} \nabla^{2n} S^{\mu\nu}], \quad (\text{C1})$$

with $\lambda \rightarrow 0$. For $n = 0$, this corresponds to the Gaussian fixed point of quadratic gravity, but it also contains

Gaussian fixed points of even higher derivatives, such as cubic gravity, which is obtained for $n = 1$. To determine the essential operators in the universality class given by the Gaussian fixed point of (C1), we make a general ansatz for possible field redefinitions,

$$\Psi_{\mu\nu}^g = \gamma_g g_{\mu\nu} + \gamma_R R g_{\mu\nu} + \gamma_S S_{\mu\nu} + \mathcal{O}(\partial^4). \quad (\text{C2})$$

Note that this is the same ansatz for the field redefinitions used in (26), truncated to second order in derivatives. Despite taking the same ansatz for the field redefinitions, the essential and inessential operators resulting from (C1) differ from those in the universality class of GR. Explicitly, we obtain for $n = 0$,

$$\begin{aligned} \Psi_{\mu\nu}^g \frac{\delta \bar{\Gamma}_k}{\delta g_{\mu\nu}} = & \int d^4x \sqrt{g} \frac{1}{\lambda} \left[\frac{1 - 12\omega}{6} \gamma_S R S_{\mu\nu} S^{\mu\nu} + 2\gamma_S S_{\mu}^{\nu} S_{\nu}^{\rho} S_{\rho}^{\mu} - 2\gamma_S S_{\mu\nu} S_{\rho\sigma} C^{\mu\rho\nu\sigma} \right. \\ & \left. + \frac{-4(1 + 12\omega)\gamma_R + (1 + 4\omega)\gamma_S}{8} R \nabla^2 R - \gamma_S S_{\mu\nu} \nabla^2 S^{\mu\nu} + \mathcal{O}(\partial^8) \right]. \end{aligned} \quad (\text{C3})$$

In $d = 4$, γ_g drops out since R^2 and S^2 are classically marginal. Thus, the right-hand side of (C3) starts at sixth order in derivatives. This means that field redefinitions in the universality class of quadratic gravity do not affect the quadratic curvature invariants. Thus, all quadratic curvature invariants become essential. This is a crucial difference from the GR universality class in which the quadratic curvature invariants are inessential, apart from the topological term. Hence, the MES, which was constructed with respect to the GR universality class, cannot be used to study quadratic gravity. Instead, the quadratic curvature invariants have to be taken into account as essential operators. Moreover, (C3) also shows that many more cubic curvature invariants are essential compared to the GR universality class. Only two cubic curvature invariants can be set to zero

using field redefinitions, meaning that all other cubic curvature invariants are essential. Thus, even though field redefinitions can still be used to construct essential schemes that can be applied to quadratic gravity, a lot fewer operators are inessential and can be set to zero in the EAA.

Similar statements apply to universality classes of even higher derivative Gaussian fixed points. As the order in derivatives is increased, fewer operators turn out to be inessential at the corresponding Gaussian fixed points.

Let us close this Section by remarking that we discussed here only universality classes that contain a Gaussian fixed point. There may very well be universality classes that do not contain any Gaussian fixed point. To classify essential operators in such classes, quantum effects of field redefinitions have to be accounted for explicitly, i.e., following (9).

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