Erratum: Noether's theorems and the energy-momentum tensor in quantum gauge theories [Phys. Rev. D 106, 125012 (2022)]

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(Received 9 January 2026; published 10 February 2026)

DOI: 10.1103/hcyz-ylrv

In this paper, an error was found in its treatment of spinor fields. In particular, it assumed that spinor fields transformed under a double cover of the group $\mathrm{GL}(4,\mathbb{R})$ under general coordinate transformations, which when restricted to the subgroup of Lorentz transformations would reproduce the usual Lorentz transformation formula for spinor fields. The formulas in Eq. (27) thereof were obtained in this manner.

However, a finite linear double cover of $\mathrm{GL}(4,\mathbb{R})$ does not exist. This has long been known, with the classic proofs given by Weyl [1] and Cartan [2]. For this reason, spinor fields are typically considered to transform like scalar fields under general coordinate transformations. The spinorial properties of spinor fields are instead accounted for within the tetrad or vierbein formalism, which is explained in-depth in Appendix J of Carroll [3], Chapter 11 of Hamilton [4], and Chapter 12 of Weinberg [5].

In light of this, Eq. (27) does not correctly describe the transformation of spinor fields under local translations. This leads to a fork in the road: either Eq. (27) must be modified to remove the $(\partial_\alpha \xi_\beta)\sigma^{\alpha\beta}$ terms, or the local translation must be supplemented with an additional internal transformation of the vierbein.

The former case (having spinor fields transform like scalar fields) is explored in Ref. [6], which additionally gives a clearer exposition of the procedure and an elementary proof that there is no finite linear double cover of $\mathrm{GL}(4,\mathbb{R})$. The end result is an energy-momentum tensor for quantum chromodynamics that is gauge invariant but asymmetric—and which coincides with the gauge-invariant kinetic energy-momentum tensor of Leader and Lorcé [7].

The latter case (supplementing local translations with an internal vierbein transformation) is explored by another recent work [8]. This combined transformation does produce the symmetric energy-momentum tensor, but this procedure differs from a pure spacetime translation.

Please see Ref. [6] for an alternate derivation in which the errors in this work are absent.

DATA AVAILABILITY

No data were created or analyzed in this study.

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