

Inflationary gravitational wave spectral shapes as a test for low-scale leptogenesis

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We study nonthermal resonant leptogenesis in a general setting, where a heavy Majoron ϕ decays to right-handed neutrinos (RHNs) whose further out-of-equilibrium decay generates the required lepton asymmetry. Domination of the energy budget of the Universe by the ϕ or the RHNs alters the evolution history of the primordial gravitational waves (GWs) of inflationary origin, which re-enter the horizon after inflation, modifying the spectral shape. The decays of ϕ and RHNs release entropy into the early Universe, while nearly degenerate RHNs facilitate low- and intermediate-scale leptogenesis. A characteristic damping of the GW spectrum resulting in kneelike features would provide evidence for low-scale nonthermal leptogenesis. We explore the parameter space for the lightest right-handed neutrino mass $M_1 \in [10^2, 10^{14}]$ GeV and washout parameter K that depends on the light-heavy neutrino Yukawa couplings λ , in the weak ($K < 1$) and strong ($K > 1$) washout regimes. The resulting novel features compatible with observed baryon asymmetry are detectable by future experiments, like LISA and Einstein Telescope. By estimating the signal-to-noise ratio for upcoming GW experiments, we investigate the effect of the Majoron mass M_ϕ and reheating temperature T_ϕ , which depends on the $\phi - N$ Yukawa couplings y_N .

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I. INTRODUCTION

The standard model (SM) of particle physics cannot explain tiny neutrino masses and the observed matter-antimatter asymmetry of the Universe. Neutrino oscillation experiments involving solar [1–6], atmospheric [7,8], and reactor [9–12] neutrinos provide evidence that neutrinos have mass and the flavor states mix due to the propagation of multiple mass eigenstates. This, however, only quantifies the squared mass differences of the neutrinos and not the absolute mass scales involved. On the other hand, the β -decay experiment KATRIN [13] gives a stringent direct limit on the absolute value of the neutrino mass scale, $m_\nu < 0.8$ eV.

On the cosmological frontiers, measurements of the cosmic microwave background (CMB) radiation by

Planck 2018 [14] and large-scale structure constrain the sum of all neutrino masses to $\sum_i m_{\nu_i} < 0.12$ eV [15,16]. On the other hand, the observed baryon asymmetry of the Universe (BAU) [15,17], often expressed in terms of the baryon-to-photon ratio is, according to the Planck data [15],

$$\eta_B^{\text{CMB}} = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.21 \pm 0.16) \times 10^{-10}. \quad (1)$$

This agrees with the values of abundances of light elements extracted from BBN (big bang nucleosynthesis) data [18].

Just augmenting the SM with two or more Majorana right-handed neutrinos (RHNs), one can explain the tiny neutrino masses, generated via the well-known type-I seesaw mechanism [19–22], while baryon asymmetry can be explained using the baryogenesis via leptogenesis mechanism [23], with everything tied together under one umbrella. However, for this mechanism of baryogenesis to be successful, the lightest RHN mass is required to exceed the Davidson-Ibarra bound, $M_1 \gtrsim 10^9$ GeV, assuming the RHN mass spectrum is hierarchical, and assuming no lepton flavor effects [24–28]. This destroys any hope of direct detection of the high-scale seesaw and leptogenesis, as they lie well beyond the energy reach of the current or foreseeable future laboratory experiments or astrophysical

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observations, making it impossible to test such new physics.

Nonetheless, certain indirect signatures of new physics, like lepton number violation processes through neutrinoless double beta decay [29] or CP violation in neutrino oscillation [30], are searched for in laboratories. On the other hand, theoretical constraints on the low-energy values of couplings, consistent with UV completion, such as $SO(10)$ grand unified theories (GUTs) [31–37], or with the requirement of electroweak vacuum (meta)stability in the early Universe [38,39], allow us to pose some bounds on the large parameter space involved in seesaw and leptogenesis.

The discovery of gravitational waves (GWs) from black hole mergers by LIGO and Virgo collaborations [40,41] and evidence for measurements of stochastic GW background by pulsar timing array (PTA) observations [42–48] have led to several new physics cases to detect GWs of primordial origins as well. Particularly in the context of baryogenesis via leptogenesis, cosmological pathways to probe such high-scale physics involve predictions of the CMB spectral indices [49] or gravitational waves from local cosmic strings [50–53], global cosmic strings [54], domain walls [55,56], nucleating and colliding vacuum bubbles [57,58], other topological defects [59], graviton bremsstrahlung [60], gravitational production of neutrinos [61], inflationary tensor perturbations propagating as GWs [62,63], primordial black holes [64–72], and exploring the oscillatory features of the primordial non-Gaussianity and curvature bispectrum and trispectrum [73,74].

Under this circumstance, we need more such probes of beyond standard model physics and form a synergy between cosmological observations and laboratory experiments involving seesaw and low-scale leptogenesis. In this paper, we take a first step in this direction and investigate a general scenario where a heavy scalar, called Majoron, denoted by ϕ , may decay to create an initial nonthermal abundance of RHNs, whose subsequent decay leads to leptogenesis, satisfying Eq. (1). This Majoron ϕ is different from inflaton. Inflaton itself is not the direct concern of this work, although we use the inflationary reheating temperature T_{RH} in our calculations. We will be concerned with gravitational waves that are generated due to quantum tensor fluctuations of the metric during inflation [75–79]. These GWs can act as a book-keeping of the expansion history of our Universe [61,80–88]. The detailed time evolution of the Hubble rate during the expansion determines various primordial gravitational wave frequencies that are redshifted to the present day, thus allowing us to investigate the standard and nonstandard thermal history of the Universe in the pre-BBN era [78,88–98]. In this context, since we are interested in low-scale seesaw and leptogenesis, we will study a scenario where ϕ dominates the energy budget of the Universe, which leads to interesting GW spectral shapes. On one hand, ϕ decay will facilitate low-scale leptogenesis and, on the other

hand, it is also responsible for the generation of GW spectral shapes that can be measured by various GW detectors. In particular, we consider an epoch of intermediate matter domination by ϕ particles affecting the postinflationary evolution of the Universe [66,99–101].

In this model, nonthermal leptogenesis is achieved via the Majoron decay to RHNs. The leptogenesis scale is brought down mainly by the resonance effect that enhances the CP parameter ϵ . RHNs are produced in an excess initial abundance after the Majoron has decayed, compared to the thermal case. Depending on the reheating temperature T_ϕ and washout parameter K , the final efficiency factor κ_f may get enhanced compared to the thermal case due to this excess initial abundance [102,103]. Unlike [62], here, the RHNs need not dominate the Universe since the Majoron is assumed to dominate the energy density of the Universe, which distorts inflationary GWs. Therefore, we do not need to stick to only the $K \ll 1$ condition of [62] for RHN domination. Note that RHNs can still dominate after ϕ decays, depending on the parameters.

The decays occur far away from thermal equilibrium and, therefore, it is bound to release a large amount of entropy dump, which in turn dilutes the energy density of primordial GWs that enter the horizon before the decay. We show that a two-step entropy injection scenario is possible if the RHNs produced are highly relativistic at production, which eventually become nonrelativistic and then decay at a lower temperature. In this case, two kneelike features occur in the GW spectrum due to the Majoron and RHN decays, which we show for the first time. We will show the capability of several upcoming GW detectors like LISA, DECIGO, BBO, etc., in probing the parameter space for such leptogenesis, particularly in regions where it is impossible to test via laboratory or astrophysical experiments.

In this work, a framework of nonthermal *resonant* leptogenesis from the decay chain $\phi \rightarrow N_{1,2} \rightarrow H + L$ includes entropy release from both ϕ and RHNs, spans RHN masses 10^2 – 10^{14} GeV and washout strengths $K \sim 10^{-20}$ – 10^4 , systematically classifies scenarios, and highlights kneelike damping and kinked GW spectra with signal-to-noise ratio (SNR) estimates across detectors. Unlike the above studies, we focus on a nearly degenerate RHN sector decoupled from the thermal bath, providing a general yet distinctive link between Majoron-induced resonant leptogenesis and observable GW features. A schematic diagram for the idea is presented in Fig. 7.¹

The paper is organized as follows. In Sec. II, we give a brief review of the thermal leptogenesis mechanism and the Davidson-Ibarra bound on heavy neutrino masses. We consider resonant leptogenesis to bring the leptogenesis scale down. In Sec. III, we discuss the Majoron model that

¹See Refs. [104–107] for ideas related to probing nonstandard expansion histories and leptogenesis through inflationary gravitational waves.

facilitates nonthermal leptogenesis in detail and the dynamics of Majoron and RHN decays. We then construct the relevant Boltzmann equations. In Sec. IV, we identify three classes of scenarios relevant to GW observations. We provide analytical approximations of the matter-radiation equilibrium temperature and dilution factor from entropy injection for these classifications. Then, in Sec. V, we discuss primordial gravitational waves from inflation with and without an intermediate matter domination epoch. We also introduce a transfer function to encapsulate a two-step entropy injection. This section is followed by our results in Sec. VI. Finally, we conclude this paper in Sec. VII. In Appendix A, we explicitly demonstrate a scenario where the washout factor can be tiny while keeping the CP asymmetry large. In Appendix B, we describe the Boltzmann equations in terms of comoving quantities.

II. REVIEW OF THERMAL LEPTOGENESIS

Before discussing the effect of Majoron field reheating on leptogenesis, let us first give a brief review of standard thermal leptogenesis. Initially proposed by Fukugita and Yanagida in 1986 [23,108], thermal leptogenesis hinges on the out-of-equilibrium decay of RHNs, which are thermally produced through scattering processes in the early Universe from the thermal bath of the primordial SM plasma. The Lagrangian is given by

$$\mathcal{L}_{\text{thermal}} = \mathcal{L}_{\text{SM}} + i\bar{N}\not{\partial}N - \left(\lambda \bar{L} \tilde{H} N + \frac{M_N}{2} \bar{N}^c N + \text{H.c.} \right), \quad (2)$$

where $\mathcal{L}_{\text{SM}}$ is the SM Lagrangian, N represents RHNs, $\tilde{H} = i\sigma_2 H^*$, where H is the SM Higgs doublet, and L is the SM leptonic doublet. Here, $M_N = \text{diag}[M_1, M_2, M_3]$ is the RHN Majorana mass matrix and λ encompasses the light-heavy Yukawa couplings. Due to the Majorana mass term of RHNs, after electroweak symmetry breaking, the familiar type-I seesaw mechanism [20] provides mass to the light neutrinos, $m_\nu = v^2 \lambda^T M_N^{-1} \lambda$, where $v \sim 174$ GeV is the vacuum expectation value (VEV) of the SM Higgs. On the other hand, RHNs of Majorana nature can decay into both leptons and antileptons through the Yukawa coupling matrix λ , thereby violating the lepton number,

$$N_1 \rightarrow H + L \quad \text{and} \quad N_1 \rightarrow H^\dagger + L^\dagger. \quad (3)$$

Typically, this scenario assumes a hierarchical heavy neutrino mass spectrum, where the masses of the heavier RHNs, M_2 and M_3 , are much greater than the lightest RHN mass, M_1 . This condition allows the lightest neutrino, N_1 , to generate the necessary lepton asymmetry when CP is violated in their decay, after washing out any asymmetry created by N_2 and N_3 . The CP asymmetry, denoted as ϵ_1 , is given by

$$\epsilon_1 = \frac{\Gamma_{N_1 \rightarrow HL} - \Gamma_{N_1 \rightarrow H^\dagger L^\dagger}}{\Gamma_{N_1}^{rf}}, \quad (4)$$

where $\Gamma_{N_1 \rightarrow Hl}$ is the decay width of the $N_1 \rightarrow H + l$ decay and

$$\Gamma_{N_1}^{rf} = \Gamma_{N_1 \rightarrow HL} + \Gamma_{N_1 \rightarrow H^\dagger L^\dagger} = \frac{|\lambda \lambda^\dagger|_{11}}{8\pi} M_1 \quad (5)$$

is the total decay width of N_1 in the rest frame of the neutrino. The lepton asymmetry can be rewritten as $B - L$ asymmetry denoted by the number density n_{B-L} , which is subsequently converted into the baryon asymmetry through $B + L$ conserving electroweak sphaleron processes [109–111]. When there is no initial $B - L$ asymmetry, the final baryon asymmetry, denoted as the baryon-to-photon number density ratio η_B , is expressed as [112]

$$\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = \frac{3}{4} \frac{a_{\text{sph}}}{f} \epsilon_1 \kappa_f \simeq 0.96 \times 10^{-2} \epsilon_1 \kappa_f, \quad (6)$$

where $a_{\text{sph}} = 28/79$ [109,110,113] is the efficiency of the sphaleron processes of converting $B - L$ asymmetry into baryon asymmetry and $f = 2387/86$ is the dilution factor due to production of photons since the onset of leptogenesis until recombination assuming an isentropic expansion of the Universe. Here, κ_f is the *efficiency parameter* [112], which is independent of ϵ_1 and parametrizes the efficiency of lepton asymmetry production depending on the initial N_1 abundance and washout. It can be calculated by solving the relevant Boltzmann equations governing the evolution of the N_1 abundance and $B - L$ asymmetry [114,115]. Its final value is normalized to unity if a thermal initial abundance of N_1 is assumed and there is no washout.

A. Conditions for departure from thermal equilibrium

One of the key conditions for successful baryogenesis, as outlined by Sakharov, is the departure from thermal equilibrium. In the context of leptogenesis, this condition is satisfied if the total decay rate of N_1 is smaller than the Hubble expansion rate of the Universe when the temperature is approximately equal to the mass of N_1 ,

$$\Gamma_{N_1}^{rf} < \mathcal{H}(T \sim M_1), \quad (7)$$

where $\mathcal{H}(T \sim M_1) \equiv \mathcal{H}(M_1) = \sqrt{\frac{8\pi^3 g_*}{90}} \frac{M_1^2}{M_{\text{pl}}}$ is the Hubble parameter with $M_{\text{pl}} = 1.22 \times 10^{19}$ GeV and g_* is the relativistic degrees of freedom, which is approximately equal to 106.75 at the high temperatures under discussion. It can be shown that the above condition is satisfied when the *effective neutrino mass*, defined by

$$\tilde{m}_1 = \frac{(m_D m_D^\dagger)_{11}}{M_1}, \quad (8)$$

where $m_D = \lambda v$ is the Dirac mass matrix with $v = \langle H \rangle \simeq 174$ GeV being the VEV of the standard model Higgs, is smaller than the *equilibrium neutrino mass*, defined as

$$m_* = \frac{16\pi^{5/2}}{3\sqrt{5}} g_*^{1/2} \frac{v^2}{M_{\text{pl}}} \simeq 1.1 \times 10^{-3} \text{ eV}. \quad (9)$$

The ratio is called the decay parameter or the washout parameter,

$$K \equiv \frac{\Gamma_{N_1}^{rf}}{\mathcal{H}(M_1)} = \frac{\tilde{m}_1}{m_*}, \quad (10)$$

which determines whether N_1 decays are in thermal equilibrium or not. The case $K < 1$ is called the *weak washout regime*, where Yukawa interactions of N_1 with light neutrinos via the SM Higgs are weak, hence washout of any initial asymmetry via reverse processes is small. Therefore, leptogenesis is unavoidable in this regime. However, in the so-called *strong washout regime* ($K > 1$), although Yukawa interactions are strong, leptogenesis can still occur and the final asymmetry depends on the interplay between decay, inverse decay, and scattering of RHNs [112].

Within the type-I seesaw framework, the Yukawa couplings λ can be expressed by the Casas-Ibarra parametrization [116],

$$\lambda = \frac{1}{v} M_N^{1/2} R m_\nu^{1/2} U_{\text{PMNS}}^\dagger, \quad (11)$$

where $m_\nu = \text{diag}(m_1, m_2, m_3)$, $M_N = \text{diag}(M_1, M_2, M_3)$, and R is a complex orthogonal matrix. The effective neutrino masses

$$\tilde{m}_i = \frac{|\lambda\lambda^\dagger|_{ii} v^2}{M_i} = \sum_j m_j |R_{ij}|^2 \quad (12)$$

correspond to the physical neutrino masses m_i if R is diagonal. This means, assuming normal ordering $m_1 < m_2 < m_3$ and $m_1 = 0$, for small off-diagonal elements of the R matrix, the effective neutrino mass $\tilde{m}_1 \rightarrow 0$, hence the washout parameter $K \rightarrow 0$. However, in this case, $\tilde{m}_2 \rightarrow 10^{-3}$ eV since $m_2 = \sqrt{m_1^2 + \Delta m_{\text{atm}}^2} \sim 8.6 \times 10^{-3}$ eV, implying the washout factor $K_2 = \tilde{m}_2/m_*$ for N_2 remains $\mathcal{O}(1)$. Similarly, K_3 also remains large. It can be shown that, in general, $\tilde{m}_1$ is larger than the lightest active neutrino mass [117],

$$\tilde{m}_1 > \text{Min}[m_1, m_2, m_3]. \quad (13)$$

It turns out the final efficiency parameter κ_f can be estimated analytically for both weak and strong washout

regimes [112]. In the strong washout regime, κ_f is estimated by

$$\kappa_f(K) \simeq (2 \pm 1) \times 10^{-2} \left(\frac{0.01 \text{ eV}}{m_* K} \right)^{1.1 \pm 0.1}, \quad (14)$$

while in the weak washout regime, κ_f depends on the initial abundance of N_1 . For thermal initial abundance, it is approximately 1 in the weak regime, while for vanishing initial abundance, it is estimated as

$$\kappa_f(K) \simeq \frac{9\pi^2 K^2}{64}, \quad (K < 1). \quad (15)$$

For thermal initial N_1 abundance, κ_f can be generically written for all K as

$$\kappa_f(K) \simeq \frac{2}{z_B(K)K} (1 - e^{\frac{1}{2}z_B(K)K}), \quad (16)$$

$$\text{where } z_B(K) \simeq 1 + \frac{1}{2} \ln \left(1 + \frac{\pi K^2}{1024} \left[\ln \left(\frac{3125\pi K^2}{1024} \right) \right]^5 \right). \quad (17)$$

In Fig. 2 we show κ_f as a function of washout parameter K for thermal (red solid line) and vanishing (blue dashed line) initial abundance of N_1 . Depending on the CP parameter ϵ_1 and K , the final baryon asymmetry can be calculated from Eq. (6).

B. CP asymmetry

The CP asymmetry ϵ_1 is determined by the interference between the tree level and one-loop diagrams [118,119] and can be calculated from $lH \rightarrow lH$ scattering processes [120]. The relevant Feynmann diagrams giving tree level, vertex, and self-energy contributions are shown in Fig. 1.

For RHN mass hierarchy $M_1 < M_2 < M_3$, the CP parameter is obtained as

$$|\epsilon_1| = \sum_{i \neq 1} \frac{3}{16\pi} \frac{M_1}{M_i} \frac{\text{Im}[(\lambda\lambda^\dagger)_{ii}]}{|\lambda\lambda^\dagger|_{11}} = \frac{3}{16\pi} \frac{M_1}{v^2} \frac{\sum_i m_i^2 \text{Im}[R_{1i}^2]}{\sum_j m_j |R_{1j}|^2} < \epsilon_1^{\text{max}}, \quad (18)$$

where the maximal asymmetry is given by [27,102,121–123]

$$\epsilon_1^{\text{max}} = \frac{3}{16\pi} \frac{M_1}{v^2} (m_3 - m_1) \simeq 10^{-6} \left(\frac{M_1}{10^{10} \text{ GeV}} \right) \left(\frac{m_3 - m_1}{0.05 \text{ eV}} \right). \quad (19)$$

Note that for small values of $\tilde{m}_1$ we require small values of $|R_{1i}|^2$, as seen from Eq. (12). However, that does not necessarily make $|\epsilon_1|$ tiny, as it depends on the ratio of off-diagonal elements of R , as seen in Eq. (18). It can be verified

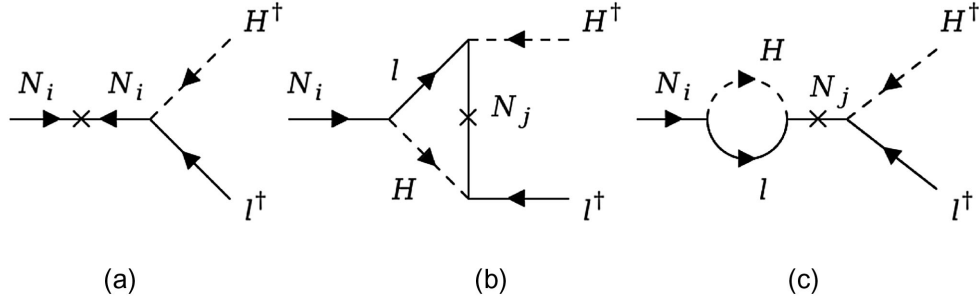


FIG. 1. Feynmann diagrams of the $N \rightarrow lH$ decay process at the (a) tree-level and (b, c) one-loop level, where (b) represents the vertex contribution and (c) represents the self-energy contribution.

that it is possible to achieve maximal CP asymmetry even if $\tilde{m}_1 \rightarrow 0$, implying $K \rightarrow 0$ (see Appendix A). For our analysis, we will consider the range $K \in [10^{-20}, 10^4]$ while keeping $K_2, K_3 > 1$.

C. Lower bounds on heavy neutrino masses

Putting the maximal CP asymmetry parameter $\epsilon_1^{\max}$ in Eq. (6) gives the maximal baryon asymmetry $\eta_B^{\max}$, which needs to be larger than the observed asymmetry η_B^{CMB} , i.e., $\eta_B^{\max} \geq \eta_B^{\text{CMB}}$. Then, one finds the Davidson-Ibarra bound on M_1 using Eq. (6) [27,112,124],

$$M_1 \geq 6.4 \times 10^8 \text{ GeV} \left(\frac{\eta_B^{\text{CMB}}}{6 \times 10^{-10}} \right) \left(\frac{0.05 \text{ eV}}{m_3 - m_1} \right) \kappa_f^{-1}. \quad (20)$$

Assuming the final efficiency factor $\kappa_f = 1$ gives the 3σ bound on the minimum reheating temperature T_{RH} at the end of inflation [122],

$$T_{\text{RH}} \gtrsim M_1 \gtrsim 4 \times 10^8 \text{ GeV}. \quad (21)$$

This stems from the condition that T_{RH} needs to be larger than the initial temperature of leptogenesis $T_i \sim M_1$ when the RHN decay process becomes out-of-equilibrium [see Eq. (7)] [125].

D. Resonant leptogenesis

Thermal leptogenesis operates at high temperatures, which in turn pushes the inflationary reheating temperature T_{RH} to a high scale, as shown in Eqs. (20) and (21). Such high scales involving RHN masses $M_i \gtrsim 10^9 \text{ GeV}$ are difficult to test in the lab. Also, a high neutrino mass conflicts with the electroweak naturalness condition, which requires RHN masses $M_i \lesssim 10^7 \text{ GeV}$ [105,126,127]. A lower reheating temperature from inflation T_{RH} is motivated in different inflationary scenarios, like in [128,129]. Even in some SUGRA models [130], the reheating temperature is found to be as low as 10^6 GeV , which contradicts a high T_{RH} , favoring low-scale leptogenesis. Motivated by these reasons, and for lab testability, the scale of leptogenesis can be brought down by several orders in the

so-called *resonant* regime, which focuses on nearly degenerate heavy neutrinos leading to pronounced CP violation in their decays. It can be shown that if the mass difference between the two lightest right-handed neutrinos is of the order of the decay widths of them, i.e., $M_2 - M_1 \sim \Gamma_{N_2}^{rf}$, the CP parameter ϵ_1 is enhanced via the self-energy contribution [123,131] in the loop level diagrams shown in Fig. 1.

The maximum CP asymmetry with resonant effect due to quasidegenerate right-handed neutrinos N_1 and N_2 can be written as [62,123]

$$\epsilon = \text{Min} \left[\epsilon_1^{\max} \frac{S_2 \cdot m_3 - m_1}{m_3 - m_1}, 1 \right], \quad \text{where } S_2 = \frac{M_2}{2\Gamma_{N_2}^{rf}} \quad (22)$$

and $M_2 - M_1 = \frac{\Gamma_{N_2}^{rf}}{2},$

where $\epsilon_1^{\max}$ is the maximal asymmetry in the nonresonant case given by Eq. (19). It was shown that within the resonant regime, the lowest RHN mass M_1 can be as low as 1 TeV [132]. In [133], this scale is shown to reach up to the electroweak symmetry breaking scale $\sim 100 \text{ GeV}$, below which sphaleron processes are suppressed and the lepton asymmetry cannot be effectively converted to baryon asymmetry. From here onwards, we will consider resonant leptogenesis with $M_1 \gtrsim 100 \text{ GeV}$ for our analysis. Note that in some alternate leptogenesis scenarios, e.g., via neutrino oscillations, BAU can be generated even for RHN mass values $\sim 100 \text{ MeV}$ [134].

III. $B-L$ SYMMETRY BREAKING AND NONTHERMAL LEPTOGENESIS

In this section, we construct a global $U(1)_{B-L}$ neutrino mass model inspired by [74] that includes all the essential components for nonthermal leptogenesis. In nonthermal leptogenesis, RHNs are produced from inflaton decay [28,102,103,135] or from a generic scalar decay [105,136]. Such a scenario can lead to an excess initial abundance of RHNs compared to their thermal abundance. As a result, the final efficiency factor κ_f can be enhanced by a factor of 100, which relaxes the Davidson-Ibarra

bound on the RHN mass given in Eq. (20). In this paper, the model extends the SM to include right-handed neutrinos N_i , where $i \in \{1, 2, 3\}$, and an additional scalar boson denoted as σ , with $B-L$ charge of $+2$. This complex scalar is responsible for generating the heavy Majorana masses of RHNs after spontaneously breaking the $U(1)_{B-L}$ symmetry. The Lagrangian is as follows:

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_{\text{SM}} + i\bar{N}\not{D}N + |\mathcal{D}_\mu\sigma|^2 \\ & - \left(\lambda\bar{L}\tilde{H}N + \frac{y_N}{2}\sigma\bar{N}^cN + \text{H.c.} \right) - V(H, \sigma), \\ V(H, \sigma) = & \lambda_H(|H|^2 - v^2)^2 + \lambda_\sigma(|\sigma|^2 - v_{B-L}^2)^2 \\ & + \lambda_{\sigma H}|\sigma|^2|H|^2, \end{aligned} \quad (23)$$

where L is the leptonic doublet, H denotes the Higgs doublet, and $\tilde{H} = i\sigma_2 H^*$, as in Eq. (2). The symbols λ , y_N , λ_H , $\lambda_{\sigma H}$, and λ_σ encompass the dimensionless coupling constants in the theory. The Yukawa coupling matrices λ and $y_N = \text{diag}[y_{N_i}]$ parametrize the strengths of the SM Higgs and the scalar couplings to the RHNs. Here, v_{B-L} is the $B-L$ symmetry breaking scale. These parameters, as we will see, play key roles in determining the GW spectral shapes, which we explore in detail. Around the $B-L$ breaking vacuum, σ can be parametrized as

$$\sigma = (v_{B-L} + \rho)e^{i\phi/v_{B-L}}, \quad (24)$$

where ϕ is the Nambu-Goldstone boson, called Majoron. After $B-L$ symmetry is broken, the N_i acquires the Majorana mass $M_i = y_{N_i}v_{B-L}$, where the extra phase is rotated away by redefining all fermions as $f \rightarrow f' = f e^{iq_f^{B-L}\phi/2v_{B-L}}$, where q_f^{B-L} is the $B-L$ charge of the fermion f . The radial component ρ acquires a mass $M_\rho = \sqrt{8\lambda_\sigma}v_{B-L}$ in the vanishing portal coupling limit, $\lambda_{\sigma H} = 0$. The Majoron ϕ couples to the RHNs through its kinetic term, which gives

$$\mathcal{L} \supset -\frac{\partial_\mu\phi}{2v_{B-L}}J_{B-L}^\mu = \frac{\phi}{2v_{B-L}}\partial_\mu J_{B-L}^\mu, \quad (25)$$

where the $B-L$ current is $J_{B-L}^\mu = \sum_f q_f^{B-L}\bar{f}\gamma^\mu f$. The second part of the equation is obtained by carrying out the integration by parts in the action and ignoring the surface term. This equation means that the Majoron couples to $B-L$ violating currents, which in the RHN sector is the Majorana mass term of the RHNs. Hence, we obtain the Majoron-RHN coupling,

$$\mathcal{L} \supset \frac{i\phi}{v_{B-L}}M_i\bar{N}_i^c N_i. \quad (26)$$

At this point, the Majoron is massless. To give it mass, we consider an asymptotically free hidden gauge sector with confinement scale $\Lambda < v_{B-L}$, and make it anomalous

under the $U(1)_{B-L}$ by adding fermions that are vectorlike under the SM but chiral under the $U(1)_{B-L}$ and charged under the hidden group. This group becomes nonperturbative at the scale Λ , and the Majoron couples to the new gauge bosons as follows:

$$\mathcal{L} \supset \frac{\phi}{v_{B-L}}\frac{g_X^2 n}{16\pi^2}G\tilde{G}, \quad (27)$$

where g_X is the new gauge coupling, n is the anomaly coefficient, and $\tilde{G}$ is the dual of the field strength tensor G of the new gauge group. This term breaks the shift symmetry of ϕ and generates a periodic potential of ϕ via instantons,

$$V(\phi) = \Lambda^4 \left(1 - \cos \frac{\phi}{v_{B-L}} \right). \quad (28)$$

The mass of the Majoron can be found by

$$M_\phi^2 = \left. \frac{\partial^2 V}{\partial \phi^2} \right|_{\phi=0} = \frac{\Lambda^4}{v_{B-L}^2}. \quad (29)$$

We will assume that $M_\rho \sim v_{B-L} \gg T_{\text{RH}}$, such that effects of ρ can be ignored. We will also assume the Majoron mass to always exceed twice the lightest RHN mass, $M_\phi > 2M_1$. Moreover, we consider a resonant leptogenesis scenario with $M_1 \sim M_2 \ll M_3$. The resonance effect makes sure that enough asymmetry is generated for all M_1 values starting from 100 GeV, to explain the BAU. For simplicity, we take $M_3 > M_\phi$. The decay width of ϕ to RH neutrinos is

$$\Gamma_{\phi \rightarrow N_i N_i} = \frac{M_\phi M_i^2}{16\pi v_{B-L}^2} \sqrt{1 - \frac{4M_i^2}{M_\phi^2}}, \quad (30)$$

where $i = 1, 2$ represents the two lightest RH neutrinos. The Majoron decay to the hidden sector through the coupling in Eq. (27) is loop suppressed or forbidden if the hidden sector particles are heavier than ϕ .

We consider the parameter space of the Majoron where it dominates the energy budget of the Universe before decaying, leading to a phase of matter-dominated pre-BBN era. The RHNs produced from Majoron decay further dominate the energy density. The decay of Majoron and RHNs injects considerable amounts of entropy into the Universe. This dampens the primordial gravitational waves of inflationary origin that re-enter the horizon during the early matter domination epochs. We show that such signature of low-scale leptogenesis in primordial gravitational waves can be detected in various GW experiments, such as BBO, DECIGO, μ -ARES, Einstein Telescope (ET), LISA, Cosmic Explorer (CE), etc.

A. Setup

After inflationary reheating ends at temperature T_{RH} , the Majorons are produced either (i) thermally via suppressed

scatterings, e.g., $lH \rightarrow \phi N$, or (ii) nonthermally from inflaton and ρ -decay or through misalignment (coherent oscillations) starting around $T_{\text{osc}} \sim \sqrt{M_\phi M_{\text{Pl}}}$. We make the coupling y_N small such that ϕ is long-lived, hence no production through N inverse decay is possible. The misalignment mechanism gives an energy density of the Majoron,

$$\rho_\phi(T_{\text{osc}}) \sim \frac{1}{2} M_\phi^2 \phi_{\text{init}}^2, \quad (31)$$

where ϕ_{init} is the initial value of ϕ at T_{osc} . We consider that ϕ had an abundance equal to the thermal abundance at temperature $T \sim M_\phi$ after the end of inflation, i.e., $\rho_\phi(M_\phi) = M_\phi n_\phi^{\text{eq}}$. We can adjust ϕ_{init} to achieve this result. In the thermal case, if scatterings freeze-out while the Majoron particles are still relativistic, we obtain the maximal ϕ abundance [100,101],

$$\left. \frac{n_\phi}{s} \right|_f = \frac{45\zeta(3)}{2\pi^4 g_{*S}}, \quad (32)$$

where subscript f denotes the time of freeze-out, n_ϕ is the number density of ϕ , and the entropy density is given by

$$s = \frac{2\pi^2}{45} g_{*S} T^3. \quad (33)$$

The effective number of degrees of freedom related to entropy is denoted by g_{*S} , which we take to be equal to $g_* = 106.75$ at high temperatures for simplicity in our analysis. In both cases, misalignment or thermal, ϕ is nonrelativistic and frozen-out at $T \sim M_\phi$, after which its energy density redshifts as matter and dominates over the radiation density starting from temperature [100,137]

$$T_{\text{dom}} \sim \frac{M_\phi}{g_*} \approx 0.01 M_\phi. \quad (34)$$

The ϕ -dominated phase starts at temperature T_{dom} and ends at decay temperature T_ϕ , calculated by considering the sudden decay assumption,

$$T_\phi = \left(\frac{90}{8\pi^3 g_*(T_\phi)} \right)^{\frac{1}{4}} \sqrt{\Gamma_\phi M_{\text{Pl}}}. \quad (35)$$

We always assume $T_\phi < T_{\text{dom}}$, such that we get a period of Majoron domination in the early Universe. The dynamics of ϕ , RHNs, and radiation can be obtained by solving the relevant Boltzmann equations, which we will describe in Sec. III B.

Now let us look at the aftermath of ϕ decay. We start with Eq. (30), the decay widths of ϕ to the lightest RHNs. As long as the mass of the Majoron $M_\phi > 2M_{1,2}$, it can decay to $N_{1,2}$. The Majorons we consider are essentially heavy

(10^5 – 10^{15} GeV) and nonrelativistic in the postinflationary scenario and their decay is in their rest frame. Hence, RHNs resulting from this can reasonably be monochromatic, where each RHN will approximately have energy $E_N \sim M_\phi/2$ at production. We are concerned only about the decay dynamics of N_1 [138]. If $M_\phi \gg M_1$, the RHNs are relativistic when they are produced at temperatures around T_ϕ . The energy of these relativistic RHNs, $E_N^{\text{rel}}(T)$, redshifts with the expansion of the Universe such that $E_N^{\text{rel}}(T)/T = E_N/T_\phi$. This gives

$$E_N^{\text{rel}}(z) \sim E_N \frac{T}{T_\phi} \sim \frac{M_\phi}{2T_\phi} \frac{M_1}{z}, \quad (36)$$

where $z = M_1/T$. The RHNs then become nonrelativistic when their energy becomes equivalent to their mass, i.e., at temperature [103]

$$T_{\text{NR}} \sim T_\phi \frac{2M_1}{M_\phi}. \quad (37)$$

Hence, the energy of each RHN after its production can be written as

$$E_N(z) \sim \begin{cases} \frac{M_\phi}{2} & \text{for } T > T_\phi \\ \frac{M_\phi}{2T_\phi} \frac{M_1}{z} & \text{for } T_\phi \geq T > T_{\text{NR}} \\ M_1 & \text{for } T \leq T_{\text{NR}}. \end{cases} \quad (38)$$

The RHNs further decay to lepton-Higgs pairs with decay widths [139]

$$\Gamma_{N_1} = \Gamma_{N_1}^{rf} \frac{M_1}{E_N(z)} \sim \mathcal{H}(M_1) K \frac{M_1}{E_N(z)}, \quad (39)$$

where $\mathcal{H}(M_1) = \sqrt{\frac{8\pi^3 g_*}{90} \frac{M_1^2}{M_{\text{Pl}}^2}}$ is the Hubble parameter with $M_{\text{Pl}} = 1.22 \times 10^{19}$ GeV. Note that the decay rates get suppressed by the Lorentz boost or time dilation factor $\gamma_N = E_N(z)/M_1$.

The decay temperatures of the RHNs are calculated assuming that the energy density of RHN is converted instantaneously into radiation [102],

$$T_{N_1} = \left(\frac{90}{8\pi^3 g_*(T_{N_1})} \right)^{\frac{1}{4}} \sqrt{\Gamma_{N_1}^{rf} M_{\text{Pl}}} = M_1 \sqrt{K}, \quad (40)$$

where $g_*(T)$ is the relativistic degrees of freedom at the corresponding temperature T , which is taken to be ~ 106.75 . The final baryon asymmetry from RHN decay can be written as

$$Y_B = \frac{n_B - n_{\bar{B}}}{s} = \frac{a_{\text{sph}} \epsilon}{s} n_N, \quad (41)$$

TABLE I. Definition of the key parameters used for our analysis in terms of couplings and masses of the Majoron and RHNs. Here, $v = 174$ GeV is the electroweak VEV, and λ is the Yukawa coupling of RHNs with active neutrinos via SM Higgs. Here, we also considered $y_{N_2} \sim y_{N_1}$.

Variable	Symbol	Definition
RHN decay width in the rest frame	$\Gamma_{N_1}^{rf}$	$\frac{ \lambda _{11}^2 M_1}{8\pi}$
Majoron decay width to RHNs	$\Gamma_{\phi \rightarrow N_i N_i}$	$\frac{y_{N_i}^2 M_\phi}{16\pi} \sqrt{1 - \frac{4M_i^2}{M_\phi^2}}$
Total decay width of Majoron	Γ_ϕ	$\Gamma_{\phi \rightarrow N_1 N_1} + \Gamma_{\phi \rightarrow N_2 N_2}$
Majoron decay temperature	T_ϕ	$\left(\frac{90}{8\pi^3 g_*(T_\phi)}\right)^{\frac{1}{4}} \sqrt{\Gamma_\phi M_{\text{pl}}}$ $\sim 0.05 y_{N_1} \left(\frac{100}{g_*(T_\phi)}\right)^{\frac{1}{4}} \sqrt{M_\phi M_{\text{pl}}}$
Washout parameter	K	$\frac{\bar{m}_1}{m_*} = \frac{\bar{m}_1}{1.1 \times 10^{-3} \text{ eV}} = \frac{ \lambda _{11}^2 v^2 / M_1}{1.1 \times 10^{-3} \text{ eV}}$
RHN decay temperature	T_{N_1}	$\left(\frac{90}{8\pi^3 g_*(T_\phi)}\right)^{\frac{1}{4}} \sqrt{\Gamma_{N_1}^{rf} M_{\text{pl}}} \sim M_1 \sqrt{K}$

where $s = g_*(2\pi^2/45)T^3$ is the entropy density and $n_B, n_{\bar{B}}$, and n_N are the number densities of baryons, antibaryons, and RHNs, respectively. Y_B is related to η_B given in Eq. (6) as $Y_B = (n_{\gamma 0}/s_0)\eta_B \simeq \eta_B/7.04$ [140], with s_0 and $n_{\gamma 0}$ being the current entropy and photon number densities, respectively. The factor $a_{\text{sph}} = 28/79$ is the efficiency of the sphaleron processes.

Using Eq. (30) in Eq. (35) and considering Eq. (34), we can write the condition $T_\phi < T_{\text{dom}}$ in terms of couplings as

$$y_{N_1} \lesssim 0.2 \sqrt{\frac{M_\phi}{M_{\text{pl}}}}. \quad (42)$$

In Table I, we summarize all the necessary conversion formulas for the parameters used in our analysis in terms of the couplings and masses of ϕ and RHNs. We consider M_ϕ, T_ϕ, M_1, K , and T_{RH} as input parameters for our analysis, and $v_{B-L}, y_{N_1}, |\lambda|_{11}^2, T_{N_1}, \Gamma_\phi$, and $\Gamma_{N_1}^{rf}$ are derived from them.

B. The Boltzmann equations

The relevant Boltzmann equations for the evolution of energy densities of the Majoron, RHNs, radiation, and $B-L$ asymmetry are

$$\begin{aligned}
\dot{\rho}_\phi &= -3\mathcal{H}\rho_\phi - \Gamma_\phi(\rho_\phi - \rho_\phi^{\text{eq}}), \\
\dot{\rho}_{N_1} &= -3\mathcal{H}(\rho_{N_1} + p_{N_1}) + \Gamma_{\phi \rightarrow N_1 N_1}(\rho_\phi - \rho_\phi^{\text{eq}}) \\
&\quad - (\Gamma_{N_1} + \Gamma_{\text{scatt}})(\rho_{N_1} - \rho_{N_1}^{\text{eq}}), \\
\dot{n}_{B-L} &= -3\mathcal{H}n_{B-L} - e\Gamma_{N_1}(n_{N_1} - n_{N_1}^{\text{eq}}) - (\Gamma_{ID} + \Gamma_{ID}^{\text{scatt}})n_{B-L}, \\
\dot{\rho}_R &= -4\mathcal{H}\rho_R + (\Gamma_{N_1} + \Gamma_{\text{scatt}})(\rho_{N_1} - \rho_{N_1}^{\text{eq}}), \quad (43)
\end{aligned}$$

where $\mathcal{H}$ is the Hubble parameter in cosmic time, the dot denotes the derivative with respect to cosmic time, and ρ_X^{eq} and n_X^{eq} represent the equilibrium energy and number density of the species X , respectively. Here, we consider the pressure of the RHN, $p_{N_1} = \rho_{N_1}/3$ for $T > T_{\text{NR}}$, where T_{NR} is defined in Eq. (37), and $p_{N_1} = 0$ for $T \leq T_{\text{NR}}$, signifying the relativistic and nonrelativistic evolution of the RHNs, respectively. We will ignore the inverse decay term $\Gamma_{\phi} \rho_\phi^{\text{eq}}$ since the Majoron decay temperature T_ϕ is much less than the Majoron mass M_ϕ , hence ϕ never comes into thermal equilibrium after it starts dominating. Γ_{ID} is the inverse decay rate. See Appendix B for details.

In the above equations, we have written terms for decays, inverse decays, and scattering processes of the nonthermally produced RHNs. Scatterings involving the Majoron $lH \leftrightarrow N\phi$ are thermally suppressed at $T \ll M_\phi$. For $T_{\text{RH}} \gg M_1$, thermal N_1 production via $\Delta L = 1$ scatterings (e.g., $N_1 l \leftrightarrow qt$) is subdominant to nonthermal production from ϕ decay, since ϕ dominates the energy density prior to decay. Scatterings of nonthermal N_1 with the plasma, described by $\Gamma_{\text{scatt}} = 2\Gamma_{Ss} + 4\Gamma_{St}$ and the associated washout $\Gamma_{ID}^{\text{scatt}} = \Gamma_{Ss} \frac{\rho_{N_1}}{\rho_{N_1}^{\text{eq}}} + 2\Gamma_{St}$ [103,112],

become relevant only when $\rho_{N_1} \sim \rho_R$, which happens at temperatures around T_ϕ and T_{N_1} . Here, Γ_{Ss} and Γ_{St} are the s - and t -channel scattering rates involving top quark and gauge bosons. We solved the above Boltzmann equations with and without the scattering terms and found that, within the parameter space of interest, the inclusion of these effects modifies the final baryon asymmetry mostly by $< 10\%$. Only at $K \sim 20$ is the change $\sim 50\%$. This means N_1 decays and inverse decays dominate the dynamics compared to scatterings. A full kinetic treatment (see, e.g., [141,142]) is beyond the scope of this paper, hence we neglect scatterings in the rest of the analysis.

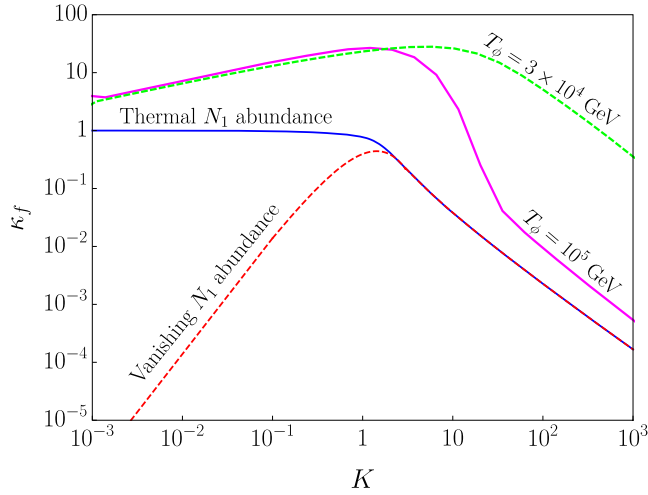


FIG. 2. Final efficiency factor κ_f as a function of washout parameter K in thermal and nonthermal leptogenesis. The solid blue and the dashed red curves represent a thermal and vanishing initial abundance of N_1 , respectively, in thermal leptogenesis. Here, we used Eqs. (16) and (17) for the blue curve, while for the red curve, we used Eq. (15) for $K \ll 1$ and interpolated it to the thermal case in the $K \gg 1$ regime. The solid magenta and dashed green lines are solutions to the Boltzmann equations in (43) for nonthermal leptogenesis with $M_1 = 10^5$ GeV, $M_\phi = 10^7$ GeV for $T_\phi = 10^5$ GeV, and 3×10^4 GeV, respectively.

We solve the Boltzmann equations from the time when the Majoron energy density overtakes the radiation energy density. Thus, the initial energy density of the Majoron is equal to the radiation density [100],

$$\rho_\phi(T_{\text{dom}}) = \rho_R(T_{\text{dom}}) = \frac{\pi^2}{30} g_*(T_{\text{dom}}) T_{\text{dom}}^4. \quad (44)$$

We take T_{dom} from Eq. (34). The initial density of the RHN is taken to be a free parameter smaller than $\rho_\phi(T_{\text{dom}})$, while initial $B - L$ asymmetry is taken to be negligible. Further details on the numerical solution of the Boltzmann equations and calculation of the final κ_f value are given in Appendix B. In Fig. 2, along with the thermal case, we show the enhancement in κ_f in the nonthermal leptogenesis scenario by solving the Boltzmann equations. The solid magenta line and dashed green line show the enhancement of the final efficiency factor as a function of the washout parameter for $M_1 = 10^5$ GeV, $M_\phi = 10^7$ GeV, and $T_\phi = 10^5, 3 \times 10^4$ GeV in the nonthermal scenario. One can compare with the thermal leptogenesis scenario (solid red and dashed blue lines). We notice that the value of washout parameter K at which κ_f is maximum increases with decreasing T_ϕ for the given parameters in Fig. 2, in contrast to results found in [102,103]. This is because, unlike [102,103], we have considered the relativistic phase of RHNs in the Boltzmann equations, and our RHN decay width given in Eq. (39) is not thermally averaged.

At this point, we can numerically calculate the final baryon asymmetry in our model. However, we are not interested in the exact amount of asymmetry generated, since our goal is studying the GW probe of leptogenesis. In the next section, we discuss the classification of scenarios based on numerical results and analytical approximations, relevant to GW sensitivity.

IV. CLASSIFICATION OF SCENARIOS

In this section, we identify three general nonthermal scenarios relevant for GW observations along the lines of [103]. We present analytical estimates of the final asymmetry, equilibrium temperature when matter domination ends, and the dilution factor from entropy injection for each of the four scenarios. In brief, the four scenarios are as follows:

- (1) Case (a): instantaneous RHN decay, portrayed in Fig. 3. We classify this scenario by requiring $\Gamma_N \gg \Gamma_\phi$, which implies $T_{N_1} \sqrt{\frac{2M_1}{M_\phi}} \gg T_\phi$ using Eq. (39). This means that the RHNs are mostly produced at temperatures around T_ϕ when the Majoron completely decays away and radiation starts dominating again. Those RHNs instantaneously decay to the SM bath because the lifetime of the highly relativistic RHNs, even after time dilation by Lorentz factor $\gamma_N = M_\phi/2M_1$, is still shorter than the lifetime of Majoron ϕ . In the nonrelativistic RHN limit, i.e., $\gamma_N \rightarrow 1$, the condition for instantaneous RHN decay becomes $T_{N_1} \gg T_\phi$. In both relativistic and nonrelativistic scenarios, RHNs do not have time to thermalize with the radiation bath even in the strong washout regime $K \gg 1$, thus the RHN decay process is strongly out-of-equilibrium. The evolutions of the energy densities of different matter and radiation are shown in Fig. 3 for both weak ($K \ll 1$) and strong ($K \gg 1$) washout regimes. We can see that for a strong washout, the RHN density is lower than the weak washout case due to a higher decay rate of the RHNs. The final asymmetry in both the weak and strong washout regions can be estimated analytically by finding the value of n_N at temperature T_ϕ by equating $\rho_\phi = \rho_R$ and $n_N = 2n_\phi$ as [103]

$$Y_B \equiv \frac{n_B - n_{\bar{B}}}{s} = \frac{3}{2} a_{\text{sph}} e \frac{T_\phi}{M_\phi}. \quad (45)$$

In this case, one can assume entropy is injected directly into the SM thermal bath at the time of Majoron decay since RHN decay is instantaneous.

- (2) Case (b): RHN radiation domination. For this scenario we need $\Gamma_N \ll \Gamma_\phi$, implying $T_{N_1} \sqrt{\frac{2M_1}{M_\phi}} \ll T_\phi$, which means the RHN decay is delayed after production. The RHNs dominate the energy budget as radiation after the Majoron ϕ decays. The RHNs

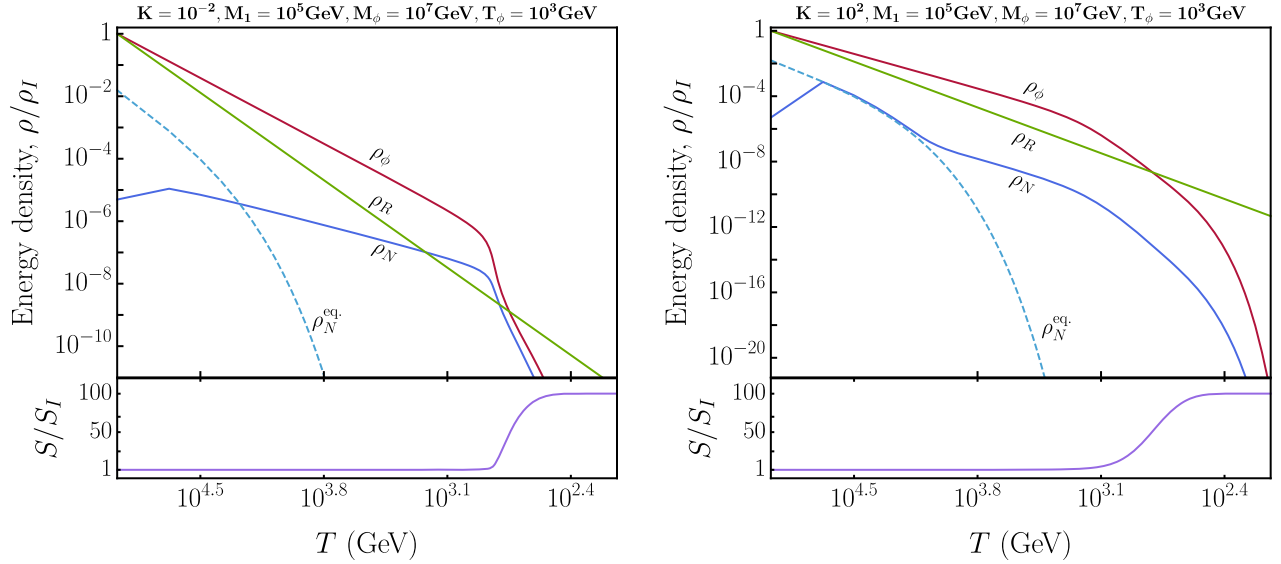


FIG. 3. The evolution of the energy densities of the Majoron ρ_ϕ , RHN $\rho_N = \rho_{N_1}$, and thermal radiation, ρ_R , with respect to the thermal radiation temperature. Here, we have taken $M_1 = 10^5$, $M_\phi = 10^7$, and $T_\phi = 10^3$ GeV for weak washout $K = 10^{-2}$ (Left) and strong washout $K = 10^2$ (Right). Both correspond to Case (a) mentioned at the beginning of Sec. IV. We can see that RHNs instantaneously decay after production. The bottom panels in each of the images show the total entropy of the Universe, $S = s_{\text{total}} a^3$, where $s_{\text{total}} = s + s_N$, with s corresponding to the thermal sector entropy density, defined in Eq. (33), and s_N being the entropy density of the decoupled RHN sector. Here, a is the scale factor.

will decay relativistically at temperature T_N^{rel} when their decay rate becomes equal to the Hubble rate, i.e., $\Gamma_N(T_N^{\text{rel}}) \sim \mathcal{H}(T_N^{\text{rel}})$. Considering $\mathcal{H}(T_N^{\text{rel}}) \sim (T_N^{\text{rel}})^2/M_{\text{pl}}$ and using Eqs. (36), (39), and (40),

we obtain the decay temperature of RHNs to be

$$T_N^{\text{rel}} \sim (T_{N_1}^2 T_{\text{NR}})^{1/3}. \quad (46)$$

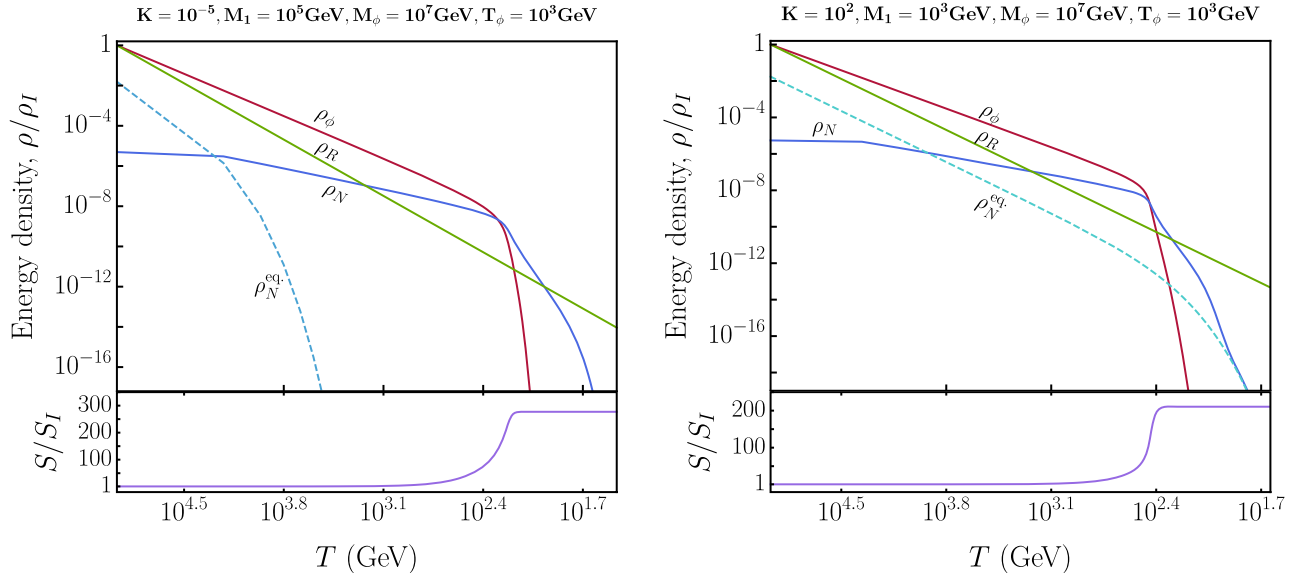


FIG. 4. Same as Fig. 3 but for the RHN radiation domination scenario, as described in Case (b) of Sec. IV, with $M_\phi = 10^7$ and $T_\phi = 10^3$ GeV assuming $M_1 = 10^5$ GeV, $K = 10^{-5}$ (Left), and $M_1 = 10^3$ GeV, $K = 100$ (Right). RHNs dominate as radiation starting from temperature T_ϕ until their decay at T_N^{rel} defined in Eq. (46). For weak washout, $K \ll 1$ (Left), RHNs never thermalize, however, the figure on the right shows RHN radiation domination for $K \gg 1$, where RHNs can thermalize later due to strong Yukawa couplings. The bottom panels show the entropy injection.

The necessary condition for RHN radiation domination is $T_{\text{NR}} < T_{N_1}$, which also implies $T_N^{\text{rel}} < T_{N_1}$, as shown in Fig. 4. The left plot shows the RHN radiation domination for $K \ll 1$, whereas the right plot shows RHN radiation domination for $K \gg 1$. In the weak washout regime ($K \ll 1$), one can calculate n_N at T_N^{rel} by considering $\rho_N = \rho_R$, where ρ_N can be written as $E_N^{\text{rel}} n_N$. Using Eq. (41), we find that the final asymmetry turns out to be the same as Case (a) above, given in Eq. (45). For $K \gg 1$, the final asymmetry can be calculated by solving the Boltzmann equations. Since the RHNs decay relativistically, there is negligible entropy injection at T_N^{rel} . However, significant entropy is injected into the RHN sector from the Majoron decay at around temperature T_ϕ .

- (3) Case (c): RHN matter domination. We again consider $\Gamma_N \ll \Gamma_\phi$ for this scenario, as in Case (b), i.e., $T_{N_1} \sqrt{\frac{2M_1}{M_\phi}} \ll T_\phi$. However, unlike Case (b), here we require $T_{N_1} \ll T_\phi$, more specifically we are interested in the scenario where $T_{N_1} < T_{\text{NR}} < T_\phi$, which translates to $\sqrt{K} < 2T_\phi/M_\phi$. Thus, strong washout $K > 1$ is not possible in this scenario in our model. Relativistic RHNs first dominate the energy budget of the Universe as radiation after the decay of the Majoron ϕ . Then, the RHNs become nonrelativistic at T_{NR} and dominate as matter until their decay at temperature T_{N_1} . Hence, there are two matter-dominated epochs separated by a radiation-dominated

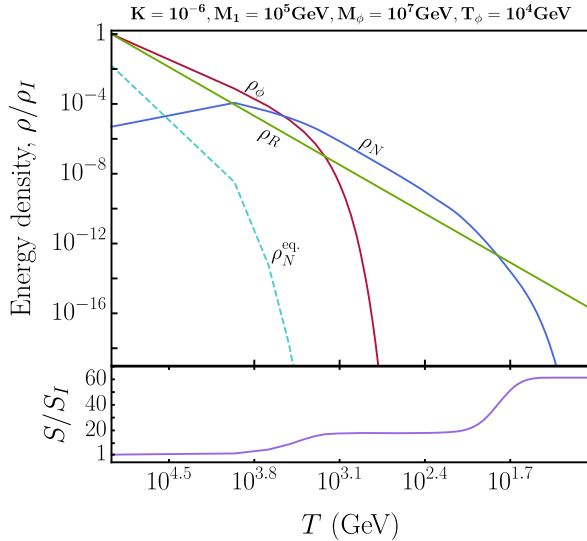


FIG. 5. Same as Fig. 3 but for the RHN matter domination scenario, described in Case (c) of Sec. IV, with $K = 10^{-6}$, $M_1 = 10^5$ GeV, $M_\phi = 10^7$ GeV, and $T_\phi = 10^4$ GeV. RHNs dominate as radiation first then as matter after temperature $T_{\text{NR}} \sim 200$ GeV, defined in Eq. (37). The bottom panels show the two stages of entropy injection.

epoch, and thus two stages of entropy injection in this case, first when the Majoron decays to relativistic RHNs at T_ϕ and second when the RHNs decay at T_{N_1} . In the first stage, entropy is injected into the decoupled RHN sector and not into the SM thermal bath. When the RHNs decay out-of-equilibrium, the final radiation domination starts. If $K \ll 1$, the RHNs never thermalize with the radiation bath, as seen in Fig. 5 (Left). The final BAU for $K \ll 1$ is estimated as [103]

$$Y_B = \frac{3}{4} a_{\text{sph}} \epsilon \frac{T_{N_1}}{M_1} \simeq 0.26 \sqrt{K} \epsilon. \quad (47)$$

In Fig. 6, we show the parameter space of the lightest RHN, spanned by M_1 and K , and the regions of successful nonthermal leptogenesis within it, i.e., $\eta_B \geq \eta_B^{\text{CMB}}$, for different values of M_ϕ and T_ϕ . The darker gray region at the bottom left is where sphaleron processes are turned off and leptogenesis is not efficient. The different colored regions correspond to different values of M_ϕ and T_ϕ . For this plot, we utilize the analytical expressions given in Eqs. (45) and (47) for Case (a), Case (c), and Case (b) $K < 1$ regions, and numerical solutions of the Boltzmann equations for the Case (b) $K > 1$ region.

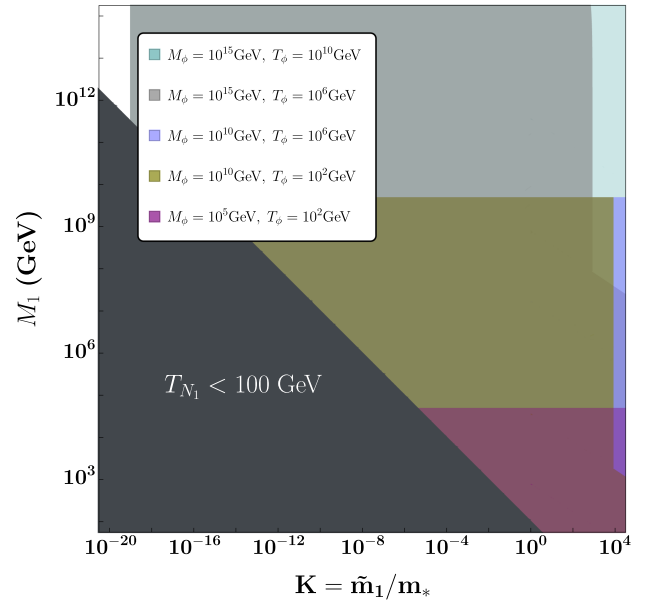


FIG. 6. Shaded regions of different colors correspond to successful nonthermal leptogenesis, $\eta_B \gtrsim \eta_B^{\text{CMB}}$, in the parameter space of the lightest RHN for given values of M_ϕ and T_ϕ . The dark gray region at the bottom left is where the RHN decay temperature is less than 100 GeV, hence sphaleron processes are suppressed and leptogenesis is not efficient.

A. Analytical estimates of equilibrium temperature

Equilibrium temperature is the temperature at which a matter-dominated epoch ends, kickstarting a radiation-dominated era. From the discussion above, we see that for Cases (a) and (b), the equilibrium temperature is around the decay temperatures T_ϕ . The exact equilibrium temperature is a little lower than the decay temperature since the decay is not instantaneous. However, for our considerations, T_ϕ is a reasonable estimate of the equilibrium temperature.

In Case (c), there are two matter-dominated epochs, hence entropy is injected into the Universe in two steps, as discussed earlier. First, the Majoron decaying to relativistic RHNs increases the total entropy of the Universe at around T_ϕ . The second stage of entropy injection is when RHNs decay at temperature T_{N_1} . There is a radiation-dominated epoch between the two matter-dominated epochs. Hence, we take the two equilibrium temperatures to be T_ϕ and T_{N_1} . Thus, we approximate the equilibrium temperatures for GW analysis as

$$T_{\text{eq}} \equiv T_{\text{dec}} \sim \begin{cases} T_\phi & \text{for Cases (a) and (b)} \\ T_\phi \text{ \& } T_{N_1} & \text{for Case (c).} \end{cases} \quad (48)$$

B. Dilution factor from entropy injection

Both Majoron and nonrelativistic RHN decay inject entropy into the Universe if the decay products are relativistic. The general formula for the dimensionless dilution factor from the entropy injection by a dominating species χ decaying to relativistic particles is given by [62,99]

$$\Delta = \frac{s(T_{\text{after}})a^3(T_{\text{after}})}{s(T_{\text{before}})a^3(T_{\text{before}})} = \left(1 + 2.95 \left(\frac{2\pi^2 \langle g_*(T) \rangle}{45} \right)^{\frac{1}{3}} \left(\frac{n_\chi}{s} M_\chi \right)^{\frac{4}{3}} \frac{1}{(M_{\text{pl}} \Gamma_\chi)^{\frac{2}{3}}} \right)^{\frac{3}{4}}, \quad (49)$$

where s is the entropy density, $T_{\text{before/after}}$ represents temperature before/after the decay process, and $\frac{n_\chi}{s}$ is the initial abundance of the particle χ .

For our analysis, we estimate the dilution factor using the formula in (49) for the four classifications we made:

Case (a): In this case, the RHNs decay out-of-equilibrium instantaneously to relativistic particles at T_ϕ . Therefore, we can assume that it is the Majoron that is directly decaying to the radiation bath releasing entropy. The dilution factor Δ can be given by Eq. (49) with χ replaced by ϕ . The initial abundance is given by Eq. (32). In the large entropy injection approximation $\Delta \gg 1$, using

Eq. (35) we can write the dilution factor to be

$$\Delta \simeq 3.7 \times 10^9 \left(\frac{M_\phi}{10^{15} \text{ GeV}} \right) \left(\frac{\text{TeV}}{T_\phi} \right). \quad (50)$$

Case (b): RHNs dominate the energy budget of the Universe as radiation after their production. Eventually, the RHNs decay relativistically at $T_N^{\text{rel}} < T_\phi$. Hence, the RHN decay injects negligible entropy into the Universe. The only stage of substantial entropy injection is when the Majoron ϕ decays at T_ϕ . The dilution factor is again estimated by Eq. (50).

Case (c): RHNs dominate the energy budget of the Universe as soon as they are produced, first as radiation, then as matter. The energy density stored in ϕ species is first transferred to the RHNs and then dumped into the radiation bath when RHNs decay at T_{N_1} . Hence, this scenario leads to two matter-dominated epochs separated by a radiation-dominated epoch.

The dilution factor from ϕ decay is again given by Eq. (50). The dilution factor from RHN decay can be estimated using Eq. (49) considering the RHN decay width Γ_N^{rf} and initial abundance after ϕ decay as

$$\frac{n_N}{s_{\text{tot}}^{\text{after}}} = \frac{2n_\phi}{\Delta_\phi s_{\text{tot}}^{\text{before}}} = \frac{2}{\Delta_\phi} \frac{n_\phi}{s} = \frac{45 \zeta(3)}{\pi^4} \frac{1}{g_{*S}} \frac{1}{\Delta_\phi}, \quad (51)$$

where s_{tot} is the total entropy of the Universe, including entropy of the decoupled RHN sector, and superscripts “before” and “after” refer to before and after ϕ decay. Total entropy after ϕ decay, $s_{\text{tot}}^{\text{after}}$, can be written as $s_{\text{tot}}^{\text{before}} \Delta_\phi$, where Δ_ϕ is given by Eq. (50). For the last equality, we use Eq. (32) since before the Majoron decays, the total entropy of the Universe is simply the entropy of the thermal bath. Note that considering only the lightest RHN N_1 is sufficient here. Again in the large entropy injection limit $\Delta_N \gg 1$, one can write the dilution factor from RHN decay as

$$\Delta_N \simeq 2000 \left(\frac{10^{-12}}{K} \right)^{1/2} \left(\frac{10^8 \text{ GeV}}{M_\phi} \right) \left(\frac{T_\phi}{100 \text{ TeV}} \right). \quad (52)$$

V. GRAVITATIONAL WAVES OF INFLATIONARY ORIGIN

A. Primordial spectrum in standard and nonstandard cosmology

Primordial inflation, the early exponential expansion of the Universe, is believed to have transitioned into an epoch known as reheating. This transition resulted in the creation of a high-energy plasma of radiation, following the dynamics of the standard model of particle physics. During inflation, GWs from quantum tensor fluctuations of the metric were generated. Initially, while outside the cosmological horizon, these GWs exhibited constant amplitudes.

We show the key events and temperatures for all the above cases in schematic diagrams in Fig. 7. These diagrams help visualize the differences in the timelines corresponding to Cases (a), (b) and (c). We show each timeline starting from the inflationary reheating temperature T_{RH} until today. All the relevant events take place above the BBN temperature T_{BBN} . However, as they re-entered the horizon in the radiation-dominated era, their amplitudes underwent damping. The propagation of GW modes that re-enter the horizon is controlled by a transfer function.

The current power spectrum of GWs, denoted as $\Omega_{\text{GW}}(k)$, can be expressed as a function of the wave number, $k = 2\pi f$, where f is the frequency [62],

$$\Omega_{\text{GW}}(k) = \frac{1}{12} \left(\frac{k}{a_0 \mathcal{H}_0} \right)^2 T_T^2(k) P_T^{\text{prim}}(k), \quad (53)$$

with $a_0 = 1$ representing the present-day scale factor and $\mathcal{H}_0 \approx 2.2 \times 10^{-4} \text{ Mpc}^{-1}$ being the current Hubble expansion rate [105]. $P_T^{\text{prim}}(k)$ is the primordial tensor power spectrum from inflation, and $T_T^2(k)$ is the transfer function that characterizes the propagation of GWs in the background of a Friedmann-Lemaître-Robertson-Walker universe after the temperature at the time of horizon re-entry, T_{in} , given by [91]

$$T_{\text{in}} = 5.8 \times 10^6 \text{ GeV} \left(\frac{106.75}{g_*(T_{\text{in}})} \right)^{1/6} \left(\frac{k}{10^{14} \text{ Mpc}^{-1}} \right). \quad (54)$$

The conventional parametrization for the inflationary tensor power spectrum, $P_T^{\text{prim}}(k)$, typically involves expressing its amplitude (A_T) and spectral index (n_T) with respect to the pivot scale $k_* = 0.05 \text{ Mpc}^{-1}$, as outlined in the Planck 2018 analysis [143],

$$P_T^{\text{prim}}(k) = A_T(k_*) \left(\frac{k}{k_*} \right)^{n_T}, \quad (55)$$

where the amplitude $A_T(k_*)$ is related to the scalar power spectrum via the tensor-to-scalar-ratio $r \leq 0.035$ [144],

$$A_T(k_*) = A_S(k_*) r, \quad A_S(k_*) = 2.0989 \times 10^{-9}, \quad (56)$$

where $A_S(k_*)$ is the scalar power spectrum [143]. For our analysis, we fix the value of $r = 0.035$. The spectral index n_T plays a critical role in characterizing the inflationary tensor power spectrum. While standard single-field slow-roll inflation predicts a red-tilted spectrum, where n_T satisfies the consistency relation $n_T = -r/8$ [145], other scenarios, including blue-tilted spectra ($n_T > 0$), are well motivated in various cosmological models [146–153].

Numerical computation and analytical fitting give the transfer function $T_T^2(k)$ as

$$T_T^2(k) = \Omega_m^2 \left(\frac{g_*(T_{\text{in}})}{g_*^0} \right) \left(\frac{g_{*S}^0}{g_{*S}(T_{\text{in}})} \right)^{4/3} \left(\frac{3j_1(z_k)}{z_k} \right)^2 F(k). \quad (57)$$

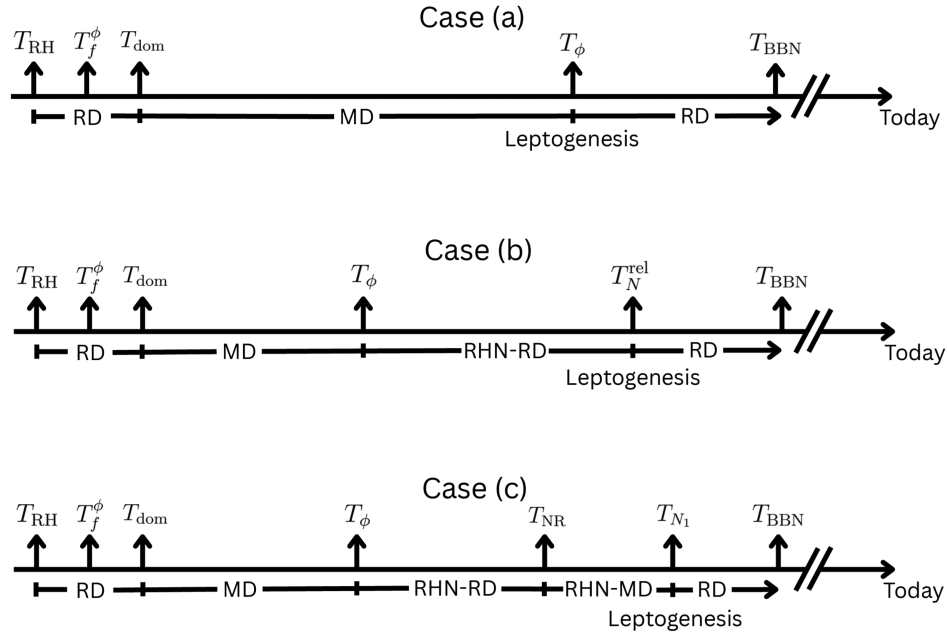


FIG. 7. Schematics of key events and temperatures for Cases (a), (b), and (c). Inflation ends at reheating temperature T_{RH} , after which ϕ remains in thermal equilibrium until its freeze-out temperature T_f^ϕ . Then, ϕ becomes nonrelativistic and we start the first matter domination epoch at $T_{\text{dom}} \sim 1\% M_\phi$. This matter domination ends when ϕ decays at T_ϕ . For Case (a), RHNs also instantaneously decay at T_ϕ and leptogenesis occurs before the BBN temperature T_{BBN} . In Case (b), RHNs dominate as radiation from T_ϕ until their decay at T_N^{rel} . In Case (c), RHNs dominate as radiation until T_{NR} , at which point they become nonrelativistic and start the second matter domination epoch. This epoch ends at RHN decay temperature T_{N_I} when leptogenesis occurs.

Here, $g_*^0 = 3.36$ and $g_{*S}^0 = 3.91$ are the present-day values of the effective degrees of freedom, and $\Omega_m = 0.31$ [105] is the total matter density. The Bessel function $j_1(z_k)$ describes the damping of GW amplitudes after horizon re-entry with $z_k \equiv k\tau_0$ and $\tau_0 = 2/\mathcal{H}_0$. In the limit $z_k \gg 1$, relevant for the frequencies of interest, we can replace $j_1(z_k)$ with $1/(\sqrt{2}z_k)$. Finally, the fitting function $F(k)$ in standard cosmology is given by [154]

$$F(k)_{\text{standard}} = T_1^2\left(\frac{k}{k_{\text{eq}}}\right) T_2^2\left(\frac{k}{k_{\text{RH}}}\right). \quad (58)$$

If there was an intermediate matter domination epoch, the fitting function is modified [154],

$$F(k)_{\text{IMD}} = T_1^2\left(\frac{k}{k_{\text{eq}}}\right) T_2^2\left(\frac{k}{k_{\text{dec}}}\right) T_3^2\left(\frac{k}{k_{\text{dec S}}}\right) T_2^2\left(\frac{k}{k_{\text{RHS}}}\right). \quad (59)$$

Here, we introduce

$$k_{\text{eq}} = 7.1 \times 10^{-2} \text{ Mpc}^{-1} \cdot \Omega_m h^2, \quad (60)$$

$$k_{\text{dec}} = 1.7 \times 10^{14} \text{ Mpc}^{-1} \left(\frac{g_{*S}(T_{\text{dec}})}{g_{*S}^0} \right)^{1/6} \left(\frac{T_{\text{dec}}}{10^7 \text{ GeV}} \right), \quad (61)$$

$$k_{\text{RH}} = 1.7 \times 10^{14} \text{ Mpc}^{-1} \left(\frac{g_{*S}(T_{\text{RH}})}{g_{*S}^0} \right)^{1/6} \left(\frac{T_{\text{RH}}}{10^7 \text{ GeV}} \right), \quad (62)$$

$$k_{\text{dec S}} = k_{\text{dec}} \Delta^{2/3}, \quad (63)$$

$$k_{\text{RHS}} = k_{\text{RH}} \Delta^{-1/3}, \quad (64)$$

where T_{dec} is defined by (48) and we set $h = 0.7$. The entropy dilution factor Δ is the dilution factor, and the fit functions read

$$T_1^2(x) = 1 + 1.57x + 3.42x^2, \quad (65)$$

$$T_2^2(x) = (1 - 0.22x^{3/2} + 0.65x^2)^{-1}, \quad (66)$$

$$T_3^2(x) = 1 + 0.59x + 0.65x^2. \quad (67)$$

At this point we can plot the spectrum $\Omega_{\text{GW}} h^2$ as a function of current frequency, using Eq. (53). We discuss the features in these plots accounting for ϕ and N domination in Sec. VI A. The suppression of the GW spectrum happens above a frequency f_{sup} , given by

$$f_{\text{sup}} \simeq 2.7 \times 10^{-8} \text{ Hz} \left(\frac{T_{\text{dec}}}{\text{GeV}} \right). \quad (68)$$

The suppression factor is given by [80]

$$R_{\text{sup}} = \frac{\Omega_{\text{GW}}^{\text{IMD}}}{\Omega_{\text{GW}}^{\text{standard}}} \simeq \frac{1}{\Delta^{4/3}}. \quad (69)$$

B. Two-step entropy injection

Our Case (c) discussed in Sec. IV has two stages of entropy injection. We could not find any previous study of the transfer function to fit two stages of entropy injection. Therefore, extrapolating on results obtained from numerical fitting in [154], we propose a modified transfer function to incorporate the two stages by assuming the form of the fitting function analogous to Eq. (59) to be

$$F(k)_{\text{IMD}}^{2\text{-step}} = T_1^2\left(\frac{k}{k_{\text{eq}}}\right) T_2^2\left(\frac{k}{k_{\text{dec}}^\phi}\right) T_3^2\left(\frac{k}{k_{\text{dec S}}^\phi}\right) T_2^2\left(\frac{k}{k_{\text{dec}}^N}\right) \times T_3^2\left(\frac{k}{k_{\text{dec S}}^N}\right) T_2^2\left(\frac{k}{k_{\text{RHS}}^{2\text{-step}}}\right), \quad (70)$$

where k_{dec}^ϕ and k_{dec}^N are related to the decay temperatures T_ϕ and T_{N_1} , respectively, and $k_{\text{dec S}}^\phi$ and $k_{\text{dec S}}^N$ are related to the domination temperatures of ϕ and N_1 , respectively. Note that in Eq. (70), we use the function $T_2^2(x)$ for the energy scales corresponding to decay temperatures and $T_3^2(x)$ for energy scales corresponding to domination temperatures, as suggested in [154]. We define the energy scales analogous to Eqs. (61)–(64) as

$$k_{\text{dec}}^\phi = 1.7 \times 10^{14} \text{ Mpc}^{-1} \left(\frac{g_{*S}(T_\phi)}{g_{*S}^0} \right)^{1/6} \left(\frac{T_\phi}{10^7 \text{ GeV}} \right), \quad (71)$$

$$k_{\text{dec}}^N = 1.7 \times 10^{14} \text{ Mpc}^{-1} \left(\frac{g_{*S}(T_{N_1})}{g_{*S}^0} \right)^{1/6} \left(\frac{T_{N_1}}{10^7 \text{ GeV}} \right), \quad (72)$$

$$k_{\text{dec S}}^\phi = k_{\text{dec}}^\phi \Delta_\phi^{2/3}, \quad (73)$$

$$k_{\text{dec S}}^N = k_{\text{dec}}^N \Delta_N^{2/3}, \quad (74)$$

$$k_{\text{RHS}}^{2\text{-step}} = k_{\text{RH}} (\Delta_\phi \Delta_N)^{-1/3}, \quad (75)$$

where Δ_ϕ and Δ_N are the dilution factors from ϕ and RHN decays, respectively. Here, in Eq. (75), we utilized the fact that the total dilution factor after ϕ and RHN decay is the product of the individual dilution factors Δ_ϕ and Δ_N corresponding to the two stages of entropy injection. A proper way to obtain the two-step transfer function requires numerical fitting, which we left as a future direction. In this case, we have two suppression frequencies corresponding to Majoron and RHN decays, analogous to Eq. (68),

$$f_{\text{sup}}^\phi \simeq 2.7 \times 10^{-8} \text{ Hz} \left(\frac{T_\phi}{\text{GeV}} \right),$$

$$f_{\text{sup}}^N \simeq 2.7 \times 10^{-8} \text{ Hz} \left(\frac{T_{N_1}}{\text{GeV}} \right). \quad (76)$$

Similar to Eq. (69), the suppression factors corresponding to the first (ϕ decay) and the second (N decay) steps of

entropy injection are given by

$$R_{\text{sup}}^{\text{1st}} \simeq \frac{1}{(\Delta_\phi \Delta_N)^{4/3}}, \quad R_{\text{sup}}^{\text{2nd}} \simeq \frac{1}{\Delta_N^{4/3}}. \quad (77)$$

C. Signal-to-noise ratio

Interferometers are instruments that measure displacements in terms of dimensionless strain noise, denoted as $h_{\text{GW}}(f)$. This strain noise is related to the amplitude of GWs and can be converted into an energy density, expressed by the formula

$$\Omega_{\text{exp}}(f)h^2 = \frac{2\pi^2 f^2}{3\mathcal{H}_0^2} h_{\text{GW}}(f)^2 h^2. \quad (78)$$

Here, $\mathcal{H}_0$ represents the present-day Hubble rate ($\mathcal{H}_0 = h \times 100 \frac{\text{km/s}}{\text{Mpc}}$). To assess the likelihood of detecting the primordial GW background, we calculate the SNR using the experimental sensitivity $\Omega_{\text{exp}}(f)h^2$, either given or projected. The SNR is determined by the following formula:

$$\text{SNR} \equiv \sqrt{\tau \int_{f_{\text{min}}}^{f_{\text{max}}} df \left(\frac{\Omega_{\text{GW}}(f)h^2}{\Omega_{\text{exp}}(f)h^2} \right)^2}. \quad (79)$$

In this analysis, we use $h = 0.7$ and an observation time of $\tau = 4$ years. A detection threshold of $\text{SNR} \geq 10$ is applied. It is important to note that this formula for SNR calculation is derived under the weak signal approximation, where the GW signal is much smaller than the instrumental noise [155]. While it may overestimate the true SNR for strong signals, we adopt it here for both weak and strong GW signals for simplicity.

In the spectrum plots, we display the sensitivity curves for different ongoing and future GW experiments. They can be grouped as follows:

- (i) Ground-based interferometers: these detectors, such as LIGO/VIRGO [40,41,156–159], aLIGO/aVIRGO [160–162], AION [163–166], ET [167,168], and CE [169,170], use interferometric techniques on the Earth’s surface to detect gravitational waves.
- (ii) Space-based interferometers: space-based detectors like LISA [171], BBO [172–174], DECIGO, U-DECIGO [175,176], AEDGE [163,177], and μ -ARES [178] are designed to detect gravitational waves from space, offering different advantages over their ground-based counterparts.
- (iii) Recasts of star surveys: surveys such as GAIA/THEIA [179] utilize astrometric data from stars to indirectly infer the presence of gravitational waves.
- (iv) Pulsar timing arrays: PTA experiments like SKA [42–44], EPTA [45,46], and NANOGrav [180–182]

use precise timing measurements of pulsars to detect gravitational wave signatures.

- (v) CMB polarization: experiments like Planck 2018 [14,143], BICEP 2/Keck [144], as computed by [183], and LiteBIRD [184] focus on detecting gravitational wave imprints in the polarization of the cosmic microwave background radiation.
 - (vi) CMB spectral distortions: projects like PIXIE, Super-PIXIE [185], and VOYAGER2050 [186] aim to detect gravitational wave signatures through spectral distortions in the cosmic microwave background.
- The blue violin lines in the spectrum plots are the most recent data from NANOGrav.

D. Bounds from BBN and CMB on dark radiation

The gravitational wave energy density should be smaller than the bound on dark radiation parametrized as the number of additional neutrinos ΔN_{eff} [187,188],

$$\int_{f_{\text{min}}}^{\infty} \frac{df}{f} \Omega_{\text{GW}}(f)h^2 \leq 5.6 \times 10^{-6} \Delta N_{\text{eff}}. \quad (80)$$

We neglect the frequency dependence of this bound and set $\Omega_{\text{GW}} \leq 5.6 \times 10^{-6} \Delta N_{\text{eff}}$ for the GW spectra that we calculate. BBN puts a bound on $\Delta N_{\text{eff}}^{\text{BBN}} \simeq 0.4$ [189]. Planck plus BAO observations set the bound $\Delta N_{\text{eff}}^{\text{Planck+BAO}} \simeq 0.28$ [14]. Projected bounds from future experiments are $\Delta N_{\text{eff}}^{\text{Proj}} = 0.014$ for CMB-HD [190], $\Delta N_{\text{eff}}^{\text{Proj}} = 0.05$ for CMB-Bharat [191], $\Delta N_{\text{eff}}^{\text{Proj}} = 0.06$ for CMB Stage IV [192] and NASA’s PICO mission [193], and $\Delta N_{\text{eff}}^{\text{Proj}} \lesssim 0.12$ for CORE [194], the South Pole Telescope [195], and Simons observatory [196].

VI. RESULTS

A. Example spectra for different cases

Figure 8 illustrates the damped GW spectra (black lines) for Case (a) and Case (b) discussed in Sec. IV, assuming $r = 0.035$, $M_1 = 10^5$ GeV, $K = 10^3$, and spectral index $n_T = 0$ (Left) and $n_T = 0.3$ (Right). Here, we demonstrate the influence of T_ϕ and M_ϕ on the GW spectra. We set the minimum T_ϕ to be approximately 100 GeV to accommodate sphaleron effects within the electroweak scale. The gray solid line marked “No IMD” is the standard spectrum with no intermediate matter-dominated phase, while the black solid and dashed lines are the spectral shapes in the presence of the matter-dominated epoch. Different colored regions correspond to sensitivity curves of current and future GW experiments. The blue violin lines are taken from the recent NANOGrav 15 year dataset [47,48]. Various horizontal lines on the top of the plot are coming from the current or projected ΔN_{eff} bounds from BBN, Planck 2018, and CMB surveys [62]. Higher frequencies in the figure correspond to earlier times, showcasing

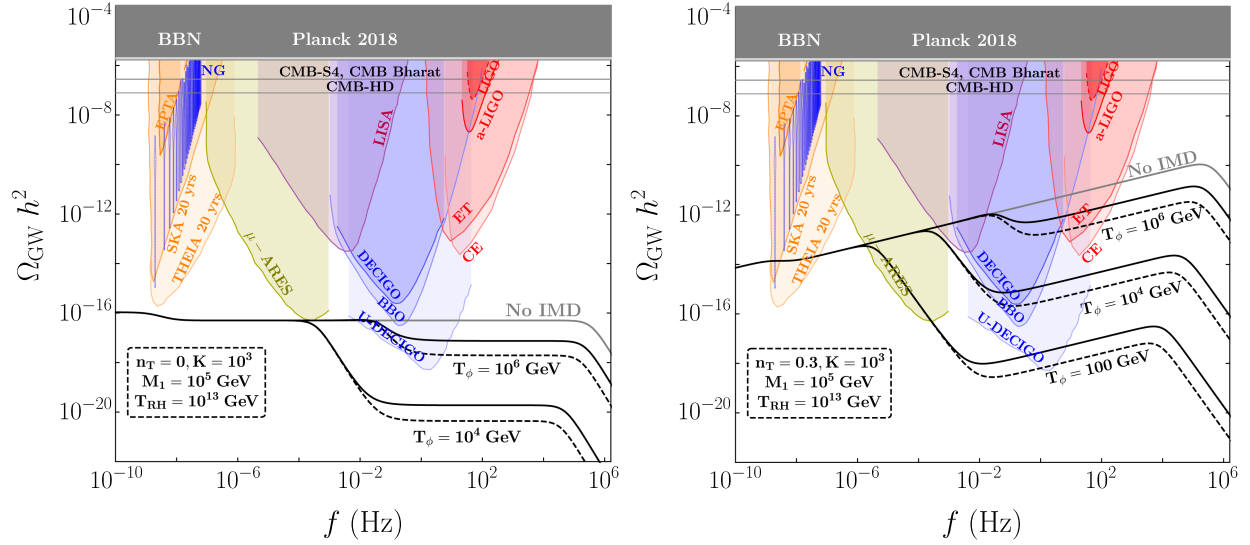


FIG. 8. Suppressed gravitational spectra of a benchmark Case (a) scenario when RHNs decay instantaneously or Case (b) where RHNs decay relativistically. Here, we have taken $r = 0.035$, $T_{\text{RH}} = 10^{13}$ GeV, $M_1 = 10^5$ GeV, $K = 10^3$, and varied T_ϕ and M_ϕ . Values of the spectral index are $n_T = 0$ (Left) and $n_T = 0.3$ (Right). The gray line shows the standard GW spectrum without any intermediate matter domination. Black solid and dashed lines correspond to $M_\phi = 10^9$ and 3×10^9 GeV, respectively. The extreme dampening at frequencies above ~ 100 Hz is due to inflaton decay.

suppression beyond the critical frequency f_{sup} due to entropy injection given by Eq. (68). Suppression at frequencies higher than $f_{\text{sup}} \sim 10^{-5} - 0.1$ Hz is attributed to Majoron and RHN decay, while suppression at frequencies higher than ~ 10 Hz is a consequence of inflaton decay terminating at the reheating temperature $T_{\text{RH}} = 10^{13}$ GeV. Lower values of T_ϕ shift the suppression frequency to the left and increase the dilution factor, as expected from

Eq. (50). The solid and dashed spectral lines represent $M_\phi = 10^9$ and 3×10^9 GeV, respectively. A higher M_ϕ means more dilution. Importantly, GW signatures remain invariant across weak and strong washout regimes, as they solely depend on the Majoron dynamics in this case.

Figure 9 presents the GW signature of long-lived RHNs in Case (c), depicting the two stages of entropy injection. We show the dependence of the spectra on parameters K and M_1

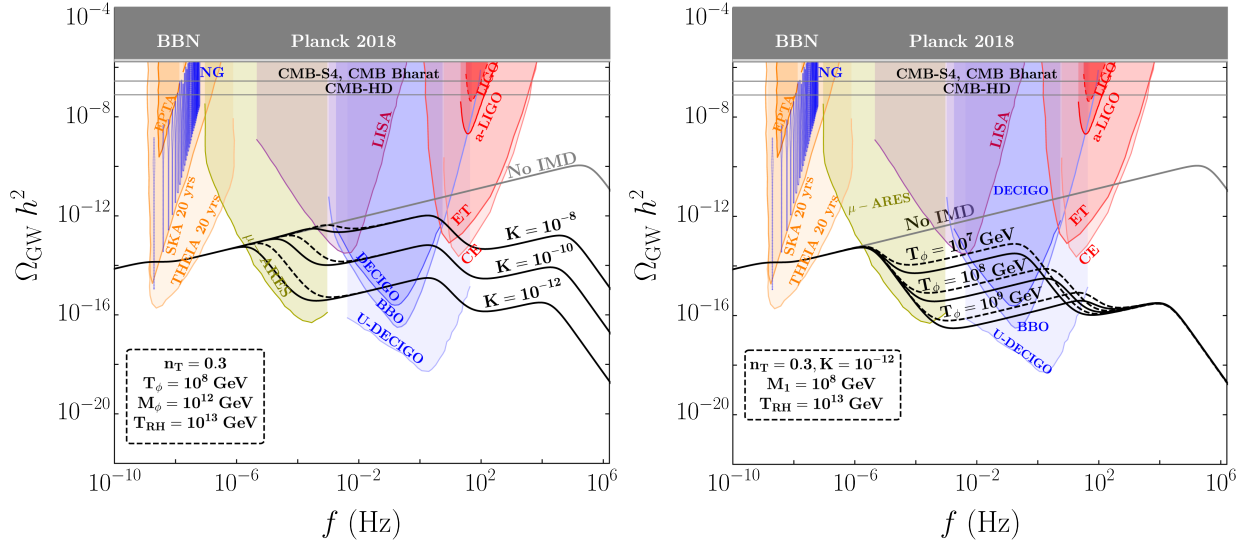


FIG. 9. Same as Fig. 8 but for a benchmark Case (c) “RHN matter domination” scenario. The left plot shows the dependence of the spectra on M_1 and K , while the right plot shows the dependence on M_ϕ and T_ϕ . In the left plot, the solid and dashed black lines represent $M_1 = 10^8$ and 3×10^8 GeV, respectively. In the right plot, the solid and dashed black lines represent $M_\phi = 10^{12}$ and 2×10^{12} GeV, respectively.

(Left) and on M_ϕ and T_ϕ (Right). The necessary condition for this case is $\sqrt{K} < 2T_\phi/M_\phi$. We choose the combination $T_{N_1} = M_1\sqrt{K}$ in a way such that $T_{N_1} \geq 100$ GeV. For these spectra, we take $r = 0.035$, $n_T = 0.3$, and use the two-step transfer function utilizing Eq. (70). Notably, the Majoron's effect manifests as an additional dip in the spectrum, a kneelike feature, positioned between the dips induced by inflaton decay and RHN decay. Thus, we have two kneelike features, one from ϕ decay and another from RHN decay. In the left plot, the solid and dashed spectral lines correspond to $M_1 = 10^8$ and 3×10^8 GeV, respectively. Here, we take $M_\phi = 10^{12}$ GeV, $T_\phi = 10^8$ GeV, and $T_{RH} = 10^{13}$ GeV. We see that a lower K and M_1 lead to higher suppression, as expected from Eq. (52). In the right plot, we fix $M_1 = 10^8$ GeV, $K = 10^{-12}$, and $T_{RH} = 10^{13}$ GeV. Here, solid and dashed spectral lines correspond to $M_\phi = 10^{12}$ and 2×10^{12} GeV, respectively. Here, we see that the overall suppression increases with increasing T_ϕ and decreasing M_ϕ , also understood from Eq. (52). This trend is opposite to the trend in Cases (a) and (b), as shown in Fig. 8.

In Case (c), the suppression factor associated with the middle “kneelike” feature is given by $R_{\text{sup}}^{\text{1st}} = (\Delta_\phi \Delta_N)^{-4/3}$, defined in Eq. (77). The minimum possible value of this factor corresponds to the maximum suppression. In this case, $R_{\text{sup}}^{\text{1st}} \sim 693.5K^{2/3}$, obtained by substituting Eqs. (50) and (52) in Eq. (77). The minimum value of the washout parameter K is obtained by considering the RHN decay

temperature $T_{N_1} = M_1\sqrt{K} \sim 100$ GeV, such that sphalerons can successfully convert lepton asymmetry to baryon asymmetry. For the maximum value of $M_1 = 10^{14}$ GeV, we obtain the minimum value of $R_{\text{sup}}^{\text{1st}} \simeq 7 \times 10^{-14}$. If we do not consider leptogenesis, then T_{N_1} can go as low as the BBN temperature, which is around 10 MeV. In that case, the minimum suppression factor is $R_{\text{sup}}^{\text{1st}} \simeq 3 \times 10^{-19}$. If in the future a two-step entropy injection with a smaller suppression factor is detected, it would rule out the non-thermal leptogenesis scenario within our model.

B. Estimates for signal-to-noise ratio

The next set of Figs. 10–13 demonstrate the projected future sensitivities of U-DECIGO, DECIGO, BBO, μ -ARES, LISA, CE, and ET experiments in terms of the SNR given in Eq. (79), in the parameter space of the RHN for different values of n_T , M_ϕ , T_ϕ , and T_{RH} . In these figures, the black dashed lines separate the Case (a), Case (b), and Case (c) regions. In each of the SNR plots, the gray-shaded region is excluded due to the requirement that the latest radiation-dominated era should start before BBN takes place at around temperatures of a few MeV. In our model, the latest radiation domination starts at the equilibrium temperature $T_{\text{eq}} = T_{\text{dec}}$ defined in Eq. (48). In all the SNR plots, the colored regions associated with different experiments can be probed by that experiment with $\text{SNR} \geq 10$. In all these SNR plots, we choose $T_{RH} = 10^{16}$ GeV to eliminate its effect on the SNR. The two solid

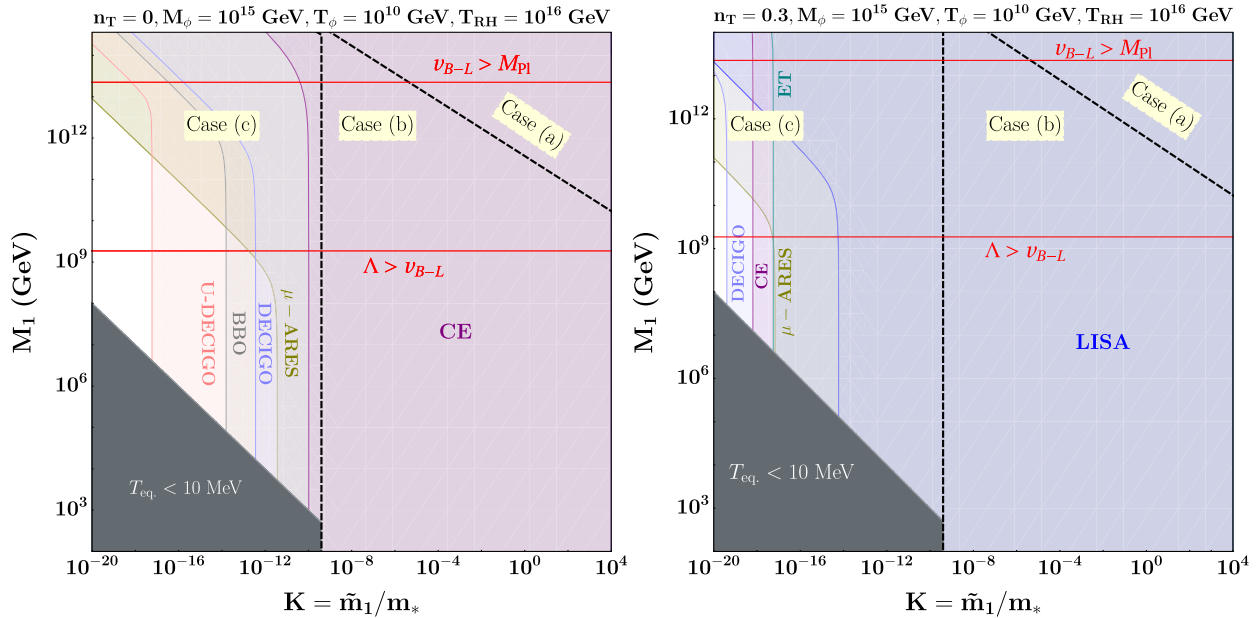


FIG. 10. Projected sensitivities of different experiments in the RHN parameter space for $n_T = 0$ (Left) and $n_T = 0.3$ (Right). Here, $\tilde{m}_1$ is the effective neutrino mass defined by Eq. (8) and $m_* = 1.1 \times 10^{-3}$ eV is the equilibrium neutrino mass defined in Eq. (9). For both plots we took $M_\phi = 10^{15}$ GeV, $T_\phi = 10^{10}$ GeV, and $r = 0.035$. The colored regions associated with each experiment imply sensitivity with $\text{SNR} > 10$. As expected, a higher n_T increases the overall SNR. Nonthermal leptogenesis is possible in this model only in the region between the two solid red lines. See the main text for a detailed description and analysis.

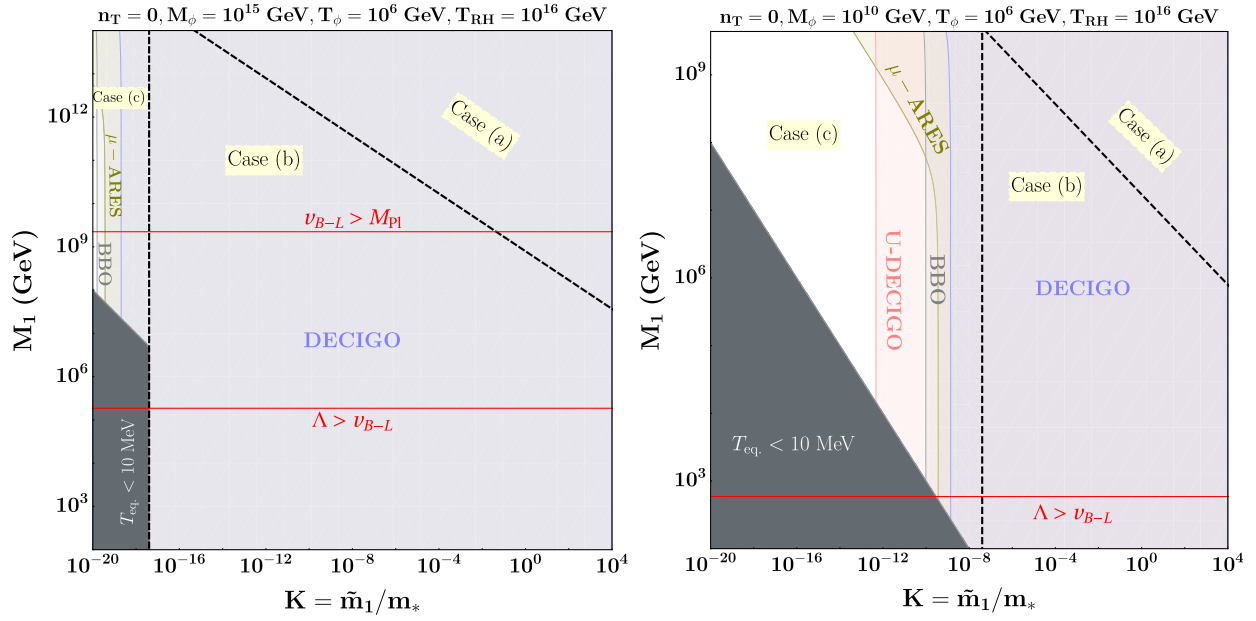


FIG. 11. Same as Fig. 10 but for $n_T = 0$, $T_\phi = 10^6$ GeV, and $M_\phi = 10^{15}$ GeV (Left) and 10^{10} GeV (Right). For a fixed T_ϕ , the left plot is expected to have smaller overall SNR values due to higher M_ϕ . Here, $m_* = 1.1 \times 10^{-3}$ eV.

red lines represent theoretical constraints on our model. In the region above the upper red line, v_{B-L} becomes larger than the Planck scale, while below the lower red line, the requirement that the confinement scale should be less than the $B-L$ breaking scale, $\Lambda < v_{B-L}$, discussed in Sec. III, is not satisfied. Hence, successful leptogenesis is possible only in the region between these two lines in our model. In the vanilla leptogenesis scenario, the vacuum stability of SM Higgs gives an upper bound on the lightest RHN mass $M_1 \lesssim 10^{14}$ GeV [197,198], and we assume it would be approximately the same in our model [54]. Hence, we take the upper limit on M_1 to be around 10^{14} GeV. On the other hand, the perturbativity limit of the model requires the Yukawa couplings $\lambda_{ij} < 4\pi$, which along with the necessity to produce at least one mass eigenstate $m_\nu = 0.05$ eV gives another bound, $M_2 \lesssim 10^{15}$ GeV. Hence, in our quasidegenerate RHN model, we find the perturbativity bound $M_1 \lesssim 10^{15}$ GeV. The perturbativity limit of the light-heavy neutrino mixing $y_{N_i} < 4\pi$ is always satisfied in our model.

In Fig. 10, we pick a high Majoron mass $M_\phi = 10^{15}$ GeV and decay temperature $T_\phi = 10^{10}$ GeV and show the projected sensitivities of different experiments for $n_T = 0$ (Left) and $n_T = 0.3$ (Right). We see that increasing the spectral index n_T also increases the GW-sensitive region by increasing the SNR, as expected. For $n_T = 0$, LISA and ET will not be able to probe leptogenesis, but for $n_T = 0.3$ both the experiments will be sensitive to a large portion of the parameter space. In the Case (a) and Case (b) regions, the SNR will not depend on M_1 and K but only on M_ϕ and T_ϕ . In the Case (c) region, RHNs decay at $T_{N_1} = M_1 \sqrt{K}$.

Hence, decreasing M_1 or K results in more suppression and lower SNR.

In Fig. 11, we show the sensitivities for $n_T = 0$, $T_\phi = 10^6$ GeV and $M_\phi = 10^{15}$ GeV (Left) and 10^{10} GeV (Right). We see that for $M_\phi = 10^{10}$ GeV, the GW-sensitive region in the parameter space is smaller compared to that for $M_\phi = 10^{15}$ GeV. Also, comparing Fig. 11 (Left) with Fig. 10, we see that the Case (a) and Case (b) regions cover more space

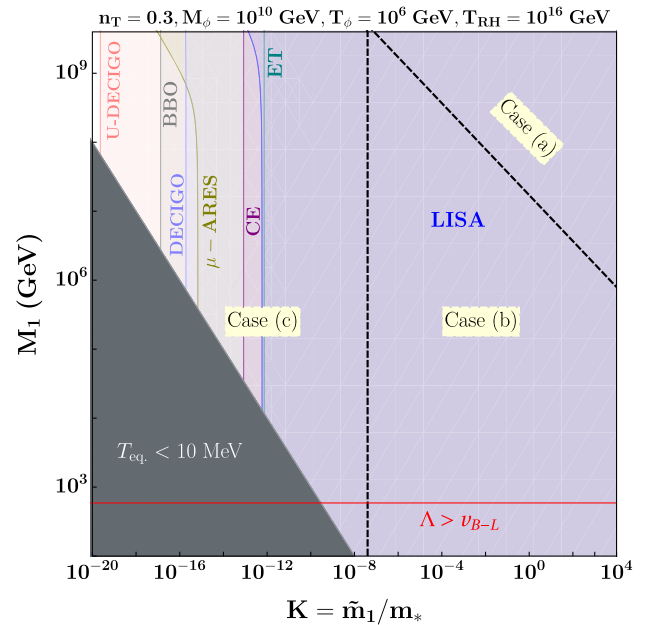


FIG. 12. Same as Fig. 11 (Right) but for $n_T = 0.3$. Here, $m_* = 1.1 \times 10^{-3}$ eV.

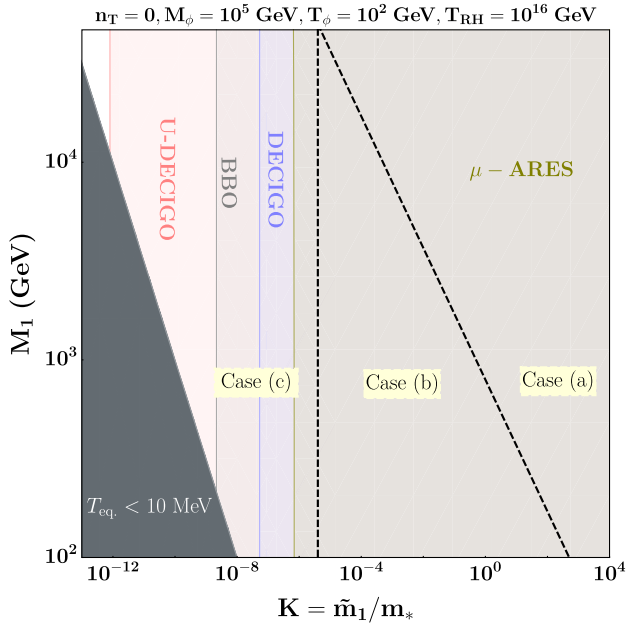


FIG. 13. Same as Fig. 10 but for $T_\phi = 100$ GeV and $n_T = 0$, $M_\phi = 10^5$ GeV. Here, $m_* = 1.1 \times 10^{-3}$ eV.

for lower T_ϕ values. We notice that CE is no longer sensitive for $n_T = 0, T_\phi = 10^6$ GeV in Fig. 11 (Left), while it is sensitive for $T_\phi = 10^{10}$ GeV in Fig. 10 (Left).

In Fig. 12, we show the SNR for $n_T = 0.3$, $T_\phi = 10^6$ GeV, and $M_\phi = 10^{10}$ GeV. Comparing with the Fig. 11 (Right) plot, again the effect of n_T is seen, and LISA, CE, and ET will be able to probe the parameter space for $n_T = 0.3$.

Figure 13 shows the shape of the sensitive regions for $M_\phi = 10^5$ GeV and $T_\phi = 100$ GeV assuming $n_T = 0$. We see that UDECIGO, DECIGO, BBO, and μ -ARES can probe leptogenesis with $T_\phi = 100$ GeV, given the small mass M_ϕ of the Majoron. A small Majoron mass injects a lower amount of entropy when it decays, as a result of which the suppression is small, hence the SNR is larger.

We calculate the SNR contours for all these GW experiments taking various values of M_ϕ and T_ϕ for $n_T = 0$ and 0.3 but do not show all of them in this paper. However, based on these results and Figs. 10–13, we summarize which key GW experiments can probe the parameter space M_1 vs K in Table II.

It is worth mentioning that there are certain models of inflation that go beyond the standard power-law description of the inflationary gravitational waves spectrum. For example, a broken power-law spectrum of IGW is suggested in [199]. The power-law is broken at a scale $k_{\text{break}} > k_*$ corresponding to the frequency $f_{\text{break}} \ll 10^{-8}$ Hz, where k_* is the pivot scale defined in Eq. (55). Below f_{break} , the spectrum is red-tilted ($n_T^{\text{CMB}} \lesssim 0$), while above f_{break} it can be blue-tilted ($n_T > 0$). Since our focus is on $f \gg 10^{-8}$ Hz, we can replace k_* with k_{break} in Eq. (55) to obtain a broken power-law assuming $n_T^{\text{CMB}} = 0$. In this case, $\Omega_{\text{GW}}(k)$ and the resulting SNR are suppressed by a factor $(k_*/k_{\text{break}})^{n_T} \sim \mathcal{O}(0.1-1)$ relative to the pure power-law case. Similarly, inflationary models with a Gauss-Bonnet term can yield a blue-tilted spectrum with a running n_T [200], also modifying the SNR. However, the effect of an intermediate matter-dominated era quantified by the SNR with and without the matter domination remains unaffected. We, therefore, do not consider such extensions further in this work.

VII. DISCUSSION AND CONCLUSION

Leptogenesis at low and intermediate energy scales provides an attractive scenario for baryogenesis, linking the observed matter-antimatter asymmetry to the neutrino sector and seesaw scale. In this analysis, we offer a probe of this scenario via observations of the stochastic GW background that originates during inflation (which is responsible for the creation of primordial density perturbation of the Universe) and propagates in the postinflationary era to reach us at present. Particularly, we construct a global $U(1)_{B-L}$ model of neutrino mass that facilitates nonthermal leptogenesis through the decay of the Goldstone boson, called Majoron, denoted by ϕ . We show that detecting a

TABLE II. Future sensitivity of different key GW experiments to probe the RHN parameter space (M_1 vs. K) with threshold $\text{SNR} > 10$, for benchmark values of M_ϕ and T_ϕ . Columns show sensitivities for $n_T = 0$ and $n_T = 0.3$. Here, the check mark implies that the particular experiment will be sensitive to nonthermal leptogenesis for a given value of n_T . Dashes imply that in the considered parameter space, $\text{SNR} < 10$ for the given experiment. We considered $T_{\text{RH}} = 10^{16}$ GeV and maximal tensor-to-scalar ratio $r = 0.035$.

M_ϕ (GeV)	T_ϕ (GeV)	U-DECIGO		BBO		μ -ARES		LISA		ET		CE	
		$n_T = 0$	$n_T = 0.3$	$n_T = 0$	$n_T = 0.3$	$n_T = 0$	$n_T = 0.3$	$n_T = 0$	$n_T = 0.3$	$n_T = 0$	$n_T = 0.3$	$n_T = 0$	$n_T = 0.3$
10^{15}	10^{10}	✓	✓	✓	✓	✓	✓	...	✓	...	✓	✓	✓
	10^6	✓	✓	✓	✓	✓	✓	...	✓	...	✓	...	✓
	10^2	...	...	...	...	...	✓	...	...	...	...	...	...
10^{10}	10^6	✓	✓	✓	✓	✓	✓	...	✓	...	✓	...	✓
	10^2	...	✓	...	✓	...	✓	...	...	...	...	...	...
	10^2	✓	✓	✓	✓	✓	✓	...	✓	...	✓	...	✓

characteristic GW spectral shape will provide an indication of a RHN or ϕ matter-dominated era in the early Universe. We present a minimal model realization of nonthermal leptogenesis, for which we find that a large fraction of the viable parameter space will be probed in future GW experiments. For the lightest RHN mass M_1 and washout parameter K involving light-heavy neutrino Yukawa couplings λ , we find M_1 in the range $100 \text{ GeV} \lesssim M_1 \lesssim 10^{14} \text{ GeV}$ and K for extremely weak washout ($K \sim 10^{-20}$) and for strong washout ($K \sim 10^4$) regimes consistent with successful baryogenesis, and novel GW shapes will be probed by U-DECIGO, DECIGO, BBO, μ -ARES, LISA, CE, ET, etc. By estimating the SNR for GW experiments, we investigate the effect of the Majoron mass M_ϕ and decay temperature T_ϕ , which depends on the $\phi - N$ Yukawa couplings y_N .

The key interpretations of our results are summarized below:

- (i) The overall SNR is larger for a larger spectral index n_T (compare, for example, Fig. 10 left and right plots).
- (ii) Nonthermal leptogenesis can be probed in future GW experiments, such as U-DECIGO, BBO, etc. (see spectra in Figs. 8 and 9 and SNR in Figs. 10–13).
- (iii) If $T_{N_1} \ll T_\phi$ in the Case (c) scenario with the two-step entropy injection, lower values of M_1 and K reduce the SNR and, therefore, it is challenging to test for all experiments (Figs. 10–13).
- (iv) For Case (a) and Case (b) scenarios discussed in Sec. IV, a lower T_ϕ value decreases the SNR and frequency of suppression f_{sup} given in Eq. (68), making it more difficult to observe (see spectra in Fig. 8). However, suitable low values of T_ϕ increase the overall GW-sensitive region in the parameter space of the lightest RHN (compare, for example, Figs. 10 and 11).
- (v) A higher M_ϕ in general means larger entropy injection, which decreases the overall SNR values for all experiments (compare solid and dashed spectra in Fig. 8). However, for Case (c), higher M_ϕ increases the SNR [solid and dashed spectra in Fig. 9 (Right)]. M_ϕ also sets the upper bound on M_1 in our model, i.e., $M_1 \leq M_\phi/2$. We are interested in the parameter space where ϕ is long-lived to dominate the energy budget of the Universe [see the discussion following Eqs. (34) and (42)].
- (vi) In Case (a) and Case (b) described in Sec. IV, the leptogenesis scale is $\sim T_\phi$, which means GW experiments can probe leptogenesis even for strong washout $K > 1$, where RHNs might eventually thermalize with the radiation bath (Figs. 10–13). This is unlike the model in [62] where the RHN domination criterion $K \lesssim 4 \times 10^{-4}$ must be satisfied for GW sensitivity.
- (vii) Table II summarizes which key experiments can probe nonthermal leptogenesis for $n_T = 0$ and 0.3,

respectively, given benchmark values of the Majoron mass M_ϕ and decay temperature T_ϕ .

If the characteristic features of the GW spectral shapes proposed in this study are observed, one may look to target additional observations to distinguish between an RHN or Majoron ϕ dominated pre-BBN era and other forms of early matter domination. Particularly in low-scale leptogenesis, RHN masses of GeV–TeV could be searched in typical heavy neutral lepton search experiments (see [201–203] for current experimental limits) at the particle physics laboratories. In this manner, we can complement GW searches with laboratory searches in the same BSM parameter space. However, such a study is beyond the reach of the present paper and we plan to explore this in future. RHNs, if they exist, should also show up in experiments, such as neutrinoless double beta decay [204,205] or lepton number violating processes [206,207], and, therefore, provide us with a myriad of pathways to independently verify the existence of an early RHN domination or Majoron domination era. This leads to a unique and exciting opportunity to form synergies between GW searches and laboratory and CMB searches.

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX A: STRUCTURE OF COMPLEX ORTHOGONAL MATRIX R

Here, we explicitly show that CP asymmetry defined in Eq. (18) can be large even for a tiny washout parameter K defined in Eq. (10). For example, take a simple form of R defined in Eq. (11),

$$R = \begin{pmatrix} \cos \omega & 0 & -\sin \omega \\ 0 & 1 & 0 \\ \sin \omega & 0 & \cos \omega \end{pmatrix}, \quad (\text{A1})$$

where $\omega = x + iy$ is a complex angle. Assuming normal ordering and $m_1 = 0$, the CP asymmetry can be written in terms of x and y as

$$\begin{aligned} |\epsilon_1| &= \frac{3}{32\pi} \frac{M_1 m_3}{v^2} \frac{\sin 2x \sinh 2y}{\sin^2 x \cosh^2 y + \cos^2 x \sinh^2 y} \\ &= \frac{3}{16\pi} \frac{M_1 m_3}{v^2} \frac{\sin 2x \sinh 2y}{\cosh 2y - \cos 2x}. \end{aligned} \quad (\text{A2})$$

When $x = y \ll 1$, it can be verified that $|\epsilon_1| = \epsilon_1^{\text{max}}$, which is the maximal CP asymmetry defined in Eq. (19). Taking

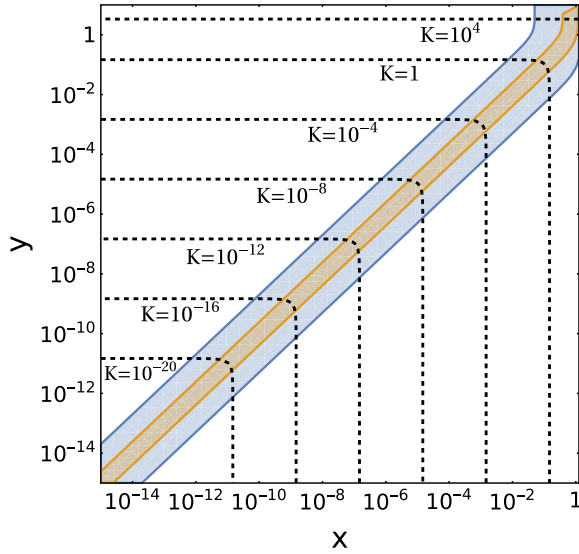


FIG. 14. CP parameter $|\epsilon_1|$ defined in Eq. (18) and contours of decay parameter K defined in Eq. (10) in the parameter space spanned by x and y . Blue and orange regions signify $|\epsilon_1| > 10^{-7}$ and 7×10^{-7} , respectively. It is seen that for $x = y \ll 1$, large $|\epsilon_1|$ can be obtained even for a tiny value of K .

$x = y = 10^{-11}$ gives $|\epsilon_1| \sim 10^{-6}$ (assuming $M_1 = 10^{10}$ GeV), while $K \sim 10^{-20}$. In Fig. 14 we show the regions where $|\epsilon_1| > 10^{-7}$ (blue) and $|\epsilon_1| > 7 \times 10^{-7}$ (orange) in the parameter space spanned by x and y . Along with it, we also show the contours of constant K values in the same parameter space. For this plot we assume normal ordering and $m_1 = 0$, $M_1 = 10^{10}$ GeV, $M_2 = 10^{11}$ GeV, and $M_3 = 10^{12}$ GeV. We see that for such high RHN mass scales, it is possible to generate $|\epsilon_1| = \epsilon_1^{\max}$ even if $\tilde{m}_1 \rightarrow 0$, hence $K \rightarrow 0$. However, K_2 and K_3 remain large.

APPENDIX B: CHANGE OF VARIABLE

It is convenient to use comoving variables while solving the Boltzmann equations. We adopt the same prescription as in [102,208],

$$E_{\phi, N_i} = \rho_{\phi, N_i} a^3, \quad \tilde{N}_1 = n_1 a^3 = \frac{\rho_{N_1}}{M_1} a^3, \\ \tilde{N}_{B-L} = n_{B-L} a^3, \quad R = \rho_R a^4, \quad S = s a^3, \quad (B1)$$

where a is the scale factor of the Universe. Note that E_{N_1} and $\tilde{N}_1$ become comoving quantities only after RHNs become nonrelativistic, i.e., at temperatures below T_{NR} defined in Eq. (37). Instead of time t , we write the Boltzmann equations in terms of the ratio of scale factor a to its initial value a_I , given by

$$y = \frac{a}{a_I}. \quad (B2)$$

We take $a_I = 1$ without loss of generality. The expansion rate can be written as

$$H = \sqrt{\frac{8\pi(a_I E_{\phi} y + a_I E_{N_1} y + R)}{3M_{pl}^2 a_I^4 y^4}}. \quad (B3)$$

The temperature can be written as

$$T = \left[\frac{30R}{\pi^2 g_* a_I^4 y^4} \right]^{\frac{1}{4}}. \quad (B4)$$

The redshift parameter z is defined in terms of M_1 ,

$$z = \frac{M_1}{T} = M_1 a_I \left[\frac{\pi^2 g_*}{30R} \right]^{\frac{1}{4}} y. \quad (B5)$$

The equilibrium energy densities are given by [102,141]

$$E_{N_1}^{\text{eq}} = \rho_{N_1}^{\text{eq}} a^3 = \frac{a_I^3 M_1^4 y^3}{\pi^2} \left[\frac{3}{z^2} K_2(z) + \frac{1}{z} K_1(z) \right], \\ \tilde{N}_1^{\text{eq}} = \frac{E_{N_1}^{\text{eq}}}{M_1}, \quad n_{N_1}^{\text{eq}} = \frac{E_{N_1}^{\text{eq}}}{M_1 y^3}, \quad n_R^{\text{eq}} = \frac{3\zeta(3)}{4\pi^2} \times 2 \left(\frac{M_1}{z(y)} \right)^3. \quad (B6)$$

The inverse decay rate can be written as [112]

$$\Gamma_{ID} = \frac{1}{2} \Gamma_{N_1} \frac{n_{N_1}^{\text{eq}}}{n_R^{\text{eq}}}. \quad (B7)$$

As described earlier, we ignore the inverse decay term of ϕ as well as the RHN scattering terms involving Γ_{scatt} and $\Gamma_{ID}^{\text{scatt}}$ in the Boltzmann equations given in Eq. (43), and we rewrite them as

$$\frac{dE_{\phi}}{dy} = -\frac{\Gamma_{\phi}}{\mathcal{H}} \frac{E_{\phi}}{y}, \\ \frac{dE_{N_1}}{dy} = \frac{\Gamma_{\phi \rightarrow N_1 N_1}}{\mathcal{H}} \frac{E_{\phi}}{y} - \frac{\Gamma_{N_1}}{\mathcal{H} y} (E_{N_1} - E_{N_1}^{\text{eq}}) - \frac{E_{N_1}}{y} \Theta(T - T_{NR}), \\ \frac{d\tilde{N}_{B-L}}{dy} = -\frac{\Gamma_{N_1}}{\mathcal{H} y} \left[\epsilon(\tilde{N}_1 - \tilde{N}_1^{\text{eq}}) + \frac{1}{2} \frac{n_{N_1}^{\text{eq}}}{n_R^{\text{eq}}} \tilde{N}_{B-L} \right], \\ \frac{dR}{dy} = \frac{\Gamma_{\phi \rightarrow R}}{\mathcal{H}} E_{\phi} + \frac{\Gamma_{N_1}}{\mathcal{H}} (E_{N_1} - E_{N_1}^{\text{eq}}),$$

where $\Theta(x)$ is the Heaviside step function separating the relativistic stage of RHNs from the nonrelativistic stage,

$$\Theta(x) = \begin{cases} 1 & \text{for } x > 1 \\ 0 & \text{otherwise.} \end{cases} \quad (B8)$$

By solving the above equations, one can calculate the efficiency parameter to be

$$\kappa_f = -\frac{4}{3} \epsilon^{-1} R^{-3/4} \tilde{N}_{B-L} \left[\frac{\pi^4 g_*^{3/4}}{30^3 \zeta(3)^4} \right]. \quad (B9)$$

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