

High quality QCD axion in the Standard Model

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Although the axion is the most compelling solution to the strong CP problem, the *ad hoc* introduced global Peccei-Quinn symmetry suffers from a severe fine-tuning problem known as the *quality problem*. In this letter, we show that the discrete gauge symmetry $\mathbb{Z}_4 \times \mathbb{Z}_3$ motivated from the internal structure of the standard model can naturally predict a high-quality axion, leading to a distinct and testable parameter space. Remarkably, this minimal framework simultaneously accounts for neutrino masses, baryon asymmetry, and dark matter.

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I. INTRODUCTION

Gauge symmetry is the powerful guiding principle for understanding the nature of elementary particles. In fact, the standard model (SM) of particle physics is built upon the gauge symmetry, $SU(3) \times SU(2) \times U(1)$, which is very successful to describe the nature of quarks, leptons, and the Higgs boson. With the establishment of the SM, a natural question is *are there more gauge symmetries naturally embedded in the SM?*

The discrete gauge symmetry $\mathbb{Z}_4$ is a well-known example motivated by the internal structure of the SM. This symmetry is anomaly-free with respect to all SM gauge interactions if all chiral fermions carry a $\mathbb{Z}_4$ charge of $+1$, since one fermion generation contains four chiral fermions contributing to the QCD anomaly and four $SU(2)_L$ doublets contributing to the electroweak anomaly. Interestingly, however, the $\mathbb{Z}_4$ symmetry is no longer anomaly-free once nonperturbative gravitational effects are included, which is known as the *Dai-Freed anomaly* [1,2]. This anomaly can be canceled by introducing three right-handed neutrinos $\bar{N}_i$ [3], with all chiral fermions being defined as left-handed Weyl fermions. The presence of the heavy Majorana fields $\bar{N}_i$ is particularly attractive, as they naturally account for the observed tiny neutrino masses via the seesaw mechanism [4–7], and their decays in the early Universe can generate the present baryon asymmetry through leptogenesis [8].

In addition, the SM has three generations of fermions, which naturally introduces another discrete gauge symmetry, $\mathbb{Z}_3$, that is anomaly-free with respect to all gauge interactions. All chiral fermions can also be assigned with a $\mathbb{Z}_3$ charge of $+1$. Similarly, this model possesses the Dai-Freed anomaly [9], which can again be canceled by introducing three chiral fermions, χ_i , whose $\mathbb{Z}_3$ charge is -1 . This fermion can serve as a candidate for dark matter (DM).

In this letter, we construct an extension of the SM with the discrete symmetry $\mathbb{Z}_4 \times \mathbb{Z}_3$ that is directly motivated by the SM itself. This framework naturally requires two Higgs doublets, which in turn elegantly give rise to a Peccei-Quinn (PQ) symmetry and an associated axion that resolves the strong CP problem. Remarkably, the resulting axion is a high-quality axion free from the fine-tuning problem and predicts a distinct parameter space testable in upcoming future experiments. Moreover, this scenario naturally predicts two-component DM, providing a solution to the dark matter problem.

II. ANOMALY-FREE DISCRETE GAUGE SYMMETRIES

We consider a minimal extension of the SM by introducing anomaly-free discrete gauge symmetries, $\mathbb{Z}_4 \times \mathbb{Z}_3$. For simplicity of notations, the SM quarks, leptons, and partially the Higgs doublets are organized into $SU(5)$ representations, as shown below. We do not, however, assume any grand unification of the SM gauge groups, nor do we impose any grand unified theory relations among the gauge or Yukawa coupling constants. We denote them as

$$T(\mathbf{10}) \equiv \{q, \bar{u}, \bar{e}\}, \quad (1a)$$

$$\bar{F}(\mathbf{5}^*) \equiv \{\bar{d}, l\}, \quad (1b)$$

$$H(\mathbf{5}^*) \equiv \{H_1, H_2\}. \quad (1c)$$

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TABLE I. The charges of fermions under $\mathbb{Z}_4$ and $\mathbb{Z}_3$ symmetries.

Fermions	T	$\bar{F}$	$\bar{N}$	χ
$\mathbb{Z}_4$	1	1	1	0
$\mathbb{Z}_3$	1	1	1	-1

TABLE II. The charges of the Higgs boson and PQ field under $\mathbb{Z}_4$ and $\mathbb{Z}_3$ symmetries.

Scalars	H_1	H_2	Φ
$\mathbb{Z}_4$	-2	-2	-1
$\mathbb{Z}_3$	2	1	-1

As outlined in the Introduction, to cancel the Dai-Freed anomaly of $\mathbb{Z}_4$ symmetry, three right-handed neutrinos labeled $\bar{N}(\mathbf{1})$ are naturally introduced. The anomaly cancellation of $\mathbb{Z}_3$ requires the existence of three extra Majorana fermions χ . The fermions in this model contain three families of $T(\mathbf{10})$, $\bar{F}(\mathbf{5}^*)$, and $\bar{N}(\mathbf{1})$. The $\mathbb{Z}_4 \times \mathbb{Z}_3$ charge assignments for all fermions are summarized in Table I. Here, all fermions are left-handed Weyl spinors.

In order to reproduce the SM fermion mass terms while preserving both discrete symmetries, a single Higgs field is insufficient.¹ Two Higgs doublets H_1 and H_2 are, therefore, naturally required, which, as we will see later, in turn give rise to a PQ symmetry. The Higgs doublets have the following Yukawa interactions:

$$\mathcal{L}_Y^{\text{SM}} \sim \frac{1}{2} T \bar{T} H_1^\dagger + T \bar{F} H_2 + \bar{N} \bar{F} H_1^\dagger + \text{H.c.} \quad (2)$$

With H_1 providing the up-type quark masses and the Dirac neutrino masses, while H_2 provides the down-type quark and charged lepton masses, it can be expanded as

$$\mathcal{L}_Y^{\text{SM}} \sim -f_u q \bar{u} H_1^\dagger - f_e \bar{e} l H_2 - f_d q \bar{d} H_2 - f_D \bar{N} l H_1^\dagger + \text{H.c.} \quad (3)$$

To generate large Majorana masses for the right-handed neutrinos $\bar{N}$ and Majorana masses for the extra fermions χ , we introduce a new Higgs field Φ whose $\mathbb{Z}_4 \times \mathbb{Z}_3$ charge is $(-1, -1)$. All the charges of Higgs fields are summarized in Table II. The allowed interactions of the Φ are given by

$$\mathcal{L}_Y^{N,\chi} \sim \frac{1}{2} \frac{1}{M_{\text{pl}}} \Phi^2 \bar{N} \bar{N} + \frac{1}{2} \frac{1}{4!} \frac{\Phi^4}{M_{\text{pl}}^3} \chi \chi + \text{H.c.} \quad (4)$$

¹If there is only one Higgs field H and $\mathcal{L} \sim T \bar{T} H^\dagger$ satisfies the discrete symmetries, $\mathcal{L} \sim T \bar{F} H$ does not.

These higher-dimensional operators are simply suppressed by Planck mass $M_{\text{pl}} \simeq 2.4 \times 10^{18}$ GeV without assuming any other scales.

Since all the required operators contain only even powers of Φ , this means that Φ naturally possesses a $\mathbb{Z}_2$ gauge symmetry. Only the field Φ is odd under the $\mathbb{Z}_2$, while all other fields transform as the $\mathbb{Z}_2$ -even. This symmetry is anomaly-free, since it does not act on the chiral fermions. Thus, we impose this exact discrete $\mathbb{Z}_2$ symmetry throughout this letter.² We will show that this minimal extension of the SM naturally processes the PQ symmetry, with the field Φ playing the role of the PQ field, and elegantly resolves the axion high-quality problem.

III. HIGH-QUALITY AXION

The existence of two Higgs fields naturally introduces a new $U(1)$ global symmetry. We can see that the present model is invariant under the following global $U(1)$ transformation:

$$\{l, q\} \rightarrow e^{-i\alpha} \{l, q\}, \quad (5a)$$

$$\{\bar{e}, \bar{u}, \bar{d}, \bar{N}\} \rightarrow e^{-i\alpha} \{\bar{e}, \bar{u}, \bar{d}, \bar{N}\}, \quad (5b)$$

$$\{H_1, H_2\} \rightarrow \{e^{-2i\alpha} H_1, e^{+2i\alpha} H_2\}, \quad (5c)$$

$$\{\Phi\} \rightarrow \{e^{+i\alpha} \Phi\}, \quad (5d)$$

$$\{\chi\} \rightarrow \{e^{-2i\alpha} \chi\}. \quad (5e)$$

This symmetry can be identified as the PQ symmetry [10,11]. It is known that the spontaneous breaking of the global $U(1)_{\text{PQ}}$ symmetry generates a light pseudoscalar Nambu-Goldstone boson, called the axion labeled by a , which provides a solution to the strong CP problem [10–13]. However, the axion solution suffers from a severe quality problem [14,15]. We show that with a simple charge assignment $(-1, -1)$ of the $U(1)_{\text{PQ}}$ field Φ under the $\mathbb{Z}_3$ and $\mathbb{Z}_4$ symmetries (see Table II), a high quality for the QCD axion is naturally guaranteed, as explained below.

A higher-dimensional operator involving Φ that couples H_1 and H_2 is allowed under the discrete symmetries and the $U(1)$ PQ symmetry,

$$\mathcal{L}_{\text{PQ}} \sim \frac{1}{4!} \frac{H_1^\dagger H_2 \Phi^{*4}}{M_{\text{pl}}^2}, \quad (6)$$

which is also suppressed by the Planck mass M_{pl} . It is surprising that this interaction preserves the PQ symmetry with the PQ charges shown in Eq. (5). Symmetry breaking occurs when the field Φ acquires a vacuum expectation

²This $\mathbb{Z}_2$ is assumed to guarantee the complete stability of the χ DM, as explained later.

value (vev) defined as $\langle \Phi \rangle \equiv F_a$, which is also commonly referred to as the axion decay constant F_a .

The QCD instantons generate an effective axion potential as $V_a = m_a^2 F_a^2 [1 - \cos(\bar{\theta} + N_D a / F_a)]$, which gives rise to the axion mass m_a . This dynamical axion field drives its potential to the minimum, that is, $N_D a / F_a = -\bar{\theta}$, thereby solving the strong CP problem. The axion mass is $m_a = [\sqrt{m_u m_d} / (m_u + m_d)] \times N_D m_\pi f_\pi / F_a = N_D \times 5.7 \mu\text{eV} \times (10^{12} \text{ GeV} / F_a)$, where up-quark mass $m_u = 2.16 \text{ MeV}$, down-quark mass $m_d = 4.67 \text{ MeV}$, pion mass $m_\pi \simeq 140 \text{ MeV}$, and pion decay constant $f_\pi = 92 \text{ MeV}$. Additionally, N_D denotes the domain wall number, which counts the number of distinct vacua and appears in the axion-gluon interaction as $\mathcal{L} = N_D a G \tilde{G} / F_a$ [16,17].

The domain wall number N_D can be obtained by summing over the PQ charges of the colored fermions as $N_D = |3 \times [(X_q - X_u) + (X_q - X_d)]|$, where X_i is the corresponding PQ charge for species i . In typical models, such as the Kim-Shifman-Vainshtein-Zakharov (KSVZ) model [18,19], one has $N_D = 1$, while in the Dine-Fischler-Srednicki-Zhitnitsky (DFSZ) model $N_D = 3$ or 6 [20,21]. In our model, due to the coupling between the Higgs fields and Φ^4 , a distinctive prediction of $N_D = 12$ is obtained. This leads to a different relationship between axion mass and the vev of Φ . In general, the observable couplings of the axion to SM particles, such as photons or nucleons, are proportional to m_a [17]. Different domain wall numbers N_D do not lead to significant differences—our model predicts the same couplings as the DFSZ model. However, the vev of Φ affects the axion quality, as will be discussed below. Therefore, our model can predict a distinctive target region in parameter space.

We are now at the key point in this letter. Quantum gravity effects, like wormholes, are believed to explicitly violate all global symmetries, including the PQ symmetry [22–25]. At energies small compared to the Planck mass M_{pl} , these symmetry-violating effects should be described by higher-dimensional potential operators of the $U(1)_{PQ}$ field Φ as $V_g(\Phi) \sim \Phi^n / M_{\text{pl}}^{n-4}$. Typically, low-dimensional operators spoil the QCD axion potential, and suppressing them requires severe fine-tuning of $\sim 10^{-100}$ for the coupling [14,15].

However, in our setup, with the charge assignment $(-1, -1)$ for the Φ , the lowest-dimensional leading term of the potential consistent with both $\mathbb{Z}_3$ and $\mathbb{Z}_4$ discrete symmetries is³

$$V_g(\Phi) = \frac{g}{12!} \frac{\Phi^{12}}{M_{\text{pl}}^8} + \text{H.c.} \quad (7)$$

³We have also allowed the higher-dimensional operator $(H_1^\dagger H_2)^2 \Phi^4$ that explicitly breaks $U(1)$ PQ symmetry. However, the contribution to the extra mass for the axion can be negligible because of the small vevs of $H_{1,2}$.

After the spontaneous PQ symmetry breaking by the Φ condensation, $\Phi = F_a e^{i \frac{a}{\sqrt{2} F_a}}$, the Planck scale physics would induce an extra potential V_a^g for the axion as [14,15]

$$V_a^g = (m_a^g F_a)^2 \left[1 - \cos \left(\frac{12a}{\sqrt{2} F_a} + \delta \right) \right], \quad (8)$$

with δ being the phase of coupling g and m_a^g is the gravity-induced axion mass,

$$(m_a^g)^2 = \frac{|g| M_{\text{pl}}^2}{12!} \left(\frac{F_a}{\sqrt{2} M_{\text{pl}}} \right)^{10}. \quad (9)$$

The gravity-induced potential, Eq. (8), would certainly shift the QCD axion potential. To solve the strong CP problem, the value of the effective phase, $\theta_{\text{eff}} \equiv \bar{\theta} + N_D \langle a \rangle / F_a$, must be smaller than $\sim 10^{-10}$ [26]. In order for the gravity-induced potential not to violate this constraint, it implies that the ratio between the gravity-induced axion mass and the QCD mass, $r \equiv (m_a^g / m_a)^2$, should also be less than 10^{-10} . For reasons of naturalness, we assume the gravity potential coupling to be $|g| = 0.01$. This requirement indicates that as long as

$$F_a \leq \left(\frac{12! \cdot 2^5 \times 10^{-10} m_\pi^2 f_\pi^2 M_{\text{pl}}^8 N_D^2}{|g|} \right)^{\frac{1}{12}} = 2 \times 10^{12} \text{ GeV}, \quad (10)$$

the axion would possess high quality to solve the strong CP problem without encountering the fine-tuning problem. This range of F_a corresponds to an axion mass of $m_a \gtrsim 3 \times 10^{-5} \text{ eV}$ with $N_D = 12$, which is a prediction in our model. Moreover, this predicted lower bound on m_a will not change much with various assumptions of coupling g , since the F_a depends only on $|g|^{-1/12}$.

Through the misalignment mechanism [27–29], axion fields can be produced in the early Universe and leave a relic abundance. Since the mass of the QCD axion is determined by a decay constant, its relic density can be fixed by m_a together with the initial misalignment angle θ_i as [30]

$$\Omega_a h^2 \simeq 0.12 \left(\frac{1.7 \times 10^{-5} \text{ eV}}{m_a} \right)^{7/6} \langle \theta_i^2 \rangle. \quad (11)$$

Compared with today's DM relic abundance $\Omega_{\text{DM}} h^2 \simeq 0.12$, the axion with $m_a \sim 10^{-5} \text{ eV}$ and $F_a \sim 10^{12} \text{ GeV}$ can constitute the dominant component of DM for $\langle \theta_i^2 \rangle \simeq 1$, or only a small fraction of it for $\langle \theta_i^2 \rangle < 1$. However, for smaller values of F_a , the axion cannot account for the main DM component. The lower bound on F_a will be discussed in the next section, where we consider the presence of the

DM component χ and derive the corresponding cosmological constraints.

IV. TWO-COMPONENT DARK MATTER

Besides the high-quality axion, our theoretical framework with anomaly-free $\mathbb{Z}_3$ and $\mathbb{Z}_4$ symmetries also predicts three generations of fermion DM components χ with charge of $(0, -1)$, as shown in Table I. They can be produced nonthermally in the early Universe, for example through inflaton decay [31], and together with the axion can account for the total DM abundance.

The Majorana fermion χ acquires its mass through the coupling with Φ^4 , as shown in Eq. (4). Due to the limit $\langle \Phi \rangle \lesssim 2 \times 10^{12}$ GeV and suppression by the Planck scale, this mass is relatively tiny as

$$m_\chi = \frac{1}{4!} \frac{\langle \Phi \rangle^4}{M_{\text{pl}}^3} = 50 \text{ eV} \times \left(\frac{\langle \Phi \rangle}{2 \times 10^{12} \text{ GeV}} \right)^4. \quad (12)$$

The fermion χ is even under the $\mathbb{Z}_2$ symmetry discussed below Eq. (4), while all the SM fermions are odd. As a result, χ has no decay channel into SM fermions. Its stability allows it to serve as a stable dark sector particle.

The main constraints on light fermionic DM arise from cosmological and astrophysical considerations. Since DM serves as the host for galaxies, it must possess a sufficiently large energy density. However, due to Fermi statistics, the number of fermions that can occupy a given phase-space volume is limited. This imposes a lower bound on the mass of fermionic DM, known as the Tremaine-Gunn bound [32]. Recent studies show that if fermions constitute all of the DM, their mass must be larger than about 200 eV [33,34]. Taking into account the fermion DM fraction f_χ and the number of fermion species g_χ , which is 3 in our case, this bound becomes

$$m_\chi^{\text{min}} = 7 \text{ eV} \times \left(\frac{3}{g_\chi} \right) \times \left(\frac{f_\chi}{0.1} \right). \quad (13)$$

Comparing Eq. (12) with Eq. (13), one can obtain that this fermion can account for at most about $\sim 70\%$ of the total DM abundance. As F_a decreases, the mass of fermion will quickly decrease because of the $\langle \Phi \rangle^4$ dependence shown in Eq. (12), resulting in a decrease in the possible fraction of DM it can contribute to. This, in turn, requires the axion to dominate in DM for $F_a \lesssim 10^{12}$ GeV.

According to Eq. (11), the maximum value of θ_i is 2π . Given $\Omega_a \simeq 0.1$, we have a lower bound of Φ vev as $F_a \gtrsim 1.7 \times 10^{11}$ GeV. However, if the axion field starts near the top of its potential and rolls down more slowly, larger DM abundance can be produced and the lower bound on F_a can be relaxed. Here, we take the lower bound as $F_a \gtrsim 10^{11}$ GeV for illustration. As a result, combined with the cosmological constraint on the two-component DM

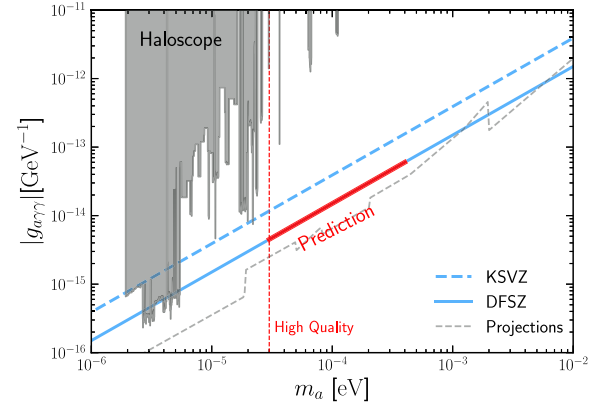


FIG. 1. Predicted parameter space of axion mass m_a and coupling $g_{a\gamma\gamma}$ (red solid line), and the current haloscope experimental limits (gray shaded region). The red dashed line shows the lower limit of axion mass with high quality. The benchmark KSVZ and DFSZ models are shown as blue dashed and solid lines, respectively. The future haloscope experimental projections are shown as gray dashed curves. All experimental limits are compiled from [46]. Some of the astrophysical constraints [47] in this mass range are not shown, as they are far from the predicted parameter space.

composed of axion and χ , our model has a clear prediction of Φ vev and axion mass range of

$$10^{11} \text{ GeV} \lesssim F_a \lesssim 2 \times 10^{12} \text{ GeV}, \quad (14a)$$

$$3 \times 10^{-5} \text{ eV} \lesssim m_a \lesssim 5 \times 10^{-4} \text{ eV}, \quad (14b)$$

shown as the red solid line in Fig. 1. Different from other benchmark models, our model has a well-defined prediction. In most of these predicted parameter spaces, axion is still the dominant DM component and can be tested by direct detection. In this mass range, haloscope experiments [35–45] targeting for the axion-photon coupling have already begun to approach the KSVZ model parameter space, and will soon be able to probe the couplings predicted by our model, which are similar to those in the DFSZ model.⁴ Future haloscope experimental projections (gray dashed lines) could provide a definitive test of our scenario [46].

Even though the light fermion DM makes up only a subdominant component, it can still induce observable astrophysical effects. Typically, nonthermally produced light DM remains a warm relic at present [48], whose distribution is modulated by the dominant cold DM, consequently affecting the spatial structure of visible matter [49].

⁴The axion-photon coupling is determined by the ratio of the electromagnetic anomaly to the color anomaly coefficients. In our model, compared with the DFSZ model, although the PQ charge of fermions is doubled, this ratio remains the same.

This could serve as another smoking gun signature of our model.

V. CONCLUSION AND DISCUSSIONS

In conclusion, the axion, as the Nambu-Goldstone boson associated with the $U(1)_{PQ}$ symmetry, has long been regarded as a compelling solution to the strong CP problem. However, it suffers from a serious quality problem, and the very existence of a global $U(1)_{PQ}$ symmetry is often introduced by hand. In this letter, we have shown that the discrete gauge symmetries $\mathbb{Z}_4$ and $\mathbb{Z}_3$, which can naturally arise within the SM structure, are sufficient to generate a high-quality PQ symmetry and QCD axion. Moreover, the right-handed neutrino required by the model simultaneously accounts for neutrino masses and the baryon asymmetry of the Universe, while the chiral fermion χ remains stable and serves as a part of DM.

Taking into account both the axion quality and cosmological constraints, this framework leads to several testable predictions. First, the QCD axion mass is predicted to lie within the range $10^{-5} \text{ eV} \lesssim m_a \lesssim 10^{-4} \text{ eV}$, and its coupling to photons can be probed by upcoming haloscope experiments. Although phenomenologically our model is similar to the DFSZ model, the detection of a lighter axion with mass $m_a \lesssim 3 \times 10^{-5} \text{ eV}$ in experiments would exclude our model. Second, a light fermionic DM component may slightly modify the matter distribution, leaving observable imprints as a smoking gun. Additionally, the Yukawa coupling of the right-handed neutrino is suppressed by the Planck scale, predicting a relative small Majorana mass $10 \text{ TeV} \lesssim M_N \lesssim 10^3 \text{ TeV}$.⁵ In such a case, the observed baryon asymmetry can be explained through the nonthermal [50] or the resonant leptogenesis [51].

However, if we consider the $B-L$ gauge symmetry instead of the $\mathbb{Z}_4$, we have to introduce a new Higgs field to generate the masses of the right-handed neutrinos. In this extension of the SM, the masses of the right-handed neutrinos are independent of the PQ breaking scale and they can be much larger than the PQ breaking scale, such as $M_{N_i} \sim 10^{15} \text{ GeV}$. See the Appendix for details of the $B-L$ extension of the SM.

Overall, we provide a novel perspective on the strong CP problem and model building: addressing the puzzles of the SM by ingredients already embedded within it. Every new degree of freedom introduced in this setup is well motivated and minimal in nature.

⁵If we assume the charges of Φ to be (2,2), the Φ can couple to $\bar{N}N$ without suppression of the M_{pl} , and the right-handed neutrinos receive masses of the order 10^{10-12} GeV . Furthermore, one of the χ 's becomes a candidate of the DM of mass 10^{4-6} GeV . However, we need the substantial suppression of an allowed potential Φ^6 to keep the high quality of the axion.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [52].

APPENDIX: $B-L$ EXTENSION

The PQ field Φ couples to the right-handed neutrinos $\bar{N}_i$, and its vacuum expectation value generates their Majorana masses $M_{\bar{N}_i}$. Consequently, the masses of $\bar{N}_i$ are related to the PQ breaking scale F_a . This feature is an appealing aspect of the $\mathbb{Z}_4 \times \mathbb{Z}_3$ extension of the SM [53].

However, if we consider the $U(1)_{B-L}$ gauge instead of introducing the $\mathbb{Z}_4$ symmetry, a new Higgs field is naturally introduced to generate the right-handed neutrino masses. In this $B-L$ extension, the masses of the right-handed neutrinos are no longer correlated with the PQ scale and may be significantly larger, e.g., $M_{N_i} \sim 10^{15} \text{ GeV}$. In this appendix, we emphasize that the high quality of the QCD axion can still be maintained even within the $B-L$ extension of the SM.

In the $U(1)_{B-L}$ extended SM, the $\mathbb{Z}_{12}$ discrete symmetry can still be anomaly-free, including the Dai-Freed anomaly [1,2], if we introduce the three left-handed fermions χ_i , too. Now, we assume $SU(3) \times SU(2) \times U(1)_Y \times U(1)_{B-L} \times \mathbb{Z}_{12}$ gauge theory. The charges of all fields are shown in

TABLE III. The charges of fermions under $U(1)_{B-L}$ and $\mathbb{Z}_{12}$ symmetries.

Fermions	T	$\bar{F}$	$\bar{N}$	χ
$B-L$	1	-3	5	0
$\mathbb{Z}_{12}$	4	4	4	-4

TABLE IV. The charges of Higgs under $U(1)_{B-L}$ and $\mathbb{Z}_{12}$ symmetries.

Scalars	H_1	H_2	Φ	Φ'
$B-L$	2	2	0	-10
$\mathbb{Z}_{12}$	-4	4	-1	4

Table III and Table IV. As for the fermion sector the discrete symmetry is equivalent to the original $\mathbb{Z}_3$.

The right-handed neutrinos $\bar{N}_i$ obtain Majorana masses M_N from the vev of Φ' , since we have $\bar{N}_i \bar{N}_i \Phi'$ coupling. On the other hand, the PQ symmetry is spontaneously broken

down by the vev of Φ , while Eq. (4) and $\chi\chi\Phi^4$ coupling in Eq. (6) are unchanged. The PQ symmetry is explicitly broken by Eq. (7), and hence we can keep the high quality of the axion, as in the $\mathbb{Z}_4 \times \mathbb{Z}_3$ model.

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- [1] X.-z. Dai and D. S. Freed, Eta invariants and determinant lines, *J. Math. Phys. (N.Y.)* **35**, 5155 (1994); **42**, 2343(E) (2001).
 - [2] K. Yonekura, Dai-Freed theorem and topological phases of matter, *J. High Energy Phys.* **09** (2016) 022.
 - [3] M. Kawasaki and T. T. Yanagida, Dai-Freed anomaly in the standard model and topological inflation, *J. High Energy Phys.* **11** (2023) 106.
 - [4] P. Minkowski, $\mu \rightarrow e\gamma$ at a rate of one out of 10^9 muon decays?, *Phys. Lett.* **67B**, 421 (1977).
 - [5] T. Yanagida, Horizontal gauge symmetry and masses of neutrinos, *Conf. Proc. C* **7902131**, 95 (1979).
 - [6] T. Yanagida, Horizontal symmetry and mass of the top quark, *Phys. Rev. D* **20**, 2986 (1979).
 - [7] M. Gell-Mann, P. Ramond, and R. Slansky, Complex spinors and unified theories, *Conf. Proc. C* **790927**, 315 (1979).
 - [8] M. Fukugita and T. Yanagida, Baryogenesis without grand unification, *Phys. Lett. B* **174**, 45 (1986).
 - [9] C.-T. Hsieh, Discrete gauge anomalies revisited, [arXiv: 1808.02881](https://arxiv.org/abs/1808.02881).
 - [10] R. D. Peccei and H. R. Quinn, Constraints imposed by CP conservation in the presence of instantons, *Phys. Rev. D* **16**, 1791 (1977).
 - [11] R. D. Peccei and H. R. Quinn, CP conservation in the presence of instantons, *Phys. Rev. Lett.* **38**, 1440 (1977).
 - [12] F. Wilczek, Problem of strong P and T invariance in the presence of instantons, *Phys. Rev. Lett.* **40**, 279 (1978).
 - [13] S. Weinberg, A new light boson?, *Phys. Rev. Lett.* **40**, 223 (1978).
 - [14] M. Kamionkowski and J. March-Russell, Planck scale physics and the Peccei-Quinn mechanism, *Phys. Lett. B* **282**, 137 (1992).
 - [15] R. Holman, S. D. H. Hsu, T. W. Kephart, E. W. Kolb, R. Watkins, and L. M. Widrow, Solutions to the strong CP problem in a world with gravity, *Phys. Lett. B* **282**, 132 (1992).
 - [16] J. E. Kim, Light pseudoscalars, particle physics and cosmology, *Phys. Rep.* **150**, 1 (1987).
 - [17] J. E. Kim and G. Carosi, Axions and the strong CP problem, *Rev. Mod. Phys.* **82**, 557 (2010); **91**, 049902(E) (2019).
 - [18] J. E. Kim, Weak interaction singlet and strong CP invariance, *Phys. Rev. Lett.* **43**, 103 (1979).
 - [19] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Can confinement ensure natural CP invariance of strong interactions?, *Nucl. Phys. B* **166**, 493 (1980).
 - [20] M. Dine, W. Fischler, and M. Srednicki, A simple solution to the strong CP problem with a harmless axion, *Phys. Lett.* **104B**, 199 (1981).
 - [21] A. R. Zhitnitsky, On possible suppression of the axion hadron interactions. (In Russian), *Sov. J. Nucl. Phys.* **31**, 260 (1980), <https://www.osti.gov/biblio/7063072>.
 - [22] S. B. Giddings and A. Strominger, Loss of incoherence and determination of coupling constants in quantum gravity, *Nucl. Phys. B* **307**, 854 (1988).
 - [23] S. R. Coleman, Why there is nothing rather than something: A theory of the cosmological constant, *Nucl. Phys. B* **310**, 643 (1988).
 - [24] G. Gilbert, Wormhole induced proton decay, *Nucl. Phys. B* **328**, 159 (1989).
 - [25] S. W. Hawking, Particle creation by black holes, *Commun. Math. Phys.* **43**, 199 (1975); **46**, 206(E) (1976).
 - [26] C. Abel *et al.*, Measurement of the permanent electric dipole moment of the neutron, *Phys. Rev. Lett.* **124**, 081803 (2020).
 - [27] J. Preskill, M. B. Wise, and F. Wilczek, Cosmology of the invisible axion, *Phys. Lett.* **120B**, 127 (1983).
 - [28] L. Abbott and P. Sikivie, A cosmological bound on the invisible axion, *Phys. Lett.* **120B**, 133 (1983).
 - [29] M. Dine and W. Fischler, The not so harmless axion, *Phys. Lett.* **120B**, 137 (1983).
 - [30] D. J. E. Marsh, Axion cosmology, *Phys. Rep.* **643**, 1 (2016).
 - [31] P. B. Greene and L. Kofman, On the theory of fermionic preheating, *Phys. Rev. D* **62**, 123516 (2000).
 - [32] S. Tremaine and J. E. Gunn, Dynamical role of light neutral leptons in cosmology, *Phys. Rev. Lett.* **42**, 407 (1979).
 - [33] V. Domcke and A. Urbano, Dwarf spheroidal galaxies as degenerate gas of free fermions, *J. Cosmol. Astropart. Phys.* **01** (2015) 002.
 - [34] J. Alvey, N. Sabti, V. Tiki, D. Blas, K. Bondarenko, A. Boyarsky, M. Escudero, M. Fairbairn, M. Orkney, and J. I. Read, New constraints on the mass of fermionic dark matter from dwarf spheroidal galaxies, *Mon. Not. R. Astron. Soc.* **501**, 1188 (2021).
 - [35] S. DePanfilis, A. C. Melissinos, B. E. Moskowitz, J. T. Rogers, Y. K. Semertzidis, W. U. Wuensch, H. J. Halama, A. G. Prodell, W. B. Fowler, and F. A. Nezrick, Limits on the abundance and coupling of cosmic axions at 4.5lt ; $m_a\text{lt}$; 5.0meV , *Phys. Rev. Lett.* **59**, 839 (1987).
 - [36] W. Wuensch, S. De Panfilis-Wuensch, Y. K. Semertzidis, J. T. Rogers, A. C. Melissinos, H. J. Halama, B. E. Moskowitz, A. G. Prodell, W. B. Fowler, and F. A. Nezrick, Results of a laboratory search for cosmic axions and other weakly coupled light particles, *Phys. Rev. D* **40**, 3153 (1989).

- [37] C. Hagmann, P. Sikivie, N. S. Sullivan, and D. B. Tanner, Results from a search for cosmic axions, *Phys. Rev. D* **42**, 1297 (1990).
- [38] C. Hagmann *et al.*, First results from a second generation galactic axion experiment, *Nucl. Phys. B, Proc. Suppl.* **51**, 209 (1996).
- [39] B. T. McAllister, G. Flower, J. Kruger, E. N. Ivanov, M. Goryachev, J. Bourhill, and M. E. Tobar, The ORGAN experiment: An axion haloscope above 15 GHz, *Phys. Dark Universe* **18**, 67 (2017).
- [40] A. P. Quiskamp, B. T. McAllister, P. Altin, E. N. Ivanov, M. Goryachev, and M. E. Tobar, Direct search for dark matter axions excluding ALPogenesis in the 63- to 67- μ eV range with the ORGAN experiment, *Sci. Adv.* **8**, abq3765 (2022).
- [41] A. Quiskamp, B. T. McAllister, P. Altin, E. N. Ivanov, M. Goryachev, and M. E. Tobar, Exclusion of axionlike-particleogenesis dark matter in a mass window above 100 μ eV, *Phys. Rev. Lett.* **132**, 031601 (2024).
- [42] S. Ahn *et al.* (CAPP Collaboration), Extensive search for axion dark matter over 1 GHz with CAPP'S main axion experiment, *Phys. Rev. X* **14**, 031023 (2024).
- [43] A. Rettaroli *et al.* (QUAX Collaboration), Search for axion dark matter with the QUAX-LNF tunable haloscope, *Phys. Rev. D* **110**, 022008 (2024).
- [44] A. P. Quiskamp, G. R. Flower, S. Samuels, B. T. McAllister, P. Altin, E. N. Ivanov, M. Goryachev, and M. E. Tobar, Near-quantum-limited axion dark matter search with the ORGAN experiment around 26 μ eV, *Phys. Rev. D* **111**, 095007 (2025).
- [45] C. Goodman *et al.* (ADMX Collaboration), ADMX axion dark matter bounds around 3.3 μ eV with Dine-Fischler-Srednicki-Zhitnitsky discovery ability, *Phys. Rev. Lett.* **134**, 111002 (2025).
- [46] C. O'Hare, Axionlimits: Data, plots and code for constraints on axions, axion-like particles, and dark photons, <https://cajohare.github.io/AxionLimits/> (2020), (accessed: 2025-10-17).
- [47] P. Carenza, M. Giannotti, J. Isern, A. Mirizzi, and O. Straniero, Axion astrophysics, *Phys. Rep.* **1117**, 1 (2025).
- [48] C. Dvorkin, T. Lin, and K. Schutz, Cosmology of sub-MeV dark matter freeze-in, *Phys. Rev. Lett.* **127**, 111301 (2021).
- [49] S.-F. Ge and L. Tan, Probing light dark matter with cosmic gravitational focusing, [arXiv:2509.21213](https://arxiv.org/abs/2509.21213).
- [50] W. Buchmuller, R. D. Peccei, and T. Yanagida, Leptogenesis as the origin of matter, *Annu. Rev. Nucl. Part. Sci.* **55**, 311 (2005).
- [51] A. Pilaftsis and T. E. J. Underwood, Resonant leptogenesis, *Nucl. Phys. B* **692**, 303 (2004).
- [52] <https://cajohare.github.io/AxionLimits/docs/ap.html>.
- [53] P. Langacker, R. D. Peccei, and T. Yanagida, Invisible axions and light neutrinos: Are they connected?, *Mod. Phys. Lett. A* **01**, 541 (1986).