

Search for the $D^*\bar{D}^*$ molecular state $X_2(4013)$ in K^-p and pp collisions

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Motivated by the interpretation of $X(3872)$ as a $D\bar{D}^*$ molecular state, heavy-quark spin symmetry predicts a spin-2 partner, $X_2(4013)$, which can be regarded as a $D^*\bar{D}^*$ molecule with quantum numbers $J^{PC} = 2^{++}$. Its experimental confirmation, however, remains elusive. In this work, we investigate the production mechanisms of $X_2(4013)$ in the reactions $K^-p \rightarrow D_s^- \Lambda_c^+ X_2(4013)$ and $pp \rightarrow \Lambda_c^+ \Lambda_c^+ X_2(4013)$ within an effective Lagrangian framework. The production processes are modeled via t -channel $D^*/\bar{D}^*$ meson exchanges, while initial-state interactions (ISIs) mediated by Pomeron and Reggeon exchanges are also taken into account. Our calculations indicate that the total cross sections can reach the pb level, suggesting that $X_2(4013)$ may be accessible at current and future experiments such as AMBER@CERN and LHCb. Inclusion of ISIs enhances the cross sections by nearly 1 order of magnitude. The differential distributions show distinct angular behaviors for the two reactions: the K^-p reaction exhibits a forward-peaked distribution, whereas the pp reaction shows a dip near central angles. This study provides a quantitative theoretical benchmark for future experimental searches of $X_2(4013)$ and highlights the importance of ISIs in high-energy particle investigations.

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I. INTRODUCTION

In recent years, the discovery of numerous hadrons containing heavy quarks has dramatically expanded our understanding of strong interactions, driven by higher-energy experiments, improved statistics, and rapid advances in detection and analysis techniques [1–6]. Many of these states lie beyond the scope of the conventional quark model, offering unique insights into the nonperturbative regime of QCD [7]. Among the various theoretical frameworks proposed to interpret near-threshold exotic states, hadronic molecular states—bound or resonant systems of two or more conventional hadrons—have emerged as particularly compelling candidates [7].

A prototypical molecular state candidate is the charmoniumlike $X(3872)$, which was first observed by the Belle Collaboration in 2003 in the $J/\psi\pi^+\pi^-$ invariant mass spectrum from $B \rightarrow KJ/\psi\pi^+\pi^-$ decays [8] and subsequently confirmed by BABAR [9], CDF [10], D0 [11], CMS [12], LHCb [13], and BESIII [14]. Notably, later measurements observed $X(3872)$ not only in the $J/\psi\pi^+\pi^-$

channel but also in several other decay modes, including $\bar{D}^0 D^0 \pi^0$ [15], $\bar{D}^{*0} D^0$ [16–18], $J/\psi\gamma$ [19–21], and $J/\psi\omega$ [22]. These observations can be naturally interpreted within a molecular framework. In particular, the $\bar{D}^0 D^0 \pi^0$ [15] and $\bar{D}^{*0} D^0$ [16–18] channels are dominated by tree-level decays of the $D\bar{D}^*$ molecular component, while pronounced isospin-violating effects further support interpreting $X(3872)$ as a $D\bar{D}^*$ molecular state with quantum numbers $J^{PC} = 1^{++}$ [23]. Its decays are primarily realized through loop processes involving the $D\bar{D}^*$ molecular component.

In addition to direct searches for the molecular nature of $X(3872)$, indirect approaches can also provide valuable insights. Interpreting $X(3872)$ as a $D\bar{D}^*$ molecular state, heavy quark symmetry [24] predicts the existence of its molecular partners, a $D^*\bar{D}^*$ state with spin-parity $J^{PC} = 2^{++}$, denoted as $X_2(4013)$. Observing $X_2(4013)$ would offer a crucial test of whether $X(3872)$ is indeed a genuine $D\bar{D}^*$ molecular state. Beyond heavy quark symmetry, numerous theoretical studies also support the existence of the $D^*\bar{D}^*$ molecular state [7,25–34]. These studies cover several widely used theoretical frameworks—including contact-range effective field theory [28], one-boson-exchange models [29], and lattice QCD [30]—and consistently predict its mass to be around 4013 MeV.

Despite strong theoretical support, experimental confirmation of $X_2(4013)$ remains lacking. In particular, direct

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analyses of the $D^*\bar{D}^*$ invariant mass spectrum have not revealed a clear resonance near 4013 MeV [35]. Although BESIII has reported potential signals in the processes $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ and $e^+e^- \rightarrow \pi^+\pi^-h_c$ [36], higher-statistics data are required for definitive confirmation. The proximity of $X_2(4013)$ to $Z_c(4020)$ suggests that complex coupled-channel dynamics may affect its observation [37], highlighting the need for novel experimental strategies. Indeed, several alternative approaches have been explored, including kinematic reconstruction of $B^+ \rightarrow K^+X_2(4013) \rightarrow K^+\bar{D}^0D^0\pi^0$ to search for $X_2(4013)$ [38]; however, isospin-breaking effects in this process may make it difficult for it to occur in practice. Radiative transitions such as $Y(4360) \rightarrow X_2(4013)\gamma$ [39] have also been considered for direct detection of $X_2(4013)$. Direct e^+e^- collision searches can only set upper limits on the production cross section [40], indicating that more sensitive and diverse production mechanisms are required.

Kaon-induced reactions at facilities such as OKA@U-70 [41], SPS@CERN [42], and the newly commissioned AMBER@CERN [43], as well as proton-proton collisions at LHC@CERN [44,45], ALICE@LHC [46], and RHIC@BNL [47], offer promising experimental environments for the discovery of such exotic states. Future high-luminosity programs at colliders like the FCC [48] further enhance these prospects. Motivated by these opportunities, we study the production cross sections of $X_2(4013)$ in the reactions $K^-p \rightarrow D_s^-\Lambda_c^+X_2(4013)$ and $pp \rightarrow \Lambda_c^+\Lambda_c^+X_2(4013)$, incorporating initial-state interactions (ISIs) of the K^-p and pp systems. This approach establishes theoretical benchmarks essential for guiding forthcoming experimental searches and maximizes the potential for observing $X_2(4013)$ across multiple channels.

This paper is organized as follows. In Sec. II, we present the theoretical formalism. In Sec. III, the numerical result and discussions are given. This is followed by conclusions in the last section.

II. THEORETICAL FORMALISM

In this work, we focus on the production of $X_2(4013)$ in the reactions $K^-p \rightarrow D_s^-\Lambda_c^+X_2(4013)$ and $pp \rightarrow \Lambda_c^+\Lambda_c^+X_2(4013)$. The corresponding tree-level Feynman diagrams are shown in Fig. 1, which illustrate the central production mechanism. At high energies, the production is dominated by free D^* and $\bar{D}^*$ mesons, whereas at lower energies, strong interactions between D^* and $\bar{D}^*$ lead to the formation of the molecular state $X_2(4013)$.

To compute the amplitudes corresponding to Fig. 1, we first define the effective Lagrangian densities for the relevant interaction vertices. The couplings for $X_2\bar{D}^*D^*$, $\Lambda_c p D^*$, and $K\bar{D}^*D_s$ interactions are given by [39,49,50]

$$\mathcal{L}_{X_2\bar{D}^*D^*} = g_{X_2} X_2^{\mu\nu}(4013) D_\mu^* \bar{D}_\nu^*, \quad (1)$$

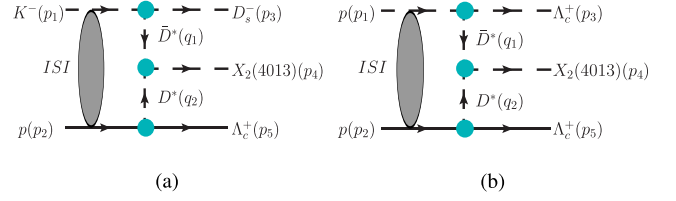


FIG. 1. Tree-level Feynman diagrams for $X_2(4013)$ production. (a) Diagram for the $K^-p \rightarrow D_s^-\Lambda_c^+X_2(4013)$ reaction via t -channel D^* exchange. (b) Diagram for the $pp \rightarrow \Lambda_c^+\Lambda_c^+X_2(4013)$ reaction via t -channel $\bar{D}^*$ exchange. The gray-shaded region indicates initial-state interactions (ISIs). Particle momenta ($p_1, p_2, p_3, p_4, p_5, q_1, q_2$) are indicated.

$$\mathcal{L}_{\Lambda_c p D^*} = g_{\Lambda_c p D^*} \bar{\Lambda}_c \gamma^\mu p D_\mu^{*0} + \text{H.c.}, \quad (2)$$

$$\mathcal{L}_{K\bar{D}^*D_s} = ig_{K\bar{D}^*D_s} \bar{D}^{*\mu} [\bar{D}_s \partial_\mu K - (\partial_\mu \bar{D}_s) K] + \text{H.c.} \quad (3)$$

Here, the coupling $g_{\Lambda_c p D^*} = -5.20$ GeV is derived from SU(4) symmetry using benchmark values $g_{\pi NN} = 13.45$ GeV and $g_{\rho NN} = 6.0$ GeV [51]. The couplings $g_{X_2} = 9.0$ GeV and $g_{K\bar{D}^*D_s} = 5.0$ GeV are adopted from Refs. [39,50].

For the diagram in Fig. 1b, the $\Lambda_c p \bar{D}^*$ vertex is described by the Lagrangian [49],

$$\mathcal{L}_{\Lambda_c p \bar{D}^*} = g_{\Lambda_c p \bar{D}^*} \bar{\Lambda}_c \gamma^\mu p \bar{D}_\mu^{*0} + \text{H.c.}, \quad (4)$$

where $g_{\Lambda_c p \bar{D}^*} = -5.20$ GeV is fixed similarly through SU(4) symmetry.

Since hadrons are not pointlike, we include form factors for the exchanged $\bar{D}^*$ and D^* mesons. We adopt a commonly used monopole form factor,

$$\mathcal{F}_i = \frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 - q_i^2}, \quad i = \bar{D}^*, D^*, \quad (5)$$

where q_i and m_i are the 4-momentum and mass of the exchanged meson, respectively. The cutoff Λ_i characterizes the hadronic size scale and is empirically chosen to exceed the meson mass by several hundred MeV, $\Lambda_i = m_i + \alpha \Lambda_{\text{QCD}}$, with $\Lambda_{\text{QCD}} = 220$ MeV. The parameter α , reflecting nonperturbative QCD effects at low energies, is treated as a free parameter to be discussed in Sec. III.

The propagators for the $\bar{D}^*$ and D^* mesons are given by

$$G_i^{\mu\nu}(q) = \frac{i(-g^{\mu\nu} + q_i^\mu q_i^\nu / m_i^2)}{q_i^2 - m_i^2}, \quad i = \bar{D}^*, D^*, \quad (6)$$

where μ and ν denote the polarization indices of the corresponding vector mesons.

Using the effective Lagrangians and propagators introduced above, the invariant scattering amplitudes for the reactions shown in Fig. 1 can be written as

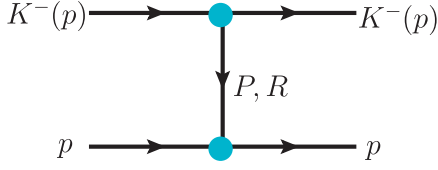


FIG. 2. The ISI mechanisms for the reactions $K^-p \rightarrow K^-p$ and $pp \rightarrow pp$ are mediated by the exchange of the Pomeron and the f_2 , a_2 , ρ , and ω Reggeons, which are denoted by P and R , respectively.

$$\mathcal{M}_a^{\text{Born}} = ig_a \bar{u}(p_5) \gamma^\mu u(p_2) \frac{-g^{\mu\nu} + q_2^\mu q_2^\nu / m_{D^*}^2}{q_2^2 - m_{D^*}^2} \epsilon^{\nu\sigma}(p_4) \times \frac{-g^{\sigma\eta} + q_1^\sigma q_1^\eta / m_{D^*}^2}{q_1^2 - m_{D^*}^2} (p_3^\eta + p_1^\eta) \mathcal{F}, \quad (7)$$

$$\mathcal{M}_b^{\text{Born}} = -g_b \bar{u}(p_5) \gamma^\mu u(p_2) \frac{-g^{\mu\nu} + q_2^\mu q_2^\nu / m_{D^*}^2}{q_2^2 - m_{D^*}^2} \gamma^\sigma \times \frac{-g^{\sigma\eta} + q_1^\sigma q_1^\eta / m_{D^*}^2}{q_1^2 - m_{D^*}^2} \mathcal{F}, \quad (8)$$

where $g_a = g_{X_2} g_{\Lambda_c p D^*} g_{K \bar{D}^* D_s}$, $g_b = g_{X_2} g_{\Lambda_c p \bar{D}^*} g_{K \bar{D}^* D_s}$, and $\mathcal{F} = \mathcal{F}_{\bar{D}^*} \mathcal{F}_{D^*}$ denotes the product of the form factors for the exchanged mesons.

To enhance the predictive reliability, we incorporate ISIs in the $K^-p \rightarrow K^-p$ and $pp \rightarrow pp$ channels, which are represented by the shaded region in Fig. 1. These ISI mechanisms are essential for a realistic description of the production dynamics. Our approach follows the simplified framework of Refs. [52,53], where the elastic $\bar{K}N \rightarrow \bar{K}N$ and $NN \rightarrow NN$ amplitudes are modeled solely through Pomeron and Reggeon exchanges. Despite its simplicity, this model reproduces elastic scattering data with high accuracy [52,53]. By employing the optical theorem, the elastic amplitude can be consistently related to the total cross section, ensuring a coherent description of both elastic and total scattering.

The tree-level Feynman diagrams in Fig. 2 provide a detailed illustration of these fundamental ISI exchange mechanisms, which include contributions from the Pomeron and the Reggeons f_2, a_2, ρ , and ω . The full scattering amplitude for the $\bar{K}N \rightarrow \bar{K}N$ and $NN \rightarrow NN$ processes is obtained as a coherent sum of these exchanges [52,53],

$$T(s, t) = A_P(s, t) + A_{f_2}(s, t) \pm A_{a_2}(s, t) + A_\omega(s, t) \pm A_\rho(s, t), \quad (9)$$

where s and t are the squared center-of-mass energy and squared 4-momentum transfer, respectively, and the energy scale is set as $s_0 = 1 \text{ GeV}^2$. The signs correspond to the specific charge channels: the upper signs apply

TABLE I. The parameters of the Pomeron and Reggeon exchanges were determined based on elastic and total cross section data in Refs. [52,53].

i	η_i	$\alpha_i(t)$	$C_i^{\bar{K}N}$ (mb)	C_i^{NN} (mb)
P	i	$1.081 + (0.25 \text{ GeV}^{-2})t$	11.82	21.70
f_2	$-0.861 + i$	$0.548 + (0.93 \text{ GeV}^{-2})t$	15.67	75.4875
ρ	$-1.162 - i$	$0.548 + (0.93 \text{ GeV}^{-2})t$	2.05	1.0925
ω	$-1.162 - i$	$0.548 + (0.93 \text{ GeV}^{-2})t$	7.055	20.0625
a_2	$-0.861 + i$	$0.548 + (0.93 \text{ GeV}^{-2})t$	1.585	1.7475

to K^-p or pp scattering, while the lower signs apply to K^-n scattering.

In the high-energy regime, each partial amplitude shown in Eq. (9) contributing to $\bar{K}N \rightarrow \bar{K}N$ or $NN \rightarrow NN$ scattering is parametrized using the standard Regge pole form [52,53],

$$A_i(s, t) = \eta_i s C_i^{\bar{K}N/NN} \left(\frac{s}{s_0} \right)^{\alpha_i(t)-1} \exp \left(\frac{B_i^{\bar{K}N/NN}}{2} t \right), \quad (10)$$

where $\alpha_i(t) = \alpha_i(0) + \alpha'_i t$ is the linear Regge trajectory, η_i is the signature factor, and $C_i^{\bar{K}N}$ (C_i^{NN}) is the residue coupling at the $\bar{K}N$ (NN) vertex. The slope parameters $B_i^{\bar{K}N/NN}$, which characterize the diffractive cone for each trajectory, are adopted from Refs. [52,53] as follows: $B_P^{\bar{K}N} = 5.5 \text{ GeV}^{-2}$, $B_R^{\bar{K}N} = 4.0 \text{ GeV}^{-2}$, $B_P^{NN} = 9.0 \text{ GeV}^{-2}$, and $B_R^{NN} = 6.0 \text{ GeV}^{-2}$. The remaining parameters— $C_i^{\bar{K}N}$, C_i^{NN} , $\alpha_i(0)$, α'_i and η_i —are taken from Ref. [52] and listed in Table I. This complete parameter set is constrained by fits to experimental data for both elastic differential and total cross sections.

After incorporating the initial-state interactions, the full amplitude can be written as [52]

$$\mathcal{M}^{\text{full}} = \mathcal{M}^{\text{Born}} + \frac{i}{16\pi^2 s} \int d^2 \vec{k}_t \mathcal{T}(s, k_t^2) \mathcal{M}^{\text{Born}}(s, k_t^2), \quad (11)$$

where $\vec{k}_t$ denotes the momentum transfer in the $K^-p \rightarrow K^-p$ or $pp \rightarrow pp$ reaction. Using this amplitude, we calculate the differential cross sections for the processes $K^-p \rightarrow D_s^- \Lambda_c^+ X_2(4013)$ and $pp \rightarrow \Lambda_c^+ \Lambda_c^+ X_2(4013)$ in the center-of-mass frame. Since these are $2 \rightarrow 3$ reactions, the cross sections follow from the total amplitude according to

$$d\sigma = \frac{m_p}{2\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}} \sum_{s_i, s_f} |\mathcal{M}^{\text{full}}|^2 \times \frac{d^3 \vec{p}_3}{2E_3} \frac{d^3 \vec{p}_4}{2E_4} \frac{m_{\Lambda_c} d^3 \vec{p}_5}{E_5} \delta^4(p_1 + p_2 - p_3 - p_4 - p_5), \quad (12)$$

Here, E_3 , E_4 , and E_5 denote the energies of D_s^- , X_2 , Λ_c^+ in the K^-p reaction and of Λ_c^+ , X_2 , Λ_c^+ in the pp reaction, respectively. The employed particle masses are $m_{K^-} = 493.68$ MeV, $m_{\Lambda_c^+} = 2286.46$ MeV, and $m_p = 938.27$ MeV.

III. RESULTS AND DISCUSSIONS

In this work, our main objective is to search for the theoretically predicted $J^{PC} = 2^{++}$ $\bar{D}^*D^*$ molecular state, for which only tentative experimental indications exist but no firm confirmation has yet been achieved [36]. We propose to study its production through the reactions $K^-p \rightarrow D_s^- \Lambda_c^+ X_2(4013)$ and $pp \rightarrow \Lambda_c^+ \Lambda_c^+ X_2(4013)$. The calculations are performed within the effective Lagrangian framework, considering the central production mechanism [52,53] with t -channel exchange of D^* and $\bar{D}^*$ mesons. The s - and u -channel contributions are not considered because they would necessitate the production of baryons containing two $c\bar{c}$ pairs. The corresponding thresholds are quite high, with the lowest possible masses being $m_{D_s^-} + m_{X_2(4013)} + m_{\Lambda_c^+} = 8267.81$ MeV for the reaction $K^-p \rightarrow D_s^- \Lambda_c^+ X_2(4013)$, and $m_{\Lambda_c^+} + m_{X_2(4013)} + m_{\Lambda_c^+} = 8585.92$ MeV for $pp \rightarrow \Lambda_c^+ \Lambda_c^+ X_2(4013)$.

The dominant source of theoretical uncertainty arises from the parameter α , which characterizes the form factor associated with D^* and $\bar{D}^*$ mesons exchange. Since α cannot be determined from first principles, it is constrained phenomenologically through fits to experimental data. Fortunately, values of $\alpha = 1.5$ and 1.7 have been obtained by fitting the processes $e^+e^- \rightarrow D\bar{D}$ [54] and $e^+e^- \rightarrow \gamma_{\text{ISR}} D\bar{D}$ [55], as detailed in Ref. [56]. In the present analysis, we adopt $\alpha = 1.5$ and 1.7 for our calculations.

Using the determined α values, the cross sections for the reactions $K^-p \rightarrow D_s^- \Lambda_c^+ X_2(4013)$ and $pp \rightarrow \Lambda_c^+ \Lambda_c^+ X_2(4013)$ are evaluated. The theoretical results, obtained with a cutoff $\alpha = 1.5$ or 1.7 , are shown in Fig. 3 for energies from near threshold up to 58 GeV. The cross sections exhibit a sharp increase near the thresholds, occurring at 8267.81 and 8585.92 MeV for the two reactions, respectively, due to the opening of phase space. Beyond the thresholds, the cross sections continue to rise more gradually and eventually stabilize at higher energies. Moreover, the production cross section of $X_2(4013)$ in the K^-p reaction is larger than that in the pp reaction. For example, at 50.27 GeV, the K^-p collision cross section stabilizes at 8.161 ($\alpha = 1.5$) and 12.881 pb ($\alpha = 1.7$), while at 50.09 GeV, the pp collision cross section stabilizes at 0.813 ($\alpha = 1.5$) and 1.315 pb ($\alpha = 1.7$).

The results also show that the line shapes of the cross sections for the $K^-p \rightarrow D_s^- \Lambda_c^+ X_2(4013)$ and $pp \rightarrow \Lambda_c^+ \Lambda_c^+ X_2(4013)$ reactions differ significantly in the energy region $\sqrt{s} = 20.77 - 36.77$ GeV. That is, in this region, the cross section for the $K^-p \rightarrow D_s^- \Lambda_c^+ X_2(4013)$ reaction increases much more rapidly than that for the

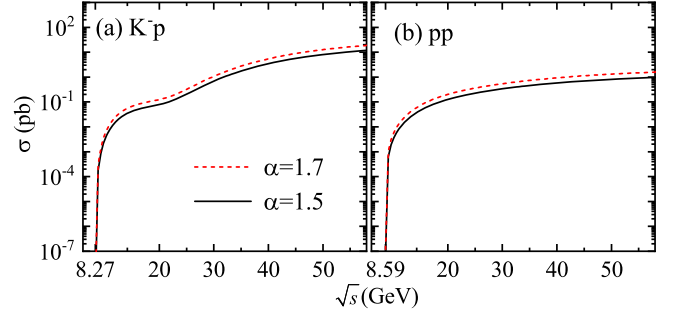


FIG. 3. Total cross sections for $X_2(4013)$ production as functions of the center-of-mass energy $\sqrt{s}$. Results are shown for the $K^-p \rightarrow D_s^- \Lambda_c^+ X_2(4013)$ (left curves) and $pp \rightarrow \Lambda_c^+ \Lambda_c^+ X_2(4013)$ (right curves) reactions, calculated with two different values of the form factor cutoff parameter $\alpha = 1.5$ (red dashed lines) and $\alpha = 1.7$ (black solid lines).

$pp \rightarrow \Lambda_c^+ \Lambda_c^+ X_2(4013)$ reaction. This difference may arise from the significant role of the Pauli exclusion principle in the pp initial state. In the pp reaction, the cross section rises smoothly above threshold due to the competition between short-range attraction and Pauli repulsion. At sufficiently high energies, the dynamics of internal quarks and gluons become dominant, leading to a saturation of the cross section. In contrast, the K^-p reaction is free from Pauli constraints, allowing wave function overlap under interaction, which enhances the production cross section (corresponding to $\sqrt{s} = 20.77 - 36.77$ GeV). At even higher energies, the reaction gradually becomes dominated by quark and gluon dynamics, and the cross section saturates in this region as well. Moreover, the production of the $\Lambda_c^+ \Lambda_c^+$ pair in the pp reaction is also subject to Pauli exclusion effects, which may partially explain why the $X_2(4013)$ production cross section in pp collisions is smaller than that in K^-p reactions.

As shown in Fig. 3, the cross sections exhibit a strong sensitivity to the form factor parameter α . For the reaction $K^-p \rightarrow D_s^- \Lambda_c^+ X_2(4013)$ at $\sqrt{s} = 30.27$ GeV, we obtain

$$\sigma(\alpha = 1.5) \approx 0.728 \text{ pb}, \quad \sigma(\alpha = 1.7) \approx 1.139 \text{ pb}.$$

Hence, the enhancement factor is

$$Br \equiv \frac{\sigma(\alpha = 1.7)}{\sigma(\alpha = 1.5)} \approx \frac{1.139}{0.728} \approx 1.564, \quad (13)$$

which indicates that the cross section for $\alpha = 1.7$ is roughly 56.4% larger than that for $\alpha = 1.5$. For the $pp \rightarrow \Lambda_c^+ \Lambda_c^+ X_2(4013)$ reaction at $\sqrt{s} = 30.09$ GeV, the α -dependent factor Br is 1.598. However, as the energy increases, this factor becomes larger. For example, we obtain $Br = 1.578$ for the $K^-p \rightarrow D_s^- \Lambda_c^+ X_2(4013)$ reaction at $\sqrt{s} = 50.27$ GeV and $Br = 1.617$ for the $pp \rightarrow \Lambda_c^+ \Lambda_c^+ X_2(4013)$ reaction at $\sqrt{s} = 50.09$ GeV. This result further indicates that particular caution must be exercised

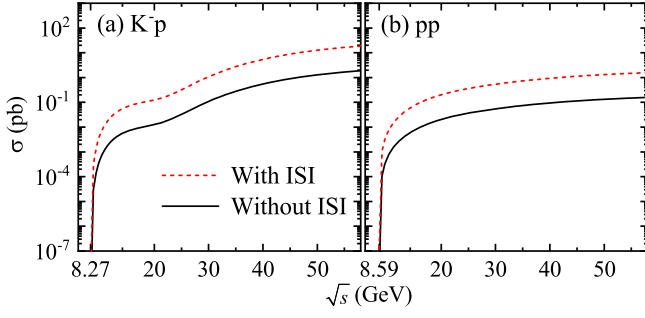


FIG. 4. Effect of initial-state interactions (ISIs) on the total cross sections for $X_2(4013)$ production, calculated with $\alpha = 1.7$. (a) Comparison of the cross sections for the $K^-p \rightarrow D_s^- \Lambda_c^+ X_2(4013)$ reaction with ISI (black solid line) and without ISI (Born term only, red dashed line). (b) Same as (a) but for the $pp \rightarrow \Lambda_c^+ \Lambda_c^+ X_2(4013)$ reaction.

when treating the form factor of the D^* and $\bar{D}^*$ mesons and highlights the importance of precise experimental determination of this parameter.

To illustrate the role of ISIs in the production of $X_2(4013)$, we compare the cross sections for K^-p and pp collisions with the cutoff parameter fixed at $\alpha = 1.7$. Figure 4(a) shows the result including K^-p ISI, while Fig. 4(b) presents the corresponding case for pp ISI. In both reactions, ISI leads to a pronounced enhancement of the cross section. For instance, at $\sqrt{s} = 50.27$ GeV, the total cross section for K^-p scattering is 12.881 pb, compared with the Born-level value of 1.304 pb, yielding an enhancement factor of about 9.878. Likewise, at $\sqrt{s} = 50.09$ GeV, the pp cross section increases from 0.133 to 1.315 pb, corresponding to a factor of roughly 9.887. Although the enhancement factors are nearly identical, the effect in pp collisions is marginally larger. This subtle difference may stem from the intrinsic distinctions in ISI dynamics between meson-baryon and baryon-baryon systems [57]. Such modifications, arising from the K^-p and pp interactions, indicate that these effects are essential and cannot be neglected in studies of heavy-quark particle production.

In addition to the total cross sections, we also calculated the differential cross sections for the reactions $K^-p \rightarrow D_s^- \Lambda_c^+ X_2(4013)$ and $pp \rightarrow \Lambda_c^+ \Lambda_c^+ X_2(4013)$ as functions of the scattering angle of the outgoing Λ_c relative to the beam direction at different kaon and proton beam energies, i.e., $P_{K^-/p} = 20.0, 30.0$, and 40.0 GeV. The theoretical results are shown in Fig. 5.

As shown in Fig. 5, for the reaction $K^-p \rightarrow D_s^- \Lambda_c^+ X_2(4013)$, the differential cross section exhibits a simple, monotonic behavior: it is strongly enhanced in the extreme forward direction ($\cos \theta \rightarrow 1$) and decreases smoothly toward the backward direction ($\cos \theta \rightarrow -1$), which is a typical signature of a t -channel contribution. This behavior is consistent with the physical processes considered in our study [see Fig. 1(a)]. Interestingly, although the reaction mechanism for $pp \rightarrow \Lambda_c^+ \Lambda_c^+ X_2(4013)$ is the same

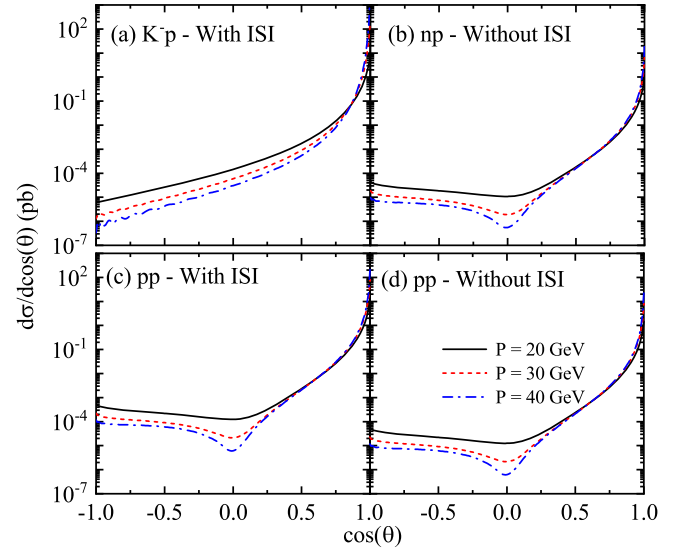


FIG. 5. Differential cross sections for (a) $K^-p \rightarrow D_s^- \Lambda_c^+ X_2(4013)$ (with ISI), (b) $np \rightarrow \Lambda_c^+ \Sigma_c(2445) X_2(4013)$, and (c)(d) $pp \rightarrow \Lambda_c^+ \Lambda_c^+ X_2(4013)$ (with and without ISI, respectively), calculated with $\alpha = 1.7$. The black solid lines, red dashed lines, and blue dot-dashed lines are obtained at beam energies $P = 20, 30$, and 40 GeV, respectively.

as that for $K^-p \rightarrow D_s^- \Lambda_c^+ X_2(4013)$ (both being central production processes mediated by t -channel D^* and $\bar{D}^*$ exchanges), the differential cross section for the pp reaction behaves quite differently. It does not display the characteristic features of a t -channel contribution; instead, it exhibits a nonmonotonic shape, with a pronounced dip near central angles ($\cos \theta \approx 0$) and a significant increase toward both the forward and backward directions. This trend becomes more pronounced as the incident proton energy increases, as shown in Fig. 5(c). Specifically, as the incident proton energy varies from 20 to 40 GeV, the dip structure becomes increasingly prominent.

It should be noted that the dip structure observed in the differential cross section of the $pp \rightarrow \Lambda_c^+ \Lambda_c^+ X_2(4013)$ reaction does not originate from the ISI of the $pp \rightarrow pp$ system. As shown in Fig. 5(d), even when the ISI contribution is switched off, a similar dip still appears around $\cos \theta = 0$. At the same time, we have also ruled out the possibility that the dip is caused by the presence of identical p and Λ_c^+ in the initial and final states, which would invoke the Pauli exclusion principle. We have further calculated the reaction $np \rightarrow \Lambda_c^+ \Sigma_c(2445) X_2(4013)$. The results show that a similar dip structure also appears in the differential cross section of $np \rightarrow \Lambda_c^+ \Sigma_c(2445) X_2(4013)$ [see Fig. 5(b)]. One possible reason is that this dip arises from a “dynamical zero” caused by helicity conservation. The detailed differences can be seen in their respective amplitudes, given by Eqs. (7) and (8). From this, we find that the K^-p system involves a relatively simple meson derivative coupling, whereas the pp system features a more complex baryon current coupling with nontrivial helicity

structure. The latter is subject to stronger constraints from helicity conservation when coupling to high-spin final-state particles, which gives rise to the distinctive central dip observed in the differential cross section, and this effect is dependent on the incident energy.

IV. CONCLUSIONS

Thanks to significant experimental progress, it is now possible to search for the $J^P = 2^{++} D^* \bar{D}^*$ molecular state $X_2(4013)$, which is predicted by heavy-quark symmetry as the spin-2 partner of the $X(3872)$, commonly interpreted as a $D\bar{D}^*$ molecule. Despite previous efforts, we propose investigating it via the reactions $K^- p \rightarrow D_s^- \Lambda_c^+ X_2(4013)$ and $pp \rightarrow \Lambda_c^+ \Lambda_c^+ X_2(4013)$. The reaction mechanisms mainly involve central production through t -channel D^* and $\bar{D}^*$ exchange, while also accounting for ISIs dominated by Pomeron and Reggeon exchanges. The only parameter in the model is the form factor α , for which we adopt the values $\alpha = 1.5$ and 1.7 , determined from fits to experimental data.

The calculated total cross sections for both reaction channels can reach the pb level, suggesting that experimental observation is feasible at facilities such as AMBER@CERN and LHCb. Specifically, at a center-of-mass energy of 50.27 GeV, the $K^- p$ collision cross section stabilizes at 8.161 pb for $\alpha = 1.5$ and 12.881 pb for $\alpha = 1.7$. Meanwhile, at 50.09 GeV, the pp collision cross section stabilizes at 0.813 ($\alpha = 1.5$) and 1.315 pb ($\alpha = 1.7$). The $K^- p$ and pp ISIs mediated by Pomeron and Reggeon exchanges play a crucial role in the search for the $X_2(4013)$, as they enhance the total cross sections by nearly an order of magnitude compared with the results without ISI. The differential cross sections exhibit distinctly different angular behaviors: the $K^- p$ reaction shows a monotonic forward-peaking distribution, typical of t -channel-dominated processes, whereas the pp reaction displays a nonmonotonic dip near $\cos \theta \approx 0$.

DATA AVAILABILITY

The data that support the findings of this article are openly available [58].

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