

Probing invisible particles with charm hadron decays

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We point out opportunities to probe invisible particles, left- and right-handed neutrinos, axionlike particles and dark photons (Z') with rare decays of charm hadrons. We employ and recast existing searches in $D \rightarrow (\pi, \omega)X$, $D^0 \rightarrow X$ and $\Lambda_c \rightarrow pX$, where X denotes one of the above invisible final states including dineutrinos. The branching ratios are clean null tests of the standard model yet are essentially unconstrained for some parameters of light new physics, limited only by weak lifetime constraints at the level of $\mathcal{O}(10^{-1})$. On the other hand, if models are probed, branching ratios still reach up to 10^{-3} (Z') and 10^{-4} (axionlike particles). Chirality-preserving operators from heavy new physics in the dimension-six standard model effective theory imply tighter upper limits, up to few $\times 10^{-5}$. Constraints on chirality-flipping heavy new physics, such as lepton number violation from dimension-seven standard model effective theory, or with light sterile neutrinos, are weaker, with branching ratios up to few $\times 10^{-4}$. Sensitivities to different couplings arise with $\Lambda_c \rightarrow pX$ and $D \rightarrow \pi\pi X$ decays, in particular in relation with the other modes. Processes can be studied at running and future experiments with high charm luminosities, BESIII, Belle II, a super-tau-charm factory and Z factories, such as the Future Circular Collider and the Circular Electron Positron Collider.

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I. INTRODUCTION

Missing energy signatures allow one to explore different physics scenarios in one go: they are sensitive to neutrino interactions, including sterile ones, as well as phenomena such as lepton number violation (LNV). Missing energy can also stem from models beyond the standard model, with long-lived new particles decaying outside of detectors or into a dark, invisible sector, opening the way to probe the latter. Looking for invisibles in rare, flavor-changing neutral current (FCNC) quark transitions combines the best of both worlds, predictivity and sensitivity, and complements precision flavor studies with leptonic and radiative final states.

In this work we consider rare, FCNC processes in the up-quark sector, of charm quarks into invisible final states. Charm physics opens a unique window into the flavor landscape, complementing existing precision programs with kaons and b hadrons. A genuine feature of $|\Delta c| = |\Delta u| = 1$ transitions is an efficient Glashow-Iliopoulos-Maiani (GIM)

suppression, which in particular renders $c \rightarrow u\ell\bar{\ell}$ transitions in the standard model (SM) too small to be observable in the foreseeable future [1]. This identifies the dineutrino modes, or more general, decays into invisible final states, as clean null tests of the SM.

To date, only two FCNC modes into invisibles have been measured, $\mathcal{B}(K^+ \rightarrow \pi^+ \nu\bar{\nu}) = (1.14^{+0.40}_{-0.33}) \times 10^{-10}$ and recently, $\mathcal{B}(B^+ \rightarrow K^+ \nu\bar{\nu}) = (2.3 \pm 0.7) \times 10^{-5}$ [2]. In charm, no such FCNC mode to invisibles has been seen. Only few upper limits on branching ratios have been reported by BESIII [3–5] and Belle [6] at the level of $10^{-5} - 10^{-4}$. On the other hand, new physics (NP)-induced branching ratios in charm can be large, especially if the quarks are $SU(2)_L$ singlets, or NP is light, as in both cases the strong down-type quark constraints can be evaded. In addition, the invisible searches are mostly subjected to kinematic cuts and, therefore, have some degree of model dependence. That means that rates in excess of even $10^{-5} - 10^{-4}$ are *a priori* not excluded, and the decays are suitable for experimental searches at present experiments Belle II and BESIII and future facilities [7–9]. A survey of the reach of NP models, taking into account most recent theoretical and experimental developments, is therefore well motivated.

We consider SM extensions with heavy and light particles that lead to signatures with missing energy in charm. Dedicated studies in standard model effective field theory (SMEFT) are available [10] and harvest the rare

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decay-high p_T frontier and synergies arising from $SU(2)_L$, linking left-handed charged leptons to the neutrinos. We also consider LNV contributions, as well as those with light, sterile neutrinos. We furthermore analyze rare charm decays to axionlike particles (ALPs) and dark photons. For previous works, see Refs. [11–24]. We aim at a systematic, comparative study of two-, three-, and four-body decays of charmed mesons and baryons and opportunities from global analysis. As searches typically involve kinematic cuts, we perform recasts which allow us to derive novel limits from existing data.

The plan of the paper is as follows: The effective field theory (EFT) and light new physics models are introduced in Sec. II. We discuss the decays of charm hadrons to invisibles and their observables in Sec. III. We perform a recast of existing searches for charm to invisibles in Sec. IV. In Sec. V we work out experimental constraints on the parameters, masses and couplings of the models. The main results of this work, predictions for achievable branching ratios in the EFT and light beyond standard model (BSM) models as well as correlations are presented in Sec. VI. In Sec. VII we conclude. Auxiliary information on form factors is given in the Appendix.

II. MODELS

We present the NP models that lead to signatures with missing energy in charm. In Sec. II A we briefly review the SMEFT framework, including also LNV operators. The SMEFT amended by right-handed (RH) neutrinos as new light degrees of freedom, the ν SMEFT, is discussed in Sec. II B. In Sec. II C we consider the weak effective field theory (WET), suitable to compute low-energy decay observables. We also give the matching of the SMEFT and ν SMEFT operators onto WET in Sec. II D. ALPs are studied in Sec. II E and light Z' 's decaying to the dark sector in Sec. II F. Both lead to decay structure significantly different from the left-handed (LH) neutrinos of the SMEFT. Observables connected to these models are discussed in Sec. III.

A. SMEFT

The SMEFT accounts for heavy NP consistent with Lorentz and $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry and linear breaking of the electroweak symmetry. The Lagrangian reads

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_d \sum_i \frac{C_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}, \quad (1)$$

where Λ is the scale of NP that is assumed to be sufficiently separated from the weak scale given by the vacuum expectation value of the Higgs, $v = (\sqrt{2}G_F)^{-1/2} \approx 246$ GeV. The Wilson coefficients $C_i^{(d)}$ of the d -dimensional

operators $\mathcal{O}_i^{(d)}$ parametrize the NP contribution and could be inferred from a UV theory or experimentally extracted from fits to data.

At leading order with dimension six in the Warsaw basis [25] the operators

$$\mathcal{Q}_{\ell q}^{(1)} = \bar{Q}\gamma_\mu Q \bar{L}\gamma^\mu L, \quad \mathcal{Q}_{\ell q}^{(3)} = \bar{Q}\gamma_\mu \tau^a Q \bar{L}\gamma^\mu \tau^a L, \quad (2)$$

$$\mathcal{Q}_{\ell u} = \bar{U}\gamma_\mu U \bar{L}\gamma^\mu L, \quad \mathcal{Q}_{\ell d} = \bar{D}\gamma_\mu D \bar{L}\gamma^\mu L \quad (3)$$

contribute to $c \rightarrow u\nu\bar{\nu}$ transitions, where Q (L) denote the left-handed quark (lepton) $SU(2)_L$ doublet and $U(D)$ the right-handed up-type (down-type) quark singlet, respectively. Pauli matrices are denoted as τ^a and for convenience we suppressed both quark- and lepton-flavor indices.

Also LNV transitions can be probed by missing energy searches. $\Delta L = 2$ requires SMEFT operators of uneven dimension with the leading contribution to rare charm decays arising at dimension seven. A single dimension-seven operator in the basis of [10,26–28]

$$\mathcal{O}_{\ell^2 q u H}^{prst} = \epsilon^{\alpha\beta} (L_{p\alpha}^T C L_{r\sigma}) (\bar{Q}_s^\sigma U_t) H_\beta \quad (4)$$

is present to give a contribution to $c \rightarrow u\nu\nu$. Here C denotes charge conjugation and the H is the Higgs $SU(2)_L$ doublet. We use Greek letters for $SU(2)_L$ indices and Latin letters for flavor indices here. This is in slight contrast to $d_s \rightarrow d_t \nu\nu$ transitions, where two operators at dimension seven are present, namely

$$\begin{aligned} \mathcal{O}_{\ell^2 dq H}^{(1),prst} &= \epsilon^{\alpha\beta} \epsilon^{\sigma\rho} (L_{p\alpha}^T C L_{r\sigma}) (\bar{D}_s Q_{t,\rho}) H_\beta, \\ \mathcal{O}_{\ell^2 dq H}^{(2),prst} &= \epsilon^{\alpha\beta} \epsilon^{\sigma\rho} (L_{p\alpha}^T C \sigma_{\mu\nu} L_{r\sigma}) (\bar{D}_s \sigma^{\mu\nu} Q_{t,\rho}) H_\beta, \end{aligned} \quad (5)$$

with $\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$. A similar operator to $\mathcal{O}_{\ell^2 dq H}^{(2)}$ with the field content $\{L, L, \bar{Q}, U, H\}$, which is allowed by Lorentz and gauge symmetry, vanishes because of different chiralities and the identity

$$\bar{\psi}_1 \sigma^{\mu\nu} P_{L(R)} \psi_2 \bar{\psi}_3 \sigma_{\mu\nu} P_{R(L)} \psi_4 = 0. \quad (6)$$

Here $P_{L(R)} = \frac{1}{2}(1 \mp \gamma_5)$ are the left- and right-handed projection operators.

B. ν SMEFT

The SMEFT is amended by right-handed sterile neutrinos N ; see Refs. [29–31]. The lowest-dimensional operators that contribute to the decays in our work arise at dimension six and read

$$\begin{aligned}\mathcal{O}_{QuNL}^{prst} &= (\bar{N}_p L_r)(\bar{Q}_s U_t), \\ \mathcal{O}_{uN}^{prst} &= (\bar{N}_p \gamma_\mu N_r)(\bar{U}_s \gamma^\mu U_t), \\ \mathcal{O}_{QN}^{prst} &= (\bar{N}_p \gamma_\mu N_r)(\bar{Q}_s \gamma^\mu Q_t),\end{aligned}\quad (7)$$

where p and r are lepton-flavor indices and s and t are quark-flavor indices.

C. WET with left- and right-handed neutrinos

We incorporate additional light RH neutrinos together with the LH ones of the SM in the WET Lagrangian [10]

$$\mathcal{L}_{\nu_i \bar{\nu}_j}^{\text{WET}} = \frac{4G_F}{\sqrt{2}} \frac{\alpha_e}{4\pi} \sum_k \mathcal{C}_k^{ij} \cdot \mathcal{Q}_k^{ij} + \text{H.c.}, \quad (8)$$

with α_e the electromagnetic fine-structure constant and G_F Fermi's constant. The indices i, j denote the neutrino flavors (mass eigenstates) and neutrinos are assumed to be Dirac fermions. LH neutrinos allow for the four-fermion operators

$$\mathcal{Q}_{L(R),L}^{ij} = (\bar{u}_{L(R)} \gamma_\mu c_{L(R)})(\bar{\nu}_{jL} \gamma^\mu \nu_{iL}). \quad (9)$$

While a SM contribution to $\mathcal{C}_{LL}^{ii}$ is induced by Z penguin and box diagrams via loop level, it is strongly GIM and Cabibbo-Kobayashi-Maskawa (CKM) suppressed in charm and entirely negligible for the purpose of this work.

For new light degrees of freedom such as light RH neutrinos or to describe LNV contributions we extend the operators basis by additional four-fermion ones

$$\begin{aligned}\mathcal{Q}_{L(R),R}^{ij} &= (\bar{u}_{L(R)} \gamma_\mu c_{L(R)})(\bar{\nu}_{jR} \gamma^\mu \nu_{iR}), \\ \mathcal{Q}_S^{(l),ij} &= (\bar{u}_{L(R)} c_{R(L)})(\bar{\nu}_j \nu_i), \\ \mathcal{Q}_P^{(l),ij} &= (\bar{u}_{L(R)} c_{R(L)})(\bar{\nu}_j \gamma_5 \nu_i), \\ \mathcal{Q}_{T(T_3)}^{ij} &= (\bar{u} \sigma_{\mu\nu} c)(\bar{\nu}_j \sigma^{\mu\nu} (\gamma_5) \nu_i).\end{aligned}\quad (10)$$

In the observables the neutrino flavors are not measured and it is therefore required to sum the flavor indices incoherently in all observables. For the branching fraction this corresponds to

$$\mathcal{B}(c \rightarrow uv\bar{\nu}) = \sum_{ij} \mathcal{B}(c \rightarrow uv\bar{\nu}_i). \quad (11)$$

For convenience we define here the combinations of Wilson coefficients that enter various observables

connected to the $c \rightarrow u$ transitions and appear in Sec. III:

$$\begin{aligned}x_{SP\pm} &= \sum_{ij} |\mathcal{C}_S^{ij} \pm \mathcal{C}_S^{\prime ij}|^2 + |\mathcal{C}_P^{ij} \pm \mathcal{C}_P^{\prime ij}|^2, \\ x_{LR\pm} &= \sum_{ij} |\mathcal{C}_{LL}^{ij} \pm \mathcal{C}_{RL}^{ij}|^2 + |\mathcal{C}_{RR}^{ij} \pm \mathcal{C}_{LR}^{ij}|^2, \\ x_T &= \sum_{ij} |\mathcal{C}_T^{ij}|^2 + |\mathcal{C}_{T_3}^{ij}|^2.\end{aligned}\quad (12)$$

D. Matching onto WET

In the SMEFT at dimension six only the operators $\mathcal{Q}_{LL}$ and $\mathcal{Q}_{RL}$ are induced [32]:

$$\mathcal{C}_{LL} = \frac{\sqrt{2}\pi}{\alpha_e G_F \Lambda^2} (\mathcal{C}_{\ell q}^{(1)} + \mathcal{C}_{\ell q}^{(3)}), \quad \mathcal{C}_{RL} = \frac{\sqrt{2}\pi}{\alpha_e G_F \Lambda^2} \mathcal{C}_{\ell u}. \quad (13)$$

At dimension seven also scalar and pseudoscalar operators $\mathcal{Q}_{S,P}$ and $\mathcal{Q}'_{S,P}$ are induced. The matching can be read off by separating the operator (4) into its $SU(2)_L$ components:

$$\begin{aligned}\frac{1}{\Lambda_{\text{LNV}}^3} \mathcal{C}_{\ell^2 quH}^{prst} \mathcal{O}_{\ell^2 quH}^{prst} &\rightarrow \frac{v_T}{\sqrt{2}\Lambda_{\text{LNV}}^3} \mathcal{C}_{\ell^2 quH}^{prst} \\ &\times \left[(\nu_{Lp}^T C \nu_{Lr})(\bar{u}_{Ls} u_{Rt}) \right. \\ &\quad \left. + (\ell_{Lp}^T C \nu_{Lr})(\bar{d}_{Ls} u_{Rt}) \right].\end{aligned}\quad (14)$$

Here v_T is the vacuum expectation value of the Higgs which in our case equals the SM Higgs vacuum expectation value v . The tree-level matching onto the scalar and pseudoscalar operators in Eq. (8) reads

$$\begin{aligned}\mathcal{C}_{S(P)}^{ij} &= \pm \sqrt{2} \frac{2\pi}{\alpha_e} \left(\frac{v}{\Lambda_{\text{LNV}}} \right)^3 \mathcal{C}_{\ell^2 quH}^{ij12}, \\ \mathcal{C}_{S(P)}^{\prime ij} &= + \sqrt{2} \frac{2\pi}{\alpha_e} \left(\frac{v}{\Lambda_{\text{LNV}}} \right)^3 \mathcal{C}_{\ell^2 quH}^{ji21*},\end{aligned}\quad (15)$$

in agreement with [28,33].

In the ν SMEFT we similarly perform a tree-level matching and obtain

$$\begin{aligned}\mathcal{C}_{LR}^{ij} &= \frac{\sqrt{2}\pi}{\alpha_e G_F \Lambda_N^2} \mathcal{C}_{QN}^{ij12}, \quad \mathcal{C}_{S(P)}^{ij} = \pm \frac{1}{2} \frac{\sqrt{2}\pi}{\alpha_e G_F \Lambda_N^2} \mathcal{C}_{QuNL}^{ij12}, \\ \mathcal{C}_{RR}^{ij} &= \frac{\sqrt{2}\pi}{\alpha_e G_F \Lambda_N^2} \mathcal{C}_{uN}^{ij12}, \quad \mathcal{C}_{S(P)}^{\prime ij} = + \frac{1}{2} \frac{\sqrt{2}\pi}{\alpha_e G_F \Lambda_N^2} \mathcal{C}_{QuNL}^{ji21*}.\end{aligned}\quad (16)$$

To summarize, in the $d = 6$ SMEFT, SMEFT at $d = 7$ with LNV and the $d = 6$ ν SMEFT the following combinations of coefficients (12) are induced:

$$\begin{aligned}
 x_{LR\pm} &\propto \sum_{ij} |C_{\ell q}^{(1)ij} + C_{\ell q}^{(3)ij} \pm C_{\ell u}^{ij}|^2, & \text{SMEFT } d=6, \\
 x_{SP\pm} &\propto \sum_{ij} |C_{\ell^2 quH}^{ij12} \pm C_{\ell^2 quH}^{ji21*}|^2, & \text{SMEFT } d=7 \\
 & & (\text{LNV}), \\
 \left. \begin{aligned}
 x_{LR\pm} &\propto \sum_{ij} |C_{QN}^{ij12} \pm C_{uN}^{ij12}|^2, \\
 x_{SP\pm} &\propto \sum_{ij} |C_{QuNL}^{ij12} \pm C_{QuNL}^{ji21*}|^2
 \end{aligned} \right\}, & \nu\text{SMEFT } d=6. \quad (17)
 \end{aligned}$$

There is no tensor generated; $x_T = 0$ in all cases. If one turns on just a single (ν)SMEFT operator, relations arise:

$$x_{LR} \equiv x_{LR+} = x_{LR-}, \quad x_{SP} \equiv x_{SP+} = x_{SP-}. \quad (18)$$

Since dineutrino modes are SM null tests, they are very sensitive to the NP scale. For $d=6$ branching ratios are $\propto 1/\Lambda^4$ and $\propto 1/\Lambda_{\text{LNV}}^6$ for $d=7$.

E. ALPs

ALPs are pseudo Nambu-Goldstone bosons originating from a spontaneously broken global $U(1)$ symmetry. For the ‘‘classical’’ QCD axion, which arises from the Peccei-Quinn symmetry to solve the CP problem of QCD, the coupling to photons and its mass are related. ALPs provide a more general framework with additional couplings, arising in superstring theory [34], supersymmetry with the R axion in [35] and in composite Higgs models in [36]. ALPs can also serve as a candidate for dark matter [37] and arise in flavor symmetry breaking [21]. We work in the effective field theory framework for ALPs described in [20,38] with a general effective Lagrangian up to dimension five at the UV scale $\Lambda = 4\pi f$. The effective ALP Lagrangian coincides with the one of the QCD axion for $m_a = 0$.

We evaluate observables below the electroweak scale and use the part of the effective Lagrangian,

$$\mathcal{L}_{\text{ALP}}^{c \rightarrow u} = \frac{\partial^\mu a}{2f} (k_{12}^V \bar{u} \gamma_\mu c + k_{12}^A \bar{u} \gamma_\mu \gamma_5 c) + \text{H.c.}, \quad (19)$$

that describes the flavor off-diagonal interaction of an ALP with quarks. Here there are two coupling constants $k_{12}^{V(A)}$, which are vector and axial-vector couplings, respectively. The flavor-changing couplings in the up sector remain scale invariant neglecting small Yukawas (i.e. y_b and smaller); see Ref. [38].

Various decay channels of ALPs are plausible depending on the choice of couplings and their existing experimental constraints. To obtain contributions to missing energy we follow the approach in Refs. [20,38] and consider only ALPs that decay outside the detector. The pseudoscalar nature and the momentum-dependent coupling are

distinguishing features of this model that set it apart from the other models we consider in this work.

F. Light Z'

We extend the SM by a light neutral Z' vector boson stemming from an additional $U(1)'$ gauge group. The interactions between the light Z' and SM particles can be described through an effective field theory; see Refs. [15,39,40]. Without loss of generality, we assume the Z' to be diagonalized to its mass-eigenstate basis and neglect kinematic mixing terms $\propto A^\mu Z'_\mu$. For the diagonalization procedure for both massive and massless Z' 's see Ref. [39]. At energies below the electroweak scale it is sufficient to consider the lowest-dimension flavor-changing operators

$$\mathcal{L}_{Z'}^{\text{eff}} \supset \frac{1}{\Lambda_{\text{eff}}} \bar{u} (C_D^{Z'} + \gamma_5 C_{D5}^{Z'}) \sigma^{\mu\nu} c Z'_{\mu\nu} + \text{H.c.}, \quad (20)$$

and for $m_{Z'} \neq 0$ in addition

$$\begin{aligned}
 \mathcal{L}_{Z'}^{\text{eff}} &\supset C_L^{Z'} \bar{u}_L \gamma^\mu c_L Z'_\mu + C_R^{Z'} \bar{u}_R \gamma^\mu c_R Z'_\mu + \text{H.c.} \\
 &+ C_V^{Z'} \bar{\chi} \gamma^\mu \chi Z'_\mu + C_A^{Z'} \bar{\chi} \gamma^\mu \gamma_5 \chi Z'_\mu, \quad (21)
 \end{aligned}$$

where χ denotes dark fermions charged under $U(1)'$ with Dirac mass m_χ . Due to the additional operators (21) the $m_{Z'} \rightarrow 0$ limit is not straightforward. This could be circumvented by scaling $C_{L/R}^{Z'} (C_{V/A}^{Z'}) \rightarrow \frac{m_{Z'}}{\Lambda_{\text{eff}}} C_{L/R}^{Z'} (C_{V/A}^{Z'})$ that guarantees the correct $m_{Z'} \rightarrow 0$ limit [15]. However, such a scaling is not unique and we leave Eq. (21) as is.

For convenience we employ the notation

$$\begin{aligned}
 x_{LR\pm}^{Z'} &= |C_L^{Z'} \pm C_R^{Z'}|^2, \\
 x_{D(5)}^{Z'} &= |C_{D(5)}^{Z'}|^2 / \Lambda_{\text{eff}}^2, \\
 x_{\text{Re(Im)}LRD}^{Z'} &= \text{Re(Im)} \left\{ \frac{(C_L^{Z'} + C_R^{Z'}) C_D^{Z'*}}{\Lambda_{\text{eff}}} \right\}, \\
 x_{\text{Re(Im)}LRD5}^{Z'} &= \text{Re(Im)} \left\{ \frac{(C_L^{Z'} - C_R^{Z'}) C_{D5}^{Z'*}}{\Lambda_{\text{eff}}} \right\}, \quad (22)
 \end{aligned}$$

for the combinations of Wilson coefficients that appear in branching fractions. The last two contributions in (22) originate from interference terms of the Lagrangians (20) and (21), which for simplicity we do not consider for phenomenology in this work.

The Z' 's contribute to missing energy signatures by either predominantly decaying to invisibles or by possessing a sufficiently large lifetime to decay outside the detector. For massless or stable Z' 's the missing energy signature is directly generated via the two-body decay topology $c \rightarrow u Z'$, where the Z' is on shell.

For light masses with $m_{Z'} \simeq 1$ GeV and a dominant decay through invisibles, the Z' 's are produced off shell.

TABLE I. Coefficients $A_k^{h_c F}$ defined in Eq. (27) for various charmed hadrons h_c and hadronic final states F . The three centre dots for $D \rightarrow \pi\pi$ indicate that form factor determinations are not available.

$h_c \rightarrow F$	$A_{SP-}^{h_c \rightarrow F}$	$A_{SP+}^{h_c \rightarrow F}$	$A_{LR-}^{h_c \rightarrow F}$	$A_{LR+}^{h_c \rightarrow F}$	$A_T^{h_c \rightarrow F}$
	[10 ⁻⁸]	[10 ⁻⁸]	[10 ⁻⁸]	[10 ⁻⁸]	[10 ⁻⁸]
$D^0 \rightarrow$	143 ± 5	0	0	0	0
$D^0 \rightarrow \pi^0$	0	3.6 ± 0.1	0	0.89 ± 0.06	8.0 ± 1.3
$D^+ \rightarrow \pi^+$	0	18.2 ± 0.7	0	3.6 ± 0.2	39 ± 7
$D_s^+ \rightarrow K^+$	0	4.0 ± 0.2	0	0.82 ± 0.05	5.9 ± 1.1
$D^0 \rightarrow \omega/\rho^0$	0.26 ± 0.04	0	0.91 ± 0.14	0.059 ± 0.008	79 ± 16
$\Lambda_c \rightarrow p$	0.84 ± 0.08	1.8 ± 0.2	1.7 ± 0.1	1.02 ± 0.09	4.3 ± 0.5
$\Xi_c^+ \rightarrow \Sigma^+$	1.5 ± 0.2	3.4 ± 0.3	3.6 ± 0.3	1.9 ± 0.2	8.8 ± 0.9
$D^0 \rightarrow \pi^+\pi^-$	...	0	0.57 ± 0.03	0.11 ± 0.01	...
$D^+ \rightarrow \pi^+\pi^0$	...	0	1.93 ± 0.08	0.47 ± 0.06	...

Specifically, we consider for simplicity the decay $Z' \rightarrow \chi\bar{\chi}$ to be the sole contribution to the decay width $\Gamma_{Z'}$. In this case the decay topology is that of a three-body decay. This assumption is plausible if either the Z' is too light to decay into two visible SM particles but heavy enough to decay into light NP particles or if couplings to SM particles are too small. For an off-shell Z' with momentum q^μ the decay width of $Z' \rightarrow \chi\bar{\chi}$ is given as

$$\Gamma_{Z'}(q^2) = \frac{\sqrt{q^2}}{12\pi} \sqrt{1 - \frac{4m_\chi^2}{q^2}} \left(|\mathcal{C}_V^{Z'\chi}|^2 \left(1 + \frac{2m_\chi^2}{q^2} \right) + |\mathcal{C}_A^{Z'\chi}|^2 \left(1 - \frac{4m_\chi^2}{q^2} \right) \right), \quad (23)$$

with on-shell width $\Gamma_{Z'} \equiv \Gamma_{Z'}(m_{Z'}^2)$. Fixing $\Gamma_{Z'}$ and the NP masses hence constrains the Wilson coefficients $\mathcal{C}_{A,V}^{Z'\chi}$.

For the phenomenological analysis we employ a benchmark (BM) Z' model

$$\text{BM}_V \ Z': (m_{Z'}, \Gamma_{Z'}, m_\chi) = (1, 0.1, 0.1) \text{ GeV} \\ \text{and } \mathcal{C}_A^{Z'\chi} = 0 \quad (24)$$

and $\mathcal{C}_V^{Z'\chi} = 1.94$ using (23).

III. CHARM DECAYS TO INVISIBLES

In this section we work out low-energy observables, the branching ratios of rare charm decays, in the NP models of Sec. II. We discuss light LH and RH neutrino observables in Sec. III A, ALP observables in Sec. III B and light Z' observables in Sec. III C. Heavy NP in the context of SMEFT or ν SMEFT contribute only to specific Wilson coefficients presented in Sec. III A as discussed by the matching in Sec. II D.

We analyze both the total branching ratios as well as the differential branching ratios of decays into missing energy, E_{miss} . The branching fraction as a differential of the missing energy E_{miss} is related to the branching fraction in the

invariant mass q^2 of the invisible particle(s) as $d\mathcal{B}/dE_{\text{miss}} = 2m_{h_c} d\mathcal{B}/dq^2$, where m_{h_c} is the mass of the decaying charm hadron. We consider the decays $D^0 \rightarrow$ invisible, $D^{0,+} \rightarrow \pi^{0,+} +$ invisible, $D_s^+ \rightarrow K^+$ invisible, $D^0 \rightarrow \rho^0(\omega) +$ invisible, $\Lambda_c \rightarrow p +$ invisible, $\Xi_c^+ \rightarrow \Sigma^+ +$ invisible and $D^{+(0)} \rightarrow \pi^+\pi^{-(0)} +$ invisible.

A. Dineutrino modes

We work out contributions to the various decays with missing energy in models with LH or RH neutrinos. We factorize the branching ratios of a charmed hadron h_c into hadronic final states F as

$$\mathcal{B}(h_c \rightarrow F \nu \bar{\nu}) = \sum_{k=\{SP\pm, LR\pm, T\}} A_k^{h_c \rightarrow F} x_k, \quad (25)$$

where $A_k^{h_c \rightarrow F}$ are decay-dependent coefficients and x_k are the shorthand notations of the combinations of Wilson coefficients defined in Eq. (12). The numerical values for the coefficients A_k with $k \in \{SP\pm, LR\pm, T\}$ are summarized in Table I. Results are consistent with Ref. [10] albeit employing different form factors for $D_{(s)} \rightarrow \pi(K)$.

For a single or two final state particle we additionally calculate the differential branching fraction via

$$\frac{d\mathcal{B}(h_c \rightarrow F \nu \bar{\nu})}{dq^2} = \sum_{k=\{SP\pm, LR\pm, T\}} a_k^{h_c \rightarrow F}(q^2) x_k, \quad (26)$$

where q^2 is the invariant mass squared of the neutrinos or more general of the invisible final state particle(s). Here $a_k^{h_c \rightarrow F}$ are q^2 and process-dependent functions related to the coefficients in Eq. (25) via

$$A_k^{h_c \rightarrow F} = \int_{q_{\text{min}}^2}^{q_{\text{max}}^2} a_k^{h_c \rightarrow F}(q^2) dq^2, \quad (27)$$

with $q_{\max}^2 = (m_{h_c} - m_F)^2$. For decays without a charged $D_{(s)}$ meson we choose the full kinematically allowed region with the lower integration limits $q_{\min}^2 = 0$, while for the charged modes we perform cuts to remove τ background following Ref. [10].

The resonance background induced via $D^+ \rightarrow \tau^+(\rightarrow \pi^+\bar{\nu})\nu$, $D_s^+ \rightarrow \tau^+(\rightarrow K^+\bar{\nu})\nu$ or $D^+ \rightarrow \tau^+(\rightarrow \pi^+\pi^0\bar{\nu})\nu$, leading to the same final states as the nonresonant contribution, can be removed with a cut [10]

$$q^2 > (m_\tau^2 - m_F^2)(m_D^2 - m_\tau^2)/m_\tau^2, \quad (28)$$

while still retaining sufficient phase space for potential NP contributions. Here $m_F = m_\pi$, m_K or $2m_\pi$, respectively,¹ and m_τ is the tau-lepton mass. We advise experiments searching for these decays to apply similar strategies and to not neglect these modes because of the resonances. For the numerical calculation we use the values

$$\begin{aligned} q_{\min, D^+ \rightarrow \pi^+}^2 &= 0.34 \text{ GeV}^2, \\ q_{\min, D_s^+ \rightarrow K^+}^2 &= 0.66 \text{ GeV}^2, \\ q_{\min, D^+ \rightarrow \pi^+\pi^0}^2 &= 0.31 \text{ GeV}^2, \end{aligned} \quad (29)$$

when we give results on total branching ratios.

For two hadronic final states F_1 and F_2 we calculate the three-differential branching fraction via

$$\frac{d\mathcal{B}(h_c \rightarrow F_1 F_2 \nu \bar{\nu})}{dq^2 dp^2 d\cos\theta_{F_1}} = \sum_{k=\{LR\pm, SP\pm, T\}} b_k^{h_c \rightarrow F_1 F_2}(q^2, p^2, \theta_{F_1}) x_k, \quad (30)$$

where the coefficient functions $b_k^{h_c \rightarrow F_1 F_2}(q^2, p^2, \theta_{F_1})$ now depend additionally on p^2 the invariant mass squared of the two final state hadrons and θ_{F_1} the angle between F_1 and the negative direction of flight of the h_c hadron in the rest frame of the two-hadron system.

The coefficients $A_k^{h_c \rightarrow F_1 F_2}$ of the integrated branching in Eq. (25) are calculated via

$$\begin{aligned} A_k^{h_c \rightarrow F_1 F_2} &= \int_{q_{\min}^2}^{q_{\max}^2} \int_{(m_{F_1} + m_{F_2})^2}^{(m_{h_c} - \sqrt{q^2})^2} \int_{-1}^1 b_k^{h_c \rightarrow F_1 F_2}(q^2, p^2, \theta_{F_1}) \\ &\times d\cos\theta_{F_1} dp^2 dq^2, \end{aligned} \quad (31)$$

¹For $D^+ \rightarrow \pi^+\pi^0\nu\bar{\nu}$ a lower cut $q^2 > (m_\tau^2 - p^2)(m_D^2 - m_\tau^2)/m_\tau^2$ is sufficient to remove resonant τ background. It however depends on p^2 , the invariant mass of the pion pair, and we therefore use the simpler p^2 -independent cut which removes only at most an additional 5% of the branching fraction.

with the appropriate cuts of Eq. (29) for $D^+ \rightarrow \pi^+\pi^0\nu\bar{\nu}$. In $D^0 \rightarrow \pi^+\pi^-\nu\bar{\nu}$ resonance backgrounds via τ decays are kinematically forbidden.

1. $D^0 \rightarrow \nu\bar{\nu}$

For $D^0 \rightarrow \nu\bar{\nu}$ decays the coefficients $A_k^{D^0 \rightarrow \nu\bar{\nu}}$ of the branching ratio in Eq. (25) vanish except for one [10]:

$$A_{SP-}^{D^0 \rightarrow \nu\bar{\nu}} = \frac{G_F^2 \alpha_e^2 f_D^2}{64\pi^3 m_{D^0}} \frac{m_{D^0}^6}{(m_u + m_c)^2}, \quad (32)$$

where $f_D = 209.0 \pm 2.4$ MeV [41] is the D -meson decay constant, m_{D^0} the mass of the D^0 meson and $m_{c(u)}$ are the masses of the charm(up) quark. The contributions from vector and axial-vector operators are helicity suppressed by two powers of the neutrino mass and the interference between pseudoscalar operators and vector and axial-vector operators is suppressed by one order. At dimension six in the SMEFT we therefore do not expect a signal for $D^0 \rightarrow \text{invisible}$. Requisite scalar or pseudoscalar operators are, on the other hand, induced at $d = 7$ or in the ν SMEFT.

2. $D \rightarrow \pi\nu\bar{\nu}$ and $D_s \rightarrow K\nu\bar{\nu}$

For $D \rightarrow \pi\nu\bar{\nu}$ the q^2 -dependent functions $a_k^{D \rightarrow \pi}$ of the differential branching fraction in Eq. (26) read [10]

$$\begin{aligned} a_{SP+}^{D^0 \rightarrow \pi^0}(q^2) &= 3N_{D\pi} q^2 \frac{(m_D^2 - m_\pi^2)^2 f_0^2(q^2)}{(m_c - m_u)^2 2}, \\ a_{LR+}^{D^0 \rightarrow \pi^0}(q^2) &= N_{D\pi} \lambda_{D\pi} \frac{f_+^2(q^2)}{2}, \\ a_T^{D^0 \rightarrow \pi^0}(q^2) &= 16N_{D\pi} q^2 \frac{\lambda_{D\pi}}{(m_D + m_\pi)^2} \frac{f_T^2(q^2)}{2}, \\ a_k^{D^+ \rightarrow \pi^+}(q^2) &= 2a_k^{D^0 \rightarrow \pi^0}(q^2), \end{aligned} \quad (33)$$

with the normalization

$$N_{D\pi} = \frac{\tau_D G_F^2 \alpha_e^2 \sqrt{\lambda_{D\pi}}}{3 \cdot 2^{10} m_\pi^3 \pi^5}, \quad (34)$$

where $\lambda_{D\pi} = \lambda(m_D^2, m_\pi^2, q^2)$, $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$ is the Källén function and contributions of other Wilson coefficients vanish. The functions $f_{+,0,T}(q^2)$ are the $D \rightarrow \pi$ form factors,² which are available from lattice QCD in [42,44]; see Appendix A 1. For $D_s^+ \rightarrow K^+$ we use the same expressions as for $D^+ \rightarrow \pi^+$

²Note that we include the isospin factor $1/\sqrt{2}$ for the neutral pion in the final state in Eq. (33) and use the same form factors from [42–44] for $D^{0(+)} \rightarrow \pi^{0(+)}\nu\bar{\nu}$ as opposed to including it in the form factor itself.

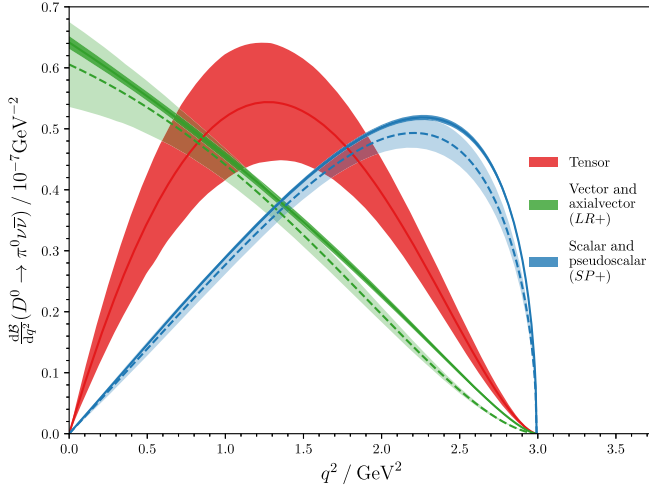


FIG. 1. The $d\mathcal{B}(D^0 \rightarrow \pi^0 \nu \bar{\nu})/dq^2$ distributions as functions of q^2 . Solid curves ($SP+$, $LR+$) are based on the form factors from Fermilab lattice [42]. The main source of uncertainty stems from the form factors and is illustrated by the bands. Also shown (dashed curves) are form factors from the ETM Collaboration [43,44], featuring larger uncertainties. The tensor form factor has only been provided by ETM and is therefore shown by the solid (red) curve. For each of the solid curves a single x_k , $k \in \{SP+, LR+, T\}$ of Eq. (12) is turned on such that $\mathcal{B}(D^0 \rightarrow \pi^0 \nu \bar{\nu}) = 10^{-7}$. Identical values of x_k have been used for the dashed and the solid curves. Distributions of $D^+ \rightarrow \pi^+ \nu \bar{\nu}$ are identical; see text for details.

throughout this paper with obvious kinematic replacements. We use the same form factors for simplicity, which is supported as a good approximation by Ref. [42]. The differential distribution of $D^0 \rightarrow \pi^0 \nu \bar{\nu}$ decays is shown in Fig. 1 by turning on a single x_k coefficient for each curve and using an arbitrary normalization. We include uncertainties from particle masses, lifetimes and form factors, with the latter being the main source of uncertainty. One should note that only the tensor contribution always increases in the direction of the $q^2 = 0$ kinematic end point and could in this way be cleanly separated from the other two contributions. Distributions of $D^+ \rightarrow \pi^+ \nu \bar{\nu}$ are identical with the only differences in the normalization caused by the lifetime and isospin factors and additional contributions of τ background in the region $q^2 < 0.34 \text{ GeV}^2$.

Because of discrepancies of the form factor $f_{0,+}$ in the high- q^2 region from different lattice calculations, see Appendix A 1 and Ref. [42], we show result in solid (dashed) using most recent (ETM) form factors. The lattice form factors of Ref. [42] have significantly smaller uncertainties than those by the ETM Collaboration from Ref. [43]. The effect of the discrepancies of the form factors on the branching ratio is partially kinematically suppressed at the end point and affects only a limited q^2 region. At most a $\sim 6\%$ difference for the total branching fraction is observed. For f_T only a computation by the

ETM collaboration is available. We emphasize to also consider f_T , when trying to resolve the deviations.

3. $D \rightarrow \rho^0(\omega) \nu \bar{\nu}$

For $D \rightarrow V \nu \bar{\nu}$ decays with $V = \rho^0(\omega)$ the functions $a_k^{D \rightarrow V}$ of the differential branching fraction in Eq. (26) are given as

$$\begin{aligned} a_{LR+}^{D \rightarrow V}(q^2) &= 2N_{DV} \frac{\lambda_{DV}}{(m_D + m_V)^2} V(q^2)^2, \\ a_{LR-}^{D \rightarrow V}(q^2) &= 2N_{DV} \left((m_D + m_V)^2 A_1(q^2)^2, \right. \\ &\quad \left. + 32 \frac{m_D^2 m_V^2}{q^2} A_{12}(q^2)^2 \right), \\ a_{SP-}^{D \rightarrow V}(q^2) &= 3N_{DV} \frac{\lambda_{DV}}{m_c^2} A_0(q^2)^2, \\ a_T^{D \rightarrow V}(q^2) &= 32N_{DV} \left(\frac{T_1(q^2)^2 \lambda_{DV} + (m_D^2 - m_V^2)^2 T_2(q^2)^2}{q^2} \right. \\ &\quad \left. + 8 \frac{m_D^2 m_V^2}{(m_D + m_V)^2} T_{23}(q^2)^2 \right), \end{aligned} \quad (35)$$

with the normalization

$$N_{DV} = \frac{G_F^2 \alpha_e^2 \tau_D \sqrt{\lambda_{DV} q^2}}{3 \times 10^{10} \pi^5 m_D^3}, \quad (36)$$

where $\lambda_{DV} = \lambda(m_D^2, m_V^2, q^2)$. The scalar or pseudoscalar contribution of x_{SP+} vanishes. We find the same expressions as Ref. [45]. The form factors of the $D \rightarrow V$ transitions are defined in Appendix A 2. We evaluate these expression for $D^0 \rightarrow \rho^0 \nu \bar{\nu}$ as well as $D^0 \rightarrow \omega \nu \bar{\nu}$ decays, where we use as an approximation the same form factors for both. Because of similar masses the resulting distributions are nearly identical and we give therefore no separate results. In Fig. 2 we show the differential distribution for different combinations of Wilson coefficients. Both the scalar contributions ($SP-$) and the vector and axial-vector contributions ($LR+$) overlap, due to similar kinematic dependence (35) and the numerically similar q^2 slopes of the form factors V and A_0 . In absolute terms, however, the distributions differ as seen by comparing the coefficients A_k in Table I.

4. $\Lambda_c \rightarrow p \nu \bar{\nu}$ and $\Xi_c^+ \rightarrow \Sigma^+ \nu \bar{\nu}$

The differential branching ratio of the baryonic three-body decay $\Lambda_c \rightarrow p \nu \bar{\nu}$ is calculated using the helicity formalism described in [46,47] and neglecting neutrino masses. It reads

$$\frac{d\mathcal{B}(\Lambda_c \rightarrow p \nu \bar{\nu})}{dq^2} = \sum_{k=\{SP\pm, LR\pm, T\}} a_k^{\Lambda_c \rightarrow p}(q^2) x_k, \quad (37)$$

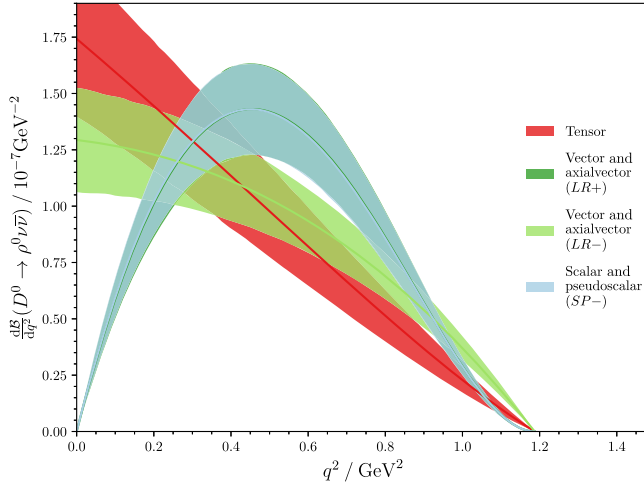


FIG. 2. The $d\mathcal{B}(D^0 \rightarrow \rho^0 \nu \bar{\nu})/dq^2$ distributions as functions of q^2 ; see Fig. 1. Central values are normalized to $\mathcal{B}(D^0 \rightarrow \rho^0 \nu \bar{\nu}) = 10^{-7}$. Due to the proximity of masses and for similar form factors the distributions for $D^0 \rightarrow \omega \nu \bar{\nu}$ are similar. The distributions proportional to x_{LR+} and x_{SP-} overlap; see text.

where the contributions $a_k(q^2)$ are defined as

$$\begin{aligned} a_{SP+}^{\Lambda_c \rightarrow p}(q^2) &= 2N_{\Lambda_c p} q^2 f_0^2 s_+ \left(\frac{m_{\Lambda_c} - m_p}{m_c - m_u} \right)^2, \\ a_{SP-}^{\Lambda_c \rightarrow p}(q^2) &= 2N_{\Lambda_c p} q^2 g_0^2 s_- \left(\frac{m_{\Lambda_c} + m_p}{m_c + m_u} \right)^2, \\ a_{LR+}^{\Lambda_c \rightarrow p}(q^2) &= \frac{2N_{\Lambda_c p}}{3} s_- (f_+^2 (m_{\Lambda_c} + m_p)^2 + 2q^2 f_\perp^2), \\ a_{LR-}^{\Lambda_c \rightarrow p}(q^2) &= \frac{2N_{\Lambda_c p}}{3} s_+ (g_+^2 (m_{\Lambda_c} - m_p)^2 + 2q^2 g_\perp^2), \\ a_T^{\Lambda_c \rightarrow p}(q^2) &= \frac{32N_{\Lambda_c p}}{3} (h_\perp^2 s_- 2(m_{\Lambda_c} + m_p)^2 \\ &\quad + h_+^2 s_- q^2 + \tilde{h}_\perp^2 s_+ 2(m_{\Lambda_c} - m_p)^2 + \tilde{h}_+^2 s_+ q^2), \end{aligned} \quad (38)$$

with the normalization

$$N_{\Lambda_c p} = \frac{G_F^2 \alpha_e^2 \tau_{\Lambda_c} \sqrt{\lambda(m_{\Lambda_c}^2, q^2, m_p^2)}}{2^{11} \pi^5 m_{\Lambda_c}^3} \quad (39)$$

and

$$s_\pm = (m_{\Lambda_c} \pm m_p)^2 - q^2. \quad (40)$$

We use the form factors $f_i, g_i, h_i, \tilde{h}_i$ of the $\Lambda_c \rightarrow p$ transition in the helicity basis introduced in Ref. [48]. Numerical values for the form factors are available from lattice QCD in Ref. [49]; see Appendix A 3. Our result agrees with calculations for $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ [47] and $\Lambda_c \rightarrow$

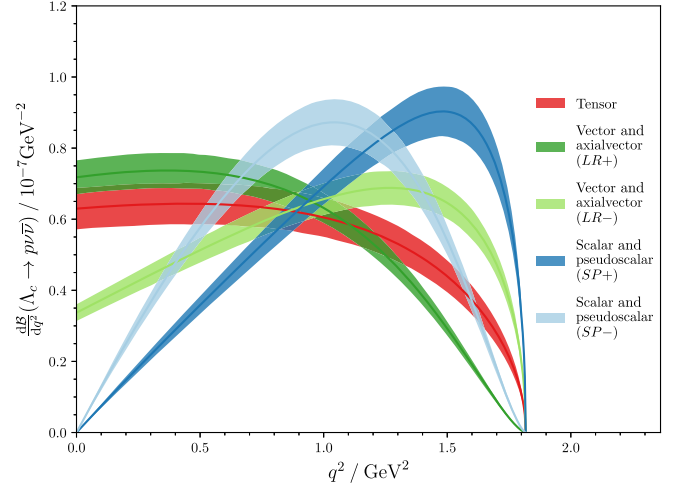


FIG. 3. The $d\mathcal{B}(\Lambda_c \rightarrow p \nu \bar{\nu})/dq^2$ distributions as functions of q^2 ; see Fig. 1. Central values are normalized to $\mathcal{B}(\Lambda_c \rightarrow p \nu \bar{\nu}) = 10^{-7}$. The uncertainty includes statistical and systematic errors of the form factors given in [49].

$p \ell^+ \ell^-$ [50] in the limit of massless leptons. For $\Xi_c^+ \rightarrow \Sigma^+$ transitions we use the same form factors and the same expressions with obvious kinematic replacements. This can be done because their respective form factors are equal in the isospin limit via flavor symmetries [10].

In Fig. 3 we show the differential branching fraction, where for each curve a single x_k of Eq. (12) is turned on. All curves are normalized to $\mathcal{B}(\Lambda_c \rightarrow p \nu \bar{\nu}) = 10^{-7}$. The uncertainties of the $\Lambda_c \rightarrow p$ form factors are the main source of uncertainty for this observable and larger than for $D \rightarrow \pi$. Figure 3 highlights the possibility to probe the chirality of light neutrinos. Scalar and pseudoscalar NP operators, only allowed with RH neutrinos, are distinguishable by their unique q^2 behavior. The curves of $x_{SP\pm}$, corresponding to scalar and pseudoscalar operators, vanish at the low q^2 end point, while others remain essentially finite with completely negligible corrections from neutrino masses. Additionally, the other curves of $x_{LR\pm}$ and x_T are below the scalar and pseudoscalar contributions in the high q^2 region.

5. $D \rightarrow \pi \pi \nu \bar{\nu}$

The functions b_k of the three-differential branching fraction (30) for $D^{0(+)} \rightarrow \pi^+ \pi^{-(0)} \nu \bar{\nu}$ decays read

$$\begin{aligned} b_{SP-}^{D \rightarrow \pi\pi}(q^2, p^2, \theta_{\pi^+}) &= \frac{\tau_D}{2} \frac{1}{(m_c + m_u)^2} |\mathcal{F}_t|^2, \\ b_{LR+}^{D \rightarrow \pi\pi}(q^2, p^2, \theta_{\pi^+}) &= \frac{\tau_D}{6} \sin^2 \theta_{\pi^+} |\mathcal{F}_\perp|^2, \\ b_{LR-}^{D \rightarrow \pi\pi}(q^2, p^2, \theta_{\pi^+}) &= \frac{\tau_D}{6} [|\mathcal{F}_0|^2 + \sin^2 \theta_{\pi^+} |\mathcal{F}_\parallel|^2], \\ b_T^{D \rightarrow \pi\pi}(q^2, p^2, \theta_{\pi^+}) &= \frac{8\tau_D}{3} [|\mathcal{F}_0^T|^2 \\ &\quad + \sin^2 \theta_{\pi^+} (|\mathcal{F}_\parallel^T|^2 + |\mathcal{F}_\perp^T|^2)]. \end{aligned} \quad (41)$$

Here $\mathcal{F}_i(q^2, p^2, \theta_{\pi^+})$ are the transversity form factors defined in Appendix A 4, in agreement with Ref. [10]. The scalar and tensor form factors $\mathcal{F}_t$ and $\mathcal{F}_t^T$ are known much less well than the vector and axial-vector ones. We therefore refrain from phenomenological analysis involving the former in this work. p^2 and q^2 distributions using (axial-)vector form factors are given in [32].

B. ALP modes

Decays to ALPs differ by their decay topologies compared to the scenarios described in Secs. III A and III C. For example instead of a three-body decay for a single hadronic final state, it is a two-body decay with a delta distribution as its differential branching fraction in q^2 . If an experimental q^2 resolution is considered, smearing effects are however necessary. The branching fraction is given as a function of the ALP mass m_a via

$$\mathcal{B}(h_c \rightarrow Fa) = \frac{|k_{12}^V|^2}{f^2} a_V^{h_c \rightarrow F}(m_a) + \frac{|k_{12}^A|^2}{f^2} a_A^{h_c \rightarrow F}(m_a) \quad (42)$$

with the two possible types of ALP couplings, where $a_k^{h_c \rightarrow F}(m_a)$ parametrize the kinematic dependence on the ALP mass.

If the ALP does not decay, its contribution to signatures with invisibles is obvious. In general however ALPs can decay to various SM particles for all values of m_a that we consider. To study its contribution to missing energy modes we therefore require the ALPs to decay outside the detector. If the decaying h_c hadron is at rest, the fraction of ALPs, which escape a detector of transverse radius $R_{\max}$, is [20]

$$F_T = \int_0^{\frac{\pi}{2}} \sin \theta \exp\left(-\frac{m_a \Gamma_a R_{\max}}{|p_{LAB}^T|}\right) d\theta \quad (43)$$

with the transverse momentum

$$p_{LAB}^T = \sin \theta \frac{1}{2m_{h_c}} \sqrt{\lambda(m_{h_c}^2, m_F^2, m_a^2)}. \quad (44)$$

The decay width Γ_a of an ALP depends in general on further couplings of the model. For further details, see Ref. [20]. For the radius we chose the outer radius of the muon identifier of the BESIII detector $R_{\max} = 2.8$ m. We multiply the branching ratio by this factor when we consider $\Gamma_a \neq 0$.

1. $D \rightarrow \pi a$ and $D_s \rightarrow Ka$

The branching fraction of $D \rightarrow \pi a$ is calculated via Eq. (42) with the functions [20]

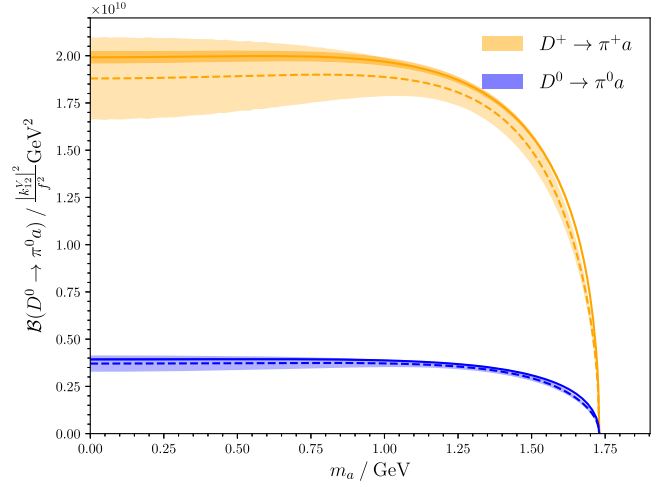


FIG. 4. Branching ratio of $D^0 \rightarrow \pi^0 a$ (lower band, blue) and $D^+ \rightarrow \pi^+ a$ (upper band, orange) with the coupling $|k_{12}^V/f|^2$ factored out, depending on the ALP mass m_a . The difference between the charged and the neutral mode is lifetime and isospin factors. The main source of uncertainty stems from the $D \rightarrow \pi$ form factors. Solid (dashed) curves are using most recent (ETM) form factors; see Sec. III A 2.

$$\begin{aligned} a_V^{D^+ \rightarrow \pi^+}(m_a) &= \frac{\tau_D m_D^3}{64\pi} |f_0^{D \rightarrow \pi}(m_a^2)|^2 \\ &\times \left(1 - \frac{m_\pi^2}{m_D^2}\right)^2 \lambda^{1/2}\left(1, \frac{m_\pi^2}{m_D^2}, \frac{m_a^2}{m_D^2}\right), \\ a_V^{D^0 \rightarrow \pi^0}(m_a) &= \frac{1}{2} a_V^{D^+ \rightarrow \pi^+}(m_a), \end{aligned} \quad (45)$$

for both modes, respectively, and the axial-vector contribution vanishes. For $D_s^+ \rightarrow K^+ a$ we use the same replacements as discussed in Sec. III A 2. The branching ratio allows us to probe the vector coupling k_{12}^V/f , while contributions of axial-vector couplings are unconstrained. This is caused by the parity conservation of QCD, which causes the hadronic matrix element of two pseudoscalar mesons to vanish for axial currents.

The branching ratio of $D^{0(+)} \rightarrow \pi^{0(+)} a$ is shown in Fig. 4 as a function of the ALP mass m_a . For differential branching ratios $D \rightarrow \pi a$ differs from $D \rightarrow \pi \nu \bar{\nu}$ as its decay topology is that of a two-body decay. The differential branching fraction would be proportional to a Dirac delta function $\delta(q^2 - m_a^2)$ and would be distinguishable from the contributions shown in Fig. 3. Note that because of this the kinematic cuts for the charged modes in Eq. (29) cannot be applied for arbitrary ALP masses and only $D^0 \rightarrow \pi^0 +$ invisible or baryon modes remain viable NP searches for $m_a^2 < q_{\text{cut}}^2$.

2. $D \rightarrow \rho(\omega) a$

For the decay to vector mesons e.g. $D \rightarrow Va$ with $V = \rho^0, \omega$, contrary to decays to pseudoscalar mesons,

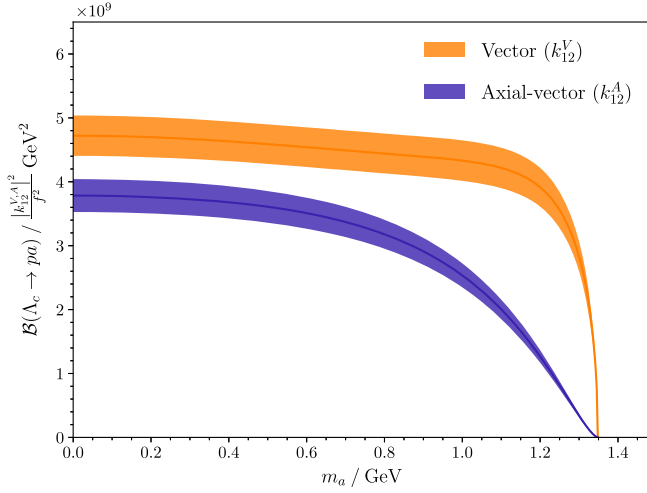


FIG. 5. Branching ratio of $\Lambda_c \rightarrow pa$ against m_a with only the axial coupling switched on (lower band, blue) and only the vector one turned on (upper band, orange). The couplings $|k_{12}^{V,A}|^2/f^2$ have been factored out, and the different sensitivity stems from different form factors, (47). The main source of uncertainty stems from the form factors, shown as bands.

the contribution from the vector coupling vanishes as a consequence of “the equations of motion.” The contribution to the branching fraction, calculated via Eq. (42), is given by

$$a_A^{D \rightarrow V}(m_a) = \frac{\tau_D}{64\pi} m_D^3 \lambda^{3/2} \left(1, \frac{m_V^2}{m_D^2}, \frac{m_a^2}{m_D^2} \right) A_0^2(m_a) \quad (46)$$

and probes the axial coupling of the ALP instead. Both $D_{(s)} \rightarrow \pi(K)a$ and $D \rightarrow V$ probe therefore orthogonal directions and are complementary.

3. $\Lambda_c \rightarrow pa$ and $\Xi_c \rightarrow \Sigma a$

The branching fraction of $\Lambda_c \rightarrow pa$ is calculated via Eq. (42) with

$$\begin{aligned} a_V^{\Lambda_c \rightarrow p}(m_a) &= N_{\Lambda_c p}^{\text{ALP}} (m_{\Lambda_c} - m_p)^2 s_+ f_0^2, \\ a_A^{\Lambda_c \rightarrow p}(m_a) &= N_{\Lambda_c p}^{\text{ALP}} (m_{\Lambda_c} + m_p)^2 s_- g_0^2, \end{aligned} \quad (47)$$

and the normalization

$$N_{\Lambda_c p}^{\text{ALP}}(m_a) = \frac{\tau_{\Lambda_c}}{64\pi m_{\Lambda_c}} \sqrt{\lambda \left(1, \frac{m_p^2}{m_{\Lambda_c}^2}, \frac{m_a^2}{m_{\Lambda_c}^2} \right)}, \quad (48)$$

where f_0 , g_0 , and $s_{\pm}$ are evaluated at $q^2 = m_a^2$. In contrast to $D \rightarrow \pi a$ decays, $\Lambda_c \rightarrow pa$ is sensitive to both vector and axial-vector ALP couplings. The branching ratio as a function of the ALP mass m_a is shown in Fig. 5. Due to different form factors involved, and $f_0 > g_0$, the sensitivity to the vector coupling is larger. For $\Xi_c^+ \rightarrow \Sigma^+ a$ we use the same formulas; see Sec. III A 4.

4. $D \rightarrow \pi\pi a$

Similar to $D \rightarrow Va$ decays the branching ratio of $D^0 \rightarrow \pi^+ \pi^- a$ and $D^+ \rightarrow \pi^+ \pi^0 a$ is sensitive to the axial ALP coupling only. The branching ratio is obtained via Eq. (42) with

$$a_A^{D \rightarrow \pi\pi} = \int_{4m_\pi^2}^{(m_D - m_a)^2} dp^2 \int_{-1}^1 d \cos \theta_{\pi^+} \frac{16\pi^4}{\alpha_e^2 G_F^2} \frac{\sqrt{p^2}}{q^2} |\mathcal{F}_I|^2. \quad (49)$$

There is however insufficient knowledge on the scalar transversity form factor $\mathcal{F}_I$ —see Appendix A 4—so we refrain from using it for phenomenology in this work.

C. Z' modes

Decays that involve a Z' decay to invisible dark sector particles χ are, besides the couplings, also heavily dependent on the masses and widths of the NP particles. We consider a light Z' with mass $m_{Z'}/2 > m_\chi$ and at most $m_{Z'} \simeq \mathcal{O}(\text{GeV})$.

For the on-shell production of a Z' in the narrow-width approximation the branching fraction of $h_c \rightarrow FZ'(\rightarrow \chi\bar{\chi})$ reads

$$\begin{aligned} \mathcal{B}(h_c \rightarrow FZ'(\rightarrow \chi\bar{\chi})) \\ \simeq \mathcal{B}(h_c \rightarrow FZ')(m_{Z'}^2) \cdot \mathcal{B}(Z' \rightarrow \chi\bar{\chi}). \end{aligned} \quad (50)$$

We can simplify this further assuming that the Z' only decays into the dark sector, that is, $\mathcal{B}(Z' \rightarrow \chi\bar{\chi}) = 1$. The result is then identical to the one for a stable Z' .

In the case of a finite Z' width, where the narrow-width approximation breaks down, the branching fraction can be approximated by [51]

$$\begin{aligned} \mathcal{B}(h_c \rightarrow FZ'(\rightarrow \chi\bar{\chi})) \\ = \int_{q_{\min}^2}^{q_{\max}^2} \frac{d\mathcal{B}(h_c \rightarrow FZ'(\rightarrow \chi\bar{\chi}))}{dq^2} dq^2 \\ \simeq \int_{q_{\min}^2}^{q_{\max}^2} \Gamma_{Z'}(q^2) BW(q^2) \mathcal{B}(h_c \rightarrow FZ')(q^2) dq^2, \end{aligned} \quad (51)$$

with Breit-Wigner shape

$$BW(q^2) = \frac{1}{\pi} \frac{\sqrt{q^2}}{(q^2 - m_{Z'}^2)^2 + \Gamma_{Z'}(q^2)^2 m_{Z'}^2} \quad (52)$$

and $\mathcal{B}(h_c \rightarrow FZ')(q^2)$ is the off-shell two-body branching fraction with momentum transfer q^2 . Here, $\Gamma_{Z'}(q^2)$ is given in Eq. (23). The q^2 integration is performed over the kinematically allowed region $4m_\chi^2 < q^2 \leq (m_{h_c} - m_F)^2$

except for $D_{(s)}^+ \rightarrow \pi^+(K^+)Z'(\rightarrow \chi\bar{\chi})$, where $q_{\min}^2 = \max(4m_\chi^2, q_{\min, D_{(s)}^+ \rightarrow \pi^+(K^+)})$ to remove the τ background (29).

1. $D^0 \rightarrow Z' \rightarrow \chi\bar{\chi}$

The Z' contribution to $D^0 \rightarrow$ invisible reads

$$\mathcal{B}(D^0 \rightarrow Z' \rightarrow \chi\bar{\chi}) = \frac{\tau_D |\mathcal{C}_A^\chi|^2 f_D^2 m_\chi^2}{8\pi(\Gamma_{Z'}^2 m_{Z'}^2 + (m_{Z'}^2 - m_D^2)^2)} \times \sqrt{1 - \frac{4m_\chi^2}{m_D^2}} x_{LR-}^{Z'}. \quad (53)$$

Similar to the EFT contribution to dineutrinos discussed in Sec. III A 1 a vector contribution, $\propto \mathcal{C}_V^{Z'\chi}$ vanishes and an axial-vector one, $\propto \mathcal{C}_A^{Z'\chi}$ is suppressed by the mass of the invisible particle, here m_χ .

2. $D \rightarrow \pi Z'(\rightarrow \chi\bar{\chi})$

The branching fraction for $D^0 \rightarrow \pi^0 Z'$ with momentum transfer q^2 is given as

$$\mathcal{B}(D^0 \rightarrow \pi^0 Z')(q^2) = \frac{\lambda^{3/2}(m_D^2, m_\pi^2, q^2)}{64\pi m_D^3 (m_D + m_\pi)^2 q^2} \times \left(x_{LR+}^{Z'} \frac{f_+^2}{2} (m_D + m_\pi)^2 + 16x_D^{Z'} \frac{f_T^2}{2} q^4 + 8x_{\text{ReLRD}}^{Z'} \frac{f_+ f_T}{2} q^2 (m_D + m_\pi) \right), \quad (54)$$

which agrees with Refs. [15,51], for $q^2 = m_{Z'}^2$. We compute the differential and total branching fraction via Eq. (51).

In Fig. 6 we show the differential branching ratio of $D^0 \rightarrow \pi^0 Z'(\rightarrow \chi\bar{\chi})$ for different couplings $x_k^{Z'}$ with $k \in \{LR+, D\}$ of Eq. (22). We use BM_V of Eq. (24) and for each of the curves a single coefficient x_k is turned on such that the integrated branching ratio is normalized to the same value. In Fig. 7 we show the same differential branching ratio but vary instead a single model parameter $m_{Z'}$, $\Gamma_{Z'}$ or m_χ and others conforming to BM_V . For all curves we assume only $x_{LR+}^{Z'} \neq 0$ and fix its value via normalization of the branching ratio. We learn that the shape of the differential branching ratio on the mass and width parameters of the Z' model is strong, while the dependence on the coupling type is rather small.

3. $D \rightarrow \rho^0(\omega)Z'(\rightarrow \chi\bar{\chi})$

For $D \rightarrow VZ'(\rightarrow \chi\bar{\chi})$ with $V = \rho^0(\omega)$ additional couplings contribute than for the pseudoscalar meson final

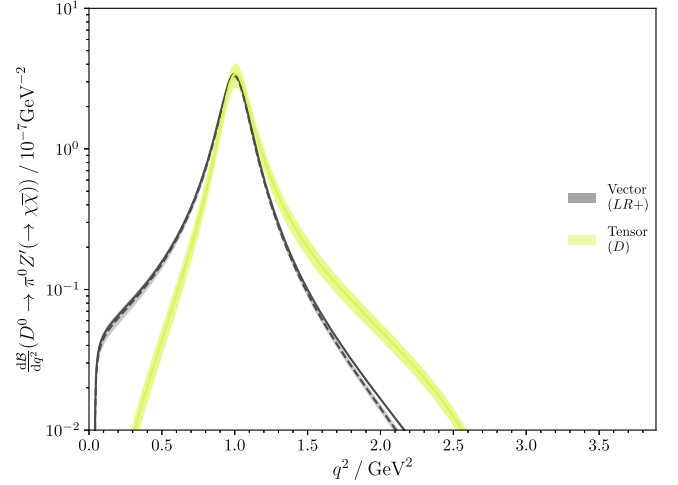


FIG. 6. Shapes of $d\mathcal{B}(D^0 \rightarrow \pi^0 Z'(\rightarrow \chi\bar{\chi}))/dq^2$, with $\mathcal{B}(D^0 \rightarrow \pi^0 \nu\bar{\nu}) = 10^{-7}$, for different couplings, obtained by switching on a single coefficient $x_k^{Z'}$, $k \in \{LR+, D\}$ of Eq. (12) for the Z' benchmark (24). Identical for $D^+ \rightarrow \pi^+ Z'(\rightarrow \chi\bar{\chi})$ with different normalization of the coefficients $x_k^{Z'}$ caused by lifetime difference and isospin factor. Solid (dashed) curves are using most recent (ETM) form factors; see Sec. III A 2.

states and the off-shell two-body branching fraction is given as

$$\mathcal{B}(D \rightarrow VZ')(q^2) = \sum_{k \in \{LR\pm, D, D5, \text{ReLRD}, \text{ReLRD5}\}} a_k^{D \rightarrow VZ'}(q^2) x_k^{Z'} \quad (55)$$

with the functions

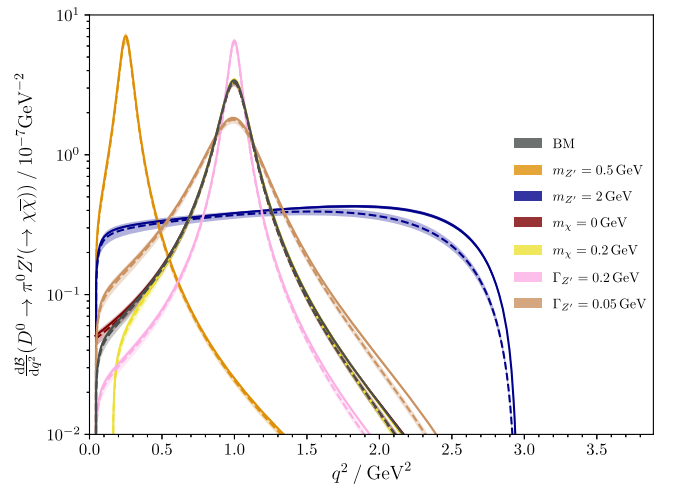


FIG. 7. Shapes of $d\mathcal{B}(D^0 \rightarrow \pi^0 Z'(\rightarrow \chi\bar{\chi}))/dq^2$, with $\mathcal{B}(D^0 \rightarrow \pi^0 \nu\bar{\nu}) = 10^{-7}$, for fixed coupling $x_{LR+}^{Z'}$ but different model parameters $m_{Z'}$, $\Gamma_{Z'}$, and m_χ departing from BM_V of Eq. (24).

$$\begin{aligned}
 a_{LR+}^{D \rightarrow VZ'}(q^2) &= N_{DV}^{Z'} \lambda_{DV} V^2, \\
 a_{LR-}^{D \rightarrow VZ'}(q^2) &= N_{DV}^{Z'} (m_D + m_V)^2 \left(32 \frac{m_D^2 m_V^2}{q^2} A_{12}^2 + (m_D + m_V)^2 A_1^2 \right), \\
 a_D^{D \rightarrow VZ'}(q^2) &= 4 N_{DV}^{Z'} \lambda_{DV} (m_D + m_V)^2 T_1^2, \\
 a_{D5}^{D \rightarrow VZ'}(q^2) &= 4 N_{DV}^{Z'} (8 m_D^2 m_V^2 q^2 T_{23}^2 + (m_D^2 - m_V^2)^2 (m_D + m_V)^2 T_2^2), \\
 a_{\text{ReLRD}}^{D \rightarrow VZ'}(q^2) &= -8 N_{DV}^{Z'} (m_D + m_V) \lambda_{DV} V T_1, \\
 a_{\text{ReLRD5}}^{D \rightarrow VZ'}(q^2) &= 8 N_{DV}^{Z'} (m_D + m_V) (16 m_D^2 m_V^2 A_{12} T_{23} + (m_D^2 - m_V^2) (m_D + m_V)^2 A_1 T_2),
 \end{aligned} \tag{56}$$

and the normalization

$$N_{DV}^{Z'} = \frac{\lambda_{DV}^{1/2}}{64 \pi m_D^3 (m_D + m_V)^2}. \tag{57}$$

For the form factors we refer to Appendix A 2. The differential branching fraction with the subsequent decay of the Z' is obtained from Eq. (51).

4. $\Lambda_c \rightarrow pZ'(\rightarrow \chi\bar{\chi})$ and $\Xi_c^+ \rightarrow \Sigma^+ Z'(\rightarrow \chi\bar{\chi})$

For $\Lambda_c \rightarrow pZ'$ the off-shell two-body branching fraction used in Eq. (51) for the differential and total branching fraction reads

$$\mathcal{B}(\Lambda_c \rightarrow pZ')(q^2) = \sum_{k=\{LR\pm, D, D5, \text{ReLRD}, \text{ReLRD5}\}} a_k^{\Lambda_c \rightarrow pZ'}(q^2) x_k^{Z'} \tag{58}$$

with the functions

$$\begin{aligned}
 a_{LR+}^{\Lambda_c \rightarrow pZ'}(q^2) &= N_{\Lambda_c p}^{Z'} s_- (2f_\perp^2 q^2 + f_+^2 (m_{\Lambda_c} + m_p)^2), \\
 a_{LR-}^{\Lambda_c \rightarrow pZ'}(q^2) &= N_{\Lambda_c p}^{Z'} s_+ (2g_\perp^2 q^2 + g_+^2 (m_{\Lambda_c} - m_p)^2), \\
 a_D^{\Lambda_c \rightarrow pZ'}(q^2) &= 16 N_{\Lambda_c p}^{Z'} s_- (2h_\perp^2 (m_{\Lambda_c} + m_p)^2 + q^2 h_+^2), \\
 a_{D5}^{\Lambda_c \rightarrow pZ'}(q^2) &= 16 N_{\Lambda_c p}^{Z'} s_+ (2\tilde{h}_\perp^2 (m_{\Lambda_c} - m_p)^2 + q^2 \tilde{h}_+^2), \\
 a_{\text{ReLRD}}^{\Lambda_c \rightarrow pZ'}(q^2) &= -8 N_{\Lambda_c p}^{Z'} (m_{\Lambda_c} + m_p) s_- (2f_\perp h_\perp + f_+ h_+), \\
 a_{\text{ReLRD5}}^{\Lambda_c \rightarrow pZ'}(q^2) &= 8 N_{\Lambda_c p}^{Z'} (m_{\Lambda_c} - m_p) s_+ (2g_\perp \tilde{h}_\perp + g_+ \tilde{h}_+), \\
 N_{\Lambda_c p}^{Z'} &= \frac{\lambda^{1/2}(m_{\Lambda_c}^2, m_p^2, q^2)}{64 \pi m_{\Lambda_c}^3},
 \end{aligned} \tag{59}$$

using the form factors in Appendix A 3. We adopt the same formulas for $\Xi_c^+ \rightarrow \Sigma^+$; see Sec. III A 4.

For massless Z' 's, using the end point relation $h_\perp(0) = \tilde{h}_\perp(0)$, we recover the branching fraction from Ref. [19]:

$$\mathcal{B}(\Lambda_c \rightarrow pZ') = \frac{\tau_{\Lambda_c} (m_{\Lambda_c}^2 - m_p^2)^3}{2 \pi m_{\Lambda_c}^3} h_\perp(0)^2 \frac{|\mathcal{C}_D^{Z'}|^2 + |\mathcal{C}_{D5}^{Z'}|^2}{\Lambda_{\text{eff}}^2}. \tag{60}$$

5. $D \rightarrow \pi\pi Z'(\rightarrow \chi\bar{\chi})$

For the decays $D^{+(0)} \rightarrow \pi^+ \pi^{0(-)} Z'(\rightarrow \chi\bar{\chi})$ the q^2 differential and total branching ratio is obtained via Eq. (51) with the off-shell branching ratio

$$\mathcal{B}(D \rightarrow \pi\pi Z')(q^2) = \int_{4m_\pi^2}^{(m_D - \sqrt{q^2})^2} dp^2 \int_{-1}^1 d \cos \theta_{\pi^+} \sum_k b_k^{D \rightarrow \pi\pi Z'}(q^2, p^2, \theta_{\pi^+}) x_k^{Z'}, \tag{61}$$

where $k \in \{LR\pm, D(5), \text{Re}LRD(5), \text{Im}LRD(5)\}$. The functions b_k are given as

$$\begin{aligned}
 b_{LR+}^{D \rightarrow \pi\pi Z'} &= \mathcal{N}_{D \rightarrow \pi\pi}^{Z'} \frac{\sqrt{p^2}}{4q^2} \sin^2 \theta_{\pi^+} |\mathcal{F}_\perp|^2, \\
 b_{LR-}^{D \rightarrow \pi\pi Z'} &= \mathcal{N}_{D \rightarrow \pi\pi}^{Z'} \frac{\sqrt{p^2}}{4q^2} (|\mathcal{F}_0|^2 + \sin^2 \theta_{\pi^+} |\mathcal{F}_\parallel|^2), \\
 b_D^{D \rightarrow \pi\pi Z'} &= \mathcal{N}_{D \rightarrow \pi\pi}^{Z'} \sqrt{p^2} \sin^2 \theta_{\pi^+} |\mathcal{F}_\perp^T|^2, \\
 b_{D5}^{D \rightarrow \pi\pi Z'} &= \mathcal{N}_{D \rightarrow \pi\pi}^{Z'} \sqrt{p^2} (|\mathcal{F}_0^T|^2 + \sin^2 \theta_{\pi^+} |\mathcal{F}_\parallel^T|^2), \\
 b_{\text{Re(Im)}LRD}^{D \rightarrow \pi\pi Z'} &= -\mathcal{N}_{D \rightarrow \pi\pi}^{Z'} \sqrt{\frac{p^2}{q^2}} \sin^2 \theta_{\pi^+} \text{Im(Re)}\{\mathcal{F}_\perp \mathcal{F}_\perp^{T*}\}, \\
 b_{\text{Re(Im)}LRD5}^{D \rightarrow \pi\pi Z'} &= -\mathcal{N}_{D \rightarrow \pi\pi}^{Z'} \sqrt{\frac{p^2}{q^2}} (\text{Im(Re)}\{\mathcal{F}_0 \mathcal{F}_0^{T*}\} \\
 &\quad + \sin^2 \theta_{\pi^+} \text{Im(Re)}\{\mathcal{F}_\parallel \mathcal{F}_\parallel^{T*}\}), \quad (62)
 \end{aligned}$$

with the normalization factor

$$\mathcal{N}_{D \rightarrow \pi\pi}^{Z'} = \frac{64\pi^4}{\alpha_e^2 G_F^2}. \quad (63)$$

The transversity form factors $\mathcal{F}_i$ are defined in Appendix A 4.

D. Distinguishing models

We discuss possibilities to distinguish models from decays to invisibles, for which we consider neutrinos from contact interactions and light new physics (ALPs, Z'), here collectively denoted by X . Depending on the width Γ_X of the latter, we have the following cases.

$\Gamma_X = 0$, *that is, infinite lifetime*. The decays such as $D \rightarrow \pi X$ are two-body decays. One can extract the mass of X , m_X , from the missing energy, $m_X^2 = 2m_D E_{\text{miss}} + m_\pi^2 - m_D^2$. To distinguish different invisibles with similar mass needs more than one decay. ALPs, Z' have different couplings to the SM, and due to spin come with different form factors. This induces in general different branching ratios for identical masses. If the mass is extracted and the branching ratio measured, the couplings can be obtained, in a given model, which provides correlations with and predictions for other decay modes. (Up to complications if more than one coupling per model is active. In that case, say with two couplings as in the ALP model one would need to measure two modes and then predict a third one.)

In Fig. 8 we show the ratio of $\mathcal{B}(D \rightarrow \pi X)$ to $\mathcal{B}(\Lambda_c \rightarrow p X)$ in the EFT, ALP, and Z' models which induce $D \rightarrow \pi X$ decays. Scenarios with $\mathcal{B}(D \rightarrow \pi X) = 0$, e.g. $x_{LR-} \neq 0$ or the axial-vector coupling of ALPs, induce $\mathcal{B}(\Lambda_c \rightarrow p X)$ and can be correlated with $D \rightarrow VX$ or $D \rightarrow \pi\pi X$. We learn that correlations can distinguish the models.

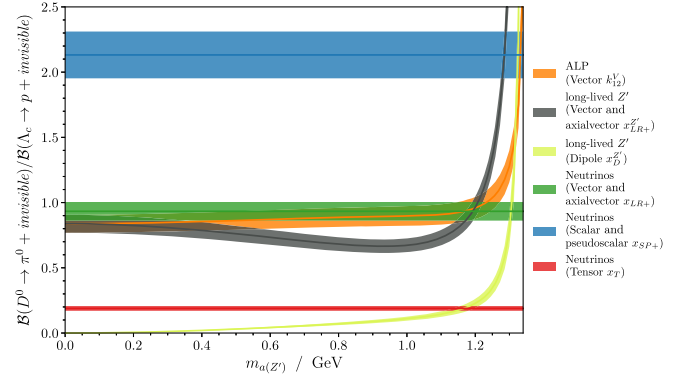


FIG. 8. Ratio of the branching fractions of $D^0 \rightarrow \pi^0 + \text{invisible}$ over $\Lambda_c \rightarrow p + \text{invisible}$ for ALP models with $\Gamma_a = 0$ (orange) and long-lived Z' models with vector coupling (gray) and dipole coupling (yellow) against the invisible mass m_X , $X = a$ or $X = Z'$. Also shown by horizontal bands are EFT predictions with vector x_{LR+} , scalar x_{SP+} and tensor x_T structure.

The other extreme case is that X decays promptly, perhaps predominantly to dark fermions. This gives rise, for instance, in $D \rightarrow \pi(X \rightarrow \chi\chi)$ to a three-body decay with a distribution in missing energy. This induces four-fermion operators, with different Dirac structure, that can give different shapes shown, for instance, in Fig. 1. Also here the models can be distinguished.

The intermediate case is when the X is a resonance. Its peak gives m_X . For similar mass, the rates and shapes of the distributions differ, as illustrated in Fig. 7.

IV. RECAST OF EXPERIMENTAL DATA

Upper bounds on branching ratios of rare charm decays with invisibles in the final state are listed in Table II. We discuss how we perform a simplified reanalysis (“recast”) of the experimental data under a different signal shape. This is necessary to reinterpret the bounds for the different models we consider.

Experimental collaborations measure upper limits on the branching ratio via a binwise distribution of the differential branching ratio in the invariant mass of the invisible particle (s) q^2 or some otherwise comparable kinematic variable. Under optimal conditions the number of binwise

TABLE II. Experimental limits on rare charm decays with dineutrinos or invisible BSM particles in the final state. [†]With τ^+ decaying to $\bar{\nu}$ plus a light meson.

Decay	Experimental limit @90% CL
$\mathcal{B}(D^0 \rightarrow \text{invisible})$	$< 9.4 \times 10^{-5}$ [6]
$\mathcal{B}(D^0 \rightarrow \pi^0 \nu \bar{\nu})$	$< 2.1 \times 10^{-4}$ [4]
$\mathcal{B}(D^0 \rightarrow \omega \gamma')$	$< 1.1 \times 10^{-5}$ [3]
$\mathcal{B}(D^+ \rightarrow \tau^+ \nu)^\dagger$	$< 1.2 \times 10^{-3}$ [52]
$\mathcal{B}(\Lambda_c \rightarrow p \gamma')$	$< 8.0 \times 10^{-5}$ [5]

background events b_i and efficiencies ϵ_i as well as the performed kinematic cuts and considered smearing effects are provided. In this case the number of signal events s_i in a q^2 bin i is given as [15]

$$s_i = \tilde{B}_i(h_c \rightarrow F + \text{invisible}) \times N_{\text{tot}} \times \epsilon_i, \quad (64)$$

where N_{tot} is the total number of h_c hadrons and $\tilde{B}_i$ denotes the branching ratio within bin i including other kinematic cuts and smearing. For our likelihood

$$\begin{aligned} \mathcal{L}(\mathcal{B}, \vec{\nu} | \vec{n}) &= \prod_{i=1}^m \text{Pois}(n_i; s_i(\mathcal{B}, \vec{\nu}) + b_i(\vec{\nu})) \\ &\times \prod_i^k \mathcal{N}(\nu_i; \mu_{\nu_i}, \sigma_{\nu_i}) \end{aligned} \quad (65)$$

we consider a product of Poisson distributions $\text{Pois}(n; \lambda) = \lambda^n e^{-\lambda} / n!$ for the binned measurements and Gaussian distributions $\mathcal{N}(x; \mu, \sigma) = e^{-\frac{(x-\mu)^2}{2\sigma^2}} / \sqrt{2\pi\sigma^2}$ for nuisance parameters $\vec{\nu}$ like the number of background events, etc. Here μ_{ν_i} (σ_{ν_i}) are the expectation value (standard deviation) of the nuisance parameter ν_i . Recast limits on the branching ratio $\mathcal{B}$ are obtained by performing a profile likelihood ratio test and by assuming for simplicity that the test statistic is χ^2 distributed.

For the measurement of $\mathcal{B}(D^0 \rightarrow \pi^0 \nu \bar{\nu})$ by Ref. [4] the signal region is $q^2 \in [1.1, 1.9] \text{ GeV}^2$ and no binwise efficiencies and no proper³ binwise background have been provided. The used signal shape corresponds to our scenario with light LH and RH neutrinos with only the coefficient x_{LR+} nonzero. To reinterpret this result we therefore can follow a more simplified approach, assuming no bin dependence of the efficiencies and by translating the bound on the full branching ratio to a bound on the branching ratio in the bin $q^2 \in [1.1, 1.9] \text{ GeV}^2$. We obtain

$$\langle \mathcal{B} \rangle (D^0 \rightarrow \pi^0 + \text{invisible})|_{q^2 \in [1.1, 1.9] \text{ GeV}^2} \lesssim 5.8 \times 10^{-5} \quad (66)$$

and constrain other signal shapes by evaluating them in the same q^2 bin.

The measurement of $D^+ \rightarrow \tau^+ \nu_\tau$ [52] can be recast to a measurement of $D^+ \rightarrow \pi^+ + \text{invisible}$ within the signal region $q^2 < 0.05 \text{ GeV}^2$ [53]. The number of measured signal events is $n = 11$ and the estimated number of background events is $b = 13.5 \pm 1.0$ in the signal region. With the total number of tagged D decays $N_{\text{tot}} = 4.6 \times 10^5$ and single pion detection efficiency $\epsilon = 0.89$ this yields [53]

$$\langle \mathcal{B} \rangle (D^+ \rightarrow \pi^+ + \text{invisible})|_{q^2 < 0.05 \text{ GeV}^2} \lesssim 8.0 \times 10^{-6} \quad (67)$$

at 90% confidence level (CL), based on the mixed Bayesian-frequentist approach given in Ref. [54]. Here, the Poisson distributed part of the likelihood in Eq. (66) (with the number of bins $m = 1$) is maximized with an estimator Q and then a subsequent Bayesian analysis of Q is performed. The estimator Q is defined as [54]

$$Q = \frac{n N_{\text{tot}} \epsilon}{N_{\text{tot}} \epsilon \langle \mathcal{B} \rangle_{\text{data}} + b} - N_{\text{tot}} \epsilon, \quad (68)$$

with $\langle \mathcal{B} \rangle_{\text{data}}$ fixed by

$$\frac{n N_{\text{tot}} \epsilon}{N_{\text{tot}} \epsilon \langle \mathcal{B} \rangle_{\text{data}} + E[b]} - N_{\text{tot}} \epsilon = 0 \quad (69)$$

and $E[b] = 13.5$, the expectation value of b . The Bayesian analysis is performed by generating a normal distributed sample of background events, $\{b\}$, and subsequently a sample of observed events, $\{n\}$ for each tested value of $\langle \mathcal{B} \rangle$ using a Poisson distribution with the expectation values $\{\mu\} = N_{\text{tot}} \epsilon \langle \mathcal{B} \rangle + \{b\}$. The limit on the branching ratio at a CL of α corresponds to the value of $\langle \mathcal{B} \rangle$ for which the fraction α of the samples has positive Q .

Following instead a purely frequentist approach by performing a profile likelihood ratio test with the likelihood in Eq. (66) using minimization routines from IMNUT [55], we obtain an upper limit on the branching ratio as 6.0×10^{-6} at 90% CL. To be conservative in the statistical approach we decide to use the bound of Eq. (67) in the subsequent analysis.

The situation for $\mathcal{B}(\Lambda_c \rightarrow p \gamma')$ and $\mathcal{B}(D^0 \rightarrow \omega \gamma')$ is less clear cut, as no per-bin efficiencies are provided. However, the modes are useful as they probe couplings that are not accessible with $D \rightarrow \pi + \text{invisible}$. Experiments do provide binwise background and observed events, albeit small signal windows. For $\Lambda_c \rightarrow p \gamma'$ a signal window has been specified as $q^2 \in [0.0, 0.1] \text{ GeV}^2$, while for $D^0 \rightarrow \omega \gamma'$ no such signal region is given. In both cases the signal shape has been smeared by a Gaussian distribution; we verified however that for our application this effect is rather small ($< 1\%$) and will not include any smearing effect for the evaluated observables. To obtain bounds for signal distributions that differ from experimental signal distributions we require the branching ratio in the region $q^2 \in [0.0, 0.1] \text{ GeV}^2$ to fulfill the bounds in Table II for the dark photon. We do this for both modes although the bound for $D^0 \rightarrow \omega \gamma'$ uses potentially a bigger region. This should be done because also for $D^0 \rightarrow \omega \gamma'$ the signal lies mostly within $q^2 \in [0.0, 0.1] \text{ GeV}^2$ and the bound given in Ref. [3] should approximately apply in this region. If we would consider signals with higher contributions outside this window, we expect higher deviations and a more detailed binwise recast is necessary but beyond the scope of this work. We conservatively use

³In Ref. [4] a per-bin MC background has been provided which however does not directly agree with data and has been adjusted by a corrections factor for the whole signal region.

$$\begin{aligned} \langle \mathcal{B} \rangle (\Lambda_c \rightarrow p + \text{invisible})|_{q^2 < 0.1 \text{ GeV}^2} &\lesssim 8.0 \times 10^{-5}, \\ \langle \mathcal{B} \rangle (D^0 \rightarrow \omega + \text{invisible})|_{q^2 < 0.1 \text{ GeV}^2} &\lesssim 1.1 \times 10^{-5}. \end{aligned} \quad (70)$$

We learn that for two-body decays into an invisible particle, such as a long-lived Z' or an ALP, two mass windows can be probed with present data, a low mass region $m_a < 0.22 \text{ GeV}$ using (67) [and $m_a < 0.3 \text{ GeV}$ using (70)] and a high mass window $1.05 < m_a < 1.38 \text{ GeV}$ using the dineutrino search (66).

Outside these mass windows, and more general for decay modes of all charmed hadrons, lifetime constraints apply. They are obtained from subtracting the sum of all reported exclusive branching ratios from unity. Using [2] we find for the unaccounted branching ratios

$$\begin{aligned} \mathcal{B}(D^0 \rightarrow \text{unacc}) &\lesssim 0.050, & \mathcal{B}(D^+ \rightarrow \text{unacc}) &\lesssim 0.082, \\ \mathcal{B}(D_s^+ \rightarrow \text{unacc}) &\lesssim 0.39, \\ \mathcal{B}(\Lambda_c \rightarrow \text{unacc}) &\lesssim 0.25, & \mathcal{B}(\Xi_c^+ \rightarrow \text{unacc}) &\lesssim 0.77. \end{aligned} \quad (71)$$

which are weak, however, provide constraints beyond the searches Eqs. (66)–(70). As any measurement is expected to give better bounds, we will not use (71) in the tables with constraints on model parameters.

V. CONSTRAINTS IN EFT AND MODELS

We work out limits on Wilson coefficients in Sec. VA and for couplings of ALP and light Z' models in Secs. VB and VC, respectively.

A. Upper limits on Wilson coefficients

Bounds on the combinations of Wilson coefficients in Eq. (12) of the EFT with LH and RH neutrinos are obtained from the measurements of Table II and recasts given in Eqs. (66)–(70). For each of the bounds we turn on only a single x_k . We use the hadronic coefficients $A_k^{h_c \rightarrow F}$ from Table I to obtain limits on the NP coefficients, presented in Table III. The bound on x_{SP-} or x_{SP} from $D^0 \rightarrow \text{invisible}$ agrees with Ref. [10]. The limit on x_{SP} from $D^0 \rightarrow \text{invisible}$ is almost 2 orders of magnitude stronger than from $D^0 \rightarrow \pi^0 \nu \bar{\nu}$. The coefficient x_{LR-} remains weakly constrained as only searches with narrow q^2 windows probe this coupling. Extending the analysis of $D^0 \rightarrow \omega + \text{invisible}$ or $\Lambda_c \rightarrow p + \text{invisible}$ to a bigger q^2 window could improve these bounds.

Besides the general WET limits on x_k presented in Table III additional constraints exist within SMEFT and ν SMEFT. Upper limits on the chirality-preserving four-fermion operators with left-handed neutrinos and right-handed neutrinos O_{QN}, O_{uN} (7) have been obtained from large missing transverse energy (MET) plus jet searches at the LHC [56,57]

TABLE III. Upper limits from the decays $h_c \rightarrow F + \text{invisible}$ on the combinations of Wilson coefficients x_k of the LH and RH neutrino model in Eqs. (12) and (18). Limits are obtained using experimental input from Table II. Blank spaces indicate that the coupling cannot be constrained by the decay. The columns with x_{SP} and x_{LR} are single SMEFT coefficient limits; see Eq. (18).

	Light LH and RH neutrino						
	x_{SP-}	x_{SP+}	x_{SP}	x_{LR-}	x_{LR+}	x_{LR}	x_T
$h_c \rightarrow F$	[10 ⁰]	[10 ³]	[10 ³]	[10 ⁴]	[10 ⁴]	[10 ⁴]	[10 ³]
$D^0 \rightarrow$	66		0.066				
$D^0 \rightarrow \pi^0$		4.6	4.6		2.2	2.2	1.8
$D^+ \rightarrow \pi^+$		112	112		0.52	0.52	18.6
$D^0 \rightarrow \omega$	13×10^4		133	9.3	58	0.92	0.083
$\Lambda_c \rightarrow p$	15×10^5	1185	659	129	10.9	5.9	2.9
Best overall	66	4.6	0.066	9.3	0.52	0.52	0.083

$$x_{LR\pm} \lesssim 2 \left(\frac{2\pi v^2}{\alpha_e \Lambda^2} \right)^2 |C_{uc}^{4F}|^2, \quad (72)$$

$$\begin{aligned} |C_{uc}^{4F}|^2 &= \sum_{ij} \left(|C_{\ell q}^{(1)ij} + C_{\ell q}^{(3)ij}|^2 + |C_{u\ell}^{ij}|^2 \right) \\ &+ \sum_{pr} \left(|C_{QN}^{pr}|^2 + |C_{uN}^{pr}|^2 \right) \end{aligned} \quad (73)$$

with $C_{uc}^{4F}/\Lambda^2 < 0.22/\text{TeV}^2$ [57,58]. Here, ij (pr) are left-handed (right-handed) neutrino flavors, which are contributing incoherently. We obtain

$$x_{LR\pm} \lesssim 270 \quad (\text{MET} + \text{jet}). \quad (74)$$

Due to the incoherent sum, all operators and all flavors are constrained. This feature makes the MET plus jet searches from the LHC and in the future from the high-luminosity LHC (HL-LHC) particularly important for invisible decays. For lepton universal (LU), charged lepton-flavor conserving (cLFC) and general flavor structures bounds exploiting the $SU(2)_L$ link between processes into charged dileptons and into dineutrinos have been obtained from Drell-Yan data [10]

$$\begin{aligned} x_{LR\pm} &\lesssim 68(\text{LU}), \\ x_{LR\pm} &\lesssim 392(\text{cLFC}), \\ x_{LR\pm} &\lesssim 1432(\text{general}). \end{aligned} \quad (75)$$

Since the MET plus jet analysis interpreted in [56,57] required a full theory simulation and in view of the sizable uncertainties we conservatively use for now (75) to estimate upper SMEFT limits. The limits from high p_T are considerably stronger than the current limits on $x_{LR\pm}$ from D decays; see Table III.

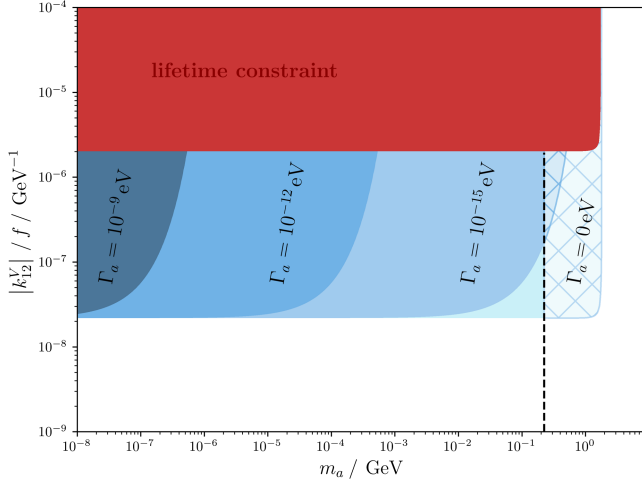


FIG. 9. Excluded regions (blue) of $|k_{12}^V|/f$ depending on the ALP mass m_a for various ALP decay widths $\Gamma_a = \{10^{-9} \text{ eV}, 10^{-12} \text{ eV}, 10^{-15} \text{ eV}, 0 \text{ eV}\}$ (from left to right). The larger the width, the weaker the constraint. The exclusion region stems from the recast (67) of $D^+ \rightarrow \pi^+ + \text{invisible}$ valid for $m_a < 0.224 \text{ GeV}$, the latter indicated by a dashed vertical line. The lighter colored, hatched region to the right of the dashed line is unconstrained. The lifetime constraint (red) of D^+ (71) is orders of magnitude weaker.

B. Constraints on ALP parameters

We show in Fig. 9 the excluded region (blue) of the coupling $|k_{12}^V|/f$ and the ALP mass for various decay widths Γ_a . The larger the latter, the weaker the constraints. For instance, for $\Gamma = 10^{-12} \text{ eV}$ we cannot probe masses $m_a \gtrsim 10^{-3} \text{ GeV}$ with the invisible decay as the ALP is expected to mostly decay inside the detector to $D^+ \rightarrow \pi^+ \gamma \gamma$ or $D^+ \rightarrow \pi^+ e^+ e^-$. ALP constraints from the latter decays are relevant but are beyond the scope of this work. The recast of $D^+ \rightarrow \pi^+ + \text{invisible}$ only applies for masses up to $m_a = 0.224 \text{ GeV}$, indicated by the dashed, vertical line. As they give the strongest constraints, we work out in the following limits for $\Gamma_a = 0$.

In Table IV we present bounds on vector and axial-vector ALP couplings for two benchmark masses $m_a = 0$ and $m_a = 1.2 \text{ GeV}$, corresponding to the mass windows available by experimental searches; see Sec. IV. All searches are so far only performed in one of the two mass windows. Entries with “-” indicate that there is no sensitivity to this coupling. The entries with “...” indicate no data but decays are in principle probing the coupling. Specifically the axial-vector coupling k_{12}^A is only constrained for small m_a . We note that the mass benchmarks are representative for their respective mass window as long as they are sufficiently far from the kinematic endpoint of large m_a at which the decay feels phase space suppression and limits weaken.

Additional constraints on the vector ALP couplings can be obtained from $D^0 \rightarrow \pi^0 a$ via Eq. (66) applicable for $1.05 < m_a/\text{GeV} < 1.38$. Our results improve on previous

TABLE IV. Upper limits on ALP couplings from charm decays to invisibles for $m_a = 1.2 \text{ GeV}$ and $m_a = 0 \text{ GeV}$, both with $\Gamma_a = 0$; see also Table III. “n.a.” indicates that presently no usable data are available, but the decay mode is in principle sensitive to the coupling, as opposed to blank spaces, where this is not the case. For $D^+ \rightarrow \pi^+ + \text{invisible}$ our results agree with Ref. [53].

	$m_a = 1.2 \text{ GeV}$		$m_a = 0 \text{ GeV}$	
	$ k_{12}^V /f$	$ k_{12}^A /f$	$ k_{12}^V /f$	$ k_{12}^A /f$
	$[10^{-7} \text{ GeV}^{-1}]$			
$D^0 \rightarrow \pi^0 + \text{invisible}$	2.4	...	...	...
$D^+ \rightarrow \pi^+ + \text{invisible}$	...	...	0.22	...
$D^0 \rightarrow \omega + \text{invisible}$	...	...	...	0.70
$\Lambda_c \rightarrow p + \text{invisible}$	...	...	1.3	2.4
Best overall	2.4	...	0.22	0.70

works [11,13] where no recast of the $D^0 \rightarrow \pi^0 \nu \bar{\nu}$ search has been performed and the experimental limit reported in Table II has been used for the whole m_a region without factoring in a different signal shape.

C. Constraints on Z' parameters

For Z' models constraints can be separated into the case of a long-lived one and an invisibly decaying one. For the former the Z' is on shell and we expect only events for $q^2 = m_{Z'}^2$, while for the latter we have an off-shell Z' and expect a q^2 distribution with shape depending on the model parameters. A clean boundary between those two is limited by the experimental resolution in q^2 .

For a long-lived Z' , and to a lesser extend also for the invisibly decaying one, the current experimental bounds are limited to roughly two mass windows; see Sec. IV. In Fig. 10 we present bounds of the coupling $x_{LR}^{Z'} \equiv x_{LR+}^{Z'} = x_{LR-}^{Z'}$ (22) as a function of the Z' mass. The mass windows of the existing searches are visible. The lifetime constraints [see Eq. (71)] have only outside these regions an impact, the strongest is coming from the D^+ . The exclusion regions from $D \rightarrow (\pi^0, \omega) + \text{inv}$ and $\Lambda_c \rightarrow p + \text{inv}$ are new results of this work.

For an invisibly decaying Z' using benchmark BM_V (24), we present bounds on the combinations of couplings, defined in Eq. (22), in Table V. As the mass $m_{Z'} = 1 \text{ GeV}$ is in the large mass search window, and $\Gamma_{Z'}/m_{Z'} = 0.1$, it follows that couplings, which are not constrained by $D^0 \rightarrow \pi^0 + \text{invisible}$ are only weakly constrained.

D. D - $\bar{D}$ -mixing contributions

We comment briefly on D -mixing constraints in the light NP models. For light ALPs $m_a \approx 0$ they are obtained as [53]

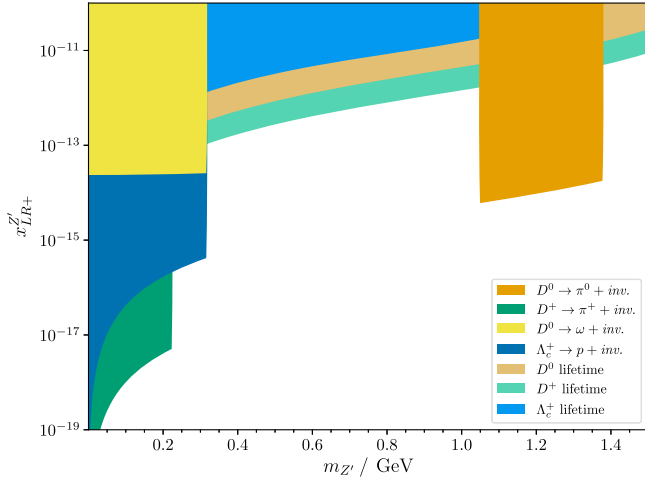


FIG. 10. Upper limits on the coefficient $x_{LR+}^{Z'} \equiv x_{LR+}^{Z'} = x_{LR-}^{Z'}$ (22) in long-lived Z' models from different decay modes, including lifetime constraints (71).

$$\frac{|k_{12}^V|}{f} \lesssim 8.3 \times 10^{-7} \text{ GeV}^{-1}, \frac{|k_{12}^A|}{f} \lesssim 4.3 \times 10^{-7} \text{ GeV}^{-1}. \quad (76)$$

Constraints allowing for CP violation are one order of magnitude stronger yet slightly weaker than the rare decay limits given in Table IV.

We naively extrapolate heavy Z' results [59,60] to obtain a rough estimate for the Z' model

$$\frac{|(\mathcal{C}_L^{Z'})^2 + (\mathcal{C}_R^{Z'})^2 - X \mathcal{C}_L^{Z'} \mathcal{C}_R^{Z'}|}{m_D^2 - m_{Z'}^2} \lesssim 6 \times 10^{-13} \text{ GeV}^{-2} \quad (77)$$

yielding

$$x_{LR\pm}^{Z'} \lesssim 2 \times 10^{-12} \quad (78)$$

for $m_{Z'} = 1 \text{ GeV}$, weaker than the rare decay bounds in Table V. The factor X includes ratios of hadronic matrix elements and RG running, which we do not consider due to the proximity of scales, $X \sim 5.1$. The constraint (77) can be evaded by tuning $\mathcal{C}_L \approx X \mathcal{C}_R$ or $\mathcal{C}_L \approx X/\mathcal{C}_R$ [59].

In addition, constraints from the lifetime difference in the $D - \bar{D}$ system on operators with light particles have been worked out in [61]. Limits are, depending on the Dirac structure of the operator, either weaker or at most comparable to the ones from rare decays presented in Table III. Exceptions arise for some operators such as tensors in scenarios with finite invisible fermion masses of several hundred MeV, which also suppress phase space in the charm decays.

VI. PREDICTIONS

We summarize predictions for the $h_c \rightarrow F + \text{invisible}$ branching ratios in the EFT, ALP, and Z' models. We use the constraints on the couplings from Sec. V. In Table VII

TABLE V. Upper limits on Z' couplings $x_k^{Z'}$ from Eq. (22) in the benchmark (24); see also Table III. The columns with $x_{LR}^{Z'}$ are single Wilson coefficient limits; see Eq. (18).

$h_c \rightarrow F$	BM _V Z'				
	$x_{LR+}^{Z'}$	$x_{LR-}^{Z'}$	$x_{LR}^{Z'}$	$x_D^{Z'}$	$x_{D5}^{Z'}$
$h_c \rightarrow F$	[10^{-14}]	[10^{-14}]	[10^{-14}]	[10^{-14} GeV^2]	
$D^0 \rightarrow$					
$D^0 \rightarrow \pi^0$	2.9		2.9	3.1	
$D^+ \rightarrow \pi^+$	85.9		85.9	5.1×10^4	
$D^0 \rightarrow \omega$	2.0×10^4	471	460	475	444
$\Lambda_c \rightarrow p$	24.6	287	132	63.3	614
Best overall	2.9	287	2.9	3.1	444

(Table VI) we give upper limits on the branching ratio for light LH and RH neutrinos (in the $d = 6$ SMEFT), in Table VIII for ALPs and in Table IX for the Z' model. For each of the limits we assume only a single combination of Wilson coefficients x_k is turned on while all others are set to zero.

For general patterns in Tables VI–IX some comments are in order. The large differences between $D^0 \rightarrow \pi^0$, $D^+ \rightarrow \pi^+$, $D_s^+ \rightarrow K^+$, $\Lambda_c \rightarrow p$, $\Xi_c^+ \rightarrow \Sigma^+$ are caused by the lifetimes $\tau_{D^0} \simeq \frac{5}{4} \tau_{D_s^+} \simeq \frac{5}{2} \tau_{D^+}$, $\tau_{\Xi_c^+} \simeq 2 \tau_{\Lambda_c}$, the isospin factor of 2 for the π^0 and to some degree the kinematic cuts of Eq. (29) for charged $D_{(s)}$ mesons. Specifically, branching ratios induced by couplings which have larger contributions at low q^2 [see e.g. x_{LR+} (green) in Fig. 1] are more strongly affected by these cuts. Note also the impact of the recast on the EFT limits given in Table VII. For instance, the upper limit on $\mathcal{B}(D^0 \rightarrow \pi^0 \nu \bar{\nu})$ with x_{SP+} turned on is stronger than

TABLE VI. Upper limits on the branching fraction of various $h_c \rightarrow F + \text{invisible}$ decays in SMEFT assuming LU, cLFC and general based on Eq. (75). Limits are in agreement with Ref. [10] with updated meson form factors in our work. *Include kinematic cuts; see Eq. (29).

$h_c \rightarrow F$	SMEFT		
	$\mathcal{B}_{\text{LU}}$ [10^{-7}]	$\mathcal{B}_{\text{cLFC}}$ [10^{-6}]	$\mathcal{B}_{\text{general}}$ [10^{-5}]
$D^0 \rightarrow$	0	0	0
$D^0 \rightarrow \pi^0$	6.0	3.5	1.3
$D^+ \rightarrow \pi^+$	24.3*	14.0*	5.1*
$D_s^+ \rightarrow K^+$	5.6*	3.2*	1.2*
$D^0 \rightarrow \omega/\rho^0$	6.6	3.8	1.4
$\Lambda_c \rightarrow p$	18.8	10.8	4.0
$\Xi_c \rightarrow \Sigma^+$	35.6	20.5	7.5
$D^0 \rightarrow \pi^+ \pi^-$	4.4	2.4	0.9
$D^+ \rightarrow \pi^+ \pi^0$	16.3*	9.4*	3.4*

TABLE VII. Upper limits on the branching ratios of $h_c \rightarrow F + \text{invisible}$ from bounds on the Wilson coefficients of LH and RH neutrino models from Table III for coupling combinations x_k switched on. The values in brackets correspond to the directly measured experimental limits. “...” indicates for $D \rightarrow \pi\pi$ that no bound is available because of insufficient information on the form factors; see text for details. *Include kinematic cuts; see Eq. (29). The columns with x_{SP} and x_{LR} are single SMEFT coefficient limits; see Eq. (18).

$h_c \rightarrow F$	Light LH and RH neutrino						
	x_{SP-}	x_{SP+}	x_{SP}	x_{LR-}	x_{LR+}	x_{LR}	x_T
	Upper limit on branching ratio $\mathcal{B}/10^{-4}$						
$D^0 \rightarrow$	(0.94)	0	(0.94)	0	0	0	0
$D^0 \rightarrow \pi^0$	0	1.8	0.025	0	0.50	0.50	1.4
$D^+ \rightarrow \pi^+$	0	8.9*	0.13*	0	2.0*	2.0*	6.9*
$D_s^+ \rightarrow K^+$	0	1.9*	0.027*	0	0.45*	0.45*	1.0*
$D^0 \rightarrow \rho^0/\omega$	0.0017	0	0.0017	0.85	0.031	0.51	20.1
$\Lambda_c \rightarrow p$	0.0056	0.87	0.017	1.6	0.53	1.4	7.6
$\Xi_c \rightarrow \Sigma^+$	0.0091	1.5	0.030	3.2	0.93	2.7	14.7
$D^0 \rightarrow \pi^+\pi^-$	...	0	...	0.53	0.060	0.35	...
$D^+ \rightarrow \pi^+\pi^0$	...	0	...	1.8*	0.25*	1.3*	...

the experimental value provided in Table II (which is based on x_{LR+}), since the extrapolation outside of the signal region depends on the NP model, for which we correct using Eq. (66).

Both Z' and ALP constraints are generically dependent on the details of the models and are shown for the Z' benchmark (24) and ALP benchmarks $m_a = \{0, 1.2\}$ GeV. The dependence on m_a is however small if $q^2 = m_a^2$ is sufficiently away from the phase space boundary $q^2 = (m_{h_c} - m_F)^2$. For $m_a = 1.2$ GeV this holds with the exception of $D^0 \rightarrow \rho^0/\omega + a$ because the latter decays are kinematically forbidden.

TABLE VIII. Upper limits on the branching ratios of $h_c \rightarrow F + \text{invisible}$ in ALPs models using the limits from Table IV for the benchmarks $m_a = 1.2$ GeV and $m_a = 0$ GeV, both with $\Gamma_a = 0$. Values in parentheses are input for the predictions given in this table.

$h_c \rightarrow F$	$m_a = 1.2$ GeV		$m_a = 0$ GeV	
	k_{12}^V	k_{12}^A	k_{12}^V	k_{12}^A
	Branching ratio $\mathcal{B}/10^{-4}$			
$D^0 \rightarrow \pi^0$	(0.58)	0	0.019	0
$D^+ \rightarrow \pi^+$	2.9	0	(0.096)	0
$D_s^+ \rightarrow K^+$	1.2	0	0.046	0
$D^0 \rightarrow \rho^0/\omega$	0	...	0	0.039
$\Lambda_c \rightarrow p$	0.61	...	0.024	(0.80)
$\Xi_c \rightarrow \Sigma^+$	1.1	...	0.054	1.8

TABLE IX. Upper limits on the branching ratios of $h_c \rightarrow F + \text{invisible}$ in Z' models for the benchmark (24) using bounds on the $x_k^{Z'}$ in Table V. For charm baryons, the branching ratio limits from $x_{D5}^{Z'}$ are weaker than lifetime constraints (71).

$h_c \rightarrow F$	BM _V Z'			
	$x_{LR-}^{Z'}$	$x_{LR+}^{Z'}$	$x_D^{Z'}$	$x_{D5}^{Z'}$
	Branching ratio $\mathcal{B}/10^{-4}$			
$D^0 \rightarrow$	0	0	0	0
$D^0 \rightarrow \pi^0$	0	2.9	1.9	0
$D^+ \rightarrow \pi^+$	0	14.7*	9.8*	0
$D_s^+ \rightarrow K^+$	0	5.5*	2.5*	0
$D^0 \rightarrow \rho^0/\omega$	24	0.014	0.59	378
$\Lambda_c \rightarrow p$	763	4.5	16.7	4.3×10^3
$\Xi_c \rightarrow \Sigma^+$	1.7×10^3	7.4	28.6	9.2×10^3
$D^0 \rightarrow \pi^+\pi^-$	86	0.23	...	...
$D^+ \rightarrow \pi^+\pi^0$	373	0.97	...	...

In Fig. 11 we show the upper limits on $\mathcal{B}(\Lambda_c \rightarrow pa)$ as a function of the ALP mass using the limits on $|k_{12}^V|/f$ from Eqs. (66) and (70) for $\Gamma_a = 0$. Our limits from $D^0 \rightarrow \pi^0 + \text{invisible}$ agree partially with Ref. [13].⁴ A similar flatness of the branching ratio with respect to m_a can be inferred also from Fig. 4 for $D \rightarrow \pi a$. Dashed lines indicate a naive extrapolation of limits outside their region of validity. We find that recasting $D^+ \rightarrow \tau^+\nu \rightarrow \pi^+ + \text{invisible}$ provides presently the strongest limit on $|k_{12}^V|/f$ [53]; however note that this channel is eventually limited to $m_a \leq 0.58$ GeV due to the two-body decay kinematics imposed by the τ resonance (29).

Models with only scalar and pseudoscalar contributions $x_{SP-} \neq 0$ and other x_k vanishing are most strongly constrained by $D^0 \rightarrow \text{invisible}$. For tensor and dipole couplings $D^0 \rightarrow \rho^0/\omega + \text{invisible}$ are golden modes, with bounds currently limited by the small signal windows of the naive recast. Future experimental limits on $\Lambda_c \rightarrow p + \text{invisible}$ for a larger q^2 region would be beneficial as this mode probes all couplings at the same time. The same holds in principle for $D \rightarrow \pi\pi + \text{invisible}$ decays; however the knowledge of the $D \rightarrow \pi\pi$ scalar and tensor form factors should be improved.

Furthermore, differential branching ratios can distinguish different NP models and couplings. In Fig. 12 we show the maximum achievable $dB/dq^2(\Lambda_c \rightarrow p + \text{inv})$ in different models using the constraints from Sec. V. Apart

⁴Reference [13] uses a modified bag model for the form factors and introduces additional operators $Q_{1(2)} \propto (\bar{u}(\gamma_5)c)a$. However, $Q_{1(2)}$ cannot give different limits on the branching ratio than $Q_{3(4)} \propto (\bar{u}\gamma_\mu(\gamma_5)c)\partial^\mu a$ (see Fig. 6 of Ref. [13]), as both sets are related by e.o.m. The limits from $Q_{3(4)}$ are of the same magnitude as ours and differences can be accounted for by the form factors.

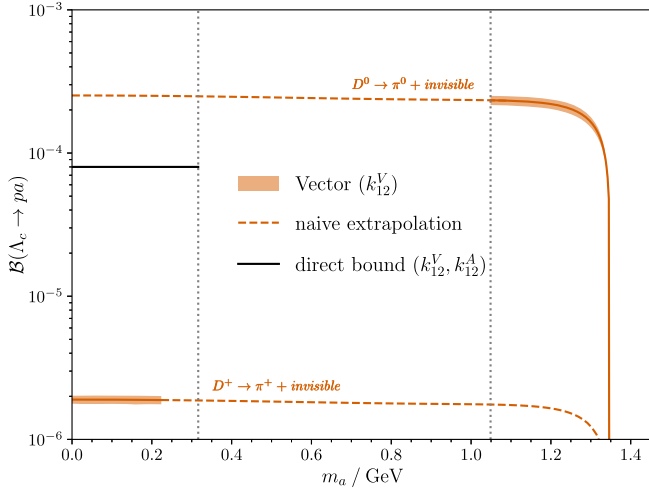


FIG. 11. Upper limit on the branching fraction of $\Lambda_c \rightarrow pa$ depending on the ALP mass m_a using $D^0 \rightarrow \pi^0 + \text{invisible}$ (66) for large m_a and $D^+ \rightarrow \pi^+ + \text{invisible}$ (67) for small m_a , both valid if only the vector coupling is present. Also shown is the direct bound (black, solid line) $\Lambda_c \rightarrow p + \text{invisible}$ (70) valid for small m_a and both couplings. Dashed lines correspond to naive extrapolations of the bounds outside their respective signal regions. Error bands indicate form factor uncertainties.

from the height, the shapes of the q^2 distribution are vastly different. We emphasize the region of $q^2 \approx 0$, in which most distributions vanish, except for the tensor (red) and the vector ones (green, lighter green); see also Figs. 1–3. In addition, the resonance structure from the Z' with finite width is a smoking gun for this model. More general, light LH and RH neutrinos allow for finite contributions at the kinematic end point $q^2 = 0$ for vector, axial-vector or tensor couplings for baryon decays and vector and axial-vector couplings for mesons. In scenarios with other

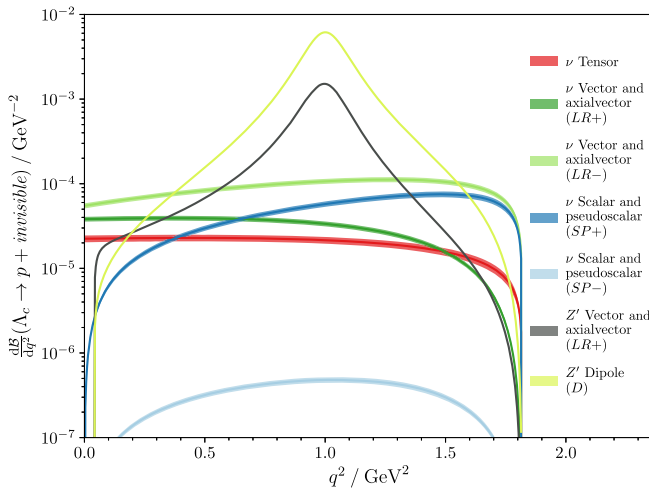


FIG. 12. Maximal achievable differential branching fraction of $\Lambda_c \rightarrow p + \text{invisible}$ in EFT (red, green, light green, blue, and light blue) and light Z' models (gray and yellow) in BM_V (24).

couplings the rate dies off toward the end point with significant slopes for low q^2 , which are also distinguishable. The behavior toward $q^2 \rightarrow 0$ is diagnostic: if observed, for instance, in Λ_c decays this would require scalar, pseudoscalar couplings, which indicate very specifically the nonstandard phenomena LNV or light sterile neutrinos. In general the whole distribution contains information. Bounds on $D^0 \rightarrow \pi^0 + \text{invisible}$ for example in the high q^2 are most constraining for x_{SP+} , while for x_{LR+} this happens at low q^2 . These features can be exploited experimentally if the q^2 binning is sufficient.

VII. CONCLUSIONS

We study rare decays of charmed hadrons into invisible final states, which arise in extensions of the SM with light and heavy degrees of freedom: dineutrinos in four-fermion $SU(2)_L \times U(1)_Y$ invariant SMEFT and νSMEFT operators induced by heavy BSM mediators, left- and right-handed light neutrinos in four-fermion WET operators from weak-scale NP, a light Z' boson which is long lived or decays into light and dark fermions, and an ALP that is sufficiently long lived to escape the detector undecayed.

We work out achievable branching ratios of two-, three- and four-body decays. We find that branching ratios reach $10^{-4} - 10^{-3}$ with chirality-flipping interactions; see Tables VII–IX. Branching ratios are more constrained in the chirality-preserving SMEFT, $\lesssim \text{few} \times 10^{-5}$ (see Table VI) due to the constraints from processes involving charged leptons. Lepton-flavor structure plays a key role in the link between neutrinos and charged leptons, making upper limits flavor structure dependent [10]. In the future MET plus jet searches in pp collisions will become important [56,57].

The $d = 7$ LNV SMEFT and the $d = 6$ νSMEFT induce chirality-flipping operators. They are presently best probed with $D^0 \rightarrow \text{invisibles}$, reaching scales as low as $\Lambda_{\text{LNV}} \gtrsim 1.5$ TeV and $\Lambda_{\nu\text{SMEFT}} \gtrsim 2.1$ TeV, allowing for NP with invisibles in $|\Delta c| = |\Delta u| = 1$ transitions to be just around the corner.

Not all couplings with RH neutrinos and in the light mediator models, Z' and ALPs, are presently experimentally constrained; see Table IV. For instance, for larger ALP masses the axial-vector coupling is not probed, leaving branching ratios of decays $D \rightarrow V + \text{invisible}$, with vectors $V = \omega, \rho, \dots$ and baryons $\Lambda_c \rightarrow p + \text{invisible}$ essentially unconstrained, up to lifetime constraints (71), which are weak, at best at the level of $\mathcal{O}(0.1)$. This highlights the importance of searches in modes beyond $D \rightarrow \pi + \text{invisible}$ due to the sensitivity to different and additional couplings.

We perform a recast of the searches in Table II (see Sec. IV), which results in improved constraints on ALP models specifically from the $D^0 \rightarrow \pi^0 \nu \bar{\nu}$ search. The methodology could be improved in the future and we

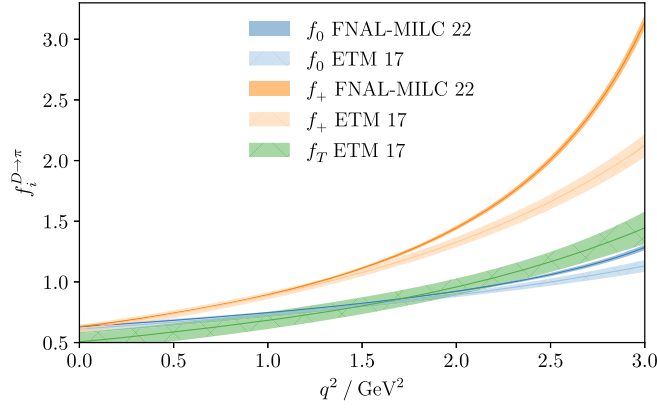


FIG. 13. Mesonic form factors $f_{+,0,T}$ of $D \rightarrow \pi$ are shown as functions of q^2 . The form factors are taken from the ETM Collaboration (lighter colors) [43,44] and from the Fermilab Lattice and MILC Collaborations [42] (darker colors).

encourage the experimental collaborations to broaden their signal regions and present results in a way that can cleanly be reinterpreted, as also suggested for the $B \rightarrow K^{(*)}\nu\bar{\nu}$ analysis [45].

Correlations enable the identification of the underlying type of NP, by comparing shapes of missing energy distributions from different couplings (Figs. 1–3 and 12) and by comparing branching ratios of different decay modes (Fig. 8). Sharper interpretations of the null test observables analyzed in this work need improved hadronic transition form factors of charmed hadrons, such as from lattice QCD. Especially limited information exists presently for $D \rightarrow V$ and $D \rightarrow \pi\pi$.

The missing energy modes are well-suited for the experiments Belle II [62], BESIII [63], and future e^+e^- colliders, such as a super-tau-charm factory [7] or the Tera-Z facilities Future Circular Collider [9,64] and Circular Electron Positron Collider [8], with sizable charm rates [10,32]. Since any observation of $c \rightarrow u + \text{invisibles}$ transitions heralds NP, with sizable branching ratios in decays of D mesons and charm baryons, systematic experimental searches are encouraged.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data that support the findings of this article are openly available [65].

APPENDIX: FORM FACTORS

We provide details on the hadronic transition form factors used in our analysis.

1. $D \rightarrow \pi$ form factors

For $D \rightarrow \pi$ transitions only three independent form factors with vector, scalar or tensor currents exist. They can be parametrized as [43]

$$\begin{aligned} \langle \pi(p_P) | \bar{u} \gamma_\mu c | D(p_D) \rangle \\ = (p_D + p_P)_\mu f_+(q^2) + (p_D - p_P)_\mu f_-(q^2), \end{aligned} \quad (\text{A1})$$

where q is the momentum transfer given by $q = p_D - p_P$ with p_D (p_P) the momenta of the D meson (pion). The scalar form factor f_0 can be related through the equations of motion and is given by

$$\begin{aligned} \langle \pi(p_P) | \bar{u} c | D(p_D) \rangle \\ = \frac{m_D^2 - m_P^2}{m_c - m_u} \underbrace{\left(f_+(q^2) + \frac{q^2}{m_D^2 - m_P^2} f_-(q^2) \right)}_{\equiv f_0(q^2)}. \end{aligned} \quad (\text{A2})$$

The form factors fulfill the end point relation $f_+(0) = f_0(0)$. The tensor form factor is parametrized as [44]

$$\begin{aligned} \langle \pi(p_P) | \bar{u} \sigma^{\mu\nu} c | D(p_D) \rangle \\ = \frac{2}{m_D + m_P} (p_P^\mu p_D^\nu - p_P^\nu p_D^\mu) f_T(q^2). \end{aligned} \quad (\text{A3})$$

The form factors from lattice QCD are shown in Fig. 13, from the ETM Collaboration [43,44] (lighter colors) and for $f_{+,0}^{D \rightarrow \pi}$ also from the Fermilab Lattice and MILC Collaborations [42] (darker colors). Both agree for lower q^2 but differ at higher q^2 as already noted in Ref. [42]. For $D_s \rightarrow K$ we use the same form factors as for $D \rightarrow \pi$, as supported by [42].

2. $D \rightarrow V$ form factors

For $D \rightarrow V$ transitions there are seven independent form factors, which we define [66–68]

$$\begin{aligned}
\langle V(p, \eta^*) | \bar{u} \gamma_5 c | D(p_D) \rangle &= -\frac{m_V(\eta^* \cdot q)}{m_c + m_u} A_0(q^2), \\
\langle V(p, \eta^*) | \bar{u} \gamma^\mu c | D(p_D) \rangle &= i \frac{2V(q^2)}{m_D + m_V} \epsilon^{\mu\nu\rho\sigma} \eta_\nu^* p_{D\rho} p_\sigma, \\
\langle V(p, \eta^*) | \bar{u} \gamma^\mu \gamma_5 c | D(p_D) \rangle &= 2m_V A_0(q^2) \frac{\eta^* \cdot q}{q^2} q^\mu + (m_D + m_V) A_1(q^2) \left(\eta^{*\mu} - \frac{\eta^* \cdot q}{q^2} q^\mu \right) \\
&\quad - A_2(q^2) \frac{\eta^* \cdot q}{m_D + m_V} \left(p_D^\mu + p^\mu - \frac{m_D^2 - m_V^2}{q^2} q^\mu \right), \\
\langle V(p, \eta^*) | \bar{u} \sigma_{\alpha\beta} \gamma_5 c | D(p_D) \rangle &= (\eta_\alpha^* (p_D + p)_\beta - \eta_\beta^* (p_D + p)_\alpha) T_1(q^2) + (\eta_\alpha^* q_\beta - \eta_\beta^* q_\alpha) \frac{m_D^2 - m_V^2}{q^2} (T_2(q^2) - T_1(q^2)) \\
&\quad + (\eta^* \cdot q) (q_\alpha (p_D + p)_\beta - q_\beta (p_D + p)_\alpha) \left(\frac{T_3(q^2)}{m_D^2 - m_V^2} + \frac{T_2(q^2) - T_1(q^2)}{q^2} \right), \tag{A4}
\end{aligned}$$

using the convention $\epsilon_{0123} = +1$. It is common to define two additional form factors via the expressions

$$\begin{aligned}
A_{12}(q^2) &\equiv -\frac{1}{16m_D m_V^2 (m_D + m_V)} (\lambda(q^2, m_D^2, m_V^2) A_2(q^2) - (m_D + m_V)^2 (m_D^2 - m_V^2 - q^2) A_1(q^2)), \\
T_{23}(q^2) &\equiv -\frac{1}{8m_D m_V^2 (m_D - m_V)} (\lambda(q^2, m_D^2, m_V^2) T_3(q^2) - (m_D^2 - m_V^2) (m_D^2 + 3m_V^2 - q^2) T_2(q^2)). \tag{A5}
\end{aligned}$$

For our numerical calculation we use the light-cone sum rules determination of the vector and axial-vector form factors from Ref. [66] and the quark-model calculation from Ref. [67] for the tensor form factors. In the heavy quark limit tensor and vector factors are related; see Ref. [69].

3. $\Lambda_c \rightarrow p$ form factors

The ten independent form factors for $\Lambda_c \rightarrow p$ decays read in the helicity-based definition [48]

$$\langle p(p_p, \lambda_p) | \bar{u} c | \Lambda_c(p_{\Lambda_c}, \lambda_{\Lambda_c}) \rangle = f_0(q^2) \frac{m_{\Lambda_c} - m_p}{m_c - m_u} \bar{u}_p(p_p, \lambda_p) u_{\Lambda_c}(p_{\Lambda_c}, \lambda_{\Lambda_c}), \tag{A6}$$

$$\langle p(p_p, \lambda_p) | \bar{u} \gamma_5 c | \Lambda_c(p_{\Lambda_c}, \lambda_{\Lambda_c}) \rangle = g_0(q^2) \frac{m_{\Lambda_c} + m_p}{m_c + m_u} \bar{u}_p(p_p, \lambda_p) \gamma_5 u_{\Lambda_c}(p_{\Lambda_c}, \lambda_{\Lambda_c}), \tag{A7}$$

$$\begin{aligned}
\langle p(p_p, \lambda_p) | \bar{u} \gamma^\mu c | \Lambda_c(p_{\Lambda_c}, \lambda_{\Lambda_c}) \rangle &= \bar{u}_p(p_p, \lambda_p) \left[f_0(q^2) (m_{\Lambda_c} - m_p) \frac{q^\mu}{q^2} + f_+(q^2) \frac{m_{\Lambda_c} + m_p}{s_+} \left(p_{\Lambda_c}^\mu + p_p^\mu - (m_{\Lambda_c}^2 - m_p^2) \frac{q^\mu}{q^2} \right) \right. \\
&\quad \left. + f_\perp(q^2) \left(\gamma^\mu - \frac{2m_p}{s_+} p_{\Lambda_c}^\mu - \frac{2m_{\Lambda_c}}{s_+} p_p^\mu \right) \right] u_{\Lambda_c}(p_{\Lambda_c}, \lambda_{\Lambda_c}), \tag{A8}
\end{aligned}$$

$$\begin{aligned}
\langle p(p_p, \lambda_p) | \bar{u} \gamma^\mu \gamma_5 c | \Lambda_c(p_{\Lambda_c}, \lambda_{\Lambda_c}) \rangle &= -\bar{u}_p(p_p, \lambda_p) \gamma_5 \left[g_0(q^2) (m_{\Lambda_c} + m_p) \frac{q^\mu}{q^2} + g_+(q^2) \frac{m_{\Lambda_c} - m_p}{s_-} \left(p_{\Lambda_c}^\mu + p_p^\mu - (m_{\Lambda_c}^2 - m_p^2) \frac{q^\mu}{q^2} \right) \right. \\
&\quad \left. + g_\perp(q^2) \left(\gamma^\mu + \frac{2m_p}{s_-} p_{\Lambda_c}^\mu - \frac{2m_{\Lambda_c}}{s_-} p_p^\mu \right) \right] u_{\Lambda_c}(p_{\Lambda_c}, \lambda_{\Lambda_c}), \tag{A9}
\end{aligned}$$

$$\begin{aligned}
 \langle p(p_p, \lambda_p) | \bar{u} i \sigma^{\mu\nu} c | \Lambda_c(p_{\Lambda_c}, \lambda_{\Lambda_c}) \rangle = & \bar{u}_p(p_p, \lambda_p) \left[2h_+(q^2) \frac{p_{\Lambda_c}^\mu p_p^\nu - p_{\Lambda_c}^\nu p_p^\mu}{s_+} + h_\perp(q^2) \left(\frac{m_{\Lambda_c} + m_p}{q^2} (q^\mu \gamma^\nu - q^\nu \gamma^\mu) \right. \right. \\
 & - 2 \left(\frac{1}{q^2} + \frac{1}{s_+} \right) (p_{\Lambda_c}^\mu p_p^\nu - p_{\Lambda_c}^\nu p_p^\mu) \Big) + \tilde{h}_+(q^2) \left(i \sigma^{\mu\nu} - \frac{2}{s_-} [m_{\Lambda_c} (p_p^\mu \gamma^\nu - p_p^\nu \gamma^\mu) \right. \\
 & - m_p (p_{\Lambda_c}^\mu \gamma^\nu - p_{\Lambda_c}^\nu \gamma^\mu) + (p_{\Lambda_c}^\mu p_p^\nu - p_{\Lambda_c}^\nu p_p^\mu) \Big) \\
 & + \tilde{h}_\perp(q^2) \frac{m_{\Lambda_c} - m_p}{q^2 s_-} ((m_{\Lambda_c}^2 - m_p^2 - q^2) (\gamma^\mu p_{\Lambda_c}^\nu - \gamma^\nu p_{\Lambda_c}^\mu) \\
 & \left. \left. - (m_{\Lambda_c}^2 - m_p^2 + q^2) (\gamma^\mu p_p^\nu - \gamma^\nu p_p^\mu) + 2(m_{\Lambda_c} - m_p) (p_{\Lambda_c}^\mu p_p^\nu - p_{\Lambda_c}^\nu p_p^\mu) \right) \right] u_{\Lambda_c}(p_{\Lambda_c}, \lambda_{\Lambda_c}).
 \end{aligned} \tag{A10}$$

The tensor matrix element is inferred from the dipole definition in Ref. [48] through the relation $\sigma^{\mu\nu} \gamma_5 = \frac{i}{2} \epsilon^{\mu\nu\alpha\beta} \sigma_{\alpha\beta}$. The following end point relations hold [50]:

$$\begin{aligned}
 f_0(0) &= f_+(0), & g_\perp(q_{\max}^2) &= g_+(q_{\max}^2), \\
 g_0(0) &= g_+(0), & \tilde{h}_\perp(q_{\max}^2) &= \tilde{h}_+(q_{\max}^2), \\
 h_\perp(0) &= \tilde{h}_\perp(0).
 \end{aligned} \tag{A11}$$

We take the form factors $f_0, g_0, f_+, g_+, f_\perp, g_\perp, h_+, h_\perp, \tilde{h}_+$ and $\tilde{h}_\perp$ from lattice QCD computations [49]. For $\Xi_c^+ \rightarrow \Sigma^+$

we use the ones from $\Lambda_c \rightarrow p$, which are related in the flavor symmetry limit [10].

4. $D \rightarrow \pi\pi$ form factors

There are seven independent transversity form factors $\mathcal{F}_i(q^2, p^2, P \cdot q)$ for $D^0 \rightarrow \pi^+ \pi^-$ transitions depending on the momentum transfer $q^\mu = p_D^\mu - p^\mu$, the momentum of the dipion system $p^\mu = p_1^\mu + p_2^\mu$ and the angle θ_{π^+} between the π^+ momentum and the negative direction of flight of the D meson in the dipion center of mass system. The angle is related to the scalar product $P \cdot q$, where $P^\mu = p_1^\mu - p_2^\mu$. Here p_1 (p_2) are the momenta of the π^+ (π^-) and p_D the momentum of the D meson.

We define the form factors as [70,71]

$$\begin{aligned}
 -i \langle \pi^+(p_1) \pi^-(p_2) | \bar{u} \gamma_\mu \gamma_5 c | D(p_D) \rangle &= \frac{2}{\mathcal{N}_{nr}^{\pi\pi} \sqrt{\lambda_D}} \left(p^\mu - \frac{q \cdot p}{q^2} q^\mu \right) \mathcal{F}_0 + \frac{\sqrt{p^2}}{\mathcal{N}_{nr}^{\pi\pi} \sqrt{q^2 \lambda_p}} \left(p^\mu - \frac{4(q \cdot p)(q \cdot P)}{\lambda_D} p^\mu \right. \\
 &\quad \left. + \frac{4p^2(q \cdot P)}{\lambda_D} q^\mu \right) \mathcal{F}_\parallel + \frac{q^\mu}{\mathcal{N}_{nr}^{\pi\pi} \sqrt{q^2 \lambda_p}} \mathcal{F}_\perp, \\
 i \langle \pi^+(p_1) \pi^-(p_2) | \bar{u} \gamma_\mu c | D(p_D) \rangle &= + \frac{4\sqrt{p^2}}{\mathcal{N}_{nr}^{\pi\pi} \sqrt{q^2 \lambda_D \lambda_p}} i \epsilon_{\mu\alpha\beta\gamma} q^\alpha p_1^\beta p_2^\gamma \mathcal{F}_\perp, \\
 \langle \pi^+(p_1) \pi^-(p_2) | \bar{u} \sigma^{\mu\nu} \gamma_5 c | D(p_D) \rangle &= \frac{2(p^\mu q^\nu - q^\mu p^\nu)}{\mathcal{N}_{nr}^{\pi\pi} \sqrt{q^2 \lambda_D}} \left(\mathcal{F}_0^T + \frac{(P \cdot q) \sqrt{p^2}}{\sqrt{q^2 \lambda_p}} \left(\mathcal{F}_\perp^T - 2 \frac{P \cdot q}{\sqrt{\lambda_D}} \mathcal{F}_\parallel^T \right) \right) \\
 &\quad - \frac{2(p^\mu P^\nu - P^\mu p^\nu) \sqrt{p^2}}{\mathcal{N}_{nr}^{\pi\pi} \sqrt{\lambda_D \lambda_p}} \mathcal{F}_\perp^T + \frac{(P^\mu q^\nu - q^\mu P^\nu) \sqrt{p^2}}{\mathcal{N}_{nr}^{\pi\pi} q^2 \sqrt{\lambda_p}} \left(\mathcal{F}_\parallel^T - 2 \frac{P \cdot q}{\sqrt{\lambda_D}} \mathcal{F}_\perp^T \right), \\
 -i \langle \pi^+(p_1) \pi^-(p_2) | \bar{u} \gamma_5 c | D(p_D) \rangle &= \frac{1}{\mathcal{N}_{nr}^{\pi\pi} \sqrt{q^2}} \frac{1}{m_c + m_u} \mathcal{F}_\perp,
 \end{aligned} \tag{A12}$$

with normalization

$$\mathcal{N}_{nr}^{\pi\pi} = \frac{G_F \alpha_e}{2^7 \pi^4 m_D} \sqrt{\pi \frac{\sqrt{\lambda_D \lambda_P}}{m_D p^2}}, \quad (\text{A13})$$

where $\lambda_P = \lambda(p^2, m_\pi^2, m_\pi^2)$ and $\epsilon_{0123} = +1$. Our definition slightly differs from Refs. [70,71] in the normalization, such that we agree with the notation in [10,32]. For brevity we omitted here the kinematic dependence of the form factors, e.g. $\mathcal{F}_i \equiv \mathcal{F}_i(q^2, p^2, P \cdot q)$. Form factors of other matrix elements can be obtained via e.o.m. or the relation $\sigma_{\mu\nu} = -\frac{i}{2} \epsilon_{\mu\nu\alpha\beta} \sigma^{\alpha\beta} \gamma_5$. For our numerical analysis we use the form factors $\mathcal{F}_{0,\perp,\parallel}$ of Ref. [32] obtained in a data-driven approach from isospin-related $D^+ \rightarrow \pi^+ \pi^- e^+ \nu_e$ measurements. We refrain from evaluating expressions with other

form factors e.g. $\mathcal{F}_i, \mathcal{F}_i^T$ as no data-driven approach is available for those and other methods show some disagreement; see Ref. [32].

For the form factors of $D^- \rightarrow \pi^- \pi^0$ we follow a similar approach as the data-driven approach in Ref. [32]. However in this case the isospin related decay is $D^0 \rightarrow \pi^- \pi^0 e^+ \nu_e$ and only the contribution from the ρ meson contributes. We fix the normalization by the measurement $\mathcal{B}(D^0 \rightarrow \pi^- \pi^0 e^+ \nu_e) = (1.439 \pm 0.033 \pm 0.027) \times 10^{-3}$ [72] and use $r_V = V(0)/A_1(0) = 1.548 \pm 0.079 \pm 0.041$ and $r_2 = A_2(0)/A_1(0) = 0.823 \pm 0.056 \pm 0.026$ [72] instead of the values in Ref. [32] for this decay only.

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