Portal matter models of kinetic mixing with two light dark gauge bosons

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The kinetic mixing (KM) portal mandates the existence of at least one new gauge boson, the dark photon (DP) based on the group $U(1)_D$, which mixes with the Standard Model (SM) photon via loops of other new heavy particles carrying both SM and dark charges called portal matter (PM). Arguments exist based on the renormalization group equations running of the $U(1)_D$ gauge coupling suggesting that at higher scales $U(1)_D$ becomes part of a more complex non-Abelian group, a simple example being just the SM-like $G_D = SU(2)_I \times U(1)_{Y_I}$. In our past analyses, it was always assumed that G_D broke in a SM-like manner directly to the DP's $U(1)_D$ which then subsequently broke at low energies $\lesssim 1$ GeV. However, this need not be the case, and G_D can instead break to $U(1)_{T_{3I}} \times U(1)_{Y_I}$, with T_{3I} being the diagonal generator of $SU(2)_I$, now producing two light gauge bosons which obtain masses at the $\lesssim 1$ GeV scale. In this paper, we explore the phenomenology of a very simple realization of this kind of alternative setup employing non-Abelian KM and having a minimal, leptonlike PM sector, demonstrating its distinctive nature in comparison to the previously examined symmetry breaking path. The effects of interference between these gauge bosons on thermal dark matter annihilation, the production of new heavy gauge, and Higgs and PM states at colliders, as well as the corresponding signatures for the light dark gauge bosons are examined. Collider signatures of this setup are found to be particularly challenging.

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I. INTRODUCTION AND OVERVIEW

The Standard Model (SM), though very successful at describing an array of experimental results, fails to address some of our basic questions and observations. Perhaps the greatest challenge is the nature of dark matter (DM): what is it, why is it so common, and how does it interact with us? To address these mysteries necessarily leads us into realms outside of the SM. Currently, the only established interaction of DM with the SM is gravity. Under most circumstances, assuming that DM does not dominantly consist of primordial black holes [1], it is very likely that for DM to achieve its observed relic abundance [2], additional, yet to be observed, interaction(s) are necessary between it and ordinary matter.

The possible scenarios for DM go back many years and were traditionally associated with the solutions to other problems, e.g., the QCD axion [3–5] and thermal Weakly Interacting Massive Particle (WIMPS) in the few GeV to ~100 TeV mass range [6–8]. Ever widening DM searches are pushing deeper into model parameter spaces but have so far come up empty [9–16]. While searches are continuing, these null results have prompted new DM scenarios wherein candidates exist over wide regimes in both masses and couplings [17–23], presenting us with significant experimental challenges. For much of this parameter space, the necessary interactions can be described by a small set of models, termed "portals." In all cases, in addition to the DM and SM fields, the existence of other new particles are required which act as mediators for this new interaction [24] but whose nature depends upon the details of a particular setup.

One of the simplest models of this kind extends the thermal WIMP setup [25–27] down into the lower DM mass range of a few MeV up to several GeV, something which is not possible in the SM due to the Lee-Weinberg bound [28,29]. In the renormalizable kinetic mixing (KM), vector portal model [30–32], a new "dark" gauge interaction is introduced based on the group $U(1)_D$, whose (generally massive) gauge boson, the dark photon (DP), V [33–35], couples to a conserved dark charge, Q_D, that is carried by the DM but not by the SM fields. The generation of the DP mass via the breaking of $U(1)_D$ is assumed to occur via the vevs of one or more Dark Higgs fields [36] as in the SM; these same vevs may also contribute to the mass of the DM itself which is assumed to lie in the general proximity of that of the DP $\lesssim 1$ GeV. The interaction between the SM with the dark sector fields is then induced (at low energy scales) by the KM of the DP and the SM photon, A, at loop-level via vacuum polarizationlike

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diagrams. For such diagrams to occur at, e.g., the 1-loop level, other new fields, presumably heavy, must also be present which carry both SM as well as $U(1)_D$ dark charges so that they can couple to both V and A. In order for such new states to have avoided detection and to satisfy Higgs coupling, unitarity, and precision electroweak constraints, they must be either complex scalars and/or vectorlife fermions (VLFs) [37–44]. We refer to such exotic particles as portal matter (PM), and they have been the subject of much recent attention by ourselves and other authors [45–64].

Models in this class must satisfy constraints arising from a suite of different observations and experiments including those from direct and indirect detection as well as accelerator searches and from both astrophysics and cosmology. While we know that to obtain the observed abundance of DM, the annihilation cross section into SM particles at freeze-out must roughly satisfy the generic requirement [25–27], $\sigma v_{\rm rel} \sim 3 \times 10^{-26}$ cm^3 sec^{-1}. We also know for DM in this mass range, $\lesssim$1 GeV, that during the time of the Cosmic Microwave Background (CMB), this cross section must be suppressed by several orders of magnitude [2,65–68] and similarly so today [69–71]. This implies that $\sigma v_{\rm rel}$ must have a significant temperature (T) dependence, excluding certain types of DM and interactions with the SM fields in this mass range. A general class of models is immediately impacted: those that involve a standard s-wave annihilation process as would be the case for Dirac fermion DM with a vector mediator, e.g., a DP, so that we must entertain other possibilities. For example, if this process were instead (i) p-wave leading to a $v^2 \sim T$ suppression at later times, as may be realized in the case of complex scalar or Majorana fermionic DM,[1] we may be able to avoid such constraints. Another possibility is that DM annihilation to SM final states may take place (ii) through the coannihilation mechanism, as would be realized by pseudo-Dirac DM with a sizable mass splitting also leading to an inelastic DM setup; this could be induced by, e.g., a $Q_D = 2$ dark Higgs vev. In such a case, the DM annihilation rate is exponentially Boltzmann suppressed, in a manner controlled by this mass splitting, due to the much lower temperatures at sufficiently later times [73–76]. As is well known, if the mass splitting is sufficiently large in the inelastic DM setup, constraints arising from both direct and indirect detection searches can also be rather easily avoided.

It has been observed that the renormalization group equations running of the $U(1)_D$ coupling into the UV may lead to the eventual loss of perturbativity at a scale not too far above the weak scale [51,54,77,78]. An obvious solution is that $U(1)_D$ must be embedded into some larger, non-Abelian gauge group, G_D, whose beta function will have the opposite sign, before this scale is reached. One may easily imagine that the PM mass scale and that associated with the breaking of G_D may be intimately related. This possibility plus the investigation of how the SM and G_D might be unified into a more UV-complete setup has been the subject of much of our recent work [45,46,48–51,53,54,57,60–62,64]. In some of these setups, it was assumed that G_D was "SM-like" in that $G_D = SU(2)_I \times U(1)_{Y_I}$ [79], where here the label "I" refers to the almost invisible interactions taking place in the dark sector, with $U(1)_D$ embedded into G_D exactly like $U(1)_{em}$ is in the case of the SM, except for it also being broken at small, $\sim$1 GeV, mass scales.[2] Paralleling the SM, it was assumed that G_D broke *directly* to only $U(1)_D$ in a single step but at the $\sim$10 TeV scale so the heavy gauge bosons analogous to the SM W, Z (here denoted as Z_I, W_I) would obtain comparable large masses as part of this process.

In this paper, we explore an alternative possibility by giving up this "SM-like" bias thus leading to a somewhat different set of phenomenological predictions. Here, it is assumed that at the $\sim$10 TeV scale *only* $SU(2)_I$ is broken by a real scalar triplet with $Y_I = 0$, Σ, whose $T_{3I} = 0$ component gets a large vev, v_T, generating a mass $g_I v_T$, for the $W_I^{\pm} \sim W_{1I} \pm iW_{2I}$ gauge bosons (analogs of the SM $W^{\pm}$) but leaving the W_{3I} field massless. Below this scale, $U(1)_{T_{3I}} \times U(1)_{Y_I}$ will remain unbroken down to $\lesssim$1 GeV. The VLF PM chosen for this setup is just an $SU(2)_I$ doublet, $SU(2)_L$ singlet, with $Y_I = 0$ but having a nonzero SM hypercharge, $Y/2 = -1$, in analogy with the SM right-handed leptons. The two $Q_{em} = -1$ states in this doublet, $E_{1,2}$, are initially degenerate in mass (by symmetry arguments), but this degeneracy is then broken by the same large vev that breaks $SU(2)_I$ down to $U(1)_{T_{3I}}$. Once this degeneracy is broken, non-Abelian KM at the 1-loop level between the SM hypercharge gauge boson, B, and the still massless W_{3I} is generated; the value of ϵ that results from this is finite and is a function of the ratio of the $E_{1,2}$ masses. A parallel setup is also possible by employing instead color-triplet, $SU(2)_L$-singlet VL quarks with similar results. Subsequent to the usual SM symmetry breaking by the Higgs doublet at $\sim$246 GeV, this KM (to leading order in the ratio of the relevant vevs) will manifest as occurring between the SM photon and the W_{3I}; once $U(1)_{T_{3I}} \times U(1)_{Y_I}$ is completely broken near $\sim$1 GeV, the photon will appear to mix with both of these light gauge boson mass eigenstates, here denoted as $Z_{1,2}$, which are linear combinations of W_{3I} and B_I. This final step of symmetry breaking at the low scale requires the introduction of a pair of dark Higgs fields with several possible choices available and which lead to somewhat different phenomenologies. We argue the merits of a specific choice of dark Higgs that leads to pseudo-Dirac DM, so that the advantages of the inelastic DM setup can be realized and

[1]See, however, [72].

[2]This possibility was originally motivated by our earlier work on E_6-type gauge models [80].

that also generates a mixing between the lighter of the E_i and, e.g., the SM e_R to allow for this PM field to decay sufficiently rapidly. Since there are now two light "dark" gauge bosons which will both couple to the SM via KM (instead of just the usual DP), interference effects will be shown to be of some relevance to the relic density calculation, especially so if they have comparable couplings and are not too different in mass, i.e., $M_{Z_2}/M_{Z_1} \lesssim 2$–$2.5$. In our case, this will open up previously disfavored parameter space regions satisfying this observational constraint. In comparison to the previously examined "SM-like" G_D breaking scenario, the minimal version of this type of setup consequently has a somewhat more limited high energy collider footprint.

The outline of this paper is as follows: In Sec. II, we present a detailed description of this alternative model setup following along with the effects of both the non-Abelian KM and the various steps of gauge symmetry breaking down to the sub-GeV scale leading to there being two light dark gauge bosons, Z_i. For this specific setup, an $SU(2)_L$ isodoublet of a SM-charged singlet VLF is chosen as the PM fields though similar results are obtained with other possible choices. We argue that a specific minimal choice of the two Higgs fields that are necessary in this final step of symmetry can be made based on other phenomenological requirements that need to be satisfied: allowing the PM fields to decay and generate pseudo-Dirac DM. This choice then fixes the relative (vector) couplings of the Z_i once their $O(1)$ mass ratio is determined; we then explore the implications of this through a set of representative benchmark models. In Sec. III, we study the impact of having these two light gauge bosons contribute to the annihilation of DM into SM final states as part of the relic density calculation in the case where the DM is assumed to be pseudo-Dirac/inelastic with a significant mass splitting to easily satisfy several existing observational and experimental constraints. We show that when the Z_i are relatively close in mass, constructive interference between the exchanges opens up a newly favorable region of parameter space, $m_{\rm DM}/M_{Z_1} \sim 0.5$–$0.9$, which had been previously disfavored. In Sec. IV, we explore some of the collider, i.e., LHC, FCC-hh, and e^+e^- signatures that arise in the current setup. The new heavy states include the SM isosinglet, leptonlike PM fermions, and E_i, as well as the $W_I^\pm$ that result from $SU(2)_I \to U(1)_{T_{3I}}$ breaking. While the E_i are pair produced as usual via SM electroweak interactions and decay into $e+$ Missing Transverse Energy (MET), the W_I's are not so easily produced as in the previously examined setup as there this relied upon (i) the PM sharing an $SU(2)_I$ representation with some SM field and/or (ii) the existence of a heavy Z_I state which is coupled to the SM as well as to W_I pairs. Neither of these conditions hold in the current model. We show that W_I production is instead ϵ^2 suppressed and so likely to be inaccessible unless the W_I is rather light and/or large center of mass energies are available. Other possible states in the dark scalar sector, most notably the heavy real scalar, Σ^0, which is left over from this same $SU(2)_I$ breaking, may be made visible via its mixing with the SM Higgs. This same mixing is also shown to generate rare Higgs decays, such as $H(125) \to \gamma + Z_i$, but with branching fractions that are likely below $\sim 10^{-8}$ and are thus invisible, far below the SM background. We also show that the production of the Z_i in e^+e^- via PM exchange is difficult to observe without a precise understanding of the SM background. Finally, a discussion and our conclusions can be found in Sec. V.

II. A MODEL WITH TWO LIGHT DARK GAUGE BOSONS: THE BASIC SETUP

In our past discussions when we have considered a dark sector gauge group of the form $G_D = SU(2)_I \times U(1)_{Y_I} = 2_I 1_{Y_I}$ [with the corresponding gauge bosons being $W_{i(=1-3)I}, B_I$ having the couplings g_I, g_{Y_I}, respectively], we always assumed that the symmetry breaking took place in a "SM-like" manner, i.e., $2_I 1_{Y_I} \to 1_D$. This was assumed to occur at the scale of a $\sim$ few TeV or more via a dark Higgs, 2_I doublet vev, with the 1_D remaining unbroken and whose gauge boson is identified with the dark photon (DP), V. This $U(1)_D$ group was subsequently broken near the $\sim$1 GeV scale via a vev of another dark Higgs that carries a nonzero dark charge, $Q_D \neq 0$. This DP then undergoes KM via PM loops with one or both of the neutral SM gauge fields, at energies far below the weak scale, then emerges as an effective kinetic mixing with just the SM photon. Of course, even if we assume that the dark gauge group, G_D, remains as above, it is of course possible that this symmetry may be broken in a non-SM-like manner leading to a different phenomenology that we investigate below in a simple setup.

We imagine that at the high scale, $\sim$ a few TeV or more, G_D is broken by the $T_{3I} = 0$ vev, v_T, of an $SU(2)_I$ triplet, Σ, carrying $Y_I = 0$. This vev generates the masses, $g_I v_T$, for the non-Hermitian gauge bosons, $W_I^\pm$,[3] but leaves both of the remaining W_{3I} and B_I gauge fields massless, i.e., $U(1)_{T_{3I}} \times U(1)_{Y_I}$ remains unbroken. The fields in Σ with $T_{3I} = \pm 1$, in the large vev limit, then become the Goldstone bosons for these now massive $W_I^\pm$ and no longer remain in the physical spectrum. Simultaneously, we also imagine the existence of an $SU(2)_I$, vectorlike doublet of PM fermion fields carrying $Y_I = 0$, which is also an $SU(2)_L$ singlet; if these fields are also chosen to be color singlets, then they are assumed to have $Y/2 = Q_{em} = -1$, analogous to the right-handed (RH)-leptons in the SM, i.e.,

$$\mathcal{E}_{L,R} = \begin{pmatrix} E_1 \\ E_2 \end{pmatrix}_{L,R}, \tag{1}$$

[3]Here, the $\pm$ label refers not to the dark or electric charge but to the corresponding dark isospin raising and lowering operators, $T_I^\pm$, to which $W_I^\pm$ couple.

which are clearly degenerate via $SU(2)_I$ and having a Dirac mass, M_E, that is assumed to be of order the G_D breaking scale,

$$M_E(\bar{E}_{1L}E_{1R} + \bar{E}_{2L}E_{2R}) + \text{H.c.}, \tag{2}$$

with $SU(2)_I$ here acting in the vertical direction. This mass term can, however, be augmented by allowing a Yukawa coupling to the previously introduced $SU(2)_I$-breaking Higgs triplet, Σ, i.e.,

$$\mathcal{L}_\Sigma = y_\Sigma \bar{\mathcal{E}}_L \sigma \cdot \Sigma \mathcal{E}_R + \text{H.c.}, \tag{3}$$

with σ being the set of Pauli matrices and y_Σ an O(1) Yukawa coupling, so that when the vev becomes nonzero, the $E_{1,2}$ states now become split in mass, i.e.,

$$M_{E_1,E_2} = M_E \pm \frac{y_\Sigma v_T}{\sqrt{2}}, \tag{4}$$

but where we still expect that the ratio of these two masses to be of order unity. Note that this 2_I PM fermion doublet does *not* contain any SM fields so that, at this point, the $W_I^\pm$ gauge fields only connect $E_1 - E_2$ and not, e.g., one of the E_i with the SM e_R as might have frequently occurred in our previously explored setups. We note that, as the induced mass splitting between these two states, $\Delta = \sqrt{2} y_\Sigma v_T$, may be larger than the mass of $W_I^\pm$ itself, $g_I v_T$, we anticipate that the 2-body decay process $E_1 \to E_2 W_I$ is somewhat likely to be allowed to occur on shell. Now, since the E_i carry $T_{3L} = Y_I/2 = 0$ but still have $T_{3I} = \pm 1/2$ as well as $Y/2 = -1$, due to their mass difference they can generate a finite and calculable, non-Abelian KM at the 1-loop level between the SM hypercharge gauge boson, $\hat{B}$, and the $SU(2)_I$ gauge boson, $\hat{W}_{3I}$, with $T_{3I} = 0$. Thus, the set of kinetic terms in the Lagrangian for this model contains an additional piece (suppressing Lorentz indices) of the form

$$\mathcal{L}_{KM} = \frac{1}{2}\hat{\beta} \cdot \hat{\mathbf{W}}_\mathbf{I} \hat{B}, \tag{5}$$

where the nonzero component of $\hat{\beta}$ points in the "3" direction, projecting out W_{3I}; we see that no analogous Abelian KM term is generated here since the PM fields have $Y_I = 0$. Since the magnitude of $\hat{\beta}$ (which we call β from now on) is expected to be quite small, this KM can be removed by the simple approximate field redefinitions,

$$\hat{B} \to B + \beta W_{3I}, \qquad \hat{W}_{3I} \to W_{3I}, \tag{6}$$

while leaving the other fields unchanged. Hence, we observe that the dark Higgs triplet field Σ plays a very special role in this setup in that it not only is the source of the (partial) breaking of the $SU(2)_I$ gauge symmetries but also simultaneously leads to the generation of non-Abelian KM by splitting the PM masses, which is itself $SU(2)_I$ breaking. Switching to the conventional normalization for the KM mixing parameter, $\beta \to \epsilon/c_w$, where $c_w = \cos\theta_w$, etc., we find that (assuming that no other PM fields are present) explicitly

$$\epsilon = \left(\frac{g_I}{e}\right)\left(\frac{\alpha}{6\pi}\right)\ln\left(\frac{M_{E_1}^2}{M_{E_2}^2}\right), \tag{7}$$

which is in the desired range, $\sim 10^{-4}$–10^{-3} assuming g_I/e is O(1). This is the simplest fermionic PM sector that one can imagine with PM fields in a nontrivial representation of 2_I which yields a finite and calculable value for the parameter ϵ. It is possible that instead of the color and $SU(2)_L$ singlet VLF PM fields, E_i, we could have chosen (or added) color triplets which are also $SU(2)_L$ singlet VLF PM fields, U_i and/or D_i. In such a case, essentially all of the analysis above would go through in exactly the same manner up to possible signs and factors of 2 arising in the expression for ϵ. Thus, the pure leptonlike case outlined here is seen to be the simplest member of a class of such models. Recalling that in the SM $g_Y = g\tan\theta_w$, then in addition to conventional coupling to $g_I T_{3I}$, W_{3I} now pick up a KM-induced coupling to the SM hypercharge which, for convenience in later discussion, we may write suggestively in the form

$$g t_w \frac{\epsilon}{c_w}\frac{Y}{2} W_{3I}, \tag{8}$$

similar to what happens in the more familiar DP model setups.

There are several observations to make about this model. First, the PM in this setup does not reside in a 2_I multiplet with any of the SM fields and is also vectorlike with respect to G_D, as it already is with respect to the SM; these two aspects are clearly linked. When any SM and PM fields both reside in the same G_D representation, they must share the same SM quantum numbers since by assumption G_D commutes with the SM gauge group. However, this implies that if this multiplet is, e.g., left-(or right-)handed, then the same multiplet with the opposite handedness cannot simultaneously exist. If it did, it would require the SM field contained within it to also be vectorlike with respect to the $2_L 1_Y$ gauge symmetries, something we know not to be the case for any of the usual fermions. Thus, the only way for PM to be vectorlike with respect to G_D is for there not to be any SM fields sharing a common multiplet with it, as is the case here. Secondly, there is another possibility where an equally simple but distinct PM setup can be constructed: consider two vectorlike, 2_I and 2_L PM singlets with different masses and having opposite values of Y_I but with common values of the SM hypercharge. In such a case, the KM will be Abelian, with the mixing being between the two hypercharge gauge bosons, but the corresponding

parameter, ϵ, would remain finite and calculable. Such a setup would completely mirror the discussion below but with several important differences; however, such considerations are beyond the scope of the present work.

To go further with the present analysis, we need to discuss the gauge symmetry breaking in more detail. Subsequent to this high scale breaking, the SM itself will break in the familiar manner via the $\simeq$246 GeV vev, $v_{\rm SM}$, of an 2_L doublet (here with $Y/2 = -1/2$) to which W_{3I} also couples via the above KM. Recalling that the SM Z mass is just $M_{Z_0} = g v_{\rm SM}/2c_w$ and that the SM photon field remains massless and decouples, the 2×2 mass-squared matrix describing the mixing of the weak basis states Z_0, W_{3I_0} can be written as

$$\mathcal{M}^2_{ewk} = M^2_{Z_0}\begin{pmatrix} 1 & -\epsilon t_w \\ -\epsilon t_w & \epsilon^2 t_w^2 \end{pmatrix}, \tag{9}$$

which is diagonalized at leading order in ϵ by the transformations

$$Z_0 \to Z + \epsilon t_w W_{3I}, \qquad W_{3I_0} \to W_{3I} - \epsilon t_w Z. \tag{10}$$

This results in a fractional shift in the SM Z mass of $O(t_w^2\epsilon^2) \sim 10^{-8}$, which can be safely neglected, while W_{3I} remains massless but picks up an additional mass-mixing induced coupling from the Z so that the interaction of W_{3I} with the SM fields in the mass eigenstate basis is given by (employing the SM relations $e = g s_w$ and $Q_{em} = T_{3L} + Y/2$)

$$g t_w \frac{\epsilon}{c_w}\frac{Y}{2} + \epsilon t_w \frac{g}{c_w}(T_{3L} - s_w^2 Q_{em}) = e\epsilon Q_{em}, \tag{11}$$

in the usual DP-like fashion. Note that at this stage, while *both* the W_{3I} and B_I remain massless, it is only W_{3I} that couples to the SM fields via KM. Any small masses for the light dark gauge bosons discussed below do not alter the coupling above up to very small mass suppressed correction terms. However, further symmetry breaking at lower scales will generally lead to $W_{3I} - B_I$ mass mixing so that both of the resulting mass eigenstates will experience KM with the SM photon as we now discuss.

At this stage, the remaining unbroken dark gauge symmetries are just $U(1)_{T_{3I}} \times U(1)_{Y_I}$, and to completely break these, we need to employ (at least) two dark Higgs fields, $H_{1,2}$, which will obtain the vevs, $v_{1,2}$, and will supply us with the Goldstone bosons needed for both the W_{3I} and B_I (or some linear combinations thereof) to obtain masses. Subject to the constraint that both gauge symmetries are indeed broken, there still remains some significant freedom in the choice of quantum numbers for these two Higgs fields. Here, we are guided by other plausible requirements based on satisfying phenomenological and experimental constraints. Note that these small vevs appearing in $H_{1,2}$ will also naturally lead to the existence of two light dark Higgs, $h_{1,2}$. In practice, these are approximately linear combinations of the light CP-even states originating from the representations $H_{1,2}$ whose masses, in the appropriate limit, are roughly set by these same vevs.

The first of these requirements, (i), arises from a simple observation. As noted above, unlike in other previously examined PM models, the $W_I^{\pm}$ here do not connect the PM fields with SM ones which have identical SM quantum numbers. Thus, while E_1 can decay to $W_I E_2$, the lighter E_2 is at this point a charged, stable field with a $\sim$TeV scale mass, which is not phenomenologically acceptable. This problem can be alleviated by choosing, e.g., H_1 to be an 2_I doublet with $Y_I = 0$, which then obtains the vev, $v_1 \lesssim 1$ GeV. One can then couple $\mathcal{E}$ to the SM right-handed charged lepton with the same SM quantum numbers, generically described as e_R, via the H_1, i.e.,

$$\mathcal{L}_{\rm mix} = y_{\rm mix}\bar{\mathcal{E}}_L e_R H_1 + \text{H.c.} \tag{12}$$

Then, e.g., the fields $E_2 - e$ will experience a small mass mixing once H_1 obtains its vev thus allowing for the decay $E_2 \to e_R W_{3I}$ (which is always kinematically allowed) as a FCNC-like process. Given the relative mass scales involved, we might expect the relevant mixing angle to be $\phi_{eE} \sim v_1/v_T \sim O(\epsilon)$ which is sufficient for this decay to the avoid the familiar experimental lifetime constraints on new heavy charged states. This is especially so due to the longitudinal couplings of the mass eigenstates formed from both W_{3I} and B_I after symmetry breaking, leading to a significant enhancement in this amplitude which can compensate for this small mixing as has been discussed in earlier work [45,51]. This is even more obvious if we take $y_{\rm mix}$ to be an O(1) coupling and consider the Goldstone component in H_1 and its coupling to, e.g., $\bar{E}_L e_R$. We also not that the heavier E_1 PM state will also obtain an additional decay channel, $E_1 \to e_R W_I$, as result of this same mixing process; this may or may not compete with the already discussed $E_1 \to E_2 W_I$ decay depending upon the exact mass eigenvalues and the size of the mixing. Note that if we had instead gone the route of color triplet VLFs as PM, this same mixing mechanism would be required to allow the lighter of the two quarklike states to mix with the right-handed SM u or d quarks; the quantum numbers of the H_1 in such a setup would be exactly the same as chosen in the present case.

Next, let us consider the DM sector of our setup which we assume here to be fermionic. As is well known and as was briefly discussed in the Introduction, the easiest way in which to avoid numerous constraints arising from, e.g., the CMB as well as direct and indirect detection DM searches, is to require that, (ii), the DM, χ, be pseudo-Dirac due to the generation of Majorana mass terms as well as a bare Dirac mass producing an inelastic DM-type scenario. These will

split χ into two states, $\chi_{1,2}$, now having masses $m_1 < m_2 = m_1(1+\delta)$, which clearly avoids direct detection for sufficiently large values of δ (which we assume is the case in the analysis below), since the coupling to the responsible gauge boson(s) is off diagonal at tree- evel. In such a case, the DM will achieve its relic abundance via coannihilation into SM final states which becomes very highly suppressed at later times as the temperature falls significantly, thus avoiding CMB constraints as well as present day indirect detection searches. To do this in the simplest possible manner, we imagine that χ lives in some vectorlike representation, $X_{L,R}$, of G_D in such a way that when H_2 develops a vev, these Majorana mass terms are generated; in this case, we can write a general set of mass terms in the Lagrangian as

$$\mathcal{L}_{Dark} = m_X \bar{X}_L X_R + \frac{1}{2}\left(y_L \bar{X}_L^c X_L + y_R \bar{X}_R^c X_R\right) H_2 + \text{H.c.}, \tag{13}$$

and, for simplicity, we assume the equality of the left and right Yukawa couplings, $y_L = y_R = y$. Now if the DM were to be in an $SU(2)_I$ singlet representation, H_2 would also need to be a singlet with $Y_I \neq 0$. In such a case, W_{3I} and B_I, would each obtain masses from one of the vevs of $H_{1,2}$, but we would then be faced with the unhappy situation that these states would never mix and so already be mass eigenstates with B_I still uncoupled to the fields of the SM by KM. The next simplest possibility, one we considered earlier [62] and which we adopt here, is that X is instead a 2_I doublet with $Y_I/2 = 1/2$ and where χ itself has $T_{3I} = 1/2$, i.e.,

$$X_{L,R} = \begin{pmatrix} \chi \\ \psi \end{pmatrix}_{L,R}. \tag{14}$$

In such a case, H_2 must be identified as an $SU(2)_I$ triplet with $Y_I/2 = 1$ to generate the appropriate additional mass terms once it obtains a vev. While the associated phenomenology of the DM sector in this case is discussed in the next section, these two Higgs fields and their corresponding vevs are sufficient for our purposes. However, we note that it is also possible to go further and also couple the doublet $X_{L,R}$ to the triplet Σ introduced above in such a way as to split the originally degenerate Dirac masses, m_X, of the χ, ψ doublet members, i.e.,

$$\mathcal{L}'_\Delta = y'_\Sigma \bar{X}_L \sigma \cdot \Sigma X_R + \text{H.c.} \tag{15}$$

If we assume that this Yukawa coupling, y'_Σ, has a magnitude somewhat similar to those of the first or second generation fermions to the SM Higgs, then ψ's Dirac mass (which is the only mass that it possesses) can be significantly increased relative to that of χ, allowing for a decay with a relatively short decay lifetime. From now on, we denote the smaller Dirac mass of the DM state, χ, by m_D for clarity.

We note that while these requirements are strongly suggestive, they are not the unique choice for the set of Higgs fields that break the low energy gauge symmetries and other possibilities may be contemplated. We do, however, adopt these specific representations and associated quantum numbers for their vevs in the discussion that follows.

Putting the requirements (i) and (ii) together from our discussion above, the resulting $W_{3I} - B_I$ mass-squared matrix that arises from these two vevs can be written as

$$\mathcal{M}^2 = \begin{pmatrix} M_A^2 + M_B^2 & t_I M_B^2 \\ t_I M_B^2 & t_I^2 M_B^2 \end{pmatrix}, \tag{16}$$

where we defined the ratio of gauge coupling $g_{Y_I}/g_I = t_I > 0$ in analogy with the SM, and for simplicity, further defined the mass-squared parameters,

$$M_A^2 = (g_I v_1)^2/2, \qquad M_B^2 = (g_I v_2)^2. \tag{17}$$

Introducing the dimensionless ratio $\rho = M_A^2/M_B^2$, this mass-squared matrix can be diagonalized via a rotation through an angle, ϕ, given by

$$\tan 2\phi = \frac{2t_I}{\rho + (1 - t_I^2)}, \tag{18}$$

yielding the mass-squared eigenvalues, $M_{1,2}^2 = M_B^2 \lambda_\mp$ (i.e., $M_1 < M_2$), where $\lambda_\mp$ are given by the expression

$$\lambda_\mp = \frac{1}{2}\left[(1+t_I^2) + \rho\right] \mp \frac{1}{2}\left[(1+t_I^2)^2 + \rho^2 + 2\rho(1-t_I^2)\right]^{1/2}. \tag{19}$$

Interestingly, we see that when $M_A^2 \to 0$ (i.e., $\rho \to 0$) then $\phi \to \theta_I$, and the eigenstate with a nonzero mass in this limit appears as the conventional DP which couples to the usual combination $Q_D = T_{3I} + Y + I/2$. Of course, the remaining gauge field is seen to be massless in this same limit. We refer to these gauge boson mass eigenstates more generally as $Z_{1,2}$ in the discussion that follows. Continuing, we see that, since W_{3I} is just a linear combination of these two mass eigenstates, i.e., $W_{3I} = Z_1 c_\phi + Z_2 s_\phi$, both $Z_{1,2}$ will now have KM-induced couplings to SM fields which takes the familiar form

$$e\epsilon Q_{em}(c_\phi Z_1 + s_\phi Z_2) = v_1^{\text{SM}} Z_1 + v_2^{\text{SM}} Z_2, \tag{20}$$

while the general gauge couplings of the $Z_{1,2}$ to dark sector fields and PM can now be written as

$$g_I[c_\phi T_{3I} - s_\phi t_I (Y_I/2)]Z_1 + g_I[s_\phi T_{3I} + c_\phi t_I (Y_I/2)]Z_2, \tag{21}$$

and we can easily check that when $\rho \to 0$, the Z_2 couplings become just those of the familiar DP.

For later convenience, we define the ratio of these physical gauge boson masses by $\lambda_R^2 = \lambda_+/\lambda_- = M_2^2/M_1^2$ which is always > 1. Using the two invariants of the $\mathcal{M}^2$ matrix, it is straightforward to obtain a bound on t_I as a function of λ_R by demanding that the ratio ρ be a real quantity assuming that both t_I and λ_R are used as physical input parameters (as they are linked to observables). To see this, we first define the dimensionless ratio of the matrix invariants,

$$\Lambda = \frac{Det\mathcal{M}^2}{(\mathrm{Tr}\mathcal{M}^2)^2} = \frac{\rho t_I^2}{[\rho + (1+t_I^2)]^2} = \frac{\lambda_R^2}{(1+\lambda_R^2)^2} < \frac{1}{4}, \tag{22}$$

and then solve for $\rho(\Lambda)$ requiring a physical, i.e., a real-valued result. From these considerations, we can then obtain a lower bound on the value of t_I as a function of λ_R, i.e.,

$$t_I^2 > \frac{4\Lambda}{1-4\Lambda} \to t_I > \frac{2\lambda_R}{\lambda_R^2 - 1}, \tag{23}$$

which we make use of in the analyses that follow. It is important to note that this lower bound weakens quite rapidly as λ_R^2 increases.

III. PHENOMENOLOGICAL IMPLICATIONS: DM AT LOW ENERGIES WITH TWO $U(1)$'S

As discussed above, the combination of the Dirac mass and the dark Higgs triplet-induced Majorana mass term for χ splits this field into two pseudo-Dirac states, $\chi_{1,2}$, having masses $m_{1,2} = m_D \mp y v_2/\sqrt{2}$. As is well known and as used in our previous work [62], these states will interact with the $Z_{1,2}$ gauge bosons in an off-diagonal manner, i.e., via

$$\mathcal{L}_{\rm int} = \frac{i g_I}{2}[\bar{\chi}_2 \gamma_\mu \chi_1 - (1 \leftrightarrow 2)](v_1^D Z_1 + v_2^D Z_2), \tag{24}$$

where the parameters v_i^D are directly obtainable from the general coupling expressions as presented by Eqs. (22) and (23) above,

$$v_1^D = \frac{1}{2}(c_\phi - t_I s_\phi), \qquad v_2^D = \frac{1}{2}(s_\phi + t_I c_\phi). \tag{25}$$

Given some input values for the ratio of the lightest (stable) DM state to that of Z_1, $r = m_1/M_1$, the relative mass splitting between χ_2 and χ_1, $m_2/m_1 = 1+\delta$, and the ratio of the squares of the dark gauge boson masses, λ_R^2, as well as the value of t_I, we can determine the DM coannihilation cross section for, e.g., $\chi_1\chi_2 \to e^+e^-$, in a rather straightforward manner up to an overall unknown coupling factor. Note that in the symmetric $y_L = y_R = y$ limit considered here that there are no "diagonal" annihilation channels into SM final states, e.g., $\chi_1\chi_1, \chi_2\chi_2 \to e^+e^-$. Further, it is assumed that any light dark Higgs fields that are present in this setup do not contribute significantly to this cross section as their couplings to the light SM fermions can only arise via mixing with the SM Higgs, where they are already found to be small due to the appearance of ratios $m_{e,\mu}/v_{\rm SM}$ in these couplings. This mixing of the light dark Higgs with the SM Higgs is also found to be quite highly constrained by the observed limit on the $H(125)$ branching fraction into invisible final states, $B_{\rm inv} < 0.10$–0.15, by both ATLAS and CMS at the LHC [81,82]. Correspondingly, in the case where the light gauge bosons may decay to leptons, there are also other constraints as well from searches involving collimated lepton pairs [83,84] which also enforce a strong suppression upon this mixing. We do, however, have a few more brief words to say about the possible role of these light dark Higgs fields in our later discussions below.

Before continuing, we recall that the value of $r = m_1/M_1$, is bounded from above in these setups. When $r > 1$, the s-wave, on-shell process, $\bar{\chi}_1\chi_1 \to 2Z_1$, $Z_1 \to$ SM, becomes kinematically allowed and thus significantly violates the CMB bound on the deposition of electromagnetic energy into the thermal bath when Z_1 decays to, say, e^+e^-. To safely avoid this mishap, we need not only require that $r < 1$ but an even stronger condition is required since the collision of the two χ's, with both being toward the top of their thermal energy distributions, can lead to a center of mass energy, $\sqrt{s}$, somewhat larger than just the naive value of $2m_1$. While the actual upper bound depends in a complex manner on of the interplay of the model parameters, here we, for simplicity, only require that $r < 0.9$ [85]. In a similar vein, we also require that small values of r are excluded, corresponding to excluding very light DM masses less than $\simeq$10 MeV to avoid any constrains coming from the BBN [86–89]; in practice, we simply require $r > 0.1$. Lastly, we also assume that λ_R^2 is sufficiently great so that the relativistic Breit-Wigner approximation remains valid[4] in that the mass separation between the Z_i resonances is always large compared to their individual total widths.

To begin, we recall that for coannihilating DM, where the $\chi_1\chi_1$ and $\chi_2\chi_2$ annihilation cross sections, $\sigma_{11,22}$, can be neglected, we may write the effective cross section weighted by the relative velocity as

$$\sigma v_{\rm rel} = \mathcal{F}^2 \frac{2R\sigma_{12}}{1+R^2}, \tag{26}$$

[4]See, for example, [90] and references therein.

where σ_{12} is the "parton-level" velocity-weighted $\chi_1\chi_2 \to$ SM cross section which occurs via the $Z_{1,2}$ exchanges, to be described below, and R is given by the expression

$$R = (1+\delta)^{3/2} e^{-\delta x_F}, \tag{27}$$

with $m_2/m_1 = 1+\delta$, as defined above and, as usual, $x_F = m_1/T_F \simeq 20$, with T_F being the thermal freeze out temperature. Finally, the overall rate for this process is set by the model-dependent numerical prefactor, $\mathcal{F}$, given by

$$\mathcal{F} = \left(\frac{\epsilon g_{\rm eff}}{10^{-4}}\right)\left(\frac{100\ {\rm MeV}}{M_{Z_1}}\right), \tag{28}$$

where we have defined the effective coupling combination

$$g_{\rm eff} = g_I c_\phi v_1^D = \frac{1}{2} g_I c_\phi (c_\phi - t_I s_\phi). \tag{29}$$

Once the thermally averaged value for $\sigma v_{\rm rel}$, i.e., $\langle \sigma v_{\rm rel}\rangle$, is determined by integration over the DM thermal distributions and then compared with that required to obtain the observed [2] DM relic abundance in this case, $\sigma_0 \simeq 4.4 \times 10^{-26}$ cm^3 sec^{-1} [25–27], we can then extract the corresponding value needed for the quantity $\mathcal{F}$ as a function of the other model parameters. The particular parameter space regions where $\mathcal{F}$ is not required to be too large (and hence more likely to already be excluded by other experiments) would then clearly be seen as the more favorable, and we employ this perspective below.

To proceed further, we need to have an expression for σ_{12} before it can be integrated over the thermal distributions of the χ's. In the Center of Mass (CoM) frame, we find that this is given by

$$\sigma_{12} = \frac{2\alpha_{em}}{s}(\epsilon g_{\rm eff})^2 \left[\frac{m_1 m_2 + e_1 e_2 + \frac{p^2}{3}}{s}\right] \times \sum_{ij} (P_{ij} \tilde{v}_i^D \tilde{v}_j^D \tilde{v}_i^{\rm SM} \tilde{v}_j^{\rm SM}), \tag{30}$$

with $e_{1,2}$ (here in lower case to avoid any confusion with the fermion PM fields introduced above) and p being the energy and momenta of the colliding $\chi_{1,2}$ particles which we can express in terms of s and the $\chi_{1,2}$ masses as

$$e_1 = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}}, \qquad e_2 = e_1(1 \leftrightarrow 2),$$
$$p_1 = p_2 = p = (e_i^2 - m_i^2)^{1/2}, \tag{31}$$

and where, since we have pulled outside of this expression into the definition of $\mathcal{F}$ both $g_{\rm eff}$ and ϵ, one finds that $\tilde{v}_1^D = \tilde{v}_1^{\rm SM} = 1$, while

$$\tilde{v}_2^D = \frac{t_\phi + t_I}{1 - t_\phi t_I} = \tan(\phi + \theta_I), \qquad \tilde{v}_2^{\rm SM} = t_\phi. \tag{32}$$

Also making an appearance in this cross-section expression is the familiar propagator factor for the Z_i exchanges,

$$P_{ij} = s^2 \frac{(s - M_i^2)(s - M_j^2) + (M_i\Gamma_i)(M_j\Gamma_j)}{[(s - M_i^2)^2 + (\Gamma_i M_i)^2][(1 \to 2)]}, \tag{33}$$

where the Γ_i are the total widths of the Z_i which are calculable after making a few assumptions. For example, when $m_1 + m_2 < M_i$, the Z_i will almost exclusively decay into $\chi_1\chi_2$ since decays to the SM final states, such as e^+e^-, are always suppressed by an additional factor of ϵ^2. In such a case, to an excellent approximation one has

$$\frac{\Gamma_i}{M_i} = \frac{g_I^2 (v_i^D)^2}{24\pi}[(1 + x_a - x_b)^2 - 4x_a]^{1/2} \times \left[2 - x_a - x_b - (x_a - x_b)^2 + 6(x_a x_b)^{1/2}\right], \tag{34}$$

with $x_{a,b} = m_{a,b}^2/M_i^2$. Whereas, if the $\chi_1\chi_2$ final state is *not* kinematically accessible and Z_i decays only to, e.g., the SM (massless) e^+e^- final state were to be allowed (which we assume to be the case here), we would instead obtain

$$\frac{\Gamma_i}{M_i} = \epsilon^2 \frac{\alpha_{em}(v_i^{\rm SM})^2}{3}. \tag{35}$$

These widths are clearly directly connected with the $r < 0.9$ constraint discussed above together with the chosen values of δ as well as λ_R^2 in the case of Z_2, as is seen by the numerical results to be presented below. It should be noted that in almost all of the cases to be discussed, this mass constraint will allow the Z_2 to almost always decay to $\chi_1\chi_2$ on shell as the kinematic requirement for this, $r(2+\delta) < \lambda_R$, is mostly always met. Assuming no other decay modes for Z_2 are allowed, this lone mode implies that the Z_2's decay will almost always be invisible unless the subsequent decay of $\chi_2 \to \chi_1 e^+ e^-$, which, as is discussed below, now occurs via virtual Z_i exchanges, is visible However, as is well known, this rate is suppressed by both ϵ^2 as well as by the 3-body phase space, so likely is not sufficiently rapid to be experimentally observed above backgrounds especially when it is highly boosted as it may be at colliders.

As noted, in the numerical analysis that follows, it is assumed that no other decay channel are open, especially for the Z_2; however, other possibilities do exist. For example, since the vevs of the two light dark Higgs representations induce mixing between the W_{3I} and B_I into the Z_i mass eigenstates, it is natural that there remain off-diagonal couplings between these gauge bosons and the two corresponding light Higgs scalars, $h_{a=1,2}$, of the generic form

$$\mathcal{L}_{o.d.} = g_I \mu_a M_2 Z_2^\nu Z_{1\nu} h_a, \tag{36}$$

where the μ_a are dimensionless parameters that are rather complex combinations of the model couplings, etc, but which are generally of order unity, resulting from the required mixings in both the gauge and scalar boson sectors to obtain the relevant mass eigenstates. Defining $r_1 = 1/\lambda_R^2 = M_1^2/M_2^2$ and $r_a = m_{h_a}^2/M_2^2$, one finds that the partial decay width for the process $Z_2 \to Z_1 h_a$ (assuming it to be kinematically allowed) to be given by

$$\frac{\Gamma_2}{M_2} = \frac{g_I^2 \mu_a^2}{192\pi} \frac{1}{r_1} [\lambda(1, r_1, r_a) + 12 r_1] \lambda^{1/2}(1, r_1, r_a), \tag{37}$$

with λ here being the familiar kinematic function

$$\lambda(1, r_1, r_a) = (1 + r_1 - r_a)^2 - 4 r_1. \tag{38}$$

In principle, especially when λ_R^2 becomes somewhat large, we see that this partial width may compete with or even exceed that for $Z_2 \to \chi_1 \chi_2$. However, as noted in the numerical study that follows, we ignore the possibility that this decay mode may be available for simplicity though we should be mindful that it may be of phenomenological relevance in some regimes of parameter space.

Given σ_{12}, expressed as a function of the dimensionless parameters r, δ, t_I and λ_R^2, we can perform the thermal averaging, compare with the necessary cross section to achieve the observed DM relic abundance, and extract the required value of $\mathcal{F}$. However, if we are to explore the effect of now having two light dark gauge bosons instead of just the usual DP, in particular, the effects associated with their interference (e.g., the so-called Bactrian Effect [91]), we first must have as a reference the "standard" result for the case where the second gauge boson decouples in the limit when $M_2^2 \to \infty$ with which to compare.

We note in passing that the effect we are seeking here is *opposite* to that explored in earlier work [91]. There, destructive interference between the gauge boson exchanges in the case of s-wave DM annihilation was shown to be sufficient to avoid the cross-section constraints from the CMB in some parameter space regions. Here, we are using the now *constructive* interference between these exchanges to overcome the already large Boltzmann suppression to obtain the observed DM relic density in otherwise exclude parameter space regions.

Returning to the present analysis, it is assumed for simplicity that the only SM final state available is just e^+e^-. This is shown in the top panel of Fig. 1 as a function of r assuming different values of the mass splitting parameter δ. Clearly, this result is, by definition, independent of both t_I and λ_R^2. As we can see from this figure, (the square root of) the "inverse" of the usual Breit-Wigner peak shows itself via the minimum in the required value of $\mathcal{F}$ but which is distorted asymmetrically around this peak (seen as the minimum) by the velocity distribution weighting of the cross section; clearly the region around the minimum is "preferred." The width of this special region and the shallowness of the distribution at the minimum is significantly affected by the chosen value of δ. As δ increases, the sum of the $\chi_{1,2}$ masses also increases pushing the minimum to lower r values. Also as δ increases, the required value of $\mathcal{F}$ needs to increase to compensate for the relative exponential suppression in the χ_2 thermal distribution. If, e.g., we demanded values of $\mathcal{F}$ less than $\simeq$ unity, then we need either require that $r \sim 0.3$–0.4 or $r \sim 0.44$–0.7 with the tighter constraint applying when δ obtains larger values. Certainly, the region above and significantly far away from the minimum appears somewhat disfavored (especially so for higher δ values) given that larger values of $\mathcal{F}$ seem to be necessary in such cases.

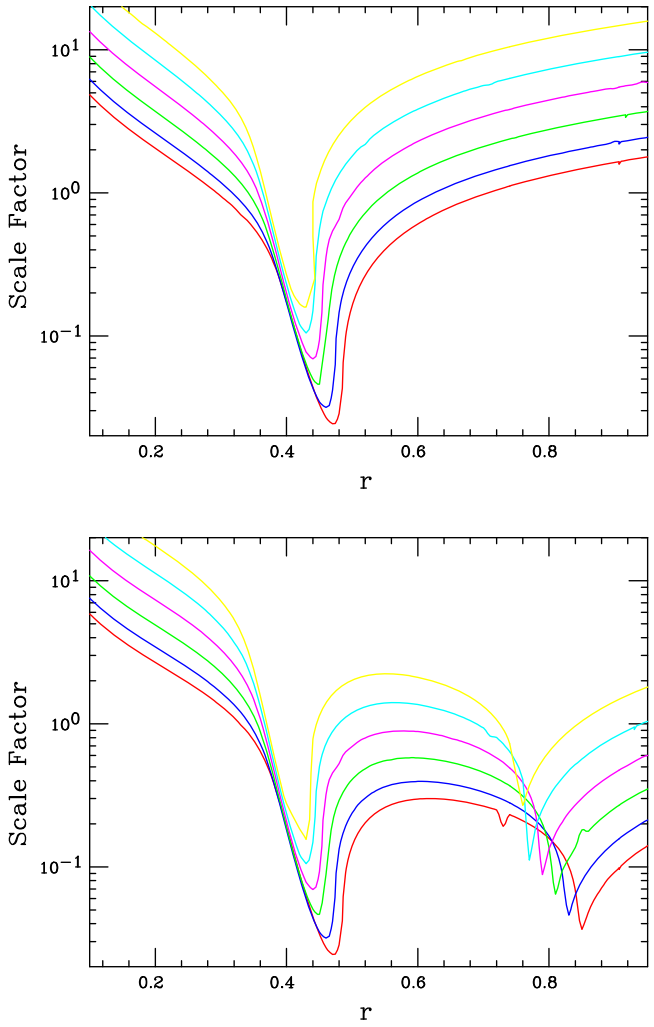


FIG. 1. Value of the coupling scaling factor, $\mathcal{F}$, as defined in the text, as a function of r for, from bottom to top on the right, $\delta = 0.05$–0.30 in steps of 0.05. The top panel assumes the familiar case of a single light dark gauge boson, whereas the bottom panel assumes two gauge bosons with $\lambda_R^2 = 3$ and $t_I = 1.8$ for purposes of demonstration.

TABLE I. Benchmark points. The benchmark λ_R^2 and t_I parameter values employed in the calculations described in the text. The value of the ratio R_2 showing the effect of the two gauge boson exchanges in the 3-body decay of $\chi_2 \to \chi_1 e^+ e^-$ is also given.

λ_R^2	t_I	R_2
3	1.8	0.55
4	1.5	1.14
5	1.2	4.54

In order to explore how these quite familiar results will change due to the existence of the Z_2 and, in particular, $Z_1 - Z_2$ interference effects, we consider the gauge boson mass ratio range $3 \le \lambda_R^2 \le 5$ beyond which we can easily imagine how the asymptotic region, essentially shown in the top panel of Fig. 1, is eventually more closely approached. This analysis requires choosing a set of values for t_I satisfying the lower bound given in Eq. (25); these benchmark values are given in Table I and are generally chosen to lie not too far above this lower bound. We expect that almost all of the modifications due to the added Z_2 exchange will take place above the Z_1, and this will be roughly borne out in the analysis that follows. As is easily seen, at the amplitude level, the magnitude of the interference of the $Z_{1,2}$ contributions, be it constructive or destructive, is essentially controlled by both the separation between the two gauge boson masses via λ_R^2 as well as the both the sign and the magnitude of ratio of the product of their couplings to the initial and final states, i.e.,

$$\mathcal{R} = \frac{v_2^{\rm SM} v_2^D}{v_1^{\rm SM} v_1^D} = t_\phi t_{\phi+\theta_I}, \tag{39}$$

which can be uniquely determined once t_I is known for a fixed value of λ_R^2 as was discussed in the previous section.

In the lower panel of Fig. 1, we consider the case of $\lambda_R^2 = 3, t_I = 1.8$ where several things are immediately apparent. First, and most obvious, is the presence of the second minimum near $r \sim 0.8$ due to the existence of the relatively close by Z_2. The fact that this minimum is visible in the range $r < 0.9$ indicates that the kinematic requirement $r(2+\delta) < \lambda_R$ needed for the on-shell decay $Z_2 \to \chi_1\chi_2$ to be allowed on shell is not always satisfied once r is larger than $\simeq 0.75$. Second, we see that in the region between the two resonances as well as just beyond the Z_2 minimum, the required value of $\mathcal{F}$ is always less than $\simeq$1–2 due to the (constructive) interference of the two amplitudes making this entire parameter region now no longer "disfavored" and certainly, at the very least, more viable thus opening up more of the parameter space for further study. Third, the effect of the Z_2 exchange is not very noticeable in the region at or immediately below the Z_1 resonance minimum (in comparison to the case of just a single exchange) as might have been expected due to the proximity of this resonance although some modest influence of destructive interference is noticeable under somewhat closer scrutiny for sufficiently small values of $r \lesssim 0.3$. The bottom line is that we see that the constructive interference due to having two nearby resonances opens up a significant region of parameter space that was previously unfavorable. Furthermore, in this region, the Z_1 is seen to always decay to SM final states. It is to be noted that the minimum corresponding to the Z_2 is somewhat more shallow than that due to Z_1. Since the couplings are comparable, this is mainly due to the fact that Z_1 is generally far more narrow as it only decays to SM final states (here, just e^+e^-), which are likely allowed, whereas the Z_2 can also decay to the χ's and so is much wider thus suppressing the height of the cross-section peak. Finally, in the region *above* the second resonance and not easily seen in this figure (but would be if, e.g., values of $\lambda_R^2 = 2$ were to be entertained), the two amplitudes will destructively interfere forcing us to chose even larger values of $\mathcal{F}$ than in the single exchange model to obtain the observed DM relic density. This region continues to be disfavored.

In Fig. 2, we can examine what happens as one slowly increases the value of λ_R^2 to 4(5) in the upper (lower) panel; significant changes from the case of $\lambda_R^2 = 3$ are clearly observed. When $\lambda_R^2 = 4$, the choice $t_I = 1.5$ still leads to a constructive interference between the two resonances, but they are now spaced further apart with at least some of the greatest activity appearing beyond the limited $r \le 0.9$ range that we consider here. This leads to a modest increase in the value of $\mathcal{F}$ required to reproduce the observed DM relic density in the region between the two resonances which, however, remains relatively preferred especially for smaller values of the parameter δ. Once we go beyond $\lambda_R^2 = 4$, the value of $\mathcal{R}$ is found to change sign for our chosen benchmark values of t_I, thus leading to a destructive interference between the $Z_{1,2}$ exchanges. This effect is not immediately obvious in the case of $\lambda_R^2 = 5$ as the $Z_{1,2}$ resonances are still spaced rather closely so that the cross section does not have enough room to dive to particularly small values and correspondingly larger values of $\mathcal{F}$. This increase in the required value of $\mathcal{F}$ to reproduce the observed relic density when going from $\lambda_R^2 = 4$ to $\lambda_R^2 = 5$ does not appear to be significantly larger that than seen when going from the case of $\lambda_R^2 = 3$ to that of $\lambda_R^2 = 4$. Here, we see that in this interference regime, values of $\delta \lesssim 0.15$–0.2 will still remain in the preferred range. Obviously, once λ_R^2 grows further, exceeding these values, one will obtain results which will appear to be very much like those seen in Fig. 1.

Clearly, we have learned from this analysis that if λ_R^2 is sufficiently small so that constructive interference occurs in the region above the Z_1 or that destructive interference remains relatively ineffective, then that range of the DM mass now becomes much more favorable in comparison to

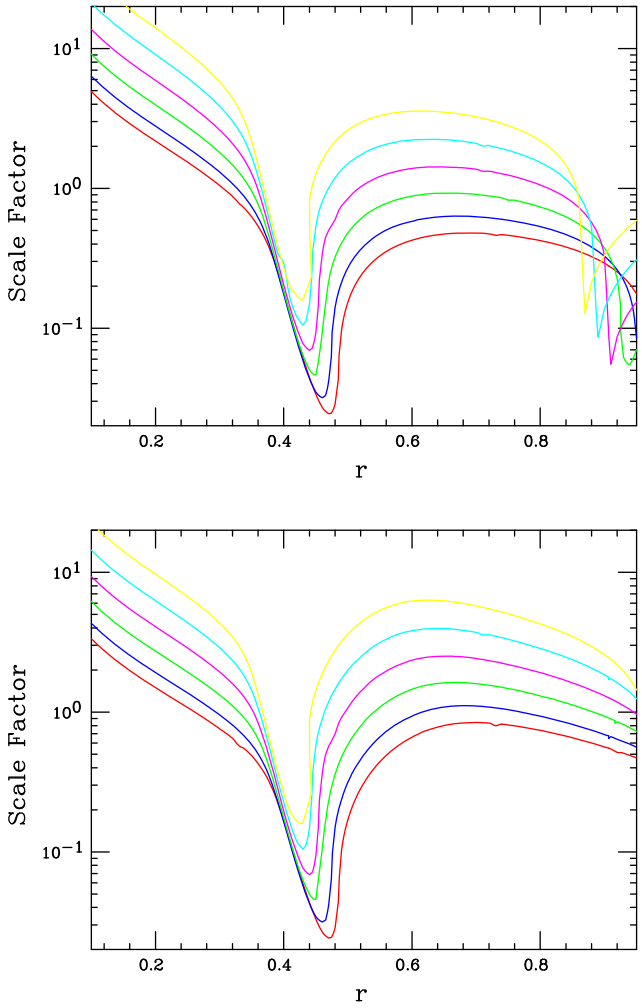


FIG. 2. Same as the bottom panel in Fig. 1 but now for (top) $\lambda_R^2 = 4$ with $t_I = 1.5$ and (bottom) $\lambda_R^2 = 5$ with $t_I = 1.2$.

what occurs in the case when there is only a single DP exchange. Of course, in the present case, we have relied on a specific setup to demonstrate this result, and we are to be reminded that other choices of the dark Higgs fields would reweight the couplings of the $Z_{1,2}$ gauge bosons in a somewhat different manner. However, our expectation is that what we have found above would be a rather qualitative general result, although the specific details would be model dependent.

Finally, we need to return to the question of how the χ_2 decays; since $r\delta \leq 0.3 < 1$ in the parameter range of interest to us here, the decay $\chi_2 \to \chi_1 Z_1$ cannot occur on shell. Also, since the light Higgs mass eigenstate fields, h_a, we encountered above only couple diagonally to the χ's, there are no allowed decays of the type $\chi_2 \to \chi_1 h_a$. Instead, one needs to focus on the 3-body decay process via the virtual exchange of the two light gauge bosons, e.g., $\chi_2 \to \chi_1 Z_{1,2}^*, Z_{1,2}^* \to e^+e^-$. In the case of such a decay being mediated by a *single* DP, V, with coupling g_D, this rate is well known [75] to be approximately given by the expression [in the limit where $(r\delta)^2 \ll 1$ is assumed so that any energy dependence in the gauge boson propagators can be ignored]

$$\Gamma_{3-body} \simeq \frac{4m_2^5}{15\pi m_V^4}\alpha_D \alpha \epsilon^2 \delta^5, \tag{40}$$

whereas in the present setup, both of the two Z_i exchanges will simultaneously contribute to this process. Of course, in general, the propagator energy dependence *cannot* be ignored so that the validity of this approximation in the present case, now with two gauge boson exchanges, cannot be taken for granted and will need to be tested here. However, *if* this approximation were to be roughly valid, one would then expect that the decay rate would be rescaled by an overall factor of

$$R_2 = \left(1 + f\frac{\mathcal{R}}{\lambda_R^2}\right)^2, \tag{41}$$

which may be quite substantial until larger λ_R^2 values are reached. Here, $f = f(r\delta, \lambda_R^2)$ is a correction factor, which we find differing from unity by at most a few 10's of percent, accounting for finite $(r\delta)^2$ effects in our parameter range of interest. The values of this ratio, R_2, for our set of benchmark points is also shown in Table I where we see that both destructive and constructive interference effects are possible depending upon our location in parameter space; note that for finite $r\delta$, this effect can be only slightly degraded. Given the value of R_2, we can then express the corresponding unboosted decay length for this χ_2 decay as

$$c\tau \simeq 4.8 \times 10^6 \text{ cm}\left[\left(\frac{M_{Z_1}}{100 \text{ MeV}}\right)\left[\frac{g_{\text{eff}}\epsilon}{10^{-4}}\right]^2\left(\frac{I}{10^{-7}}\cdot R_2\right)\right]^{-1}. \tag{42}$$

Figure 3 shows the numerical results for the 3-body phase space integral, I, appearing in the formula above as a function of r for different values of δ in the limit that $m_e^2 \to 0$. Here, we can see, among other things, the rough validity of the $\sim(r\delta)^5$ scaling that we might expect from the approximate decay width formula above and that I takes on a very wide range of values, $\sim 10^{-12}$–10^{-3}, as both r and δ are varied over their allowed ranges. However, violations of this scaling at the level of ~25% are found but are essentially invisible in this figure. The anticipated approximated parameter scaling can be seen in both the power law behavior with r as well as the approximately constant spacing between the curves corresponding to different values of δ. Note that for the parameter ranges that we consider here I is such that one obtain lifetimes values corresponding to macroscopic unboosted decay lengths so that χ_2 will appear as either just MET or MET + a displaced

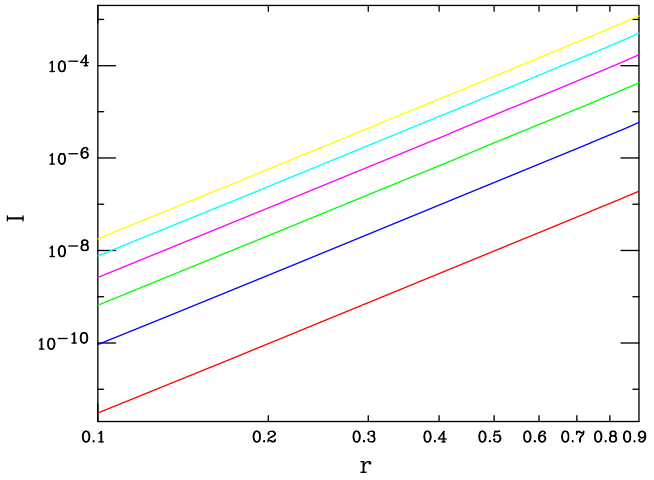


FIG. 3. The phase space integral, I, as a function of r for different values of δ as described in the text. The curves from bottom to top correspond to different choices of δ from 0.05 to 0.30 in steps of 0.05, respectively.

e^+e^- pair in a detector depending upon how it is produced even when constructive interference between the two Z_i is present and interference effects are relevant.

IV. PHENOMENOLOGICAL IMPLICATIONS: PRESENT AND FUTURE HIGH ENERGY COLLIDERS

As discussed earlier, unlike in the PM models previously considered, the high energy collider phenomenology of the present model is somewhat limited. The reasons for this are twofold: (i) First, the two PM fermions in the current setup are paired alone together within their own $SU(2)_I$ representation which, in the absence of very small mixing, is completely isolated from the SM. This means that the non-Hermitian $W_I^{\pm}$ gauge bosons do not couple directly to any of the SM fields. (ii) Both of the light Hermitian gauge bosons, $Z_{1,2}$, only couple to the SM via suppressed KM interactions. Thus, e.g., W_I pair production via a TeV scale, and potentially resonant, Z_I exchange, where the Z_I previously had unsuppressed couplings to the SM quarks or leptons in the initial state, is now absent. This is unfortunate as new neutral heavy gauge bosons can provide an important window onto new physics and are relatively accessible [92–94]. A reaction similar to this via *light* Z_i exchanges, but which is thus never resonant, *does* occur but its rate is $\sim\epsilon^2$ suppressed as we discuss briefly below. The other possible channel, W_I associated production with a heavy PM fermion via, e.g., gq-fusion, is also absent here since, again, the PM does not share any representation with the SM fermions. On top of this, all of the new Higgs fields needed to break the various non-SM gauge symmetries are dark and so will not couple directly to any of the SM particles in the absence of mixing which for the most part cannot be too large due to the experimental constraints on the H(125) properties. Just how bad this situation is examined below.

The PM fermions themselves, here assumed to be $SU(2)_L$ isosinglet charged leptons for concreteness, can only be produced by the usual electroweak interactions via γ, Z exchanges, at relatively suppressed rates, and so it is more than likely that only the lighter of these two states, E_2, might be accessible to the LHC and possibly at future colliders. In fact, as we see in the figures discussed below, increasing the PM mass by ∼50% results in a reduction of the production cross section by more than an order of magnitude. As discussed above, the mixing of this state with a SM isosinglet, RH-lepton, denoted generically by e_R, leads to the dominant decays $E_2 \to eZ_{1,2}$. If the Z_i decay invisibly or are sufficiently boosted, the pair production of E_2's will appear as opposite-sign dileptons plus MET (qualitatively similar to that for isosinglet sleptons), which suffers a significant SM backgrounds from W^+W^- pair production with both W's simultaneously decaying into same flavor charged leptons plus neutrinos. The pair production cross sections for these states at the LHC and at FCC-hh with various center of mass energies are shown for reference in the top and bottom panels of Fig. 4, respectively, and are smaller than those for all other possible charged VL fermionic PM states implying that only a relatively poor PM search reach is obtainable.

Using 139 fb^{-1} of integrated luminosity at the 13 TeV LHC, the authors of Ref. [41] have recast the then existing ATLAS and CMS SUSY searches to obtain limits on the pair production of isosinglet vectorlike leptons decaying with just this pattern of interest, i.e., $E^+E^- \to e^+e^- +$ MET, obtaining a lower bound on the mass of such a particle of i$M_{E_2} > 895$ GeV. These same authors have also extrapolated these results to obtain possible exclusion (not discovery) reaches at the HL-LHC with an luminosity of 3 ab^{-1} of 1.45 TeV (but still assuming that $\sqrt{s} = 13$ TeV) and at the $\sqrt{s} = 100$ TeV FCC-hh, also with an assumed integrated luminosity of 3 ab^{-1}, of 3.33 TeV. An increase in the integrated luminosity by an order of magnitude may be able to push this mass reach of the FCC-hh up into the neighborhood of ≃4.5 TeV, whereas a reduction of $\sqrt{s}$ to 60(80) TeV may result in a decreased reach of roughly ≃30(13)% or so as indicated by this figure.

Of course, if we had chosen the PM fields to be color triplets, as was mentioned above, i.e., U_i and/or D_i, then these fields would be pair produced via QCD interactions, and the relevant final state signature would then be 2 jets + MET, possibly with flavor tagging, leading to somewhat greater search reaches at hadron colliders by factors of roughly ≃1.5–3 [51] depending upon the assumed collider energy and luminosity, e.g., ≃10 TeV or more at the $\sqrt{s} =$ 100 TeV FCC-hh with 30 ab^{-1} of integrated luminosity.

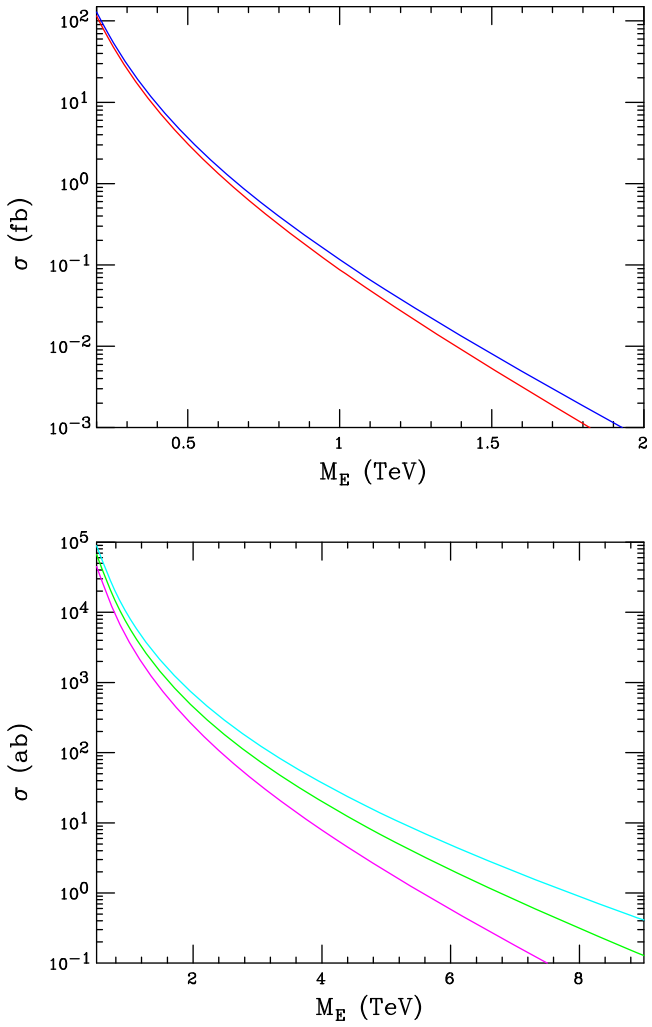


FIG. 4. Pair production cross sections for the SM isosinglet, charged leptonlike PM fermion at hadron colliders via s-channel γ, Z exchange: (top) at the $\sqrt{s} = 13(14)$ TeV LHC represented by the red (blue) curve, respectively, and (bottom) at the $\sqrt{s} = 100(80, 60)$ TeV FCC-hh, represented by the cyan (green, magenta) curve, respectively.

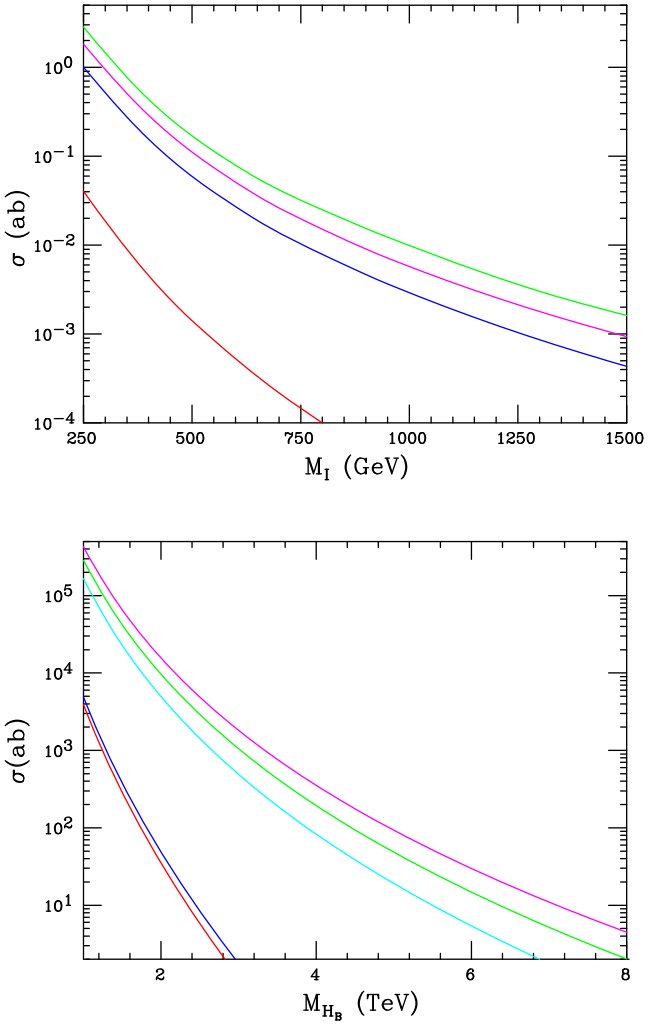


FIG. 5. Top: $W_I W_I^\dagger$ production cross section as a function of its mass at the $\sqrt{s} = 14$ TeV LHC and the $\sqrt{s} = 100(80, 60)$ TeV FCC-hh corresponding to the green(magenta, blue) curves, respectively, assuming that $g_I\epsilon = 10^{-4}$, as discussed in the text. Bottom: H_B production cross section as a function of its mass via gg-fusion mediated by top quark loops assuming that $s_\lambda^2 = 0.025$ for purposes of illustration as discussed in the text. The red (blue) curve is for the $\sqrt{s} = 13(14)$ TeV LHC, whereas the magenta (green, cyan) curve corresponds to the $\sqrt{s} = 100(80, 60)$ TeV FCC-hh, respectively.

As noted above, the heavy W_I's may be pair produced in a nonresonant manner from SM initial states via the s-channel exchange of the light $Z_{1,2}$ gauge bosons but with rates which are necessarily ϵ^2 suppressed. In fact, the mass of the W_I is so large compared to those of the Z_i that we can obtain the desired cross section simply from that due to the exchange of a single, massless W_{3I} state (since the longitudinal modes do not couple to the initial state), thus removing much of the apparent model dependency. Except for the W_I mass itself and the numerical value of the product, $g_I\epsilon$, the (subprocess) differential cross section for $\bar{f}f \to W_I W_I^\dagger$ is completely determined and is provided by the PDG [95] after some slight modifications. Assuming for demonstration purposes that $g_I\epsilon = 10^{-4}$, similar to the discussion above, we obtain the result for this cross section at the LHC and FCC-hh for different assumed center of mass energies as shown in the upper panel of Fig. 5. Here, we see that, independently of how these states might decay, the pair production cross section for W_I is far too small to be observable at the HL-LHC even if M_I were to be as low as only 250 GeV. The $\sqrt{s} = 100$ TeV FCC-hh, however, fares somewhat better but even there, with an integrated luminosity of 30 ab^{-1}, does not provide a mass reach beyond, perhaps, ∼500 GeV, a value which might be

further eroded if the center of mass energy is lowered and depending upon how the W_I's decay itself might be tagged. It seems more than likely that any direct access to the physics of the W_I must be obtained by other means than via this previously standard process.

Although the SM Higgs is highly constrained not to mix with light dark Higgs that may yield H(125) decays to invisible final states at too large of a rate, the mixing of the usual Higgs with the *heavier* dark Higgs living at the $\sim v_T$ scale is a bit less constrained. Those Higgs scalars, to lowest order, will only couple to the heavy dark sector fields, $W_I, E_{1,2}$, etc., which are all kinematically inaccessible to H(125). Thus, we see that their mixing with the SM Higgs will essentially only lead to an overall reduction in the H(125) total width and will leave all of the branching fractions unaltered, something which is a bit more subtle to detect by the LHC experiments. The real, $T_{3I} = Y_I = 0$ scalar, Σ^0, is an example of such a field which may experience a small $\sim v_{\rm SM}/v_T$-scale mixing with the SM Higgs and may be able to be produced at a visible rate via this mixing. We recall that the other scalars in Σ carrying $T_{3I} = \pm 1$, in the ratio of large vevs limit, no longer remain in the physical spectrum as they are essentially the Goldstone bosons which are eaten by the $W_I^\pm$.

If we denote the heavy scalar mass eigenstate (assuming only mixing with the SM Higgs in obvious notation) by $H_B = c_\lambda \Sigma^0 + s_\lambda h_{\rm SM}$, [with $h = H(125)$ then being identified with the lighter, orthogonal combination], we can employ, e.g., the process $gg \to H_B$ via the top quark loop (assuming that any heavy colored PM fields are absent in the present setup) to estimate this cross section at hadron colliders; this is shown in the lower panel of Fig. 5 for the $\sqrt{s} = 13$, 14 LHC and also for the FCC-hh with $\sqrt{s} = 60$, 80, or 100 TeV. Here, for purposes of demonstration, we have assumed a 2.5% *overall* suppression of the H(125) couplings (i.e., $s_\lambda^2 = 0.025$) relative to the SM which still remains relatively safe within the experimentally allowed region [96–99] from current fits to the H(125) coupling data. Note that unitarity constraints on electroweak singlet heavy Higgs fields from SM WW scattering further require that [100] $M_{H_B}|s_\lambda| \lesssim 1$ TeV which in our numerical example roughly corresponds to the bound $M_{H_B} \lesssim 6.3$ TeV. Note further that we have ignored the possible corresponding production of H_B employing the WW fusion process as it is quite sensitive to both M_{H_B} as well as the specific value of s_λ^2 for just this reason. Thus, the cross section we display here is actually the *minimum* result for this chosen value of s_λ^2. Of course, this situation may change significantly if heavy colored PM fields coupling to H_B are also/were instead present in an augmented version of this setup; this would be the case if we had chosen color triplet instead of color singlet PM fields.

Since $H_B \simeq \Sigma^0$ couples to pairs of W_I's (as well as the E_i's as noted), their production via this intermediary is far likely dominant, especially so if the $H_B \to W_I W_I^\dagger$ decay can occur on shell as it can also be the dominant decay mode. If these heavy modes are not kinematically accessible on shell, then decays to SM final states, in particular, W^+W^-, due to the rather strong coupling that yields the unitarity bound mentioned above, may become of significant relevance and will dominate. Although dependent upon the details of the particular mass spectrum of these dark states, it is reasonable that the HL-LHC reach may exceed ~1 TeV while that for the 100 TeV FCC-hh may approach ~5–6 TeV.

One can imagine other processes that can be induced via the mixing between the SM Higgs and Σ^0 at loop level, e.g., $h \to \gamma + Z_i$, as generated by the leptonlike PM fields. However, the rate for such a process is not only suppressed by an overall loop factor, $\sim\alpha^3/\pi^2$ and by small mixings, $s_\lambda^2 \sim 0.025$, but also by the small mass ratios, e.g., $m_h^2/M_I^2 \lesssim 10^{-2}$. A rough estimate of the sum of partial widths for these processes is given to leading order in the small mass ratios, i.e., $M_{Z_i}^2 \ll m_h^2 \ll M_{E_i}^2$, by

$$\sum_i \Gamma(h \to \gamma + Z_i) \simeq \frac{m_h \alpha^3}{288\pi^2} s_\lambda^2 \left(\frac{\alpha_I}{\alpha}\right)^2 \left(\frac{m_h}{M_I}\right)^2 \left[\frac{M_{E_1}^2 - M_{E_2}^2}{M_{E_1} M_{E_2}}\right]^2, \tag{43}$$

where employing typical values of the various parameters and using the expected SM Higgs total width of ≃4.1 MeV leads to a branching fraction of $B \sim 10^{-9}$–10^{-8} which is quite inaccessible. A rather similar result, up to factors of a few, would also be found to hold if color triplet PM fields as discussed above had instead been chosen.

As noted above, in the limit that $v_T^2 \gg v_{\rm SM}^2 \gg v_{1,2}^2$, the only other (beyond the SM Higgs and Σ^0) remaining physical Higgs field degrees of freedom that are *not* eaten as Goldstone bosons are those solely arising from the dark $SU(2)_I$ doublet and triplet scalars, $H_{1,2}$, respectively, none of which carry SM quantum numbers. Needless to say, this makes studying any of the physics associated with them rather difficult to explore directly. This, however, also happens in a similar fashion in the more conventional "SM-like" symmetry breaking pattern as there the heavy Z_I eats an additional Goldstone boson occurring in the $SU(2)_I$-breaking Higgs doublet employed therein. Thus, it behooves us to further explore indirect collider signatures for the new physics induced by the setup above.

As a step in this direction, we consider the pair production of the light dark gauge bosons and/or light dark Higgs at future e^+e^- colliders paralleling the analysis procedure as employed in Ref. [61]. To be specific, we consider the ten (sub-)processes $e^+e^- \to Z_i Z_j, h_a h_b, Z_i h_a$,

with $i = 1, 2; a = 1, 2$, which we assume produce an invisible final state via their decays to DM. These processes dominantly arise from the $t-, u-$ channel exchanges of the leptonlike PM fermions which are induced by the couplings resulting from the required $E_2 - e_R$ mixing, described in Eq. (14), via the Yukawa coupling $y_{\rm mix}$. In what follows, we assume that $y_{\rm mix}$ takes the value of unity as our results are then easily scalable. Since these particles are all invisible, we perform the usual trick of tagging this final state via an initial state photon [i.e., Initial State Radiation (ISR)], and so we are actually looking for the well-known $e^+e^- \to \gamma +$ "nothing" mode. Noting that the masses of the Z_i and h_a are all far smaller than the nominal $\sqrt{s} = 1$ TeV value that we consider below, we can treat them as (essentially) massless and employ the Goldstone boson equivalence theorem [101] to perform our cross-section calculation so that their individual contributions can all be easily summed together. This greatly simplifies the result as individual dependencies on the various mixing angles will vanish when this sum is performed so that only the overall scale set by $y_{\rm mix}$ remains an unknown. To calculate the relevant cross section, as in earlier work, we make use of an improved radiator function, $\mathcal{R}_\gamma$, as discussed in, e.g., Refs. [102,103] so that we can write the $e^+e^- \to \gamma$ plus light dark boson cross section as

$$\frac{d\sigma}{dx_\gamma d\cos\theta_\gamma} = \sigma_D(s \to \hat{s})\mathcal{R}_\gamma(x_\gamma, \theta_\gamma), \tag{44}$$

where the parameter $a = 1 + 2M_E^2/s$ so that $\sigma_D(s)$ is just given by (after summing over the various final states with their proper mixing angles and weighting factors)

$$\sigma_D = \frac{y_{\rm mix}^4}{64\pi s}\left[\frac{(3a^2-1)}{a}\tanh^{-1}\left(\frac{1}{a}\right) - 3\right]. \tag{45}$$

Note that this cross section rapidly falls as $\sim(\sqrt{s}/M_E)^8 \to 0$ for small center of mass energies thus requiring us to look toward measurements at higher energy colliders and not at experiments such as, e.g., BELLE-II. In the expression above, $x_\gamma = 2E_\gamma/\sqrt{s}$ is the scaled energy of the ISR photon so that $\hat{s} = (1 - x_\gamma)s$ is the e^+e^- → light dark boson pair subprocess center of mass energy squared. Here, θ_γ is the photon's scattering angle as measured from the e^- beam direction in the center of mass frame. The radiator function that we again use, in the limit of vanishing electron mass, is given by [102,103]

$$\mathcal{R}_\gamma(x_\gamma, \theta_\gamma) = \frac{\alpha_{em}}{\pi}\frac{1}{x_\gamma}\left[\frac{1 + (1-x_\gamma)^2}{1 - \cos^2\theta_\gamma} - \frac{x_\gamma^2}{2}\right], \tag{46}$$

so that the integration over $z = \cos\theta_\gamma$ is trivial once the relevant experimental detector angular acceptance cuts, $-z_0 \le z \le z_0$, are imposed. As is well known, this final state suffers from a huge SM background coming from the subprocess $e^+e^- \to$ neutrino pairs that arises from both s-channel Z and, dominantly, t-channel W exchanges; we make use of the analytical results for this SM cross section that are found in Refs. [104–107] to estimate this background. We also recall that this SM contribution arises almost completely from the left-handed electron polarization state, whereas the new physics signal we seek arises instead from *right-handed* electrons. This then behooves us to focus on linear colliders, such as the ILC, C^3,

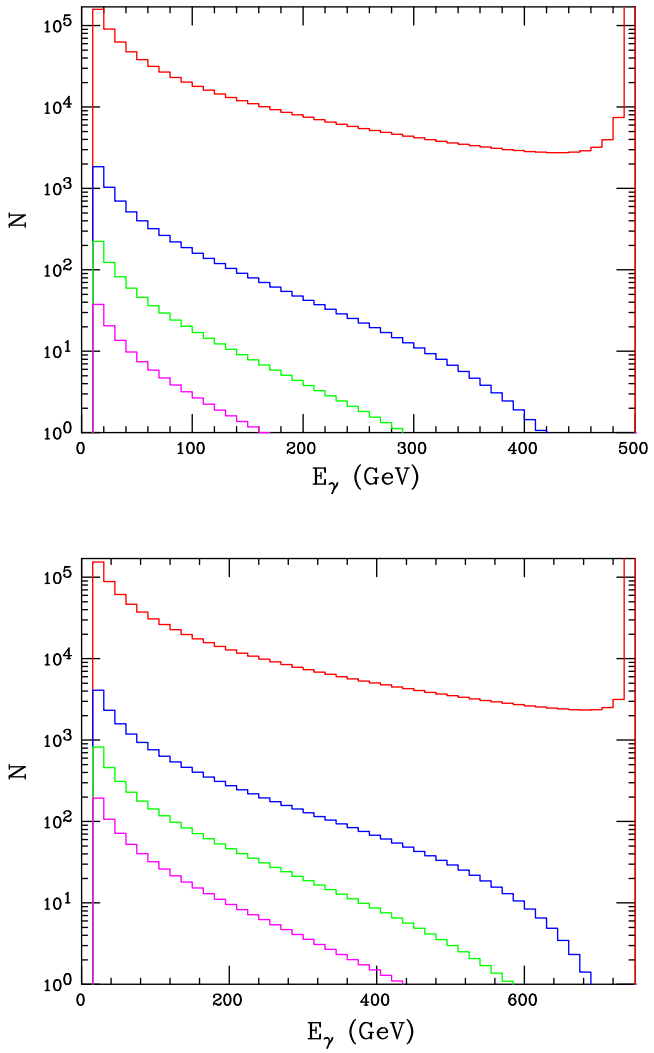


FIG. 6. Top: SM background (red) as well as the signal event rates in 10 GeV photon energy bins for the process $e^+e^- \to \gamma +$ nothing arising here from dark Z_i and dark Higgs production as described in the text assuming $\sqrt{s} = 1$ TeV, an integrated luminosity, L, of 1 ab^{-1}, $P_{e^-} = 0.8, P_{e^+} = 0.3$, $z_0 = 0.995$, $E_\gamma > 10$ GeV, and with $M_{E_2} = 1(1.5, 2)$ TeV, corresponding to the blue (green, magenta) histogram for purposes of demonstration. Bottom: same as the top panel but now for $\sqrt{s} =$ 1.5 TeV in 15 GeV photon energy bins and assuming $E_\gamma > 15$ GeV.

or CLIC [108–112] as they will allow for highly polarized $e^{\pm}$ beams, e.g., $|P_{e^-}| = 0.8$ and $|P_{e^+}| = 0.3$, to reduce this sizable SM background. Making a judicious choice of these polarizations, a purely LH-coupling, e.g., $e_L^- e_R^+$, as that induced by the SM W coupling[5] can then be suppressed by a factor of $(1-0.8)(1-0.3) = 0.14$ while with this same choice a purely RH coupling, e.g., $e_R^- e_L^+$, as occurs here in the PM-electron interaction, can be simultaneously enhanced by a factor of $(1+0.8)(1+0.3) = 2.34$; this has been noted by the authors of other BSM studies employing this particular final state [113–117].

To make a direct comparison of both the signal and background we require (as in earlier work) that $E_\gamma > 10$ GeV and assume detector coverage down to very small angles, i.e., $z_0 = 0.995$. Further, we ignore the (small) additional contribution of the heavier PM state, E_1, to the total rate with $M_{E_2} \gtrsim 1$ TeV given the current search bound discussed above and the rapid fall off of the cross section with increasing PM mass (that we soon observe). Making these assumptions as well as the preferred polarization choice above, we arrive at the results shown in upper panel of Fig. 6 for both the signal and background. Here. we see that, unfortunately, in the best case scenario, even at lower photon energies the signal falls short of the SM background by over two orders of magnitude when $M_{E_2} = 1$ TeV and then falls off quite rapidly as either the photon energy or the PM mass increases. The only possible solution is to move closer to the PM pair production threshold, and to that end, we consider going to case of $\sqrt{s} = 1.5$ TeV with the result as shown in the lower panel of this same figure. Here, we see that, while this certainly goes in the right direction by increasing S/B by a factor of a few, the signal still remains relatively small. Clearly, unless this SM background is extremely well understood so that percent level deviations are discernible, it is unlikely that the signal will be at all visible.

As we have learned from the discussion above, while the opportunities to directly and/or indirectly probe the physics of the current setup at future colliders are somewhat limited, some interesting possibilities remain. Further studies of these and other processes not discussed here are clearly warranted.

V. DISCUSSION AND CONCLUSION

The KM of the SM photon with a light DP associated with a dark gauge group, $U(1)_D$, offers an interesting and flexible window into the possibility of light DM and how it achieved its observed abundance. This mechanism requires the existence of a new set of heavy fields, which are either vectorlike fermions with respect to the SM gauge symmetries and/or complex scalars, that carry both $U(1)_D$ as well as SM couplings, termed portal matter. For a range of parameters, the running of the $U(1)_D$ gauge coupling, especially in the case of pseudo-Dirac DM, argues for a possible more UV-complete version of this setup not far from the ∼10 TeV scale. There, $U(1)_D$ is embedded into an, at least partially, non-Abelian group structure, G_D, in order to maintain perturbativity by having the opposite sign for its β function, with the SM-like $G_D = SU(2)_I \times U(1)_{Y_I}$ being a simple example. Although we have examined various aspects of the phenomenology associated with this type of setup in earlier work, it has always been assumed that G_D was broken in a SM-like manner, i.e., directly to $U(1)_D$ via a dark Higgs doublet, which then is subsequently broken at a scale ∼1 GeV; this need not be the case. In this paper, we have considered the possibility that this G_D will be broken instead by a real dark Higgs triplet so that $U(1)_{Y_I} \times U(1)_{T_{3I}}$ remains intact until the low mass scale is reached, and thus, there are two light dark gauge bosons. If the PM is chosen to transform as an $SU(2)_I$ VLF doublet carrying nonzero SM hypercharge but which is an $SU(2)_L$ singlet, then the same vev which breaks $SU(2)_I$ splits the mass degeneracy in this doublet leading to a non-Abelian KM between the (still massless) $U(1)_{T_{3I}}$ gauge boson, W_{3I}, and the familiar hypercharge gauge field, B, of the SM. This PM could be "leptonlike," in which case it carries $Q_{em} = -1$ and/or "quarklike," in which case it carries $Q_{em} = 2/3$ or $-1/3$. Here, we have concentrated on the leptonlike case for simplicity but either choice leads to essentially the same calculable value for the KM mixing strength, ϵ, of the desired magnitude. This PM sector is more minimal than previously encountered, being atypical for several reasons, and so the resulting phenomenology is also found to be somewhat different.

After shifting the fields to remove the KM and the SM symmetry is broken, the $U(1)_{Y_I} \times U(1)_{T_{3I}}$ remains and requires two Higgs fields to generate the appropriate masses and mix the W_{3I} and B_I gauge bosons. Here, these representations were chosen to also perform other tasks, e.g., to allow the lighter PM field to decay, so it must mix with an analogous SM field with the same hypercharge. In the case of leptonlike (quarklike) PM, this is just the right-handed charged lepton, e_R (or quark, u_R, d_R). The first of these dark Higgs, H_1 carrying a vev, v_1, is thus chosen to be a real $SU(2)_I$ doublet which does this necessary task for us. To generate the Majorana mass term, which together with a Dirac mass for the DM, leads to a pseudo-Dirac realization with large mass splitting, δ, hence avoiding multiple direct and indirect search constraints as well as those arising from the CMB, a second dark Higgs, H_2, is required. Since our DM fermion lies in an $SU(2)_I$ doublet, the required dark Higgs in this case, H_2, is correspondingly an isotriplet with $Y_I/2 = 1$ whose $T_{3I} = 1$ member obtains the vev, v_2. These two vevs, which are assumed to be of comparable value, are then sufficient to completely break the remaining gauge symmetries leaving us with the two light dark gauge

[5]The much smaller Z exchange process contributes roughly equally to both left- and right-handed polarizations.

bosons, $Z_{1,2}$, having somewhat complex coupling structures to dark sector fields. However, they only interact with SM fields via KM, and thus, their couplings only depend only upon ϵ and the mixing angle, ϕ, needed to diagonalize the $W_{3I} - B_I$ mass-squared matrix. Demanding that the corresponding mass-squared parameters appearing in this matrix be real was found to lead to an additional constraint on the ratio of the $SU(2)_I$ and $U(1)_{Y_I}$ gauge couplings, g_{Y_I}/g_I, which leads to some important phenomenological implications.

The fact that both Z_i will contribute to the DM relic density calculation (which happens via coannihilation) was found to open up newly favored regions of model parameter space for the ratio of the DM to Z_1 masses due to the (mostly) constructive interference between the two exchanges. This region was previously found to require large and so disfavored values of $g_I\epsilon$ in order to overcome the relative suppression associated with the χ_2 thermal distribution. Now it is found that this suppression can be overcome without such large couplings when the masses of both Z_i's are not too dissimilar, $\lambda_R^2 = M_2^2/M_1^2 \lesssim 4$–5, with this constructive interference naturally occurring in this parameter regime due to the consistency constraints within the setup. However, the size of this newly allowed region is also found to depend to some extent upon the assumed values of $\chi_2 - \chi_1$ DM relative mass spitting, δ, with smaller values remaining preferred. These same interference effects were also shown to lead to O(1) or larger modifications to the anticipated χ_2 decay lifetime.

The new particles in this model were found to be more difficult to access than in the SM-like setup for several reasons, the most important being that the PM in the present case does not share an $SU(2)_I$ representation with any of the SM fermions. Thus, e.g., the heavy W_I can no longer be efficiently pair produced via the exchange of a heavy Z_I gauge boson (absent in the current setup) that couples with electroweak strength ($\sim g_I$) to the SM fermions, nor can the W_I be made via associated production together with PM as it does not couple to the SM fermions at tree level. The W_I can be pair produced via KM suppressed couplings, but these cross sections are quite small resulting in poor search reaches. The PM itself can, of course, be pair produced via SM interactions which, in the leptonlike case, occurs as usual via s-channel γ and Z exchanges and will lead to the sleptonlike, opposites-sign dilepton + MET signature at colliders. Existing limits from the LHC are rather poor for these states ($\sim$0.9 TeV) due to their small production cross sections (as they are SM isosinglets) and the correspondingly large W-pair backgrounds. More serious constraints are, of course, obtainable in the case that the PM is quarklike as then it is produced via the strong interactions, but in either case, future hadron colliders will be able to significantly extend the search reaches for these states. In the case of the scalar sector, the light dark Higgs fields at the $\lesssim$1 GeV scale are constrained from mixing with H(125) as they easily lead to a significant branching fraction for decay into the invisible final state. The heavier dark Higgs that remains from the real $SU(2)_I$ triplet responsible for the breaking of $SU(2)_I$, Σ^0, however, is allowed to have a somewhat larger mixing with the SM Higgs as the resulting effects are somewhat more subtle, reducing the H(125) width while leaving branching fractions unaltered. This was shown to lead to the possibility that this still small mixing may be sufficiently large to allow for substantial Σ^0 production at the LHC (for $\lesssim$1 TeV masses) as well as at future colliders by, e.g., gg-fusion via the conventional top quark loop while still avoiding unitarity and other experimental constraints on H(125) properties. Rare decays of the H(125) into the dark light gauge bosons plus a photon were also shown, however, to be highly suppressed. Finally, indirect signatures, e.g., $e^+e^- \to \gamma +$ "nothing" at future TeV-scale lepton colliders with $e^\pm$ beam polarization, as can arise from the production of the light dark gauge and Higgs bosons in the current setup, were explored but were unfortunately found to fall short of discovery unless the rather large SM background were known to a rather high precision. Other mechanisms to obtain information about this sector of the model will require further investigation as collider signatures for this setup are clearly challenging.

PM models generally require the existence of extended gauge and Higgs sectors which can lead to many additional new fields and other links between the SM and the dark sector beyond just the required KM. Hopefully, signals of the dark sector will soon be discovered.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

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