

$D_{(s)}(2S)$ and $D_{(s)}^*(2S)$ production in nonleptonic $B_{(s)}$ weak decaysZhi-Jie Sun,¹ Yong-Jin Sun,¹ Zhi-Qing Zhang^{2,*} You-Ya Yang,³ and Si-Yang Wang⁴¹*Bingtuan Xingxin Vocational and Technical College, Tiemenguan, Xinjiang, 841007, China*²*School of Physics and Advanced Energy, Henan University of Technology, Zhengzhou, Henan 450001, China*³*Physics Department, College of Physics and Optoelectronic Engineering, Jinan University, Guangzhou 510632, China*⁴*Huanghe Jiaotong University, Jiaozuo, Henan 454950, China*

(Received 18 November 2025; accepted 5 February 2026; published 26 February 2026)

Recently, many new excited states of heavy mesons have been discovered in recent experiments, including radially excited states. The production processes of these states from the $B_{(s)}$ meson have drawn significant interest. In this paper, we use the covariant light-front approach to study the nonleptonic $B_{(s)}$ meson decays to the first radially excited states $D_{(s)}(2S)$ and $D_{(s)}^*(2S)$. Our results reveal that many channels exhibit large branching ratios in the range 10^{-5} – 10^{-4} , even up to 10^{-3} for individual channels, which are detectable by current experiments. Our predictions for the decays $B_{(s)} \rightarrow D_{(s)}^*(2S)(\pi, \rho, K^{(*)})$ are larger than those given by the Bethe-Salpeter (BS) equation method, but agree well with the relativistic quark mode (RQM) and the relativistic independent quark model (RIQM) calculations. For comparison, we also present the branching ratios of the decays $B_{(s)} \rightarrow D_{(s)}^*(1S)(\pi, \rho, K^{(*)})$, which are comparable with other theoretical results and the data. Although the branching ratios of the decays $B_{(s)} \rightarrow D_{(s)}^*(1S)(\rho, K^*)$ are much larger than those of the decays $B_{(s)} \rightarrow D_{(s)}^*(2S)(\rho, K^*)$, the polarization properties between them are similar, that is, the longitudinal polarization fractions are dominant and can amount roughly to 90%.

DOI: [10.1103/h7p6-dkhd](https://doi.org/10.1103/h7p6-dkhd)**I. INTRODUCTION**

A wealth of experimental data on $B_{(s)}$ meson decays have been collected, allowing studies of both ground and radially excited final states. Due to their heavy masses, $B_{(s)}$ meson exhibit rich decay modes [1]. These decays offer an opportunity to search for the radially excited states and clarify their properties. For example, $\eta_c(2S)$ was discovered in exclusive decays $B \rightarrow KK_S K^- \pi^+$ [2] more than two decades ago. The well-determined radially excited charmonium states, such as $\psi(2S)$ and $\eta_c(2S)$, exhibit a significant production yield in the nonleptonic decays of B meson. One can find that many ratios $\frac{\text{Br}(B \rightarrow \psi(2S)M)}{\text{Br}(B \rightarrow J/\psi M)}$ with M being a light pseudoscalar or vector meson are around or even larger than 50% [1]. What happens if charmonium states are replaced by charmed mesons? Although many first radially excited (2S) state candidates of the ground

charmed mesons $D^{(*)}$ and $D_s^{(*)}$ have been found in experiments, they are still not well-determined. Usually, $D(2550)^0$ is classified as 2S state 2^1S_0 in the D meson family. It was first observed by BABAR in the $e^+e^- \rightarrow D^{*+}\pi^-$ channel in 2010 [3], and was confirmed by LHCb with significance using pp collision data. The assignment of $D(2^1S_0)$ to $D(2550)^0$ was supported by various quark models [4–6]. In addition, another unnatural state $D_J(2580)^0$ found by LHCb may also be a candidate for $D(2^1S_0)$, since $D_J(2580)^0$ and $D(2550)^0$ have similar resonance parameters. $D_{s0}(2590)^+$ with $J^P = 0^-$ newly observed by LHCb in the $D^+K^+\pi^-$ invariant mass spectrum of the decay $B^0 \rightarrow D^+D^+K^+\pi^-$ is suggested for a candidate of $D_s(2^1S_0)$ state [7]. Certainly, this suggestion has difficulties accounting for the mass and width of $D_{s0}(2590)^+$. Some theoretical works suggest that $D_{s0}(2590)^+$ may not be a pure $D_s(2^1S_0)$ state but rather has D^*K component [8,9]. Regarding the neutral radially excited 2S state $D(2^3S_1)$, there are several candidates, such as $D_1^*(2680)^0$, $D^*(2650)^0$ and $D_1^*(2600)^0$ observed by LHCb [10–12]. There is still some uncertainty in the measurements, and whether they are identical cannot be determined, but the spin-parity of $D_1^*(2680)^0$ and $D_1^*(2600)^0$ can be confirmed as 1^- . The mass of $D^*(2640)^\pm$ discovered by Delphi is

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consistent with the prediction of the charged state $D(2^3S_1)$ [13]. Unfortunately, this discovery has not been confirmed in any other experiments and its spin-parity numbers have not been identified up to now. The $D_{s1}^*(2700)^\pm$ can be assigned to the state $D_s(2^3S_1)$, which was first observed by BABAR [14] and whose quantum numbers J^P were determined as 1^- by Belle [15]. Certainly, except for the assignment of $D_s(2^3S_1)$ to $D_{s1}^*(2700)^\pm$, $D_s(1^3D_1)$ [13] and a mixture of $D_s(2^3S_1)$ and $D_s(1^3D_1)$ [16] have also been proposed. In a word, we will assume the $D(2550)^0$ as neutral state $D(2^1S_0)$, the $D_{s0}(2590)^\pm$ as $D_s(2^1S_0)$, the $D_1^*(2600)^0$, $D^*(2640)^\pm$ as $D(2^3S_1)$, the $D_{s1}^*(2700)^\pm$ as $D_s(2^3S_1)$, respectively. It is noticed that no candidate for the charged $D(2^1S_0)$ state has been observed in experiments up to now.

For the nonleptonic decays, the hadronic transition matrix element between the initial and final mesons is most crucial for the theoretical calculations. The factorization assumption based on the vacuum saturation approximation is often used to simplify the calculations. Specifically, the matrix elements are factorized into a product of two single matrix elements of currents, where one is parametrized by the decay constant of the emitted light meson and the other is represented by the transition form factor. As for the form factors, they can be extracted from data or relied on some nonperturbative and perturbative methods, such as the Bauer-Stech-Wirbel (BSW) model [17], the QCD light-cone sum

rules (LCSR) [18–23], the lattice QCD (LQCD) [24–26], the relativistic quark model (RQM) [27,28], the QCD sum rules (QCDSR) [29,30], the Bethe-Salpeter (BS) method [31], the covariant light front quark model (CLFQM) [32] and the perturbative QCD (PQCD) approach [33].

This paper is organized as follows. The formalism of the CLFQM, the hadronic matrix elements and the helicity amplitudes combined via form factors are listed in Sec. II. In addition to the numerical results for the $B_{(s)} \rightarrow D_{(s)}(1S, 2S)$ and $B_{(s)} \rightarrow D_{(s)}^*(1S, 2S)$ transition form factors, the branching ratios, the longitudinal (transverse parallel) polarization fractions $f_L(f_\parallel)$ of the corresponding decays are presented in Sec. III. Detailed comparisons with other theoretical values and relevant discussions are also included. The summary is presented in Sec. IV. Some specific rules when performing the p^- integration and the analytical expressions for the $B_{(s)} \rightarrow D_{(s)}(1S, 2S)$ and $B_{(s)} \rightarrow D_{(s)}^*(1S, 2S)$ transition form factors are collected in Appendixes A and B, respectively.

II. FORMALISM

A. The form factors

The Bauer-Stech-Wirbel (BSW) form factors for the $B_{(s)} \rightarrow D_{(s)}$ and $B_{(s)} \rightarrow D_{(s)}^*$ transitions are defined as follows,

$$\langle D_{(s)}(P'') | V_\mu | B_{(s)}(P') \rangle = \left(P_\mu - \frac{m_{B_{(s)}}^2 - m_{D_{(s)}}^2}{q^2} q_\mu \right) F_1^{B_{(s)}D_{(s)}}(q^2) + \frac{m_{B_{(s)}}^2 - m_{D_{(s)}}^2}{q^2} q_\mu \cdot F_0^{B_{(s)}D_{(s)}}(q^2), \quad (1)$$

$$\begin{aligned} \langle D_{(s)}^*(P'', \epsilon''^*) | V_\mu - A_\mu | B_{(s)}(P') \rangle = & -\epsilon_{\mu\nu\alpha\beta} \epsilon''^{\nu*} P^\alpha q^\beta \frac{V(q^2)}{m_{B_{(s)}} + m_{D_{(s)}^*}} - i \frac{2m_{D_{(s)}^*} \epsilon''^* \cdot P}{q^2} q_\mu A_0(q^2) \\ & - i \epsilon''^* \left(m_{B_{(s)}} + m_{D_{(s)}^*} \right) A_1(q^2) + i \frac{\epsilon''^* \cdot P}{m_{B_{(s)}} + m_{D_{(s)}^*}} P_\mu A_2(q^2) \\ & + i \frac{2m_{D_{(s)}^*} \epsilon''^* \cdot P}{q^2} q_\mu A_3(q^2), \end{aligned} \quad (2)$$

where $P = P' + P''$, $q = P' - P''$ and the convention $\epsilon_{0123} = 1$ is adopted.

In order to calculate the amplitudes of the transition form factors, we need the following Feynman rules for the meson-quark-antiquark vertices

$$i\Gamma'_p = H'_p \gamma_5, \quad (3)$$

$$i\Gamma''_p = \gamma_0 H''_p \gamma_5 \gamma_0, \quad (4)$$

$$i\Gamma''_V = i\gamma_0 H''_V \left[\gamma_\mu - \frac{1}{W'_V} (p'_1 - p_2)_\mu \right] \gamma_0, \quad (5)$$

where the last two formulas are for the final state mesons. The results of the lowest-order transition form factors could be obtained by calculating the right Feynman diagram in Fig. 1, where the left panel is for the decay amplitude of $B_{(s)}$ meson. In the covariant quark model, the treatment of transition form factors is relatively covariant throughout the calculation process, where the light-front coordinates of a momentum p are used $p = (p^-, p^+, p_\perp)$ with $p^\pm = p^0 \pm p_z$, $p^2 = p^+ p^- - p_\perp^2$. The incoming (outgoing) meson has the mass M' (M'') with the momentum $P' = p'_1 + p_2$ ($P'' = p''_1 + p_2$), where p''_1 and p_2 are the momenta of the quark and antiquark inside the incoming

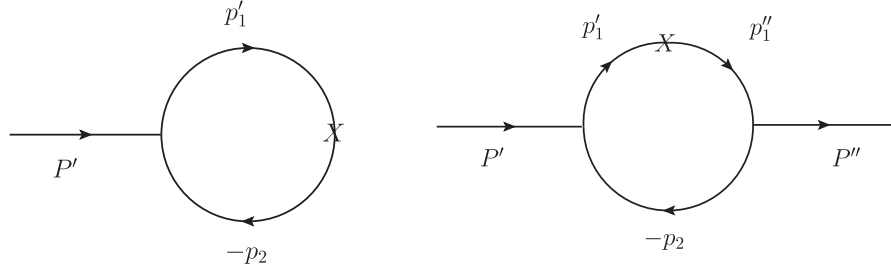


FIG. 1. Feynman diagrams for $B_{(s)}$ decay (left) and transition (right) amplitudes, where $P^{(n)}$ is the incoming (outgoing) meson momentum, $p_1^{(n)}$ is the quark momentum, p_2 is the antiquark momentum and X denotes the vector or axial-vector transition vertex.

(outgoing) meson with the mass $m_1^{(n)}$ and m_2 , respectively. Here we use the same notations as those in Refs. [34,35]. These momenta can be expressed in terms of the internal variables $(x_i, p'_\perp)$ as

$$p_{1,2}^+ = x_{1,2}P'^+, \quad p_{1,2\perp} = x_{1,2}P'_\perp \pm p'_\perp \quad (6)$$

with $x_1 + x_2 = 1$. Using these internal variables, we can define some quantities for the incoming meson which will be used in the following calculations

$$\begin{aligned} M_0^2 &= (e'_1 + e_2)^2 = \frac{p_\perp^2 + m_1^2}{x_1} + \frac{p_\perp^2 + m_2^2}{x_2}, \\ \tilde{M}'_0 &= \sqrt{M_0^2 - (m'_1 - m_2)^2}, \\ e_i^{(\prime)} &= \sqrt{m_i^{(\prime)2} + p_\perp^2 + p_z^2}, \\ p'_z &= \frac{x_2 M'_0}{2} - \frac{m_2^2 + p_\perp^2}{2x_2 M'_0}, \end{aligned} \quad (7)$$

where M'_0 is the kinetic invariant mass of the incoming meson and can be expressed as the energies of the quark and the antiquark $e_i^{(\prime)}$. It is similar to the case of the outgoing meson. For the general $P \rightarrow P$ transition, the amplitude for the lowest order is

$$\mathcal{B}_\mu^{PP} = -i^3 \frac{N_c}{(2\pi)^4} \int d^4 p'_1 \frac{H'_P H''_P}{N'_1 N''_1 N_2} S_\mu^{PP}, \quad (8)$$

where $N_1^{(n)} = p_1^{(n)2} - m_1^{(n)2}$, $N_2 = p_2^2 - m_2^2$ arise from the quark propagators, and the trace S_μ^{PP} can be obtained directly by using the Lorentz contraction,

$$S_\mu^{PP} = \text{Tr}[\gamma_5(\not{p}'_1 + m'_1)\gamma_\mu(\not{p}'_1 + m'_1)\gamma_5(-\not{p}_2 + m_2)], \quad (9)$$

where the analytical expression for S_μ^{PP} is listed in Appendix B. It is similar for the $P \rightarrow V$ transition amplitude,

$$\mathcal{B}_\mu^{PV} = -i^3 \frac{N_c}{(2\pi)^4} \int d^4 p'_1 \frac{H'_P (iH''_V)}{N'_1 N''_1 N_2} S_\mu^{PV} \varepsilon^{*\nu}, \quad (10)$$

where

$$\begin{aligned} S_{\mu\nu}^{PV} &= (S_V^{PV} - S_A^{PV})_{\mu\nu} \\ &= \text{Tr} \left[\left(\gamma_\nu - \frac{1}{W''_V} (p''_1 - p_2)_\nu \right) (\not{p}'_1 + m'_1) (\gamma_\mu - \gamma_\mu \gamma_5) \right. \\ &\quad \left. \times (\not{p}'_1 + m'_1) \gamma_5 (-\not{p}_2 + m_2) \right]. \end{aligned} \quad (11)$$

In practice, we use the light-front decomposition of the Feynman loop momentum and integrate out the minus component through the contour method. If the covariant vertex functions are not singular when performing integration, the transition amplitudes will pick up the singularities in the antiquark propagators. The integration then leads to

$$\begin{aligned} N_1^{(n)} &\rightarrow \hat{N}_1^{(n)} = x_1 (M'^{(n)2} - M_0'^{(n)2}), \\ H_M^{(n)} &\rightarrow h_M^{(n)} \\ W''_V &\rightarrow w''_V \\ \int \frac{d^4 p'_1}{N'_1 N''_1 N_2} H'_P H''_M S^{PM} &\rightarrow -i\pi \int \frac{dx_2 d^2 p'_\perp}{x_2 \hat{N}_1' \hat{N}_1''} h'_P h''_M \hat{S}^{PM}, \end{aligned} \quad (12)$$

where

$$M_0'^2 = \frac{p_\perp'^2 + m_1'^2}{x_1} + \frac{p_\perp'^2 + m_2^2}{x_2}, \quad (13)$$

with $p_\perp'' = p'_\perp - x_2 q_\perp$. The explicit forms of h''_M and w''_V are given by [34]

$$h''_P = h''_V = (M'^2 - M_0'^2) \sqrt{\frac{x_1 x_2}{N_c}} \frac{1}{\sqrt{2\tilde{M}'_0}} \varphi'', \quad (14)$$

$$w''_V = M_0'' + m'_1 + m_2, \quad (15)$$

where φ'' is the light-front momentum distribution amplitude for S -wave mesons,

$$\varphi'' = \varphi''(x_2, p''_\perp) = 4 \left(\frac{\pi}{\beta^2} \right)^{\frac{3}{4}} \sqrt{\frac{dp''_z}{dx_2}} \exp \left(-\frac{p''_z{}^2 + p''_\perp{}^2}{2\beta^2} \right). \quad (16)$$

where β is a phenomenological parameter and can be fixed by fitting the corresponding decay constant. Regarding the first radially excited charmed mesons $D_{(s)}^*(2S)$ and $D_{(s)}(2S)$, the distribution function is given as

$$\begin{aligned} \varphi''(2S) &= 4 \left(\frac{\pi}{\beta^2} \right)^{\frac{3}{4}} \sqrt{\frac{dp''_z}{dx_2}} \exp \left(-\frac{p''_z{}^2 + p''_\perp{}^2}{2\beta^2} \right) \\ &\times \frac{1}{\sqrt{6}} \left(-3 + 2 \frac{p''_z{}^2 + p''_\perp{}^2}{\beta^2} \right). \end{aligned} \quad (17)$$

Using the formulas provided above and taking the integration rules given in Refs [34,35], we obtain the expressions of the $B_{(s)} \rightarrow D_{(s)}(1S, 2S)$ and $B_{(s)} \rightarrow D_{(s)}^*(1S, 2S)$ transition form factors, which are listed in Appendix B.

B. Hadronic matrix elements

We present the formulas for the nonleptonic decays $B \rightarrow DM$ and $B \rightarrow D^*M^1$ with $M = \pi, K, \rho, K^*$. The effective Hamiltonian which describes such processes can be written as [36]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{uq} \{C_1 Q_1 + C_2 Q_2\} \quad (18)$$

where G_F is the Fermi coupling constant, $V_{cb}^* V_{uq}$ is the product of the CKM matrix elements with $q = s, d$, and $C_{1,2}$ are the Wilson coefficients. The local tree-level four-quark operators $Q_{1,2}$ are defined as

$$Q_1 = [\bar{c}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta] [\bar{q}_\beta \gamma^\mu (1 - \gamma_5) u_\alpha], \quad (19)$$

$$Q_2 = [\bar{c}_\alpha \gamma_\mu (1 - \gamma_5) b_\alpha] [\bar{q}_\beta \gamma^\mu (1 - \gamma_5) u_\beta], \quad (20)$$

where α and β are color indices. In the following calculations, the combination of Wilson coefficients $a_1 = C_2 + C_1/3$ will be used. Using these form factors, we can obtain the partial widths for our considered nonleptonic decays, which are written as

$$\begin{aligned} \Gamma(B \rightarrow DP) &= \frac{G_F^2 (m_B^2 - m_D^2)^2 |\vec{p}_c|}{16\pi m_B^2} |V_{uq}^* V_{cb}|^2 |a_1|^2 f_P^2 F_0^2(m_P^2), \end{aligned} \quad (21)$$

$$\Gamma(B \rightarrow D^*P) = \frac{G_F^2 |\vec{p}_c|^3}{4\pi} |V_{uq}^* V_{cb}|^2 |a_1|^2 f_P^2 A_0^2(m_P^2), \quad (22)$$

$$\Gamma(B \rightarrow DV) = \frac{G_F^2 |\vec{p}_c|^3}{4\pi} |V_{uq}^* V_{cb}|^2 |a_1|^2 f_V^2 F_+^2(m_V^2), \quad (23)$$

$$\begin{aligned} \Gamma(B \rightarrow D^*V) &= \frac{G_F^2 |\vec{p}_c|}{16\pi m_B^2} |V_{uq}^* V_{cb}|^2 (|H_0|^2 + |H_+|^2 + |H_-|^2), \end{aligned} \quad (24)$$

where $\vec{p}_c$ is the momentum of either of the two final state mesons in the B rest frame and $H_{0,\pm}$ are the helicity amplitudes,

$$\begin{aligned} H_0 &= \frac{if_V a_1}{2m_{D^*}} \left[(m_B^2 - m_{D^*}^2 - m_V^2)(m_B + m_{D^*}) A_1^{BD^*}(m_V^2) \right. \\ &\quad \left. - \frac{4m_B^2 p_c^2}{m_B + m_{D^*}} A_2^{BD^*}(m_V^2) \right], \end{aligned} \quad (25)$$

$$\begin{aligned} H_\pm &= if_V m_V a_1 \left[-(m_B + m_{D^*}) A_1^{BD^*}(m_V^2) \right. \\ &\quad \left. \mp \frac{2m_B p_c}{m_B + m_{D^*}} V^{BD^*}(m_V^2) \right]. \end{aligned} \quad (26)$$

The polarization fractions are defined as

$$f_{L,\parallel,\perp} = \frac{H_{0,\parallel,\perp}}{H_0 + H_\parallel + H_\perp} \quad (27)$$

where $H_\parallel$ and $H_\perp$ are parallel and perpendicular amplitudes, respectively, and can be obtained through $H_{\parallel,\perp} = \frac{(H_\pm \pm H_\mp)}{\sqrt{2}}$.

III. NUMERICAL RESULTS AND DISCUSSIONS

A. Transition form factors

The input parameters, including the masses of the initial and final mesons (quarks), the CKM matrix elements, the lifetimes of the decaying mesons, the decay constants of related mesons and the shape parameters fitted by the decay constants are listed in Table I. It is noticed that in the absence of the observed data on the masses and the decay constants of these first radially excited charmed and strange-charmed mesons, we take the corresponding predictions from established theoretical approaches [37–41]. Based on the input parameters listed in Table I, the numerical results of the transition form factors at $q^2 = 0$ can be obtained, as shown in Tables II and III.

All computations are carried out within the reference frame $q^+ = 0$, where the form factors can only be obtained at spacelike momentum transfers $q^2 = -q_\perp^2 \leq 0$. We need to know the form factors in the timelike region for the physical decay processes. Here we use the following double-pole approximation to parametrize the form factors in the spacelike region and then extend to the timelike region,

¹It is similar for the decays $B_s \rightarrow D_s M$ and $B_s \rightarrow D_s^* M$.

TABLE I. The values of the input parameters [1,11,37–41].

Mass (GeV)	$m_b = 4.8$	$m_c = 1.4$	$m_s = 0.37$	$m_{u,d} = 0.25$
	$m_\pi = 0.140$	$m_K = 0.494$	$m_\rho = 0.775$	$m_{K^*} = 0.892$
	$m_D = 1.8695$	$m_{D(2S)} = 2.549$	$m_{D_s} = 1.96835$	$m_{D_s(2S)} = 2.591$
	$m_{D_s^\pm} = 2.1066$	$m_{D^{*0}} = 2.0068$	$m_{D^{*\pm}} = 2.0102$	$m_B = 5.279$
	$m_{D_s^\pm(2S)} = 2.732$	$m_{D^{*0}(2S)} = 2.627$	$m_{D^{*\pm}(2S)} = 2.637$	$m_{B_s^0} = 5.367$
CKM	$V_{cb} = 0.0408 \pm 0.0014$	$V_{us} = 0.2243 \pm 0.0008$	$V_{ud} = 0.97373 \pm 0.00031$	
Decay constants (GeV)	$f_\pi = 0.132$	$f_K = 0.16$	$f_\rho = 0.209$	
	$f_{K^*} = 0.217$	$f_{D_s} = 0.2499 \pm 0.0005$	$f_D = 0.2058 \pm 0.0089$	
	$f_{D_s(2S)} = 0.161 \pm 0.020$	$f_{D(2S)} = 0.117 \pm 0.020$	$f_{D_s^\pm} = 0.272^{+0.039}_{-0.038}$	
	$f_{D^{*0}} = 0.339 \pm 0.022$	$f_{D^{*\pm}} = 0.341 \pm 0.023$	$f_{D_s^\pm(2S)} = 0.312 \pm 0.017$	
	$f_{D^{*0}(2S)} = 0.289 \pm 0.016$	$f_{D^{*\pm}(2S)} = 0.290 \pm 0.016$	$f_B = 0.190 \pm 0.025$	
	$f_{B_s} = 0.253 \pm 0.008 \pm 0.007$			
Shape parameters (GeV)	$\beta'_{D_s^\pm(1S)} = 0.436^{+0.039}_{-0.040}$	$\beta'_{D^{*0}(1S)} = 0.500^{+0.140}_{-0.187}$	$\beta'_D = 0.466^{+0.022}_{-0.021}$	
	$\beta'_{D_s^\pm(2S)} = 0.473^{+0.041}_{-0.041}$	$\beta'_{D^{*0}(2S)} = 0.456^{+0.004}_{-0.003}$	$\beta'_{D_s} = 0.5416^{+0.001}_{-0.002}$	
	$\beta'_{D^{*\pm}(1S)} = 0.502^{+0.041}_{-0.041}$	$\beta'_{D^{*\pm}(2S)} = 0.453^{+0.004}_{-0.003}$	$\beta'_{B_s^0} = 0.626^{+0.045}_{-0.045}$	
	$\beta'_{D(2S)} = 0.297^{+0.041}_{-0.041}$	$\beta'_{D_s(2S)} = 0.422^{+0.026}_{-0.025}$	$\beta'_B = 0.555^{+0.060}_{-0.060}$	
Mean life ($10^{-12}s$)	$\tau_{B^0} = 1.517 \pm 0.004$	$\tau_{B_s^0} = 1.516 \pm 0.006$	$\tau_{B^\pm} = 1.638 \pm 0.004$	

TABLE II. The $B_{(s)} \rightarrow D_{(s)}(1S, 2S)$ transition form factors in the CLFQM. The uncertainties are from the decay constants of $B_{(s)}$ and final state mesons.

F	F(0)	$F(q_{\max}^2)$	a	b
F_1^{BD}	$0.66^{+0.00+0.01}_{-0.01-0.01}$	$0.81^{+0.01+0.01}_{-0.00-0.01}$	$0.80^{+0.01+0.04}_{-0.02-0.04}$	$0.86^{+0.01+0.02}_{-0.01-0.02}$
F_0^{BD}	$0.66^{+0.00+0.01}_{-0.01-0.01}$	$0.70^{+0.02+0.01}_{-0.03-0.01}$	$0.46^{+0.04+0.00}_{-0.02-0.01}$	$0.77^{+0.01+0.05}_{-0.01-0.05}$
$F_1^{BD(2S)}$	$0.26^{+0.01+0.01}_{-0.02-0.02}$	$0.34^{+0.02+0.01}_{-0.03-0.03}$	$0.99^{+0.04+0.16}_{-0.10-0.18}$	$0.66^{+0.07+0.17}_{-0.12-0.15}$
$F_0^{BD(2S)}$	$0.26^{+0.01+0.02}_{-0.01-0.02}$	$0.32^{+0.03+0.02}_{-0.00-0.04}$	$0.65^{+0.03+0.05}_{-0.01-0.04}$	$-0.24^{+0.02+0.01}_{-0.03-0.03}$
$F_1^{B_s D_s}$	$0.67^{+0.00+0.01}_{-0.00-0.01}$	$0.81^{+0.00+0.00}_{-0.01-0.01}$	$0.82^{+0.00+0.02}_{-0.01-0.02}$	$0.96^{+0.01+0.03}_{-0.02-0.03}$
$F_0^{B_s D_s}$	$0.67^{+0.00+0.01}_{-0.00-0.01}$	$0.71^{+0.01+0.01}_{-0.02-0.01}$	$0.48^{+0.00+0.08}_{-0.01-0.08}$	$0.85^{+0.02+0.06}_{-0.02-0.06}$
$F_1^{B_s D_s(2S)}$	$0.26^{+0.02+0.02}_{-0.02-0.02}$	$0.30^{+0.02+0.01}_{-0.02-0.02}$	$0.58^{+0.12+0.05}_{-0.16-0.01}$	$0.33^{+0.11+0.04}_{-0.13-0.02}$
$F_0^{B_s D_s(2S)}$	$0.26^{+0.01+0.01}_{-0.02-0.02}$	$0.25^{+0.02+0.01}_{-0.02-0.02}$	$-0.10^{+0.22+0.22}_{-0.14-0.26}$	$-0.07^{+0.13+0.18}_{-0.21-0.08}$

$$F(q^2) = \frac{F(0)}{1 - aq^2/m^2 + bq^4/m^4}, \quad (28)$$

where m represents the initial meson mass and $F(q^2)$ denotes the different form factors F_1, F_0, V, A_0, A_1 , and A_2 . The values of a and b can be obtained by performing a 3-parameter fit to the form factors in the range $-15 \text{ GeV}^2 \leq q^2 \leq 0$, which are collected in Tables II and III. The uncertainties arise from the decay constants of the initial and the final mesons.

An estimate of the $SU(3)_F$ breaking effects can be obtained by studying the form factor ratios between the transitions $B_s \rightarrow D_s^{(*)}(1S, 2S)$ and $B \rightarrow D^{(*)}(1S, 2S)$, and the results are listed as

$$\frac{F_1(B_s \rightarrow D_s)}{F_1(B \rightarrow D)} = 1.02, \quad \frac{F_1(B_s \rightarrow D_s(2S))}{F_1(B \rightarrow D(2S))} = 0.76, \quad (29)$$

TABLE III. The $B \rightarrow D^*(1S, 2S)$ and $B_s \rightarrow D_s^*(1S, 2S)$ transition form factors in the CLFQM. The uncertainties are from the decay constants of $B_{(s)}$ and final state mesons.

F	F(0)	$F(q_{\max}^2)$	a	b
V^{BD^*}	$0.77^{+0.00+0.01}_{-0.00-0.01}$	$0.94^{+0.00+0.01}_{-0.00-0.01}$	$0.78^{+0.00+0.03}_{-0.01-0.03}$	$0.82^{+0.04+0.14}_{-0.08-0.16}$
$A_0^{BD^*}$	$0.75^{+0.00+0.06}_{-0.01-0.02}$	$0.79^{+0.00+0.05}_{-0.02-0.02}$	$0.17^{+0.01+0.01}_{-0.01-0.00}$	$0.12^{+0.07+0.02}_{-0.06-0.03}$
$A_1^{BD^*}$	$0.67^{+0.00+0.01}_{-0.01-0.11}$	$0.76^{+0.00+0.01}_{-0.01-0.01}$	$0.38^{+0.01+0.01}_{-0.01-0.01}$	$0.19^{+0.05+0.09}_{-0.05-0.09}$
$A_2^{BD^*}$	$0.58^{+0.00+0.00}_{-0.01-0.02}$	$0.69^{+0.01+0.01}_{-0.01-0.02}$	$0.68^{+0.02+0.00}_{-0.02-0.00}$	$0.66^{+0.25+0.10}_{-0.12-0.16}$
$V^{BD^*(2S)}$	$0.19^{+0.05+0.01}_{-0.06-0.01}$	$0.19^{+0.01+0.03}_{-0.04-0.01}$	$0.11^{+0.05+0.03}_{-0.06-0.05}$	$0.34^{+0.00+0.03}_{-0.04-0.06}$
$A_0^{BD^*(2S)}$	$0.27^{+0.04+0.01}_{-0.05-0.00}$	$0.28^{+0.00+0.01}_{-0.02-0.00}$	$0.24^{+0.05+0.00}_{-0.06-0.00}$	$-0.07^{+0.15+0.03}_{-0.29-0.06}$
$A_1^{BD^*(2S)}$	$0.16^{+0.04+0.01}_{-0.05-0.00}$	$0.15^{+0.05+0.02}_{-0.04-0.02}$	$-0.23^{+0.04+0.00}_{-0.04-0.00}$	$0.17^{+0.03+0.01}_{-0.01-0.01}$
$A_2^{BD^*(2S)}$	$-0.05^{+0.04+0.01}_{-0.04-0.00}$	$0.01^{+0.00+0.00}_{-0.00-0.00}$	$-1.10^{+0.02+0.01}_{-0.01-0.01}$	$-0.63^{+0.02+0.01}_{-0.01-0.00}$
$V^{B_s D_s^*}$	$0.78^{+0.01+0.01}_{-0.01-0.01}$	$0.87^{+0.00+0.00}_{-0.00-0.01}$	$0.86^{+0.01+0.04}_{-0.01-0.04}$	$1.11^{+0.01+0.02}_{-0.01-0.02}$
$A_0^{B_s D_s^*}$	$0.74^{+0.01+0.01}_{-0.01-0.01}$	$0.69^{+0.01+0.01}_{-0.01-0.00}$	$0.23^{+0.01+0.01}_{-0.01-0.01}$	$0.21^{+0.00+0.01}_{-0.00-0.01}$
$A_1^{B_s D_s^*}$	$0.66^{+0.01+0.01}_{-0.02-0.02}$	$0.95^{+0.00+0.00}_{-0.00-0.01}$	$0.81^{+0.01+0.02}_{-0.01-0.02}$	$0.93^{+0.01+0.01}_{-0.01-0.01}$
$A_2^{B_s D_s^*}$	$0.57^{+0.00+0.00}_{-0.01-0.00}$	$0.68^{+0.01+0.00}_{-0.01-0.00}$	$0.80^{+0.00+0.03}_{-0.01-0.03}$	$0.96^{+0.01+0.03}_{-0.01-0.02}$
$V^{B_s D_s^*(2S)}$	$0.26^{+0.03+0.04}_{-0.03-0.04}$	$0.28^{+0.00+0.02}_{-0.09-0.10}$	$0.25^{+0.02+0.04}_{-0.01-0.04}$	$0.30^{+0.03+0.04}_{-0.01-0.00}$
$A_0^{B_s D_s^*(2S)}$	$0.31^{+0.02+0.01}_{-0.03-0.02}$	$0.33^{+0.00+0.08}_{-0.01-0.07}$	$0.21^{+0.01+0.03}_{-0.05-0.00}$	$-0.09^{+0.04+0.10}_{-0.02-0.14}$
$A_1^{B_s D_s^*(2S)}$	$0.21^{+0.02+0.03}_{-0.03-0.03}$	$0.20^{+0.00+0.01}_{-0.07-0.06}$	$-0.16^{+0.20+0.04}_{-0.17-0.02}$	$0.12^{+0.00+0.12}_{-0.01-0.07}$
$A_2^{B_s D_s^*(2S)}$	$-0.01^{+0.02+0.06}_{-0.02-0.06}$	$0.01^{+0.00+0.03}_{-0.01-0.01}$	$-4.03^{+0.51+0.75}_{-0.47-1.38}$	$4.31^{+0.00+2.13}_{-0.00-2.44}$

$$\frac{V(B_s \rightarrow D_s^*)}{V(B \rightarrow D^*)} = 1.01, \quad \frac{V(B_s \rightarrow D_s^*(2S))}{V(B \rightarrow D^*(2S))} = 1.37, \quad (30)$$

$$\frac{A_1(B_s \rightarrow D_s^*)}{A_1(B \rightarrow D^*)} = 0.99, \quad \frac{A_1(B_s \rightarrow D_s^*(2S))}{A_1(B \rightarrow D^*(2S))} = 1.31, \quad (31)$$

$$\frac{A_2(B_s \rightarrow D_s^*)}{A_2(B \rightarrow D^*)} = 0.98, \quad \frac{A_2(B_s \rightarrow D_s^*(2S))}{A_2(B \rightarrow D^*(2S))} = 0.20. \quad (32)$$

The significant $SU(3)_F$ breaking effects are connected with the unique dynamic characteristics of the radially excited states. The light-cone distribution amplitudes (LCDAs) of the radially excited states $D^*(2S)$ and $D_s^*(2S)$ have distinct node structures compared to those of the corresponding ground states. For the $B_{(s)} \rightarrow D_{(s)}^*(2S)$ transitions, the quark $u/d(s)$ with a mass of 0.25(0.37) GeV causes different node positions, which lead to the different results in the overlap integral of the wave functions of the initial and final mesons. So the ratios between the $B_s \rightarrow D_s^*(2S)$ and $B \rightarrow D^*(2S)$ transition form factors usually reflect a more pronounced $SU(3)_F$ symmetry breaking effects compared to those between the $B_s \rightarrow D_s^*$ and $B \rightarrow D^*$ transition form factors. But the ratio $\frac{A_2(B_s \rightarrow D_s^*(2S))}{A_2(B \rightarrow D^*(2S))} = 0.20$ is an exception.

It is noticed that the values of $A_2(B \rightarrow D^*(2S))$ and $A_2(B_s \rightarrow D_s^*(2S))$ are very small, in particular, the latter is extremely close to 0, as shown in Fig. 3(d). A tiny deviation can cause a significant change in the ratio. Furthermore, the influence of the form factor A_2 on the branching ratio (physical observable) is very small. In a word, the ratio $\frac{A_2(B_s \rightarrow D_s^*(2S))}{A_2(B \rightarrow D^*(2S))}$ is not suitable for measuring the $SU(3)_F$ symmetry breaking effects.

TABLE IV. The $B \rightarrow D$ transition form factor. As a comparison, we also present other theoretical results.

	F_1^{BD}
This work	$0.66^{+0.01}_{-0.01}$
LCSR [19]	$0.648^{+0.067}_{-0.063}$
MC [19] ^a	0.652 ± 0.023
PQCD [45]	$0.52^{+0.12}_{-0.10}$
LCSR [20]	$0.653^{+0.004}_{-0.011}$
LCSR [21]	$0.673^{+0.038}_{-0.041}$
LCSR [22]	0.552 ± 0.216
LCSR [23]	$0.659^{+0.029}_{-0.032}$
LQCD [24]	0.658 ± 0.017
LQCD [25]	0.672 ± 0.027
HPQCD [26]	0.664 ± 0.034
RQM [28]	0.696

^aRefers to the Monte Carlo method.

TABLE V. The $B \rightarrow D^*$ and $B_s \rightarrow D_s^{(*)}$ transition form factors at $q^2 = 0$, together with other theoretical results for comparison.

Transition	Reference	$F_0(0)$	$V(0)$	$A_0(0)$	$A_1(0)$	$A_2(0)$
$B \rightarrow D^*$	This work	...	0.77	0.75	0.67	0.58
	RQM [28]	...	0.92	0.81	0.73	0.63
	CLFQM [48]	...	0.77	0.68	0.60	0.61
$B_s \rightarrow D_s^{(*)}$	This work	0.67	0.78	0.74	0.66	0.57
	RQM [27]	0.74	0.95	0.67	0.70	0.75
	CLFQM [48]	0.67	0.75	0.66	0.62	0.57
	QCDSR [42]	0.70	0.63	0.52	0.62	0.75
	BS [47]	0.57	0.57	...	0.65	0.67
	LCSR [18]	0.86	...	...	...	...
	LFQM [49]	...	0.74	0.63	0.61	0.59
	PQCD [44]	0.52	0.64	0.48	0.50	0.53
	PQCD [43]	0.55	0.62	0.47	0.49	0.52
	CCQM [46]	0.77	0.74	0.72	0.68	0.63
	RQM [28]	0.66	0.93	0.63	0.67	0.72

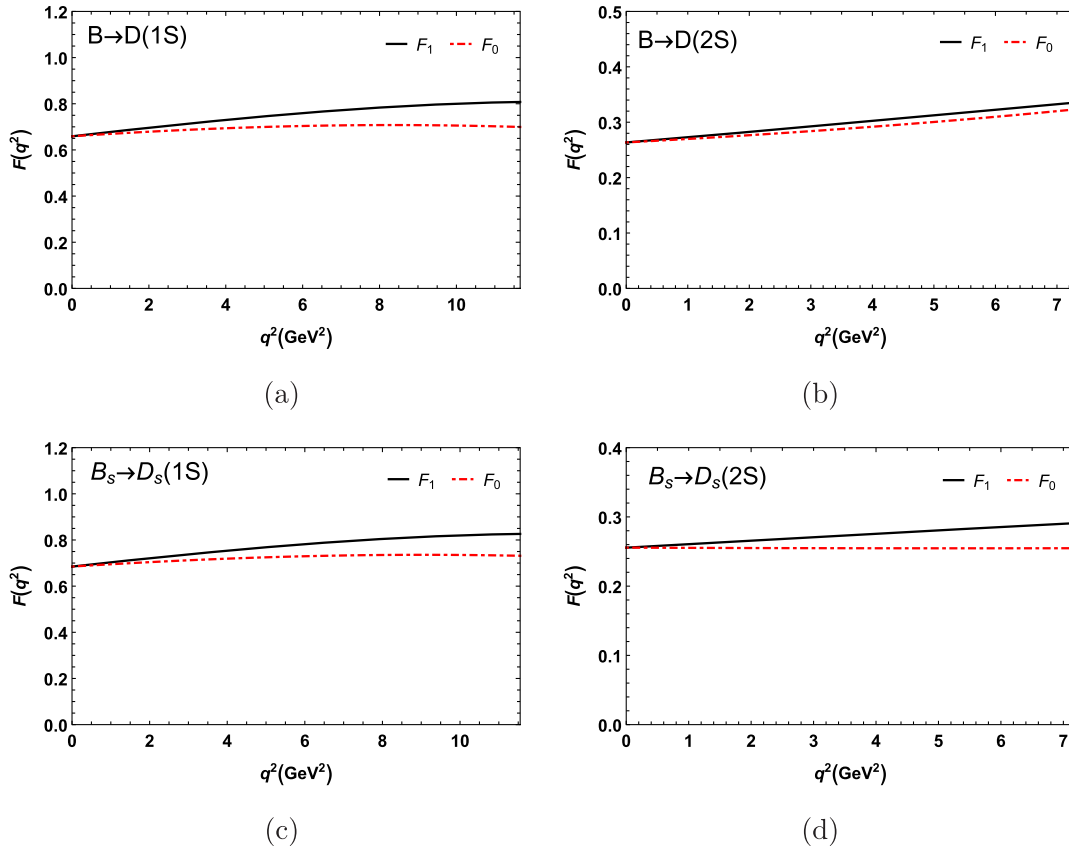
In Tables IV and V, we compare the form factors of the transitions $B_{(s)} \rightarrow D_{(s)}, D_{(s)}^*$ at maximum recoil ($q^2 = 0$) with those obtained within the QCD LCSR [18–23], LQCD [24–26], RQM [27,28], QCDSR [42], PQCD [43–45], CCQM [46], BS equation [47] and the previous CLFQM [48,49]. Upon comparison, we find that our predictions for

the form factors of the transitions $B_{(s)} \rightarrow D_{(s)}, D_{(s)}^*$ are consistent with most of the other theoretical results, especially for the values of F_1^{BD} . In a word, these dramatically different theoretical calculations lead to rather close values of the transition form factors. Unfortunately, research on the $B_{(s)} \rightarrow D_{(s)}(2S), D_{(s)}^*(2S)$ transition form factors is still very limited, and we look forward to more studies on them in the future.

We plot the q^2 -dependence of the $B_{(s)} \rightarrow D_{(s)}(1S, 2S)$ and $B_{(s)} \rightarrow D_{(s)}^*(1S, 2S)$ transition form factors in Figs. 2 and 3. In Fig. 2, the form factors of the transitions $B_{(s)} \rightarrow D_{(s)}$ are approximately 2.5 times larger than those of the transitions $B_{(s)} \rightarrow D_{(s)}(2S)$. The difference in the form factors between the transitions $B_{(s)} \rightarrow D_{(s)}^*$ and $B_{(s)} \rightarrow D_{(s)}^*(2S)$ is also very obvious, as shown in Fig. 3.

B. Nonleptonic decays

In this section, using the form factors obtained in the preceding section, we give the branching ratios of the nonleptonic decays $B \rightarrow D(1S, 2S)(\pi, \rho, K^{(*)})$, $B_s \rightarrow D_s(1S, 2S)(\pi, \rho, K^{(*)})$, $B \rightarrow D^*(1S, 2S)(\pi, \rho, K^{(*)})$ and $B_s \rightarrow D_s^*(1S, 2S)(\pi, \rho, K^{(*)})$, which are listed in Tables VI–IX, respectively. Here, uncertainties arise from the $B_{(s)}$ meson lifetime, the decay constants of initial and


 FIG. 2. Form factors $F_1(q^2)$ and $F_0(q^2)$ of the transitions $B \rightarrow D(1S)$ (a), $B \rightarrow D(2S)$ (b), $B_s \rightarrow D_s(1S)$ (c), and $B_s \rightarrow D_s(2S)$ (d).

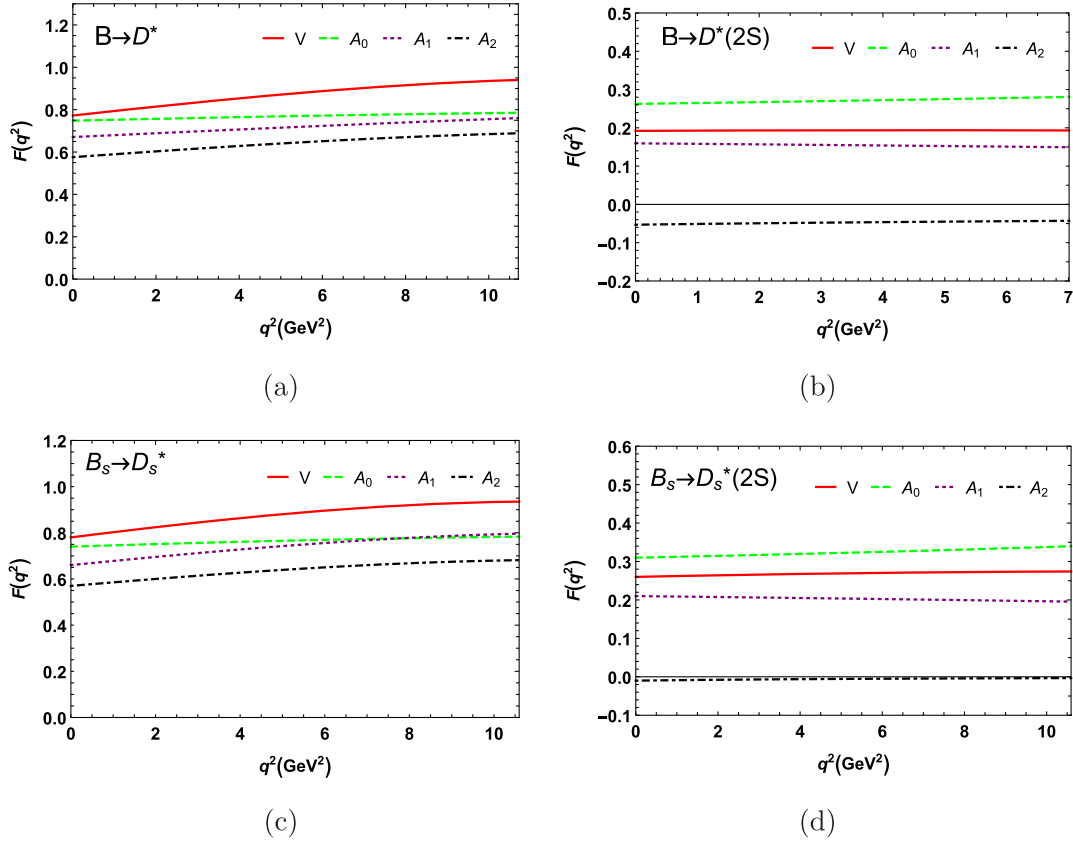


FIG. 3. Form factors $V(q^2)$, $A_0(q^2)$, $A_1(q^2)$, and $A_2(q^2)$ of the transitions $B \rightarrow D^*(1S)$ (a), $B \rightarrow D^*(2S)$ (b), $B_s \rightarrow D_s^*(1S)$ (c), and $B_s \rightarrow D_s^*(2S)$ (d).

final state mesons, respectively. We compare our predictions with the results from the LCSRs [18], the RQM [27], the PQCD approach [33], the QCDSRs [42], the BS equation method [47,50], the three-point QCDSRs [51], the QCDF approach [52,53] and the relativistic independent quark model (RIQM) [54]. The available experimental data [1] are also included. We adopt the Wilson coefficient $a_1 = 1.1$ in the calculations. The following are some comments.

- (1) From Table VI, one can find that the branching ratios of the decays $B \rightarrow D(1S)(\pi, \rho, K^{(*)})$ fall within the range of 10^{-4} – 10^{-3} , which are about one order larger than those of the corresponding decays $B \rightarrow D(2S)(\pi, \rho, K^{(*)})$. The small branching ratios for the latter are related to the node structure of the wave function of $D(2S)$ meson. When calculated the overlap integral of the wave functions, contributions from the positive and negative parts of the $D(2S)$ meson wave function cancel each other out, which result in the small decay branching fractions.
- (2) The branching ratios of the neutral decays $\bar{B}_{(s)}^0 \rightarrow D_{(s)}(1S)^+(\pi, \rho, K^{(*)})^-$ are larger than the experimental data [1], as shown in Tables VI and VII. A similar situation also occurs in the calculations from the QCDF approach in Refs. [52,53], where

although the contributions from more comprehensive amplitudes [53] and the next-to-next-to-leading-order (NNLO) vertex corrections [52] were considered, the predictions for the branching ratios of the decays $\bar{B}_{(s)}^0 \rightarrow D_{(s)}(1S)^+(\pi, \rho, K^{(*)})^-$ still exceed the data. By comparison, the results from the PQCD approach given in [33] are closer to the experimental data for these neutral $B_{(s)}$ decays, where except for the factorizable emission diagram amplitude, the non-factorizable emission diagram amplitude may also give a significant contribution. Furthermore, these two kinds of amplitudes partially cancel each other. It is strange that the difference between our predictions and experimental data for the charged decays $B^- \rightarrow D(1S)^0(\pi, \rho, K^{(*)})^-$ is not significant.

- (3) Our predictions for the branching ratios of the decays with ground state $D_{(s)}$ or $D_{(s)}^*$ meson involved are comparable with those given by the BS method [50]. While if replaced the ground state charmed meson with the radially excited one in these decays, the predicted results between these two approaches show significant discrepancies. For example, $\text{Br}(B \rightarrow D(2S)(\pi, \rho, K^{(*)}))$ are about one order larger than those given by the BS method [50]. Such significant difference can be clarified by future experiments.

TABLE VI. The branching ratios of the decays $B \rightarrow D(1S, 2S)(\pi, \rho, K^{(*)})$.

Modes	This work	[50]	[33]	[52]	[53]	Exp.[1]	Unit
$\bar{B}^0 \rightarrow D^+(1S)\pi^-$	$4.36^{+0.02+0.01+0.12}_{-0.01-0.08-0.12}$	3.24	2.69	3.93	3.582	2.51	(10^{-3})
$\bar{B}^0 \rightarrow D^+(1S)K^-$	$0.34^{+0.00+0.00+0.01}_{-0.00-0.01-0.01}$	0.245	0.245	0.245	0.245	0.245	
$\bar{B}^0 \rightarrow D^+(1S)\rho^-$	$10.09^{+0.04+0.02+0.27}_{-0.01-0.02-0.27}$	7.91	6.96	10.42	...	7.6	
$\bar{B}^0 \rightarrow D^+(1S)K^{*-}$	$0.56^{+0.00+0.00+0.01}_{-0.00-0.01-0.02}$	0.431	0.407	0.525	...	0.45	
$\bar{B}^0 \rightarrow D^+(2S)\pi^-$	$4.76^{+0.01+0.48+0.33}_{-0.01-0.62-0.57}$	0.458	...	...	...	...	(10^{-4})
$\bar{B}^0 \rightarrow D^+(2S)K^-$	$0.37^{+0.00+0.04+0.04}_{-0.00-0.05-0.03}$	0.034	...	...	...	...	
$\bar{B}^0 \rightarrow D^+(2S)\rho^-$	$10.54^{+0.03+1.06+0.73}_{-0.03-1.37-1.26}$	1.03	...	...	...	...	
$\bar{B}^0 \rightarrow D^+(2S)K^{*-}$	$0.58^{+0.00+0.06+0.04}_{-0.00-0.08-0.07}$	0.0557	...	...	...	...	
$B^- \rightarrow D^0(1S)\pi^-$	$4.71^{+0.01+0.00+0.12}_{-0.01-0.01-0.13}$	3.49	5.11	...	...	4.61	(10^{-3})
$B^- \rightarrow D^0(1S)K^-$	$0.37^{+0.00+0.00+0.01}_{-0.00-0.01-0.01}$	0.264	0.40	...	...	0.364	
$B^- \rightarrow D^0(1S)\rho^-$	$10.88^{+0.03+0.00+0.27}_{-0.03-0.02-0.31}$	8.40	11.3	...	...	9.7	
$B^- \rightarrow D^0(1S)K^{*-}$	$0.60^{+0.00+0.00+0.02}_{-0.00-0.01-0.02}$	0.466	0.649	...	...	0.53	
$B^- \rightarrow D^0(2S)\pi^-$	$5.14^{+0.01+0.67+0.36}_{-0.01-0.67-0.61}$	0.488	...	...	...	...	(10^{-4})
$B^- \rightarrow D^0(2S)K^-$	$0.40^{+0.00+0.05+0.03}_{-0.00-0.05-0.05}$	0.0362	...	...	...	...	
$B^- \rightarrow D^0(2S)\rho^-$	$11.36^{+0.03+1.48+0.79}_{-0.03-1.48-1.35}$	1.09	...	...	...	...	
$B^- \rightarrow D^0(2S)K^{*-}$	$0.62^{+0.00+0.08+0.04}_{-0.00-0.08-0.07}$	0.0595	...	...	...	...	

 TABLE VII. The branching ratios of the decays $\bar{B}_s^0 \rightarrow D_s^+(1S, 2S)(\pi, \rho, K^{(*)})^-$, together with other theoretical results and data for comparison.

Modes	This work	[50]	[27]	[55]	[42]	[47]	[33]	[51]	[18]	[53]	[52]	Exp.[1]	Unit
$\bar{B}_s^0 \rightarrow D_s^+(1S)\pi^-$	$4.56^{+0.01+0.45+0.17}_{-0.01-0.31-0.15}$	2.92	3.5	5.3	5	2.7	2.13	1.42	1.7	4.717	4.39	2.98 ± 0.14	(10^{-3})
$\bar{B}_s^0 \rightarrow D_s^+(1S)K^-$	$0.36^{+0.00+0.04+0.01}_{-0.00-0.02-0.01}$	0.221	0.28	0.4	0.4	0.21	0.171	0.103	0.13	0.3575	0.334	0.225 ± 0.012	
$\bar{B}_s^0 \rightarrow D_s^+(1S)\rho^-$	$10.52^{+0.03+1.06+0.42}_{-0.03-0.76-0.38}$	7.04	9.4	12.6	13	6.4	5.1	...	4.2	...	11.30	6.8 ± 1.4	
$\bar{B}_s^0 \rightarrow D_s^+(1S)K^{*-}$	$0.58^{+0.00+0.06+0.02}_{-0.00-0.04-0.02}$	0.392	0.47	0.8	0.6	0.38	0.302	0.05	0.24	...	0.564	...	
$\bar{B}_s^0 \rightarrow D_s^+(2S)\pi^-$	$4.69^{+0.02+0.82+0.67}_{-0.02-0.47-0.43}$	1.13	7	...	...	...	...	...	...	...	...	...	(10^{-4})
$\bar{B}_s^0 \rightarrow D_s^+(2S)K^-$	$0.36^{+0.00+0.06+0.04}_{-0.00-0.05-0.03}$	0.084	0.5	...	...	...	...	...	...	...	...	...	
$\bar{B}_s^0 \rightarrow D_s^+(2S)\rho^-$	$10.41^{+0.04+1.87+1.53}_{-0.04-1.00-0.09}$	2.49	17	...	...	...	...	...	...	...	...	...	
$\bar{B}_s^0 \rightarrow D_s^+(2S)K^{*-}$	$0.57^{+0.00+0.10+0.08}_{-0.00-0.05-0.05}$	0.134	0.8	...	...	...	...	...	...	...	...	...	

(4) A similar situation also occurs in the decays $B \rightarrow D^*(1S, 2S)(\pi, \rho, K^{(*)})$ as shown in Table VIII, where the branching ratios of the neutral decays $\bar{B}^0 \rightarrow D^{*0}(1S)(\pi, \rho, K^{(*)})$ are about 2 times as large as the data, while the difference between our predictions and experimental measurements for the charged decays $B^- \rightarrow D^{*0}(1S)(\pi, \rho, K^{(*)})^-$ is not significant. Our predictions are comparable with those given by the BS approach [50] for the ground state $D^*(1S)$ case, but much larger for the excited state $D^*(2S)$ case. Note that our results are

consistent well with the RIQM calculations [54] in both $D^*(1S)$ and $D^*(2S)$ cases.

(5) Compared to the experimental data, the similar situation occurs again in the decays $\bar{B}_s^0 \rightarrow D_s^{*+}(1S)(\pi, \rho, K)^-$, that is the predictions for their branching ratios have an obvious exceedance, as shown in Table IX. Certainly, our predictions for the branching ratios of the decays $\bar{B}_s^0 \rightarrow D_s^{*+}(2S)(\pi, \rho, K^{(*)})^-$ are also larger than those given by the BS approach [50], but agree well the RQM [27] and the RIQM results [54].

TABLE VIII. The branching ratios (10^{-3}) of the decays $B \rightarrow D^*(1S, 2S)(\pi, \rho, K^{(*)})$, together with other theoretical results and data for comparison.

Modes	$\bar{B}^0 \rightarrow D^{*+}(1S)\pi^-$	$\bar{B}^0 \rightarrow D^{*+}(1S)K^-$	$\bar{B}^0 \rightarrow D^{*+}(1S)\rho^-$	$\bar{B}^0 \rightarrow D^{*+}(1S)K^{*-}$
This work	$5.25^{+0.02+0.06+0.18}_{-0.01-0.02-0.42}$	$0.40^{+0.00+0.00+0.00}_{-0.00-0.01-0.03}$	$14.32^{+0.06+0.15+0.05}_{-0.02-0.48-0.07}$	$0.83^{+0.00+0.01+0.00}_{-0.00-0.03-0.06}$
[50]	3.80	0.281	8.73	0.758
[54]	...	...	14.24	0.83
[33]	2.60	0.237	7.94	0.488
Exp.[1]	2.66	0.216	6.8	0.33
Modes	$\bar{B}^0 \rightarrow D^{*+}(2S)\pi^-$	$\bar{B}^0 \rightarrow D^{*+}(2S)K^-$	$\bar{B}^0 \rightarrow D^{*+}(2S)\rho^-$	$\bar{B}^0 \rightarrow D^{*+}(2S)K^{*-}$
This work	$0.45^{+0.00+0.16+0.01}_{-0.00-0.16-0.01}$	$0.03^{+0.00+0.01+0.00}_{-0.00-0.01-0.00}$	$1.09^{+0.00+0.41+0.02}_{-0.00-0.41-0.02}$	$0.06^{+0.00+0.02+0.00}_{-0.00-0.02-0.00}$
[50]	0.038	0.00274	0.0267	0.00162
[54]	...	...	1.12	0.07
Modes	$B^- \rightarrow D^{*0}(1S)\pi^-$	$B^- \rightarrow D^{*0}(1S)K^-$	$B^- \rightarrow D^{*0}(1S)\rho^-$	$B^- \rightarrow D^{*0}(1S)K^{*-}$
This work	$5.67^{+0.01+0.05+0.98}_{-0.01-0.19-2.53}$	$0.43^{+0.00+0.00+0.07}_{-0.00-0.01-0.19}$	$15.45^{+0.04+0.13+2.30}_{-0.04-0.53-6.52}$	$0.89^{+0.00+0.01+0.13}_{-0.00-0.03-0.37}$
[50]	4.11	0.304	8.73	0.846
[54]	...	...	15.39	0.90
[33]	5.04	0.398	11.7	0.682
Exp.[1]	5.17	0.419	9.8	0.81
Modes	$B^- \rightarrow D^{*0}(2S)\pi^-$	$B^- \rightarrow D^{*0}(2S)K^-$	$B^- \rightarrow D^{*0}(2S)\rho^-$	$B^- \rightarrow D^{*0}(2S)K^{*-}$
This work	$0.46^{+0.00+0.16+0.01}_{-0.00-0.17-0.01}$	$0.03^{+0.00+0.01+0.00}_{-0.00-0.01-0.00}$	$1.10^{+0.00+0.42+0.02}_{-0.00-0.42-0.02}$	$0.06^{+0.00+0.02+0.00}_{-0.00-0.02-0.00}$
[50]	0.041	0.00295	0.0287	0.00173
[54]	...	...	1.18	0.07

TABLE IX. The branching ratios (10^{-3}) of the decays $\bar{B}_s^0 \rightarrow D_s^*(1S, 2S)(\pi, \rho, K^{(*)})$, together with other theoretical results and data for comparison.

Modes	This work	[27]	[42]	[47]	[51]	[33]	[50]	[54]	Exp.[1]
$\bar{B}_s^0 \rightarrow D_s^{*+}(1S)\pi^-$	$4.03^{+0.02+1.30+0.59}_{-0.02-1.18-0.45}$	2.7	2	3.1	2.11	2.42	3.37	...	1.9
$\bar{B}_s^0 \rightarrow D_s^{*+}(1S)K^-$	$0.31^{+0.00+0.09+0.05}_{-0.00-0.09-0.03}$	0.21	0.2	0.24	0.159	0.165	0.249	...	0.132
$\bar{B}_s^0 \rightarrow D_s^{*+}(1S)\rho^-$	$11.19^{+0.04+3.34+1.55}_{-0.04-3.01-1.15}$	8.7	13	9.0	...	5.69	7.26	11.73	9.5
$\bar{B}_s^0 \rightarrow D_s^{*+}(1S)K^{*-}$	$0.65^{+0.00+0.19+0.09}_{-0.17-0.01-0.07}$	0.48	0.6	0.56	0.163	0.347	0.688	0.69	...
$\bar{B}_s^0 \rightarrow D_s^{*+}(2S)\pi^-$	$0.61^{+0.00+0.12+0.07}_{-0.00-0.10-0.07}$	0.8	...	...	...	...	0.108	...	...
$\bar{B}_s^0 \rightarrow D_s^{*+}(2S)K^-$	$0.05^{+0.00+0.01+0.01}_{-0.00-0.01-0.01}$	0.06	...	...	...	...	0.00777	...	...
$\bar{B}_s^0 \rightarrow D_s^{*+}(2S)\rho^-$	$1.51^{+0.01+0.30+0.22}_{-0.01-0.26-0.22}$	2.2	...	...	...	...	0.0475	1.05	...
$\bar{B}_s^0 \rightarrow D_s^{*+}(2S)K^{*-}$	$0.09^{+0.00+0.02+0.01}_{-0.00-0.01-0.01}$	0.12	...	...	...	...	0.00332	0.06	...

- (6) When the same initial state $B_{(s)}$ decays to the same final states $D_{(s)}^{(*)}(nS)$, $n = 1, 2$ with emission of different types of light mesons, the branching ratios exhibit a clear hierarchical pattern, primarily due to the hierarchical structure of the CKM factors, that is $V_{ud} \gg V_{us}$,

$$\begin{aligned}
\mathcal{B}r(B \rightarrow D(nS)\pi) &\gg \mathcal{B}r(B \rightarrow D(nS)K), \\
\mathcal{B}r(B \rightarrow D(nS)\rho) &\gg \mathcal{B}r(B \rightarrow D(nS)K^*), \\
\mathcal{B}r(B \rightarrow D^*(nS)\pi) &\gg \mathcal{B}r(B \rightarrow D^*(nS)K), \\
\mathcal{B}r(B \rightarrow D^*(nS)\rho) &\gg \mathcal{B}r(B \rightarrow D^*(nS)K^*).
\end{aligned}$$

The upper relationships remain valid when B and $D^{(*)}$ are replaced with B_s and $D_s^{(*)}$, respectively.

To test the factorization hypothesis, as well as the SU(3) relations in B meson decays, we can define the ratios of

the branching fractions for our considered decays and list the values in Table X, together with other theoretical results and experimental data. From Table X, one can find that our predictions are consistent with the results from the

TABLE X. The ratios of the decays $\bar{B}(s) \rightarrow D_{(s)}^{(*)+}(1S, 2S)(\pi, \rho, K^{(*)})^-$. The results from the QCDF and the BS equation approaches and the available data are also listed for comparison.

Ratios	This work	[50]	[58]	Exp.[1]	Ratios	This work	[50]
$\frac{\text{Br}(\bar{B}_d \rightarrow D^{*+} \pi^-)}{\text{Br}(\bar{B}_d \rightarrow D^+ \pi^-)}$	1.20	1.17	0.90	1.06	$\frac{\text{Br}(\bar{B}_d \rightarrow D^{*+}(2S) \pi^-)}{\text{Br}(\bar{B}_d \rightarrow D^+(2S) \pi^-)}$	0.95	0.83
$\frac{\text{Br}(\bar{B}_d \rightarrow D^{*+} \rho^-)}{\text{Br}(\bar{B}_d \rightarrow D^+ \rho^-)}$	2.31	2.44	2.61	3.03	$\frac{\text{Br}(\bar{B}_d \rightarrow D^{*+}(2S) \rho^-)}{\text{Br}(\bar{B}_d \rightarrow D^+(2S) \rho^-)}$	2.21	2.25
$\frac{\text{Br}(\bar{B}_d \rightarrow D^{*+} K^-)}{\text{Br}(\bar{B}_d \rightarrow D^+ K^-)}$	1.92	2.08	2.91	2.86	$\frac{\text{Br}(\bar{B}_d \rightarrow D^{*+}(2S) K^-)}{\text{Br}(\bar{B}_d \rightarrow D^+(2S) K^-)}$	2.34	2.71
$\frac{\text{Br}(\bar{B}_d \rightarrow D^{*+} K^{*-})}{\text{Br}(\bar{B}_d \rightarrow D^+ K^{*-})}$	1.18	1.63	0.89	1.05	$\frac{\text{Br}(\bar{B}_d \rightarrow D^{*+}(2S) K^{*-})}{\text{Br}(\bar{B}_d \rightarrow D^+(2S) K^{*-})}$	0.81	0.81
$\frac{\text{Br}(\bar{B}_d \rightarrow D^{*+} K^{*-})}{\text{Br}(\bar{B}_d \rightarrow D^+ K^{*-})}$	1.65	1.76	1.72	2.20	$\frac{\text{Br}(\bar{B}_d \rightarrow D^{*+}(2S) K^{*-})}{\text{Br}(\bar{B}_d \rightarrow D^+(2S) K^{*-})}$	1.57	1.64
$\frac{\text{Br}(\bar{B}_d \rightarrow D^{*+} K^{*-})}{\text{Br}(\bar{B}_d \rightarrow D^+ K^{*-})}$	1.40	1.08	1.94	2.08	$\frac{\text{Br}(\bar{B}_d \rightarrow D^{*+}(2S) K^{*-})}{\text{Br}(\bar{B}_d \rightarrow D^+(2S) K^{*-})}$	1.93	2.03
$\frac{\text{Br}(\bar{B}_d \rightarrow D^{*+} K^-)}{\text{Br}(\bar{B}_d \rightarrow D^+ \pi^-)}$	0.078	0.076	0.076	0.082	$\frac{\text{Br}(\bar{B}_d \rightarrow D^{*+}(2S) K^-)}{\text{Br}(\bar{B}_d \rightarrow D^+(2S) \pi^-)}$	0.078	0.074
$\frac{\text{Br}(\bar{B}_d \rightarrow D^{*+} K^-)}{\text{Br}(\bar{B}_d \rightarrow D^+ \rho^-)}$	0.076	0.074	0.075	0.081	$\frac{\text{Br}(\bar{B}_d \rightarrow D^{*+}(2S) K^-)}{\text{Br}(\bar{B}_d \rightarrow D^+(2S) \rho^-)}$	0.067	0.072
$\frac{\text{Br}(\bar{B}_d \rightarrow D^{*+} K^{*-})}{\text{Br}(\bar{B}_d \rightarrow D^+ K^{*-})}$	0.056	0.054	0.050	0.059	$\frac{\text{Br}(\bar{B}_d \rightarrow D^{*+}(2S) K^{*-})}{\text{Br}(\bar{B}_d \rightarrow D^+(2S) K^{*-})}$	0.055	0.054
$\frac{\text{Br}(\bar{B}_s \rightarrow D_s^{*+} \pi^-)}{\text{Br}(\bar{B}_s \rightarrow D_s^+ \pi^-)}$	13.41	11.92	13.30	14.54	$\frac{\text{Br}(\bar{B}_s \rightarrow D_s^{*+}(2S) \pi^-)}{\text{Br}(\bar{B}_s \rightarrow D_s^+(2S) \pi^-)}$	12.68	33.24
$\frac{\text{Br}(\bar{B}_s \rightarrow D_s^{*+} \pi^-)}{\text{Br}(\bar{B}_s \rightarrow D_s^+ \rho^-)}$	1.05	0.90	1.01	1.19	$\frac{\text{Br}(\bar{B}_s \rightarrow D_s^{*+}(2S) \pi^-)}{\text{Br}(\bar{B}_s \rightarrow D_s^+(2S) \rho^-)}$	0.99	2.47

TABLE XI. Polarization fractions (%) of the decays $B_{(s)}^0 \rightarrow D_{(s)}^{*+}(1S, 2S)(\rho, K^*)^-$. f_L and $f_{\parallel}$ refer to the longitudinal and transverse parallel polarization fractions, respectively. In Ref. [47], BS + FA refers that the form factors are obtained in the BS equation approach and the amplitudes are evaluated under the factorization approximation (FA). BS + PQCD refers that the nonfactorizable and annihilation contributions are further included under the PQCD approach.

Channel	$\bar{B}^0 \rightarrow D^{*+}(1S) \rho^-$	$\bar{B}^0 \rightarrow D^{*+}(1S) K^{*-}$	$\bar{B}^0 \rightarrow D^{*+}(2S) \rho^-$	$\bar{B}^0 \rightarrow D^{*+}(2S) K^{*-}$
f_L [%]	90.59	88.11	93.52	91.56
$f_{\parallel}$ [%]	8.27	10.47	6.42	8.36
Channel	$B^- \rightarrow D^{*0}(1S) \rho^-$	$B^- \rightarrow D^{*0}(1S) K^{*-}$	$B^- \rightarrow D^{*0}(2S) \rho^-$	$B^- \rightarrow D^{*0}(2S) K^{*-}$
f_L [%]	90.60	88.12	93.25	91.21
$f_{\parallel}$ [%]	8.26	10.46	6.68	8.70
Channel	$\bar{B}_s^0 \rightarrow D_s^{*+}(1S) \rho^-$	$\bar{B}_s^0 \rightarrow D_s^{*+}(1S) K^{*-}$	$\bar{B}_s^0 \rightarrow D_s^{*+}(2S) \rho^-$	$\bar{B}_s^0 \rightarrow D_s^{*+}(2S) K^{*-}$
f_L [%]	89.13	86.44	92.08	89.84
BS+FA [47]	87.40	84.10	...	...
BS+PQCD [47]	85.4	85.7	...	...
PQCD [59]	87	83	...	...
Belle [60]	105	...	...	...
$f_{\parallel}$ [%]	9.11	11.47	7.65	9.90
BS+FA [47]	10.4	13.3	...	...
BS+PQCD [47]	11.3	10.4	...	...

QCDF [52] and the BS equation [50] approaches and the available data [1]. The ratios $\frac{\text{Br}(\bar{B}_s \rightarrow D_s^+ \pi^-)}{\text{Br}(\bar{B}_d \rightarrow D^+ K^-)}$ and $\frac{\text{Br}(\bar{B}_s \rightarrow D_s^+ \pi^-)}{\text{Br}(\bar{B}_d \rightarrow D^+ \pi^-)}$ are used to determine the ratio of fragmentation functions f_d/f_s , which is an important quantity for precise measurements of absolute B_s decay rates in experiments [56,57]. As for the case with the first radially excited charm meson involved, except for the ratios $\frac{\text{Br}(\bar{B}_s \rightarrow D_s^+(2S) \pi^-)}{\text{Br}(\bar{B}_d \rightarrow D^+(2S) K^-)}$ and $\frac{\text{Br}(\bar{B}_s \rightarrow D_s^+(2S) \pi^-)}{\text{Br}(\bar{B}_d \rightarrow D^+(2S) \pi^-)}$, which show relatively large differences, the results of other ratios agree well with the results given by the BS equation [50]. So we recommend future experiments measuring these two ratios to clarify which method is more reliable.

The polarization fractions for the decays $\bar{B}_{(s)}^0 \rightarrow D_{(s)}^{*+}(1S, 2S)(\rho, K^*)^-$ are listed in Table XI, where the results of the BS equation [47] and the PQCD [59] approaches and the available data [60] are also listed for comparison. From Table XI, it could be found that our predictions are comparable to the data and other theoretical results. Although there exist significant discrepancies between the form factors of the transitions $B_{(s)} \rightarrow D_{(s)}^*$ and $B_{(s)} \rightarrow D_{(s)}^*(2S)$, similar polarization behaviors can be observed in these decays $B_{(s)} \rightarrow D_{(s)}^*(1S, 2S)(\rho, K^*)$, that is, the longitudinal polarization is dominant, reaching approximately 90%, while the transverse parallel and perpendicular polarization fractions are only a few percent or roughly 10%. It shows that the form factors have little impact on polarization.

IV. SUMMARY

In this article, we have provided a detailed study of the nonleptonic decays $B_{(s)} \rightarrow D_{(s)}^*(1S, 2S)(\pi, \rho, K^*)$ in the framework of the covariant light-front approach. Except for the predictions for neutral decays $B_{(s)} \rightarrow D_{(s)}^*(1S)(\pi, \rho, K^*)$, which have some excess compared to the data, overall the branching ratios for the decays involving a ground state $D^{(*)}$ or $D_s^{(*)}$ meson are consistent with the experimental measurements. As for the decays $B_{(s)} \rightarrow D_{(s)}^*(2S)(\pi, \rho, K^*)$, most of their branching ratios lie in the range 10^{-5} – 10^{-4} , which are likely to be detected by the present LHCb and Belle II experiments. Our predictions for the decays with the first radially excited $D^{(*)}(2S)$ or $D_s^{(*)}(2S)$ meson involved are larger than the results given by the BS equation approach, but agree well with the RQM and RIQM calculations. We hope that future experiments can clarify these discrepancies. Although there exists an obvious difference in the branching ratios between the decays $B_{(s)} \rightarrow D_{(s)}^*(1S)(\rho, K^*)$ and $B_{(s)} \rightarrow D_{(s)}^*(2S)(\rho, K^*)$, similar polarization behaviors can be observed in them, that is, the longitudinal polarization is dominant, reaching approximately 90%, while the

transverse parallel and perpendicular polarization fractions are only a few percent or roughly 10%. These studies are of great significance for our understanding of the first radially excited charmed mesons, even the spectrum of charmed mesons.

ACKNOWLEDGMENTS

This work is partly supported by the National Natural Science Foundation of China under Grant No. 11347030 and the Natural Science Foundation of Henan Province under Grants No. 232300420116, No. 252300421302.

DATA AVAILABILITY

The data that support the findings of this article are openly available [1].

APPENDIX A: SOME SPECIFIC RULES UNDER THE p^- INTEGRATION

When performing the integration, we need to include the zero-mode contribution. It amounts to performing the integration in a proper way in the CLFQM. Specifically, we use the following rules given in Refs. [34,35]

$$\hat{p}'_{1\mu} \doteq P_\mu A_1^{(1)} + q_\mu A_2^{(1)}, \quad (\text{A1})$$

$$\begin{aligned} \hat{p}'_{1\mu} \hat{p}'_{1\nu} &\doteq g_{\mu\nu} A_1^{(2)} + P_\mu P_\nu A_2^{(2)} + (P_\mu q_\nu + q_\mu P_\nu) A_3^{(2)} \\ &+ q_\mu q_\nu A_4^{(2)}, \end{aligned} \quad (\text{A2})$$

$$\begin{aligned} Z_2 &= \hat{N}'_1 + m_1^2 - m_2^2 + (1 - 2x_1)M^2 \\ &+ (q^2 + q \cdot P) \frac{p'_\perp \cdot q_\perp}{q^2}, \end{aligned} \quad (\text{A3})$$

$$\hat{p}'_{1\mu} \hat{N}_2 \rightarrow q_\mu \left[A_2^{(1)} Z_2 + \frac{q \cdot P}{q^2} A_1^{(2)} \right], \quad (\text{A4})$$

$$\hat{p}'_{1\mu} \hat{p}'_{1\nu} \hat{N}_2 \rightarrow g_{\mu\nu} A_1^{(2)} Z_2 + q_\mu q_\nu \left[A_4^{(2)} Z_2 + 2 \frac{q \cdot P}{q^2} A_2^{(1)} A_1^{(2)} \right], \quad (\text{A5})$$

$$\begin{aligned} A_1^{(1)} &= \frac{x_1}{2}, \quad A_2^{(1)} = A_1^{(1)} - \frac{p'_\perp \cdot q_\perp}{q^2}, \\ A_3^{(2)} &= A_1^{(1)} A_2^{(1)}, \end{aligned} \quad (\text{A6})$$

$$\begin{aligned} A_4^{(2)} &= (A_2^{(1)})^2 - \frac{1}{q^2} A_1^{(2)}, \quad A_1^{(2)} = -p_\perp^2 - \frac{(p'_\perp \cdot q_\perp)^2}{q^2}, \\ A_2^{(2)} &= (A_1^{(1)})^2. \end{aligned} \quad (\text{A7})$$

APPENDIX B: EXPRESSIONS OF $B_{(s)} \rightarrow P, V$ FORM FACTORS

$$\begin{aligned}
 S_{\mu\nu}^{PV} &= (S_V^{PV} - S_A^{PV})_{\mu\nu} \\
 &= \text{Tr} \left[\left(\gamma_\nu - \frac{1}{W_V''} (p_1'' - p_2)_\nu \right) (p_1'' + m_1') (\gamma_\mu - \gamma_\mu \gamma_5) (\not{p}_1' + m_1') \gamma_5 (-\not{p}_2 + m_2) \right] \\
 &= -2i\epsilon_{\mu\nu\alpha\beta} \{ p_1'^\alpha P^\beta (m_1'' - m_1') + p_1'^\alpha q^\beta (m_1' + m_1' - 2m_2) + q^\alpha P^\beta m_1' \} \\
 &\quad + \frac{1}{W_V''} (4p_{1\nu}' - 3q_\nu - P_\nu) i\epsilon_{\mu\alpha\beta\rho} p_1'^\alpha q^\beta P^\rho \\
 &\quad + 2g_{\mu\nu} \{ m_2(q^2 - N_1' - N_1'' - m_1'^2 - m_1''^2) - m_1'(M'^2 - N_1'' - N_2 - m_1'^2 - m_2^2) \\
 &\quad - m_1''(M'^2 - N_1' - N_2 - m_1'^2 - m_2^2) - 2m_1' m_1'' m_2 \} \\
 &\quad + 8p_{1\mu}' p_{1\nu}' (m_2 - m_1') - 2(P_\mu q_\nu + q_\mu P_\nu + 2q_\mu q_\nu) m_1' + 2p_{1\mu}' P_\nu (m_1' - m_1'') \\
 &\quad + 2p_{1\mu}' q_\nu (3m_1' - m_1'' - 2m_2) + 2P_\mu p_{1\nu}' (m_1' + m_1'') + 2q_\mu p_{1\nu}' (3m_1' + m_1'' - 2m_2) \\
 &\quad + \frac{1}{2W_V''} (4p_{1\nu}' - 3q_\nu - P_\nu) \{ 2p_{1\mu}' [M'^2 + M''^2 - q^2 - 2N_2 + 2(m_1' - m_2)(m_1'' + m_2)] \\
 &\quad + q_\mu [q^2 - 2M'^2 + N_1' - N_1'' + 2N_2 - (m_1 + m_1'')^2 + 2(m_1' - m_2)^2] \\
 &\quad + P_\mu [q^2 - N_1' - N_1'' - (m_1' + m_1'')^2] \}. \tag{B1}
 \end{aligned}$$

$$\begin{aligned}
 S_{\mu}^{PP} &= \text{Tr} [\gamma_5 (\not{p}_1'' + m_1'') \gamma_\mu (\not{p}_1' + m_1') \gamma_5 (-\not{p}_2 + m_2)] \\
 &= 2p_{1\mu}' [M'^2 + M''^2 - q^2 - 2N_2 - (m_1' - m_2)^2 - (m_1'' - m_2)^2 + (m_1' - m_1'')^2] \\
 &\quad + q_\mu [q^2 - 2M'^2 + N_1' - N_1'' + 2N_2 + 2(m_1' - m_2)^2 - (m_1' - m_1'')^2] \\
 &\quad + P_\mu [q^2 - N_1' - N_1'' - (m_1' - m_1'')^2], \tag{B2}
 \end{aligned}$$

The following are the analytical expressions of the $B_{(s)} \rightarrow D_{(s)}^{(*)}(1S, 2S)$ transition form factors in the covariant light-front quark model

$$\begin{aligned}
 F^{B_{(s)}D_{(s)}}(q^2) &= \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{h'_{B_{(s)}} h''_{D_{(s)}}}{x_2 \hat{N}_1' \hat{N}_1''} [x_1 (M_0'^2 + M_0''^2) + x_2 q^2 - x_2 (m_1' - m_1'')^2 \\
 &\quad - x_1 (m_1' - m_2)^2 - x_1 (m_1'' - m_2)^2] \tag{B3}
 \end{aligned}$$

$$\begin{aligned}
 F_0^{B_{(s)}D_{(s)}}(q^2) &= F_1^{B_{(s)}D_{(s)}}(q^2) + \frac{q^2}{(q \cdot P)} \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{2h'_{B_{(s)}} h''_{D_{(s)}}}{x_2 \hat{N}_1' \hat{N}_1''} \left\{ -x_1 x_2 M'^2 - p_\perp'^2 - m_1' m_2 \right. \\
 &\quad + (m_1'' - m_2)(x_2 m_1' + x_1 m_2) + 2 \frac{q \cdot P}{q^2} \left(p_\perp'^2 + 2 \frac{(p'_\perp \cdot q_\perp)^2}{q^2} \right) + 2 \frac{(p'_\perp \cdot q_\perp)^2}{q^2} \\
 &\quad \left. - \frac{p'_\perp \cdot q_\perp}{q^2} [M''^2 - x_2 (q^2 + q \cdot P) - (x_2 - x_1) M'^2 + 2x_1 M_0'^2 - 2(m_1' - m_2)(m_1' + m_1'')] \right\}, \tag{B4}
 \end{aligned}$$

$$\begin{aligned}
 V^{B_{(s)}D_{(s)}^*}(q^2) &= \frac{N_c (M' + M'')}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{2h'_{B_{(s)}} h''_{D_{(s)}^*}}{x_2 \hat{N}_1' \hat{N}_1''} \left\{ x_2 m_1' + x_1 m_2 + (m_1' - m_1'') \frac{p'_\perp \cdot q_\perp}{q^2} + \frac{2}{w_{D_{(s)}^*}''} \left[p_\perp'^2 + \frac{(p'_\perp \cdot q_\perp)^2}{q^2} \right] \right\}, \tag{B5}
 \end{aligned}$$

$$\begin{aligned}
A_1^{B(s)D(s)^*}(q^2) = & -\frac{1}{M' + M''} \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{h'_{B(s)} h''_{D(s)^*}}{x_2 \hat{N}'_1 \hat{N}''_1} \left\{ 2x_1(m_2 - m'_1)(M_0'^2 + M_0''^2) - 4x_1 m'_1 M_0'^2 \right. \\
& + 2x_2 m'_1 q \cdot P + 2m_2 q^2 - 2x_1 m_2 (M'^2 + M''^2) + 2(m'_1 - m_2)(m'_1 + m''_1)^2 + 8(m'_1 - m_2) \\
& \times \left[p_\perp'^2 + \frac{(p'_\perp \cdot q_\perp)^2}{q^2} \right] + 2(m'_1 + m''_1)(q^2 + q \cdot P) \frac{p'_\perp \cdot q_\perp}{q^2} - 4 \frac{q^2 p_\perp'^2 + (p'_\perp \cdot q_\perp)^2}{q^2 w''_{D(s)^*(nS)}} \\
& \times \left[2x_1(M'^2 + M_0'^2) - q^2 - q \cdot P - 2(q^2 + q \cdot P) \frac{p'_\perp \cdot q_\perp}{q^2} - 2(m'_1 - m''_1)(m'_1 - m_2) \right] \Big\}, \quad (B6)
\end{aligned}$$

$$\begin{aligned}
A_2^{B(s)D(s)^*}(q^2) = & \frac{N_c(M' + M'')}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{2h'_{B(s)} h''_{D(s)^*}}{x_2 \hat{N}'_1 \hat{N}''_1} \left\{ (x_1 - x_2)(x_2 m'_1 + x_1 m_2) - \frac{p'_\perp \cdot q_\perp}{q^2} [2x_1 m_2 + m'_1 \right. \\
& + (x_2 - x_1) m'_1] - 2 \frac{x_2 q^2 + p'_\perp \cdot q_\perp}{x_2 q^2 w''_{D(s)^*}} [p'_\perp \cdot p''_\perp + (x_1 m_2 + x_2 m'_1)(x_1 m_2 - x_2 m''_1)] \Big\}, \quad (B7)
\end{aligned}$$

$$\begin{aligned}
A_0^{B(s)D(s)^*}(q^2) = & \frac{M' + M''}{2M''} A_1^{B(s)D(s)^*}(q^2) - \frac{M' - M''}{2M''} A_2^{B(s)D(s)^*}(q^2) - \frac{q^2}{2M''} \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{h'_{B(s)} h''_{D(s)^*}}{x_2 \hat{N}'_1 \hat{N}''_1} \\
& \times \left\{ 2(2x_1 - 3)(x_2 m'_1 + x_1 m_2) - 8(m'_1 - m_2) \times \left[\frac{p_\perp'^2}{q^2} + 2 \frac{(p'_\perp \cdot q_\perp)^2}{q^4} \right] - [(14 - 12x_1) m'_1 \right. \\
& - 2m''_1 - (8 - 12x_1) m_2] \frac{p'_\perp \cdot q_\perp}{q^2} + \frac{4}{w''_{D(s)^*}} [(M'^2 + M''^2 - q^2 + 2(m'_1 - m_2)(m''_1 + m_2))] \\
& \times (A_3^{(2)} + A_4^{(2)} - A_2^{(1)}) + Z_2(3A_2^{(1)} - 2A_4^{(2)} - 1) + \frac{1}{2} [x_1(q^2 + q \cdot P) - 2M'^2 - 2p'_\perp \cdot q_\perp \\
& - 2m'_1(m''_1 + m_2) - 2m_2(m'_1 - m_2)] (A_1^{(1)} + A_2^{(1)} - 1) q \cdot P \left[\frac{p_\perp'^2}{q^2} + \frac{(p'_\perp \cdot q_\perp)^2}{q^4} \right] (4A_2^{(1)} - 3) \Big\}. \quad (B8)
\end{aligned}$$

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- [1] S. Navas *et al.* (Particle Data Group), *Phys. Rev. D* **110**, 030001 (2024).
[2] S. K. Choi *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **89**, 102001 (2002); **89**, 129901(E) (2002).
[3] P. del Amo Sanchez *et al.* (BABAR Collaboration), *Phys. Rev. D* **82**, 111101 (2010).
[4] J. B. Liu and M. Z. Yang, *Chin. Phys. C* **40**, 073101 (2016).
[5] A. M. Badalian and B. L. G. Bakker, *Phys. Rev. D* **84**, 034006 (2011).
[6] Q. T. Song, D. Y. Chen, X. Liu, and T. Matsuki, *Phys. Rev. D* **92**, 074011 (2015).
[7] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **126**, 122002 (2021).
[8] J. M. Xie, M. Z. Liu, and L. S. Geng, *Phys. Rev. D* **104**, 094051 (2021).
[9] P. G. Ortega, J. Segovia, D. R. Entem, and F. Fernandez, *Phys. Lett. B* **827**, 136998 (2022).
[10] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. D* **101**, 032005 (2020).
[11] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. D* **94**, 072001 (2016).
[12] R. Aaij *et al.* (LHCb Collaboration), *J. High Energy Phys.* **09** (2013) 145.
[13] S. Godfrey and I. T. Jardine, *Phys. Rev. D* **89**, 074023 (2014).
[14] B. Aubert *et al.* (BABAR Collaboration), *Phys. Rev. Lett.* **97**, 222001 (2006).
[15] J. Brodzicka *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **100**, 092001 (2008).
[16] D. Ebert, R. N. Faustov, and V. O. Galkin, *Eur. Phys. J. C* **66**, 197–206 (2010).
[17] R. Dhir, R. Verma, and A. Sharma, *Adv. High Energy Phys.* **2013**, 706543 (2013).
[18] R. H. Li, C. D. Lu, and Y. M. Wang, *Phys. Rev. D* **80**, 014005 (2009).

- [19] S. B. Wu, H. J. Tian, Y. L. Yang, W. Cheng, H. B. Fu, and T. Zhong, *Eur. Phys. J. C* **85**, 552 (2025).
- [20] H. B. Fu, X. G. Wu, H. Y. Han, Y. Ma, and T. Zhong, *Nucl. Phys.* **B884**, 172 (2014).
- [21] Y. Zhang, T. Zhong, X. G. Wu, K. Li, H. B. Fu, and T. Huang, *Eur. Phys. J. C* **78**, 76 (2018).
- [22] J. Gao, T. Huber, Y. Ji, C. Wang, Y. M. Wang, and Y. B. Wei, *J. High Energy Phys.* **05** (2022) 024.
- [23] T. Zhong, Y. Zhang, X. G. Wu, H. B. Fu, and T. Huang, *Eur. Phys. J. C* **78**, 937 (2018).
- [24] D. L. Yao, P. Fernandez-Soler, F. K. Guo, and J. Nieves, *Phys. Rev. D* **101**, 034014 (2020).
- [25] J. A. Bailey *et al.* (MILC Collaboration), *Phys. Rev. D* **92**, 034506 (2015).
- [26] H. Na *et al.* (HPQCD Collaboration), *Phys. Rev. D* **92**, 054510 (2015); **93**, 119906(E) (2016).
- [27] R. N. Faustov and V. O. Galkin, *Phys. Rev. D* **87**, 034033 (2013).
- [28] R. N. Faustov, V. O. Galkin, and X. W. Kang, *Phys. Rev. D* **106**, 013004 (2022).
- [29] Y. M. Wang, H. Zou, Z. T. Wei, X. Q. Li, and C. D. Lu, *Eur. Phys. J. C* **54**, 107 (2008).
- [30] Y. M. Wang, H. Zou, Z. T. Wei, X. Q. Li, and C. D. Lu, *Eur. Phys. J. C* **55**, 607 (2008).
- [31] T. Wang, Y. Jiang, H. Yuan, K. Chai, and G. L. Wang, *J. Phys. G* **44**, 045004 (2017).
- [32] Z. Q. Zhang, Z. J. Sun, Y. C. Zhao, Y. Y. Yang, and Z. Y. Zhang, *Eur. Phys. J. C* **83**, 477 (2023).
- [33] R. H. Li, C. D. Lu, and H. Zou, *Phys. Rev. D* **78**, 014018 (2008).
- [34] H. Y. Cheng, C. K. Chua, and C. W. Hwang, *Phys. Rev. D* **69**, 074025 (2004).
- [35] W. Jaus, *Phys. Rev. D* **60**, 054026 (1999).
- [36] M. Neubert and B. Stech, *Adv. Ser. Dir. High Energy Phys.* **15**, 294 (1998).
- [37] S. Godfrey and K. Moats, *Phys. Rev. D* **93**, 034035 (2016).
- [38] G. L. Wang, *Phys. Lett. B* **633**, 492 (2006).
- [39] DELPHI Collaboration, *Phys. Lett. B* **426**, 231 (1998).
- [40] P. Ball and R. Zwicky, *Phys. Rev. D* **71**, 014029 (2005).
- [41] D. Becirevic, B. Blossier, A. Gerardin, A. Le Yaouanc, and F. Sanfilippo, *Nucl. Phys.* **B872**, 313 (2013).
- [42] P. Blasi, P. Colangelo, G. Nardulli, and N. Paver, *Phys. Rev. D* **49**, 238 (1994).
- [43] Y. Y. Fan, W. F. Wang, and Z. J. Xiao, *Phys. Rev. D* **89**, 014030 (2014).
- [44] X. Q. Hu, S. P. Jin, and Z. J. Xiao, *Chin. Phys. C* **44**, 053102 (2020).
- [45] Y. Y. Fan, W. F. Wang, S. Cheng, and Z. J. Xiao, *Chin. Sci. Bull.* **59**, 125 (2014).
- [46] N. R. Soni, A. Issadykov, A. N. Gadaria, Z. Tyulemissov, J. J. Patel, and J. N. Pandya, *Eur. Phys. J. Plus* **138**, 163 (2023).
- [47] X. J. Chen, H. F. Fu, C. S. Kim, and G. L. Wang, *J. Phys. G* **39**, 045002 (2012).
- [48] R. C. Verma, *J. Phys. G* **39**, 025005 (2012).
- [49] G. Li, F. L. Shao, and W. Wang, *Phys. Rev. D* **82**, 094031 (2010).
- [50] T. Zhou, T. Wang, Y. Jiang, L. Huo, and G. L. Wang, *J. Phys. G* **48**, 055006 (2021).
- [51] K. Azizi, R. Khosravi, and F. Falahati, *Int. J. Mod. Phys. A* **24**, 5845 (2009).
- [52] T. Huber, S. Kränkl, and X. Q. Li, *J. High Energy Phys.* **09** (2016) 112.
- [53] Q. Chang, S. Xu, and L. Chen, *Nucl. Phys.* **B921**, 454 (2017).
- [54] K. Dash, P. C. Dash, R. Panda, L. Nayak, S. Kar, and N. Barik, *Eur. Phys. J. C* **83**, 1163 (2023).
- [55] C. Albertus, *Few Body Syst.* **55**, 1017 (2014).
- [56] R. Fleischer, N. Serra, and N. Tuning, *Phys. Rev. D* **83**, 014017 (2011).
- [57] R. Fleischer, N. Serra, and N. Tuning, *Phys. Rev. D* **82**, 034038 (2010).
- [58] F. M. Cai, W. J. Deng, X. Q. Li, and Y. D. Yang, *J. High Energy Phys.* **10** (2021) 235.
- [59] R. H. Li, X. X. Wang, A. I. Sanda, and C. D. Lu, *Phys. Rev. D* **81**, 034006 (2010).
- [60] R. Louvot *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **104**, 231801 (2010).