

Understanding the enhanced $\phi\eta'$ decay mode of the $\phi(2170)$ through strange-meson loops

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(Received 17 August 2026; accepted 21 September 2026; published 5 October 2026)

The nature of the strangeoniumlike $\phi(2170)$ remains controversial. Recent measurements of $e^+e^- \rightarrow \phi\eta^{(\prime)}$ reveal a striking puzzle: a broad $\phi(2170)$ -like structure appears in the $\phi\eta'$ channel but not in $\phi\eta$, despite the strong phase-space suppression of $\phi\eta'$. We investigate this unexpectedly large $\phi\eta'$ decay fraction within the excited-strangeonium assignment. A combined analysis of the $e^+e^- \rightarrow \phi\eta$ and $e^+e^- \rightarrow \phi\eta'$ cross sections is performed by including short-distance amplitudes from vacuum quark-pair creation and long-distance transitions mediated by strange-meson loops. The short-distance mechanism predicts too small a $\phi\eta'/\phi\eta$ strength ratio to explain the data. In contrast, strange-meson loops naturally enhance this ratio because the SU(3)-flavor factor at the $K^{(*)}K^{(*)}\eta^{(\prime)}$ vertex suppresses the transition to $\phi\eta$ relative to $\phi\eta'$. The resulting overall description of both cross sections supports an important role for long-distance dynamics in hidden-strangeness decays of excited-vector strangeonia. More intriguingly, although the excited-strangeonium contributions have been included, the current high-precision $\phi\eta$ data still favor the existence of an extra narrow vector state near 2.15 GeV with a width of about 25 MeV. If confirmed, it would be a promising exotic-hadron candidate in the light-vector sector. Furthermore, we test $\Gamma(Y \rightarrow \phi\eta)/\Gamma(Y \rightarrow \phi\eta') = 0.25, 1$, and 4 as fit inputs and obtain similarly good descriptions of the cross sections in all three cases. The present data therefore do not allow this ratio to discriminate among different internal configurations of the narrow state.

DOI: 10.1103/h4mj-2fj9

I. INTRODUCTION

The identification of higher vector ϕ mesons remains an open issue in light-hadron spectroscopy. The $\phi(2170)$, also referred to as the $Y(2175)$ [1], has attracted continuous attention since its first observation in the initial-state-radiation process $e^+e^- \rightarrow \gamma_{\text{ISR}}\phi f_0(980)$ [2]. It has

subsequently been studied in many other hidden-strangeness [3–9] and open-strangeness processes [10–16]. Although the $\phi(1020)$ and $\phi(1680)$ are commonly identified as the $1S$ and $2S$ $s\bar{s}$ vector states, respectively, the nature of the $\phi(2170)$ remains unsettled. Proposed interpretations include conventional excited $s\bar{s}$ states [17–24], strangeonium hybrids [25–29], fully strange tetraquarks [30–40], ϕf_0 molecular or $\phi K\bar{K}$ three-body configurations [41–46], $\Lambda\bar{\Lambda}$ molecules [47–49], and triangle-singularity effects [50]. A reliable identification of the $\phi(2170)$ therefore cannot be based on its mass alone, but should also account for correlated information from different decay channels and production mechanisms.

For a long time, the appearance of the $\phi(2170)$ in the $\phi f_0(980)$ mode played a central role in its theoretical

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interpretation. This prominence of a hidden-strangeness channel motivated exotic pictures such as a $\phi f_0(980)$ hadronic molecule and $\phi K\bar{K}$ three-body dynamics [41,42]. Recent Beijing Spectrometer III (BESIII) measurements of electron-positron annihilation into open-strange channels have provided an important update [10,11], with the $\phi(2170)$ also observed in channels such as K^+K^- , $K_1(1270)K$, $K_1(1400)K$, and $KK(1460)$. Since open-strange modes are expected to dominate the decays of excited-vector $s\bar{s}$ states, these experimental advances make it natural to first examine the $\phi(2170)$ within the excited-strangeonium picture. A useful strategy is to test whether conventional $s\bar{s}$ assignments can coherently describe the available experimental information. If the main features can be accommodated in this framework, the present data do not require an exotic interpretation. If sizable discrepancies remain, the case for nonconventional dynamics becomes more compelling. Along this line, a combined analysis of seven open-strange processes showed that all cross sections can be described simultaneously through interference among the $\phi(3S)$, $\phi(2D)$, and nonresonant amplitudes [17]. That analysis also determined the resonance parameters of the relevant $\phi(3S)$ and $\phi(2D)$ states, providing a well-constrained excited-strangeonium baseline for studying additional decay processes associated with the $\phi(2170)$.

Recent measurements of $e^+e^- \rightarrow \phi\eta$ [7] and $e^+e^- \rightarrow \phi\eta'$ [9] reveal a particularly striking puzzle. BESIII observed a broad structure near 2.18 GeV with a width of about 150 MeV in the $\phi\eta'$ cross section [9]. This width is consistent with that of the $\phi(2170)$ observed in $\phi f_0(980)$ and multiple open-strange modes [10–13]. By contrast, the structure reported in $e^+e^- \rightarrow \phi\eta$ is much narrower, with a width of only several tens of MeV [7], and no evident broad structure appears in the $\phi\eta$ line shape. This difference suggests that the two fitted structures should not be regarded directly as the same resonance contribution. The central puzzle is therefore the unexpectedly large $\phi\eta'$ decay fraction of the broad structure associated with the $\phi(2170)$. The $\phi\eta'$ channel is strongly phase-space suppressed, with phase space favoring $\phi\eta$ by a factor of about 4.3. This decay pattern might suggest a strangeness-rich configuration inside the $\phi(2170)$, such as a fully strange tetraquark component [33]. A recent theoretical study also emphasized that the ratio of the $\phi\eta$ and $\phi\eta'$ partial widths may constrain the internal structure of the $\phi(2170)$ [42]. That study found that the ratio predicted for a $\phi K\bar{K}$ molecular state can agree with some experimental solutions within uncertainties. However, as noted above, these experimental solutions should not be interpreted directly as the $\phi\eta'/\phi\eta$ decay ratio of the $\phi(2170)$. This motivates us to analyze the measured $e^+e^- \rightarrow \phi\eta$ and $e^+e^- \rightarrow \phi\eta'$ cross sections directly, rather than use the resonance parameters and branching fractions extracted separately in the experimental analyses as theoretical inputs.

In this work, we examine whether the unexpectedly large $\phi\eta'$ decay fraction of the $\phi(2170)$ can be understood within the excited-strangeonium picture. We perform a combined analysis of the measured $e^+e^- \rightarrow \phi\eta$ and $e^+e^- \rightarrow \phi\eta'$ cross sections. Specifically, the $\phi(2170)$ is described as a manifestation of interference between the $\phi(3S)$ and $\phi(2D)$ states, whose resonance parameters are fixed by the previous open-strange analysis [17]. The $\phi(1680)$ contribution and nonresonant backgrounds are also included to describe the full line shapes of both processes. The key issue is the mechanism by which these vector strangeonium states decay into the $\phi\eta$ and $\phi\eta'$ final states. We consider short-distance transitions described by quark-pair creation from the vacuum and long-distance transitions induced by strange-meson loops. The former are the direct decay amplitudes of the excited $s\bar{s}$ states, whereas the latter arise from their couplings to open-strange meson pairs followed by rescattering into $\phi\eta^{(\prime)}$.

The comparison of these two mechanisms reveals a clear difference. The short-distance contribution alone predicts a much smaller $\phi\eta'/\phi\eta$ strength and therefore cannot account for the unexpected prominence of the phase-space-suppressed $\phi\eta'$ mode. More importantly, the strange-meson-loop contribution changes this relative strength through the SU(3)-flavor structure of the $K^{(*)}K^{(*)}\eta^{(\prime)}$ vertices, which suppresses the transition to $\phi\eta$ relative to that to $\phi\eta'$. The large $\phi\eta'$ decay fraction can therefore be traced to long-distance strange-meson-loop dynamics within the same excited-strangeonium framework that describes the open-strange line shapes. With both decay mechanisms included, the $e^+e^- \rightarrow \phi\eta$ and $e^+e^- \rightarrow \phi\eta'$ cross sections can be described well overall. Nevertheless, the $\phi\eta$ data still indicate a possible additional narrow vector structure near 2.15 GeV. This structure is a compelling candidate for an exotic state and warrants further high-precision experimental study. We also show that the available cross-section data do not uniquely determine its relative couplings to $\phi\eta'$ and $\phi\eta$. Its $\phi\eta'/\phi\eta$ partial-width ratio alone therefore cannot decisively determine its internal structure.

The remainder of this paper is organized as follows: Sec. II reviews the experimental status of the $\phi(2170)$. Section III presents the theoretical framework for the $e^+e^- \rightarrow \phi\eta^{(\prime)}$ cross sections. Section IV gives the combined analysis of the $\phi\eta$ and $\phi\eta'$ cross sections and discusses the dynamical origin of the enhanced $\phi\eta'$ decay fraction of the $\phi(2170)$. Section V examines the possible additional narrow vector structure and the implications of its $\phi\eta'/\phi\eta$ partial-width ratio. Section VI provides the summary.

II. EXPERIMENTAL STATUS OF THE $\phi(2170)$

The present understanding of the $\phi(2170)$ relies on a broad set of measurements rather than on a single discovery

channel. It is therefore necessary to review the relevant experimental information in more detail before turning to the analysis of the $\phi\eta^{(\prime)}$ cross sections.

The vector state denoted the $Y(2175)$ is currently listed by the Particle Data Group (PDG) as the $\phi(2170)$, with quantum numbers $I^G(J^{PC}) = 0^-(1^{--})$ [1]. The 2026 PDG averages of its resonance parameters are $m = (2164 \pm 5)$ MeV and $\Gamma = 88^{+26}_{-21}$ MeV. The uncertainty on the width is large because the individual measurements are mutually inconsistent [1]. This already points to substantial channel dependence among the reported resonance parameters associated with $\phi(2170)$.

The first evidence was reported by *BABAR* in 2006 using the initial-state-radiation (ISR) process $e^+e^- \rightarrow \gamma_{\text{ISR}}\phi f_0(980)$ with $\phi \rightarrow K^+K^-$ and $f_0(980) \rightarrow \pi\pi$ [2]. The fitted mass and width were $m = (2175 \pm 10 \pm 15)$ MeV, $\Gamma = (58 \pm 16 \pm 20)$ MeV. Subsequent *BABAR* analyses of the charged- and neutral-dipion modes gave compatible masses near 2.17–2.19 GeV, although these results were based on closely related data samples [51]. *BABAR* also investigated the $\phi\eta$ channel and obtained a lower mass, $2125 \pm 22 \pm 10$ MeV, albeit with limited statistical precision [6]. Independently, using the BESII detector, the BES Collaboration observed an enhancement in $J/\psi \rightarrow \eta Y$, $Y \rightarrow \phi f_0(980)$ with $m = (2186 \pm 10 \pm 6)$ MeV and $\Gamma = (65 \pm 23 \pm 17)$ MeV, providing the first confirmation through a production mechanism other than direct e^+e^- annihilation [52].

In 2009, Belle studied the ISR processes $e^+e^- \rightarrow \gamma_{\text{ISR}}\phi\pi^+\pi^-$ and $\gamma_{\text{ISR}}\phi f_0(980)$ [3]. The resulting resonance parameters, $m = 2079 \pm 13^{+79}_{-28}$ MeV and $\Gamma = 192 \pm 23^{+25}_{-61}$ MeV, corresponded to a lower mass and considerably broader structure than those observed by *BABAR* and BESII. The large asymmetric model uncertainties, however, substantially limit the interpretation of this result. A higher-statistics *BABAR* analysis subsequently obtained $m = (2180 \pm 8 \pm 8)$ MeV, $\Gamma = (77 \pm 15 \pm 10)$ MeV from the $\phi f_0(980)$ line shape, including interference with the $\phi(1680)$ [4]. BESIII later observed the state with a significance exceeding 10σ in $J/\psi \rightarrow \eta\phi\pi^+\pi^-$, obtaining $m = (2200 \pm 6 \pm 5)$ MeV, $\Gamma = (104 \pm 15 \pm 15)$ MeV [53].

A different production mechanism was investigated by BESIII through $e^+e^- \rightarrow \eta Y(2175)$, $Y(2175) \rightarrow \phi f_0(980)$ at c.m. energies between 3.7 and 4.6 GeV [54]. The measured resonance parameters, $m = (2135 \pm 8 \pm 9)$ MeV and $\Gamma = (104 \pm 24 \pm 12)$ MeV, gave a mass lower than those obtained in most direct-production measurements in electron-positron collisions. In the same period, a structure was observed in the $e^+e^- \rightarrow K^+K^-$ cross section with $m = (2239.2 \pm 7.1 \pm 11.3)$ MeV, $\Gamma = (139.8 \pm 12.3 \pm 20.6)$ MeV [10]. This enhancement is also referred to as the $Y(2240)$. Since the electromagnetic $K\bar{K}$ amplitude can contain both isoscalar and isovector components, it may include contributions from the $\phi(2170)$, ω -like, and ρ -like states, and is therefore not

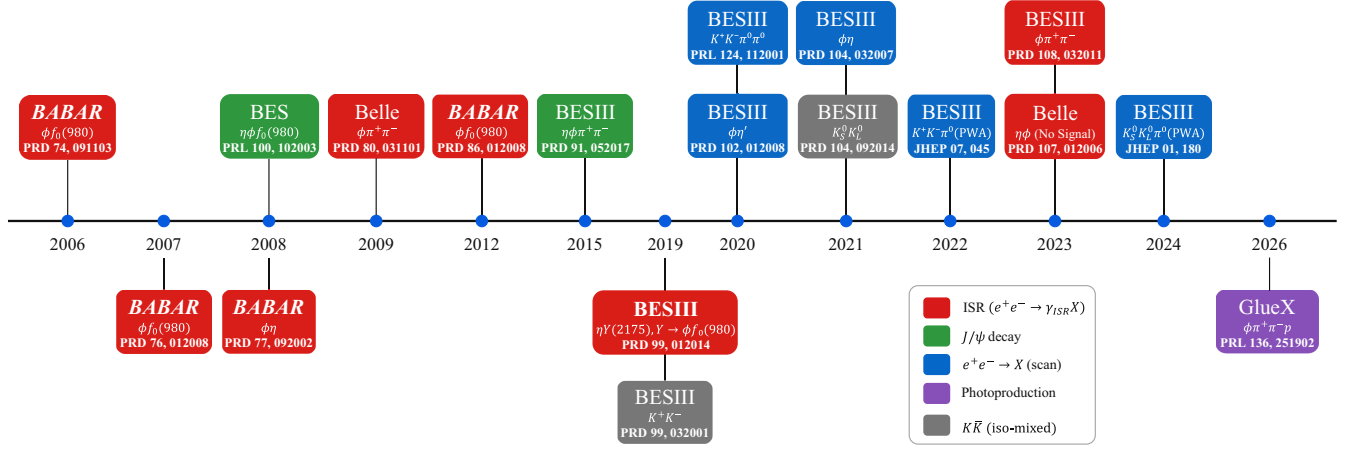
included in the PDG averages of $\phi(2170)$. For the $K\bar{K}$ channel, the authors of Ref. [17] calculated the contributions from the nearby excited ρ , ω , and ϕ states by combining their mass spectra, $K\bar{K}$ decay widths, and dileptonic couplings. The structure around 2.2 GeV was found to be dominated by the interference structure from the $\phi(3S)$ and $\phi(2D)$ contributions.

The BESIII energy scan data have enabled measurements in several additional hidden- and open-strangeness channels. A partial-wave analysis of $e^+e^- \rightarrow K^+K^-\pi^0\pi^0$ gave $m = (2126.5 \pm 16.8 \pm 12.4)$ MeV, $\Gamma = (106.9 \pm 32.1 \pm 28.1)$ MeV [11]. In the $\phi\eta'$ channel, BESIII observed a prominent structure with $m = (2177.5 \pm 4.8 \pm 19.5)$ MeV and $\Gamma = (149.0 \pm 15.6 \pm 8.9)$ MeV [9]. Its mass is close to the nominal $\phi(2170)$ value, but its width is substantially larger than those obtained in the early measurements of the $\phi f_0(980)$ channel. By contrast, the $e^+e^- \rightarrow \phi\eta$ measurement yielded one of the narrowest reported line shapes, $m = (2163.5 \pm 6.2 \pm 3.0)$ MeV, $\Gamma = (31.1^{+21.1}_{-11.6} \pm 1.1)$ MeV [7].

A structure near 2.27 GeV was also observed in $e^+e^- \rightarrow K_S^0 K_L^0$, with $m = (2273.7 \pm 5.7 \pm 19.3)$ MeV, $\Gamma = (86 \pm 44 \pm 51)$ MeV [12]. Its mass is compatible within uncertainties with that of the $Y(2240)$ observed in the K^+K^- channel, whereas the width has a smaller central value, although the two widths remain consistent within the large uncertainties. A partial-wave analysis of $e^+e^- \rightarrow K^+K^-\pi^0$, dominated by the $K^*(892)^+K^-$ and $K_2^*(1430)^+K^-$ intermediate states, gave $m = (2190 \pm 19 \pm 37)$ MeV, $\Gamma = (191 \pm 28 \pm 60)$ MeV [13]. This is one of the broadest measurements included in the current PDG average. The high-statistics BESIII energy scan of $e^+e^- \rightarrow \phi\pi^+\pi^-$ obtained $m = (2178 \pm 20 \pm 5)$ MeV and $\Gamma = (140 \pm 36 \pm 16)$ MeV in the $\phi f_0(980)$ -enriched sample [5]. This result favors a broad $\phi f_0(980)$ line shape, consistent with the broad structure associated with the $\phi(2170)$ in $\phi\eta'$ and multiple open-strange modes. The smaller widths obtained in the early ISR measurements of the $\phi f_0(980)$ channel are likely due to their limited precision. The BESIII energy scan instead favors a broad line shape. In the $\phi\eta$ channel, Belle found no significant $\phi(2170)$ contribution in its higher-statistics ISR measurement [8]. When the mass and width of the narrow structure reported by BESIII were fixed in the fit, its significance was only 1.7σ .

The most recent BESIII result included in the 2026 PDG average is the partial-wave analysis of $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$. Evidence for a structure in the $K^*(892)^0 \bar{K}^0$ line shape was found with $m = (2164.7 \pm 9.1 \pm 3.1)$ MeV, $\Gamma = (32.4 \pm 21.0 \pm 1.8)$ MeV [14]. Its mass and width are consistent with the narrow structure observed in the $\phi\eta$ channel, although the statistical uncertainty remains large.

Complementary information has recently been obtained from photoproduction. GlueX searched for the $\phi(2170)$

FIG. 1. Timeline of experimental studies of the $\phi(2170)$.

state in $\gamma p \rightarrow \phi \pi^+ \pi^- p$. A line shape fixed to the nominal $\phi(2170)$ parameters showed a significance of only 1.9σ after systematic uncertainties were included, together with the upper limit $\sigma[\gamma p \rightarrow \phi(2170)p] \mathcal{B}[\phi(2170) \rightarrow \phi \pi^+ \pi^-] < 272$ pb at the 90% confidence level [55]. A free fit gave $m = (2249 \pm 12 \pm 10)$ MeV, $\Gamma = (153 \pm 54 \pm 76)$ MeV, suggesting that the higher-mass enhancement structure may be distinct from the nominal $\phi(2170)$, although a dedicated partial-wave analysis is still required.

Several general features emerge from the existing measurements. First, the fitted mass is more stable than the width in channels containing a $\phi(2170)$ -like state. The peak positions of most such measurements cluster between 2.16 and 2.20 GeV, whereas the principal lower-mass results occur near 2.12–2.14 GeV. The structures observed in the $K \bar{K}$ channels and in GlueX photoproduction instead tend to lie near 2.24–2.27 GeV.

Second, the reported Breit-Wigner widths exhibit a strong channel dependence. The $\phi \eta$ and neutral $K^*(892)^0 \bar{K}^0$ measurements give narrow widths of about 30 MeV. The $\phi \eta'$, $\phi \pi^+ \pi^-$, and charged $K^*(892)^+ K^-$ analyses instead favor widths of 140–190 MeV, while the early $\phi f_0(980)$ measurements lie between these two groups. The six measurements entering the PDG width average have $\chi^2 = 30.1$, corresponding to a confidence level below 10^{-4} [1]. The PDG value therefore summarizes a set of channel-dependent Breit-Wigner widths and should not be interpreted as a universal resonance width. Such a pattern can arise when several nearby vector states contribute to different channels with channel-dependent relative strengths and interference phases, so that each cross section exhibits only a single visible structure but with a distinct line shape. This provides a natural explanation for the channel dependence of the fitted $\phi(2170)$ parameters. The analysis of open-strange channels in Ref. [17], which includes interference between the $\phi(3S)$ and $\phi(2D)$ states, provides a concrete example.

In summary, the $\phi(2170)$ has been investigated in ISR production, direct $e^+ e^-$ energy scans, J/ψ decays, and photoproduction, as summarized in Fig. 1. Most observed structures associated with $\phi(2170)$ in various final states have masses between 2.16 and 2.20 GeV, while lower-mass structures near 2.12–2.14 GeV and higher-mass enhancements near 2.24–2.27 GeV have also been reported. The fitted widths span approximately 30–190 MeV. Recent BESIII measurements have substantially extended the experimental information in both hidden- and open-strange channels, but sizeable differences among the reported resonance parameters remain.

III. THEORETICAL FRAMEWORK

A. Cross section formula for $e^+ e^- \rightarrow \phi \eta^{(\prime)}$

In general, the $e^+ e^- \rightarrow \phi \eta^{(\prime)}$ cross section receives two types of contributions: a direct continuum background and resonant amplitudes from intermediate strangeonium-like states. It can therefore be modeled as their coherent sum [1],

$$\sigma_f(s) = \left| \mathcal{M}_f^{\text{Con}}(s) + \sum_k e^{i\varphi_{fk}} \mathcal{M}_k(s) \right|^2. \quad (1)$$

Here, φ_{fk} is the relative phase between the k th resonant amplitude and the continuum amplitude, and $f = 1$ (2) denotes the $\phi \eta$ ($\phi \eta'$) channel. The continuum amplitude $\mathcal{M}_f^{\text{Con}}$ is parametrized as

$$\mathcal{M}_f^{\text{Con}} = a_f \cdot e^{-b_f(E_{\text{cm}} - m_{\phi} - m_{\eta^{(\prime)}})} \sqrt{\Phi_2(s)}, \quad (2)$$

where $\Phi_2(s)$ is the two-body phase-space factor, and a_f and b_f are free parameters that govern the normalization and energy dependence of the continuum amplitude, respectively. The contribution from an intermediate strangeonium-like state R_k is described by a relativistic Breit-Wigner amplitude [1,56], as follows:

$$\mathcal{M}_k = \frac{\sqrt{12\pi\Gamma_{R_k}^{e^+e^-}\mathcal{B}(\phi\eta^{(l)})\Gamma_{R_k}^{\text{tot}}}}{s - M_{R_k}^2 + iM_{R_k}\Gamma_{R_k}^{\text{tot}}} \sqrt{\frac{\Phi_2(s)}{\Phi_2(M_{R_k}^2)}}. \quad (3)$$

M_{R_k} , $\Gamma_{R_k}^{\text{tot}}$, and $\Gamma_{R_k}^{e^+e^-}$ denote the mass, total width, and dileptonic width of the k th intermediate state, respectively, while $\mathcal{B}(\phi\eta^{(l)})$ is its branching fraction to $\phi\eta^{(l)}$. Throughout this work, $\eta^{(l)}$ denotes either η or η' .

B. The mechanisms of excited-strangeonium decays to $\phi\eta^{(l)}$

In this work, we consider two decay mechanisms for the excited-strangeonium states into $\phi\eta^{(l)}$, as illustrated in Fig. 2. Figure 2(a) represents the Okubo-Zweig-Iizuka (OZI)-allowed tree-level contribution, where an excited-strangeonium state directly decays into $\phi\eta^{(l)}$ via the quark-pair-creation (QPC) mechanism. Figure 2(b) depicts the long-distance rescattering contribution, in which the initial strangeonium state couples to intermediate $K^{(*)}\bar{K}^{(*)}$ meson pairs, followed by their rescattering into the hidden-strange final state $\phi\eta^{(l)}$. Hadronic loop mechanism had been widely employed in studies of hidden-flavor transitions of excited charmonia and bottomonia [57]. A representative example is the hadronic-loop prediction that the branching fractions of $\Upsilon(5S) \rightarrow \eta\Upsilon_J(1D)$ can be of order 10^{-3} [58], which was subsequently confirmed by Belle [59].

Here, we employ an effective Lagrangian approach to describe the interaction vertices involved in Fig. 2. The relevant effective Lagrangians are given by [60–62]

$$\mathcal{L}_{\phi^{(*)}\phi\eta^{(l)}} = g_{\phi^*\phi\eta^{(l)}}\epsilon_{\mu\nu\alpha\beta}\partial^\mu\phi^{*\nu}\partial^\alpha\phi^\beta\eta^{(l)}, \quad (4)$$

$$\begin{aligned} \mathcal{L}_{\phi K^{(*)}K^{(*)}} = & -ig_{\phi KK}\phi^\mu(\bar{K}\overleftrightarrow{\partial}_\mu K) \\ & -g_{\phi K^*K}\epsilon^{\mu\nu\alpha\beta}\partial_\mu\phi_\nu(\bar{K}\partial_\alpha K_\beta^* + \partial_\alpha\bar{K}_\beta^*K) \\ & -ig_{\phi K^*K^*}(\bar{K}_\nu^*\overleftrightarrow{\partial}_\mu K^{*\nu}\phi^\mu + \bar{K}^{*\mu}K_\nu^*\overleftrightarrow{\partial}_\mu\phi^\nu \\ & + \phi_\nu\overleftrightarrow{\partial}_\mu\bar{K}^{*\nu}K^{*\mu}), \end{aligned} \quad (5)$$

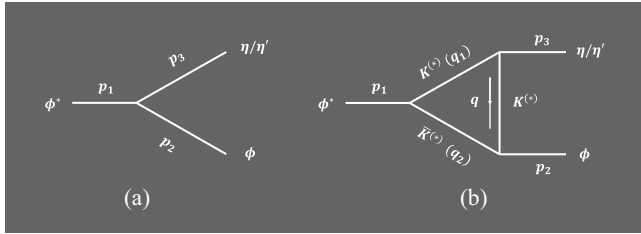


FIG. 2. Two decay mechanisms for highly excited-strangeonium states into $\phi\eta^{(l)}$. (a) OZI-allowed tree-level process. (b) Loop-induced transition through intermediate $K^{(*)}\bar{K}^{(*)}$ loops.

$$\begin{aligned} \mathcal{L}_{K^{(*)}K^{(*)}\eta^{(l)}} = & ig_{K^*K\eta^{(l)}}[(\bar{K}^{*\mu}K\partial_\mu\eta^{(l)} - \bar{K}^{*\mu}\partial_\mu K\eta^{(l)}) \\ & + (\partial_\mu\bar{K}K^{*\mu}\eta^{(l)} - \bar{K}K^{*\mu}\partial_\mu\eta^{(l)})] \\ & + g_{K^*K^*\eta^{(l)}}\epsilon_{\mu\nu\alpha\beta}\partial^\mu\bar{K}^{*\nu}\partial^\alpha K^{*\beta}\eta^{(l)}. \end{aligned} \quad (6)$$

Here, ϕ^* , ϕ , $\eta^{(l)}$, and $K^{(*)}$ denote an excited-vector strangeonium, the ground state $\phi(1020)$, the $\eta^{(l)}$, and the kaon fields, respectively.

Using these effective Lagrangians, the tree-level amplitude for the process in Fig. 2(a) is

$$\mathcal{M}_{\text{Tree}} = g_{\phi^*\phi\eta^{(l)}}\epsilon^\mu(p_1)\epsilon_{\nu\mu\alpha\beta}(-ip_1^\nu)(ip_2^\alpha)\epsilon^{*\beta}(p_2). \quad (7)$$

The loop amplitude for the process in Fig. 2(b) receives contributions from the six allowed intermediate $K^{(*)}\bar{K}^{(*)}$ configurations shown in the Appendix, as follows:

$$\mathcal{M}_{\text{Loop}} = 4 \sum_{i=1}^6 \mathcal{M}_{\text{Loop}}^{(i)}. \quad (8)$$

The factor of 4 accounts for the charge-conjugate and isospin-related intermediate kaon loops. Explicit expressions for $\mathcal{M}_{\text{Loop}}^{(i)}$ are given in Eqs. (A1)–(A6). To account for the off-shell effect of the exchanged meson and regulate the UV behavior of the loop integral, we introduce the phenomenological dipole form factor $\mathcal{F}(q, m_{K^{(*)}}^2)$ [63–65], as follows:

$$\mathcal{F}(q, m_{K^{(*)}}^2) = \left(\frac{m_{K^{(*)}}^2 - \Lambda^2}{q^2 - \Lambda^2} \right)^2. \quad (9)$$

The cutoff is parametrized as $\Lambda = m_{K^{(*)}} + \alpha_\Lambda\Lambda_{\text{QCD}}$, with $\Lambda_{\text{QCD}} = 0.22$ GeV. The dimensionless parameter α_Λ , which is usually taken to be in the range 1–5 [65–68], is the only free parameter in the decay amplitudes. The total amplitude for $\phi^* \rightarrow \phi\eta^{(l)}$ is then

$$\mathcal{M}_{\text{Total}} = \mathcal{M}_{\text{Tree}} + \mathcal{M}_{\text{Loop}}. \quad (10)$$

The branching fraction for an excited-strangeonium state decaying into $\phi\eta^{(l)}$ is

$$\mathcal{B}(\phi^* \rightarrow \phi\eta^{(l)}) = \frac{1}{3} \frac{1}{\Gamma_{\text{tot}}} \frac{|\vec{p}_2|}{8\pi m_1^2} |\mathcal{M}_{\text{Total}}|^2. \quad (11)$$

Here, the factor $1/3$ averages over the polarization of the initial state, and $\vec{p}_2$ is the momentum of the final-state ϕ meson in the rest frame of the decaying ϕ^* .

To evaluate these branching fractions, we first determine the coupling constants in the decay amplitudes. The value $g_{\phi KK} = 4.57$ is extracted from the measured partial width of $\phi(1020) \rightarrow K^+K^-$ [1]. In the SU(3)-flavor limit, the

remaining ground-state couplings are related to $g_{\phi KK}$ as follows [69]:

$$\begin{aligned} g_{\phi K^* K} &= g_{VVP}, & g_{\phi K^* K^*} &= g_{\phi KK}, \\ g_{K^* K \eta} &= (\alpha + \gamma) g_{\phi KK}, & g_{K^* K \eta'} &= (\beta + \delta) g_{\phi KK}, \\ g_{K^* K^* \eta} &= (\alpha + \gamma) g_{VVP}, & g_{K^* K^* \eta'} &= (\beta + \delta) g_{VVP}. \end{aligned} \quad (12)$$

Here, $g_{VVP} = 3g_{\phi KK}^2/(4\pi^2 f_\pi)$ and the pion decay constant is $f_\pi = 132$ MeV [60]. The parameters α , β , γ , and δ are determined by the flavor decomposition of the pseudoscalar nonet. In the octet-singlet basis, the pseudoscalar nonet matrix is written as [42,70,71]

$$\mathcal{P} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} \end{pmatrix}. \quad (13)$$

To display explicitly the flavor weights of the η and η' states, we define

$$\begin{aligned} \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} &\equiv \alpha\eta + \beta\eta', \\ -\frac{2\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} &\equiv \gamma\eta + \delta\eta'. \end{aligned} \quad (14)$$

We adopt the single-angle octet-singlet mixing convention [69],

$$\begin{aligned} |\eta\rangle &= \cos \theta |\eta_8\rangle - \sin \theta |\eta_1\rangle, \\ |\eta'\rangle &= \sin \theta |\eta_8\rangle + \cos \theta |\eta_1\rangle. \end{aligned} \quad (15)$$

Then, substituting Eq. (15) into Eq. (14) gives

$$\begin{aligned} \alpha &= \frac{\cos \theta - \sqrt{2} \sin \theta}{\sqrt{6}}, & \beta &= \frac{\sqrt{2} \cos \theta + \sin \theta}{\sqrt{6}}, \\ \gamma &= \frac{-2 \cos \theta - \sqrt{2} \sin \theta}{\sqrt{6}}, & \delta &= \frac{\sqrt{2} \cos \theta - 2 \sin \theta}{\sqrt{6}}. \end{aligned} \quad (16)$$

For the mixing angle $\theta = -14.1^\circ$ [71,72], these relations yield

$$\frac{g_{K^{(*)} K^{(*)} \eta}}{g_{K^{(*)} K^{(*)} \eta'}} = \frac{\alpha + \gamma}{\beta + \delta} = \frac{-\cos \theta - 2\sqrt{2} \sin \theta}{2\sqrt{2} \cos \theta - \sin \theta} \simeq -0.094. \quad (17)$$

This small coupling ratio suppresses the $\phi\eta$ loop amplitude by about 1 order of magnitude relative to the $\phi\eta'$

amplitude, corresponding to roughly 2 orders of magnitude at the rate level. The triangle loop mechanism can therefore overcome the smaller phase space of $\phi\eta'$, in marked contrast to the QPC amplitude, which favors $\phi\eta$ for the spectroscopic inputs used below. The couplings of the excited-vector strangeonia to $K^{(*)}K^{(*)}$ and $\phi\eta^{(\prime)}$ are extracted from the corresponding partial widths:

$$g_{\phi^* KK} = \left[\frac{48\pi m_{\phi^*}^5 \mathcal{B}(KK) \Gamma_{\phi^*}}{\lambda^{\frac{3}{2}}(m_{\phi^*}^2, m_K^2, m_K^2)} \right]^{\frac{1}{2}}, \quad (18)$$

$$g_{\phi^* K^* K} = \left[\frac{96\pi m_{\phi^*}^3 \mathcal{B}(K^* K) \Gamma_{\phi^*}}{\lambda^{\frac{3}{2}}(m_{\phi^*}^2, m_{K^*}^2, m_K^2)} \right]^{\frac{1}{2}}, \quad (19)$$

$$\begin{aligned} g_{\phi^* K^* K^*} &= \left[\frac{192\pi m_{\phi^*}^5 m_{K^*}^4 \mathcal{B}(K^* K^*) \Gamma_{\phi^*}}{\lambda^{\frac{3}{2}}(m_{\phi^*}^2, m_{K^*}^2, m_{K^*}^2)} \right. \\ &\quad \left. \times \frac{1}{(12m_{K^*}^4 + 20m_{\phi^*}^2 m_{K^*}^2 + m_{\phi^*}^4)} \right]^{\frac{1}{2}}, \end{aligned} \quad (20)$$

$$g_{\phi^* \phi\eta^{(\prime)}} = \left[\frac{96\pi m_{\phi^*}^3 \mathcal{B}(\phi\eta^{(\prime)}) \Gamma_{\phi^*}}{\lambda^{\frac{3}{2}}(m_{\phi^*}^2, m_{\phi}^2, m_{\eta^{(\prime)}}^2)} \right]^{\frac{1}{2}}. \quad (21)$$

Here, $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ is the Källén function. Γ_{ϕ^*} denotes the total width of the corresponding excited-strangeonium state, and the branching fractions are taken from the QPC calculations of Ref. [17], which use the same model parameters.

IV. THE UNIFIED DESCRIPTION FOR THE CROSS SECTIONS OF $e^+e^- \rightarrow \phi\eta'$ AND $\phi\eta$

To examine whether the conventional $s\bar{s}$ strangeonium framework can provide a unified description of the measured $e^+e^- \rightarrow \phi\eta$ [7] and $e^+e^- \rightarrow \phi\eta'$ [9] line shapes, we perform a simultaneous fit to the two cross-section datasets. The resonant amplitudes are constructed from the $\phi(2S)$, $\phi(3S)$, and $\phi(2D)$ states. We identify the $\phi(2S)$ with the $\phi(1680)$ and take its mass and total width from the PDG [1], while the $\phi(3S)$ and $\phi(2D)$ parameters are fixed to those obtained from the open-strange analysis of Ref. [17]. The dileptonic widths are also taken from Ref. [17], while the QPC branching fractions for the $\phi\eta$ and $\phi\eta'$ decays are calculated here using the same model and parameter set. All three states are included in the $\phi\eta$ amplitude. For the $\phi\eta'$ channel, the $\phi(2S)$ lies far below threshold and its off-shell contribution can be absorbed into the continuum background, so only the $\phi(3S)$ and $\phi(2D)$ amplitudes are retained. Once these inputs are fixed, the loop contributions in

TABLE I. Resonance parameters and decay properties of the excited-strangeonium states. The mass and total width of the $\phi(2S)$ are taken from the PDG [1], while those of the $\phi(3S)$ and $\phi(2D)$ and the dileptonic widths are taken from Ref. [17]. The $\phi\eta$ and $\phi\eta'$ branching fractions are calculated with the QPC model using the same parameter set as Ref. [17].

Parameters	$\phi(2S)$	$\phi(3S)$	$\phi(2D)$
Mass (MeV)	1680 ± 20	2183 ± 1	2290 ± 3
Γ_{Total} (MeV)	150 ± 50	185 ± 4	312 ± 6
$\Gamma_{e^+e^-}$ (eV)	285	106	17
$\mathcal{B}(KK)$	0.108	0.059	0.087
$\mathcal{B}(K^*K)$	0.851	0.242	0.067
$\mathcal{B}(K^*K^*)$	...	0.007	0.105
$\mathcal{B}(\phi\eta)$	0.041	0.011	0.003
$\mathcal{B}(\phi\eta')$	...	0.0003	0.0001

both channels are controlled by the common cutoff parameter α_Λ . The resonance parameters and decay inputs are summarized in Table I.

The resulting fitted curves are shown in Fig. 3, and the fitted parameters are listed in Table II. The fit gives $\chi^2/\text{d.o.f.} = 1.023$, indicating that the framework provides a satisfactory overall description of the available $e^+e^- \rightarrow \phi\eta$ and $e^+e^- \rightarrow \phi\eta'$ data. As shown in Fig. 3(a), the $e^+e^- \rightarrow \phi\eta$ cross section at low energies is governed predominantly by the $\phi(2S)$ contribution and the nonresonant continuum. The $\phi(2D)$ and $\phi(3S)$ contribute in the energy region above approximately 2.1 GeV. However, the broad $\phi(2D)$ and $\phi(3S)$ contributions do not appear as an obvious structure in the $e^+e^- \rightarrow \phi\eta$ cross section. For the $\phi\eta'$ channel, shown in Fig. 3(b), the broad enhancement in the $\phi(2170)$ region is reproduced by the coherent interplay

TABLE II. Parameters in the combined fit. The index $i = 1$ (2) denotes the $\phi\eta$ ($\phi\eta'$) channel. The a_i and b_i are the continuum parameters in Eq. (2), and φ_{ik} are the relative phases in Eq. (1).

Parameters	Values
a_1 ($\times 10^{-3}$)	2.03 ± 0.02
b_1 (GeV^{-1})	1.66 ± 0.01
φ_{11} (rad)	1.15 ± 0.03
φ_{12} (rad)	3.99 ± 0.07
φ_{13} (rad)	1.84 ± 0.59
a_2 ($\times 10^{-3}$)	0.50 ± 0.02
b_2 (GeV^{-1})	1.15 ± 0.04
φ_{22} (rad)	0.78 ± 0.07
φ_{23} (rad)	4.64 ± 0.18
α_Λ	4.23 ± 0.24
$\chi^2/\text{d.o.f.}$	1.023

of the $\phi(2D)$, $\phi(3S)$, and continuum amplitudes. The measured structure should therefore not be interpreted simply as the contribution of a single isolated resonance. Instead, it can arise from interference between the two excited-strangeonium states.

Table III lists the loop-induced and total branching fractions of the excited-vector strangeonium states to $\phi\eta$ and $\phi\eta'$, evaluated at the fitted value $\alpha_\Lambda = 4.23 \pm 0.24$. The loop-induced quantities are obtained from $|\mathcal{M}_{\text{Loop}}|^2$, whereas the total branching fractions are calculated from $|\mathcal{M}_{\text{Tree}} + \mathcal{M}_{\text{Loop}}|^2$. A comparison of Tables I and III shows that, for both $\phi(3S)$ and $\phi(2D)$, although the tree-level QPC mechanism strongly favors $\phi\eta$, the strange-meson loop contribution to $\phi\eta'$ exceeds that to $\phi\eta$ by approximately 1 to 2 orders of magnitude. This enhancement arises primarily from the SU(3)-flavor structure of

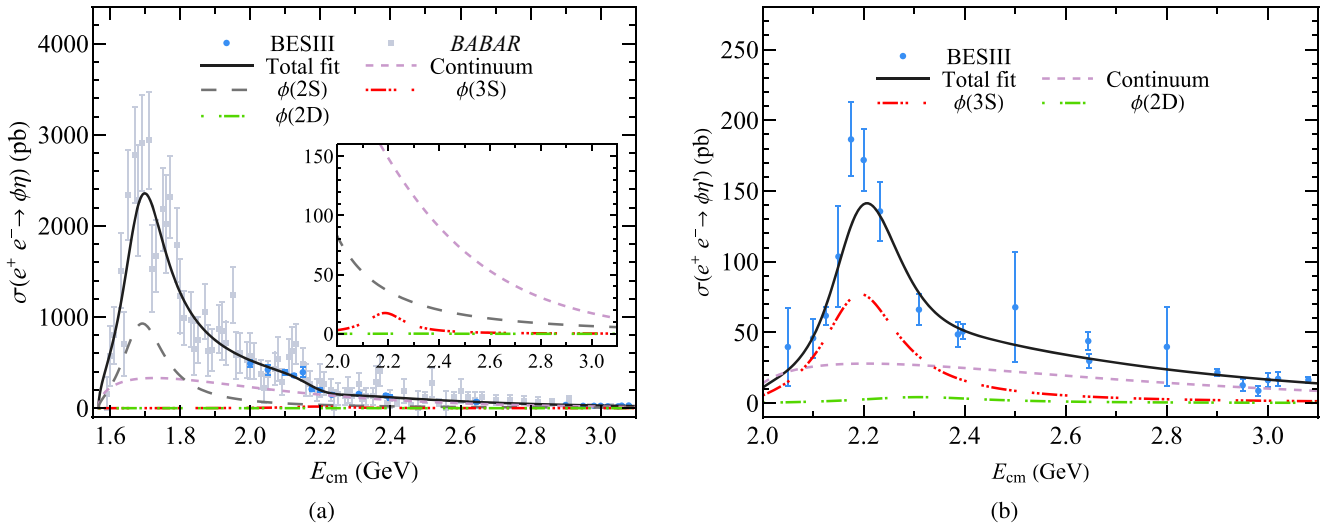


FIG. 3. Combined fit to the $e^+e^- \rightarrow \phi\eta$ [6,7] and $e^+e^- \rightarrow \phi\eta'$ [9] cross sections, shown in panels (a) and (b), respectively.

TABLE III. Loop-induced and total branching fractions for the decays of the excited-vector strangeonium states into $\phi\eta^{(\prime)}$, evaluated at the fitted value $\alpha_\Lambda = 4.23 \pm 0.24$. The loop-induced results are obtained from $|\mathcal{M}_{\text{Loop}}|^2$, whereas the total results are calculated from $|\mathcal{M}_{\text{Tree}} + \mathcal{M}_{\text{Loop}}|^2$.

Channels	$\phi(2S)$		$\phi(3S)$		$\phi(2D)$	
	$\mathcal{B}_{\text{Loop}}(\%)$	$\mathcal{B}_{\text{Total}}(\%)$	$\mathcal{B}_{\text{Loop}}(\%)$	$\mathcal{B}_{\text{Total}}(\%)$	$\mathcal{B}_{\text{Loop}}(\%)$	$\mathcal{B}_{\text{Total}}(\%)$
$\phi\eta$	0.79 ± 0.12	6.59 ± 0.18	0.13 ± 0.02	0.99 ± 0.01	0.05 ± 0.01	0.10 ± 0.01
$\phi\eta'$	$\dots$	$\dots$	4.04 ± 0.58	4.30 ± 0.61	1.76 ± 0.30	2.03 ± 0.30

the $K^{(*)}\bar{K}^{(*)}\eta^{(\prime)}$ vertices, for which the $K^{(*)}\bar{K}^{(*)}\eta'$ coupling is about 1 order of magnitude larger than the $K^{(*)}\bar{K}^{(*)}\eta$ coupling. This coupling hierarchy allows the strange-meson loop contribution to overcome the smaller phase space of the $\phi\eta'$ channel. Consequently, after the loop-induced amplitudes are included, the total branching fraction for $\phi\eta'$ becomes substantially larger than that for $\phi\eta$, indicating that long-distance strange-meson loop dynamics may be dominant in the hidden-strange decays of excited-vector strangeonia.

We therefore conclude that the enhanced $\phi\eta'$ decay mode of the $\phi(2170)$ can be understood by including the $\phi(3S)$ and $\phi(2D)$ states as the relevant intermediate resonances, together with strange-meson loop contributions to their $\phi\eta^{(\prime)}$ transitions. The measured $e^+e^- \rightarrow \phi\eta$ and $e^+e^- \rightarrow \phi\eta'$ cross sections can thus be reproduced within the conventional $s\bar{s}$ strangeonium baseline. This interpretation is consistent with the open-strange analysis of Ref. [17] and provides a unified description of the open- and hidden-strangeness processes.

V. THE HINTS OF AN ADDITIONAL NARROW STRUCTURE NEAR THE $\phi(2170)$

Although the combined fit in Sec. IV has provided a satisfactory overall description of both cross sections of $e^+e^- \rightarrow \phi\eta$ and $e^+e^- \rightarrow \phi\eta'$, it does not reproduce a high-precision BESIII data point near 2.16 GeV shown in Fig. 3(a). Since the broad structure associated with the $\phi(2170)$ is already described by the excited-strangeonium baseline, this localized deviation motivates us to test an additional, much narrower vector resonance contribution based on the baseline fit in Sec. IV. We denote this new state by Y . In Ref. [42], the relative decay strengths to $\phi\eta$ and $\phi\eta'$ have been proposed as a probe of the internal structure of the $\phi(2170)$ state. Inspired by this work, we fix the ratio $\mathcal{R}_Y \equiv \Gamma(Y \rightarrow \phi\eta)/\Gamma(Y \rightarrow \phi\eta')$ to 0.25, 1, and 4 as schemes I, II, and III, respectively, in the following

exploratory fit analysis. In each scheme, M_Y , Γ_Y , and $\Gamma_{e^+e^-}^Y \mathcal{B}(Y \rightarrow \phi\eta)$ are free parameters in the combined fit, together with the continuum parameters, relative phases, and common cutoff parameter α_Λ .

The resulting fitted line shapes are shown in Fig. 4, and the fitted parameters are listed in Table IV. The values of $\chi^2/\text{d.o.f.}$ and the visual comparison in Fig. 4 show that the inclusion of an additional narrow resonance contribution near 2.15 GeV is favored. The fitted Y widths of about 25 MeV can also be compared with the structure reported by BESIII in $e^+e^- \rightarrow K^*(892)^0 \bar{K}^0 + \text{c.c.}$, for which $M = (2164.7 \pm 9.1 \pm 3.1) \text{ MeV}$ and $\Gamma = (32.4 \pm 21.0 \pm 1.8) \text{ MeV}$ were obtained [14]. The widths are consistent within uncertainties, although the fitted Y masses are somewhat lower.

Our analysis indicates a preference for an additional narrow resonance contribution but does not by itself establish a new resonance due to the large bin size of the present measured energy points. More precise measurements, particularly a finer energy scan in the 2.1–2.2 GeV region, together with a combined analysis of the $\phi\eta$, $\phi\eta'$, and open-strange final states, will be essential for determining whether an additional vector state is necessary for depicting the observed line shape behavior. If a narrow vector state near 2.15 GeV is eventually confirmed, it would be a particularly interesting exotic-hadron candidate in the light-vector sector, because its fitted width, $\Gamma_Y = 22.5\text{--}25.8 \text{ MeV}$, is smaller by approximately a factor of 7–14 than the widths of the nearby conventional $\phi(3S)$ and $\phi(2D)$ states listed in Table I. Additionally, the resonance parameters of the additional state and the value of the fitted $\chi^2/\text{d.o.f.}$ are relatively insensitive to the assumed values of $\mathcal{R}_Y$. The present data therefore constrain the mass and width of the additional resonance more strongly than its relative couplings to $\phi\eta$ and $\phi\eta'$. Consequently, the ratio $\mathcal{R}_Y$ cannot presently distinguish among its potential exotic interpretations such as a hadronic molecule, a compact

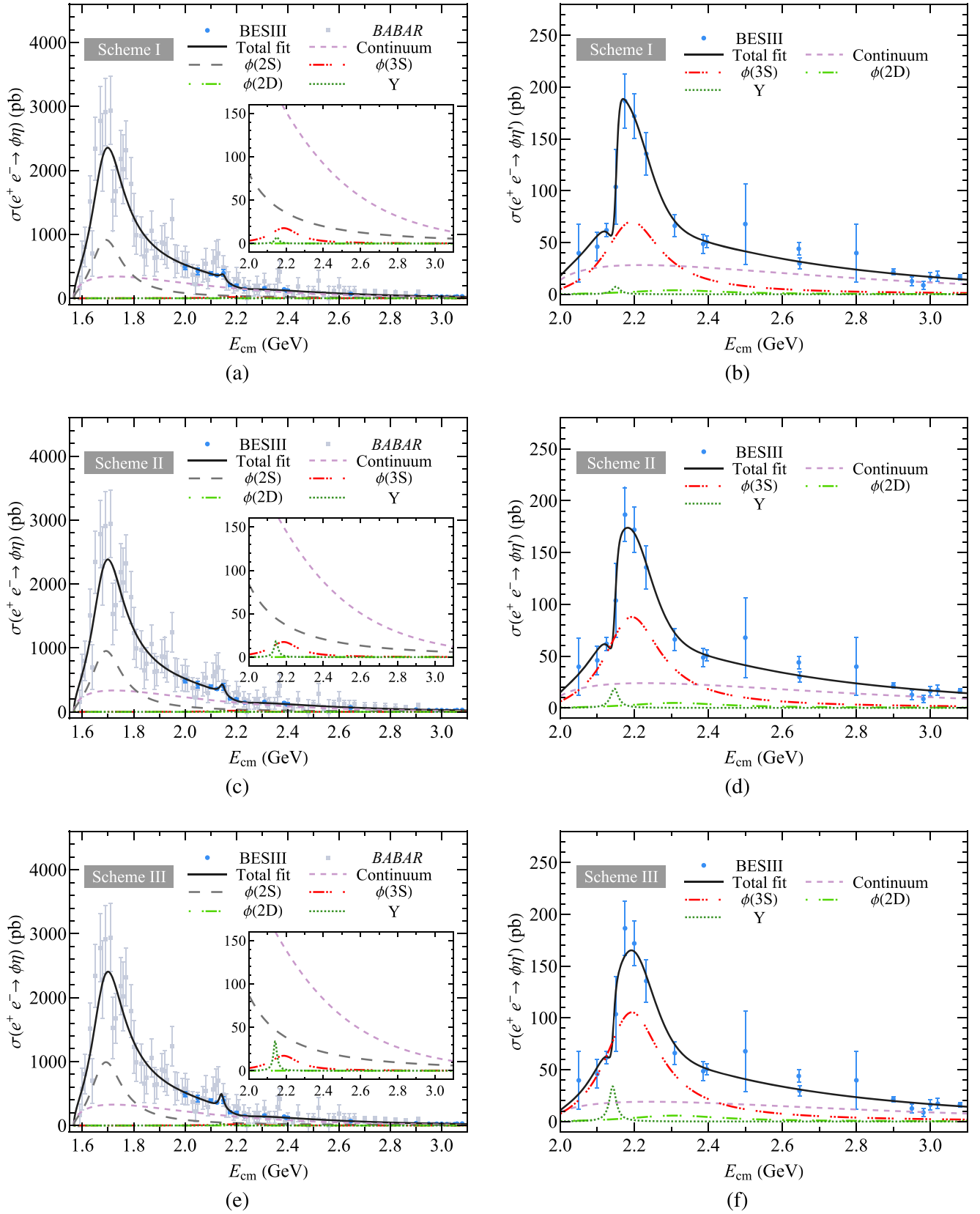


FIG. 4. Combined fits to the $e^+e^- \rightarrow \phi\eta$ [6,7] and $e^+e^- \rightarrow \phi\eta'$ [9] cross sections by adding an extra Y state under the three decay-ratio schemes.

TABLE IV. Parameters from the combined fits including the additional Y amplitude. Schemes I, II, and III fix the partial-width ratio $\mathcal{R}_Y \equiv \Gamma(Y \rightarrow \phi\eta)/\Gamma(Y \rightarrow \phi\eta')$ to 0.25, 1, and 4, respectively.

Parameters	Scheme I	Scheme II	Scheme III
	$\mathcal{R}_Y = 0.25$	$\mathcal{R}_Y = 1$	$\mathcal{R}_Y = 4$
M_Y (MeV)	2150.69 ± 2.87	2146.25 ± 2.73	2141.67 ± 2.10
Γ_Y (MeV)	25.83 ± 11.17	25.72 ± 7.30	22.53 ± 5.19
$\Gamma_{ee}^Y \mathcal{B}(Y \rightarrow \phi\eta)$ (eV)	0.060 ± 0.018	0.150 ± 0.040	0.239 ± 0.064
a_1 ($\times 10^{-3}$)	2.07 ± 0.03	2.06 ± 0.03	2.04 ± 0.03
b_1 (GeV^{-1})	1.67 ± 0.01	1.70 ± 0.01	1.72 ± 0.01
φ_{11} (rad)	1.15 ± 0.04	1.10 ± 0.04	1.06 ± 0.04
φ_{12} (rad)	4.18 ± 0.07	4.33 ± 0.08	4.39 ± 0.08
φ_{13} (rad)	2.05 ± 0.49	2.24 ± 0.55	2.35 ± 0.59
φ_{1Y} (rad)	2.47 ± 0.31	2.17 ± 0.18	1.96 ± 0.13
a_2 ($\times 10^{-3}$)	0.49 ± 0.01	0.45 ± 0.01	0.40 ± 0.01
b_2 (GeV^{-1})	1.10 ± 0.04	1.05 ± 0.04	1.01 ± 0.05
φ_{22} (rad)	1.46 ± 0.06	1.31 ± 0.06	1.07 ± 0.07
φ_{23} (rad)	5.57 ± 0.13	5.45 ± 0.12	5.26 ± 0.11
φ_{2Y} (rad)	5.60 ± 0.21	5.23 ± 0.27	4.94 ± 0.41
α_Λ	4.08 ± 0.14	4.46 ± 0.13	4.81 ± 0.13
$\chi^2/\text{d.o.f.}$	0.928	0.926	0.935

tetraquark, a strangeonium hybrid, and other exotic configurations.

VI. SUMMARY

The observation of a broad enhancement associated with the $\phi(2170)$ in the phase-space-suppressed $e^+e^- \rightarrow \phi\eta'$ channel, but not in $e^+e^- \rightarrow \phi\eta$, poses an important puzzle for understanding the nature of this vector state. We investigate whether this decay behavior can be understood through the decay dynamics of excited strangeonia without introducing an exotic component into the $\phi(2170)$ state. For this purpose, we adopt the conventional strangeonium baseline established by the previous analysis of the cross sections for e^+e^- annihilation into open-strange final states [17], in which the corresponding $\phi(2170)$ -like resonant structures are uniformly described by introducing the interference among $\phi(3S)$, $\phi(2D)$, and nonresonant amplitudes.

With the resonance parameters and dileptonic widths of the $\phi(3S)$ and $\phi(2D)$ fixed by the open-strange analysis, we explore the dynamics governing their decays into $\phi\eta^{(\prime)}$. Both short-distance QPC transitions and long-distance strange-meson loop contributions are included in a simultaneous cross-sections analysis of the e^+e^- annihilation to the $\phi\eta$ and $\phi\eta'$. The short-distance amplitudes strongly favor $\phi\eta$ and therefore cannot explain the prominence of the $\phi\eta'$ mode. However, we find that this phenomenon can be resolved by strange-meson loops. The SU(3)-flavor structure of the $K^{(*)}\bar{K}^{(*)}\eta^{(\prime)}$ vertices strongly enhances the loop transition to $\phi\eta'$ relative to that to $\phi\eta$, overcoming the suppressed phase space. The measured cross sections

can consequently be described well overall. This analysis indicates that the broad enhancement associated with $\phi(2170)$ in $e^+e^- \rightarrow \phi\eta'$ can still be attributed to the $\phi(3S)$, $\phi(2D)$ contributions and their interference effect as found in the open-strange analysis, which further supports this conventional strangeonium baseline for understanding $\phi(2170)$.

Although this framework reproduces the overall behavior of both the cross sections of $e^+e^- \rightarrow \phi\eta$ and $\phi\eta'$, it does not account for one high-precision BESIII $\phi\eta$ data point near 2.16 GeV. We therefore explore whether this deviation may indicate an additional narrow vector contribution, which we denote by Y . Since introducing this contribution is only intended as an exploratory test, we consider three representative inputs, $\Gamma(Y \rightarrow \phi\eta)/\Gamma(Y \rightarrow \phi\eta') = 0.25$, 1, and 4, covering substantially different relative decay strengths. All three schemes improve the cross section description and favor the inclusion of a narrow resonance with a mass near 2.15 GeV and a width of about 25 MeV, while giving similarly good fit qualities. The present results show that its $\phi\eta/\phi\eta'$ partial-width ratio cannot serve as an effective discriminator of its internal structure, because the major part of the anomalously large $\phi\eta'$ strength has already been explained by the strange-meson loop dynamics of the broad excited-strangeonium contributions.

Our results highlight the importance of long-distance strange-meson loops in understanding the hidden-strange decays of excited strangeonia. Such contributions should be taken into account before unusual decay patterns are attributed to exotic internal structures. The possible additional narrow contribution near 2.15 GeV motivates more precise measurements of the $e^+e^- \rightarrow \phi\eta$ and $e^+e^- \rightarrow \phi\eta'$

cross sections in the future. It also calls for improved measurements of open-strange channels, particularly $e^+e^- \rightarrow K^*(892)^0 \bar{K}^0 + \text{c.c.}$, where BESIII reported evidence for a similar narrow resonance with a mass near 2.165 GeV and a width of about 32 MeV. If an additional narrow vector state distinct from the broad $\phi(2170)$ is confirmed, it cannot be accommodated by the strangeonium baseline used here and would therefore be a compelling candidate for an exotic hadron in the light-vector sector.

ACKNOWLEDGMENTS

Q.-S. Z. is supported by the Natural Science Foundation of Inner Mongolia Autonomous Region (Grant No. 2025QN01045), the Research Support Program for High-Level Talents at the Autonomous Region level in the Inner Mongolia Autonomous Region (Grant No. 13100-15112049), the Research Startup Project of Inner Mongolia University (Grant No. 10000-23112101/101), and the National Natural Science Foundation of China (Grant No. 12247101), the Fundamental Research Funds for the Central Universities (Grant No. lzujbky-2025-jdxx07), the Natural Science Foundation of Gansu Province (Grants No. 22JR5RA389 and No. 25JRRA799), and the “111 Center” under Grant No. B20063. J.-Z. W. is supported by the National Natural Science Foundation of China under

Grants No. 12405088 and No. 12547101, and the Start-up Funds of Chongqing University. D. G. is supported by the Yanshan University Research Start-up Fund No. 8190891.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX: THE STRANGE-MESON LOOP AMPLITUDES FOR $\phi^* \rightarrow \phi\eta^{(\prime)}$

The loop contribution in Eq. (8) is obtained by summing the six triangle diagrams shown in Fig. 5. The initial excited strangeonium $\phi^*(p_1)$ first couples to an allowed $K^{(*)}(q_1)\bar{K}^{(*)}(q_2)$ pair, which subsequently rescatters into $\phi(p_2)\eta^{(\prime)}(p_3)$ through the exchange of a kaon or vector kaon with momentum q . The six diagrams exhaust the combinations of pseudoscalar and vector kaons permitted by the effective Lagrangians in Sec. III. Charge-conjugate and isospin-related loops are included through the overall factor in Eq. (8), as follows:

$$\mathcal{M}_{\text{Loop}}^{(1)} = \int \frac{d^4 q}{(2\pi)^4} (-ig_{\phi^*KK}) \epsilon_\mu(p_1) (iq_1^\mu - iq_2^\mu) (ig_{K^*K\eta^{(\prime)}}) (-iq_1^\nu - ip_3^\nu) (-g_{\phi K^*K}) \epsilon^{\kappa\lambda\theta\delta}(ip_{2\kappa})(-iq_{\theta}) \times \frac{1}{q_1^2 - m_K^2} \frac{1}{q_2^2 - m_K^2} \frac{\tilde{g}_{\chi\delta}(q)}{q^2 - m_{K^*}^2} \epsilon_\lambda^*(p_2) \mathcal{F}^2(q, m_{K^*}), \quad (\text{A1})$$

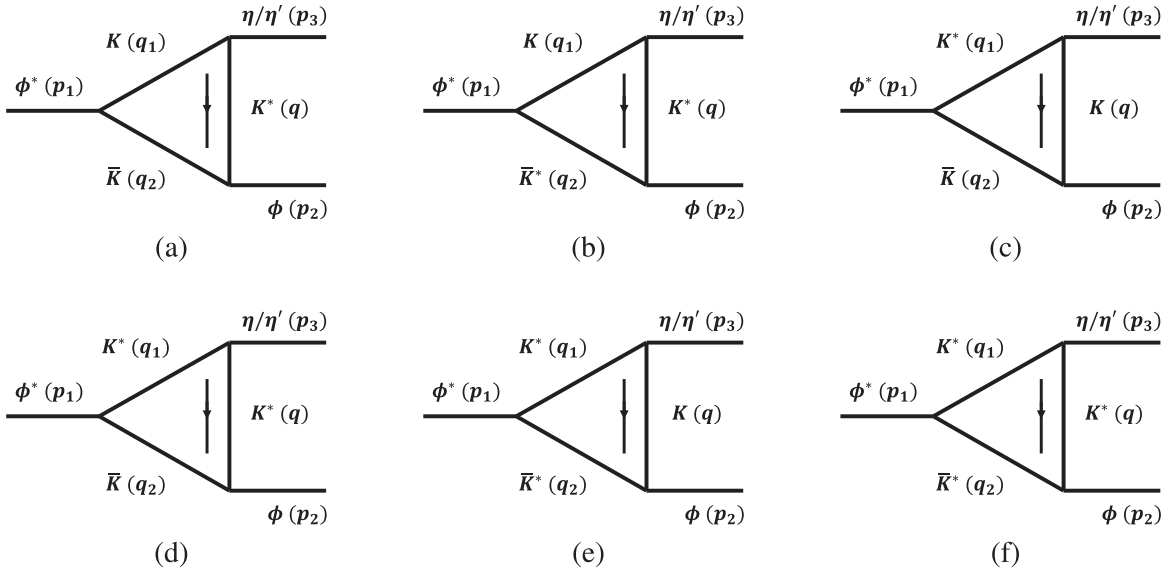


FIG. 5. Triangle loop topologies contributing to $\phi^*(p_1) \rightarrow \phi(p_2)\eta^{(\prime)}(p_3)$. The intermediate mesons and the exchanged meson are (a) $K\bar{K}$ with K^* exchange, (b) $K\bar{K}^*$ with K^* exchange, (c) $K^*\bar{K}$ with K exchange, (d) $K^*\bar{K}$ with K^* exchange, (e) $K^*\bar{K}^*$ with K exchange, and (f) $K^*\bar{K}^*$ with K^* exchange, respectively. The momenta of the intermediate mesons are denoted by q_1 and q_2 , and q is the momentum of the exchanged meson.

$$\mathcal{M}_{\text{Loop}}^{(2)} = \int \frac{d^4 q}{(2\pi)^4} (-g_{\phi^* K^* K}) \varepsilon_\mu(p_1) \epsilon^{\nu\mu\alpha\beta} (-ip_{1\nu}) (iq_{2\alpha}) (ig_{K^* K\eta^{(\nu)}}) (-iq_1^\chi - ip_3^\chi) (-ig_{\phi K^* K^*}) \\ \times [-ig^{\delta\phi}(q_2^\lambda - q^\lambda) + ig^{\lambda\phi}(p_2^\delta + q_2^\delta) - ig^{\lambda\delta}(q^\phi + p_2^\phi)] \frac{1}{q_1^2 - m_K^2} \frac{\tilde{g}_{\beta\phi}(q_2)}{q_2^2 - m_{K^*}^2} \frac{\tilde{g}_{\chi\delta}(q)}{q^2 - m_{K^*}^2} \varepsilon_\lambda^*(p_2) \mathcal{F}^2(q, m_{K^*}), \quad (\text{A2})$$

$$\mathcal{M}_{\text{Loop}}^{(3)} = \int \frac{d^4 q}{(2\pi)^4} (-g_{\phi^* K^* K}) \varepsilon_\mu(p_1) \epsilon^{\nu\mu\rho\sigma} (-ip_{1\nu}) (iq_{1\rho}) (ig_{K^* K\eta^{(\nu)}}) (ip_3^t - iq^t) (-ig_{\phi KK}) (-iq_2^\lambda + iq^\lambda) \\ \times \frac{\tilde{g}_{\sigma t}(q_1)}{q_1^2 - m_{K^*}^2} \frac{1}{q_2^2 - m_K^2} \frac{1}{q^2 - m_K^2} \varepsilon_\lambda^*(p_2) \mathcal{F}^2(q, m_K), \quad (\text{A3})$$

$$\mathcal{M}_{\text{Loop}}^{(4)} = \int \frac{d^4 q}{(2\pi)^4} (-g_{\phi^* K^* K}) \varepsilon_\mu(p_1) \epsilon^{\nu\mu\rho\sigma} (-ip_{1\nu}) (iq_{1\rho}) (g_{K^* K^* \eta^{(\nu)}}) \epsilon^{\gamma\mu\varphi\chi} (-iq_{1\gamma}) (iq_\varphi) (-g_{\phi K^* K}) \epsilon^{\kappa\lambda\theta\delta} (ip_{2\kappa}) (-iq_\theta) \\ \times \frac{\tilde{g}_{\sigma t}(q_1)}{q_1^2 - m_{K^*}^2} \frac{1}{q_2^2 - m_K^2} \frac{\tilde{g}_{\chi\delta}(q)}{q^2 - m_{K^*}^2} \varepsilon_\lambda^*(p_2) \mathcal{F}^2(q, m_{K^*}), \quad (\text{A4})$$

$$\mathcal{M}_{\text{Loop}}^{(5)} = \int \frac{d^4 q}{(2\pi)^4} (-ig_{\phi^* K^* K^*}) \varepsilon_\mu(p_1) [ig^{\sigma\beta}(q_1^\mu - q_2^\mu) - ig^{\mu\sigma}(p_1^\beta + q_1^\beta) + ig^{\mu\beta}(q_2^\sigma + p_1^\sigma) (ig_{K^* K\eta^{(\nu)}}) (ip_3^t - iq^t)] \\ \times (-ig_{\phi K^* K}) \epsilon^{\kappa\lambda\eta\phi} (ip_{2\kappa}) (-iq_{2\eta}) \frac{\tilde{g}_{\sigma t}(q_1)}{q_1^2 - m_{K^*}^2} \frac{\tilde{g}_{\beta\phi}(q_2)}{q_2^2 - m_{K^*}^2} \frac{1}{q^2 - m_K^2} \varepsilon_\lambda^*(p_2) \mathcal{F}^2(q, m_K), \quad (\text{A5})$$

$$\mathcal{M}_{\text{Loop}}^{(6)} = \int \frac{d^4 q}{(2\pi)^4} (-ig_{\phi^* K^* K^*}) \varepsilon_\mu(p_1) [ig^{\sigma\beta}(q_1^\mu - q_2^\mu) - ig^{\mu\sigma}(p_1^\beta + q_1^\beta) + ig^{\mu\beta}(q_2^\sigma + p_1^\sigma)] (g_{K^* K^* \eta^{(\nu)}}) \epsilon^{\gamma\mu\varphi\chi} (-iq_{1\gamma}) (iq_\varphi) (-ig_{\phi K^* K^*}) \\ \times [-ig^{\delta\phi}(q_2^\lambda - q^\lambda) + ig^{\lambda\phi}(p_2^\delta + q_2^\delta) - ig^{\lambda\delta}(q^\phi + p_2^\phi)] \frac{\tilde{g}_{\sigma t}(q_1)}{q_1^2 - m_{K^*}^2} \frac{\tilde{g}_{\beta\phi}(q_2)}{q_2^2 - m_{K^*}^2} \frac{\tilde{g}_{\chi\delta}(q)}{q^2 - m_{K^*}^2} \varepsilon_\lambda^*(p_2) \mathcal{F}^2(q, m_{K^*}). \quad (\text{A6})$$

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