

Bayesian inference favors quark matter in neutron star interiors

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We perform a physics-informed Bayesian analysis of the equation of state (EOS) of hybrid neutron stars that incorporates color-flavor-locked quark matter modeled by a three-flavor nonlocal Nambu–Jona-Lasinio framework with vector repulsion and diquark pairing. Contrary to the model-agnostic Bayesian analyses, our scheme allows for distinguishing between the scenarios of neutron stars with quark cores and without them. The used quark model realizes asymptotic conformality at high densities in accordance with perturbative QCD. The hadronic sector is described by the density-dependent relativistic functional DD2Y-T, which satisfies chiral effective field theory constraints and includes hyperonic degrees of freedom. We construct a large set of candidate hybrid EOSs by varying the vector and diquark couplings and apply a Maxwell construction for the quark–hadron phase transition. Observational constraints from recent NICER pulsar mass–radius measurements and tidal deformability from GW170817 are incorporated into the likelihood. Depending on whether the observational data from the black widow pulsar PSR J0952-0607 and the HESS J1731-347 object are included to the analysis or not, the posterior distribution favors vector and diquark couplings around $(\eta_V, \eta_D) \simeq (0.82, 0.40)$ or $(\eta_V, \eta_D) \simeq (0.64, 0.36)$, respectively. This corresponds to equations of state that support two-solar-mass neutron stars with superconformal speed of sound and relatively low onset densities for deconfinement. Our findings indicate that the most probable hybrid EOSs are statistically preferred over the purely hadronic baseline. The corresponding probabilities of agreeing with the observational data differ by one or two orders of magnitude depending on the data set used. This suggests that quark cores may exist in all observed neutron stars.

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I. INTRODUCTION

Neutron stars (NSs) can be considered as astrophysical laboratories for the investigation of strongly interacting matter under conditions of high baryon densities and low temperatures, which are not accessible in terrestrial experiments, e.g., with ultrarelativistic heavy-ion collisions. One of the key questions still debated controversially is whether the inner core of NSs may consist of deconfined quark matter. In other words, is the critical mass M_{onset} , where

quark deconfinement sets in, smaller than the maximum mass of the sequence of stable NSs for a given equation of state? A positive answer to this question would imply that the critical density for the onset of deconfinement under NS conditions is low enough to be attained.

The most striking and unambiguous signal for a strong phase transition in the NS interior would be the observation of mass twin NSs; see Refs. [1,2] and references therein. The detection of such a pair of stars with almost identical masses but significantly different radii requires the ability to measure the mass and radius of pulsars simultaneously with sufficient accuracy. The best instrument of such a kind presently operating is the NICER X-ray observatory mounted on the International Space Station. While no twins have yet been detected, the information on masses and radii of millisecond pulsars in a wide range of masses from $\sim 1.4M_{\odot}$ to $\sim 2.0M_{\odot}$ together with the compactness constraints from the tidal deformability measurement on the binary NS merger

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GW170817 by the LIGO-Virgo Collaboration [3] provides fragments of the mass-radius relation for NSs, which by inversion of the Tolman-Oppenheimer-Volkoff (TOV) equations is equivalent to measuring the NS equation of state (EOS). In order to infer this EOS from the still error-prone data for the mass and radius constraints, Bayesian analysis methods are the appropriate tool. According to the agnostic scheme, where the most likely EOS is selected as a posterior among a huge number of algorithmically sampled curves in the pressure vs energy density plane [see, e.g., [4,5]], in the physics-informed scheme a generic form of the EOS with physically meaningful parameters is suggested and just these parameters are varied within reasonable limits (meta-modeling); see Refs. [6–8] for examples that implement a hadron-to-quark matter phase transition.

In the present work, we apply the physics-informed Bayesian analysis scheme developed in [6] to investigate the question whether a hybrid EOS is preferable over a baseline hadronic EOS under the current mass-radius constraints for pulsars. The main advantage of this scheme compared to the widely used model-agnostic Bayesian analyses [4,5,9–11] is the possibility to distinguish between the scenarios of NSs with quark cores and without them. These scenarios can be characterized by the posterior probabilities of agreeing with the observational data. Comparing these probabilities allows one to quantitatively answer the question whether the NS cores may contain deconfined quark matter. Below we analyze these probabilities for the first time and show that according to the existing observational data on the mass-radius relation of NSs and their tidal deformability the existence of quark cores in NSs is from one to two orders of magnitude more probable than the absence of them.

Compared to the previous physics-informed Bayesian analysis performed in [6], we apply the following qualitative improvements of the scheme towards a more realistic description of dense NS matter. For the quark matter EOS we employ a recently developed three-flavor nonlocal chiral quark model of the Nambu–Jona-Lasinio (NJL) type with color-flavor-locked (CFL) color superconductivity and a nonlocal vector repulsion of the mean-field type, which assures that at asymptotically large densities the conformal limit is reached [12]. As the hadronic baseline EOS we use the relativistic density functional approach DD2npY-T recently developed by Stefan Typel [13]. For the Bayesian analysis, we include the new NICER mass-radius constraint for the $1.4M_\odot$ pulsar PSR J0614–3329 by Mauviard *et al.* [14].

The paper is organized as follows. In Sec. II, we introduce the nonlocal chiral quark model for the CFL phase of quark matter with special emphasis on analyzing its high density asymptotic, consistent with the conformal limit of QCD. In Sec. III, we describe the set of EOSs used in the analysis, while the Bayesian framework is outlined in

Sec. IV. In Sec. V we present our results and their discussion; Sec. VI gives the conclusions.

II. ASYMPTOTICALLY CONFORMAL CFL QUARK MATTER

In this work we consider the scenario of hybrid NSs with hadronic outer layers and a quark inner core. The EOS of hadron matter is modeled within the DD2Y-T model supplemented with a crust EOS [13]. The degrees of freedom of this model include nucleons, hyperons and electrons, which provide electric neutrality of the hadronic EOS. These components exist in β -equilibrium. The model respects the low density constraint of the chiral effective field theory (χ EFT) [15] and reproduces the most important properties of the nuclear matter ground state.

The EOS of quark cores is modeled within the recently proposed three-flavor nonlocal NJL model with vector repulsion and CFL color-superconductivity [12] treated within the separable approximation [16,17]. The model is formalized by the Lagrangian

$$\mathcal{L} = \bar{q}(i\cancel{D} - m + \mu\gamma_0)q + G_S \sum_{a=0,8} s_a s_a - G_V j_\mu j^\mu + 3G_D \sum_{a,b=2,5,7} d_{ab}^+ d_{ab}, \quad (1)$$

where $q = (u, d, s)^T$ is three-flavor quark field, m denotes the current mass being the same for all quark flavors and G_S , G_V and G_D stand for the coupling constants controlling the strength of interaction in the scalar, vector and diquark pairing channels, respectively. These channels have four-quark form and include nonlocal quark currents

$$s_a(x) = \int dz g(z) \bar{q}\left(x + \frac{z}{2}\right) \tau_a q\left(x - \frac{z}{2}\right), \quad (2)$$

$$j_\mu(x) = \int dz g(z) \bar{q}\left(x + \frac{z}{2}\right) \gamma_\mu q\left(x - \frac{z}{2}\right), \quad (3)$$

$$d_{ab}(x) = \int dz g(z) \bar{q}\left(x + \frac{z}{2}\right) i\gamma_5 \tau_a \lambda_b q^c\left(x - \frac{z}{2}\right), \quad (4)$$

where $\tau_0 = \sqrt{2/3}$ is introduced along with the flavor Gell-Mann matrices $\tau_{a \geq 1}$ for the sake of unifying the notations, λ_b are color Gell-Mann matrices, $q^c = i\gamma_2 \gamma_0 \bar{q}^T$ represents the charge conjugated quark field and $g(z)$ is a space-time-dependent form factor. In Ref. [12] this form factor was chosen in the instantaneous Gaussian form so that its time dependence reduces to $\propto \delta(x_0)$, while its three-momentum-dependent Fourier transform reads as

$$g_{\mathbf{k}} = \exp(-\mathbf{k}^2/\Lambda^2), \quad (5)$$

where constant Λ defines the scale separating low-momentum nonperturbative and high momentum perturbative modes.

It is important to stress that the chirally symmetric form of the Lagrangian (1) also includes pseudoscalar current densities. It can be obtained by replacing τ_a in Eq. (2) by $i\gamma_5\tau_a$. However, the corresponding currents vanish under the mean-field approximation considered in Ref. [12]. As a result, the pseudoscalar channel does not play a role in using the present model to construct the EOS of quark matter. Therefore, it is omitted to shorten the notation.

The described model suggests a simplified picture of three-flavor quark matter when light and strange quarks degenerate in mass, which corresponds to the so-called CFL-light (CFL) quark matter [18]. This simplification is related to the possibility of finding analytically the single-quark energies as eigenvalues of the 72×72 Nambu-Gorkov propagator [see Ref. [19] for details]. At the same time, high physical masses of strange quarks can lead to their suppression relative to light quarks and consequent destruction of the CFL color-superconductivity, which requires equal amounts of all quark flavors. In this case a phase of two-flavor color-superconductivity (2SC) is formed [19,20], while the CFL phase onsets later as a result of a first-order 2SC-CFL phase transition [21,22]. In Ref. [12] the stability of quark matter with respect to “unlocking” of strange quarks from the CFL phase was analyzed based on a simple kinematic criterion [23–25] and proven to be the case for the physically relevant values of the model parameters. This allows us to conclude that the proposed model of the CFL color-superconductivity provides a reasonably accurate description of cold three-flavor quark matter. It is also important to stress that equality of masses of light and strange quarks provides electric neutrality and β -equilibrium of the CFL quark matter even in the absence of electrons.

The thermodynamic potential of the present model can be found by performing the sequence of the steps described in Ref. [12]. For the convenience of the readers we briefly repeat them in Appendix A. The first of them corresponds to bosonizing the Lagrangian (1) by applying the Hubbard-Stratonovich transformation of the partition function. This introduces to the consideration auxiliary bosonic fields σ_a , ω_μ and Δ_{ab} , which are conjugated to the scalar, vector and diquark interaction channels, respectively. At the second step the mean field approximation is applied; i.e., the bosonic fields are replaced by their expectation values found from the corresponding averaged Euler-Lagrange equations. Only the zeroth scalar field does not vanish under the averaging, i.e., $\langle\sigma_a\rangle = \delta_{a0}\sigma$. It is related to the spontaneous breaking and dynamical restoration of chiral symmetry since σ enters the effective quark

mass $M_{\mathbf{k}} = m + \sigma g_{\mathbf{k}}^1$ as the corresponding mass gap. Furthermore, only the zeroth component of the vector field, which quantifies the effects of vector repulsion, survives so that $\langle\omega_\mu\rangle = g_{\mu 0}\omega$. Similarly, only the diagonal diquark fields attain the same nonvanishing expectation value $\Delta \equiv \langle\Delta_{11}\rangle + \langle\Delta_{22}\rangle + \langle\Delta_{33}\rangle$. This value plays the role of the diquark pairing gap and serves as the order parameter of the phase transition between normal quark matter and its CFL phase. With this the mean-field thermodynamic potential of cold CFL quark matter reads as

$$\Omega = -\sum_{j,a} d_j \int \frac{d\mathbf{k}}{(2\pi)^3} \left[\frac{1}{2} - \theta(-\epsilon_{j\mathbf{k}}^a) \right] \epsilon_{j\mathbf{k}}^a + \frac{\sigma^2}{4G_S} - \frac{\omega^2}{4G_V} + \frac{\Delta^2}{4G_D}. \quad (6)$$

It includes the single particle energies of quarks ($a = +$) and antiquarks ($a = -$) shifted by the quark chemical potential

$$\epsilon_{j\mathbf{k}}^a = \text{sign}(\epsilon_{j\mathbf{k}}^a) \sqrt{\epsilon_{\mathbf{k}}^2 + \Delta_{j\mathbf{k}}^2}. \quad (7)$$

Here $\epsilon_{\mathbf{k}}^\pm = \sqrt{\mathbf{k}^2 + M_{\mathbf{k}}^2} \mp \mu \mp \omega g_{\mathbf{k}}$ and $\Delta_{j\mathbf{k}} = \zeta_j \Delta g_{\mathbf{k}}$ with $\zeta_{\text{sing}} = 2$ and $\zeta_{\text{oct}} = 1$. The subscript index j distinguishes between the singlet and octet states [12,18,19]. The corresponding degeneracy factors are $d_{\text{sing}} = d/9$ and $d_{\text{oct}} = 8d/9$ with $d = 2N_c N_f$ being the spin-color-flavor degeneracy of quarks.

Equivalently to averaging the Euler-Lagrange equations, the expectation values of the scalar, vector and diquark fields can be found by minimizing the thermodynamic potential (6). Having these expectation values found, the regularized thermodynamic potential Ω^{reg} is obtained by subtracting from Ω the constant diverging thermodynamic potential of free quarks in the vacuum, i.e.,

$$\Omega^{\text{reg}} = \Omega + d \int \frac{d\mathbf{k}}{(2\pi)^3} \sqrt{\mathbf{k}^2 + m^2}. \quad (8)$$

The effective bag pressure B_{eff} is a well known phenomenological parameter of quark matter. Within the chiral quark models of the NJL type it can be obtained as the negative of the vacuum value of the regularized thermodynamic potential [see e.g. Ref. [19]]. For the present model this gives

¹As explained in the erratum [26], Ref. [12] includes a rescaling of the scalar coupling G_S by the factor $2/3$, which is not mentioned but, nevertheless, does not affect the model results. Without this rescaling the effective quark mass reads $M_{\mathbf{k}} = m + \sqrt{2/3}\sigma g_{\mathbf{k}}$. In this work we follow the notations of Ref. [12].

$$B_{\text{eff}} = d \int \frac{d\mathbf{k}}{(2\pi)^3} \left[\sqrt{M_{0,\mathbf{k}}^2 + \mathbf{k}^2} - \sqrt{m^2 + \mathbf{k}^2} \right] - \frac{(M_{0,\mathbf{k}=0} - m)^2}{4G_S}. \quad (9)$$

Hereafter the subscript index “0” denotes the quantities defined in the vacuum. Equation (9) allows us to connect the vacuum value of the effective quark mass at vanishing momentum to the effective bag pressure. Higher values of $M_{0,\mathbf{k}=0}$ correspond to higher values of B_{eff} , which is a reflection of the fact that dynamical generation of the quark mass gap and confining bag pressure are related through the phenomenon of spontaneous chiral symmetry breaking.

The pressure, baryon and energy densities of cold CFL quark matter can be obtained from the regularized thermodynamic potential as $p = -\Omega^{\text{reg}} - B_{\text{eff}}$, $n_B = \partial p / \partial \mu_B$ and $\varepsilon = \mu_B n_B - p$, respectively. This definition provides the vanishing vacuum pressure, while μ_B is the baryonic chemical potential. Below we also consider the squared speed of sound $c_S^2 = \partial p / \partial \varepsilon$ and the dimensionless interaction measure $\delta = 1/3 - p/\varepsilon$, which are used to quantify the stiffness of the NS matter and its deviation from the conformal behavior.

The typical strategy of fixing the parameters of chiral quark models of the NJL type is to fit masses and decay constants of the pseudoscalar mesons [19,27–34]. However, this strategy cannot be directly applied in a three-flavor model with mass degeneration of all the flavors. Therefore, in Ref. [12] an alternative scheme, aiming at finding reasonable values of the model parameters rather than at their ultimate determination, was suggested. The current quark mass was fixed as the mean value of the current masses of u - and d -quarks from the Review of Particle Physics [35], i.e. $m = (m_u + m_d)/2 = 3.5$ MeV. The momentum scale Λ and scalar coupling G_S were determined according to the vacuum values of the effective quark mass at vanishing momentum $M_{0,\mathbf{k}=0}$ and chiral condensate per flavor $\langle \bar{f}f \rangle_0$. Following Ref. [12], we fix $\langle \bar{f}f \rangle_0 = -(250 \text{ MeV})^3$, which is a compromise between the values of the QCD sum rules at the renormalization scale of 1 GeV, i.e., $-(229 \pm 33 \text{ MeV})^3$ [36] and $-(242 \pm 15 \text{ MeV})^3$ [37], and the prediction made according to the Gell-Mann-Oakes-Renner relation [38] with the pion mass and decay constant from the Review of Particle Physics [35] and the current quark mass mentioned above, i.e., $-(280 \pm 12 \text{ MeV})^3$. Given a gauge dependence of the momentum-dependent quark mass extracted from their propagator and strong impact of the current quark mass, we use the lattice QCD result $M_{0,\mathbf{k}=0} = 300\text{--}500$ MeV [39–42] as a guidance and consider the mean value of the vacuum effective quark mass at vanishing momentum, i.e., $M_{0,\mathbf{k}=0} = 400$ MeV. The corresponding values of the model parameters along with the effective bag pressure are shown in Table I. The vector and diquark couplings are parametrized by the

TABLE I. Values of the model parameters used in this work and the resulting effective mass at zero momentum, chiral condensate per flavor in the vacuum and the effective bag pressure.

m (MeV)	Λ (MeV)	G_S (GeV ⁻²)	$M_{0,\mathbf{k}=0}$ (MeV)	$ \langle \bar{f}f \rangle_0 ^{1/3}$ (MeV)	B_{eff} (MeV/fm ³)
3.5	564	9.62	400	250	81.3

dimensionless parameters $\eta_V = G_V/G_S$ and $\eta_D = G_D/G_S$, which below are referred to as couplings. Having the parameters given in Table I fixed, the EOS of quark matter is fully determined by η_V and η_D .

At high densities typical quark momenta significantly exceed Λ . In this case interaction among quarks is strongly suppressed by the form factor (5). The quark masses also can be neglected at high densities. As a result, dense CFL quark matter behaves as a gas of weakly interacting massless particles. Therefore, the CFL quark matter exhibits asymptotically conformal behavior with the speed of sound and dimensionless interaction measure, behaving as $c_S^2 = 1/3 - 3m^2/\mu_B^2$ and $\delta = 3m^2/\mu_B^2$ [12]. Remarkably, the obtained asymptotic expressions coincide with the ones corresponding to cold QCD matter at zero baryon density but finite isospin asymmetries [43] if $\mu_B/3$ is treated as the quark chemical potential. At the same time, if the vector repulsion among quarks is treated locally, then $c_S^2 \rightarrow 1$ for constant vector coupling [44] and $c_S^2 \rightarrow 1/3 + 0$ in the case when the latter is medium dependent [45]. Thus, accounting for the nonlocal character of interaction among quarks is important for reaching the conformal limit of QCD in agreement with the results of perturbative calculations [46–49], i.e., as $c_S^2 \rightarrow 1/3 - 0$ and $\delta \rightarrow +0$.

Figure 1 shows the speed of sound and dimensionless interaction measure of several hybrid EOSs obtained by matching the CFL EOS to the DD2Y-T hadron EOS by means of the Maxwell construction. At small densities the NS matter is constituted by hadrons with small c_S^2 and $\delta \lesssim 1/3$. The mixed quark-hadron phase is characterized by a constant pressure, which leads to vanishing speed of sound and dimensionless interaction measure being hyperbolic in energy density. Above the mixed phase CFL quarks dominate the NS matter. In this case, the speed of sound and dimensionless interaction measure exceed their conformal values up to densities of about 2 GeV fm^{-3} . As is shown in Ref. [12] and in Sec. IV, this range of ε covers the densities accessible in the NS interiors. This means that the EOS of quark matter, which can exist in NS, is rather stiff and is characterized by superconformal speed of sound. This superconformal character provides the color-superconducting quark cores with the ability to maintain the existence of NSs with the masses exceeding two solar masses. As is seen, at the densities typical for NSs the scenario of color-superconducting quark cores in their interiors is consistent with the constraints on c_S^2 and δ

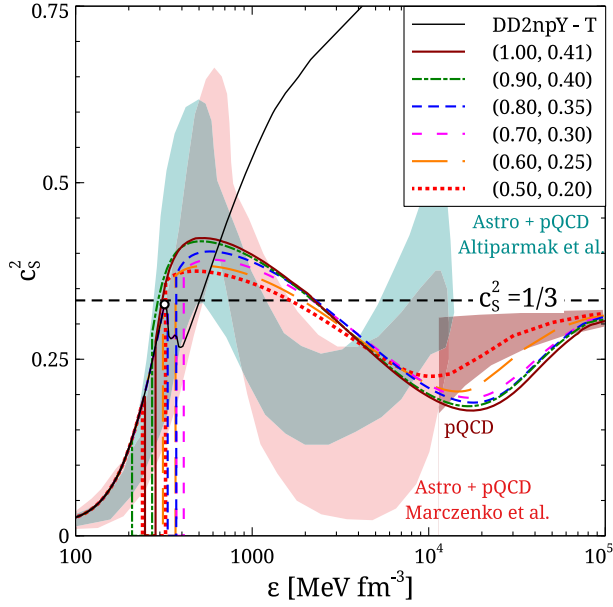


FIG. 1. Speed of sound c_s^2 (upper panel) and dimensionless interaction measure δ (lower panel) as functions of energy density ϵ obtained for the vector and diquark couplings indicated in the legend by the pairs of numbers (η_V, η_D) . The empty white circles indicate onset of hyperons in pure hadron matter. The black dashed lines represent the conformal values $c_s^2 = 1/3$ and $\delta = 0$. The astrophysical [10,11] and perturbative QCD [47] constraints represented by the shaded areas are discussed in the text.

extracted from the observational data [10,11]. It is also seen in Fig. 1 that c_s^2 and δ of the CFL quark matter simultaneously attain nearly conformal values at the energy density $\epsilon \simeq 2 \text{ GeV fm}^{-3}$. However, as is explained in Ref. [12], this does not necessarily signal approximately conformal behavior of the CFL quark matter. Above $\epsilon \simeq 2 \text{ GeV fm}^{-3}$, c_s^2 becomes subconformal, while δ exceeds its conformal value. Figure 1 shows that the CFL quark matter asymptotically reaches the conformal limit of QCD. It is also in line with the asymptotes of c_s^2 and δ found in Ref. [12], which agree with the results of perturbative QCD [47].

III. THE SET OF EOSs

Performing a reliable Bayesian analysis of the available observational data on NSs requires generating a full set of EOSs of NSs. To generate this set we first produced a collection of 31×36 quark EOSs labeled by the values of the vector and diquark couplings of the microscopic Lagrangian. These couplings are varied in the wide ranges from 0.20 to 0.50 for η_V and from 0.50 to 1.20 for η_D with increments of 0.01 and 0.02, respectively; i.e., we use $\eta_D = 0.20(0.01)0.50$ and $\eta_V = 0.50(0.02)1.20$. This ensures the generality and inclusive character of our analysis, which accounts for many physically motivated realizations of the asymptotically conformal quark EOS with vector repulsion and CFL color superconductivity.

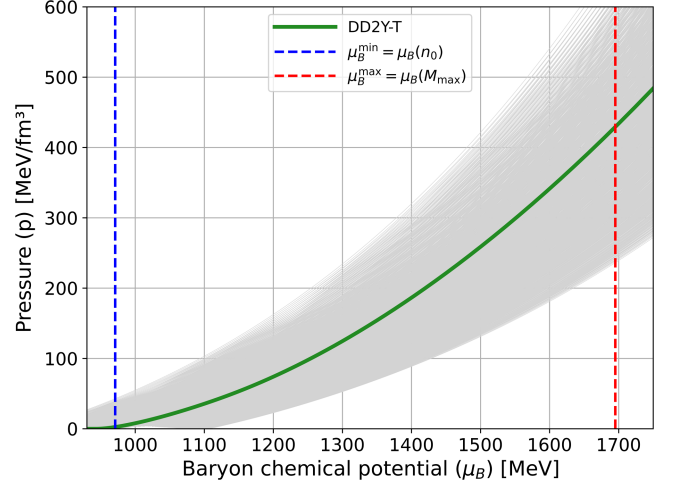


FIG. 2. Pressure p as a function of baryon chemical potential μ_B of all the generated quark EOSs (thin gray curves) and the DD2Y-T hadronic EOS (bold green curve). The red and blue vertical dashed lines indicate the minimal ($\mu_B^{\min}$) and maximal ($\mu_B^{\max}$) values of the baryon chemical potential, corresponding to the nuclear saturation density and the central density in the star with the largest maximum mass, respectively.

The mentioned range of the diquark coupling also respects the requirement that the vacuum state is stable against formation of color superconductivity, i.e., $\eta_D \leq 0.765$ [12]. The gray curves in Fig. 2 demonstrate the generated quark EOSs compared to the baseline hadronic DD2Y-T EOS [13] depicted by the green curve. The vertical red and blue dashed lines indicate the minimal values ($\mu_B^{\min}$) and maximal values ($\mu_B^{\max}$) of the baryon chemical potential, corresponding to the nuclear saturation density and the central density in the NS configuration with the highest maximum mass, respectively.

Later, it is checked which of the generated quark EOSs provide crossings with the hadronic DD2Y-T EOS in the plane of pressure p versus baryon chemical potential μ_B . This corresponds to the Maxwell construction of a first-order quark-hadron transition with a discontinuous jump of density. This construction is a partial realization of the Gibbs criterion of phase equilibrium providing continuity of the baryon chemical potential across the phase boundary but yielding a discontinuous electric chemical potential. Accounting for its continuity within the Gibbs criterion smoothens the quark-hadron transition and provides a continuous behavior of the density, i.e. the Glendenning construction of phase transition [50]. Both Maxwell and Glendenning constructions are limiting cases of a more general mixed phase construction which allows for the formation of structures with different forms and sizes (pasta phases) that minimize the free energy while accounting for the effects of surface tension and Coulomb interaction [51–53]. While the Maxwell construction would correspond to the case of asymptotically large surface tension which

separates the two coexisting phases, the Glendenning construction is obtained for vanishing surface tension between them, thus allowing for unphysically fine suspension of one phase into the other while fulfilling the constraint to obey an optimal volume fraction.

For realistic values of the surface tension parameter, however, inhomogeneous pasta structures appear in the mixed phase region [54]. They lead to a smooth quark-hadron transition and can be modeled by various interpolation schemes [55–57]. It is remarkable that already at the surface tension about 30–50 MeV/fm² the mixed quark-hadron phase of inhomogeneous pastas is indistinguishable from the one obtained with the Maxwell construction [54]. Furthermore, as was shown in Ref. [58], smoothing the phase transition in the interiors of NSs valuably affects only a narrow region of their mass-radius diagram in the vicinity of the quark onset mass, while keeping such NS characteristics as their maximum mass almost unchanged. Thus, the details of the quark-hadron transition can play only a small role in the analysis of the observational data. Therefore, in this paper, we prefer to keep the analysis economic and focus on the Maxwell construction of the quark-hadron transition, which serves as a reliable approximation of inhomogeneous pastas with realistic surface tension.

It is obvious that many quark EOSs do not provide the Maxwell crossing with the chosen hadron EOS. They are not considered in the further analysis. Some of the quark EOSs have Maxwell crossings so that the slopes of their pressures as a function of baryon chemical potential are smaller than the corresponding slopes of the hadron EOS. In this case the baryon density experiences a negative jump across the phase transition if hadron matter converts to the quark one with increasing the chemical potential. For these quark EOSs a positive density jump across the phase transition is possible only if quark matter is favored at small chemical potentials, where it has a higher pressure, while the hadronic matter is the dominating phase at high chemical potentials. This picture contradicts the commonly accepted phenomenology of QCD, and therefore, the corresponding quark EOSs are also excluded from the analysis. The remaining 307 quark EOSs provide a positive density jump at the Maxwell crossing with the DD2Y-T EOS. Each of these EOSs has a specific onset density of quark matter. We require it to exceed one nuclear saturation density, which corresponds to the minimal value of the onset chemical potential $\mu_B^{\min} = 971$ MeV. We also restrict the onset density to the values which are below the central density in the heaviest NS with purely hadronic DD2Y-T EOS, i.e., 1221 MeV/fm³. This is equivalent to requiring that the baryon chemical potential does not exceed the maximum onset value $\mu_B^{\max} = 1695$ MeV. The latter requirement is necessary for the stability of the hybrid star branches of the mass-radius relations of NSs with quark cores. Below we use the standard definition of TOV

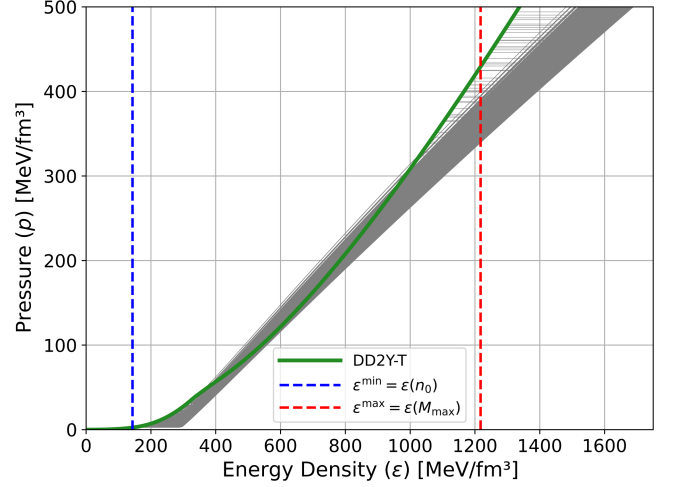


FIG. 3. Pressure p as a function of the energy density ε of the hybrid EOSs with a proper Maxwell crossing and the transition at baryon chemical potentials μ_B above $\mu_B^{\min}$ (thin gray curves). The blue and red dashed vertical lines correspond to those in Fig. 2. The bold green curve represents the DD2Y-T hadronic EOS.

stability according to which a stable (hybrid) NS branch has a positive squared zero mode radial oscillation frequency, $\omega_0^2 = dM/d\varepsilon_c > 0$. The absence of TOV stability is signaled by a negative slope of the NS mass as a function of its central energy density, which is found as the corresponding solution of the TOV equations. In the TOV-unstable case quark branches of the mass-radius relation appear after the maximum mass. This definition of the TOV stability assumes fast conversion of hadron and quark phases and ignores the possibility of the existence of stable stellar configurations even after the maximum mass if the conversion is slow [59].

Figure 3 shows the EOSs consistent with the above requirements in the plane of pressure versus energy density. As is seen, at $\varepsilon \simeq 400$ – 1000 MeV/fm³ some hybrid EOSs are stiffer than the purely hadronic one. These EOSs allow reaching the maximum NS masses about $2.2M_\odot$, which is not possible with the DD2Y-T hadronic EOS.

The squared speed of sound of the hybrid EOSs is shown in Fig. 4. The general structure discussed in Sec. II holds. It is also seen that in the density range typical for the NS interiors the quark branch of the hybrid EOS has superconformal speed of sound. This provides the NS matter with the stiffness required to maintain the existence of the objects heavier than two solar masses.

The 269 combinations of η_V and η_D , which provide a Maxwell crossing with a positive density jump at $\mu_B^{\min} \leq \mu_B \leq \mu_B^{\max}$ are shown in Fig. 5. As is seen, the corresponding region does not include all the possible values of the vector and diquark couplings. In general, the higher the vector coupling the higher the chemical potential of the quark-hadron transition, while changing the diquark coupling causes the opposite effect. Increasing η_D at a

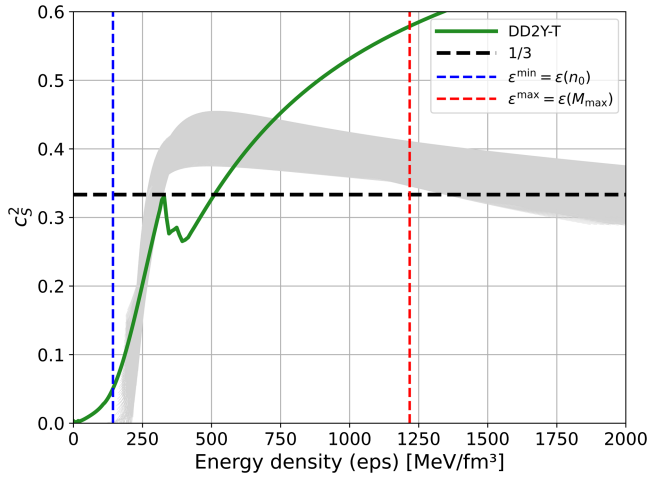


FIG. 4. Squared speed of sound c_s^2 as a function of energy density ε of the EOSs shown in Fig. 2 along with the same minimal and maximal allowed values of the energy density of the onset of quark matter (see text for details).

given value of η_V shifts the phase transition toward small chemical potentials until it reaches $\mu_B^{\min}$. This corresponds to the lower right boundary of the region shown in Fig. 5.

The hybrid EOSs represented by the blue dots located along this boundary satisfy the Seidov criterion of gravitational instability due to a large jump in energy density across the phase transition [60]. If at higher densities after the transition the gravitational stability is regained due to a significant stiffening of quark EOS, then a new stable branch of hybrid NS appears, which is disconnected from the purely hadronic branch of NS mass-radius relations. These branches include pairs of NSs of the same mass but different radii, so-called twin stars [61–63]. The hybrid star branches of the mass-radius relations of the NSs with these EOSs are TOV-stable. Recently, the possibility of the existence of twin NSs due to a first-order 2SC-CFL phase transition was concluded from the study based on a renormalization-group-consistent version of the NJL model [64]. However, the subsequent study did not find a region in the parameter space, which simultaneously provides existence of twin NSs and consistency with the observational data [65]. This can be due to either the different nature of the phase transition (2SC-CFL instead of quark-hadron) or the excessive stiffness of the corresponding EOS. It is a caveat of this EOS that it does not account for the nonlocality of quark interactions, which would facilitate reaching the conformal limit of QCD [43].

The green dots in Fig. 5 represent the EOSs, which also provide the TOV stability of their quark branches but break the Seidov criterion. Decreasing η_D at constant η_V shifts the phase transition toward higher chemical potentials. At high values of the vector coupling this happens until the phase transition disappears at $\mu_B < \mu_B^{\max}$ since the quark EOS gets too stiff. This corresponds to the $\eta_V \gtrsim 0.87$ part of the upper left boundary of the region shown in Fig. 5. At $\eta_V \lesssim 0.87$

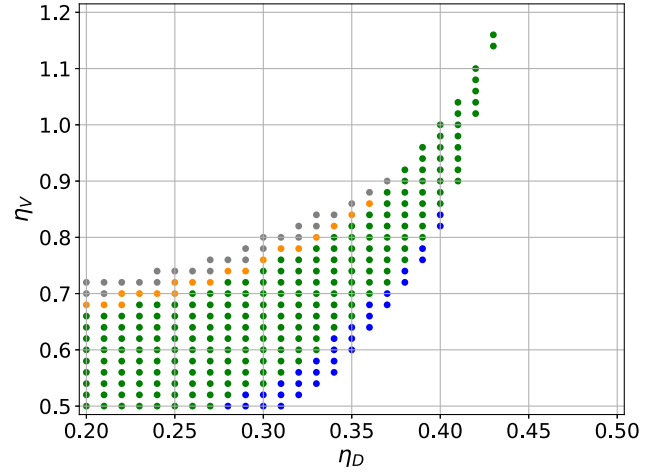


FIG. 5. Region in the plane of the vector η_V versus diquark η_D couplings providing the existence of hybrid quark-hadron EOSs with a positive density jump across the phase transition. The green and blue dots represent the EOSs, which support the TOV stability of the quark branches of the NS mass-radius relations, while the orange and gray dots do not provide such stability. The blue dots correspond to the EOSs fulfilling the Seidov criterion of the existence of twin NSs. The gray dots show the EOSs with a phase transition at $\mu_B > \mu_B^{\max}$, when the onset density exceeds the central density in the maximum-mass hadronic configurations.

this boundary is defined by higher chemical potentials. The orange and gray dots located along this boundary represent the quark EOSs, which provide a proper Maxwell crossing but generate mass-radius relations with unstable quark branches. The corresponding regions of the $\eta_V - \eta_D$ plane are separated by the condition $\mu_B = \mu_B^{\max}$. In other words, a Maxwell crossing with a positive density jump happens at $\mu_B < \mu_B^{\max}$ for the orange dots and at $\mu_B > \mu_B^{\max}$ for the gray ones. The corresponding EOSs are not used in the further analysis.

Thus, only 269 quark EOSs, which provide a Maxwell crossing with a positive density jump above the nuclear saturation density and the TOV stability of the quark branches of the mass-radius relations of the corresponding NSs are used to perform the Bayesian analysis of the available observational data. In Fig. 5 these 250 EOSs are represented by the green and blue dots. The stiffness of quark EOS increases when moving from the lower left to the upper right part of the corresponding region in the $\eta_V - \eta_D$ plane.

IV. THE BAYESIAN FRAMEWORK

To infer the posterior probability distribution of the EOS parameters from observational data, we employ a Bayesian approach. According to the Bayes theorem, the posterior probability of describing the data set $\mathcal{D}$ with the EOS constructed at given values of couplings η_V and η_D takes the form of

$$p((\eta_D, \eta_V)|\mathcal{D}) = \frac{p(\mathcal{D}|\eta_D, \eta_V)p(\eta_D, \eta_V)}{p(\mathcal{D})}, \quad (10)$$

where $p(\mathcal{D}|\eta_D, \eta_V)$ denotes the likelihood, $p(\eta_D, \eta_V)$ represents the prior probability distribution, and $p(\mathcal{D})$ is the evidence. We adopt the uniform prior

$$p(\eta_D, \eta_V) = \frac{1}{|(\eta_D, \eta_V)|} = \frac{1}{N}, \quad (11)$$

where $N = 269$ denotes the total number of the EOSs analyzed.

The total likelihood is obtained as a product over all independent observational data D_α ,

$$p(\mathcal{D}|\eta_D, \eta_V) = \prod_\alpha p(D_\alpha|\eta_D, \eta_V). \quad (12)$$

The likelihood associated with the lower bound on the maximum mass is modeled by a normal cumulative distribution function $F_{\mathcal{N}}$,

$$p(D_M|\eta_D, \eta_V) = F_{\mathcal{N}}(M_{\max}(\eta_D, \eta_V); \mu_M, \sigma_M), \quad (13)$$

with μ_M and σ_M denoting the mean and uncertainty of the constraint taken from Refs. [66].

For mass-radius data the likelihood is evaluated by curvilinear integration over the mass-radius curve obtained for the corresponding EOS:

$$p(D_{MR}|\eta_D, \eta_V) = \int_{MR} f_{MR}(M(\eta_D, \eta_V), R(M)). \quad (14)$$

When disconnected branches of the mass-radius relation occur, this integral must be evaluated separately over each branch and the contributions summed. The functions f_{MR} are constructed via kernel density estimation [67] from the Zenodo data for PSR J0030 + 0451 [68], PSR J0740 + 6620 [69], PSR J0437–4715 [70], PSR J0614–3329 [71], and HESS J1731–347 [72].

The gravitational wave (GW) likelihood is defined in an analogous way, i.e., through the curvilinear integral over the relation between the tidal deformabilities of two NSs with the proper chirp mass:

$$p(D_{GW}|\eta_D, \eta_V) = \int_{\Lambda_1 \Lambda_2} f_{GW}(\Lambda_1(\eta_D, \eta_V), \Lambda_2(\Lambda_1)), \quad (15)$$

where f_{GW} is based on the GW170817 data [73].

The evidence follows from marginalization over all possible parameter values,

$$p(\mathcal{D}) = \sum_{(\eta_D, \eta_V)} p(\mathcal{D}|\eta_D, \eta_V)p(\eta_D, \eta_V). \quad (16)$$

We perform the Bayesian analysis under two different scenarios. The first one, denoted as the *basic set*, includes all currently available NICER mass–radius constraints:

- (i) $M = 1.44^{+0.15}_{-0.14} M_\odot$ and $R = 13.02^{+1.24}_{-1.06}$ km for PSR J0030 + 0451 [74],
- (ii) $M = 2.08^{+0.07}_{-0.07} M_\odot$ and $R = 12.92^{+2.09}_{-1.13}$ km for PSR J0740 + 6620 [75], which refines the earlier NICER measurement of [76],
- (iii) $M = 1.418^{+0.037}_{-0.037} M_\odot$ and $R = 11.36^{+0.95}_{-0.63}$ km for PSR J0437–4715 [77], and
- (iv) $M = 1.44^{+0.06}_{-0.07} M_\odot$ and $R = 10.29^{+1.01}_{-0.86}$ km for PSR J0614–3329 [14].

In addition, this set incorporates the tidal deformability constraint from GW170817 [78].

The second scenario, referred to as the *full set*, extends the basic set by including two further measurements:

- (i) $M = 2.35^{+0.17}_{-0.17} \times f(\Omega) M_\odot$ for the “black widow” pulsar PSR J0952–0607 [66],² which provides a lower limit on the maximum mass, and
- (ii) $M = 0.77^{+0.20}_{-0.17} M_\odot$ and $R = 10.04^{+0.86}_{-0.78}$ km for HESS J1731–347 [83].

We note that historically Bayesian analyses have tried to implement as many precise mass and radius constraints as were available to infer the most likely EOS of NS matter. Following this strategy, a problem arises when a mass constraint like the one for the $2M_\odot$ pulsar J0348 + 0432 [84] with unconstrained radii is used together with the NICER constraint for PSR J0740 + 6620 that excludes too compact high-mass stars with $R_{2.0} < 11.79$ km. We argue that once the NICER constraint is applied, the mass constraint for PSR J0348 + 0432 should not be used. Otherwise, the selective power of the NICER constraint of the $R_{2.0}$ measurement will be eliminated. For a detailed discussion and the effect on the credibility regions in Fig. 7, see Appendix B.

V. RESULTS AND DISCUSSION

Figure 6 shows the posterior probability as a function of the vector and diquark couplings evaluated for the basic and full sets of the observational data. The probability peaks around $\eta_V \simeq 0.64$ and $\eta_D \simeq 0.36$ for the basic set. Not too high values of the most probable value of the vector coupling correspond to not too stiff EOSs. This does not

²Here we apply a correction factor $f(\Omega) = [1 + 0.2(\Omega/\Omega_K)^2]^{-1}$ because the pulsar is one of the fastest spinning pulsars at about half of its Kepler frequency, $\Omega \sim 0.5\Omega_K$, so that the well known 20% increase of the mass for a pulsar spinning at the Kepler frequency Ω_K [79–82] together with the quadratic scaling amounts to a down correction of the mass constraint by 5% when it is used as a constraint for mass-radius relations on nonrotating stars from the solutions of the TOV equations.

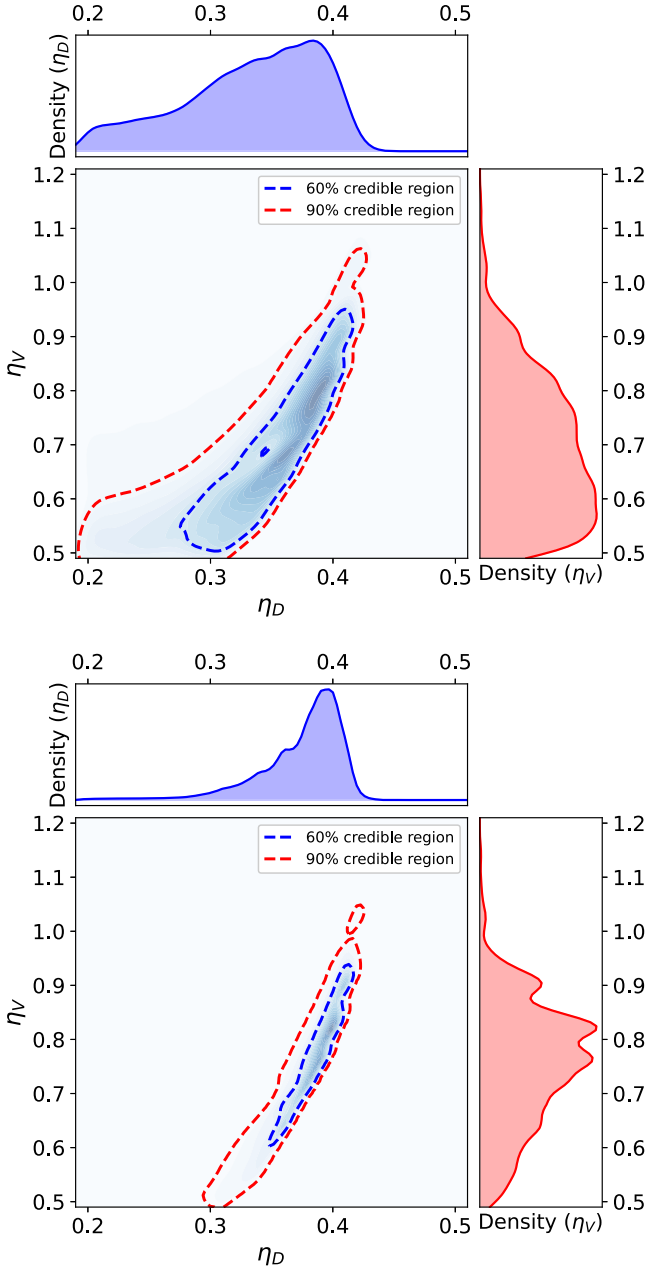


FIG. 6. Posterior probability as a function of the vector η_V and diquark η_D couplings obtained from the Bayesian analysis of the basic (upper panel) and full (lower panel) sets of the observational data.

allow the maximum NS masses to reach the high value of the black widow pulsar PSR J0952-0607 [66]. Therefore, in the full set case the maximum of the posterior probability is located at higher vector coupling around $\eta_V \simeq 0.82$ and $\eta_D \simeq 0.40$. In this case, the most probable diquark coupling is also higher than for the basic set. This is due to the fact that the data from GW170817 [73] and HESS J1731-347 [72] prefer relatively small radii of the intermediate mass NSs, which can be provided by phase transition before $1M_\odot$ requiring higher values of the diquark coupling if the

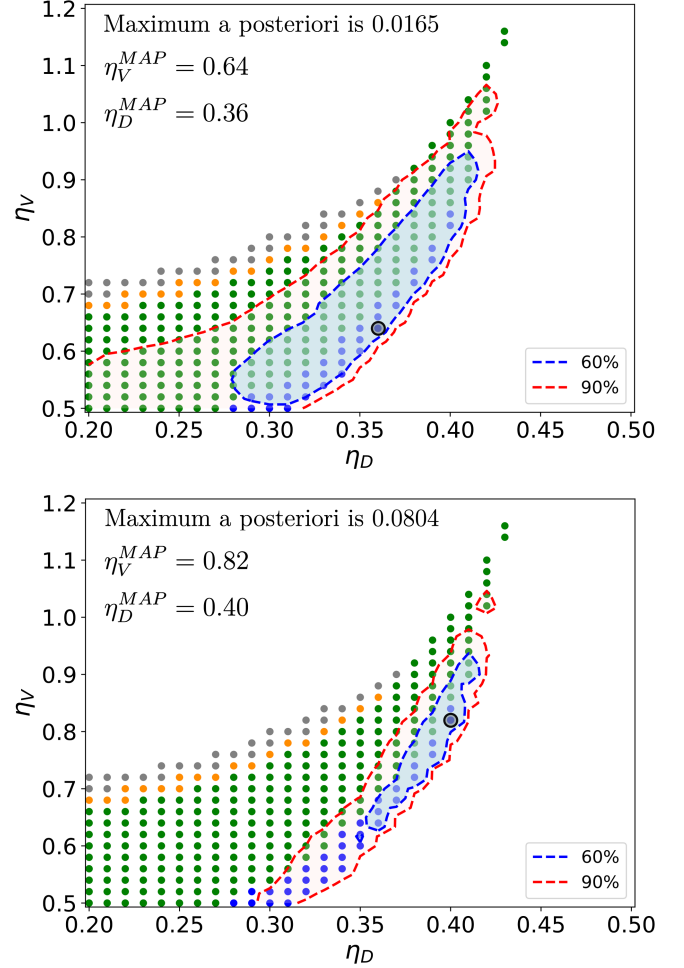


FIG. 7. The same as in Fig. 5 but with overlaid 60% and 90% credibility contours obtained for the basic (upper panel) and full (lower panel) sets of constraints from the observational data depicted in Fig. 6. The maximum *a posteriori* (MAP) sets of the parameters η_V^{MAP} and η_D^{MAP} are given in the legends and shown as fat dots on both panels of the figure.

vector one is also high. Based on this we conclude that the most probable and physically interesting values of the vector and diquark couplings are $\eta_V \simeq 0.82$ and $\eta_D \simeq 0.40$.

Figure 6 also shows the 90% and 60% confidence regions. As is seen, analyzing the full data set shifts these regions toward higher η_V and η_D , and narrows them compared to the case of the basic set. It is notable that compared to the recent results of the physically-educated Bayesian analysis [6] performed with the two-flavor NJL model with local vector repulsion [85], the 60% and 90% confidence regions obtained in this work do not include small values of the vector coupling $\eta_V \lesssim 0.5$ and $\eta_V \lesssim 0.6$ corresponding to the basic and full sets, respectively. Such small values are disfavored by the observational data since they lead to an excessive softness of quark EOSs that cannot be compensated by the vector repulsion, which in the nonlocal case is subdominant at high densities [12].

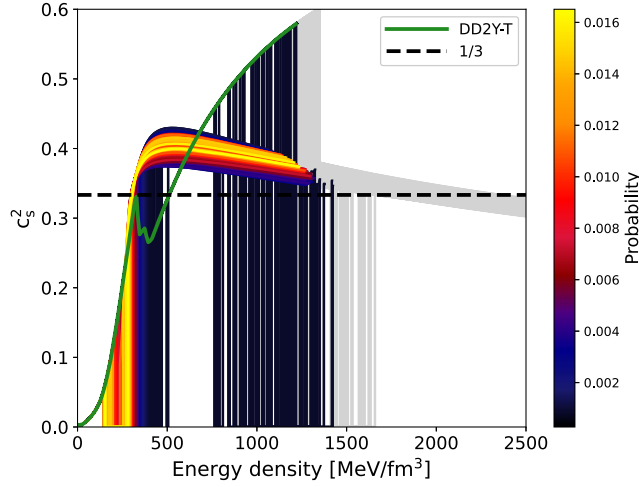


FIG. 8. Speed of sound squared c_s^2 of the NS matter as a function of energy density ε . The color map corresponds to the posterior probability evaluated using the full set of the observational data. The gray parts of the curves correspond to the densities beyond the central density in the centers of the heaviest NSs.

Figure 7 demonstrates the 60% and 90% confidence contours overplayed in the $\eta_V - \eta_D$ region showing the combinations of the couplings, which provide the existence of hybrid quark-hadron EOSs with a positive density jump across the phase transition. As is seen, the couplings leading to the quark onset density above the central density in the heaviest purely hadronic NS (gray dots in Fig. 7) are beyond these contours. In other words, most of the hybrid EOSs provide significantly better agreement with the observational data on NSs than the benchmark DD2Y-T EOS. This is a statistical evidence in favor of the scenario of NSs with quark cores compared to the purely hadronic scenario. The robustness of this evidence can be quantified by comparing the value of the posterior probability at its global maximum to the probability of the hadronic DD2Y-T EOS. The corresponding ratio is 18.73 and 276.15 for the basic and full sets of the observational data, respectively.

Figure 8 shows the squared speed of sound of the EOSs used for the Bayesian analysis of the full set of the observational data. Note, the results corresponding to the basic set are not shown since visually they are almost identical. The vertical lines indicate the boundaries of the mixed phases, which have constant pressures and, consequently, vanishing speed of sound. Such structure of c_s^2 as a function of ε is convenient for localizing quark deconfinement and is also seen in Fig. 1. Based on this, Fig. 8 allows us to make the first important conclusion that the most probable EOSs have a first-order phase transition at the energy density $\varepsilon = 150\text{--}300 \text{ MeV/fm}^3$. This range agrees well with the recent results of the Bayesian analysis with the two-flavor chiral quark model with local vector repulsion [6]. The values of ε within this range are rather small and

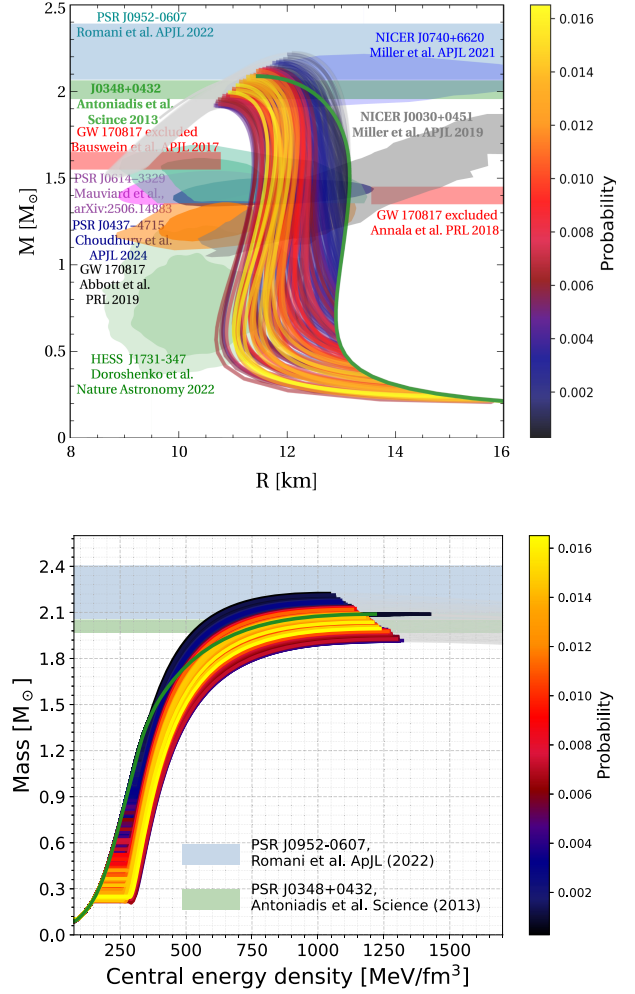


FIG. 9. Dependence of the mass M of NS on their radius R (upper panel) and central energy density ε_c (lower panel). The color map corresponds to the posterior probability evaluated using the full set of the observational data. The gray parts of the curves correspond to unstable configurations which have central energy densities exceeding the value for the maximum mass configuration. The shaded areas represent the observational constraints from Refs. [74,76,78,84,94–99].

correspond to 1–2 nuclear saturation densities. Similar small values of the onset density of deconfinement have also been reported in Refs. [86,87]. They should not be perceived as a contradiction to the fact that the experiments on relativistic heavy ion collisions (HIC) do not indicate a phase transition up to the densities about 5 nuclear saturation ones [88–91]. The resolution of this apparent paradox is related to a strong dependence of the onset density of quark matter on isospin asymmetry [92]. The very recent analysis of this dependence indicates that the onset density of deconfinement in the symmetric matter created in HIC can exceed the onset density in strongly asymmetric NS matter by a factor 3–4 [93].

Figure 8 indicates the absence of the quark-hadron phase transition at the energy densities $\varepsilon = 500\text{--}750 \text{ MeV/fm}^3$.

To explain this we notice that in the DD2Y-T EOS hyperons onset at the energy density about $310 \text{ MeV}/\text{fm}^3$. As is seen from Figs. 3 and 4, this softens the hadronic EOS. Figure 3 also shows that at $\varepsilon = 500\text{--}750 \text{ MeV}/\text{fm}^3$ this softening is so strong that a positive energy density jump from the DD2Y-T EOS is impossible even for the softest quark EOSs. This disables the possibility of the phase transition in the mentioned range of energy densities.

It is also seen from Fig. 8 that in the most probable scenarios, the NS matter becomes superconformal immediately after the phase transition. The corresponding speed of sound reaches its maximal value $c_s^2 \simeq 0.40$ at the energy density $\varepsilon \simeq 500 \text{ MeV}/\text{fm}^3$. Later, the squared speed of sound gradually decreases reaching the value $c_s^2 \simeq 0.38$ at the density $\varepsilon \simeq 1250 \text{ MeV}/\text{fm}^3$, which is the central density of the heaviest NSs. Thus, the behavior of the NS matter is likely to be nonconformal since its squared speed of sound significantly exceed the conformal value $c_s^2 = 1/3$.

The superconformal speed of sound of the NS matter ensures its stiffness needed to reach the masses of the PSR J0348 + 0432 [84] and PSR J0470 + 6620 [69]. This is seen from Fig. 9, which shows the dependence of the NS mass on its radius and central density. For the most probable scenarios the maximum mass is $2.0 - 2.2 M_\odot$. These values are smaller than in the case of the two-flavor chiral quark model with local vector repulsion [6]. This is due to the subdominant character of the nonlocal vector repulsion at high densities, which softens the quark EOS. Nevertheless, the most probable scenarios agree with the high mass of the black widow pulsar PSR J0952-0607 [66] corrected according to its rotation frequency (see footnote 2). An important conclusion, which can be drawn from Fig. 9, is that the most probable onset masses of quark matter in NSs are below $1 M_\odot$. This is a direct consequence of the small onset density of quark matter discussed above. Phenomenologically, this means that all the observed NSs contain quark cores in their interiors. It is remarkable that this scenario is statistically favorable compared to the purely hadronic NSs. Our findings suggest that the most probable radius of a $1.4 M_\odot$ canonical NS is $R_{1.4} = 11.0\text{--}12.0 \text{ km}$. It is also seen from Fig. 9 that the central density of the heaviest NSs is likely to not exceed $\varepsilon \simeq 1250 \text{ MeV}/\text{fm}^3$, which is close to $\varepsilon \simeq 1221 \text{ MeV}/\text{fm}^3$ corresponding to the benchmark scenario of purely hadronic NSs. Figure 9 also demonstrates that some hybrid EOSs extend the range of energy densities reached in the NS interiors without changing the maximum mass compared to the purely hadronic case. The NSs with these EOSs have very small TOV-stable quark cores of sizes not exceeding a few hundreds of meters. Fig. 10 shows the energy density profile for one of such NSs. Their small quark cores make almost no contribution to the total masses, which exceed the maximum mass of purely hadronic NSs just slightly. This produces a tiny bump in the dependence of the total mass of NSs on their central density. The corresponding quark onset density is

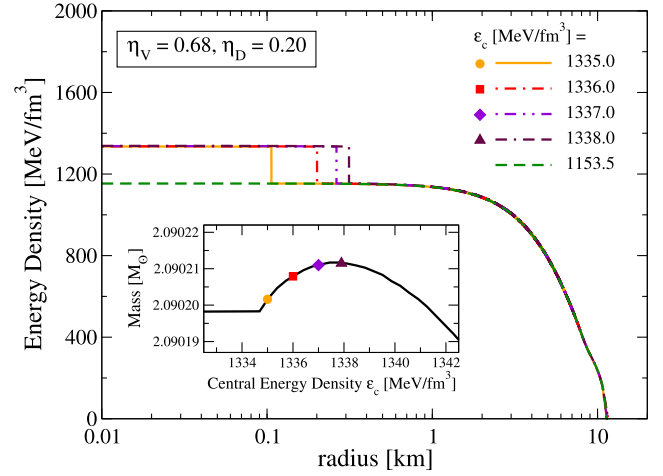


FIG. 10. Energy-density profiles of stable hybrid-star configurations for the case $\eta_V = 0.68$ and $\eta_D = 0.20$ in which the phase transition occurs very close to the central energy density of the heaviest purely hadronic DD2Y-T configuration and jumps above this value (such EOS are shown by the orange point in Fig. 7). Profiles are given for central energy densities of 1335, 1336, 1337, and 1338 MeV/fm^3 . The green dashed line shows the purely hadronic profile with central energy density equal to onset energy density. The inset displays the gravitational mass as a function of the central energy density for the same EOS, with colored markers indicating the masses of the configurations whose profiles are plotted in the upper panel.

just slightly below the central energy density in the heaviest purely hadronic NS, $1221 \text{ MeV}/\text{fm}^3$, while, as is seen from lower panel of Fig. 9 and the inset in Fig. 10, ε is significantly increased by its jump at the phase transition. As a result, the mass-central energy density diagrams of such hybrid NSs with small quark cores look just like an extension of the purely hadronic diagram to higher energy densities up to $1423 \text{ MeV}/\text{fm}^3$.

VI. CONCLUSIONS

We have performed a physics-informed Bayesian analysis of the observational data on the masses, radii, and tidal deformability of NSs within the scenario when their interiors undergo a first-order phase transition to quark matter modeled within the framework of Maxwell construction of hybrid EOS. The scheme used allows us to distinguish between the scenarios of NSs with quark cores and without them, which is its important advantage compared to the model-agnostic Bayesian analysis. The results of the analysis are confronted to the baseline scenario of purely hadronic NSs and are shown to favor hybrid NSs with quark cores in their interiors.

To exclude systematic uncertainty due to unreliable modeling of hadronic matter, we utilized the microscopic DD2Y-T EOS, which agrees with the low-density constraints of χEFT , reproduces the properties of the ground state of normal nuclear matter and includes hyperonic

degrees of freedom. The three-flavor quark matter is modeled within a version of the NJL model with nonlocal interaction. In the deconfinement region the most important interaction channels are strong vector repulsion and diquark pairing controlled by the dimensionless couplings η_V and η_D of the microscopic Lagrangian. Although vector repulsion ensures the stiffness of dense matter required to reach NS masses as high as at least $2M_\odot$, the pairing of diquarks leads to the formation of a color-superconducting CFL phase. The latter is important to provide not too late onset of deconfinement even in the case of very stiff quark matter, which is required for the TOV-stability of the quark branch of the NS mass-radius relation. For simplicity, the analysis is limited to the case where the three quark flavors are degenerate in mass. However, this does not introduce much uncertainty since in the deconfinement region bare quark masses do not play an important phenomenological role [12].

A new aspect of the study is the asymptotically conformal behavior of the quark matter EOS, which is provided by the nonlocal character of the interaction among quarks. This element of the model automatically provides its consistency with the high-density constraints of perturbative QCD. Moreover, regardless of the values of the vector and diquark couplings, the conformal value of the squared speed of sound is reached from below, in agreement with the results of perturbative QCD.

The vector and diquark couplings, which provide a proper Maxwell crossing of the quark and hadron EOSs, are found to belong to a wedge-shaped region in the $\eta_V - \eta_D$ plane. This region is limited from right-below by the condition that the quark-hadron phase transition should not occur below the nuclear saturation density. A narrow region in the vicinity of this low-density boundary represents the twin configurations of NSs. The upper left boundary of the wedge-shaped region occurs because the phase transition is not possible there. In a narrow region in the vicinity of this high-density boundary the phase transition either occurs at energy densities beyond the value in the center of the maximum mass configuration of the purely hadronic NSs, or at densities close to it where the quark branch of the corresponding mass-radius relation is TOV-unstable.

The Bayesian analysis of two sets of the observational data allowed us to find the posterior probability distribution of the vector and diquark couplings. The basic set is given by the mass-radius data from the PSR J0030 + 0451 [68], PSR J0740 + 6620 [69], PSR J0437–4715 [70], and the tidal deformability data from the GW170817 [73], while the full set additionally includes the mass-radius data from the PSR J0952–0607 [66] and HESS J1731–347 [72]. Similar to the previous findings of Refs. [86,87,100], the found 60% and 90% credibility regions elongate in the $\eta_V - \eta_D$ plane along contours with positive slopes. For the basic and full sets of the observational data, these regions concentrate around the most likely values of the

couplings $\eta_V \simeq 0.64$ and $\eta_D \simeq 0.36$, and $\eta_V \simeq 0.82$ and $\eta_D \simeq 0.40$, respectively. Unlike the NJL model with local vector repulsion [6], the 60% and 90% credibility regions do not include $\eta_V \lesssim 0.5$ for the basic and $\eta_V \lesssim 0.6$ for the full sets. This is because in the nonlocal case considered in this work the vector repulsion is subdominant at high densities, which makes small vector couplings inconsistent with high NS masses. The main modifications related to using the full set of observational data compared to the basic set correspond to increasing the most probable values of the vector and diquark couplings and narrowing the 60% and 90% credibility regions.

The most probable EOSs, which belong to the 60% credibility region, have quark-hadron phase transitions at the energy density $150 - 300 \text{ MeV/fm}^3$, i.e., at baryon densities not exceeding twice the nuclear saturation density. It is remarkable that the corresponding onset masses of quark matter in NSs are well below $1.0M_\odot$. This suggests that all the observed NSs have quark cores. The squared speed of sound of the most probable EOSs exhibits a peak of the amplitude $c_s^2 \simeq 0.40$ located in the quark phase at $\varepsilon \simeq 500 \text{ MeV/fm}^3$. The central densities of the heaviest NSs do not exceed 1250 MeV/fm^3 , when $c_s^2 \simeq 0.38$. This signals that the behavior of quark matter, which can exist in the NS interiors, is firmly superconformal. This allows NSs with CFL cores to reach maximum masses with the most probable values around $2.2M_\odot$.

The most important finding of the work is that the posterior probabilities of the most likely hybrid quark-hadron EOSs significantly exceed the probability corresponding to the purely hadronic benchmark EOS. The ratio of these probabilities is 18.73 for the basic data set and 276.15 for the entire set of observational data. This provides statistical evidence that the scenario of NSs with CFL quark cores is favored compared to the scenario of purely hadronic NSs.

Despite the fact that the analysis performed within this work is based on one hadronic EOS only, the main conclusion can be considered as robust. The reason is that the most probable NS sequences have an early onset of deconfinement at densities not exceeding two nuclear saturation densities. Within this density range the properties of nuclear matter are firmly constrained both experimentally and theoretically. Therefore, all realistic and reliable hadronic EOSs that could be used to complement the benchmark one we used here would result in very similar values for the most probable densities of the onset of deconfinement in NSs. This ensures the robustness of the conclusions drawn in this work against reasonable variations of the hadronic EOS.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [101], embargo periods may apply.

APPENDIX A: THERMODYNAMIC POTENTIAL OF THE THREE-FLAVOR NONLOCAL NJL MODEL

The Hubbard-Stratonovich transformation of the partition function $\mathcal{Z}$ is given by the identity

$$\mathcal{Z} = \int \mathcal{D}\bar{q}\mathcal{D}q \exp\left(\int dx \mathcal{L}\right) = \int \mathcal{D}\bar{q}\mathcal{D}q \mathcal{D}\sigma_a \mathcal{D}\omega_\mu \mathcal{D}d_{ab}^* \mathcal{D}d_{ab} \exp\left(\int dx \mathcal{L}^{\text{bos}}\right). \quad (\text{A1})$$

The exponential in this expression includes the integration over the four-space of the volume βV , where $\beta = 1/T$ is the inverse temperature and V stands for the three-volume. The auxiliary bosonic fields σ_a , ω_μ and d_{ab} are conjugated to the quark bilinears in the scalar, vector and diquark interaction channels, and enter the bosonized Lagrangian as

$$\mathcal{L}^{\text{bos}} = \bar{q}(i\not{\partial} - m + \mu\gamma_0)q - \sum_{a=0,8} \left(s_a \sigma_a + \frac{\sigma_a \sigma_a}{4G_S} \right) + j_\mu \omega^\mu + \frac{\omega_\mu \omega^\mu}{4G_V} - \sum_{a,b=2,5,7} \left(\frac{d_{ab}^+ \Delta_{ab} + \Delta_{ab}^* d_{ab}}{2} + \frac{\Delta_{ab}^* \Delta_{ab}}{12G_D} \right). \quad (\text{A2})$$

It gives a direct access to the Euler-Lagrange equations for the bosonic fields. Within the mean field approximation applied in Ref. [12], these fields are replaced by the expectation values obtained from the Euler-Lagrange equations. The corresponding functional integrations are suppressed. Equivalently, these expectation values can be obtained by minimizing the thermodynamic potential.

Noticing that charge conservation requires the commutation of the mean-field Lagrangian with the charge matrix $\hat{Q} = \text{diag}(2/3, -1/3, -1/3)$ [102], we conclude the absence of the nondiagonal flavor group generators in the mean-field Lagrangian, i.e. vanishing of the expectation values of the corresponding scalar currents and scalar fields. This is also the case for the scalar fields connected to the traceless flavor generators τ_3 and τ_8 . Thus, $\langle \sigma_a \rangle = \delta_{a0} \sigma$ under the mean field approximation. Applying the proper Lorentz transformation, we bring the expectation value of

the vector field to the form $\omega_\mu = g_{\mu 0} \omega$. Only three diquark fields survive the averaging in the ground state due to the locking of the color and flavor indexes of quarks in the CFL matter [103]. We utilize a color-flavor rotation, at which the surviving fields have a diagonal structure $a = b$ [19]. Furthermore, given the flavor symmetry of the CFL phase, we conclude that the expectation values of these diquark fields coincide. Based on this we introduce $\Delta \equiv \langle \Delta_{11} \rangle + \langle \Delta_{22} \rangle + \langle \Delta_{33} \rangle$ so that $\langle \Delta_{ab} \rangle = \delta_{ab} \Delta/3$. In general, Δ is complex. But it enters the mean-field Lagrangian via the product with its complex conjugate Δ^* . As a result, only the modulus of this quantity appears in the expression for the thermodynamic potential. Therefore, below Δ stands for this modulus, while its sign is suppressed for shortening the notations. With this the mean-field Lagrangian becomes

$$\mathcal{L}_{\text{MF}} + \mu q^+ q = \bar{Q} \hat{S}^{-1} Q - \frac{\sigma^2}{4G_S} + \frac{\omega^2}{4G_V} - \frac{\Delta^2}{4G_D}. \quad (\text{A3})$$

Here $Q^T = (q, q^c)/\sqrt{2}$ and $\hat{S}$ are the Nambu-Gorkov bispinor and propagator, respectively, while $q^+ q$ stands for the conserved quark number density conjugated to the quark chemical potential μ . In the momentum representation the inverse Nambu-Gorkov propagator reads as

$$\hat{S}^{-1} = \begin{pmatrix} S_+^{-1} & i\Delta g_k \gamma_5 \mathcal{O} \\ i\Delta^* g_k \gamma_5 \mathcal{O} & S_-^{-1} \end{pmatrix}, \quad (\text{A4})$$

where $S_\pm^{-1} = \not{k} - M_{\mathbf{k}} \pm \gamma_0(\mu + \omega g_{\mathbf{k}})$, the zeroth component of the four-momentum k represents the Matsubara frequencies of fermions, i.e. $k_0 = i(2n+1)\pi T$, $M_{\mathbf{k}} = m + \sigma g_{\mathbf{k}}$ is the effective momentum-dependent quark mass introduced in Sec. II, and the operator $\mathcal{O} = \tau_2 \lambda_2 + \tau_5 \lambda_5 + \tau_7 \lambda_7$ acts in the color-flavor space and is introduced for shortening the notations.

The single-quark energies can be found by solving equation $\det \hat{S} = 0$ with respect to k_0 . This is equivalent to analyzing the nonvanishing elements of the diagonalized Nambu-Gorkov propagator. Such a diagonalization is provided by a rotation in the color-flavor space. The corresponding quark basis is not given by their color-flavor states but by an octet of degenerate states and a singlet state [19]. The single-particle energies and degeneracies of these octet and singlet states, which are introduced in Sec. II, are labeled by the subscript index “ j ”.

The mean-field Lagrangian (A3) is quadratic in quark fields, which allows integrating them out. The resulting statistical partition is

$$\ln \mathcal{Z} = \ln \int \mathcal{D}\bar{Q} \mathcal{D}Q \exp\left(\int dx (\mathcal{L}_{\text{MF}} + \mu \bar{q} q)\right) = \frac{1}{2} \text{tr} \ln \hat{S} - \beta V \left(\frac{\sigma^2}{4G_S} - \frac{\omega^2}{4G_V} + \frac{\Delta^2}{4G_D} \right), \quad (\text{A5})$$

where the factor $1/2$ in the first term compensates the artificial Nambu-Gorkov doubling of degrees of freedom. Performing the trace over the Nambu-Gorkov, Dirac, color, flavor, three-momentum, and Matsubara indexes, and using the thermodynamic identity $\Omega = -\ln \mathcal{Z}/\beta V$, we arrive at the thermodynamic potential

$$\Omega = -\sum_{j,a} d_j \int \frac{d\mathbf{k}}{(2\pi)^3} \left[\frac{\epsilon_{jk}^a}{2} - T \ln(1 - f_{jk}^a) \right] + \frac{\sigma^2}{4G_S} - \frac{\omega^2}{4G_V} + \frac{\Delta^2}{4G_D}, \quad (\text{A6})$$

where $f_{jk}^a = (1 + e^{-\epsilon_{jk}^a})^{-1}$ is the single particle distribution function of quarks ($a = +$) and antiquarks ($a = -$). The final step in deriving Eq. (6) corresponds to taking the zero-temperature limit. For this, we notice that

$$T \ln(1 - f_{jk}^a)|_{T \rightarrow 0} = \epsilon_{jk}^a \theta(-\epsilon_{jk}^a). \quad (\text{A7})$$

Inserting this identity into Eq. (A6) we arrive at the zero temperature thermodynamic potential from Sec. II.

APPENDIX B: COMPETING CONSTRAINTS AT $2M_\odot$

In this appendix, we want to illustrate the effect that using the precise mass measurement of the $2M_\odot$ pulsar J0348 + 0432 [84], which comes without a constraint for its radius, along with the NICER combined mass-radius measurement for the other $2M_\odot$ pulsar J0740 + 6620, has on the 60% and 90% confidence regions in the $\eta_V - \eta_D$ parameter plane. We show the result in Fig. 11.

Comparison of this result with both panels of Fig. 7 in Sec. V reveals the counter-intuitive effect that adding another mass constraint does not narrow the posterior confidence regions, as one could naively expect, but actually widens it! This concerns in particular the 90% confidence region (shown by a red dashed line), which now includes all gray and yellow marked parameter regions that would correspond to purely hadronic NS sequences in the

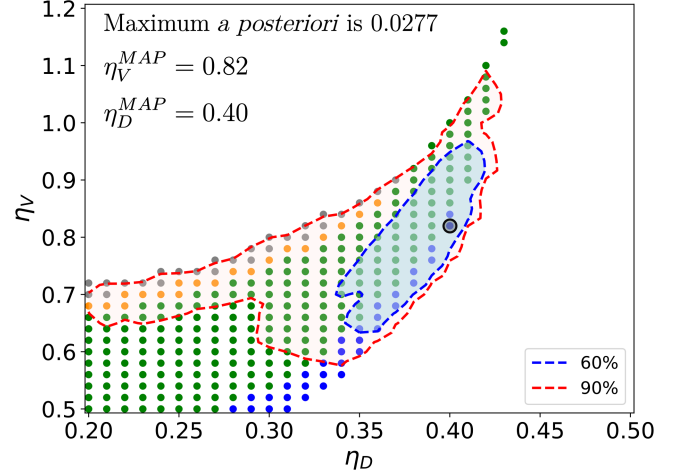


FIG. 11. The same as in Fig. 7 but with contours of the 60% and 90% confidence intervals obtained from the Bayesian analysis of the full set of the observational data supplemented with the constraint from the precise mass measurement on the $2M_\odot$ pulsar J0348 + 0432 [84] which comes without a constraint for its radius. The maximum *a posteriori* (MAP) set of the parameters η_V^{MAP} and η_D^{MAP} is given in the legend and shown as a fat dot in the figure.

$M - R$ diagram. In Fig. 7, these purely hadronic sequences are not included in the 90% confidence region, and thus are clearly disfavored against the hybrid star sequences. How do we explain this widening of the confidence region? Adding a $2M_\odot$ pulsar mass that is not constrained with respect to the radius, the hadronic baseline sequences at high mass, which did not satisfy the NICER constraint on the minimal radius $R_{2.0} < 11.79$ km at that mass, gain an additional probability from the less restrictive mass-only constraint of the PSR J0348 + 0432. We, therefore, suggest that once a more restrictive combined mass-radius measurement like the ones provided by NICER exists, one should not neutralize the selective power of this measurement by an unconstrained mass-only measurement, even if it is rather precise.

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