

Gluon TMDs for tensor polarized deuterons in a spectator model

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We present a model calculation of the transverse-momentum-dependent distributions (TMDs) for gluons in a tensor-polarized deuteron. Our model is based on the assumption that an on-shell deuteron can emit a timelike off-shell gluon, while the remaining system is treated as a single on-shell spectator particle whose mass spans a continuous real-value distribution parametrized via a spectral function. For spin-1 hadrons, the polarization is characterized not only by a spin vector S but also by a symmetric traceless spin tensor T . The deuteron-gluon-spectator coupling is described by an effective vertex containing three form factors. We obtain analytical expressions for thirteen T-even gluon TMDs. We also provide numerical results for the x -dependence and k_T -dependence of these TMDs. Our analysis reveals non-negligible results of these gluon TMDs, especially for tensor-polarized hadrons, which could potentially be explored in future experimental measurements.

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I. INTRODUCTION

Transverse-momentum-dependent parton distributions (TMDs) are fundamental quantities that encode novel information about the internal momentum and spin structure of hadrons, extending beyond the knowledge obtained from high-precision measurements of conventional parton distribution functions (PDFs). TMDs describe the parton distributions as functions of both the partonic transverse momentum k_T and the longitudinal momentum fraction x . Their significance stems from their unique sensitivity to the dynamics of color charges in quantum chromodynamics (QCD), as the transverse momentum dependence is intimately connected with gauge links and, consequently, with the nonperturbative behavior of color fields, which are otherwise difficult to access due to color confinement.

Due to their intrinsically nonperturbative nature, TMDs cannot be directly computed from first principles within perturbative QCD alone. Describing hadron structure thus requires solving QCD in the nonperturbative regime. The state-of-the-art first-principle tool for this purpose is lattice QCD (LQCD), which has achieved impressive progress in recent decades [1–4]. Phenomenological models [5–11], which serve as essential tools for gaining insights into hadron structure, play a complementary role to lattice

studies. They are typically employed to provide insights into the underlying dynamics and to offer predictions for quantities that are currently challenging to access via lattice simulations. While TMDs have been extensively investigated for spin-1/2 nucleons [12–14], studies of TMDs and structure functions for spin-1 hadrons remain relatively underdeveloped.

On the experimental side, no new measurements of polarized structure functions of spin-1 hadrons have been reported since the HERMES measurement of the $b_1(x)$ function for the deuteron in 2005 [15]. However, this situation is expected to change with several upcoming or proposed experimental programs, including those at the JLab [16], the Fermilab [17], the NICA [18], the LHC-spin [19], and the electron-ion colliders (EIC, EicC) [20,21]. Theoretically, the $b_1(x)$ structure function can be estimated, for example, using a convolution model of nucleon light-cone momentum distributions. Predictions from such simple models, however, often deviate significantly from the HERMES data, highlighting the need for more sophisticated theoretical approaches.

A particularly intriguing observable is the gluon structure function $\Delta_T g(x)$ [22], which corresponds to the PDF $h_{1TT}(x)$. This function is associated with the double-helicity flip amplitude in processes involving hadrons with spin ≥ 1 . Specifically, $h_{1TT}(x)$ describes the distribution of linearly polarized gluons within a transversely tensor-polarized target. It is sometimes referred to as “gluon transversity,” a term that can be misleading, as it suggests a transverse polarization of the gluon itself, whereas it actually specifies the polarization state of the target hadron. The experimental observation of a nonzero $\Delta_T g(x)$ would be a clear signal of non-nucleonic degrees of freedom

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within the deuteron, as such a distribution cannot arise from a simple system of bound nucleons. Therefore, measurements of unique spin-1 observables like $b_1(x)$ and $\Delta_T g(x)$ offer a potential gateway to unraveling new aspects of hadron physics in the nuclear domain.

This work employs a spectator model to calculate leading-twist T-even gluon TMDs for a tensor-polarized deuteron. In this picture, the deuteron is assumed to emit a timelike, off-shell gluon, while the remaining system is effectively treated as a single on-shell spectator particle. The mass of this spectator is not fixed but instead takes a continuous range of values, as described by a spectral function. To fully characterize the polarization of a spin-1 hadron, its density matrix involves not only a spin vector S^μ but also a symmetric, traceless rank-2 spin tensor $T^{\mu\nu}$. The deuteron-gluon-spectator vertex is modeled via an effective coupling that incorporates three independent form factors. This spectator approach has been successfully applied in previous studies to compute quark TMDs for nucleons [5,23–27] and pions [28–30], as well as gluon TMDs and TMD fragmentation functions for the proton [31–35]. Here, we extend this formalism to the case of a tensor-polarized deuteron. At leading twist, the T-even gluon TMDs consist of the unpolarized distribution f_1 , three vector-polarized distributions ($h_1^\perp$, g_1 , g_{1T}), and nine tensor-polarized distributions (f_{1LL} , f_{1LT} , f_{1TT} , $h_{1LL}^\perp$, $h_{1LT}^\perp$, $h_{1TT}^\perp$, $h_{1LL}^\perp$, $h_{1LT}^\perp$, $h_{1TT}^\perp$). The model parameters are determined by fitting to the unpolarized gluon TMD f_1 , ensuring a realistic description of the nonpolarized baseline.

The structure of this paper is as follows. In Sec. II, we present the spectator model formalism for gluon TMDs. We derive the tree-level gluon-gluon correlator and extract the leading-twist T-even TMDs via appropriate projection operators. In Sec. III, we determine the model parameters by fitting to the unpolarized TMD f_1 and then present numerical predictions for all T-even gluon TMDs. We summarize our findings and conclude in Sec. IV.

II. ANALYTIC CALCULATION OF THE T-EVEN TMDs OF TENSOR POLARIZED DEUTERON

We work with the light-cone vectors $n_+^\mu \equiv [1, 0, \mathbf{0}_T]$ and $n_-^\mu \equiv [0, 1, \mathbf{0}_T]$, satisfying $n_+ \cdot n_- = 1$ and $n_\pm^2 = 0$. Any four-vector a^μ admits the light-cone decomposition $a^\mu = a^+ n_+^\mu + a^- n_-^\mu + a_T^\mu$, with $a^\pm \equiv a \cdot n_\mp$ and $a_T^\mu \equiv (0, 0, \mathbf{a}_T)$ denoting its transverse component. In a frame where the hadron carries no transverse momentum, the hadron momentum P^μ and the parton momentum k^μ are given by

$$P^\mu = P^+ n_+^\mu + \frac{M^2}{2P^+} n_-^\mu, \quad (1)$$

$$k^\mu = xP^+ n_+^\mu + \frac{k^2 + \mathbf{k}_T^2}{2xP^+} n_-^\mu + \mathbf{k}_T, \quad (2)$$

where M is the hadron mass and $x = k^+/P^+$ is the longitudinal momentum fraction carried by the parton.

For a spin- J particle, the polarization density matrix is parametrized in terms of irreducible spin tensors up to rank $2J$. Following Ref. [36], a complete description of the polarization of a spin-1 hadron requires both a spin vector S^μ and a symmetric traceless rank-2 spin tensor $T^{\mu\nu}$. Subject to the constraints $P \cdot S = 0$ and $P_\mu T^{\mu\nu} = 0$, they are decomposed as

$$S^\mu = S_L \frac{P^+}{M} n_+^\mu - S_L \frac{M}{2P^+} n_-^\mu + S_T^\mu, \quad (3)$$

$$T^{\mu\nu} = \frac{1}{2} \left[\frac{4}{3} S_{LL} \frac{(P \cdot n_-)^2}{M^2} n_+^\mu n_+^\nu - \frac{2}{3} S_{LL} (n_+^\mu n_-^\nu + n_-^\mu n_+^\nu) - g_T^{\mu\nu} \right. \\ \left. + \frac{1}{3} S_{LL} \frac{M^2}{(P \cdot n_-)^2} n_-^\mu n_-^\nu + \frac{P \cdot n_-}{M} n_+^\mu S_{LT}^\nu \right. \\ \left. - \frac{M}{2P \cdot n_-} n_-^\mu S_{LT}^\nu + S_{TT}^{\mu\nu} \right], \quad (4)$$

where $g_T^{\mu\nu} = g^{\mu\nu} - n_+^\mu n_-^\nu$ is the symmetric transverse metric tensor, and the notation $a^{\{\mu} b^{\nu\}} \equiv a^\mu b^\nu + a^\nu b^\mu$ denotes symmetrization of the indices.

The polarization sum for a spin-1 hadron can be expressed in terms of S^μ and $T^{\mu\nu}$ as

$$\epsilon^{*\mu}(P, \lambda) \epsilon^\nu(P, \lambda) = -\frac{1}{3} \left(g^{\mu\nu} - \frac{P^\mu P^\nu}{M^2} \right) + \frac{i}{2M} \epsilon^{\mu\nu\alpha\beta} P_\alpha S_\beta - T^{\mu\nu}. \quad (5)$$

The gauge-invariant gluon-gluon correlator for spin-1 hadrons is expressed as [14,37]

$$\Phi^{\mu\nu;\rho\sigma}(x, \mathbf{k}_T; S, T) = \frac{1}{xP^+} \int \frac{d\xi^+ d\xi_T}{(2\pi)^3} e^{ik \cdot \xi} \\ \times \langle P, S, T | F^{\mu\nu}(0) U_{[0,\xi]} F^{\rho\sigma}(\xi) U'_{[\xi,0]} | P, S, T \rangle \Big|_{\xi^+=0}, \quad (6)$$

where a summation over color indices is implicit, and the two process-dependent gauge links $U_{[0,\xi]}$ and $U'_{[\xi,0]}$ ensure color gauge invariance.

At leading twist, we focus on the correlator $\Phi^{+,i,j} \equiv \Phi^{ij}$, with i, j being transverse spatial indices. This correlator can be parametrized in terms of the leading-twist gluon TMDs as [38]

$$\Phi_U^{ij}(x, \mathbf{k}_T) = \frac{1}{2} \left[-g_T^{ij} f_1(x, \mathbf{k}_T^2) + \frac{k_T^i k_T^j}{M^2} h_1^\perp(x, \mathbf{k}_T^2) \right], \quad (7)$$

$$\Phi_L^{ij}(x, \mathbf{k}_T) = \frac{1}{2} \left[i e_T^{ij} S_L g_1(x, \mathbf{k}_T^2) + \frac{\epsilon_{T\alpha}^{i\alpha} k_T^\alpha}{2M^2} S_L h_{1L}^\perp(x, \mathbf{k}_T^2) \right], \quad (8)$$

$$\Phi_T^{ij}(x, \mathbf{k}_T) = \frac{1}{2} \left[-\frac{g_T^{ij} \epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x, \mathbf{k}_T^2) + \frac{i \epsilon_T^{ij} \mathbf{k}_T \cdot \mathbf{S}_T}{M} g_{1T}(x, \mathbf{k}_T^2) - \frac{\epsilon_T^{k_T \{i} S_T^{j\}} + \epsilon_T^{S_T \{i} k_T^{j\}}}{4M} h_1(x, \mathbf{k}_T^2) - \frac{\epsilon_{Ta}^{i} k_T^{j\} \alpha S_T}{2M^3} h_{1T}^\perp(x, \mathbf{k}_T^2) \right], \quad (9)$$

$$\Phi_{LL}^{ij}(x, \mathbf{k}_T) = \frac{1}{2} \left[-g_T^{ij} S_{LL} f_{1LL}(x, \mathbf{k}_T^2) + \frac{k_T^{ij} S_{LL}}{M^2} h_{1LL}^\perp(x, \mathbf{k}_T^2) \right], \quad (10)$$

$$\Phi_{LT}^{ij}(x, \mathbf{k}_T) = \frac{1}{2} \left[-\frac{g_T^{ij} \mathbf{k}_T \cdot \mathbf{S}_{LT}}{M} f_{1LT}(x, \mathbf{k}_T^2) + \frac{i \epsilon_T^{ij} \epsilon_T^{S_{LT} k_T}}{M} g_{1LT}(x, \mathbf{k}_T^2) + \frac{S_{LT}^{i} k_T^{j\} \alpha}{M} h_{1LT}(x, \mathbf{k}_T^2) + \frac{k_T^{ij} S_{LT\alpha}}{M^3} h_{1LT}^\perp(x, \mathbf{k}_T^2) \right], \quad (11)$$

$$\Phi_{TT}^{ij}(x, \mathbf{k}_T) = \frac{1}{2} \left[-\frac{g_T^{ij} k_T^{\alpha\beta} S_{TT\alpha\beta}}{M^2} f_{1TT}(x, \mathbf{k}_T^2) + \frac{i \epsilon_T^{ij} \epsilon_T^{\beta} k_T^{\gamma\alpha} S_{TT\alpha\beta}}{M^2} g_{1TT}(x, \mathbf{k}_T^2) + S_{TT}^{ij} h_{1TT}(x, \mathbf{k}_T^2) + \frac{S_{TT\alpha}^{i} k_T^{j\} \alpha}{M^2} h_{1TT}^\perp(x, \mathbf{k}_T^2) + \frac{k_T^{ij\alpha\beta} S_{TT\alpha\beta}}{M^4} h_{1TT}^{\perp\perp}(x, \mathbf{k}_T^2) \right]. \quad (12)$$

Here, $\epsilon_T^{ij} = \epsilon^{n_+ n_- ij} = \epsilon^{-+ij}$ is the antisymmetric transverse tensor, with the nonzero components $\epsilon_T^{12} = 1$. The symmetric traceless tensors $k_T^{i_1 \dots i_n}$ are defined as

$$k_T^{ij} \equiv k_T^i k_T^j + \frac{1}{2} k_T^2 g_T^{ij}, \quad (13)$$

$$k_T^{ijk} \equiv k_T^i k_T^j k_T^k + \frac{1}{4} k_T^2 (g_T^{ij} k_T^k + g_T^{jk} k_T^i + g_T^{ki} k_T^j), \quad (14)$$

$$k_T^{ijkl} \equiv k_T^i k_T^j k_T^k k_T^l + \frac{1}{6} k_T^2 (g_T^{ij} k_T^{kl} + g_T^{ik} k_T^{jl} + g_T^{il} k_T^{jk} + g_T^{jk} k_T^{il} + g_T^{jl} k_T^{ik} + g_T^{kl} k_T^{ij}) - \frac{1}{8} k_T^4 (g_T^{ij} g_T^{kl} + g_T^{ik} g_T^{jl} + g_T^{il} g_T^{jk}), \quad (15)$$

satisfying the traceless conditions

$$g_{Tij} k_T^{ij} = g_{Tij} k_T^{ijk} = g_{Tij} k_T^{ijkl} = 0. \quad (16)$$

Table I summarizes the gluon TMDs appearing in Eqs. (7)–(12), classified according to hadron and parton

TABLE I. The columns indicate the gluon polarization, unpolarized (U), circularly polarized (Circ), and linearly polarized (Lin). The rows indicate the hadron polarization, unpolarized (U), vector polarized (L or T), and tensor polarized (LL, LT, or TT).

$H \backslash g$	U	Circ	Lin
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1 h_{1T}^\perp$
LL	f_{1LL}		$h_{1LL}^\perp$
LT	f_{1LT}	g_{1LT}	$h_{1LT} h_{1LT}^\perp$
TT	f_{1TT}	g_{1TT}	$h_{1TT} h_{1TT}^\perp h_{1TT}^{\perp\perp}$

polarizations. Among the 19 functions, six are odd under time reversal (T-odd): $f_{1T}^\perp$, $h_{1L}^\perp$, h_1 , $h_{1T}^\perp$, g_{1LT} , and g_{1TT} . In this work, we restrict ourselves to tree-level contributions and neglect effects arising from gauge links and their process dependence. At tree level, the correlator does not yield nonvanishing T-odd TMDs, as these typically require the interference of two amplitudes with distinct imaginary parts. To generate the necessary imaginary parts in the scattering amplitude, one must take into account gluon loop diagrams, including those arising from the gauge-link expansion.

In the spectator model, the deuteron state $|P, S, T\rangle$, characterized by momentum P , spin vector S , and spin tensor T , is assumed to split into an emitted gluon with momentum k , while the remaining system is treated as a single on-shell spectator particle of spin-1, denoted by $|P - k\rangle$, with momentum $P - k$ and mass M_S .

The tree-level gluon-gluon correlator in this model is given by

$$\Phi^{ij}(x, \mathbf{k}_T; S, T) = \frac{1}{xP^+} \frac{1}{(2\pi)^3} \frac{1}{2(1-x)P^+} \times [\bar{\mathcal{M}}_{0a}^i(x, \mathbf{k}_T; S, T) \mathcal{M}_{0a}^j(x, \mathbf{k}_T; S, T)]. \quad (17)$$

The tree-level gluon-gluon correlator in this model is given by

$$\begin{aligned} \mathcal{M}_{0a}^j(x, \mathbf{k}_T; S, T) &= \langle P - k | F_a^{+j} | P, S, T \rangle \\ &= \epsilon_c^{*\nu}(P - k, \lambda_S) G_{ab}^{j\alpha}(k, k) \mathcal{Y}_{\mu\nu\alpha, bc} \epsilon^\mu(P, \lambda). \end{aligned} \quad (18)$$

Here, $\epsilon^\mu(P, \lambda)$ is the deuteron polarization vector, and $\epsilon_c^{*\nu}(P - k, \lambda_S)$ is the polarization vector of the spectator with color index c . The factor

$$G_{ab}^{j\alpha}(p, k) = -\frac{i p^+}{k^2} \left(g^{j\alpha} - \frac{k^j n_-^\alpha}{p^+} \right) \quad (19)$$

corresponds to the Feynman rule for the field strength tensor $-i(p^\mu g^{\nu\rho} - p^\nu g^{\mu\rho})\delta_{ab}$ [39,40]. The effective vertex $\mathcal{Y}_{\mu\nu\alpha,bc}$ is modeled as

$$\mathcal{Y}_{bc}^{\mu\nu\alpha} = \left\{ \left[-g^{\mu\nu} g_1(k^2) - \frac{2\bar{P}^\mu \bar{P}^\nu}{M^2} g_3(k^2) \right] (P^\alpha + (P-k)^\alpha) + 2(\bar{P}^\nu g^{\alpha\mu} + \bar{P}^\mu g^{\alpha\nu}) g_2(k^2) \right\} \delta_{bc}, \quad (20)$$

with $\bar{P} = (P + (P-k))/2$. The functions $g_{1,2,3}(k^2)$ are the deuteron-gluon-spectator couplings. This parametrization follows the conventional form factor decomposition of the vector current for a spin-1 particle [41–43]. Various functional forms for $g_i(k^2)$ exist in the literature [26]. The simplest choice is a point-like coupling (i.e., constants g_i). However, this form leads to divergences upon integration over k_T . To regularize these divergences, we adopt an exponential form factor,

$$g_{1,2,3}(k^2) = \kappa_{1,2,3} e^{k^2/\Lambda_S^2}, \quad (21)$$

where $\kappa_{1,2,3}$ are free parameters and Λ_S is a cut-off scale.

Since the spectator is on-shell, $(P-k)^2 = M_S^2$, the virtuality of the gluon is given by

$$k^2 = -\frac{k_T^2}{1-x} - \frac{xM_S^2}{1-x} + xM^2. \quad (22)$$

Using Eqs. (17) and (18), the tree-level spectator approximation to the gluon-gluon correlator can be written as

$$\begin{aligned} \Phi^{ij}(x, k_T; S, T) &= \frac{1}{xP^+} \frac{1}{(2\pi)^3} \frac{1}{2(1-x)P^+} \\ &\times G_{ab'}^{i\beta*}(k, k) G_{ab}^{j\alpha}(k, k) \mathcal{Y}_{\mu'\nu'\beta,b'c'}^* \mathcal{Y}_{\mu\nu\alpha,bc} \\ &\times \epsilon_c^{*\nu}(P-k, \lambda_S) \epsilon_{c'}^{\nu'}(P-k, \lambda_S) \\ &\times \epsilon^{\mu'*}(P, \lambda) \epsilon^\mu(P, \lambda), \end{aligned} \quad (23)$$

where the summation over all polarization states of the spectator yields

$$\begin{aligned} \sum_{\lambda_S} \epsilon_c^{*\nu}(P-k, \lambda_S) \epsilon_{c'}^{\nu'}(P-k, \lambda_S) \\ = -g^{\nu\nu'} + \frac{(P-k)^\nu (P-k)^{\nu'}}{M_S^2}. \end{aligned} \quad (24)$$

By projecting the correlator Φ^{ij} onto the tensors g_T^{ij} , ϵ_T^{ij} , k_T^i , k_T^j , etc., we obtain the expressions for the leading-twist T-even gluon TMDs, which are collected in Appendix A. We observe that the coupling $g_2(k^2)$ in Eq. (20) contributes significantly to the final expressions. In the “ g_2 -vanishing approximation,” where $g_2(k^2)$ is set to zero, the TMDs

$g_1(x, k_T^2)$ and $g_{1T}(x, k_T^2)$ vanish, while the remaining TMDs satisfy the following relations:

$$f_1^{(g_{1,3})} = \frac{k_T^2}{2M^2} h_1^{\perp(g_{1,3})}, \quad (25)$$

$$f_{1LL}^{(g_{1,3})} = \frac{k_T^2}{2M^2} h_{1LL}^{\perp(g_{1,3})}, \quad (26)$$

$$f_{1LT}^{(g_{1,3})} = h_{1LT}^{(g_{1,3})} = -\frac{k_T^2}{4M^2} h_{1LT}^{\perp(g_{1,3})}, \quad (27)$$

$$\begin{aligned} h_{1TT}^{(g_{1,3})} &= \frac{k_T^2}{2M^2} [f_{1TT}^{(g_{1,3})} - h_{1TT}^{\perp(g_{1,3})}] \\ &= \left(\frac{k_T^2}{2M^2} \right)^2 h_{1TT}^{\perp\perp(g_{1,3})}. \end{aligned} \quad (28)$$

In this approximation, the gluon TMDs saturate the positivity bounds derived in Ref. [44],

$$\frac{k_T^2}{2M^2} |h_1^\perp - h_{1LL}^\perp| \leq f_1 - f_{1LL}, \quad (29)$$

$$\begin{aligned} \frac{k_T^4}{16M^4} [4(h_{1L}^\perp)^2 + (2h_1^\perp + h_{1LL}^\perp)] \\ \leq \left(f_1 + \frac{f_{1LL}}{2} + g_1 \right) \left(f_1 + \frac{f_{1LL}}{2} - g_1 \right). \end{aligned} \quad (30)$$

Furthermore, the tensor polarized TMDs $F_{(T)}^{(g_{1,3})}$ in the g_2 -vanishing approximation exhibit the same functional dependence on the couplings g_1 and g_3 ,

$$\begin{aligned} F_{(T)}^{(g_{1,3})} &\sim [4M^2(1-x)g_1 + (k_T^2 + (M(1-x) - M_S)^2)g_3] \\ &\times [4M^2(1-x)g_1 + (k_T^2 + (M(1-x) + M_S)^2)g_3]. \end{aligned} \quad (31)$$

Finally, in our model, the spectator mass M_S is not fixed but is allowed to take a continuous range of real values, weighted by a spectral function $\rho(M_S)$,

$$\rho(M_S) = \mu^{2a} \left[\frac{A}{B + \mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_S - D)^2}{\sigma^2}} \right], \quad (32)$$

where $\mu^2 = M_S^2 - M^2$, and $\{A, B, a, b, C, D, \sigma\}$ are free parameters. This spectral function models the spectator mass distribution as a smooth background with a superimposed resonance peak, which is phenomenological and is chosen to provide sufficient flexibility for describing different kinematic regions of the gluon TMDs. The asymptotic behavior of the spectral function is determined by the difference $a - b$, since at large M_S one has approximately $\rho(M_S) \sim M_S^{2(a-b)}$. The Lorentzian component, controlled mainly by the parameters A , B , and b ,

governs the large M_S behavior of the spectral function and therefore primarily affects the small x tail of the gluon distributions [45]. The Gaussian component, characterized by the parameters C , D , and σ , generates a resonance-like structure at intermediate spectator masses and mainly influences the moderate x region. In addition, the overall power-law factor μ^{2a} modulates the interplay between these two components and provides additional flexibility across the full x -range. The gluon TMDs are then weighted by the spectral function $\rho(M_S)$,

$$F(x, \mathbf{k}_T^2) = \int_M^\infty dM_S \rho(M_S) F(x, \mathbf{k}_T^2; M_S), \quad (33)$$

which provides an effective way to incorporate $q\bar{q}$ contributions to the spectator system that become energetically accessible at large M_S .

III. NUMERICAL RESULTS

In this section, we present the numerical results for the gluon TMDs of the tensor-polarized deuteron. Our model contains 11 free parameters: seven from the spectral function (a , b , A , B , C , D , σ) and four from the form factors (κ_1 , κ_2 , κ_3 , Λ_S). To determine these parameters, we perform a fit to the integrated unpolarized TMD $f_1(x)$ defined as

$$f_1(x) = \int d\mathbf{k}_T^2 f_1(x, \mathbf{k}_T^2), \quad (34)$$

which reproduces the collinear unpolarized gluon PDF at an initial scale. The $\mathbf{k}_T$ -integrated TMDs are also weighted by the spectral function as in Eq. (33). Additionally, we define the n th moment of a TMD as

$$F^{(n)}(x) = \int d\mathbf{k}_T^2 \left(\frac{\mathbf{k}_T^2}{2M^2} \right)^n F(x, \mathbf{k}_T^2). \quad (35)$$

We fix our model parameters by fitting the nNNPDF1.0 parametrization of $f_1(x)$ [46] at the scale $Q_0 = 2$ GeV. We avoid using a lower energy scale, as the parametrization in the small- x region at lower scales exhibits large uncertainties, along with effects not included in our model. Since the nNNPDF1.0 distribution $f_1^N(x_N)$ is defined as the average bound-nucleon PDF within the deuteron, a kinematic and normalization alignment is required to match our deuteron-level model $f_1^D(x_D)$, where x_D and x_N denote the parton momentum fractions relative to the entire deuteron and the single nucleon, respectively. To ensure both momentum conservation and parton-density consistency in changing the scaling variable, the relation between the two definitions obeys $x_D f_1^D(x_D) = 2x_N f_1^N(x_N)$, which directly yields $f_1^N(x_N) = f_1^D(x_N/2)/4$ with $x_D = x_N/2$. Furthermore, since the nucleon PDF $f_1^N(x_N)$ vanishes beyond $x_N = 1$, this fitting procedure effectively constrains our model up to

$x_D = 0.5$; for the kinematic region of $0.5 < x_D < 1$, the deuteron PDF in our model is assumed to be negligible or treated as zero, as it corresponds to the extreme Fermi-motion region that is not covered by the single-nucleon data. Here, we emphasize once again that, consistent with the definition established earlier in this paper, the variable x strictly represents the momentum fraction relative to the entire deuteron.

The fit is performed over the range $0.001 < x_N < 1$. For $x_N f_1^N(x_N)$, a grid of 70 points distributed logarithmically in the range $0.001 < x_N < 0.1$ and 30 points distributed linearly for $x_N > 0.1$ is selected. To account for uncertainties in the fitting procedure, we employ the bootstrap method: we create $N = 100$ replicas of the nNNPDF1.0 central value, each modified by adding random Gaussian noise with the same variance as the uncertainty of the original parametrization. For each replica, we perform a separate fit to obtain N parameter sets. The 68% uncertainty band of our fit is constructed by rejecting the largest and smallest 16% of the N values of any prediction.

The values of the model parameters are shown in Table II. The second column lists the parameter values and their uncertainties, calculated as the average and the uncertainty given by the semi-difference between the upper and lower limits of the distribution. We obtain a total $\chi^2/\text{d.o.f.} = 1.53 \pm 0.23$. The final column, labeled “replica 65,” shows the most representative parameter set, which has the minimal deviation from the mean values.

In Fig. 1, we show the numerical results of our fit for $x f_1(x)$ at $Q_0 = 2$ GeV and the integrated vector polarized gluon TMDs as functions of x . The results indicate that the effects of these TMDs are significant and could be probed by future experimental measurements. The band with dashed borders represents the nNNPDF1.0 parametrization result. The solid line corresponds to the fit from replica 65, and the cyan band shows the 68% uncertainty of the spectator model fit. The size of $x g_1(x)$ and $x g_{1T}^{(1)}(x)$ is several times smaller than that of $x f_1(x)$. As discussed in

TABLE II. Central column: mean values and uncertainties of the fitted model parameters. Rightmost column: corresponding values for replica 65.

Parameter	Mean	Replica 65
κ_1	0.081 ± 0.011	0.074
κ_2	0.157 ± 0.149	0.060
κ_3	16.31 ± 1.20	15.87
Λ_S	0.507 ± 0.023	0.509
a	1.588 ± 0.363	1.382
b	39.96 ± 17.34	35.66
A	110 ± 4	105
B	0.002 ± 0.0003	0.002
C	235 ± 229	236
D	3.355 ± 0.576	0.242
σ	0.224 ± 0.146	0.109

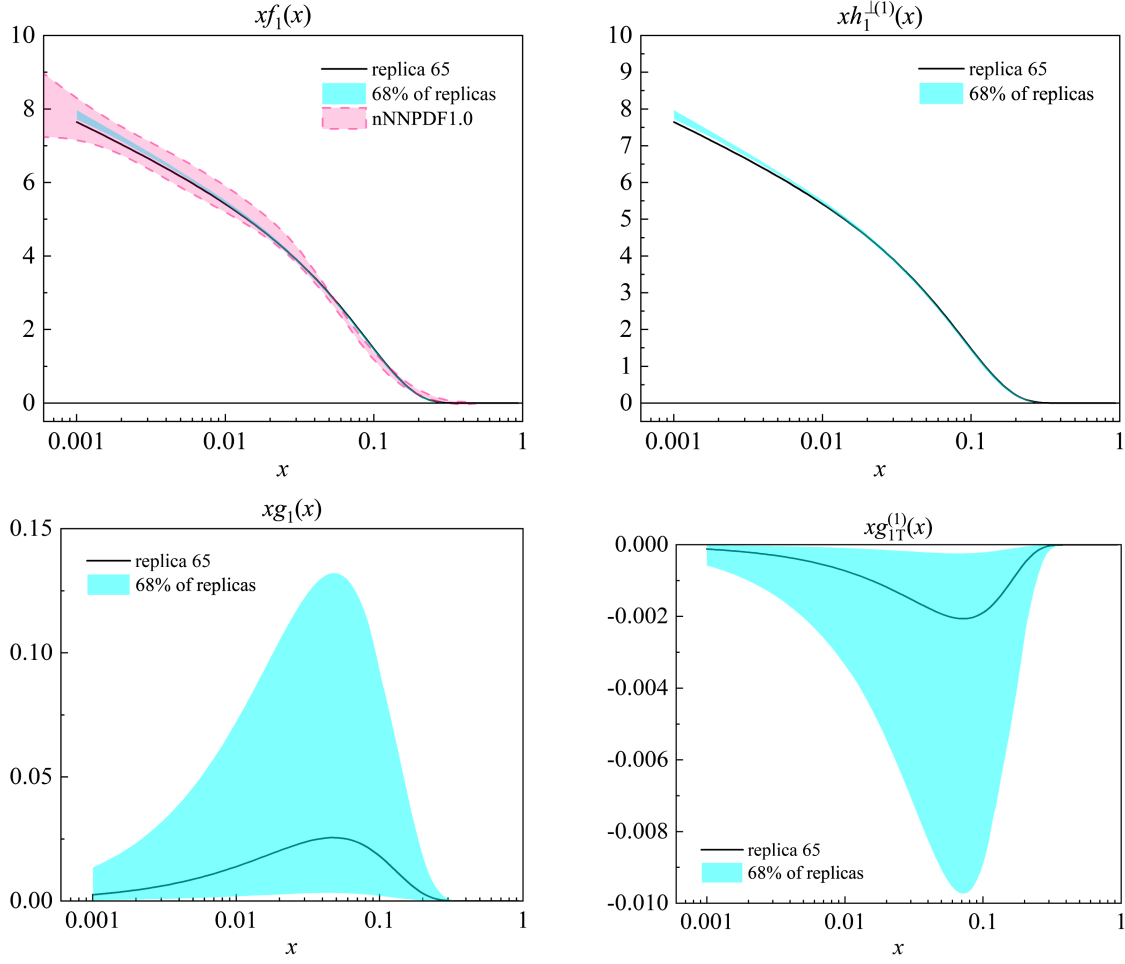


FIG. 1. Upper-left panel: Fit of the integrated unpolarized gluon TMD $xf_1(x)$ for the deuteron at $Q_0 = 2$ GeV in the range $0.001 < x < 1$. The band with dashed borders corresponds to the nNNPDF1.0 parametrization of xf_1 [46]. The solid line shows the result for replica 65. The cyan band represents the 68% uncertainty band of the spectator model fit. The other three panels display the x -dependence of $xh_1^{\perp(1)}$, xg_1 , and $xg_{1T}^{(1)}$ obtained from the spectator model using the parameters in Table II.

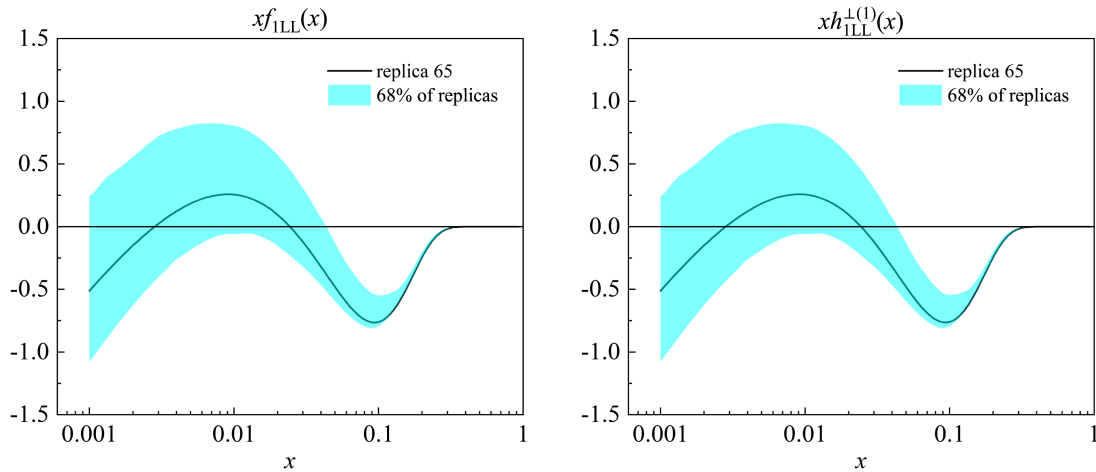


FIG. 2. The integrated LL tensor-polarized TMDs as functions of x at $Q_0 = 2$ GeV. The band represents the 68% uncertainty of the TMDs, and the solid line corresponds to replica 65.

Sec. II, this indicates that the coupling $g_2(k^2)$ in Eq. (20) has a small but necessary impact on the numerical results of gluon TMDs for the deuteron. The deuteron momentum and spin fractions carried by gluons at the scale $Q_0 = 2$ GeV in our model are predicted as

$$\langle x \rangle_g = \int_0^1 dx x f_1(x) = 0.407 \pm 0.007, \quad (36)$$

$$\langle 1 \rangle_g = \int_0^1 dx g_1(x) = 0.188 \pm 0.194. \quad (37)$$

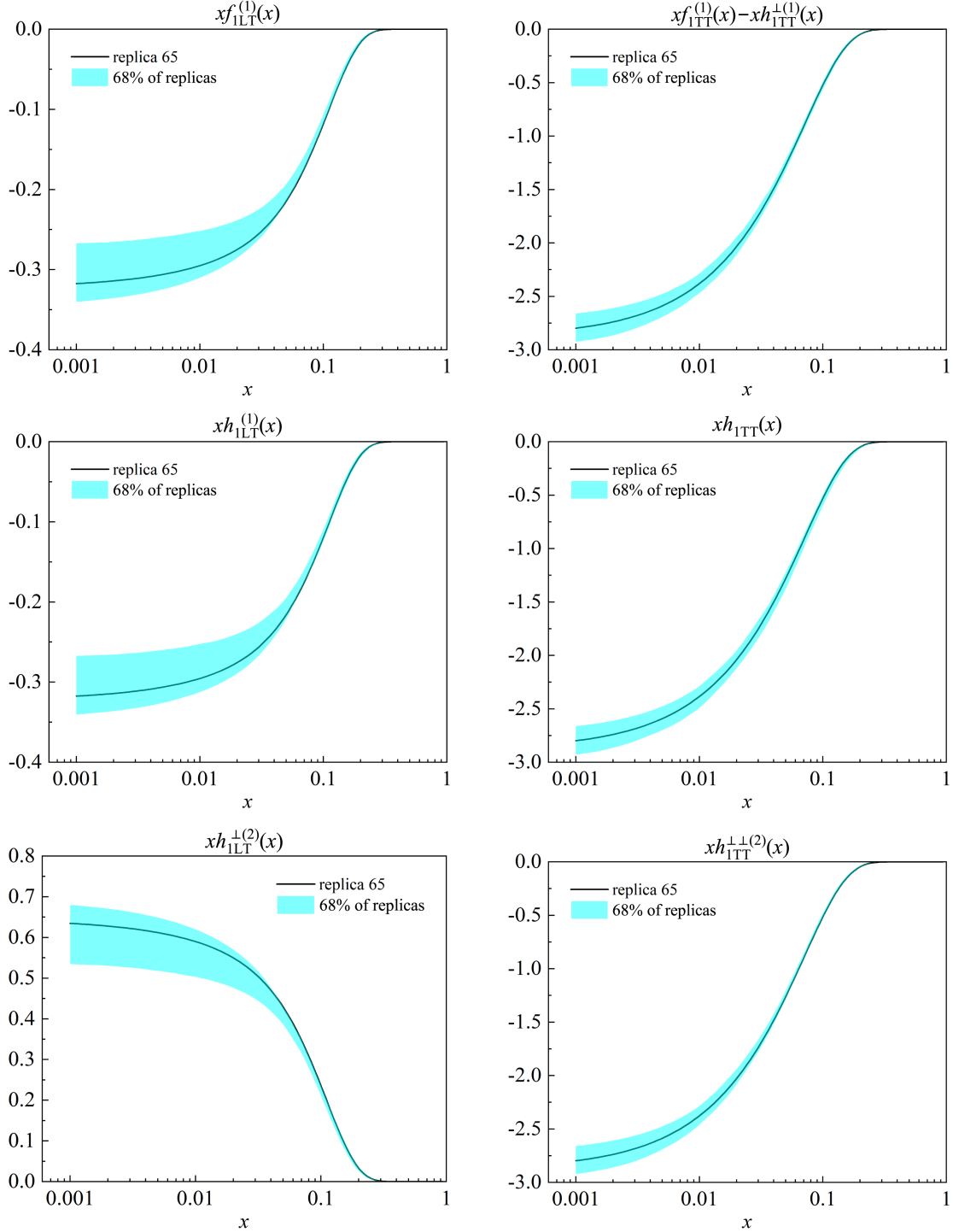


FIG. 3. The integrated LT and TT tensor-polarized TMDs as functions of x at $Q_0 = 2$ GeV. Conventions as in Fig. 2. Left column: LT-polarized PDFs $xf_{1LT}^{(1)}$, $xh_{1LT}^{(1)}$, and $xh_{1LT}^{\perp(2)}$. Right column: TT-polarized PDFs $xf_{1TT}^{(1)} - xh_{1TT}^{\perp(1)}$, xh_{1TT} , and $xh_{1TT}^{\perp\perp(2)}$.

We anticipate future parametrizations of g_1 that enable simultaneous fitting of both f_1 and g_1 , thereby improving the predictive power of the model. The notable dependence of the TMDs $g_1(x)$ and $g_{1T}^{(1)}(x)$ on the coupling $g_2(k^2)$ underscores the need for more comprehensive future studies.

In Figs. 2 and 3, we display the integrated tensor-polarized gluon TMDs as functions of x at the scale

$Q_0 = 2$ GeV, without including the evolution effects. For the deuteron with the same polarization, the gluon TMDs exhibit a similar tendency. The TMDs $xh_1^{\perp(1)}(x)$, $xg_1(x)$, and $xh_{1LT}^{\perp(2)}$ remain positive in the entire x region. In contrast, $xf_{1LL}(x)$ and $xh_{1LL}^{\perp(1)}(x)$ display more intricate behaviors. Upon integrating the TMD correlator in Eq. (6) over transverse momentum, only $f_{1LL}(x)$ and $h_{1TT}(x)$ do

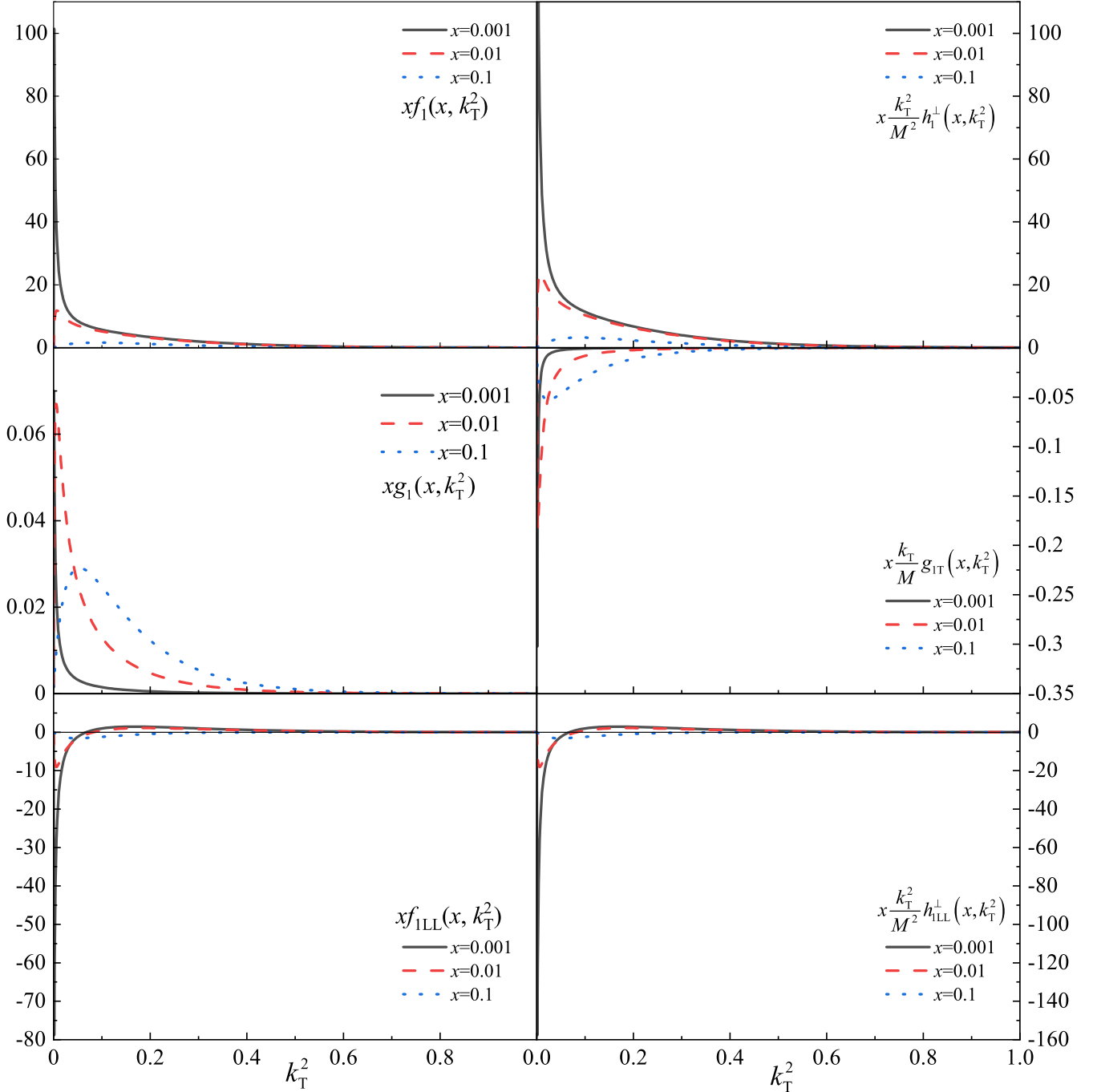


FIG. 4. The unpolarized, vector-polarized, and LL tensor-polarized TMDs (replica 65) as functions of k_T^2 for $x = 0.001, 0.01$, and 0.1 at $Q_0 = 2$ GeV. Solid line: $x = 0.001$; dashed line: $x = 0.01$; dotted line: $x = 0.1$.

not vanish. Notably, the tensor-polarized gluon distribution $xf_{\text{ILL}}(x)$ exhibits a distinctive nodal structure. It starts with positive values at small x , crosses zero around $x \sim 0.02$, and then develops a pronounced negative minimum at larger x . This characteristic wave-like behavior is in excellent agreement with the well-known tendency of the deuteron structure $b_1(x)$ (or $xf_{\text{ILL}}(x)$ for quarks) reported by the HERMES Collaboration [15]. Another integral observable is the gluon transversity $\Delta_T g(x) (\equiv -h_{\text{1TT}}(x))$, which does not exist in the spin-1/2 nucleons, as a change of two spin

units is required for the helicity-flip amplitude. In our model, $xh_{\text{1TT}}(x)$ increases with x . Although nonzero gluon transversity is often associated with non-nucleonic or genuinely gluonic degrees of freedom in spin-1 systems, the present result is obtained within a phenomenological spectator-model framework. Therefore, the nonvanishing $h_{\text{1TT}}(x)$ found here should be interpreted as a model-dependent indication rather than direct evidence of such effects.

In Figs. 4 and 5, the T-even gluon TMDs multiplied by xk_T^n/M^n are displayed as functions of k_T^2 at $x = 10^{-3}$ (solid),

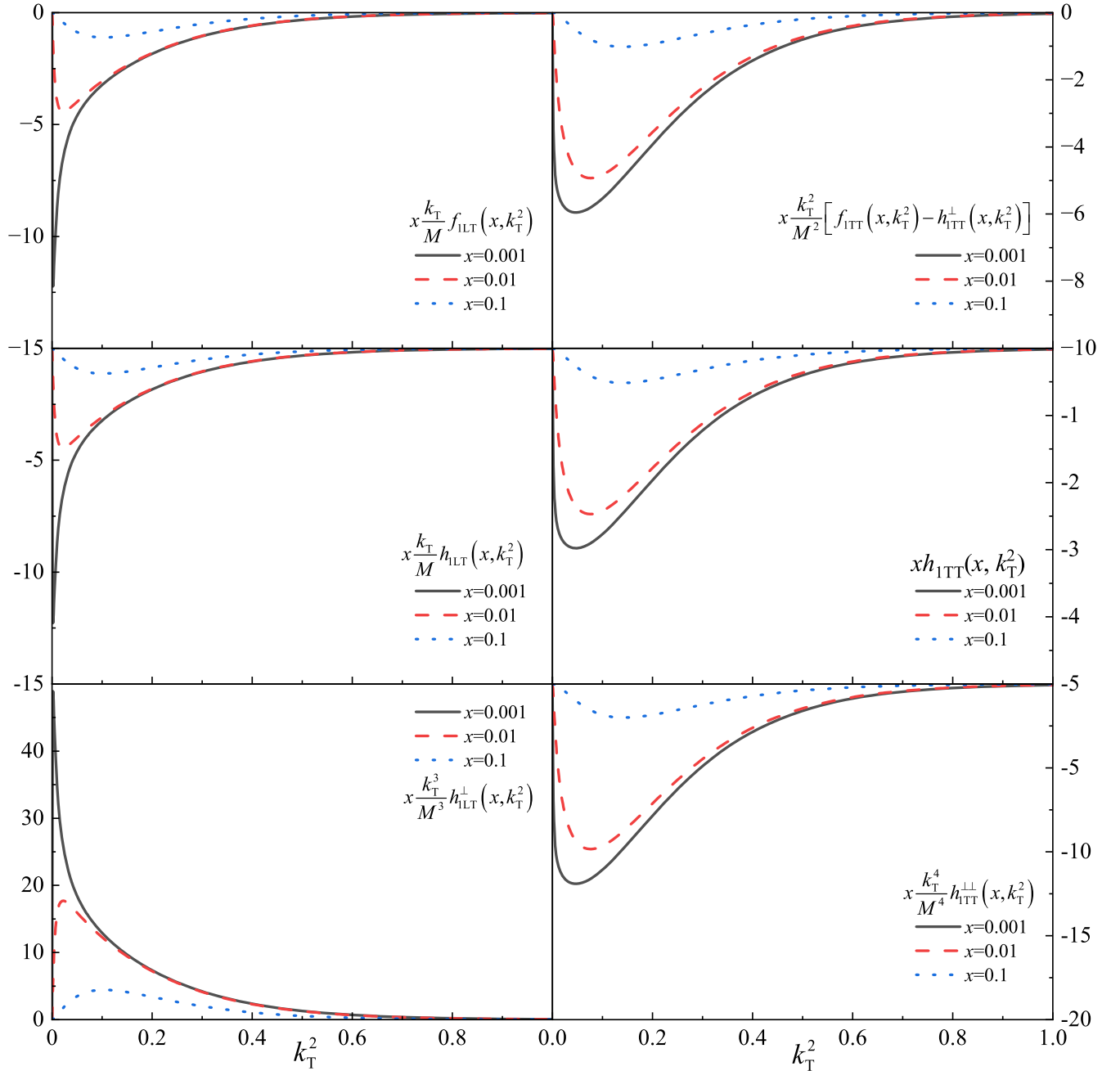


FIG. 5. The LT and TT tensor-polarized TMDs (replica 65) as functions of k_T^2 for $x = 0.001, 0.01$, and 0.1 at $Q_0 = 2$ GeV. Line conventions as in Fig. 4.

$x = 10^{-2}$ (dashed), and $x = 10^{-1}$ (dotted) at the scale $Q_0 = 2$ GeV. The parameters are taken from the most representative replica 65. The absolute magnitudes of $xg_1(x, k_T^2)$ and $xk_T/M g_{1T}(x, k_T^2)$ are smaller. These TMDs share a common trend: they exhibit a non-Gaussian shape in k_T^2 , with a long tail as k_T^2 increases. They are predominantly concentrated in the small k_T^2 region and display a peak that shifts to larger k_T^2 and becomes flatter as x increases. Simultaneously, for the tensor-polarized TMDs $xf_{1LL}(x, k_T^2)$ and $xk_T^2/M^2 h_{1LL}^\perp(x, k_T^2)$, a distinct sign-inversion behavior from negative to positive values occurs across all investigate x as k_T^2 increases. However, the relative proportions of the negative and positive contributions vary significantly with x . This x -dependent shift explains the macroscopic wave-like nodal structure observed in the integrated PDFs $xf_{1LL}(x)$ and $xh_{1LL}^{\perp(1)}(x)$. Notably, $xf_1(x, k_T^2)$ do not vanish at $k_T^2 \rightarrow 0$ GeV², indicating significant contribution from gluon orbital angular momentum $L = 1$.

To further investigate the model dependence associated with the effective deuteron-gluon-spectator vertex, we

study the logarithmic sensitivities

$$S_{\kappa_i}^F = \frac{\partial \ln F}{\partial \ln \kappa_i} = \frac{\kappa_i \partial F}{F \partial \kappa_i}, \quad (38)$$

which quantify the relative response of a given gluon TMD to variations of the vertex parameters κ_i . In Fig. 6, we show the sensitivities of three representative gluon TMDs, namely, the unpolarized distribution f_1 , the helicity distribution g_1 , and the tensor-polarized distribution h_{1TT} , as functions of k_T^2 at $x = 0.001$ (solid), $x = 0.2$ (dashed), and $x = 0.5$ (dotted). The parameters are taken from the most representative replica 65. Several characteristic features can be observed. First, in the small k_T^2 region, many TMDs become dominated by the κ_2^2 structure. In particular, at $k_T^2 = 0$, $S_{\kappa_1}^F$ and $S_{\kappa_3}^F$ vanish for several TMDs, while the dependence on κ_2 remains finite $S_{\kappa_2}^F \rightarrow 2$. As a consequence, the corresponding sensitivities to κ_i exhibit a rapid rise or fall as k_T^2 increase from the origin.

The numerical results also show that the helicity distribution g_1 is significantly more sensitive to the parameter

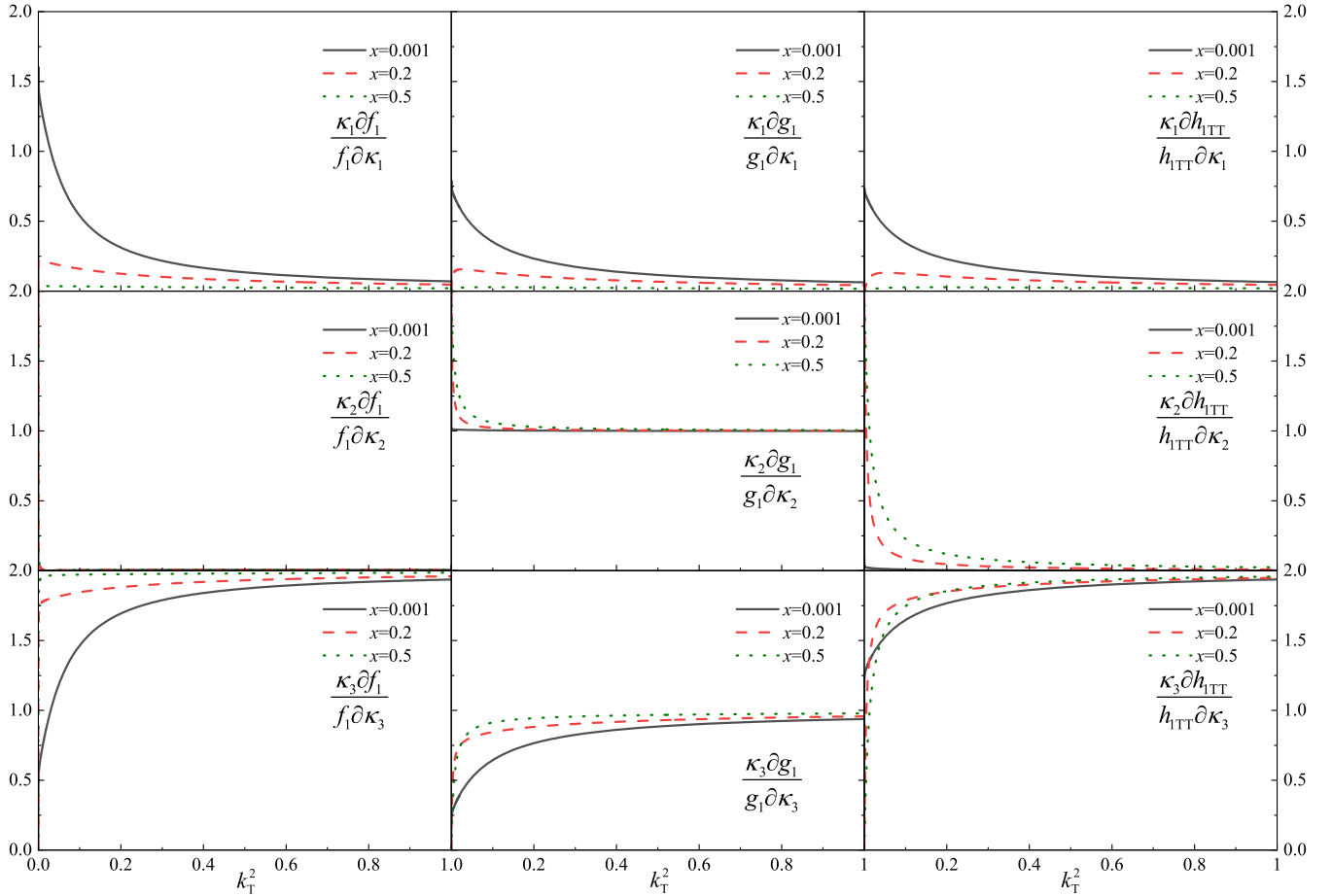


FIG. 6. The logarithmic sensitivities $S_{\kappa_i}^{f_1}$, $S_{\kappa_i}^{g_1}$, and $S_{\kappa_i}^{h_{1TT}}$ (replica 65) as functions of k_T^2 for $x = 0.001$, $x = 0.2$, and $x = 0.5$. Solid line: $x = 0.001$; dashed line: $x = 0.2$; dotted line: $x = 0.5$.

κ_2 than the other TMDs considered here. By contrast, the sensitivities of f_1 and h_{1TT} with respect to κ_2 rapidly decrease toward zero as k_T^2 increases. This feature indicates that the helicity sector is more strongly affected by the structure associated with the κ_2 term in the vertex. Since the present parameter determination is mainly based on the fit to the unpolarized gluon distribution f_1 , the sensitivity patterns further suggest that additional experimental constraints, especially on a helicity-dependent observable such as g_1 , will be important for improving the determination of the effective vertex structure in future phenomenological studies.

For the κ_1 and κ_3 structures, all three TMDs exhibit nonvanishing sensitivities over a broad range of transverse momentum k_T^2 . The sensitivity to κ_1 become noticeably larger in the small- x region, indicating that the corresponding vertex structure plays a more important role at small longitudinal momentum fractions. In particular, for h_{1TT} , the sensitivity to κ_3 shows a nontrivial dependence on both x and k_T^2 . In the low k_T^2 region, the sensitivity increases with increasing x , whereas at larger transverse momentum the sensitivity decreases with increasing x . This indicates that the tensor-polarized sector receives contributions from several competing vertex structures in the intermediate- k_T region.

In addition, the large k_T^2 region exhibits relatively stable asymptotic sensitivity patterns. The sensitivities of f_1 and h_{1TT} with respect to κ_3 approach values close to two, suggesting that these distributions are dominated asymptotically by quadratic contributions associated with the κ_3 vertex structure. By contrast, the helicity distribution g_1 exhibits approximately linear sensitivities to both κ_2 and κ_3 , indicating that interference-type contributions proportional to $\kappa_2\kappa_3$ become important in this sector.

Finally, we verify that our model results satisfy the positivity bounds outlined in Appendix B, which provide important model-independent constraints for both TMD and collinear cases [44], while neglecting T-odd gluon TMDs.

IV. CONCLUSIONS

In this paper, we presented a model calculation of leading-twist T-even gluon TMDs for the tensor-polarized deuteron within a spectator model framework. The model is based on the assumption that an on-shell deuteron can emit a timelike off-shell gluon, with the remaining system treated as a single on-shell spectator particle whose mass covers a continuous spectrum of real values governed by a spectral function. For spin-1 hadrons, the polarization description requires not only a spin vector S but also a

symmetric traceless spin tensor T to characterize tensor polarized states. The deuteron-gluon-spectator vertex is modeled to the vector current for a spin-1 particle, and we adopted an exponential form factor for the deuteron-gluon-spectator couplings.

We derived analytical expressions for thirteen T-even gluon TMDs and provided numerical results for their x -dependence and k_T -dependence. In the fit procedure, we employed the bootstrap method to account for the uncertainties, fixing the model parameters by fitting the nNNPDF1.0 parametrization for $f_1(x)$ at the scale $Q_0 = 2$ GeV. We also verified that our results satisfy the positivity bounds for both the gluon TMDs and their integrated counterparts. Our analysis reveals non-negligible impact of these gluon TMDs, particularly for tensor polarized hadrons, which could be explored in future experimental measurements.

As is well known, gluon TMDs exhibit a more intricate dependence on the structure of the color flow (gauge link), which in turn introduces a dependence on the involved process. There are two main classes of gluon TMDs: the so-called Weizsäcker-Williams (WW) gluon TMDs (f -type) and the dipole gluon TMDs (d -type). In future work, we plan to extend our model to include leading-twist T-odd gluon TMDs for the tensor polarized deuteron. It would be particularly interesting to investigate the differences between f -type and d -type gluon TMDs within this framework.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: FULL EXPRESSIONS OF GLUON TMDs

In the following, we list the final expressions of the gluon TMDs in Eqs. (7)–(12). The unpolarized gluon TMD f_1 is given by

$$\begin{aligned}
 f_1(x, \mathbf{k}_T^2) &= -g_T^{ij} \Phi_U^{ij}(x, \mathbf{k}_T) \\
 &= -\{ -8g_3 M^2(x-1) \mathbf{k}_T^2 [2\mathbf{k}_T^2 (M^2(x-1)^2 + M_S^2) + \mathbf{k}_T^4 + (M_S^2 - M^2(x-1)^2)^2] [g_1(\mathbf{k}_T^2 + M^2(x-1)^2 \\
 &\quad + M_S^2) - g_2(\mathbf{k}_T^2 + x(M^2(x-1) + M_S^2))] + g_3^2 \mathbf{k}_T^2 [2\mathbf{k}_T^2 (M^2(x-1)^2 + M_S^2) + \mathbf{k}_T^4 + (M_S^2 \\
 &\quad - M^2(x-1)^2)^2] + 8M^4(x-1)^2 [-4g_1 g_2 \mathbf{k}_T^2 (\mathbf{k}_T^2 + M^2(x-1)^2 + M_S^2) (\mathbf{k}_T^2 + x(M^2(x-1) + M_S^2)) \\
 &\quad + 2g_1^2 \mathbf{k}_T^2 (2M_S^2(\mathbf{k}_T^2 + 5M^2(x-1)^2) + (\mathbf{k}_T^2 + M^2(x-1)^2)^2 + M_S^4) + g_2^2 (\mathbf{k}_T^4 (M^2(x(5x-6) + 2) \\
 &\quad + (x(x+2) + 2)M_S^2) + 2x\mathbf{k}_T^2 (M^4(x-1)^2(2x-1) + M^2x((x-1)x+1)M_S^2 + (x+1)M_S^4) + 2\mathbf{k}_T^6 \\
 &\quad + (M^2 + M_S^2)(M^2(x-1)^2x - xM_S^2)^2)] \} \\
 &\quad \times \{ 768\pi^3 M^6(x-1)^3 x M_S^2 (\mathbf{k}_T^2 + x(M^2(x-1) + M_S^2))^2 \}^{-1}.
 \end{aligned} \tag{A1}$$

The vector polarized gluon TMDs are given by

$$\begin{aligned}
 h_1^\perp(x, \mathbf{k}_T^2) &= \frac{4M^2 k_T^{ij}}{\mathbf{k}_T^4} \Phi_U^{ij}(x, \mathbf{k}_T) \\
 &= -\{ -8g_3 M^2(x-1) [2\mathbf{k}_T^2 (M^2(x-1)^2 + M_S^2) + \mathbf{k}_T^4 + (M_S^2 - M^2(x-1)^2)^2] \\
 &\quad \times [g_1(\mathbf{k}_T^2 + M^2(x-1)^2 + M_S^2) - g_2(\mathbf{k}_T^2 + x(M^2(x-1) + M_S^2))] + g_3^2 (2\mathbf{k}_T^2 (M^2(x-1)^2 + M_S^2) \\
 &\quad + \mathbf{k}_T^4 + (M_S^2 - M^2(x-1)^2)^2 + 16M^4(x-1)^2 [g_1^2 (2M_S^2(\mathbf{k}_T^2 + 5M^2(x-1)^2) \\
 &\quad + (\mathbf{k}_T^2 + M^2(x-1)^2)^2 + M_S^4) - 2g_2 g_1 (\mathbf{k}_T^2 + M^2(x-1)^2 + M_S^2) (\mathbf{k}_T^2 + x(M^2(x-1) + M_S^2)) \\
 &\quad + g_2^2 (\mathbf{k}_T^2 + M^2(x-1)^2 + M_S^2) (\mathbf{k}_T^2 + x(M^2(x-1) + M_S^2))] \} \\
 &\quad \times \{ 384\pi^3 M^4(x-1)^3 x M_S^2 (\mathbf{k}_T^2 + x(M^2(x-1) + M_S^2))^2 \}^{-1},
 \end{aligned} \tag{A2}$$

$$\begin{aligned}
 g_1(x, \mathbf{k}_T^2) &= -\frac{i\epsilon_T^{ij}}{S_L} \Phi_L^{ij}(x, \mathbf{k}_T) \\
 &= -g_2 \{ g_2 M^2 [2\mathbf{k}_T^2 (M^2(2x-1)(x-1)^2 + (2(x-1)x+1)M_S^2) + (3x-2)\mathbf{k}_T^4 + x(M_S^2 - M^2(x-1)^2)^2] \\
 &\quad + \mathbf{k}_T^2 [g_3 (2M_S^2(\mathbf{k}_T^2 - M^2(x-1)^2) + (\mathbf{k}_T^2 + M^2(x-1)^2)^2 + M_S^4) - 4g_1 M^2(x-1) \\
 &\quad \times (\mathbf{k}_T^2 + M^2(x-1)^2 + (3-2x)M_S^2)] \} \\
 &\quad \times \{ 64\pi^3 M^2(x-1) M_S^2 (\mathbf{k}_T^2 + x(M^2(x-1) + M_S^2))^2 \}^{-1},
 \end{aligned} \tag{A3}$$

$$\begin{aligned}
 g_{1T}(x, \mathbf{k}_T^2) &= -\frac{i\epsilon_T^{ij} M}{\mathbf{k}_T \cdot \mathbf{S}_T} \Phi_T^{ij}(x, \mathbf{k}_T) \\
 &= -g_2 \{ 4g_1 M^2(x-1) (-\mathbf{k}_T^2 + M^2(x-1)^2 - M_S^2) (\mathbf{k}_T^2 + M^2(x-1)^2 + (3-2x)M_S^2) + 4g_2 M^2(x-1) \\
 &\quad \times [M_S^2((x+1)\mathbf{k}_T^2 - M^2(x-1)^2x) + M^2(x-1)^2 \mathbf{k}_T^2 + \mathbf{k}_T^4 + xM_S^4] - g_3 (-\mathbf{k}_T^2 + M^2(x-1)^2 - M_S^2) \\
 &\quad \times (\mathbf{k}_T^2 + (M_S + M(x-1))^2) (\mathbf{k}_T^2 + (M_S + M(-x) + M)^2) \} \\
 &\quad \times \{ 128\pi^3 M^2(x-1)^2 M_S^2 (\mathbf{k}_T^2 + x(M^2(x-1) + M_S^2))^2 \}^{-1}.
 \end{aligned} \tag{A4}$$

Then, the LL tensor polarized gluon TMDs are given by

$$\begin{aligned}
 f_{\text{ILL}}(x, \mathbf{k}_T^2) &= \frac{-g_T^{ij}}{S_{\text{LL}}} \Phi_{\text{LL}}^{ij}(x, \mathbf{k}_T) \\
 &= \{ \mathbf{k}_T^2 [2\mathbf{k}_T^2 (M_S^2 - 2M^2(x-1)^2) + \mathbf{k}_T^4 + (M_S^2 - M^2(x-1)^2)^2] [g_3(\mathbf{k}_T^2 + (M_S + M(x-1))^2) \\
 &\quad - 4g_1 M^2(x-1)] [g_3(\mathbf{k}_T^2 + (M_S + M(-x) + M)^2) - 4g_1 M^2(x-1)] - 4g_2^2 M^4(x-1)^2 \\
 &\quad \times [\mathbf{k}_T^4 (M^2(x(5x-6) + 2) - 2(x(x+2) + 2)M_S^2) + 2x\mathbf{k}_T^2 (M^4(x-1)^2(2x-1) \\
 &\quad + M^2(2x(2(x-1)x-1) + 3)M_S^2 - 2(x+1)M_S^4) - 4\mathbf{k}_T^6 + (M^2 - 2M_S^2)(M^2(x-1)^2x - xM_S^2)^2] \\
 &\quad - 4g_2 M^2(x-1)\mathbf{k}_T^2 [g_3(\mathbf{k}_T^4 (M^2(x-1)(3x-2) - 2(x+2)M_S^2) + 2\mathbf{k}_T^2 (M^4(x-1)^3(3x-2) \\
 &\quad + M^2(1-2x)^2(x-1)M_S^2 - (2x+1)M_S^4) - 2\mathbf{k}_T^6 + x(M^2(x-1) - 2M_S^2)(M_S^2 - M^2(x-1)^2)^2) \\
 &\quad - 4g_1 M^2(x-1)(\mathbf{k}_T^2 (M^2(x-1)(5x-4) - 2(x+1)M_S^2) - 2\mathbf{k}_T^4 \\
 &\quad + x(M^4(x-1)^3 + M^2(x-1)(4x-3)M_S^2 - 2M_S^4))] \} \\
 &\quad \times \{ 384\pi^3 M^6(x-1)^3 x M_S^2 (\mathbf{k}_T^2 + x(M^2(x-1) + M_S^2))^2 \}^{-1}, \tag{A5}
 \end{aligned}$$

$$\begin{aligned}
 h_{\text{ILL}}^\perp(x, \mathbf{k}_T^2) &= \frac{4M^2 k_T^{ij}}{\mathbf{k}_T^4 S_{\text{LL}}} \Phi_{\text{LL}}^{ij}(x, \mathbf{k}_T) \\
 &= \{ g_3^2 (\mathbf{k}_T^2 + (M_S + M(x-1))^2) (\mathbf{k}_T^2 + (M_S - xM + M)^2) (2\mathbf{k}_T^2 (M_S^2 - 2M^2(x-1)^2) + \mathbf{k}_T^4 \\
 &\quad + (M_S^2 - M^2(x-1)^2)^2) + 16g_1^2 M^4(x-1)^2 [2\mathbf{k}_T^2 (M_S^2 - 2M^2(x-1)^2) + \mathbf{k}_T^4 + (M_S^2 - M^2(x-1)^2)^2] \\
 &\quad - 8g_2^2 M^4(x-1)^2 [\mathbf{k}_T^2 (M^2(x-1)(2x-1) - 2(x+1)M_S^2) - 2\mathbf{k}_T^4 + x(M^4(x-1)^3 \\
 &\quad + M^2(x-1)(4x-3)M_S^2 - 2M_S^4)] - 4g_2 g_3 M^2(x-1) [\mathbf{k}_T^4 (M^2(x-1)(3x-2) - 2(x+2)M_S^2) \\
 &\quad + 2\mathbf{k}_T^2 (M^4(x-1)^3(3x-2) + M^2(1-2x)^2(x-1)M_S^2 - (2x+1)M_S^4) - 2\mathbf{k}_T^6 \\
 &\quad + x(M^2(x-1) - 2M_S^2)(M_S^2 - M^2(x-1)^2)^2] - 8g_1 M^2(x-1) [g_3(\mathbf{k}_T^2 + M^2(x-1)^2 + M_S^2) \\
 &\quad \times (2\mathbf{k}_T^2 (M_S^2 - 2M^2(x-1)^2) + \mathbf{k}_T^4 + (M_S^2 - M^2(x-1)^2)^2) - 2g_2 M^2(x-1) \\
 &\quad \times (\mathbf{k}_T^2 (M^2(x-1)(5x-4) - 2(x+1)M_S^2) - 2\mathbf{k}_T^4 + x(M^4(x-1)^3 \\
 &\quad + M^2(x-1)(4x-3)M_S^2 - 2M_S^4))] \} \\
 &\quad \times \{ 192\pi^3 M^4(x-1)^3 x M_S^2 (\mathbf{k}_T^2 + x(M^2(x-1) + M_S^2))^2 \}^{-1}. \tag{A6}
 \end{aligned}$$

The LT tensor polarized gluon TMDs are given by

$$\begin{aligned}
 f_{\text{ILT}}(x, \mathbf{k}_T^2) &= \frac{M}{2((\mathbf{k}_T \cdot \mathbf{S}_{\text{LT}})^2 - \mathbf{k}_T^2 S_{\text{LT}}^2)} \left(S_{\text{LT}}^{\{i} \mathbf{k}_T^{j\}} + \frac{-3(\mathbf{k}_T \cdot \mathbf{S}_{\text{LT}})^2 + 3\mathbf{k}_T^2 S_{\text{LT}}^2}{\mathbf{k}_T \cdot \mathbf{S}_{\text{LT}}} g_T^{ij} + \frac{-4(\mathbf{k}_T \cdot \mathbf{S}_{\text{LT}})^2 + 2\mathbf{k}_T^2 S_{\text{LT}}^2}{\mathbf{k}_T \cdot \mathbf{S}_{\text{LT}}} \frac{\mathbf{k}_T^i \mathbf{k}_T^j}{\mathbf{k}_T^2} \right) \Phi_{\text{LT}}^{ij}(x, \mathbf{k}_T) \\
 &= -\{ 8M^4(x-1)^2 [g_2^2 (xM_S^2((2x+1)\mathbf{k}_T^2 - 2M^2(x-1)^2x) + \mathbf{k}_T^4 + 2x^2 M_S^4) + g_1 g_2 \mathbf{k}_T^2 (-3\mathbf{k}_T^2 \\
 &\quad + M^2(x-1)^2 - (2x+1)M_S^2) + 2g_1^2 \mathbf{k}_T^2 (\mathbf{k}_T^2 - M^2(x-1)^2 + M_S^2)] - 2g_3 M^2(x-1)\mathbf{k}_T^2 \\
 &\quad \times [4g_1 ((\mathbf{k}_T^2 + M_S^2)^2 - M^4(x-1)^4) + g_2 (-2\mathbf{k}_T^2 (M^2(x-1)^2 + (2x+1)M_S^2) - 3\mathbf{k}_T^4 \\
 &\quad + M^4(x-1)^4 + 2M^2(2x-1)(x-1)^2 M_S^2 + (1-4x)M_S^4)] + g_3^2 [\mathbf{k}_T^6 (M^2(x-1)^2 + 3M_S^2) \\
 &\quad - \mathbf{k}_T^2 (M^2(x-1)^2 - M_S^2)^3 + \mathbf{k}_T^4 (-M^4(x-1)^4 - 2M^2(x-1)^2 M_S^2 + 3M_S^4) + \mathbf{k}_T^8] \} \\
 &\quad \times \{ 256\pi^3 M^4(x-1)^2 x M_S^2 (\mathbf{k}_T^2 + x(M^2(x-1) + M_S^2))^2 \}^{-1}, \tag{A7}
 \end{aligned}$$

$$\begin{aligned}
 h_{\text{ILT}}(x, \mathbf{k}_T^2) &= \frac{M}{2((\mathbf{k}_T \cdot \mathbf{S}_{\text{LT}})^2 - \mathbf{k}_T^2 \mathbf{S}_{\text{LT}}^2)} \left(-S_{\text{LT}}^{\{i} \mathbf{k}_T^{j\}} + \frac{(\mathbf{k}_T \cdot \mathbf{S}_{\text{LT}})^2 - \mathbf{k}_T^2 \mathbf{S}_{\text{LT}}^2}{\mathbf{k}_T \cdot \mathbf{S}_{\text{LT}}} g_{\text{T}}^{ij} - \frac{-4(\mathbf{k}_T \cdot \mathbf{S}_{\text{LT}})^2 + 2\mathbf{k}_T^2 \mathbf{S}_{\text{LT}}^2}{\mathbf{k}_T \cdot \mathbf{S}_{\text{LT}}} \frac{\mathbf{k}_T^i \mathbf{k}_T^j}{\mathbf{k}_T^2} \right) \Phi_{\text{LT}}^{ij}(x, \mathbf{k}_T) \\
 &= -\{ -g_3^2 \mathbf{k}_T^2 (-\mathbf{k}_T^2 + M^2(x-1)^2 - M_S^2) (\mathbf{k}_T^2 + (M_S + M(x-1))^2) (\mathbf{k}_T^2 + (M_S - xM + M)^2) \\
 &\quad + 8g_2^2 M^4 (x-1) [x\mathbf{k}_T^2 (M^2(x-1)^2 + 2xM_S^2) + (2x-1)\mathbf{k}_T^4 + x^2 M_S^2 (M_S^2 - M^2(x-1)^2)] \\
 &\quad - 2g_2 g_3 M^2 [\mathbf{k}_T^4 (-M^2(3x-2)(x-1)^2 - (4x^2 + x - 2)M_S^2) + \mathbf{k}_T^2 (M^4(2x-1)(x-1)^4 \\
 &\quad + 2M^2(2(x-1)x+1)(x-1)^2 M_S^2 + (-4x^2 + 2x-1)M_S^4) + (3-4x)\mathbf{k}_T^6 + x(M^2(x-1)^2 - M_S^2)^3] \\
 &\quad + 8g_1 M^2(x-1) [g_2 M^2 (M_S^2((-4x^2 + x+1)\mathbf{k}_T^2 + 2M^2 x(x-1)^3) + M^2(x-1)^3 \mathbf{k}_T^2 + (3-4x)\mathbf{k}_T^4 \\
 &\quad + M^4 x(x-1)^4 + (1-2x)xM_S^4) - g_3 \mathbf{k}_T^2 ((\mathbf{k}_T^2 + M_S^2)^2 - M^4(x-1)^4)] \\
 &\quad - 16g_1^2 M^4 (x-1)^2 \mathbf{k}_T^2 (-\mathbf{k}_T^2 + M^2(x-1)^2 - M_S^2) \} \\
 &\quad \times \{ 256\pi^3 M^4 (x-1)^2 x M_S^2 (\mathbf{k}_T^2 + x(M^2(x-1) + M_S^2))^2 \}^{-1}, \tag{A8}
 \end{aligned}$$

$$\begin{aligned}
 h_{\text{ILT}}^\perp(x, \mathbf{k}_T^2) &= \frac{-4M^3}{2\mathbf{k}_T^2((\mathbf{k}_T \cdot \mathbf{S}_{\text{LT}})^2 - \mathbf{k}_T^2 \mathbf{S}_{\text{LT}}^2)} \left(-S_{\text{LT}}^{\{i} \mathbf{k}_T^{j\}} + \frac{3(\mathbf{k}_T \cdot \mathbf{S}_{\text{LT}})^2 - 3\mathbf{k}_T^2 \mathbf{S}_{\text{LT}}^2}{\mathbf{k}_T \cdot \mathbf{S}_{\text{LT}}} g_{\text{T}}^{ij} + \frac{8(\mathbf{k}_T \cdot \mathbf{S}_{\text{LT}})^2 - 6\mathbf{k}_T^2 \mathbf{S}_{\text{LT}}^2}{\mathbf{k}_T \cdot \mathbf{S}_{\text{LT}}} \frac{\mathbf{k}_T^i \mathbf{k}_T^j}{\mathbf{k}_T^2} \right) \Phi_{\text{LT}}^{ij}(x, \mathbf{k}_T) \\
 &= \{ -8(2g_1 - g_2)M^4(x-1)^2 (g_2(\mathbf{k}_T^2 + xM_S^2) - g_1(\mathbf{k}_T^2 - M^2(x-1)^2 + M_S^2)) - 2g_3 M^2(x-1) \\
 &\quad \times [4g_1((\mathbf{k}_T^2 + M_S^2)^2 - M^4(x-1)^4) + g_2(-2\mathbf{k}_T^2(M^2(x-1)^2 + (2x+1)M_S^2) - 3\mathbf{k}_T^4 + M^4(x-1)^4 \\
 &\quad + 2M^2(2x-1)(x-1)^2 M_S^2 + (1-4x)M_S^4)] + g_3^2 [\mathbf{k}_T^4 (M^2(x-1)^2 + 3M_S^2) + \mathbf{k}_T^2 (-M^4(x-1)^4 \\
 &\quad - 2M^2(x-1)^2 M_S^2 + 3M_S^4) + \mathbf{k}_T^6 - (M^2(x-1)^2 - M_S^2)^3] \} \\
 &\quad \times \{ 64\pi^3 M^2(x-1)^2 x M_S^2 (\mathbf{k}_T^2 + x(M^2(x-1) + M_S^2))^2 \}^{-1}. \tag{A9}
 \end{aligned}$$

The TT tensor polarized gluon TMDs are given by

$$\begin{aligned}
 f_{\text{ITT}}(x, \mathbf{k}_T^2) - h_{\text{ITT}}^\perp(x, \mathbf{k}_T^2) &= \frac{-M g_{\text{T}}^{ij}}{k_{\text{T}}^{\alpha\beta} S_{\text{TT}}^{\alpha\beta}} \Phi_{\text{TT}}^{ij}(x, \mathbf{k}_T) \\
 &= \{ 2g_3 M^2 [g_2(2\mathbf{k}_T^2 (M^2(2x-1)(x-1)^2 + (2(x-1)x+1)M_S^2) + (3x-2)\mathbf{k}_T^4 + x(M_S^2 - M^2(x-1)^2)^2) \\
 &\quad - 4g_1(x-1)\mathbf{k}_T^2 (\mathbf{k}_T^2 + M^2(x-1)^2 + M_S^2)] + g_3^2 \mathbf{k}_T^2 (2\mathbf{k}_T^2 (M^2(x-1)^2 + M_S^2) + \mathbf{k}_T^4 + (M_S^2 - M^2(x-1)^2)^2) \\
 &\quad + 4M^4(x-1) [g_2^2((2x-1)\mathbf{k}_T^2 + M^2 x(x-1)^2 + x(2x^2-1)M_S^2) - 2g_1 g_2((3x-2)\mathbf{k}_T^2 + M^2 x(x-1)^2 \\
 &\quad + x(2x-1)M_S^2) + 4g_1^2(x-1)\mathbf{k}_T^2] \} \\
 &\quad \times \{ 128\pi^3 M^2(x-1)x M_S^2 (\mathbf{k}_T^2 + x(M^2(x-1) + M_S^2))^2 \}^{-1}, \tag{A10}
 \end{aligned}$$

$$\begin{aligned}
 h_{\text{ITT}}(x, \mathbf{k}_T^2) &= \left(\frac{2\mathbf{k}_T^{ij}}{S_{\text{TT}}^{\alpha\beta} k_{\text{T}\alpha\beta}} - \frac{\mathbf{k}_T^4 S_{\text{TT}}^{\alpha\beta} S_{\text{TT}\alpha\beta}}{2S_{\text{TT}}^{\alpha\beta} k_{\text{T}\alpha\beta} - \mathbf{k}_T^4 S_{\text{TT}}^{\alpha\beta} S_{\text{TT}\alpha\beta}} \left(\frac{S_{\text{TT}}^{ij}}{S_{\text{TT}}^{\alpha\beta} S_{\text{TT}\alpha\beta}} + \frac{2\mathbf{k}_T^{ij\alpha} k_{\text{T}\alpha}}{\mathbf{k}_T^2 S_{\text{TT}}^{\alpha\beta} k_{\text{T}\alpha\beta}} \right) \right) \Phi_{\text{TT}}^{ij}(x, \mathbf{k}_T) \\
 &= \{ 4g_2 M^2 \mathbf{k}_T^2 [g_3(\mathbf{k}_T^2 (M^2(3x-1)(x-1)^2 + (x(2x-1)+1)M_S^2) + (2x-1)\mathbf{k}_T^4 + x(M_S^2 - M^2(x-1)^2)^2) \\
 &\quad - 4g_1 M^2(x-1)((2x-1)(\mathbf{k}_T^2 + xM_S^2) + M^2 x(x-1)^2)] + \mathbf{k}_T^4 [-2g_3 M_S^2 (M^2(x-1)(g_3(x-1) + 4g_1) - g_3 \mathbf{k}_T^2) \\
 &\quad + (M^2(x-1)(4g_1 - g_3(x-1)) - g_3 \mathbf{k}_T^2)^2 + g_3^2 M_S^4] + 4g_2^2 M^4 [2xM_S^2((2(x-1)x+1)\mathbf{k}_T^2 - M^2(x-1)^2 x) \\
 &\quad + ((2x-1)\mathbf{k}_T^2 + M^2 x(x-1)^2)^2 + x^2 M_S^4] \} \\
 &\quad \times \{ 256\pi^3 M^4(x-1)x M_S^2 (\mathbf{k}_T^2 + x(M^2(x-1) + M_S^2))^2 \}^{-1}, \tag{A11}
 \end{aligned}$$

$$\begin{aligned}
h_{1\text{TT}}^{\perp\perp}(x, \mathbf{k}_\text{T}^2) &= \frac{4M^4 S_{\text{TT}}^{\alpha\beta} S_{\text{TT}\alpha\beta}}{2S_{\text{TT}}^{\alpha\beta} k_{\text{T}\alpha\beta} - \mathbf{k}_\text{T}^4 S_{\text{TT}}^{\alpha\beta} S_{\text{TT}\alpha\beta}} \left(\frac{S_{\text{TT}}^{ij}}{S_{\text{TT}}^{\alpha\beta} S_{\text{TT}\alpha\beta}} + \frac{2k_{\text{T}}^{ij\alpha} k_{\text{T}\alpha}}{\mathbf{k}_\text{T}^2 S_{\text{TT}}^{\alpha\beta} k_{\text{T}\alpha\beta}} \right) \Phi_{\text{TT}}^{ij}(x, \mathbf{k}_\text{T}) \\
&= \{-4g_3 M^2 (x-1) [(2g_1 - g_2)(\mathbf{k}_\text{T}^2 + M^2(x-1)^2) + (g_2(1-2x) + 2g_1)M_S^2] + g_3^2 [2\mathbf{k}_\text{T}^2(M^2(x-1)^2 + M_S^2) \\
&\quad + \mathbf{k}_\text{T}^4 + (M_S^2 - M^2(x-1)^2)^2] + 4(g_2 - 2g_1)^2 M^4 (x-1)^2\} \\
&\quad \times \{64\pi^3 (x-1)xM_S^2(\mathbf{k}_\text{T}^2 + x(M^2(x-1) + M_S^2))^2\}^{-1}.
\end{aligned} \tag{A12}$$

APPENDIX B: POSITIVITY BOUNDS

At leading twist, the positivity bounds are the following nine inequalities:

$$\frac{\mathbf{k}_\text{T}^2}{2M^2} |h_1^\perp - h_{1\text{LL}}^\perp| \leq f_1 - f_{1\text{LL}}, \tag{B1}$$

$$\frac{\mathbf{k}_\text{T}^4}{16M^4} [4(h_{1\text{LL}}^\perp)^2 + (2h_1^\perp + h_{1\text{LL}}^\perp)^2] \leq \left(f_1 + \frac{f_{1\text{LL}}}{2} + g_1\right) \left(f_1 + \frac{f_{1\text{LL}}}{2} - g_1\right), \tag{B2}$$

$$\frac{\mathbf{k}_\text{T}^2}{2M^2} (h_1^2 + 4h_{1\text{LT}}^2) \leq (f_1 - f_{1\text{LL}}) \left(f_1 + \frac{f_{1\text{LL}}}{2} + g_1\right), \tag{B3}$$

$$\frac{\mathbf{k}_\text{T}^6}{8M^6} [(h_{1\text{T}}^\perp)^2 + (h_{1\text{LT}}^\perp)^2] \leq (f_1 - f_{1\text{LL}}) \left(f_1 + \frac{f_{1\text{LL}}}{2} - g_1\right), \tag{B4}$$

$$\frac{\mathbf{k}_\text{T}^2}{2M^2} [(f_{1\text{T}}^\perp + g_{1\text{LT}})^2 + (f_{1\text{LT}} + g_{1\text{T}} + h_{1\text{LT}})^2] \leq (f_1 - f_{1\text{LL}}) \left(f_1 + \frac{f_{1\text{LL}}}{2} + g_1\right), \tag{B5}$$

$$\frac{\mathbf{k}_\text{T}^2}{2M^2} [(f_{1\text{T}}^\perp - g_{1\text{LT}})^2 + (f_{1\text{LT}} - g_{1\text{T}} + h_{1\text{LT}})^2] \leq (f_1 - f_{1\text{LL}}) \left(f_1 + \frac{f_{1\text{LL}}}{2} - g_1\right), \tag{B6}$$

$$|h_{1\text{TT}}| \leq \frac{1}{2} \left(f_1 + \frac{f_{1\text{LL}}}{2} + g_1\right), \tag{B7}$$

$$\frac{\mathbf{k}_\text{T}^4}{2M^4} |h_{1\text{TT}}^{\perp\perp}| \leq \left(f_1 + \frac{f_{1\text{LL}}}{2} - g_1\right), \tag{B8}$$

$$\frac{\mathbf{k}_\text{T}^4}{M^4} [g_{1\text{TT}}^2 + (f_{1\text{TT}} - h_{1\text{TT}}^\perp)^2] \leq \left(f_1 + \frac{f_{1\text{LL}}}{2} + g_1\right) \left(f_1 + \frac{f_{1\text{LL}}}{2} - g_1\right). \tag{B9}$$

For integrated TMDs case, there are the following three bounds:

$$|g_1| \leq f_1 + \frac{f_{1\text{LL}}}{2}, \tag{B10}$$

$$f_{1\text{LL}} \leq f_1, \tag{B11}$$

$$|h_{1\text{TT}}| \leq \frac{1}{2} \left(f_1 + \frac{f_{1\text{LL}}}{2} + g_1\right). \tag{B12}$$

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