

Electron-muon conversion in nuclei and rare decays induced by a lepton-flavor-violating dark photon

Alexey S. Zhevlakov^{1,2,*} Sergey Kuleshov^{3,4,†} Valery E. Lyubovitskij^{5,3,‡} and Evgenie O. Oleynik¹

¹*Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Russia*

²*Matrosov Institute for System Dynamics and Control Theory SB RAS,*

Lermontov strasse, 134, 664033 Irkutsk, Russia

³*Millennium Institute for Subatomic Physics at the High-Energy Frontier (SAPHIR) of ANID, Fernández Concha 700, Santiago, Chile*

⁴*Center for Theoretical and Experimental Particle Physics, Facultad de Ciencias Exactas, Universidad Andres Bello, Fernandez Concha 700, Santiago, Chile*

⁵*Institut für Theoretische Physik, Universität Tübingen, Kepler Center for Astro and Particle Physics, Auf der Morgenstelle 14, D-72076 Tübingen, Germany*



(Received 29 July 2025; accepted 23 December 2025; published 23 January 2026)

We study lepton-flavor-violation (LFV) effects occurring in the $e - \mu$ conversion in nuclei and in the rare radiative decays $\eta(\eta') \rightarrow \gamma\mu e$ of the η and η' mesons with a special impact of the sub-GeV mass vector mediator (dark photon). In the case of the lepton conversion, we make estimates for running and future experiments operating with electron beams at fixed target. Obtained results are implemented in analysis of the LFV decays $\eta(\eta') \rightarrow \gamma\mu e$.

DOI: [10.1103/h3qf-m7mb](https://doi.org/10.1103/h3qf-m7mb)

I. INTRODUCTION

Study of lepton flavor violation (LFV) is one of the promising directions for searching effects beyond the Standard Model (SM). During the last decades, certain experimental and theoretical progress has been achieved in understanding and deeper studying of LFV physics: neutrino oscillations, lepton conversion in nuclei, rare decays, etc. In particular, the most stringent upper limit on the branching ratio of the $\mu \rightarrow e\gamma$ decay comes from data collected at PSI (Switzerland) by the MEG and MEG II Collaborations: $\text{Br}(\mu \rightarrow e\gamma) < 3.1 \times 10^{-13}$ [1]. Before 2026, the MEG II experiment is designed to reach a sensitivity of $\text{Br}(\mu \rightarrow e\gamma) < 6 \times 10^{-14}$. The process $\mu \rightarrow 3e$ was studied by the SINDRUM (PSI) Collaboration giving the upper limit for the branching ratio $\text{Br}(\mu \rightarrow 3e) < 1 \times 10^{-12}$ [2]. More precise measurement of the $\mu \rightarrow 3e$ decay at PSI was proposed by the Mu3e Collaboration, which aims for the ultimate sensitivity of $\text{Br}(\mu \rightarrow 3e) < 10^{-16}$ [3,4].

Lepton conversion in nuclei is another very important tool for searching for LFV effects with high accuracy. There

are two main regimes for studying lepton conversion: lepton scattering off nuclei at small and large energies. Coherent $\mu^- - e$ conversion in nuclei or in muonic atoms at small energies has been studied by the SINDRUM II Collaboration [5] at PSI. Reference [5] set the current measured upper limits for the ratio

$$R_{\mu e} = \frac{\Gamma[\mu^- + (A, Z) \rightarrow e^- + (A, Z)]}{\Gamma[\mu^- + (A, Z) \rightarrow \nu_\mu + (A, Z - 1)]}, \quad (1)$$

which is $R_{\mu e}^{\text{Au}} < 7 \times 10^{-13}$ at 90% confidence level for the gold stopping target. Planned experiment MECO [6] at BNL (U.S.) is expected to improve the SINDRUM II upper limit by at least 3–4 orders of magnitude for $R_{\mu e}^{\text{Al}} < 5 \times 10^{-17}$ at 90% confidence for the aluminum stopping target. The goal of two other experiments being prepared by the COMET Collaboration at J-PARC (Japan) [3] and the Mu2e Collaboration at FermiLab (U.S.) [7] is to reach the accuracy of $R_{\mu e}^{\text{Al}} < 3 \times 10^{-17}$ at 90% confidence for the aluminum stopping target. Deep inelastic lepton conversion $e + p \rightarrow \tau + X$ was searched for by the ZEUS Collaboration at HERA (DESY) [8]. In the future, it is planned to study high-energy lepton conversion at CERN SPS [9], EIC, FCC, and LHeC colliders [10].

From the theoretical side, different models have been tested in the context of future experimental studies of LFV effects in the NA64 experiment at CERN SPS [9,11], FASER ν 2 detector at the envisioned Forward Physics Facility [12], and at future colliders (EIC, FCC, LHeC)

*Contact author: zhevlakov@theor.jinr.ru

†Contact author: sergey.kuleshov@unab.cl

‡Contact author: valeri.lyubovitskij@uni-tuebingen.de

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

[10]. There exists in literature a wide class of models and hypotheses aiming to describe LFV phenomena, e.g., such as high-dimensional operators [9,13], scalar (pseudoscalar) [13–15], axion-like particles [11,16], vector mediators [13,15,17–21], etc. Part of research works proposed use the NA64 experiment to study LFV physics, in particular, lepton conversion [9,11,20]. These studies are possible after LS3 in the NA64 experiment [22,23].

In the present paper, we focus on a role of the vector mediator (dark photon) in description of LFV effects. In particular, we have the following main objectives: (1) We derive the bounds on the diagonal and nondiagonal couplings governing the LFV $\mu - e$ transition considering different rare processes. We note that in Refs. [20,24] parameter spaces of dark photon mediator producing LFV effects were studied in processes with a possible search for radiation of this dark state. (2) We concentrate on the study of fixed-target experiments using electron beams and allowing production of a dark photon with mass in sub-GeV region. (3) We implement bounds for rare LFV light pseudoscalar meson decays from the dark photon model.

The study of light pseudoscalar meson decays is a one of the important tools for searching dark matter and studying rare physics, including C and CP violation [25] and LFV [26]. In particular, decays of light pseudoscalar mesons are used to search for invisible [27,28] and visible [29] dark photon modes. A broad scientific program, including LFV decays and rare decays of the η and η' mesons, is indicated in the proposals of the REDTOP [30] and eta-HIAF [26] factories. The eta-HIAF factory is expected to yield approximately $10^{12} \div 10^{13}$ η mesons during one year [26,31] that will be a good test of rare decay physics. In according to interest for study LFV η and η' meson decays at HIAF factory we made the estimates based on current limits for vector portal. We shown that such rare decays can be more sensitive to LFV effects than lepton conversion at GeV energies of electron beams.

The paper is organized as follows. In Sec. II, we present details of inclusion of dark photons in our quantum field formalism. In Sec. III, we discuss kinematical and dynamical aspects of the LFV physical processes, which are studied in our paper. In particular, we focus on two- and three-body rare decays of muon and $e - \mu$ lepton conversion in nuclei using electron beams at fixed-target experiments. Later, in Sec. IV, we briefly discuss what follows from results for the LFV rare decays of the η and η' mesons. In Sec. V, we summarize our results and findings. Additionally, some complementary formulas are included in the Appendix.

II. INCLUSION OF DARK PHOTONS IN QUANTUM FIELD FORMALISM

In our paper, we follow quantum field formalism allowing one to include the dark photon developed in Ref. [19]. In particular, the phenomenological Lagrangian

involves singlet scalar field σ , dark photon A' , and Dirac dark fermion χ . The mass of the dark photon $m_{A'}$ is generated via the Stueckelberg mechanism (Stueckelberg portal) [32] by extension of the derivative acting on the scalar field σ to covariant one containing mass of the A' . The corresponding model Lagrangian reads [19]

$$\mathcal{L}'_{\text{DS}} = -\frac{1}{4}A'_{\mu\nu}A'^{\mu\nu} + \frac{m_{A'}^2}{2}A'_\mu A'^\mu + \bar{\chi}(i\not{D}_\chi - m_\chi)\chi - \frac{1}{2\xi}(\partial_\mu A'^\mu)^2 + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \xi\frac{m_{A'}^2}{2}\sigma^2, \quad (2)$$

where ξ is the gauge-fixing parameter and $A'_{\mu\nu}$ is strength tension of dark photon. Note, the interaction of the dark photon with charged SM fermions can include both diagonal and nondiagonal couplings,

$$\mathcal{L}_{A'\psi} = e e A'_\mu(x) \sum_{i,j=e,\mu,\tau} \bar{\psi}_i(x) \gamma^\mu G_{ij} \psi_j(x),$$

$$G_{ij} = -\delta_{ij} + g_{ij}^V + g_{ij}^A \gamma_5, \quad (3)$$

where g_{ij}^V and g_{ij}^A are the vector and axial-vector dimensionless couplings, respectively; ϵ is the QED-dark-photon mixing parameter. The first term comes from general kinetic mixing of the photon and dark photon after shift of photon field [33]. Coupling proportional to $e e g_{ll}^V$ can be induced by direct interaction of the dark photon with matter based on a different mechanism. For example, such scenarios have been proposed in Refs. [19,34]. In the following, we use the following notations $g_{ll} = e\epsilon(-\delta_{ll} + g_{ll}^V)$ and $g_{lf} = e e g_{lf}^{A,V}$ for diagonal and nondiagonal couplings, respectively, that is needed for analysis. The δ_{ij} is delta Kronecker symbol. In our applications, we focus on the use of the vector couplings.

In the framework of this work, we consider different scenerios of interaction dark photon with SM matter, for diagonal couplings:

- (i) The general dark photon portal: This portal is connected only with general kinetic mixing term [35] $\mathcal{L} = \frac{\epsilon}{2} F_{\mu\nu} F'^{\mu\nu}$ where ϵ is the kinetic mixing parameter between the two $U(1)$ gauge symmetries, and $F_{\mu\nu}$ and $F'^{\mu\nu}$ are the strength tensors of SM electromagnetic and dark matter (DM) dark gauge (dark photon) fields, respectively. By shift of the electromagnetic (EM) field $A_\mu \rightarrow A_\mu - \epsilon A'_\mu$, we can obtain interaction dark photon with charge current of lepton or quark fields. In results $g_{ll} = e\epsilon(-\delta_{ll} + g_{ll}^V)$ includes interaction with all fermions, in general.
- (ii) The nonminimal leptophilic dark photon portal: In this scenario, the dark photon couples directly only to leptons, with no tree-level kinetic mixing. The direct coupling is parametrized as $g_{ll} = e\epsilon(g_{ll}^V)$. Interactions with quarks are not present at tree level, but are generated radiatively through fermion loops involving leptons and a virtual photon. This loop-induced coupling is suppressed by a factor of $\sim e^2/(16\pi^2)$ and

exhibits a logarithmic dependence on the lepton mass scale, $\ln(m_l^2/\mu^2)$.

- (iii) The nonminimal hadrophilic dark photon portal: This is the similar case of interaction, but where the dark photon couples directly only to quarks, gain with no tree-level kinetic mixing. The direct coupling is $g_{qq} = e\epsilon(g_{qq}^V)$. Interactions with leptons are generated indirectly through quark-loop diagrams and are suppressed by $\sim e^2/(16\pi^2)$, with a logarithmic dependence on the quark mass scale, $\ln(m_q^2/\mu^2)$.

A different scenario will give a different dependence in interaction, including vector meson and dark photon mixing, which is considered here at estimation LFV decays of η and η' mesons. In this time exist upper experimental limits on different scenarios of dark photon models which will be used here in calculation.

III. LFV DECAYS

In this section, we briefly discuss kinematical and dynamical aspects of the following LFV processes: decays of muons $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ and $e - \mu$ conversion in nuclei. Later, we will see that rare decays give strong bounds for parameter space of models.

A. Decay $\mu \rightarrow e\gamma$

A gauge-invariant set of Feynman diagrams induced by the dark photon mediator and contributing to the $\mu \rightarrow e\gamma$ decay is shown in Fig. 1. Corresponding manifestly gauge-invariant leptonic current in momentum space has the form

$$J^\mu(p, p') = \bar{u}_e(p')\Gamma^\mu(p, p')u_\mu(p),$$

$$\Gamma^\mu(p, p') = \gamma_\perp^\mu F_1^{e\mu}(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{m_\mu + m_e} F_2^{e\mu}(q^2), \quad (4)$$

where $\gamma_\perp^\mu = \gamma^\mu - \frac{q^\mu \not{q}}{q^2}$ is the Dirac matrix orthogonal to the photon momentum $q = p - p'$ with $q_\mu \gamma_\perp^\mu = 0$.

Expression for the form factors $F_i^{e\mu}(q^2)$ is given in the Appendix. In the limit $m_\mu = m_e$ and $m_{A'} \rightarrow 0$, the $F_2^{e\mu}(0)$ is equal to $\frac{g_{ll}g_{ff}}{4\pi^2}$, which is the LFV analog of the Schwinger contribution $\frac{\alpha}{2\pi}$ to the anomalous magnetic moment of the charged fermion with spin 1/2 in QED.

The decay width of the $\mu \rightarrow e\gamma$ transition reads

$$\Gamma(\mu \rightarrow e\gamma) = (g_{\mu e}g_{ll})^2 \frac{\alpha}{2} m_\mu \left(1 - \frac{m_e}{m_\mu}\right)^2 \left[F_2^{e\mu}(0)\right]^2. \quad (5)$$

Note, the Dirac form factor $F_1^{e\mu}(q^2)$ is finite for $q^2 \neq 0$, while it has divergence at $q^2 = 0$. This divergence is removed by introducing an appropriate counterterm in analogy with QED. Therefore, the renormalized Dirac form factor has the form $F_{1,r}^{e\mu}(q^2) = F_1^{e\mu}(q^2) - F_1^{e\mu}(0)$. It means that $F_{1,r}^{e\mu}(0) = 0$; i.e., LFV is absent for real photon emission. The expression for the $F_{1,r}^{e\mu}(q^2)$ is presented in the Appendix. In the limit $m_e = m_\mu$ and $m_{A'} = 0$, we reproduce the well-known QED result for the one-loop vertex correction [36].

B. Decay $\mu \rightarrow 3e$

The diagram contributing to the $\mu \rightarrow 3e$ decay, which is induced by the nondiagonal coupling of the dark photon with SM leptons, is shown in Fig. 2. The $\mu \rightarrow 3e$ decay width in the limit of massless electron $m_e = 0$ is given by

$$\Gamma(\mu \rightarrow 3e) = \frac{(g_{ll}g_{\mu e})^2}{256\pi^3 m_\mu^3} \int_0^{m_\mu^2} ds_2 \int_0^{m_\mu^2 - s_2} ds_1 \frac{1}{2} \sum_{\text{pol}} |M_{\text{inv}}|^2, \quad (6)$$

where $\frac{1}{2} \sum_{\text{pol}} |M_{\text{inv}}|^2$ is the square of the invariant matrix element of the $\mu \rightarrow 3e$ decay averaged over the muon spin and polarizations of fermions,

$$\frac{1}{2} \sum_{\text{pol}} |M_{\text{inv}}|^2 = 8 \left(\frac{(s_1 + s_2)(m_\mu^2 - s_1 - s_2) + s_2(m_\mu^2 - s_2)}{2(s_1 - m_{A'}^2)^2} + \frac{(s_1 + s_2)(m_\mu^2 - s_1 - s_2) + s_1(m_\mu^2 - s_1)}{2(s_2 - m_{A'}^2)^2} - \frac{(s_1 + s_2)(m_\mu^2 - s_1 - s_2)}{(s_1 - m_{A'}^2)(s_2 - m_{A'}^2)} \right). \quad (7)$$

Here we use the Mandelstam variables,

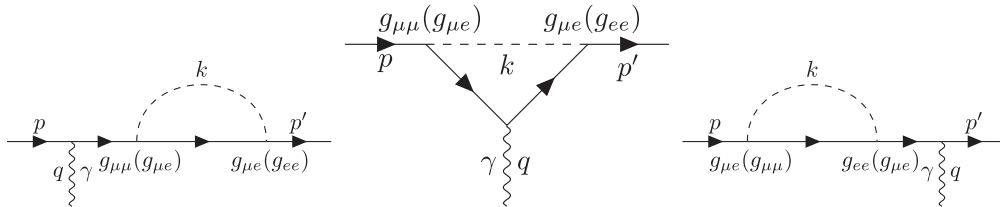


FIG. 1. Gauge-invariant set of Feynman diagrams induced by dark photon mediator and contributing to the $\mu \rightarrow e\gamma$ decay.

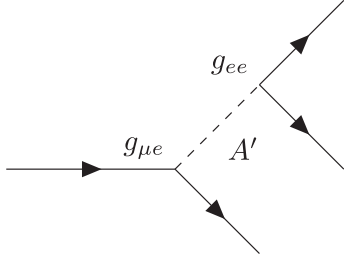


FIG. 2. Diagram contributing to the $\mu \rightarrow 3e$ decay generated by the dark photon mediator due to its nondiagonal coupling with SM leptons.

$$\begin{aligned} s_1 &= (p_1 + p_2)^2 = (p - p_3)^2, \\ s_2 &= (p_2 + p_3)^2 = (p - p_1)^2, \\ s_3 &= (p_1 + p_3)^2 = (p - p_2)^2, \\ s_1 + s_2 + s_3 &= s = m_\mu^2. \end{aligned} \quad (8)$$

C. Lepton conversion in fixed-target experiments

In this section, we focus on the process of lepton $e - \mu$ conversion. Here our main purpose is to understand possible opportunities for searching this process in fixed-target experiments. In particular, we will study the constraints for experiments, which are able to produce lepton beams with huge intensity and aim to detect feebly interactive particles of dark matter or mediators between SM and DM. One can expect that in fixed-target experiments $e - \mu$ conversion should have a good signal, because a produced muon will have an energy close to beam energy, and therefore it can be registered. Production of such an energetic muon will be the tag for the $e - \mu$ conversion. Note, the use of muon beam in fixed-target experiments for searching $\mu - e$ conversion is more problematic due to small radiation and dissipation lengths of electron in a target. The $\mu - \tau$ and $e - \tau$ conversions have difficulties in isolating the signal from τ lepton decays.

One of the best candidates to study $e - \mu$ conversion in the regime of fixed target by using electron beam is the NA64_e experiment at CERN SPS [37,38]. At present, the NA64_e setup is able to collect $\sim 10^{12}$ electrons on target. Additionally, in the future it is planned to search for dark matter using electron beams with different types of targets in the experiments LDMX at SLAC (U.S.) [39–41], BDX at FermiLab (U.S.) [42], and DarkSHINE at SHINE (China)

[43]. The experiments NA64_e, LDMX, and DarkSHINE are based on the use of missing energy and momentum techniques. The BDX experiment for searching dark matter is based on beam dump with a possibility to observe visible decay after the wall [42]. All experiments have a calorimeter system and track system, which can register energetic muons inside. We collect the main parameters of the fixed-target experiments NA64_e, LDMX, BDX, and DarkSHINE in Table I.

The number of produced muons in the reaction of the $e - \mu$

$$N_\mu \simeq \text{EoT} \cdot \frac{\rho_T N_A}{A} L_T \int dx \frac{d\sigma(E)_{e \rightarrow \mu}}{dx}, \quad (9)$$

where N_A is the Avogadro number, and $d\sigma_{e \rightarrow \mu}/dx$ is the differential cross section of the lepton conversion $eN_Z \rightarrow \mu N_Z$ on nucleus N_Z with atomic number A and charge Z . Other parameters are collected in Table I.

Kinematics of lepton conversion is specified by the following set of the Mandelstam variables:

$$\begin{aligned} s &= (k_1 + p_1)^2 = (k_2 + p_2)^2, \\ t &= (k_2 - k_1)^2 = (p_2 - p_1)^2, \\ u &= (k_1 - p_2)^2 = (k_2 - p_1)^2, \\ s + t + u &= 2M_{N_Z}^2 + m_\mu^2 + m_e^2, \end{aligned} \quad (10)$$

where p_1 and p_2 , k_1 and k_2 are the initial and final nuclear momenta and the electron and muon momenta, respectively; M_{N_Z} is the nucleus mass.

In general, the interaction of an electromagnetic field A_μ and nucleus can be effectively represented as [44]

$$\mathcal{L}_{N_Z}^{\text{eff}} = -eA_\nu J_{N_Z}^\nu, \quad (11)$$

where $J_{N_Z}^\nu$ is nuclear current. For heavy nucleus, one can exploit a spin-0 form factor with a good accuracy [45]. Thus, the hadronic current in the momentum space reads as [45,46]

$$J_{N_Z}^\nu = F_{N_Z}(t)(p_1 + p_2)^\nu. \quad (12)$$

Lepton conversion in nuclei is contributed by two diagrams shown in Fig. 3: (left panel) diagram induced

TABLE I. Parameters of the fixed-target experiments NA64_e, LDMX, BDX, and darkSHINE: target parameters (A , Z), energy of scattering beam E , target density ρ_T , first radiation length X_0 , effective thickness of the target (L_T), and planned accumulate of electrons on target (EoT).

	e -conv.	A (Z)	E (GeV)	ρ_T (g cm ⁻³)	X_0 (cm)	L_T (cm)	Projected EoT
NA64 _e :	$eN \rightarrow N\mu$	207 (82)	100	11.34	0.56	0.5	5×10^{12}
LDMX:	$eN \rightarrow N\mu$	27 (13)	16	2.7	8.9	3.56	10^{18}
BDX:	$eN \rightarrow N\mu$	27 (13)	11	2.7	8.9	3.56	10^{22}
darkSHINE:	$eN \rightarrow N\mu$	184 (74)	8	19.3	0.35	0.35	10^{16}

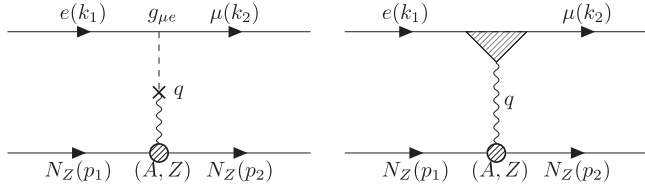


FIG. 3. Diagrams contributed to the lepton $e - \mu$ conversion in nuclei due to LFV dark photon: diagram induced by mixing of QED and dark photon (left); diagram induced by form factor relevant for the $\mu \rightarrow e\gamma$ transition (right). Dashed triangle blob in right diagram represents the vertex function $\Gamma_\nu(k_1, k_2)$.

by mixing of QED and dark photon; (right panel) diagram induced by form factor relevant for the $\mu \rightarrow e\gamma$ transition. Dashed triangle blob in right diagram represents the vertex function $\Gamma_\nu(k_1, k_2)$. Matrix element of lepton conversion is given by the sum of two terms M_1^{conv} and M_2^{conv} corresponding to diagrams shown in Fig. 3,

$$M^{\text{conv}} = M_1^{\text{conv}} + M_2^{\text{conv}}, \quad (13)$$

where

$$M_1^{\text{conv}} = \frac{e^2 g_{ll} g_{e\mu}}{q^2 - m_{A'}^2} \bar{u}_e(k_2) \gamma_\nu u_\mu(k_1) J_{N_Z}^\nu, \quad (14)$$

$$M_2^{\text{conv}} = \frac{e^2 g_{ll} g_{e\mu}}{q^2 - m_{A'}^2} \bar{u}_e(k_2) \Gamma_\mu(k_1, k_2) u_\mu(k_1) J_{N_Z}^\nu. \quad (15)$$

In M_1^{conv} the mixing is induced by lepton loop between dark photon and external photon coming from nuclei. Here it is proposed that direct interaction of dark photon with baryons does not exist and only occurs via mixing generated by the lepton loop. Lepton-loop contribution is collected in the product of two coupling eg_{ll} . Differential cross section is given by

$$\frac{d\sigma}{dt} = \frac{1}{2} \frac{\sum_{\text{pol}} |M_{\text{conv}}|^2}{16\pi(s - M_{N_Z}^2)}. \quad (16)$$

The mixing parameter ϵ is considered as a free parameter. The interaction with nucleus is parametrized by taking into account a general electric form factor which includes Z^2 and Z terms describing “elastic” and “inelastic” effects, respectively. The form of this nuclear form factor is parametrized by Tsai [47,48] and Bjorken *et al.* [49,50] as

$$F_{N_Z}^2(t) = \left(\frac{a^2 t}{1 + a^2 t} \right)^2 \left(\frac{1}{1 + \frac{t}{d}} \right)^2 Z^2 + \left(\frac{a^2 t}{1 + a^2 t} \right)^2 \left(\frac{1 + \frac{t}{4m_p^2} (\mu_p^2 - 1)}{(1 + \frac{t}{0.71 \text{ GeV}^2})^4} \right)^2 Z, \quad (17)$$

where the parameters of nuclear form factor are specified: $a = 111Z^{-1/3}/m_e$ and $d = 0.164A^{-2/3} \text{ GeV}^2$ are the

screening and nucleus size parameters, respectively [49]; $a' = 773Z^{-2/3}/m_e$ and $\mu_p = 2.793$ (proton magnetic moment) [51] are parameters for inelastic scattering.

IV. RESULTS

In this section, we present our results for the bounds on the LFV couplings extracted from analysis of rare LFV decays. We hope that our predictions will be useful for running and future experiments with huge intensity electron beams in fixed-target experiments. Unfortunately, the bounds on the LFV couplings derived from study of the $e - \mu$ conversion in nuclei using electron beam with exchange by vector boson between LFV lepton current and nuclear current are less stringent than existing bounds from LFV muon decays. Obtained bounds for diagonal and nondiagonal couplings as functions of mass of dark photon $m_{A'}$ are shown in Fig. 4. It seems that currently in fixed-target experiments it is impossible to get the level of sensitivity achieved by decay studies.

A. Bounds for LFV couplings from rare η and η' meson decays

Using obtained results for combination of diagonal and nondiagonal coupling, we have a possibility to estimate branchings of the LFV decays of η and η' mesons. An experimental study of these decays is planned in the η and η' factory REDTOP at FermiLab (USA) [30]. The vector mediator is a simple model, which can give bounds for the three-body LFV decay widths of $\eta(\eta') \rightarrow \gamma\mu e$ using existing limits from LFV decay and lepton conversion. In general, such bound should be weakly changed in dependence of type of mediator. For other LFV decays,

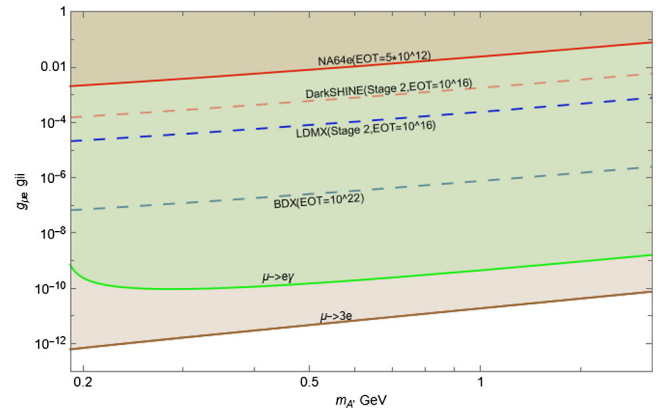


FIG. 4. $m_{A'}$ dependence of bounds for the products of diagonal and nondiagonal couplings of dark photon with SM fermions. Shaded region corresponds to available data from limits on rare muon LFV decays and for current statistics of the NA64 experiment. Dashed lines correspond to estimates of future beam dump experiments with electron beams. Parameters used here are presented in Table I.

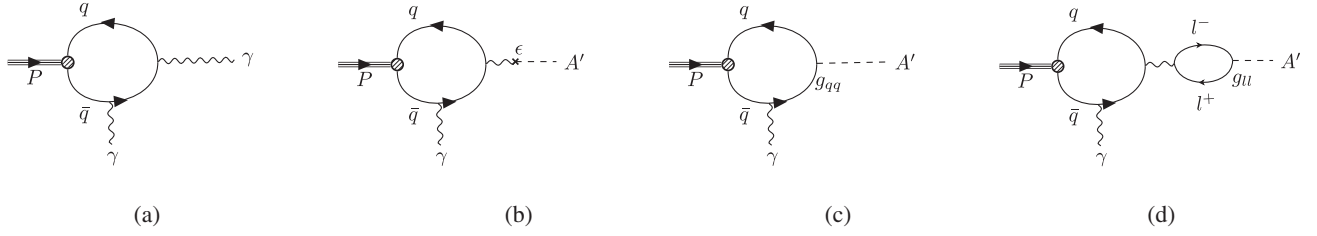


FIG. 5. Feynman diagrams that show vertices of decay pseudoscalar meson into two photons (a) or one photon and a dark photon in different scenarios: (b) general, (c) hadrophilic, and (d) leptophilic.

such as two-body decays $\eta \rightarrow e\mu$ and $\eta' \rightarrow e\mu$, strict limits from conversion were obtained in Ref. [52].

The LFV decays $\eta(\eta') \rightarrow \gamma l^+ l^-$ are related to the Dalitz lepton-flavor-conserving decays $\eta(\eta') \rightarrow \gamma l^+ l^-$ because matrix elements of both types of processes contain anomalous pseudoscalar-vector-vector couplings: $P\gamma A'$ and $P\gamma\gamma$ (the Wess-Zumino-Witten chiral anomaly), respectively. A nice point is that we can relate $P\gamma A'$ and $P\gamma\gamma$ couplings using the QED-dark-photon mixing. In particular, starting with phenomenological Lagrangian governing the chiral anomaly,

$$\mathcal{L}_{P\gamma\gamma} = \frac{e^2}{8} g_{P\gamma\gamma} \epsilon_{\mu\nu\alpha\beta} P(x) F^{\mu\nu}(x) F^{\alpha\beta}(x), \quad (18)$$

where $F^{\mu\nu}$ is the stress tensor of electromagnetic field, $\epsilon_{\mu\nu\alpha\beta}$ is the antisymmetric Levi-Civita tensor, and $g_{P\gamma\gamma}$ is the $P\gamma\gamma$ coupling. This vertex is shown in Fig. 5(a). We derive phenomenological Lagrangian describing the coupling of pseudoscalar meson P with QED and dark photons,

$$\mathcal{L}_{P\gamma A'} = \frac{e^2}{8} g_{P\gamma A'} \epsilon_{\mu\nu\alpha\beta} P(x) F^{\mu\nu}(x) F'^{\alpha\beta}(x), \quad (19)$$

where $g_{P\gamma A'}$ is the $P\gamma A'$ coupling, which for three different scenarios [corresponding Feynman diagrams are shown in Figs. 5(b)–5(d)] are fixed as:

- (1) General scenario: $g_{P\gamma A'} = \epsilon g_{P\gamma\gamma}$. In this scenario, we base on the kinetic mixing of the QED and dark photons [35], which after the shift of one of the EM field $A_\mu \rightarrow A_\mu - \epsilon A'_\mu$ leads to the coupling of the pseudoscalar meson pair of QED and dark photons.
- (2) Hadrophilic scenario: $g_{P\gamma A'} = (g_{qq}/e) g_{P\gamma\gamma}$ [53,54]. In this scenario, the dark photon directly interacts with quarks.
- (3) Leptophilic scenario: $g_{P\gamma A'} = \frac{e g_{ll}}{16\pi} \ln(m_l^2/\mu^2) g_{P\gamma\gamma}$ [55]. In this scenario, the dark photon directly interacts with leptons, and therefore its coupling with the QED photon occurs via the lepton loop, where μ is a renormalization scale and m_l is the mass of lepton propagating in the loop.

Note, the decay width of the anomaly process $P \rightarrow \gamma\gamma$ in terms of coupling $g_{P\gamma\gamma}$ reads

$$\Gamma(P \rightarrow \gamma\gamma) = \frac{\pi\alpha^2}{4} g_{P\gamma\gamma}^2 m_P^3, \quad (20)$$

where m_P is the mass of the respective pseudoscalar meson. $P \rightarrow \gamma A'$ decay width can be written as

$$\Gamma(P \rightarrow \gamma A') = \frac{\epsilon^2}{2} (1 - x_{A'P})^3 \Gamma(P \rightarrow \gamma\gamma), \quad (21)$$

where $x_{ij} = m_i^2/m_j^2$ is the ratio of squared masses of corresponding particles.

With taking into account intermediate vector mesons $V = \rho^0, \omega, \phi$ we get for the decay width

$$\Gamma(P \rightarrow \gamma\mu e) = \frac{\alpha g_{\mu e}^2}{1024\pi m_P^3} \int_{m_\mu^2}^{m_\eta^2} \frac{dq^2}{q^4} \times \frac{A(m_P^2, m_\mu^2, q^2)}{(m_{A'}^2 - q^2)^2 + \Gamma_{A' \rightarrow \mu e}^2 m_{A'}^2} g_{P\gamma A'}^2, \quad (22)$$

where

$$\begin{aligned} A(m_P^2, m_\mu^2, q^2) &= (m_\mu^2 - q^2)((m_P^2 - q^2)^3(q^2 - m_\mu^2)^2 \\ &\quad - (m_P^2 - q^2)^3(q^4 - 2m_\mu^4) \\ &\quad - (m_P^6 - q^6)m_\mu^2 q^2). \end{aligned} \quad (23)$$

In the case with including intermediate vector meson states, the coupling $g_{P\gamma A'}$ is changed as

$$g_{P\gamma A'}^2 \rightarrow \left[g_{P\gamma A'}^2 + 2g_{P\gamma A'} \text{Re}(g_{P\gamma A'}^R(q^2)) + |g_{P\gamma A'}^R(q^2)|^2 \right],$$

with mixing photon and vector meson

$$g_{P\gamma A'}^R(q^2) = \sum_{V=\rho^0, \omega, \phi} \frac{q^2 g_{VP\gamma} g_{VA'}}{q^2 - m_V^2 + i\Gamma_V m_V}, \quad (24)$$

where coupling $g_{P\gamma A'}$ for different scenarios was specified after Eq. (19), and coupling $g_{VA'}$ defining the $A' \rightarrow V$ transition is fixed for three scenarios as:

- (1) $g_{VA'} = \epsilon g_{V\gamma}$ for general dark photon scenario;
- (2) $g_{VA'} = g_{V\gamma}(g_{qq}/e)$ for the hadrophilic scenario [53,54];

- (3) $g_{VA'} = \frac{g_{\mu e}}{16\pi^2} \ln(m_l^2/\mu^2) g_{V\gamma}$ for the leptophilic scenario.

The hadrophilic and leptophilic scenarios are specified by direct interaction of dark photon only with quarks or leptons, respectively. The general scenario is based on the general mixing mechanism allowing direct interaction of the dark photon with both quarks and leptons.

The $g_{\rho^0\eta\gamma}$ coupling is fixed from decay width

$$\Gamma(V \rightarrow P\gamma) = \frac{\alpha}{24} g_{VP\gamma}^2 m_V^3 (1 - x_{PV})^3. \quad (25)$$

The couplings $g_{VP\gamma}$ are fixed from data on the corresponding $P \rightarrow V\gamma$ decay widths

$$\Gamma(P \rightarrow V\gamma) = \frac{\alpha}{8} g_{VP\gamma}^2 m_P^3 (1 - x_{VP})^3. \quad (26)$$

Numerical values of the coupling $g_{P\gamma\gamma}$ and $g_{PV\gamma}$ fixed from data [51] are

$$\begin{aligned} g_{\eta\gamma\gamma} &= 0.274 \text{ GeV}^{-1}, & g_{\eta'\gamma\gamma} &= 0.341 \text{ GeV}^{-1}, \\ g_{\rho^0\eta\gamma} &= 1.55 \text{ GeV}^{-1}, & g_{\rho^0\eta'\gamma} &= 2.73 \text{ GeV}^{-1}. \end{aligned} \quad (27)$$

The specific neutral vector meson $g_{V\gamma}$ is equal to $g_{\rho\gamma} = 0.202$ for ρ^0 , $g_{\omega\gamma} = 0.059$ for ω , and $g_{\phi\gamma} = 0.075$ for ϕ .

In the limit $m_e \rightarrow 0$ the decay width $\Gamma_{A' \rightarrow \mu e}$ is given by

$$\Gamma_{A' \rightarrow \mu e} = \frac{g_{\mu e}^2}{12\pi} m_{A'} \sqrt{1 - y_\mu^4} (1 - 2y_\mu^2 - 2y_\mu^4), \quad (28)$$

where $y_l = m_l/m_{A'}$. The decay width of the dark photon into a lepton-antilepton pair of the same flavor is given by

$$\Gamma_{A' \rightarrow \bar{l}l} = \frac{g_{ll}^2}{12\pi} m_{A'} \sqrt{1 - 4y_l^2} (1 + 2y_l^2), \quad (29)$$

where g_{ll} will be dependent on scenario.

For heavy mass of the dark photon $m_{A'} > m_\eta$, the decay width $\Gamma(P \rightarrow \gamma\mu e) \sim m_P (g_{ll} g_{\mu e})^2$. Current bounds for production of diagonal and nondiagonal coupling are at level 10^{-10} ; hence $\text{Br}(\eta(\eta') \rightarrow \gamma\mu e)$ will be at level $\lesssim 10^{-18}$. Maximal value for this decay width can be reached for the dark photon mass below η meson mass, i.e., in the region $m_{A'} < m_\eta$, where resonance production is possible. In sub-GeV mass region, the decay width for the dark photon is of order $\Gamma_{A' \rightarrow \mu e} \sim m_P \times O(g_{\mu e}^2)$, and decay width $\Gamma(\eta(\eta') \rightarrow \gamma\mu e)$ can be computed using known narrow-width approximation [51,56,57],

$$\begin{aligned} \int_{s_-}^{s_+} ds \frac{f(s)}{(s - m^2)^2 + m^2 \Gamma^2} &\approx \int_{s_-}^{s_+} ds \frac{\pi}{m\Gamma} f(s) \delta(s - m^2) \\ &= \frac{\pi}{m\Gamma} f(m^2) \end{aligned} \quad (30)$$

for $\Gamma \ll m$ and $s_- < m^2 < s_+$. The Γ is decay width, which is the sum of all channels and final results will be dependent upon which mode is dominant.

For the upper limit, we used in narrow-width approximation decay width $A' \rightarrow \mu e$ if nondiagonal coupling is dominant. As a result, $\Gamma(P \rightarrow \gamma\mu e)$ is proportional to g_{ll}^2 . In this case, the general, hadrophilic, or leptophilic scenario for diagonal interaction couplings could occur. Other scenarios are described in Ref. [58]. All scenarios have bounds from experiments that studied dark matter or can be recalculated from one to another. For the leptophilic scenario, there exist bounds from invisible mode from the BABAR [59] and NA64 [38] experiments. For the hadrophilic scenario, current limits for $m_{A'} < m_\eta$ are established from the COHERENT [29] or MiniBooNE [60,61] experiments. Upper bounds can be derived using narrow-width approximation by taking into account existing bounds for dark photon mixing for mass $m_{A'} \sim m_\mu$ and without taking into account intermediate vector meson states. In the hadrophilic case, we can obtain limits for decay branching $\text{Br}(\eta \rightarrow \gamma\mu e) < 0.08 \times (\alpha_B/\alpha)$ and $\text{Br}(\eta' \rightarrow \gamma\mu e) < 0.76 \times (\alpha_B/\alpha)$, where $\alpha_B = g_{qq}^2/4\pi$. From the COHERENT experiment, one gets $\alpha_B \sim 10^{-7} \div 10^{-9}$ for $m_\mu < m_A < m_P$. Finally, using the bound for the hadrophilic scenario, we obtain $\text{Br}(\eta \rightarrow \gamma\mu e) \lesssim 2.7 \times 10^{-7}$ and $\text{Br}(\eta' \rightarrow \gamma\mu e) \lesssim 2 \times 10^{-8}$. For the general case of dark vector boson scenario, we can use bounds from the BABAR [59] and NA64 [38] experiments, $\lesssim 2 \times 10^{-7}$ and $\lesssim 10^{-9}$, respectively. Note, the branchings for the leptophilic case are suppressed by a factor $\alpha/64\pi^3$ in comparison with the general dark photon scenario. If dark photon mass $m_{A'}$ dark photon is heavier than mass of decaying meson, then we obtain that the branching ratio is more suppressed due to existing bounds from LFV muon decays. The contribution with taking into account that intermediate meson states produce less contribution are omitted here, as they do not provide new strict upper bounds.

For the case when diagonal coupling is dominant, $\Gamma(P \rightarrow \gamma\mu e)$ is proportional to $m_P \times g_{\mu e}^2$. For the general case of dark vector boson scenario, we can use bounds from BABAR and NA64 [20] experiments for the LFV case, $\lesssim 2 \times 10^{-12}$ and $\lesssim 10^{-13}$, respectively, for η and η' mesons decays.

V. SUMMARY

We discussed a possibility to study $e - \mu$ conversion on nuclear in experiments on fixed target in a phenomenological approach with vector boson mediator (dark photon). Obtained results solidify the impossibility of such study due to the requirement to increase statistics by orders of magnitude. The strict bounds from LFV muon decay were analyzed. Branchings of the η and η' meson decays will be suppressed when the vector meson mediator is heavier than η and η' states. In the resonance case, obtained upper bounds will be $\text{Br}(\eta \rightarrow \gamma\mu e) < 2 \times 10^{-7}$ and $\text{Br}(\eta' \rightarrow \gamma\mu e) < 2 \times 10^{-8}$ based on data limits collected from search

for the dark photon in different experiments. This case also looks problematic for study at future η and η' factories, but possible with planned meson yield [26], if the corresponding processes involve feebly interacting vector mediator (*à la* dark photon). Additionally, decay into $\eta \rightarrow \gamma\mu e$ has a specific signature (with one muon in final) which can be detected and should be analyzed in super- η factories. For the case of dominance of diagonal coupling, the bounds will be more stringent at level $\lesssim 2 \times 10^{-12}$ and $\lesssim 10^{-13}$ for η and η' meson decays, respectively. It will require high statistics of meson yield. For the case of dominance of dark mode, study of such rare decays will be unattainable.

ACKNOWLEDGMENTS

We would like to thank Dmitry Kirpichnikov and Xurong Chen for discussions. This work was funded by FONDECYT (Chile) under Grant No. 1240066 and by ANID-Millennium Program-ICN2019_044 (Chile). The work of A. S. Z. is supported by the Foundation for the

Advancement of Theoretical Physics and Mathematics “BASIS” and by the PIFI CAS Grant No. 2024PVB0070. A. S. Z. also is grateful to Xurong Chen (IMP CAS, China) for the warm hospitality in Southern Center for Nuclear Science Theory, Institute of Modern Physics in Huizhou, where part of this work was done.

DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX: FORM FACTORS DESCRIBING INTERACTION OF EXTERNAL ELECTROMAGNETIC FIELD WITH LFV CURRENT

$$F_2^{lf}(q^2) = \int_0^1 dx dy dz \delta(1-x-y-z) \frac{(m_f + m_l)x}{8\pi^2} \left(\left[\frac{m_l(1+y) + m_f(z-1)}{D(m_l, m_f, y, z, t)} \right] + [m_l \leftrightarrow m_f, y \leftrightarrow z] \right), \quad (\text{A1})$$

with $D(m_l, m_f, y, z, t) = m_l^2 x - z(m_f^2 x + ty) + m_l^2(y^2 + z + yz)$,

$$F_{1;r}^{lf}(q^2) = \int_0^1 dx dy dz \delta(1-x-y-z) \frac{1}{8\pi^2} \left(\left[\frac{2(m_l m_f x(2-x) + m_l^2(x-y-z))}{D(m_l, m_f, y, z, 0)} - \frac{2(m_l^2(x-y-z) + m_f m_l x(2-x) + t(x-yz))}{D(m_l, m_f, y, z, t)} + \ln \left(\frac{D(m_l, m_f, y, z, 0)}{D(m_l, m_f, y, z, t)} \right) \right] + [m_l \leftrightarrow m_f, y \leftrightarrow z] \right). \quad (\text{A2})$$

-
- [1] K. Afanaciev *et al.* (MEG II Collaboration), *Eur. Phys. J. C* **84**, 216 (2024); **84**, 1042(E) (2024).
 - [2] U. Bellgardt *et al.* (SINDRUM Collaboration), *Nucl. Phys. B* **299**, 1 (1988).
 - [3] M. Aoki *et al.* (COMET, MEG, Mu2e and Mu3e Collaborations), [arXiv:2503.22461](https://arxiv.org/abs/2503.22461).
 - [4] R. Abramishvili *et al.* (COMET Collaboration), *Prog. Theor. Exp. Phys.* **2020**, 033C01 (2020).
 - [5] W. H. Bertl *et al.* (SINDRUM II Collaboration), *Eur. Phys. J. C* **47**, 337 (2006).
 - [6] J. L. Popp (MECO Collaboration), *Nucl. Instrum. Methods Phys. Res., Sect. A* **472**, 354 (2000).
 - [7] R. H. Bernstein (Mu2e Collaboration), *Front. Phys.* **7**, 1 (2019).
 - [8] S. Chekanov *et al.* (ZEUS Collaboration), *Phys. Rev. D* **65**, 092004 (2002).
 - [9] S. Gninenko, S. Kovalenko, S. Kuleshov, V. E. Lyubovitskij, and A. S. Zhevlakov, *Phys. Rev. D* **98**, 015007 (2018).
 - [10] S. Banerjee, V. Cirigliano, M. Dam, A. Deshpande, L. Fiorini, K. Fuyuto, C. Gal, T. Husek, E. Mereghetti, K. Monsálvez-Pozo *et al.*, [arXiv:2203.14919](https://arxiv.org/abs/2203.14919).
 - [11] B. Batell, H. Davoudiasl, R. Marcarelli, E. T. Neil, and S. Trojanowski, *Phys. Rev. D* **110**, 075039 (2024).
 - [12] J. L. Feng, F. Kling, M. H. Reno, J. Rojo, D. Soldin, L. A. Anchordoqui, J. Boyd, A. Ismail, L. Harland-Lang, K. J. Kelly *et al.*, *J. Phys. G* **50**, 030501 (2023).
 - [13] C. O. Dib, T. Gutsche, S. G. Kovalenko, V. E. Lyubovitskij, and I. Schmidt, *Phys. Rev. D* **99**, 035020 (2019).
 - [14] A. Faessler, T. Gutsche, S. Kovalenko, V. E. Lyubovitskij, and I. Schmidt, *Phys. Rev. D* **72**, 075006 (2005).
 - [15] M. Gonzalez, T. Gutsche, J. C. Helo, S. Kovalenko, V. E. Lyubovitskij, and I. Schmidt, *Phys. Rev. D* **87**, 096020 (2013).

- [16] A. Davidson and K. C. Wali, *Phys. Rev. Lett.* **48**, 11 (1982).
- [17] A. Faessler, T. Gutsche, S. Kovalenko, V. E. Lyubovitskij, I. Schmidt, and F. Simkovic, *Phys. Rev. D* **70**, 055008 (2004); *Phys. Lett. B* **590**, 57 (2004); T. Gutsche, J. C. Helo, S. Kovalenko, and V. E. Lyubovitskij, *Phys. Rev. D* **81**, 037702 (2010); **83**, 115015 (2011).
- [18] J. Heeck, *Phys. Lett. B* **758**, 101 (2016).
- [19] A. Kachanovich, S. Kovalenko, S. Kuleshov, V. E. Lyubovitskij, and A. S. Zhevlakov, *Phys. Rev. D* **105**, 075004 (2022).
- [20] A. S. Zhevlakov, D. V. Kirpichnikov, and V. E. Lyubovitskij, *Phys. Rev. D* **109**, 015015 (2024).
- [21] T. Araki, K. Asai, H. Otono, T. Shimomura, and Y. Takubo, *J. High Energy Phys.* **01** (2023) 145.
- [22] P. Crivelli, [arXiv:2301.09905](#).
- [23] Y. M. Andreiev *et al.* (NA64 Collaboration), [arXiv:2505.14291](#).
- [24] T. Araki, K. Asai, and T. Shimomura, *J. High Energy Phys.* **11** (2021) 082.
- [25] J. Shi, J. Liang, and S. Gardner, *Phys. Rev. D* **110**, 055039 (2024).
- [26] X. Chen, X. H. He, Q. Hu, D. X. Lin, Y. Liu, H. Qiu, X. Sun, Y. Tian, R. Wang, H. L. Zhang *et al.*, *Nucl. Sci. Tech.* **36**, 137 (2025).
- [27] S. N. Gninenko, D. V. Kirpichnikov, S. Kuleshov, V. E. Lyubovitskij, and A. S. Zhevlakov, *Phys. Rev. D* **109**, 075021 (2024).
- [28] Y. M. Andreiev *et al.* (NA64 Collaboration), *Phys. Rev. Lett.* **133**, 121803 (2024).
- [29] D. Y. Akimov *et al.* (COHERENT Collaboration), *Phys. Rev. D* **106**, 052004 (2022).
- [30] J. Elam *et al.* (REDTOP Collaboration), [arXiv:2203.07651](#).
- [31] Z. Mushtaq, R. Wang, X. Kang, Y. Liu, Y. Tian, X. He, H. Qiu, and X. Chen, [arXiv:2506.11733](#).
- [32] E. C. G. Stueckelberg, *Helv. Phys. Acta* **11**, 225 (1938).
- [33] M. Pospelov, *Phys. Rev. D* **80**, 095002 (2009).
- [34] A. Belyaev, A. Deandrea, S. Moretti, L. Panizzi, D. A. Ross, and N. Thongyoi, *Phys. Rev. D* **108**, 095001 (2023).
- [35] B. Holdom, *Phys. Lett.* **166B**, 196 (1986).
- [36] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, New York, 1995), p. 842.
- [37] S. Andreas, S. V. Donskov, P. Crivelli, A. Gardikiotis, S. N. Gninenko, N. A. Golubev, F. F. Guber, A. P. Ivashkin, M. M. Kirsanov, N. V. Krasnikov *et al.*, [arXiv:1312.3309](#).
- [38] Y. M. Andreiev *et al.* (NA64 Collaboration), *Phys. Rev. Lett.* **131**, 161801 (2023).
- [39] A. Berlin, N. Blinov, G. Krnjaic, P. Schuster, and N. Toro, *Phys. Rev. D* **99**, 075001 (2019).
- [40] T. Åkesson *et al.* (LDMX Collaboration), [arXiv:1808.05219](#).
- [41] A. M. Ankowski, A. Friedland, S. W. Li, O. Moreno, P. Schuster, N. Toro, and N. Tran, *Phys. Rev. D* **101**, 053004 (2020).
- [42] M. Battaglieri *et al.* (BDX Collaboration), [arXiv:1607.01390](#).
- [43] J. Chen *et al.* (DarkSHINE Collaboration), [arXiv:2411.09345](#).
- [44] M. D. Schwartz, *Quantum Field Theory and the Standard Model* (Cambridge University Press, Cambridge, England, 2014), p. 870.
- [45] T. Beranek, H. Merkel, and M. Vanderhaeghen, *Phys. Rev. D* **88**, 015032 (2013).
- [46] C. F. Perdrisat, V. Punjabi, and M. Vanderhaeghen, *Prog. Part. Nucl. Phys.* **59**, 694 (2007).
- [47] Y. S. Tsai, *Rev. Mod. Phys.* **46**, 815 (1974); **49**, 421(E) (1977).
- [48] K. J. Kim and Y. S. Tsai, *Phys. Rev. D* **8**, 3109 (1973).
- [49] J. D. Bjorken, R. Essig, P. Schuster, and N. Toro, *Phys. Rev. D* **80**, 075018 (2009).
- [50] J. D. Bjorken, S. Ecklund, W. R. Nelson, A. Abashian, C. Church, B. Lu, L. W. Mo, T. A. Nunamaker, and P. Rassmann, *Phys. Rev. D* **38**, 3375 (1988).
- [51] S. Navas *et al.* (Particle Data Group), *Phys. Rev. D* **110**, 030001 (2024).
- [52] M. Hoferichter, J. Menéndez, and F. Noël, *Phys. Rev. Lett.* **130**, 131902 (2023).
- [53] R. Escribano, S. González-Solís, and E. Royo, *Phys. Rev. D* **106**, 114007 (2022).
- [54] S. Tulin, *Phys. Rev. D* **89**, 114008 (2014).
- [55] T. Gherghetta, J. Kersten, K. Olive, and M. Pospelov, *Phys. Rev. D* **100**, 095001 (2019).
- [56] R. S. Chivukula, P. Ittisamai, K. Mohan, and E. H. Simmons, *Phys. Rev. D* **96**, 055043 (2017).
- [57] C. F. Uhlemann and N. Kauer, *Nucl. Phys.* **B814**, 195 (2009).
- [58] M. Bauer, P. Foldenauer, and J. Jaeckel, *J. High Energy Phys.* **07** (2018) 094.
- [59] J. P. Lees *et al.* (BABAR Collaboration), *Phys. Rev. Lett.* **119**, 131804 (2017).
- [60] A. A. Aguilar-Arevalo *et al.* (MiniBooNE Collaboration), *Phys. Rev. Lett.* **118**, 221803 (2017).
- [61] A. A. Aguilar-Arevalo *et al.* (MiniBooNE DM Collaboration), *Phys. Rev. D* **98**, 112004 (2018).