

Probing standard-model-like di-Higgs production at photon-photon colliders in the I(1+2)HDM type-I

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In this paper, pair production of standard model (SM)-like Higgs bosons, hh , is studied through $\gamma\gamma$ scattering at future electron-positron colliders, in the framework of the inert doublet model with two active doublets—i.e., the I(1 + 2)HDM for short. The relevance of the process $\gamma\gamma \rightarrow hh$ for such a beyond-the-SM (BSM) scenario stems from the fact that it is a one-loop process at lowest order, wherein inert charged states $\chi^\pm$ contribute alongside $W^\pm$, $H^\pm$, and heavy fermions (primarily, bottom and top quarks), crucially, at the same perturbative order. Given that $\chi^\pm/H^\pm$ masses and hS^+S^- ($S^\pm = \chi^\pm, H^\pm$) couplings are very mildly constrained, there exist regions of the parameter space of the I(1 + 2)HDM where the former can be rather light and the latter rather large. After imposing up-to-date theoretical and experimental constraints on the I(1 + 2)HDM, it is found that the production rates of such processes at future $\gamma\gamma$ machines can be enhanced by up to a factor of ≈ 50 with respect to the SM, significantly exceeding typical yields of conventional two-Higgs-doublet models (2HDMs). Further, thanks to the level of control that one can attain at such facilities on the photon kinematics, leading to excellent invariant mass resolution of the incoming photon pairs, we show how it is possible to extract from this process the value of the $\chi^\pm$ mass (along with those of the active $H^\pm$ states) with high precision, and whichever of the decays of the hh pair, both with and without beam polarization.

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I. INTRODUCTION

The standard model (SM) particle spectrum has been completed with the discovery of the Higgs boson (h) on 4 July 2012 by the ATLAS and CMS experiments at the Large Hadron Collider (LHC) [1,2]. Furthermore, this discovery has confirmed the Higgs mechanism of electroweak symmetry breaking (EWSB) and consequent mass generation. The two collaborations also carried out several Higgs coupling measurements at LHC Run I [3] and Run II [4–10], such as those to fermions and gauge bosons, with uncertainties at most of 30%–50% and 20%, respectively.

The aforementioned measurements will be improved at future experiments such as LHC Run III and the high-luminosity LHC option (HL-LHC) [11,12]. Herein, for example, the $hb\bar{b}$, $h\tau^+\tau^-$, and hZZ couplings will be measured with 4%–7%, 2%–5%, and 2%–4% precision, respectively. In addition, the above experimental uncertainties will be further reduced in the clean environment offered by future e^+e^- colliders, such as the International Linear Collider (ILC) [13], the Circular Electron-Positron Collider (CEPC) [14], the Future Circular Collider operating in e^+e^- mode (FCC-ee) [15,16], the Linear Collider Facility (LCF) at CERN [17,18], or else the Compact Linear Collider (CLIC) [19–21]. Again, for example, at the ILC, the $hb\bar{b}$, $h\tau^+\tau^-$, and hZZ couplings will be measured with 1.5%, 1.9%, and 0.6% accuracy, respectively [22].

A reason for measuring the couplings of the discovered Higgs state (with mass 125 GeV) is to understand whether these signal the presence of some new physics (NP) beyond the SM (BSM), as the SM is plagued with several flaws (see, e.g., Ref. [23] for a review of these). From the experimental side, the SM is unable to account for neutrino masses, explain the matter-antimatter asymmetry in the Universe, or

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provide a candidate for dark matter (DM). From the theoretical side, the SM suffers from the hierarchy problem (i.e., the inability to reconcile the EW scale with that of gravity without unnatural fine-tuning of its parameters). However, despite a flurry of NP models present in literature, there is no conclusive evidence in data for any of these yet. The end of Run III at the LHC and the next phase at the CERN machine (the aforementioned HL-LHC) will hopefully bring some signals of BSM physics (or, at least, hints of it), but this is not a certainty. That is, one may need to wait for the next generation of accelerators. Among the latter, the e^+e^- ones mentioned above have the advantage of providing a very clean environment wherein to search even for subtle hints of NP, compared to hadronic machines, wherein the QCD backgrounds (including those associated with the remnants from the initial state) are formidable. Such electron-positron machines also have the option of running in $\gamma\gamma$ mode, thereby effectively being $\gamma\gamma$ colliders, wherein the photon beams are generated from Compton backscattering of laser light [24–28]. Specifically, high-energy electrons/positrons from the main accelerator collide with intense laser beams, effectively converting the electrons/positrons into (similar) high-energy photons. These photons are then directed to interact with each other in a separate (from that of the e^+e^- beams) interaction point (IP) to produce $\gamma\gamma$ collisions.

Of particular interest for our purposes, which include studying the effects of beam polarization, are linear colliders, so we concentrate here on the ILC and CLIC prototypes, with the energy of the photon-photon scattering ranging from 250 to 1400 GeV. This will be instrumental in testing the possibility of NP entering the Higgs sector, which can be put under intense scrutiny in $\gamma\gamma$ collisions at CLIC, through both precise measurements of the discovered SM-like Higgs state and direct searches for BSM Higgs particles [29]. Aside from the obvious task of providing a direct measurement for the coupling of the SM-like Higgs boson to two photons, $\gamma\gamma h$ (i.e., at production rather than at decay level), a cornerstone of the physics program at the $\gamma\gamma$ option of CLIC is the exploration of trilinear (or triple) Higgs couplings, through the $\gamma\gamma \rightarrow hh$ process. Such a loop-induced channel has been studied within the SM in Refs. [30–34] and found to be rather small, while in scenarios with additional particle content it can be significantly enhanced: e.g., in supersymmetric constructs like the minimal supersymmetric SM (MSSM) and the next-to-MSSM (NMSSM) [35–39], or in non-supersymmetric ones like two-Higgs-doublet models (2HDMs) [40–46], wherein some significant enhancements have been seen in the total cross section because of charged particles from NP entering the loops (i.e., charged Higgs states, $H^\pm$, and new scalars or fermions).

In this paper, we will study how one such model of Higgs sector extensions—namely, the inert doublet model with two active doublets [abbreviated as the $I(1+2)$ HDM] of

Ref. [47]—can alter the phenomenology of the $\gamma\gamma \rightarrow hh$ process. Such a BSM scenario is well motivated, as it provides a viable DM candidate as well as extra CP -violating phases for the explanation of the Universe’s matter-antimatter asymmetry. Furthermore, extended scalar sectors such as the $I(1+2)$ HDM can help to alleviate the so-called “little hierarchy” problem, in the sense that additional scalar degrees of freedom contribute to radiative corrections to the Higgs mass parameter, potentially allowing for partial cancellations and reducing the amount of fine-tuning required compared to the SM. This effect has been discussed in the context of inert and multi-Higgs doublet extensions of the SM [48–51]. In fact, such a BSM scenario has also been found to be a viable explanation of two present anomalies from Higgs data, showing excesses at 95 and 650 GeV [52,53]. In such a NP scenario, after electroweak symmetry breaking (EWSB), one is left with a spectrum of eight physical Higgs bosons: four of which are charged (so that they enter the $\gamma\gamma \rightarrow hh$ process at one loop), two active ($H^\pm$) and two inert ($\chi^\pm$) ones. In the end, we will show that the contribution of the latter can be significantly larger than that of the former, so that the cross section for this channel can increase by several orders of magnitude above that of the SM (and also exceed the typical ones predicted in the aforementioned BSM scenarios.)

This paper is organized as follows: In Sec. II, a brief review of the $I(1+2)$ HDM is given, crucially, including formulas for trilinear and quartic Higgs self-couplings, also considering both experimental and theoretical constraints on its parameter space. In Sec. III, a discussion of the diagrammatic components of the $\gamma\gamma \rightarrow hh$ process is presented, where, after describing our computational framework, we illustrate our numerical results. The summary is finally given in Sec. IV.

II. THE $I(1+2)$ HDM FRAMEWORK

In what follows, we briefly review the $I(1+2)$ HDM setup, discuss the main theoretical and experimental constraints on its parameter space, and present the trilinear and quartic Higgs self-couplings that are directly relevant to di-Higgs production in photon-photon collisions.

A. Model setup

Our theoretical framework is built upon the $I(1+2)$ HDM of Refs. [47,54] (see also Ref. [55]). In this model, alongside the 2HDM active weak doublets, represented as $\Phi_i \sim (1, 2, 1/2)$ ($i = 1, 2$), an additional inert doublet, denoted as $\eta \sim (1, 2, 1/2)$, is added. This extension of the 2HDM has been extensively studied and is considered to be one of the most well-motivated ones. This primarily stems from the DM problem, since the inert doublet naturally provides a stable candidate protected by a discrete $\mathbf{Z}_2$ symmetry, under which the inert doublet transforms as $\eta \rightarrow -\eta$ while all other fields remain invariant under it. In

parallel, a softly broken $\mathbf{Z}'_2$ symmetry is introduced to avoid tree-level flavor-changing neutral current (FCNC) processes, under which $\Phi_1 \rightarrow +\Phi_1$ and $\Phi_2 \rightarrow -\Phi_2$.

Assuming charge/parity (CP) conservation in the Higgs sector, the most general gauge-invariant and renormalizable scalar potential that respects the above discrete symmetry,

$\mathbf{Z}_2 \times \mathbf{Z}'_2$, can thus be divided into three parts, as follows [52,56]:

$$V(\Phi_1, \Phi_2, \eta) = V(\Phi_1, \Phi_2) + V(\eta) + V(\Phi_1, \Phi_2, \eta). \quad (2.1)$$

The first part describes the 2HDM scalar sector and reads

$$V(\Phi_1, \Phi_2) = -\frac{1}{2} \left\{ m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{H.c.}] \right\} + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{1}{2} [\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{H.c.}]. \quad (2.2)$$

The second part corresponds to the inert doublet self-interactions:

$$V(\eta) = m_\eta^2 \eta^\dagger \eta + \frac{\lambda_\eta}{2} (\eta^\dagger \eta)^2. \quad (2.3)$$

Finally, the third part encodes the interaction terms between η and $\Phi_{1,2}$ fields that are defined as

$$V(\Phi_1, \Phi_2, \eta) = \lambda_{11\eta\eta} (\Phi_1^\dagger \Phi_1)(\eta^\dagger \eta) + \lambda_{22\eta\eta} (\Phi_2^\dagger \Phi_2)(\eta^\dagger \eta) + \lambda_{1\eta\eta 1} (\Phi_1^\dagger \eta)(\eta^\dagger \Phi_1) + \lambda_{2\eta\eta 2} (\Phi_2^\dagger \eta)(\eta^\dagger \Phi_2) + \frac{1}{2} [\lambda_{1\eta 1\eta} (\Phi_1^\dagger \eta)^2 + \text{H.c.}] + \frac{1}{2} [\lambda_{2\eta 2\eta} (\Phi_2^\dagger \eta)^2 + \text{H.c.}], \quad (2.4)$$

wherein, without loss of generality, all incorporated dimensionless parameters, λ_{1-5} , λ_η , $\lambda_{i\eta\eta}$, $\lambda_{i\eta\eta i}$ and $\lambda_{i\eta i\eta}$, with $i = 1, 2$, are assumed to be real.

The three complex $SU_L(2)$ doublet scalar fields, Φ_1 , Φ_2 , and η , are defined by

$$\Phi_i = \begin{pmatrix} \phi_i^+ \\ (v_i + \eta_i + iz_i)/\sqrt{2} \end{pmatrix}, \quad i = 1, 2, \quad \eta = \begin{pmatrix} \chi^+ \\ (\chi + i\chi_a)/\sqrt{2} \end{pmatrix}, \quad (2.5)$$

where v_1 and v_2 refer to the vacuum expectation values (VEVs) of the Higgs fields Φ_1 and Φ_2 , respectively.

To avoid CP violation in the Higgs sector, all parameters of the scalar potential are assumed to be real. Moreover, and due to the $\mathbf{Z}_2$ symmetry, the inert scalar doublet η does not mix with the two active ones Φ_1 and Φ_2 . Consequently, the physical scalar spectrum splits into two sectors.

- (1) *Active sector*: equivalent to that of a 2HDM, with two CP -even states (h and H), one CP -odd (A), and a pair of charged Higgs ($H^\pm$), with masses $m_h < m_H$, m_A , and $m_{H^\pm}$, respectively. At the tree level, the mass relations of this sector remain the same as in the 2HDM. Furthermore, the mixing between weak and mass eigenstates is controlled by the angles β (in the charged and CP -odd sectors) and α (in the CP -even sector). The transitions between the nonphysical fields and physical scalars are done as follows:

$$\begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix} = \mathcal{R}_\beta \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}, \quad \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \mathcal{R}_\beta \begin{pmatrix} G^0 \\ A \end{pmatrix},$$

and

$$\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \mathcal{R}_\alpha \begin{pmatrix} H \\ h \end{pmatrix}, \quad (2.6)$$

where $\mathcal{R}_\theta = \{\{\cos \theta, -\sin \theta\}, \{\sin \theta, \cos \theta\}\}$ is the usual rotation 2×2 matrix. Here, G^0 and $G^\pm$ are the neutral and charged Goldstone bosons absorbed as the longitudinal components of the Z and $W^\pm$, respectively.

Starting with ten real parameters in the scalar potential in Eq. (2.2), and taking into account the two minimization conditions together with the EW relation that fixes the $W^\pm$ mass, $2m_{W^\pm} = gv$ (with $v = \sqrt{v_1^2 + v_2^2} = 246$ GeV), we are left with six free parameters:

$$\Omega_1 = \{m_A, m_H, m_{H^\pm}, m_{12}^2, \tan \beta, \cos(\beta - \alpha)\}, \quad (2.7)$$

while the mass of the SM-like Higgs boson is fixed to its experimentally measured value, $m_h = 125$ GeV.

- (2) *Inert sector*: from which four additional scalars arise—namely, χ , χ_a , and $\chi^\pm$. For simplicity, we assume that the inert doublet η couples symmetrically to both active doublets Φ_1 and Φ_2 . This natural assumption significantly reduces the number of free parameters of the model. As a result, the analysis becomes more tractable while still preserving the

essential features of the $I(1+2)$ HDM. To ensure this, we have implemented the following identifications:

$$\lambda_a \equiv \lambda_{11\eta\eta} = \lambda_{22\eta\eta}, \quad \lambda_b \equiv \lambda_{1\eta\eta 1} = \lambda_{2\eta\eta 2},$$

and $\lambda_c \equiv \lambda_{1\eta 1\eta} = \lambda_{2\eta 2\eta},$ (2.8)

which leave us with only five extra parameters. Consequently, the squared masses of the inert scalars can be expressed as follows:

$$m_{\chi^\pm}^2 = m_\eta^2 + \frac{1}{2}\lambda_a v^2, \quad (2.9)$$

$$m_\chi^2 = m_\eta^2 + \frac{1}{2}(\lambda_a + \lambda_b + \lambda_c)v^2 = m_{\chi^\pm}^2 + \frac{1}{2}(\lambda_b + \lambda_c)v^2, \quad (2.10)$$

$$m_{\chi_a}^2 = m_\eta^2 + \frac{1}{2}(\lambda_a + \lambda_b - \lambda_c)v^2 = m_{\chi^\pm}^2 + \frac{1}{2}(\lambda_b - \lambda_c)v^2. \quad (2.11)$$

The above couplings λ_a , λ_b , and λ_c can be expressed using the square masses as follows:

$$\lambda_a = \frac{2(m_{\chi^\pm}^2 - m_\eta^2)}{v^2}, \quad (2.12)$$

$$\lambda_b = \frac{m_{\chi_a}^2 - 2m_{\chi^\pm}^2 + m_\chi^2}{v^2}, \quad (2.13)$$

$$\lambda_c = \frac{-m_{\chi_a}^2 + m_\chi^2}{v^2}. \quad (2.14)$$

We finally take the inert physical parameter basis defined in terms of the following five inputs:

$$\Omega_2 = \{m_\chi, m_{\chi_a}, m_{\chi^\pm}, m_\eta^2, \lambda_\eta\}. \quad (2.15)$$

Thus, altogether, the $I(1+2)$ HDM parameter space will be described by the 12 independent parameters given by $\Omega = \Omega_1 + \Omega_2$.

Before concluding this section, let us note that the presence of the inert doublet does not affect the interactions with either fermions or gauge bosons. Indeed, since η is odd under the imposed $\mathbf{Z}_2$ symmetry, it does not couple directly to fermions and/or gauge fields. As a result, the Yukawa and gauge structures of the $I(1+2)$ HDM remain identical to those of the conventional 2HDM, and the couplings of the physical Higgs bosons h , H , A , and $H^\pm$ to both fermions and gauge bosons are unchanged [57]. However, as we shall see (specifically, here, for the case of the charged states), the χ , χ_a , and $\chi^\pm$ fields can enter at loop level. In what follows, for the active Higgs sector, we assume that we have a 2HDM Yukawa texture of type I, where only the second doublet Φ_2 interacts with all the fermions [57].

B. Theoretical and experimental constraints

To assess the phenomenology of the model, the above parameters of the $I(1+2)$ HDM scalar potential are scanned randomly over the following ranges:

$$m_h = 125.09 \text{ GeV}, \quad m_H \in [130, 10^3] \text{ GeV}, \quad m_A, m_{H^\pm} \in [100, 10^3] \text{ GeV}, \quad m_{12}^2 \in [0, 10^6] \text{ GeV}^2,$$

$$\tan\beta \in [1, 25] \quad \text{and} \quad \sin(\beta - \alpha) \in [0.96, 1],$$

$$m_\chi, m_{\chi_a}, m_{\chi^\pm} \in [80, 10^3] \text{ GeV}, \quad m_\eta^2 \in [-10^6, 10^6] \text{ GeV}^2, \quad \text{and} \quad \lambda_\eta \in [0, 4\pi],$$

and have to satisfy the following constraints. First, we require the following theoretical constraints to be met:

- (1) *Perturbativity*: ensures that all quartic couplings λ_i in the scalar potential should be restricted to $\leq 4\pi$ to prevent nonperturbative behaviors [54].
- (2) *Boundedness from below (BFB)*: requires the Higgs potential to have a stable ground state (vacuum) [47].
- (3) *Vacuum stability*¹ (*ensuing from the above*): requires the current vacuum being the lowest energy state

¹We note that negative values of the mass parameter m_η^2 do not necessarily spoil the inert nature of the model. In particular, the vacuum structure is determined by the full scalar potential, and the inert phase is preserved as long as the minimum of the potential satisfies $\langle \eta \rangle = 0$. Throughout our parameter scan, we have verified that all the squared masses of all physical scalar states are positive, and that no deeper charge-breaking or inert-breaking minima exist, in the same way as with the standard results available in the literature [58,59].

(absolute stability) or a long-lived one (metastability).

- (4) *Tree-level perturbative unitarity*: which must be preserved in all the $2 \rightarrow 2$ scalar scattering processes involving scalars and/or gauge bosons [54].

Second, we scrutinize the remaining sample against the following precision and search bounds:

- (1) *Oblique parameters*: we investigate to what extent the S , T , and U precision observables [60] may constrain the $I(1+2)$ HDM parameter space, and thus, the scalars involved. In general, these parameters receive contributions from both active and inert doublets [58,61–63]. In the following, we perform the fit in the (S, T) plane assuming $U = 0$, and we require $\chi_{ST}^2 < 5.99$ for consistency with the current best-fit values [64]:

$$S = 0.05 \pm 0.08, \quad T = 0.09 \pm 0.07, \quad (2.16)$$

where the correlation factor among these parameters has been set to $\rho_{ST} = 0.92$.

(2) *Constraints from LEP-I* [65]:

$$m_\chi + m_{\chi^\pm} > m_{W^\pm}, \quad m_{\chi_a} + m_{\chi^\pm} > m_{W^\pm}, \quad (2.17)$$

$$m_\chi + m_{\chi_a} > m_Z, \quad 2m_{\chi^\pm} > m_Z, \quad (2.18)$$

$$m_\chi < 80 \text{ GeV}, \quad m_{\chi_a} < 80 \text{ GeV}, \quad m_{\chi_a} - m_\chi > 8 \text{ GeV}. \quad (2.19)$$

(3) *Constraints from LEP-II* [66–68]:

$$m_{\chi^\pm} > 70 \text{ GeV}. \quad (2.20)$$

(4) *DM search limits* from relic density, as well as (in) direct detection constraints via micrOMEGAs [69].

(5) *Flavor constraints* using the public C++ code SuperIso [70], by considering the most sensitive FCNC processes, such as $\text{BR}(B \rightarrow X_s \gamma)$ [71], $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ [72,73], and $\text{BR}(B \rightarrow \tau \nu)$ [71].

Lastly, in order to test compatibility of the active Higgs sector with collider data, we proceeded as follows:

We used HiggsTools [74], embedding HiggsSignals-3 [75] and HiggsBounds-6 [76,77], test the $I(1+2)$ HDM parameter space against both precision measurements of the 125 GeV Higgs boson and exclusion limits from additional Higgs searches at LEP, Tevatron, and the LHC. For each scanned parameter point, we compute the Higgs signal strength χ^2_{125} and retain only points satisfying

$$\Delta\chi^2 \equiv \chi^2_{125} - \min(\chi^2_{125}) \leq 6.18, \quad (2.21)$$

corresponding to a 95% confidence level (CL), where χ^2_{125} is the value computed by HiggsSignals-3 for the 125 GeV Higgs signal strength measurements, while $\min(\chi^2_{125})$ denotes the minimum value obtained over the full scanned parameter space. To assess the performance of the $I(1+2)$ HDM relative to the SM, we compute $\Delta\chi^2_{125} = \chi^2_{\text{SM}} - \min(\chi^2) = 152.54 - 159.36 = -6.82$, indicating a slightly better fit for the SM. Nevertheless, the reduced chi-square, $\chi^2_{\text{min}}/\text{n.d.f.} = 159.36/159 \approx 1.002$ (where $\text{n.d.f.} = 159$ corresponds to the number of observables analyzed by HiggsTools), still reflects a very good agreement with current Higgs precision data.

C. Triple and quartic Higgs self-couplings relevant for hh production

As mentioned in Sec. I, the study of the di-Higgs production processes at high-energy photon-photon colliders provides a unique opportunity to scrutinize BSM scenarios and probe the structure of its scalar potential. In particular, such processes are highly sensitive to the trilinear and quartic Higgs self-interactions, which encode direct information about the scalar potential responsible for EWSB.

In the SM, these couplings are completely fixed in terms of the Higgs boson mass m_h and the VEV v , leading to the well-known expressions listed in Table I. (Notice, however, that the quartic coupling of the SM, λ_{hhhh} , does not enter our process.) It is also well known that $\gamma\gamma \rightarrow hh$ in the SM is rather small and difficult to measure directly—hence, we explore here how the additional scalar states within the $I(1+2)$ HDM can significantly alter the structure and magnitude of such SM interactions and produce others.

TABLE I. Trilinear $h_i h_j h_k$ and quartic $h_i h_j h_k h_l$ couplings in the SM and $I(1+2)$ HDM (wherein the h is the discovered SM-like Higgs state) entering the $\gamma\gamma \rightarrow hh$ process.

Model	Couplings	Expressions
SM	λ_{hhh}	$-3m_h^2/v$
[78]	λ_{hhhh}	$-3m_h^2/v^2$
$I(1+2)$ HDM	λ_{hhh}	$-3[(2c_{\alpha+\beta} + s_{2\alpha}s_{\beta-\alpha})m_h^2 - 2c_{\alpha+\beta}c_{\beta-\alpha}^2m_{12}^2]/s_{2\beta}/v$
	λ_{HHh}	$-[(2m_h^2 + m_H^2 - 3m_{12}^2)s_{2\alpha} + m_{12}^2s_{2\beta}]c_{\beta-\alpha}/s_{2\beta}/v$
	$\lambda_{hH^\pm H^\mp}$	$-[(3m_h^2 + 2m_{H^\pm}^2 - 4m_{12}^2)c_{\alpha+\beta} + (m_h^2 - 2m_{H^\pm}^2)(c_{\alpha+\beta} - 2s_{2\beta}s_{\beta-\alpha})]/s_{2\beta}/(2v)$
	$\lambda_{h\chi^\pm \chi^\mp}$	$-2[m_{\chi^\pm}^2 - m_\eta^2]s_{\beta-\alpha}/v$
	$\lambda_{HH^\pm H^\mp}$	$-[(3m_H^2 + 2m_{H^\pm}^2 - 4m_{12}^2)s_{\alpha+\beta} + (m_H^2 - 2m_{H^\pm}^2)(s_{\alpha+\beta} - 2s_{2\beta}c_{\beta-\alpha})]/s_{2\beta}/(2v)$
	$\lambda_{H\chi^\pm \chi^\mp}$	$-2[m_{\chi^\pm}^2 - m_\eta^2]c_{\beta-\alpha}/v$
	$\lambda_{hhH^\pm H^\mp}$	$-[m_h^2c_\alpha^4t_\beta^{-2} + 2m_{H^\pm}^2c_\beta^2s_\alpha^2 + c_\alpha s_\alpha(4m_{12}^2 - (m_h^2 - m_H^2)t_\beta^{-1}s_\alpha^2) - m_{H^\pm}^2s_{2\alpha}s_{2\beta} - (m_h^2 - m_H^2)c_\alpha^3s_\alpha t_\beta - s_\alpha^2(m_{12}^2 + m_{12}^2t_\beta^{-4} - m_h^2t_\beta^{-1}s_\alpha^2)t_\beta^3 + c_\alpha^2(2m_{H^\pm}^2s_\beta^2 - (1 + t_\beta^{-4})(m_{12}^2t_\beta - m_H^2s_\alpha^2)t_\beta^2)]/v^2$
	$\lambda_{hh\chi^\pm \chi^\mp}$	$+m_{12}^2s_{2\alpha}s_{2\beta} - m_H^2c_\alpha^4t_\beta^2 + s_\alpha c_\alpha((m_h^2 - m_H^2)s_\alpha^2t_\beta - 4m_{H^\pm}^2s_\beta c_\beta) + c_\alpha^2(c_\beta^2(m_{12}^2 - 2m_{H^\pm}^2) + t_\beta^2(m_{12}^2s_\beta^2 - s_\alpha^2(1 + t_\beta^{-4})m_h^2))/v^2$
		$-2[m_{\chi^\pm}^2 - m_\eta^2]/v^2$

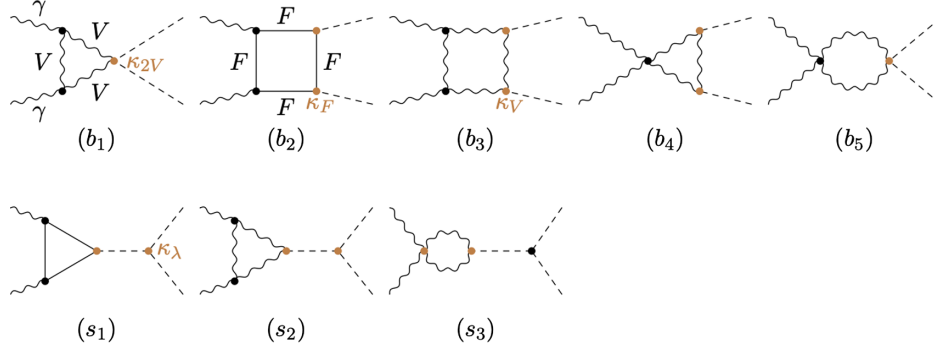


FIG. 1. Generic Feynman diagrams for di-Higgs production, in the unitary gauge, at a photon-photon collider within the SM. The incoming wavy lines correspond to photons, while outgoing dashed lines denote (SM-like) Higgs bosons. The loops are mediated either by fermions F (top or bottom quarks and FP ghosts: solid lines) or gauge bosons V ($W^\pm$: wavy internal lines). Herein, we distinguish the diagrams in terms of their propagators as s -channel diagrams (s_{1-3}) and others (b_{1-5}) (hereafter denoted by “Others” in plots). Black and brown blobs refer here to the reduced couplings $\kappa_{V,F,\lambda}$ or κ_{2V} .

The corresponding scalar spectrum of the $I(1+2)$ HDM introduces new trilinear and quartic couplings among the Higgs fields, many of which appear at the same perturbative order as the SM ones in loop-induced processes such as $\gamma\gamma \rightarrow hh$. Hence, taking into account the theoretical considerations above, the modified and additional trilinear and quartic Higgs self-interactions added to the SM can be extracted from the scalar potential as follows:

$$\mathcal{V} = \lambda_{ijk} h_i h_j h_k + \lambda_{ijkl} h_i h_j h_k h_l + \dots, \quad (2.22)$$

where λ_{ijk} and λ_{ijkl} denote the effective trilinear and quartic couplings in the mass basis, which are functions of the underlying Lagrangian parameters λ_{1-5} , λ_a , as well as the mixing angles α and β . At tree level, the corresponding couplings can be written down as shown in Table I, where the shorthand notations c_θ and s_θ stand, respectively, for $\cos(\theta)$ and $\sin(\theta)$, while $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$.

Importantly, these couplings exhibit two key features. First, the triple (λ_{hhh}) and quartic (λ_{hhhh}) Higgs self-couplings deviate from their SM values due to the mixing of the light h state (the SM-like one) with the heavy CP -even Higgs H . Second, the emergence of new vertices such as $\lambda_{hH^\pm H^\mp}$, $\lambda_{h\chi^\pm \chi^\mp}$, $\lambda_{hhH^\pm H^\mp}$, and $\lambda_{hh\chi^\pm \chi^\mp}$, which have no analogues in the SM, provide distinctive signatures that can strongly enhance or suppress the di-Higgs production rates, depending on the $I(1+2)$ HDM parameter choices. In particular, the couplings involving inert charged scalars $\chi^\mp$ are of great phenomenological interest, since their size is governed by the parameter λ_a , which remains relatively unconstrained by current data, making it a smoking-gun feature of the $I(1+2)$ HDM framework.

²Notice that this coupling does not enter our process of interest.

III. DI-HIGGS PRODUCTION

Before analyzing the predictions of the $I(1+2)$ HDM, it is instructive to briefly recall the structure of the process $\gamma\gamma \rightarrow hh$ in the SM. This will serve as a reference for identifying possible deviations induced by the extended scalar sector. In particular, we outline the loop-induced nature of the process and the relevant virtual particles involved, as well as the computational framework used to obtain the amplitudes. In this context, it is worth noticing that our numerical analyses were performed using the following set of parameters: $m_t = 171.4$ GeV, $m_b = 4.75$ GeV, $m_Z = 91.187$ GeV, and $m_{W^\pm} = 80.39$ GeV. The Weinberg angle $s_W \equiv \sin \theta_W$ is defined in the on-shell scheme as $s_W^2 = 1 - m_W^2/m_Z^2$, and for the fine-structure constant we use $\alpha = 1/137.035989$ in the Thomson limit. Subsequently, the one-loop amplitudes are generated by the public code `FeynArts` [79], written in the Feynman gauge and computed at leading order (LO) using `FormCalc` [80], adopting dimensional regularization. The outputs are generated in terms of Passarino-Veltman scalar integrals [81], which are computed using the `LoopTools` package [82]. We have checked that the total amplitude is ultraviolet (UV) finite and also renormalization-scale independent, all of this providing a good check of our calculations. The cross section for $e^+e^- \rightarrow \gamma\gamma \rightarrow hh$ is obtained by convolution of the Compton backscattered photon spectra with the partonic $\gamma\gamma \rightarrow hh$ cross section. The photon spectra are taken from the `CompAZ` library [24,83–87], which provides the photon energy spectrum for diverse beam energies, as well as the average photon polarization for any photon energy.

A. The $\gamma\gamma \rightarrow hh$ process in the SM

In the SM, the process $\gamma\gamma \rightarrow hh$ is mediated at one loop and receives contributions from s -channel diagrams (s_{1-3}) and the graphs (b_{1-5}), as one can see in Fig. 1. (We stress here that we present the Feynman diagrams in the unitary

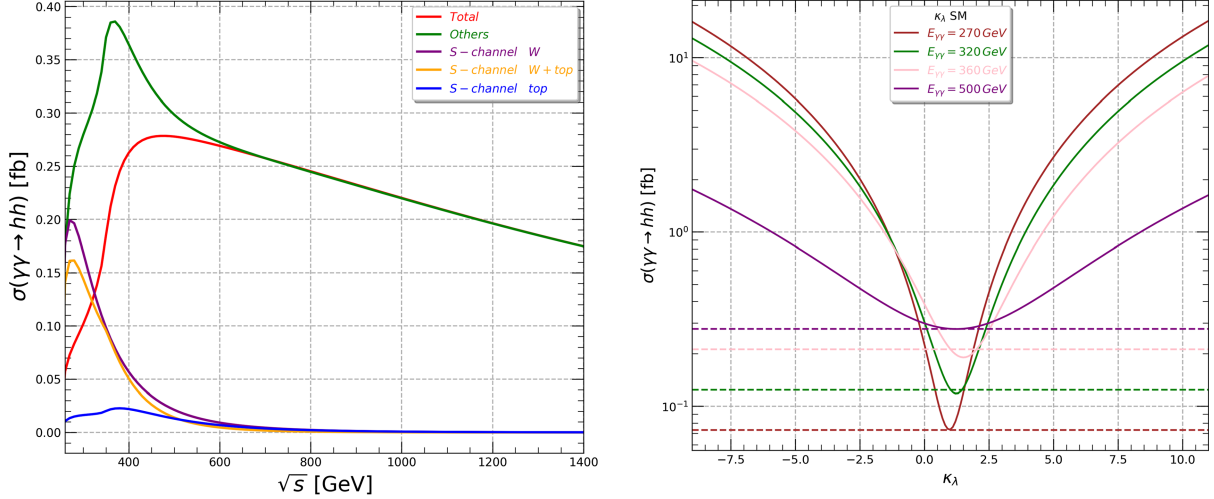


FIG. 2. *Left*: Cross section for $\gamma\gamma \rightarrow hh$ in the SM as a function of the c.m. energy. The total rate (solid red) is decomposed into two contributions: s -channel loops (solid orange) and other loops (solid green). The contribution from the s -channel loops is further decomposed into those from top quarks (solid blue) and those from $W^\pm$ bosons (solid purple). *Right*: Total cross section for $\gamma\gamma \rightarrow hh$ as a function of κ_λ for several c.m. energies. The horizontal lines are the SM cross sections ($\kappa_\lambda = 1$).

gauge in order to avoid showing all topologies with charged Goldstone bosons, but the calculation is performed in the Feynman gauge using dimensional regularization.) The virtual particles that contribute to this process, as generically depicted therein, involve the exchange of fermions (quarks and leptons), $W^\pm$ gauge bosons, $G^\pm$ charged Goldstones, and the charged Fadeev-Popov (FP) ghosts. The contribution of light quarks and charged leptons is, in general, negligible, because it is proportional to fermion masses. It is clear that the process $\gamma\gamma \rightarrow hh$ is sensitive to several reduced couplings such as $\kappa_{V,t,\lambda}$ and κ_{2V} , which are of phenomenological interest. However, given that κ_V and κ_t are now quite well measured at the LHC and consistent with the SM predictions, we focus in what follows on the sensitivity to κ_λ . It is also worth mentioning that the triangle contribution denotes all s -channel diagrams with a $1/s$ dependance (the s_{1-3} diagrams in Fig. 1), while the box contribution refers to all remaining diagrams.

We first analyze the total cross section of the partonic process $\gamma\gamma \rightarrow hh$ as a function of the $\gamma\gamma$ c.m. energy $\sqrt{s}$, while decomposing the full result in all the involved contributions, from which one can see that the terms other than the s -channel graphs almost entirely dominate (in turn led by the $W^\pm$ loops over the top ones), as seen in Fig. 2 (the additional s -channel contributions are negligible) [30–34]. Hence, at low c.m. energy, a pronounced destructive interference pattern between the SM contributions is maximized, leading to a tremendous peak-dip structure, as can be noted from the left side of Fig. 2. This happens near the $t\bar{t}$ threshold, so that the diagrams contributing significantly towards this trend are those with loops of top (anti)quarks [i.e., diagrams (s_1) and (b_2) with the top as the internal particle]. Typically, the total cross section, exhibited by a red line, increases rapidly after the $t\bar{t}$ threshold and reaches its

maximum at a collision energy of $\sqrt{s} = 450$ –500 GeV. After that, it slightly decreases at high energies, where the (b_{1-5}) topologies dominate the (s_{1-3}) ones. In the right panel of Fig. 2, we illustrate the total cross section of $\gamma\gamma \rightarrow hh$ as a function of κ_λ [34], which is defined as $\lambda_{hhh} = \kappa_\lambda \times \lambda_{hhh}^{\text{SM}}$, for several c.m. energies. By doing so, we test how the NP contribution can modify the partonic cross section $\sigma(\gamma\gamma \rightarrow hh)$ through radiative corrections³ that may significantly modify the trilinear Higgs coupling in extended scalar sectors, such as the 2HDM [43,88–91]. The SM value is obtained for $\kappa_\lambda = 1$ and is represented by a horizontal line for different energies. At $\sqrt{s} = 320$ and 360 GeV, in the case of $\kappa_\lambda < 1$, there is an enhancement, while for the case $1 < \kappa_\lambda < 1.5$, there can be a small suppression of the SM cross section. For energies around 270 GeV and 360 GeV, the cross section is enhanced for all values of κ_λ . One can then conclude that the optimal c.m. energy to study di-Higgs production at photon-photon colliders would be 270 GeV or 450 GeV (and above). Such values of κ_λ are currently constrained by di-Higgs searches at the LHC via $gg \rightarrow hh$ [92–94] to lie in the range $-1.6 < \kappa_\lambda < 7.2$, where all couplings are SM-like except for λ_{hhh} . It is clear that, by scaling λ_{hhh} through a κ_λ factor, the total cross section is enhanced with respect to the SM yield by more than 1 order of magnitude, particularly for a low c.m. energy, between $\sqrt{s} = 270$ and 360 GeV. This strong dependence of the cross section $\gamma\gamma \rightarrow hh$ on κ_λ is due

³We note, however, that such radiative corrections can even exceed the tree-level prediction. While incorporating these effects would formally correspond to two-loop contributions in the $\gamma\gamma \rightarrow hh$ process and therefore falls beyond the scope of the present lowest-order analysis, their potential impact should be kept in mind when interpreting the sensitivity to κ_λ .

to the fact that the s -channel contribution (from $W^\pm$ and top quark loops) is rather important only before the opening of the top threshold at $\sqrt{s} = 350$ GeV, so that crossing this latter results in a sharp decrease in the s -channel contribution, and hence a mild enhancement of the cross section as a function of κ_λ .

In the analysis that follows, we investigate how the $I(1+2)$ HDM can alter the $\gamma\gamma \rightarrow hh$ process compared to the SM expectations. Herein, the value of κ_λ is a derived quantity emerging from the λ_i 's in the corresponding scalar potential after EWSB. However, alongside the κ_λ rescaling effect, there will be new Feynman diagrams to consider: chiefly, those involving new charged scalars (both active and inert) entering the loops, in turn triggering new trilinear (i.e., $\lambda_{hS^\pm S^\pm}$) and (now also) quartic ($\lambda_{hhS^\pm S^\pm}$) self-couplings involving charged scalars ($S^\pm = \chi^\pm, H^\pm$). Furthermore, the interactions of the SM-like Higgs boson will be modified due to the mixing with other (active) scalars by the following scaling factors:

$$\kappa_X = \frac{\lambda_{hXX}^{I(1+2)\text{HDM}}}{\lambda_{hXX}^{\text{SM}}} \quad \text{with } X = V \text{ or } F, \quad (3.1)$$

$$\kappa_{2V} = \frac{\lambda_{hhVV}^{I(1+2)\text{HDM}}}{\lambda_{hhVV}^{\text{SM}}}, \quad \kappa_\lambda = \frac{\lambda_{hhh}^{I(1+2)\text{HDM}}}{\lambda_{hhh}^{\text{SM}}},$$

denoted as black and brown blobs in diagrams (s_{1-3}) and (b_{1-5}) of Fig 1.

B. $I(1+2)$ HDM results

In this subsection, we analyze the results for the production process $\gamma\gamma \rightarrow hh$ in our NP scenario, the $I(1+2)$ HDM. To start with, we exhibit in Fig. 3 the additional Feynman diagrams that contribute to Higgs pair production in $\gamma\gamma$ collisions, originating either from the active sector, represented by H and $H^\pm$, or from the inert one, denoted by $\chi^\pm$.

To illustrate the phenomenological impact of the new Feynman diagrams and all rescaled couplings, we conduct

an extensive scan of the $I(1+2)$ HDM parameter space according to the intervals mentioned in Sec. II B, while we further assume that χ is the lightest particle—i.e., $m_\chi < m_{\chi_a}$ and $m_\chi < m_{\chi^\pm}$ —while taking into account all the constraints described above. The upper panels in Fig. 4 illustrate the correlations between m_A and m_H (left) and between m_χ and m_η (right), with color coding according to the charged active and inert scalar masses, $m_{H^\pm}$ and $m_{\chi^\pm}$, respectively. It is clear that the pattern of correlations is nearly the same in the inert and active sectors. Furthermore, both exhibit the feature that a light charged scalar with $m_{H^\pm}, m_{\chi^\pm} \lesssim 500\text{--}600$ GeV generally implies that the neutral states H , A , and χ are also relatively light, with masses not exceeding $700\text{--}800$ GeV. We then consider the trilinear couplings λ_{hhh} and $\lambda_{hH^+H^-}$, which are useful for consistency checks of the $I(1+2)$ HDM parameter space, provided they can be measured precisely enough. The variation of their reduced values with respect to the SM prediction, projected onto the (m_H, m_A) plane, is shown in the lower panels in Fig. 4. One can then see that the coupling $\lambda_{hH^+H^-}^{I(1+2)\text{HDM}}$ can be up to 6 times larger than the corresponding SM value. In fact, in some regions of the parameter space, $\lambda_{hH^+H^-}$ could even be of opposite sign with respect to the SM value. We then emphasize here the significance of the charged scalars ($H^\pm$ and $\chi^\pm$) in the box diagrams B_2 and B_3 . In fact, the coupling of the SM-like Higgs state to such particles enters the amplitude of B_2 and B_3 in quadrature—i.e., proportionally to $\lambda_{hH^+H^-}^2$ and $\lambda_{h\chi^+\chi^-}^2$. In contrast, the charged scalar contributions to S_2 and S_4 are proportional to $\lambda_{hS^\pm S^\pm} \times \lambda_{hhh}$ or $\lambda_{HS^\pm S^\pm} \times \lambda_{Hhh}$ ($S^\pm = \chi^\pm$ or $H^\pm$), respectively, which are suppressed by a small $\cos(\beta - \alpha)$.

In the light of these results, we select four benchmark points (BPs) based on the mass hierarchy between the new (both active and inert) scalar bosons, as follows:

- BP1: $m_H < m_{\chi^\pm} < m_{H^\pm}$,
- BP2: $m_H < m_{\chi^\pm} \approx m_{H^\pm}$ (degenerate charged states),
- BP3: $m_{H^\pm} < m_H < m_{\chi^\pm}$,
- BP4: $m_{\chi^\pm} < m_H < m_{H^\pm}$, and

the parameters of which are given in Table II.

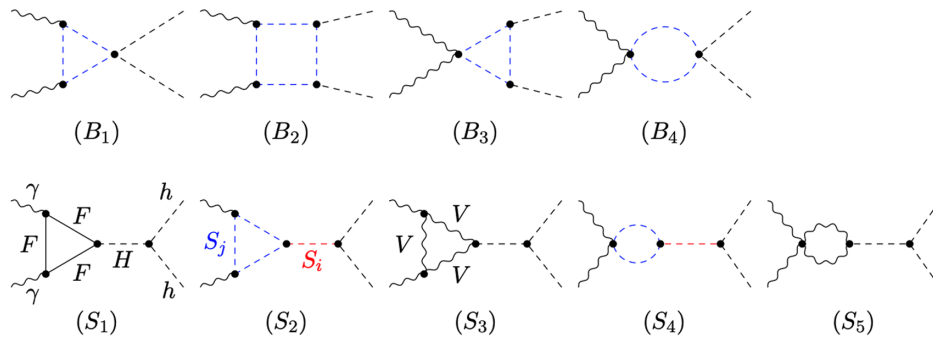


FIG. 3. Generic additional Feynman diagrams for di-Higgs production at a photon-photon collider within the $I(1+2)$ HDM with respect to the SM. Here, we adopt similar graph structures to those introduced for the SM: the s -channel ones (S_{1-5}), where the internal black (red) [blue] dashed lines represent H ($S_i = h, H$) [$S_j = \chi^\pm, H^\pm$], plus other contributions (B_{1-4}).

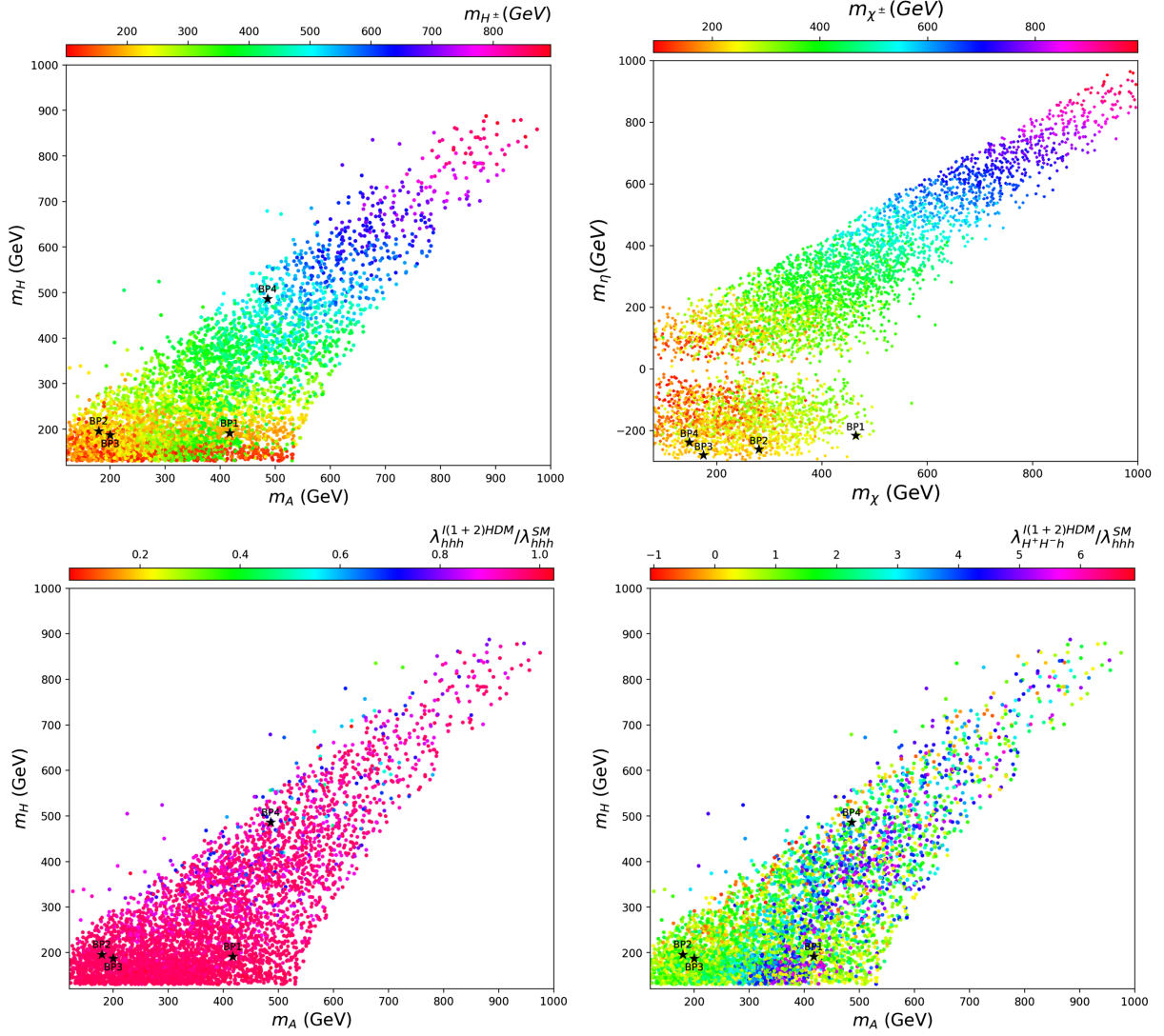


FIG. 4. Allowed parameter space in the $I(1+2)$ HDM. The upper panels show the (m_H, m_A) and (m_χ, m_η) planes, color-coded by the charged active and inert scalar masses, $m_{H^\pm}$ and $m_{\chi^\pm}$, respectively. The lower panels illustrate the (m_H, m_A) plane, with the color scale representing the reduced scalar couplings $\lambda_{hhh}^{I(1+2)HDM}/\lambda_{hhh}^{SM}$ (left) and $\lambda_{H^+H^-h}^{I(1+2)HDM}/\lambda_{hhh}^{SM}$ (right).

The diagrams in Fig. 3 show that the presence of additional (both active and inert) scalar states of the $I(1+2)$ HDM with respect to the SM will manifest themselves differently depending on whether they are neutral or charged. Of the neutral ones, only the active H state enters, and this will appear as an s -channel Breit-Wigner (BW) resonance, so long as $\sqrt{s}$ samples an interval

containing m_H . Of the charged ones, both the active $H^\pm$ and inert $\chi^\pm$ states will enter, and these will appear as loop thresholds, so long as $\sqrt{s}$ samples an interval containing $2m_{H^\pm}$ or $2m_{\chi^\pm}$, respectively.

All such roles are evident in Fig. 5 (left), where the total unpolarized partonic cross section $\sigma(\gamma\gamma \rightarrow hh)$ is presented for the selected BPs as a function of the c.m. energy $\sqrt{s}$.

TABLE II. Input parameters for the selected BPs in the $I(1+2)$ HDM. All masses (squared) are in $\text{GeV}^{(2)}$.

BPs	m_h	m_H	m_A	$m_{H^\pm}$	m_{12}^2	m_χ	m_{χ_a}	$m_{\chi^\pm}$	m_η^2	λ_η	$\tan\beta$	$\sin(\beta - \alpha)$
BP1	125.09	191	417	327	5625	464	324	290	-46947	4.33	2.73	0.993
BP2		196	179	229	4650	280	155	222	-68469	4.06	3.08	0.997
BP3		187	196	110	8124	175	175	192	-78450	0.29	3.08	0.998
BP4		486	486	543	31685	148	153	169	-56761	1.98	7.38	0.999

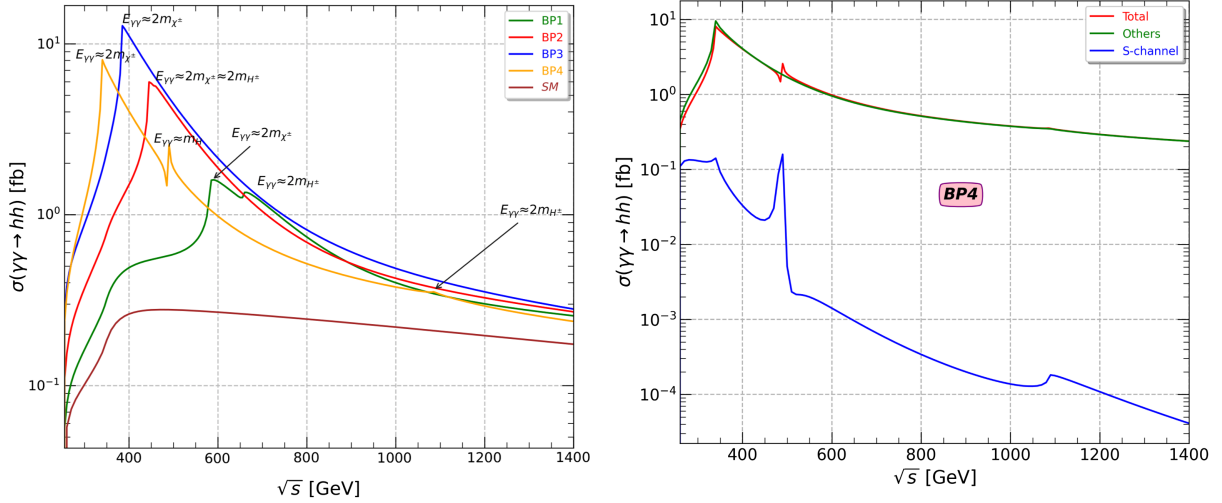


FIG. 5. *Left:* SM and I(1 + 2)HDM total cross section $\sigma(\gamma\gamma \rightarrow hh)$ as a function of the collision energy $\sqrt{s}$ for unpolarized beams for each BP. *Right:* I(1 + 2)HDM cross section $\sigma(\gamma\gamma \rightarrow hh)$ as a function of the collision energy $\sqrt{s}$ for unpolarized beams for BP4 decomposed in the two subchannels described in the text.

Herein, the aforementioned threshold effect is observed for each BP when $E_{\gamma\gamma} \approx 2m_{\chi^\pm}$ or $2m_{H^\pm}$, corresponding to the opening of one of the charged Higgs pair channels: $\gamma\gamma \rightarrow \chi^+\chi^-$ or $\gamma\gamma \rightarrow H^+H^-$, respectively. All these kinematic configurations amplify the cross section, which reaches a maximum of about 12.77 fb for BP3 at $\sqrt{s} \approx 385$ GeV. Similarly, a significant enhancement may occur in the inverted hierarchy case, where $m_{\chi^\pm} < m_{H^\pm}$ (BP4), leading to a cross section of approximately 8 fb at lower $\sqrt{s}$. For BP1 and BP2, the enhancements with respect to the SM case are also evident. The BW resonance effect is here only present for BP4, as only in this case does the $\sqrt{s}$ reach m_H , so that the heavy CP -even (active) state H can decay⁴ into hh . However, such a resonance is not expected to be very large given the fact that the λ_{Hhh} coupling is proportional to $\cos(\beta - \alpha)$, which is very small, as dictated by LHC data (see Table I). More generally, the cross section for $\gamma\gamma \rightarrow hh$ in the I(1 + 2)HDM can attain substantially larger values compared to the SM prediction, by factors ranging from 10 to almost 50—hence, above and beyond what is obtained by changing κ_λ alone. We also note that, as the charged scalar masses increase, their loop effects decouple, hence causing the cross section to eventually drop, so as to asymptotically approach the SM result. This behavior highlights the complementarity between direct searches for charged Higgs bosons and indirect probes via the $\gamma\gamma \rightarrow hh$ process.

Similarly to the SM, in the I(1 + 2)HDM there is a destructive interference between the s -channel graphs and

the others. This is illustrated in Fig. 5 (right), where we can see that at low energy, there is a slight reduction of the other contributions, while at high energy, they fully dominate over the s -channel ones despite the appearance of a resonant effect from $H \rightarrow hh$ decays. Such an interference again leads to the characteristic pick-dip structure that appears just above the hh kinematic threshold. As $\sqrt{s}$ increases, the other contributions decrease smoothly, whereas the triangle terms rapidly fall off due to the s -channel propagator suppression. These results can be traced back to the structure of the scalar interactions in our NP model—in particular, the trilinear couplings $\lambda_{h\chi^+\chi^-}$ and $\lambda_{hH^+H^-}$, together with the quartic couplings $\lambda_{hh\chi^+\chi^-}$ and $\lambda_{hhH^+H^-}$, which are relatively large, as can be seen from Table III. These enhanced couplings amplify the other contributions compared to the s -channel ones, thereby shaping the overall energy dependence of the total cross section.

TABLE III. Triple and quartic scalar couplings in the I(1 + 2)HDM, normalized to their SM values, for our BPs.

Coupling ratio	BP1	BP2	BP3	BP4
$\lambda_{Hhh}^{I(1+2)HDM} / \lambda_{hhh}^{SM}$	0.014	−0.002	−0.056	0.42
$\lambda_{hhh}^{I(1+2)HDM} / \lambda_{hhh}^{SM}$	0.97	0.97	0.97	0.95
$\lambda_{hH^+H^-}^{I(1+2)HDM} / \lambda_{hhh}^{SM}$	4.07	1.86	−0.36	3.88
$\lambda_{h\chi^+\chi^-}^{I(1+2)HDM} / \lambda_{hhh}^{SM}$	1.44	1.49	0.41	0.04
$\lambda_{hh\chi^+\chi^-}^{I(1+2)HDM} / \lambda_{hhh}^{SM}$	5.45	4.91	4.81	3.57
$\lambda_{hhH^+H^-}^{I(1+2)HDM} / \lambda_{hhh}^{SM}$	0.60	0.37	−0.27	0.12
$\lambda_{H^+H^-hh}^{I(1+2)HDM} / \lambda_{hhh}^{SM}$	4.04	1.81	−0.34	3.96
$\lambda_{\chi^+\chi^-hh}^{I(1+2)HDM} / \lambda_{hhh}^{SM}$	5.59	5.01	4.91	3.64

⁴That is, the heavy scalar H can be on shell in the hh channel for BP4. Here, the di-Higgs enhancement is therefore resonant and driven by the λ_{Hhh} coupling: any loop correction to the latter can enhance or suppress the cross-section rate.

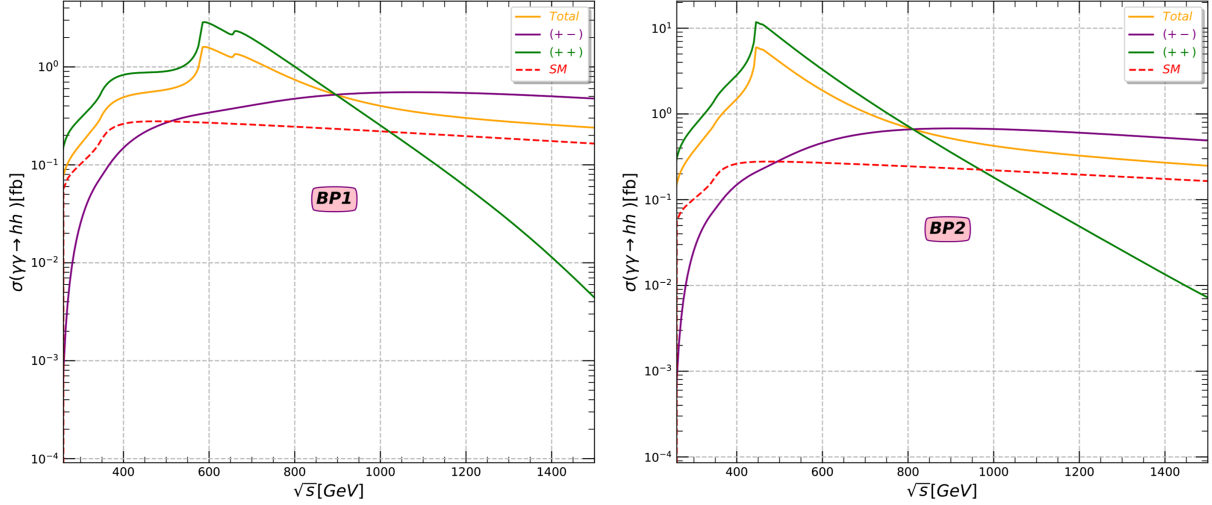


FIG. 6. SM and I(1 + 2)HDM total cross section $\sigma(\gamma\gamma \rightarrow hh)$ as a function of the c.m. energy $\sqrt{s}$. For the I(1 + 2)HDM case, we also show the corresponding values for photons with opposite helicity (+-) and identical helicity (++) for BP1 (left) and BP2 (right).

Another, equally important, aspect in the study of the $\gamma\gamma \rightarrow hh$ cross section involves the polarization of the incoming photon beams. In fact, exploiting the polarization of the photons is a vital technique for enhancing the sensitivity to the di-Higgs signal by suppressing backgrounds that stem from different helicity states of the photons. To address this issue in our study, we illustrate in Fig. 6—e.g., for BP1 and BP2—the total cross section for the process $\gamma\gamma \rightarrow hh$ in both the SM and I(1 + 2)HDM as a function of the c.m. energy $\sqrt{s}$ for different photon-photon helicity configurations.⁵ The orange curve represents the unpolarized case, while the purple and green lines correspond to the opposite-helicity (+-) and same-helicity (++) photon states, respectively. Interestingly, in agreement with previous studies, the $J_z = 0$ configuration (++) provides the dominant contribution to the di-Higgs production rate, since it couples to the CP -even component of the effective $\gamma\gamma hh$ interaction generated by loop diagrams involving charged particles. In contrast, the $J_z = 2$ configuration (+-) is suppressed and becomes more relevant only at higher energies due to non- s -channel loop contributions. In this context, and for all the BPs considered, the (++) channel exhibits in the I(1 + 2)HDM a noticeable enhancement relative to the SM, especially near the various threshold and resonance regions, reflecting the impact of modified self-interactions and additional charged scalars.

Concerning this last point, Fig. 7 illustrates the sensitivity of the normalized cross section, $\sigma^{I(1+2)HDM}/\sigma^{SM}$, to the charged scalar masses $m_{H^\pm}$ (left) and $m_{\chi^\pm}$ (right), for several

values of the c.m. energy $\sqrt{s}$. As can be seen, the corresponding rate exhibits a pronounced enhancement, particularly near the kinematic thresholds, followed by a rapid suppression once these thresholds are crossed—i.e., for $m_{H^\pm}, m_{\chi^\pm} \gtrsim \sqrt{s}/2$. This behavior reflects the opening and subsequent decoupling of charged scalar pair production channels in the loop contributions, $\gamma\gamma \rightarrow H^+H^-, \chi^+\chi^-$, which strongly affect the di-Higgs production amplitude.

So far, we have treated the two photons in the initial state as real objects with fixed energies. In a realistic $\gamma\gamma$ collider setup, however, the photons are produced via Compton backscattering of laser photons off high-energy electron and positron beams, resulting in a nontrivial photon energy spectrum and polarization distribution.⁶ To account for these effects, we convolute our partonic $\gamma\gamma \rightarrow hh$ cross sections with a photon-photon luminosity function obtained using the CompAZ parametrization,⁷ providing both the photon energy spectra and the average photon polarizations, which has been widely used in photon collider studies. Using this setup, we show in the left panel of Fig. 8 how the total cross section for the process $e^+e^- \rightarrow \gamma\gamma \rightarrow hh$ decreases with increasing $m_{\chi^\pm}$ for different values of $\sqrt{s}$, illustrated here for BP3. (The corresponding SM cross section as a function of $\sqrt{s}$ can be read from the right panel of the same figure.) For instance, at $\sqrt{s} = 500$ GeV and $m_{\chi^\pm} = 100$ GeV, the I(1 + 2)HDM cross section reaches a value of 5.68 fb. Hence, the predicted rate is comparable to, or even significantly exceeds, the SM expectation in certain regions

⁵We define here the total angular momentum along the beam axis, J_z , in terms of the helicities of the two incoming photons ($i = 1, 2$) as follows: $J_z = |\lambda_i^1 - \lambda_i^2|$, where $\lambda_i^j = \pm 1$. However, the role of photon polarization and the relative importance of the $J_z = 0$ and $J_z = 2$ channels in di-Higgs production have been extensively discussed in the literature [31,42,95].

⁶Note that quantum electrodynamics (QED) interactions preserve helicity, so that the polarization of the $e^\pm$ beams is largely transferred to the backscattered photons.

⁷Such parametrization models the Compton backscattering of optical laser photons for a standard linear collider configuration.

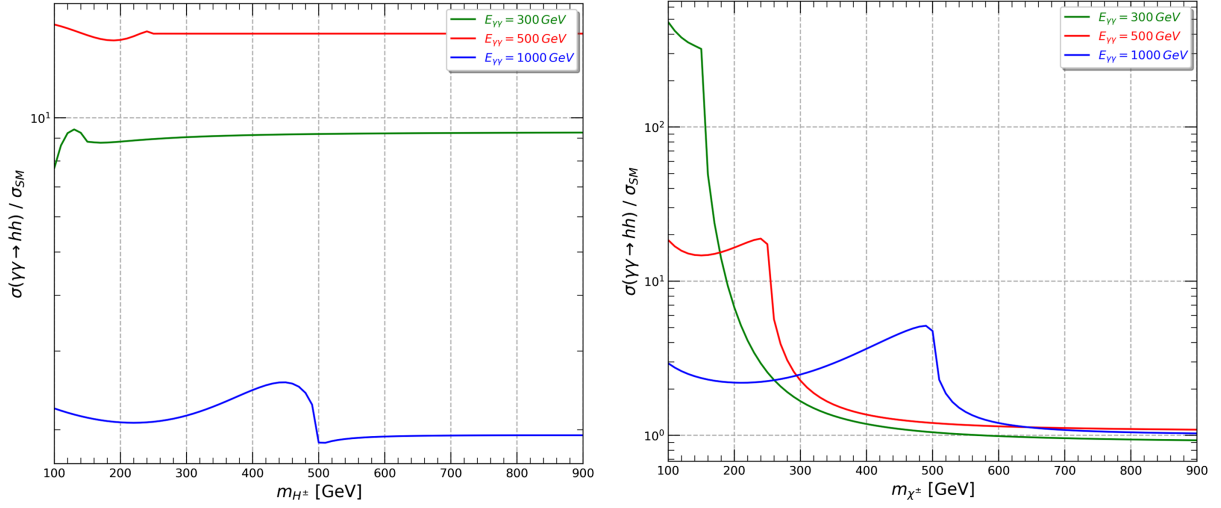


FIG. 7. Total cross section for $\gamma\gamma \rightarrow hh$, normalized to the SM prediction, as a function of the charged scalar masses, $m_{H^\pm}$ (left) and $m_{\chi^\pm}$ (right), for different values of $\sqrt{s}$.

of the parameter space—in particular, for higher values of $\sqrt{s}$ (see right panel of Fig. 8).

To end our study, we discuss some kinematical distributions for the partonic di-Higgs production, in the $\gamma\gamma$ c.m. system, before the convolution with the photon energy spectrum emerging from the e^+e^- beams. (The results for these observables are not drastically different between the two descriptions.) The three panels of Fig. 9 display the differential cross section as a function of the SM-like Higgs transverse-momentum $p_T(h)$ (left), rapidity $y(h)$ (middle), and polar-angle $\cos\theta$ (right), for two representative c.m. energies, $\sqrt{s} = 500$ GeV and 1000 GeV. We note here that both Higgs bosons have the same energy $E_h = \sqrt{s}/2$ and are in a back-to-back configuration (as we are not convoluting the photon beams with the electron/positron structure functions). As a result, by comparing the SM

predictions with the $I(1+2)$ HDM findings, here illustrated for BP3, it is clearly visible that the transverse-momentum distribution exhibits a characteristic rise near the production threshold, followed by a sharp decrease at higher $p_T(h)$ values, as expected from the kinematic limit at $\sqrt{s}/2$. The $I(1+2)$ HDM curves consistently lie above the SM ones, particularly at lower $p_T(h)$, where the enhancement arises from additional charged scalar loop effects. The rapidity distribution peaks in the central region, indicating that most SM-like Higgs bosons are produced in the transverse direction with respect to the beam axis. A similar feature is obtained in the angular distribution, which is forward-backward symmetric, as required by CP conservation in the scalar sector, which remains nearly flat, thus indicating that the $\chi^\pm$ and $H^\pm$ scalar contributions dominate over the SM and substantially reduce the angular dependence. This

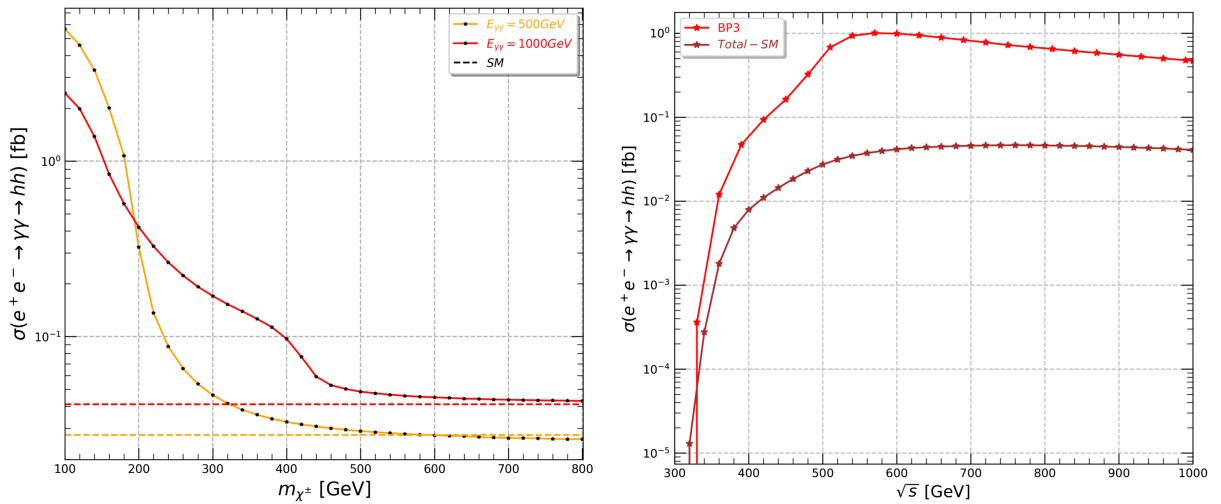


FIG. 8. The total cross section $\sigma(e^+e^- \rightarrow \gamma\gamma \rightarrow hh)$ as a function of the inert charged scalar mass $m_{\chi^\pm}$ (left), and as a function of the collision energy $\sqrt{s}$ (right) for unpolarized beams in the case of BP3.

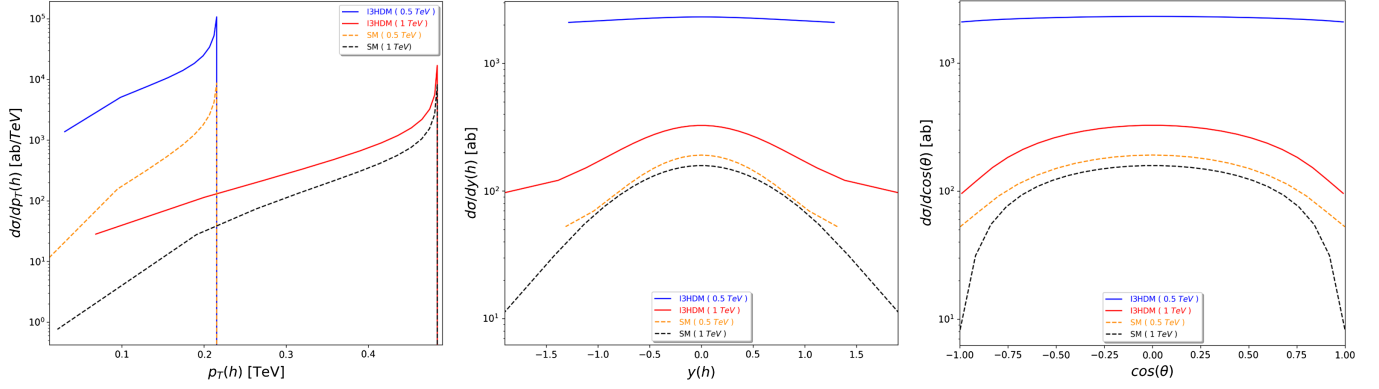


FIG. 9. The transverse-momentum (left), rapidity (center), and polar-angle (right) distributions of either h state in the process $\gamma\gamma \rightarrow hh$ within the I(1 + 2)HDM, for two values of the $\sqrt{s}$, 500 GeV and 1000 GeV, for BP3.

effect is most pronounced at $\sqrt{s} = 500$ GeV, but still visible at 1000 GeV, thus highlighting also at the differential level the sensitivity of the $\gamma\gamma \rightarrow hh$ process to both charged scalar sectors.

IV. SUMMARY

We have carried out studies of pair production of SM-like Higgs bosons at future $\gamma\gamma$ linear colliders, wherein photons are obtained via Compton backscattering of laser light from e^+e^- beams, within the framework of the I(1 + 2)HDM, a distinctive BSM scenario that includes both active (Higgs) and inert charged scalar (i.e., spin-0) states entering such a process at lowest order, alongside the SM particles. Crucially, since the $\gamma\gamma \rightarrow hh$ process is mediated at the one-loop level in the SM, this is an ideal place to look for NP effects, as the latter enter at the same perturbative order as the former. The relevant one-loop amplitudes have been computed here in the Feynman gauge using dimensional regularization, and the intervening graphs in both the SM and I(1 + 2)HDM have been grouped in such a way as to render the underpinning physics most evident.

In evaluating both the SM and I(1 + 2)HDM contributions, we have emphasized, in particular, the effects of the extra charged particles of our BSM scenario. This has been done in part because both of the latter—the customary $H^\pm$ states of the active 2HDM sector and the new $\chi^\pm$ states of the inert sector—play a significant role in explaining current data anomalies in Higgs boson searches. However, above and beyond this present motivation, we were keen to assess whether it would be possible to see their presence in the cross section of the above process as loop thresholds, given the ability that $\gamma\gamma$ colliders have of scanning over the beam energy with high precision. With this in mind, numerical results have been presented for the (partonic) cross section $\gamma\gamma \rightarrow hh$, as well as the total (photonic) one $e^+e^- \rightarrow \gamma\gamma \rightarrow hh$.

After imposing theoretical and experimental constraints, we have found in the I(1 + 2)HDM enhancements of up

to 2 orders of magnitude compared to the SM, with, indeed, very characteristic effects near the charged scalar pair production thresholds, which offset the underlying destructive interferences between the s -channels and other diagrams. Furthermore, we have also shown that the amplitude $\gamma\gamma \rightarrow hh$ would have a $\lambda_{hS^+S^-}^2$ ($S^\pm = \chi^\pm, H^\pm$) dependence through the latter. This would, in turn, enable one to extract the value of $m_{H^\pm}$, $m_{\chi^\pm}$ and also $\lambda_{hS^+S^-}$, so that, if the HL-LHC can extract the triple Higgs coupling modifier κ_λ entering the λ_{hhh} interaction, one could use $\gamma\gamma \rightarrow hh$ production to shade some information on the trilinear scalar self-couplings $\lambda_{hH^+H^-}$ and $\lambda_{h\chi^+\chi^-}$ better than one could do in $h \rightarrow \gamma\gamma$ decay while also being able to access the $\lambda_{hhH^+H^-}$ and $\lambda_{hh\chi^+\chi^-}$ couplings, which are precluded at the decay level. Finally, we have shown that the polarization of the initial electron/positron and (in turn) photon beams further increases the I(1 + 2)HDM cross sections relative to the SM.

All these features thus make $\gamma\gamma \rightarrow hh$ an excellent channel to test both trilinear and quartic scalar self-couplings and to determine the charged scalar spectrum with high precision. Specifically, our results highlight the strong potential of future $\gamma\gamma$ colliders to uncover distinctive signatures of extended Higgs sectors, such as in the I(1 + 2)HDM, at future electron-positron linear colliders: in particular, we have presented numerical results here applicable to the ILC and CLIC prototypes at both the integrated and differential levels.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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