

Study of $B_c \rightarrow J/\psi$ ($a_1(1260)$, $b_1(1235)$, $a_2(1320)$, $K_2^*(1430)$) decays with a perturbative QCD approach

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Motivated by systematic observations of B_c decays at LHCb, we analyze $B_c^+ \rightarrow J/\psi(A, T)$ decays [where A denotes axial-vector mesons $a_1(1260)^+$, $b_1(1235)^+$, and T denotes tensor mesons $a_2(1320)^+$, $K_2^*(1430)^+$] within the perturbative QCD (pQCD) factorization framework. The framework employs light-cone distribution amplitudes to regulate nonperturbative dynamics, utilizing Sudakov suppression to control endpoint divergences. Our predictions indicate branching ratios of 10^{-3} – 10^{-2} for $B_c^+ \rightarrow J/\psi A$ and 10^{-5} – 10^{-4} for $B_c^+ \rightarrow J/\psi T$ decays, which seem to be in the reach of future experiments. We also provide predictions for the polarization fractions of these decays. Based on quark helicity analysis, the longitudinal polarization amplitudes are expected to dominate the branching ratios.

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I. INTRODUCTION

The B_c meson was first discovered experimentally in 1998 by the CDF Collaboration in proton-antiproton ($p\bar{p}$) collisions at a center-of-mass energy of $\sqrt{s} = 1.8$ TeV at the Fermilab Tevatron collider [1]. This landmark discovery was made through the analysis of the semileptonic decay channel $B_c \rightarrow J/\psi \ell \nu$ (where J/ψ is the charmonium state $c\bar{c}$, and ℓ denotes a lepton such as an electron or muon). A distinctive feature of this decay process is the reconstruction of J/ψ via its dilepton final states ($\mu^+\mu^-$ or e^+e^-), while the lepton ℓ and neutrino ν provide key signatures of weak interactions. Up to now, the LHCb Collaboration has experimentally established a comprehensive set of B_c meson decay channels, such as $B_c^+ \rightarrow J/\psi \ell^+ \nu_\ell$ (where $\ell = e, \mu, \tau$) [2,3], $B_c^+ \rightarrow \chi_{c0} \pi^+$ [4], $B_c^+ \rightarrow J/\psi \pi^+$ [5], $B_c^+ \rightarrow J/\psi D_s^{(*)+}$ [6], $B_c^+ \rightarrow \psi(2S) \pi^+$ [7], $B_c^+ \rightarrow J/\psi \pi^+ \pi^- \pi^+$ [8], $B_c^+ \rightarrow J/\psi K^+$ [9], $B_c^+ \rightarrow J/\psi K^+ K^- \pi^+$ [10], and $B_c^+ \rightarrow B_s^0 \pi^+$ [11] for the first time. At the LHC with a proton-proton collision energy of $\sqrt{s} = 14$ TeV, the inclusive production cross section of B_c mesons has a theoretical prediction of 1 μb . With an integrated luminosity of 1 fb^{-1} , this corresponds to the production of $\sim 1 \times 10^9$ B_c mesons [12] before detector acceptance and branching fraction corrections, providing a

robust statistical sample for precision studies of their properties. Leveraging the LHCb experiment's high-precision detection capabilities and future upgrades for the High-Luminosity LHC (HL-LHC), B_c physics will remain pivotal in probing strong interactions, weak decay dynamics, and potential beyond-Standard-Model phenomena.

Compared to the B_u , B_d , and B_s mesons, the B_c meson stands out as a unique probe in particle physics. Composed of two heavy quarks with distinct flavors ($\bar{b}$ and c), the B_c meson decays exclusively through weak interactions, with strong and electromagnetic annihilation processes being kinematically forbidden. The presence of two heavy constituents allows for a distinctive decay mechanism: when one quark decays, the other acts as a passive spectator. This dual-heavy nature results in a significantly shorter lifetime for the B_c meson compared to other b -flavored mesons [7], underscoring the pivotal role of the c quark in its decay dynamics. With its diverse decay pathways, the B_c meson serves as an exceptional laboratory for investigating nonleptonic weak decays of heavy mesons, probing the limits of the Standard Model, and uncovering potential signatures of new physics beyond current theories [13].

Various theoretical approaches have been utilized to explore decays of B_c mesons to charmonium states, including pQCD [14], generalized factorization [15], QCD factorization [16], QCD sum rules [17], the Bethe-Salpeter equation [18], relativistic quark models [19], nonrelativistic QCD (NRQCD) [20], and the covariant light-front approach [21–24]. In the B_c meson system, the applicability of the pQCD approach originates from its distinct double heavy-flavor structure, which establishes a clear hierarchy of hard energy scales. Although the B_c^+ meson consists of a bottom quark and a charm

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antiquark ($\bar{b}c$), with the charm quark mass being significant, the ratio $m_c/m_{B_c} \sim 0.2$ indicates that the dominant energy carrier is the much heavier b quark. This justifies approximating the B_c as a heavy-light system. Consequently, the k_T -factorization theorem, successfully applied to B_u and B_d meson decays, can be extended to b -quark decays within the B_c framework. The specific decay process involves three characteristic scales: M (the B_c mass), m_2 (the charmonium mass), and $\bar{\Lambda}$, which characterizes the nonperturbative binding energy. These scales obey the hierarchy $\bar{\Lambda} \ll m_2 \ll M$, enabling a consistent systematic power expansion in m_2/M and $\bar{\Lambda}/m_2$. Within the heavy-quark and large-recoil limits, and based on the k_T -factorization theorem, the decay amplitude factorizes into a convolution of a hard kernel with the meson wave functions. The hard kernel is calculable perturbatively at the leading order in the α_s expansion, corresponding to single-gluon exchange. Higher-order radiative corrections generate single logarithmic divergences, which are absorbed into the meson wave functions. The double logarithmic divergences from overlapping collinear and soft divergences are summed to all orders into a Sudakov factor, effectively suppressing long-distance interactions. Nonperturbative effects, such as final-state interactions, are systematically incorporated into the universal wave functions of the initial and final state mesons. Although these wave functions are not calculable from first principles, their universality allows them to be constrained from other experimental processes and utilized as nonperturbative inputs. Therefore, the existence of hard scales ensures the validity of the perturbative expansion. The combination of the factorization theorem, the Sudakov mechanism, and the parametrization of wave functions establishes a comprehensive pQCD framework for systematically handling nonperturbative effects, including final-state interactions, in B_c meson decays.

Up to now, the two-body hadronic decays of B_c mesons have been extensively studied in theoretical literature [17–19, 23, 25–44]. The decays of B_c mesons into final states that include a charmonium meson hold distinctive research importance. Primarily, such decay channels offer a sensitive probe for investigating strong interaction dynamics within heavy meson systems. Furthermore, these processes entail two distinct energy scales characterized by the bottom quark mass m_b and the charm quark mass m_c . Within the framework of quantum chromodynamics (QCD), the calculation of higher-order corrections is formulated in terms of a systematic expansion in the mass ratio m_c/m_b . This approach differs from the standard perturbative expansion that uses Λ_{QCD}/m_b as the small parameter (where Λ_{QCD} denotes the QCD scale). Given that m_c/m_b is not negligibly small, this expansion may induce substantial correction effects, thus necessitating more refined theoretical treatment. This fundamental observation requires a theoretical framework distinct from that used for

ordinary B mesons. Specifically, the work addresses this by adopting a nonrelativistic B_c wave function, which inherently incorporates the double-heavy mass effects, providing a more physical starting point than LCDAs for light mesons. Additionally, by reconstructing the properties of charmonium states through their decays to known final states, these B_c decays offer a direct probe into the characteristics of charmonium. In this work, we calculate the decay amplitudes and branching fractions for $B_c^+ \rightarrow J/\psi(A, T)$, where A denotes axial-vector mesons ($a_1(1260)^+$ and $b_1(1235)^+$) and T represents tensor mesons ($a_2(1320)^+$ and $K_2^*(1430)^+$). For simplicity, the following abbreviations will be used throughout this work unless otherwise specified: a_1 denotes the axial-vector meson $a_1(1260)$, b_1 denotes the axial-vector meson $b_1(1235)$, a_2 denotes the tensor meson $a_2(1320)$, and K_2^* denotes the tensor meson $K_2^*(1430)$.

The axial-vector mesons a_1 and b_1 correspond to $^3P_1$ and $^1P_1$ states respectively, which cannot mix in the SU(3) limit due to G-parity constraints. While $^3P_1$ mesons have symmetric LCDAs similar to vector mesons [45, 46], $^1P_1$ states exhibit antisymmetric amplitudes with vanishing decay constants in this limit. For tensor mesons with quantum numbers $J^P = 2^+$, a factorizable amplitude proportional to the matrix element $\langle T | j_\mu | 0 \rangle$, where the current j_μ is of the $(V \pm A)$ or $(S \pm P)$ type, does not contribute, as this matrix element vanishes due to constraints from Lorentz covariance [47, 48]. This makes tensor meson emission strictly forbidden in naive factorization, explaining why such predictions significantly underestimate experimental results and highlighting the importance of nonfactorizable diagrams and annihilation processes. We extend previous pQCD analyses of $B_c \rightarrow J/\psi a_1(1260)$ [49] by incorporating several improvements: (i) updating CKM matrix elements [50] and hadronic parameters, (ii) including intrinsic b dependence in the B_c meson wave function to better account for transverse momentum effects [42], (iii) expanding the study to previously unexplored channels including $B_c \rightarrow J/\psi b_1, a_2$, and K_2^* decays within the pQCD framework. These specific decay channels have also been investigated in a recent study [51] using a pQCD-based approach. However, our work differs fundamentally in the treatment of charm-quark mass effects and the wave function model for the B_c meson. The cited study introduces a modified Sudakov factor, $s_c(Q, b)$, into the traditional pQCD framework, aiming to explicitly include the charm quark mass m_c in the resummation of soft gluon radiation. In contrast, our analysis employs the conventional pQCD approach with a distinct strategy to address the m_c/m_b challenge. We use a nonrelativistic approximation for the B_c light-cone wave function through a delta function $\delta(x - m_c/M_{B_c})$, which fixes the longitudinal momentum fraction of the charm quark. This approach naturally incorporates the nonrelativistic nature of the heavy-heavy B_c system and reduces sensitivity to endpoint

uncertainties in the light meson distribution amplitudes. Thus, while both studies describe the same physical processes, their main difference lies in the theoretical treatment of heavy-quark dynamics: the improved pQCD modifies the evolution kernel via a new Sudakov factor, while our approach optimizes the initial-state wave function within the standard pQCD formalism. These complementary approaches provide valuable insights for both experimental tests at LHCb and Belle II and further understanding of nonperturbative QCD effects.

This work is organized as follows: Section II outlines the theoretical foundation of the pQCD framework and describes the essential wave functions employed in the computation of $B_c \rightarrow J/\psi(A, T)$ decay amplitudes. Section III provides the numerical outcomes and analyses. Section IV summarizes the core findings of this study. For completeness, the full explicit expressions of all helicity amplitudes are compiled in the Appendix.

II. THE THEORETICAL FRAMEWORK AND HELICITY AMPLITUDES

A. The wave functions

The effective Hamiltonian of the B_c nonleptonic decays into charmonium and axial-vector mesons or tensor mesons is given by [52]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{uq} [C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu)], \quad (1)$$

where $q \in \{s, d\}$ denotes a light down-type quark. G_F is the Fermi constant, and V_{cb}^* , V_{uq} represent CKM matrix elements. The Wilson coefficients $C_{1,2}(\mu)$ (encoding short-distance effects above scale μ) and their corresponding four-quark operators $O_{1,2}(\mu)$ are defined as

$$\begin{aligned} O_1(\mu) &= \bar{b}_\alpha \gamma^\nu (1 - \gamma_5) c_\beta \otimes \bar{u}_\beta \gamma_\nu (1 - \gamma_5) q_\alpha, \\ O_2(\mu) &= \bar{b}_\alpha \gamma^\nu (1 - \gamma_5) c_\alpha \otimes \bar{u}_\beta \gamma_\nu (1 - \gamma_5) q_\beta, \end{aligned}$$

where α and β are color indices (summation over repeated indices implied). Since the Hamiltonian involves four distinct quark flavors, these decays are free from penguin operator contamination, hence direct CP violation is naturally absent.

The calculation is performed in the B_c meson rest frame. The momenta are defined in light-cone coordinates as follows: P_1 for the B_c meson, P_2 for the recoiling charmonium, and P_3 for the emitted light meson,

$$\begin{aligned} P_1 &= \frac{M}{\sqrt{2}} (1, 1, \mathbf{0}_T), \\ P_2 &= \frac{M}{\sqrt{2}} (1 - r_3^2, r_2^2, \mathbf{0}_T), \\ P_3 &= \frac{M}{\sqrt{2}} (r_3^2, 1 - r_2^2, \mathbf{0}_T). \end{aligned} \quad (2)$$

The mass ratio $r_{2,3}$ is defined as $m_{2,3}/M$, where M (m_2) denotes the mass of the B_c meson (J/ψ meson), while m_3 represents the mass of the axial-vector (tensor) meson. The momentum of the valence quarks $k_{1,2,3}$ is represented by the parametrization:

$$\begin{aligned} k_1 &= x_1 P_1 + \mathbf{k}_{1T}, \\ k_2 &= x_2 P_2 + \mathbf{k}_{2T}, \\ k_3 &= x_3 P_3 + \mathbf{k}_{3T}, \end{aligned} \quad (3)$$

where parameters k_{iT} and x_i are defined as the transverse momentum and longitudinal momentum fraction, respectively, of the quark/antiquark confined within the meson. When the final state includes a vector charmonium in conjunction with an axial-vector meson, the formalism employs the longitudinal polarization vector ϵ_L and the transverse polarization vector ϵ_T , which serve to describe the polarization states as follows:

$$\begin{aligned} \epsilon_{2L} &= \frac{1}{\sqrt{2(1-r_3^2)r_2}} (1 - r_3^2, -r_2^2, \mathbf{0}_T), \\ \epsilon_{2T} &= (0, 0, \mathbf{1}_T); \\ \epsilon_{3L} &= \frac{1}{\sqrt{2(1-r_2^2)r_3}} (-r_3^2, 1 - r_2^2, \mathbf{0}_T), \\ \epsilon_{3T} &= (0, 0, \mathbf{1}_T). \end{aligned} \quad (4)$$

The polarization vectors satisfy the normalization conditions $\epsilon_L^2 = -1, \epsilon_T^2 = 1$ and the orthogonality relations $\epsilon_{2L} \cdot P_2 = \epsilon_{3L} \cdot P_3 = 0$.

For the spin-2 polarization tensor $\epsilon^{\mu\nu}(\lambda)$ satisfying the transverse condition $\epsilon_{\mu\nu} P_2^\nu = 0$ [48,53], its helicity components $\lambda = 0, \pm 1, \pm 2$ can be constructed from the polarization vectors ϵ of vector mesons as follows:

$$\begin{aligned} \epsilon^{\mu\nu}(\pm 2) &= \epsilon^\mu(\pm 1) \epsilon^\nu(\pm 1), \\ \epsilon^{\mu\nu}(\pm 1) &= \frac{1}{\sqrt{2}} (\epsilon^\mu(\pm 1) \epsilon^\nu(0) + \epsilon^\mu(0) \epsilon^\nu(\pm 1)), \\ \epsilon^{\mu\nu}(0) &= \frac{1}{\sqrt{6}} (\epsilon^\mu(+1) \epsilon^\nu(-1) + \epsilon^\mu(-1) \epsilon^\nu(+1)) \\ &\quad + \sqrt{\frac{2}{3}} \epsilon^\mu(0) \epsilon^\nu(0), \end{aligned} \quad (5)$$

where $\epsilon(\pm 1) = \epsilon_{3T}$ represents the transverse polarization vector, and $\epsilon(0) = \epsilon_{3L}$ represents the longitudinal polarization vector. For clarity and computational efficiency in subsequent calculations, we introduce another polarization vector $\epsilon_{\bullet\mu}(\lambda) = m_T \frac{\epsilon_{\mu\nu}(\lambda) v^\nu}{P_3 \cdot v}$ [54], which satisfies

$$\begin{aligned}
\epsilon_{\bullet\mu}(\pm 2) &= 0, \\
\epsilon_{\bullet\mu}(\pm 1) &= m_T \sqrt{\frac{1}{2}} \frac{\epsilon_{3L} \cdot v}{P_3 \cdot v} \epsilon_{3T\mu}, \\
\epsilon_{\bullet\mu}(0) &= m_T \sqrt{\frac{2}{3}} \frac{\epsilon_{3L} \cdot v}{P_3 \cdot v} \epsilon_{3L\mu}.
\end{aligned} \quad (6)$$

In this work, within the nonrelativistic framework, we employ the B_c meson distribution amplitude defined in Ref. [55],

$$\Phi_{B_c}(x) = \frac{if_B}{4N_c} \left[(\not{p} + M_{B_c}) \gamma_5 \delta\left(x - \frac{m_c}{M_{B_c}}\right) \right] \exp\left(-\frac{\omega_B^2 b^2}{2}\right) \quad (7)$$

with ω_B being the shape parameter, N_c the color number, and the last exponent term representing the k_T dependence. In our calculations, we adopt the shape parameter $\omega_B = (0.60 \pm 0.05)$ GeV [55], consistent with our previous analyses of double-charm decays of the B_c meson.

For J/ψ charmonium states, wave functions have been comprehensively studied within the NRQCD framework. The quantum wave functions of the vector J/ψ meson are characterized by distinct longitudinal (Φ^L) and transverse (Φ^T) components, expressed as [42]

$$\Phi_{J/\psi}^L(x) = \frac{1}{\sqrt{2N_c}} \{m_{J/\psi} \not{\epsilon}_L \phi^L(x) + \not{\epsilon}_L \not{P}_2 \phi^T(x)\}_{\alpha\beta}, \quad (8)$$

$$\Phi_{J/\psi}^T(x) = \frac{1}{\sqrt{2N_c}} \{m_{J/\psi} \not{\epsilon}_T \phi^V(x) + \not{\epsilon}_T \not{P}_2 \phi^T(x)\}_{\alpha\beta}. \quad (9)$$

Here, ϵ_L and ϵ_T are the longitudinal and transverse polarization vectors of the J/ψ meson, respectively; $\phi^L(x)$ and $\phi^T(x)$ are the twist-2 distribution amplitudes, while $\phi^T(x)$ and $\phi^V(x)$ correspond to the twist-3 distribution amplitudes, which can be expressed as [56]

$$\phi^L(x) = \phi^T(x) = 9.58 \frac{f_{J/\psi}}{2\sqrt{2N_c}} x(1-x) \left[\frac{x(1-x)}{1-2.8x(1-x)} \right]^{0.7}, \quad (10)$$

$$\phi^t(x) = 10.94 \frac{f_{J/\psi}}{2\sqrt{2N_c}} (1-2x)^2 \left[\frac{x(1-x)}{1-2.8x(1-x)} \right]^{0.7}, \quad (11)$$

$$\phi^V(x) = 1.67 \frac{f_{J/\psi}}{2\sqrt{2N_c}} [1 + (2x-1)^2] \left[\frac{x(1-x)}{1-2.8x(1-x)} \right]^{0.7}. \quad (12)$$

For axial-vector mesons (a_1 and b_1), their wave functions involve one longitudinal polarization state (L) and two transverse polarization states (T). These can be expressed as [57]

$$\Phi_A^L(x) = \frac{1}{\sqrt{6}} \gamma_5 \{m_A \not{\epsilon}_A^* \phi_A(x) + \not{\epsilon}_A^* \not{P} \phi_A^t(x) + m_A \phi_A^s(x)\}_{\alpha\beta}, \quad (13)$$

$$\Phi_A^T(x) = \frac{1}{\sqrt{6}} \gamma_5 \{m_A \not{\epsilon}_A^* \phi_A^v(x) + \not{\epsilon}_A^* \not{P} \phi_A^T(x) + m_A i \epsilon_{\mu\nu\rho\sigma} \gamma_5 \gamma^\mu \epsilon_T^{*\nu} n^\rho v^\sigma \phi_A^a(x)\}_{\alpha\beta} \quad (14)$$

where x represents the light-cone momentum fraction of a valence quark within the meson, with $n = (1, 0, \mathbf{0}_T)$ and $v = (0, 1, \mathbf{0}_T)$ being dimensionless lightlike basis vectors. Here we adopt the convention $\epsilon^{0123} = 1$ for the Levi-Civita tensor.

The leading-twist (twist-2) distribution amplitudes for axial-vector mesons with orbital configuration $^3P_1$ and $^1P_1$ can be expressed through distinct functional forms for longitudinal ($J_z = 0$) and transverse ($J_z = \pm 1$) polarization states [57]:

$$\begin{aligned}
\phi_A(x) &= \frac{3f_A}{\sqrt{2N_c}} x(1-x) \left[a_{0A}^{\parallel} + 3a_{1A}^{\parallel} (2x-1) \right. \\
&\quad \left. + a_{2A}^{\parallel} \frac{3}{2} (5(2x-1)^2 - 1) \right],
\end{aligned} \quad (15)$$

$$\begin{aligned}
\phi_A^T(x) &= \frac{3f_A}{\sqrt{2N_c}} x(1-x) \left[a_{0A}^{\perp} + 3a_{1A}^{\perp} (2x-1) \right. \\
&\quad \left. + a_{2A}^{\perp} \frac{3}{2} (5(2x-1)^2 - 1) \right],
\end{aligned} \quad (16)$$

where f_A is the decay constant. Here, the definitions of these distribution amplitudes satisfy the following relationships:

$$\begin{aligned}
\int_0^1 \phi_{^3P_1}(x) dx &= \frac{f_{^3P_1}}{2\sqrt{2N_c}}, & \int_0^1 \phi_{^3P_1}^T(x) dx &= a_{0,^3P_1}^{\perp} \frac{f_{^3P_1}}{2\sqrt{2N_c}}; \\
\int_0^1 \phi_{^1P_1}(x) dx &= a_{0,^1P_1}^{\parallel} \frac{f_{^1P_1}}{2\sqrt{2N_c}}, & \int_0^1 \phi_{^1P_1}^T(x) dx &= \frac{f_{^1P_1}}{2\sqrt{2N_c}}.
\end{aligned} \quad (17)$$

For axial vector mesons, the twist-3 distribution amplitudes adopt these functional forms [57]:

$$\begin{aligned}
\phi'_A(x) &= \frac{3f_A}{2\sqrt{2N_c}} \left\{ a_{0A}^\perp (2x-1)^2 \right. \\
&\quad \left. + \frac{1}{2} a_{1A}^\perp (2x-1)(3(2x-1)^2 - 1) \right\}, \\
\phi_A^s(x) &= \frac{3f_A}{2\sqrt{2N_c}} \frac{d}{dx} \{ x(1-x)(a_{0A}^\perp + a_{1A}^\perp(2x-1)) \}; \\
\phi_A^v(x) &= \frac{3f_A}{4\sqrt{2N_c}} \left\{ \frac{1}{2} a_{0A}^\parallel (1 + (2x-1)^2) + a_{1A}^\parallel (2x-1)^3 \right\}, \\
\phi_A^a(x) &= \frac{3f_A}{4\sqrt{2N_c}} \frac{d}{dx} \left\{ x(1-x) \left(a_{0A}^\parallel + a_{1A}^\parallel (2x-1) \right) \right\}. \quad (18)
\end{aligned}$$

Here, x denotes the quark's momentum fraction. The Gegenbauer moments $a_{iA}^{(\perp)}$ have been extensively studied in the literature; in this work, we adopt the following values [58]:

$$\begin{aligned}
a_{0,3P_1}^\parallel &= 1, \quad a_{0,1P_1}^\perp = 1; \\
a_{0,b_1}^\parallel &= 0.0028 \pm 0.0026, \\
a_{0,a_1}^\perp &= 0; \\
a_{1,b_1}^\parallel &= -1.95 \pm 0.35, \\
a_{1,a_1}^\perp &= -1.04 \pm 0.34; \\
a_{2,a_1}^\parallel &= -0.02 \pm 0.02, \\
a_{2,b_1}^\perp &= 0.03 \pm 0.19. \quad (19)
\end{aligned}$$

The intrinsic b dependence is incorporated in the heavy-meson wave function ϕ_B but excluded for the light axial-vector meson ϕ_A . Given the demonstrated insignificance of such dependence in light mesons, we preliminarily assume it is similarly nonessential for the currently undetermined a_1 and b_1 wave functions.

For light tensor mesons with $J^P = 2^+$, the five polarization states ($\lambda = \pm 2, \pm 1, 0$) are described by a rank-2 tensor $\epsilon_{\mu\nu}^{(\lambda)}$, where angular momentum conservation restricts decays to only the longitudinal and transverse polarizations ($\lambda = 0, \pm 1$), while forbidding the maximal helicity states ($\lambda = \pm 2$). Consequently, the generic tensor meson wave function can be defined as [47]

$$\begin{aligned}
\Phi_T^L &= \frac{1}{\sqrt{6}} \left[m_T \not{\epsilon}_L \not{\phi}_T(x) + \not{\epsilon}_L \not{P} \not{\phi}_T^L(x) + m_T^2 \frac{\epsilon_\bullet \cdot v}{P \cdot v} \not{\phi}_T^s(x) \right], \\
\Phi_T^\perp &= \frac{1}{\sqrt{6}} [m_T \not{\epsilon}_\perp \not{\phi}_T^v(x) + \not{\epsilon}_\perp \not{P} \not{\phi}_T^T(x) \\
&\quad + m_T i \epsilon_{\mu\nu\rho\sigma} \gamma_5 \gamma^\mu \not{\epsilon}_\perp^\nu n^\rho v^\sigma \not{\phi}_T^a(x)], \quad (20)
\end{aligned}$$

where the distribution amplitudes have the following explicit forms [54]:

$$\begin{aligned}
\phi_T(x) &= \frac{f_T}{2\sqrt{2N_c}} \phi_\parallel(x), \quad \phi_T^t(x) = \frac{f_T^T}{2\sqrt{2N_c}} h_\parallel^{(t)}(x), \\
\phi_T^s(x) &= \frac{f_T^T}{4\sqrt{2N_c}} \frac{d}{dx} h_\parallel^{(s)}(x), \quad \phi_T^T(x) = \frac{f_T^T}{2\sqrt{2N_c}} \phi_\perp(x), \\
\phi_T^v(x) &= \frac{f_T}{2\sqrt{2N_c}} g_\perp^{(v)}(x), \quad \phi_T^a(x) = \frac{f_T}{8\sqrt{2N_c}} \frac{d}{dx} g_\perp^{(a)}(x). \quad (21)
\end{aligned}$$

The twist-2 LCDA can be expanded in Gegenbauer polynomials $C_n^{3/2}(2x-1)$ with Gegenbauer moments as coefficients. In particular, its asymptotic form is

$$\phi_{\parallel,\perp}(x) = 30x(1-x)(2x-1). \quad (22)$$

Using the QCD equations of motion, the two-particle twist-3 distribution amplitudes can be expressed in terms of both twist-2 LCDAs and three-particle twist-3 LCDAs. In the Wandzura-Wilczek approximation (where three-particle terms are neglected), the asymptotic forms of twist-3 LCDAs are derived as [54]

$$\begin{aligned}
h_\parallel^{(t)}(x) &= \frac{15}{2} (2x-1)(1-6x+6x^2), \\
h_\parallel^{(s)}(x) &= 15x(1-x)(2x-1), \\
g_\perp^{(a)}(x) &= 20x(1-x)(2x-1), \quad g_\perp^{(v)}(x) = 5(2x-1)^3. \quad (23)
\end{aligned}$$

B. Helicity amplitudes

Based on the effective Hamiltonian in Eq. (1), the decay $B_c \rightarrow J/\psi(a_1, b_1)$ can be described by the four types of Feynman diagrams shown in Fig. 1. These diagrams give rise to the factorizable emission amplitude $\mathcal{F}_e$ and the

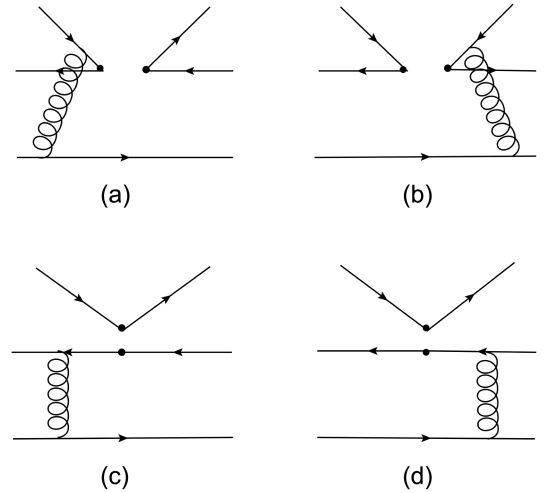


FIG. 1. Feynman diagrams for $B_c \rightarrow J/\psi A$ decays. (a), (b) The nonfactorizable emission diagrams. (c), (d) The factorizable emission diagrams.

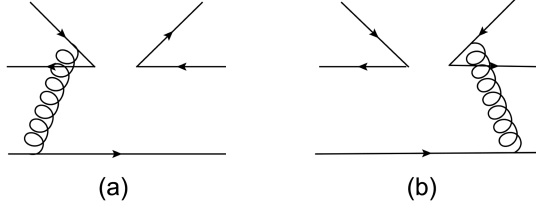


FIG. 2. Feynman diagrams for $B_c \rightarrow J/\psi T$ decays. (a), (b) The nonfactorizable emission diagrams.

nonfactorizable emission amplitude $\mathcal{M}_e$. Since the operators $\mathcal{O}_{1,2}$ are of $(V-A)(V-A)$ current type, these diagrams collectively yield the total decay amplitude:

$$\mathcal{A}(B_c \rightarrow (J/\psi)A) = V_{cb}^* V_{ud(s)} \left[\left(C_2 + \frac{1}{3} C_1 \right) \mathcal{F}_e + C_1 \mathcal{M}_e \right]. \quad (24)$$

Here, the Wilson coefficients C_1 and C_2 are convolved with the amplitudes $\mathcal{F}_e$ and $\mathcal{M}_e$. For the $B_c \rightarrow J/\psi(a_1, b_1)$ decay, the amplitude exhibits distinct polarization characteristics, comprising one longitudinal component and two transverse components. The amplitude can be decomposed as

$$\mathcal{A} = \mathcal{A}^L + \mathcal{A}^N \epsilon_2^T \cdot \epsilon_3^T + i \mathcal{A}^T \epsilon_{\alpha\beta\rho\sigma} n^\alpha v^\beta \epsilon_2^{T\rho} \epsilon_3^{T\sigma}. \quad (25)$$

Here, ϵ_2^T and ϵ_3^T are the transverse polarization vectors for the vector meson J/ψ and the axial-vector meson (a_1 or b_1), respectively. $\mathcal{A}^L$ corresponds to the longitudinal polarization contribution, while $\mathcal{A}^N$ and $\mathcal{A}^T$ describe the normal and transverse polarization contributions respectively. The total amplitudes $\mathcal{A}^L$, $\mathcal{A}^N$, and $\mathcal{A}^T$ share the same structure as Eq. (24). The factorization formulas for all polarization components (longitudinal, normal, and transverse) are provided in the Appendix.

In the decay $B_c \rightarrow J/\psi T$, the tensor meson cannot be directly produced through standard weak operators due to spin constraints, resulting in a vanishing factorizable amplitude. Consequently, the total decay is entirely dominated by nonfactorizable emission diagrams in Fig. 2.

III. NUMERICAL RESULTS AND DISCUSSIONS

The branching ratios for the decay process of the B_c meson in its rest frame can be expressed as

$$\mathcal{BR}(B_c \rightarrow J/\psi(A, T)) = \frac{G_F^2 \tau_{B_c}}{32\pi M_B} \sqrt{1 - (r_{A(T)} - r_{J/\psi})^2} \times \sqrt{1 - (r_{A(T)} + r_{J/\psi})^2} |\mathcal{A}|^2, \quad (26)$$

where τ_{B_c} is the lifetime of the B_c meson, and the decay amplitudes $\mathcal{A}$ for each channel have been explicitly given in the Appendix. When the final states involve a vector charmonium state and a axial-vector meson or tensor meson, the decay amplitude can be decomposed into three polarized components,

$$|\mathcal{A}|^2 = |\mathcal{A}_0|^2 + |\mathcal{A}_\parallel|^2 + |\mathcal{A}_\perp|^2 \quad (27)$$

where the amplitudes $\mathcal{A}_0$, $\mathcal{A}_\parallel$, and $\mathcal{A}_\perp$ denote the longitudinal, parallel, and perpendicular polarization amplitudes in the transverses basis, respectively. These relate to the helicity-basis amplitudes $\mathcal{A}^L$, $\mathcal{A}^N$, and $\mathcal{A}^T$ in Eq. (25) through a transformation relation,

$$\mathcal{A}_0 = \mathcal{A}^L, \quad \mathcal{A}_\parallel = \sqrt{2} \mathcal{A}^N, \quad \mathcal{A}_\perp = \sqrt{2} \mathcal{A}^T. \quad (28)$$

Turning to the polarizations for $B_c^+ \rightarrow J/\psi(A, T)$ decays, we usually define the polarization fractions

$$f_\lambda = \frac{|\mathcal{A}_\lambda|^2}{|\mathcal{A}_0|^2 + |\mathcal{A}_\parallel|^2 + |\mathcal{A}_\perp|^2}.$$

The polarization fractions satisfy the normalization constraint $\sum_\lambda f_\lambda = 1$, where f_0 , $f_\parallel$, and $f_\perp$ correspond to the longitudinal, parallel, and perpendicular polarization components, respectively.

In Table I, we list the input parameters used in the calculations, including meson masses and decay constants, the lifetime of the B_c meson, and the Wolfenstein parameters of the CKM matrix.

Our numerical results for the branching ratios of the $B_c \rightarrow J/\psi A$ and $B_c \rightarrow J/\psi T$ decay processes are listed in

TABLE I. Various parameters involved in the calculation process.

Mass of mesons	$M_W = 80.41 \text{ GeV}$	$M_{B_c} = 6.275 \text{ GeV}$	$m_b = 4.8 \text{ GeV}$
	$m_c = 1.5 \text{ GeV}$	$m_{J/\psi} = 3.097 \text{ GeV}$	$m_{a_1} = 1.23 \text{ GeV}$
	$m_{b_1} = 1.21 \text{ GeV}$	$m_{a_2} = 1.317 \text{ GeV}$	$m_{K_2^*(1430)} = 1.427 \text{ GeV}$
Decay constants of mesons	$f_{B_c} = 489 \pm 4 \pm 3 \text{ MeV}$	$f_{J/\psi} = 405 \pm 14 \text{ MeV}$	$f_{a_1} = 238 \text{ MeV}$
	$f_{b_1} = 180 \text{ MeV}$	$f_{a_2} = 107 \pm 6 \text{ MeV}$	$f_{K_2^*(1430)} = 118 \pm 5 \text{ MeV}$
Lifetime of meson	$\tau_{B_c} = 0.507 \text{ ps}$		
Wolfenstein parameters	$A = 0.826$	$\lambda = 0.22501$	
	$\bar{\rho} = 0.1591$	$\bar{\eta} = 0.3523$	

TABLE II. Branching ratios for $B_c \rightarrow J/\psi(A, T)$. The errors for these entries correspond respectively to uncertainties in hadronic shape parameters, decay constants, and scale dependence.

Decay modes	B_0	$B_{\parallel}$	$B_{\perp}$	B_{total}
$B_c^+ \rightarrow J/\psi a_1^+$	$7.63^{+1.10+0.44+0.33}_{-0.99-0.42-0.31} \times 10^{-3}$	$1.75^{+0.26+0.10+0.07}_{-0.22-0.10-0.08} \times 10^{-3}$	$1.68^{+0.21+0.08+0.07}_{-0.21-0.09-0.06} \times 10^{-3}$	$11.06^{+1.57+0.62+0.47}_{-1.42-0.61-0.45} \times 10^{-3}$
$B_c^+ \rightarrow J/\psi b_1^+$	$1.36^{+0.17+0.11+0.06}_{-0.14-0.09-0.06} \times 10^{-3}$	$0.09^{+0.01+0.01+0.01}_{-0.02-0.01-0.01} \times 10^{-3}$	$0.06^{+0.01+0.01+0.00}_{-0.01-0.00-0.00} \times 10^{-3}$	$1.51^{+0.19+0.13+0.07}_{-0.17-0.10-0.07} \times 10^{-3}$
$B_c^+ \rightarrow J/\psi a_2^+$	$2.77^{+0.51+0.16+0.15}_{-0.47-0.12-0.14} \times 10^{-4}$	$0.07^{+0.10+0.02+0.03}_{-0.04-0.02-0.04} \times 10^{-4}$	$0.05^{+0.07+0.01+0.02}_{-0.02-0.01-0.02} \times 10^{-4}$	$2.89^{+0.68+0.19+0.20}_{-0.53-0.15-0.20} \times 10^{-4}$
$B_c^+ \rightarrow J/\psi K_2^{*+}$	$4.26^{+0.40+0.04+0.09}_{-0.33-0.04-0.09} \times 10^{-5}$	$0.12^{+0.03+0.01+0.01}_{-0.03-0.01-0.01} \times 10^{-5}$	$0.10^{+0.02+0.00+0.00}_{-0.01-0.00-0.00} \times 10^{-5}$	$4.48^{+0.45+0.05+0.10}_{-0.37-0.05-0.10} \times 10^{-5}$

Table II. The uncertainties in our calculation results originate from three primary sources: the first kind stems from hadronic parameters $\omega_B = (0.60 \pm 0.05)$ GeV [55] for the B_c meson and $\omega = (0.60 \pm 0.10)$ GeV [42] for the J/ψ meson, the second kind pertains to decay constants of the B_c meson and final-state charmonium mesons, which can be found in Table I, and the third kind arises from $\Lambda_{\text{QCD}} = (0.25 \pm 0.05)$ GeV and a 20% variation in the hard scale $t_{\text{max}} = (1.0 \pm 0.2)t$ as defined in the Appendix, while other uncertainties such as those from CKM matrix elements V related to $\bar{\eta}$ and $\bar{\rho}$ angles of the unitary triangle are found to be negligible.

In Table II, we list the branching ratios for $B_c \rightarrow J/\psi A$ and $B_c \rightarrow J/\psi T$ decays, where A denotes axial-vector mesons (a_1, b_1) and T denotes tensor mesons (a_2, K_2^*). The key observations are as follows:

- (i) For the decay processes $B_c^+ \rightarrow J/\psi a_1^+$ and $B_c^+ \rightarrow J/\psi b_1^+$, both final states contain axial-vector mesons and exhibit relatively large branching ratios. This is primarily because a_1^+ and b_1^+ share the same quark composition (mainly $u\bar{d}$), and their decay amplitudes at the tree level benefit from the enhanced CKM matrix element contribution ($|V_{cb}V_{ud}^*| \sim 10^{-2}$). In Table II, we can see

$$\mathcal{B}(B_c^+ \rightarrow J/\psi b_1^+) < \mathcal{B}(B_c^+ \rightarrow J/\psi a_1^+).$$

The key reason lies in the effective decay constant of the b_1^+ meson, $f_{b_1^+} = f_{b_1} \cdot a_{0b_1}^{\parallel} \sim 0.0005$, which is nearly vanishing [51]. This strongly suppresses the factorizable emission diagram contributions that depend on this constant. As a result, the branching ratio of $B_c^+ \rightarrow J/\psi b_1^+$ arises almost entirely from nonfactorizable emission decay amplitudes. Nevertheless, the $b_1(1235)$, as a $^1P_1$ state, possesses a leading-twist distribution amplitude with odd-parity antisymmetric behavior. This symmetry characteristic alters the quantum interference pattern between specific Feynman diagrams (e.g., those involving quark-antiquark momentum exchange) in the nonfactorizable emission topology: in $B_c^+ \rightarrow J/\psi a_1^+$ (where a_1 is a $^3P_1$ state with a symmetric wave function), the nonfactorizable diagrams exhibit

destructive interference; while in $B_c^+ \rightarrow J/\psi b_1^+$, the same nonfactorizable topology undergoes constructive interference due to the antisymmetric wave function. This crucial interference transition effectively compensates for the suppression caused by the extremely small decay constant, allowing the branching ratio of the b_1^+ channel to remain at the order of 10^{-3} .

- (ii) To facilitate future experimental investigations, we define the ratios of branching fractions using $\mathcal{B}(B_c^+ \rightarrow J/\psi \pi^+)$ from Ref. [42] as a benchmark. The benchmark decay $B_c^+ \rightarrow J/\psi \pi^+$ is chosen because it is theoretically clean and provides a well-normalized reference within the factorization approach. The calculated ratios are as follows:

$$\begin{aligned}
R_{a_1/\pi} &\equiv \frac{\mathcal{B}(B_c^+ \rightarrow J/\psi a_1^+)}{\mathcal{B}(B_c^+ \rightarrow J/\psi \pi^+)} \approx 4.75, \\
R_{b_1/\pi} &\equiv \frac{\mathcal{B}(B_c^+ \rightarrow J/\psi b_1^+)}{\mathcal{B}(B_c^+ \rightarrow J/\psi \pi^+)} \approx 0.65, \\
R_{a_2/\pi} &\equiv \frac{\mathcal{B}(B_c^+ \rightarrow J/\psi a_2^+)}{\mathcal{B}(B_c^+ \rightarrow J/\psi \pi^+)} \approx 0.124, \\
R_{K_2^*/\pi} &\equiv \frac{\mathcal{B}(B_c^+ \rightarrow J/\psi K_2^{*+})}{\mathcal{B}(B_c^+ \rightarrow J/\psi \pi^+)} \approx 0.019. \quad (29)
\end{aligned}$$

These four ratios reveal a distinct hierarchy:

$$R_{a_1/\pi} \gg R_{b_1/\pi} > R_{a_2/\pi} \gg R_{K_2^*/\pi}.$$

The large value of $R_{a_1/\pi}$ reflects the dominance of factorizable contributions, enhanced by the sizable decay constant of the axial-vector a_1^+ meson. In contrast, the significantly smaller $R_{b_1/\pi}$ highlights the suppression associated with nonfactorizable dynamics and the peculiar $J^{PC} = 1^{+-}$ quantum number of the b_1^+ meson. The even smaller ratios for the tensor mesons, $R_{a_2/\pi}$ and $R_{K_2^*/\pi}$, indicate further suppression in decays to higher-spin ($J = 2$) final states, which involve more intricate helicity structures and receive contributions from subleading power corrections. Notably, the strange tensor

meson K_2^{*+} exhibits an order-of-magnitude further suppression relative to the nonstrange a_2^+ , a pattern attributable to the Cabibbo-suppressed $b \rightarrow s$ transition and to possible differences between the distribution amplitudes of strange and nonstrange tensor mesons. These ratios provide a sensitive set of observables for testing the interference between color-favored and color-suppressed amplitudes, the role of nonfactorizable dynamics, and the spin-dependent hadronization mechanisms in B_c decays. Precise future measurements of these channels are important for several reasons: first, they offer a direct test of pQCD frameworks in describing nonfactorizable contributions in processes involving heavy quarkonia; second, comparison with theoretical predictions will help to constrain hadronic parameters, especially the distribution amplitudes of the B_c meson and the light mesons; and finally, any substantial discrepancy between experiment and theory could signal physics beyond the Standard Model, making these decays a valuable platform for indirect searches for new physics. The inclusion of the tensor meson channels further enriches this program by probing spin-dependent dynamics and the factorization properties of higher-twist distribution amplitudes.

- (iii) A comparison with existing literature highlights significant refinements in both the theoretical framework and input parameters employed in this study. For the decay $B_c^+ \rightarrow J/\psi a_1(1260)^+$, the branching ratio reported in Ref. [49] is $(1.02^{+0.04+0.03+0.01}_{-0.08-0.05-0.01}) \times 10^{-3}$, which differs from our computed value for the same channel. These discrepancies stem primarily from two sources. First, at the level of theoretical framework, we have incorporated an intrinsic transverse-momentum b dependence into the wave function model of the B_c meson, as given in Eq. (7). This improvement allows a more accurate description of transverse-momentum effects in the hadronic wave function, leading to enhanced precision in handling both factorizable and nonfactorizable diagrams, especially those involving soft-gluon exchange, thereby reducing systematic uncertainties originating from model simplifications. Second, regarding input parameters, we have updated the CKM matrix elements, meson masses, decay constants, and hadronic shape parameters in accordance with the latest experimental data and theoretical developments. This update ensures that our predictions are based on the most reliable available inputs, making the results not only a test of the pQCD framework but also a timely and competitive theoretical reference for future experimental measurements. Therefore, the observed differences signify a substantive evolution in the theoretical methodology. This methodological advancement,

utilizing an improved wave function and contemporary parameters, yields more reliable predictions. These results provide a refined benchmark for high-precision experiments at the upgraded LHCb and will deepen the understanding of heavy-flavored hadron decay dynamics.

- (iv) Compared with Ref. [51], the branching-ratio predictions for $B_c^+ \rightarrow J/\psi A$ and $B_c^+ \rightarrow J/\psi T$ in the present work exhibit differences, yet the results of both approaches lie within the same order of magnitude. This consistency in the overall scale indicates that, despite deviations due to distinct theoretical treatments, the two methods yield compatible estimates of the basic branching ratio features for B_c decays, thereby reinforcing the robustness of the theoretical predictions. The numerical differences mainly stem from the fundamental distinction between the two theoretical frameworks. In Ref. [51], a modified Sudakov factor $s_c(Q, b)$ is introduced to explicitly incorporate the charm-quark mass scale m_c into the evolution kernel, thereby addressing radiative corrections associated with the ratio m_c/m_b . In contrast, the present study employs a B_c meson wave function of the form $\delta(x - m_c/M_{B_c})$, which fixes the longitudinal momentum fraction of the charm quark and naturally embeds the double-heavy flavor nature of the B_c meson at the level of the initial-state wave function. Theoretically, the two schemes improve the description accuracy from complementary perspectives: the δ -type wave function adopted here reduces the sensitivity to the endpoint behavior ($x \rightarrow 0, 1$) of the light-meson distribution amplitudes by locking the parton momentum fraction, while the improved Sudakov factor $s_c(Q, b)$ in Ref. [51] explicitly includes the m_c contribution in the resummation, allowing for a more precise treatment of soft-gluon radiation at the charm-quark mass scale. These complementary treatments enhance the descriptive power of the theoretical framework for the double-heavy B_c system from the angles of the initial-state wave function and the evolution kernel, respectively. Given that the two methods emphasize different aspects of the theoretical treatment, the present work and Ref. [51] together constitute two complementary realizations of the pQCD approach in B_c physics. The cross-validation of their theoretical architectures and numerical outcomes provides a more comprehensive and reliable theoretical reference for future precision measurements at experiments such as LHCb and Belle II, and extends our understanding of non-perturbative QCD effects at the charm-quark mass scale from different perspectives.
- (v) To quantitatively probe the distinct decay dynamics between the vector ρ meson and the axial-vector a_1 and b_1 mesons, we define the following two ratios,

normalized to the theoretically established branching fraction of the $B_c^+ \rightarrow J/\psi \rho^+$ decay as a reference [42]:

$$\begin{aligned} R_{a_1/\rho} &\equiv \frac{\mathcal{B}(B_c^+ \rightarrow J/\psi a_1^+)}{\mathcal{B}(B_c^+ \rightarrow J/\psi \rho^+)} \approx 1.35, \\ R_{b_1/\rho} &\equiv \frac{\mathcal{B}(B_c^+ \rightarrow J/\psi b_1^+)}{\mathcal{B}(B_c^+ \rightarrow J/\psi \rho^+)} \approx 0.18. \end{aligned} \quad (30)$$

The $B_c^+ \rightarrow J/\psi \rho^+$ channel serves as a well-understood benchmark within the factorization framework, being dominated by factorizable emission diagrams with clear vector-current contributions. This choice provides a theoretically clean and reliable baseline, enabling a quantitative comparison and interpretation of the more intricate dynamics governing the decays into axial-vector final states. In the Standard Model, the axial-vector a_1 meson and the vector meson ρ exhibit similar QCD hadronic dynamics [45]. Hence, the decay pattern of $B_c^+ \rightarrow J/\psi a_1^+$ is expected to resemble that of $B_c^+ \rightarrow J/\psi \rho^+$. The ratio $R_{a_1/\rho}$ reveals that the branching ratio $\mathcal{B}(B_c^+ \rightarrow J/\psi a_1^+)$ is approximately 35% larger than $\mathcal{B}(B_c^+ \rightarrow J/\psi \rho^+)$. This enhancement reflects the dominance of factorizable contributions in the a_1 channel, likely attributable to its larger decay constant and similar vector-current-driven dynamics. In contrast, $R_{b_1/\rho}$ indicates that $\mathcal{B}(B_c^+ \rightarrow J/\psi b_1^+)$ is suppressed by about 82% relative to $\mathcal{B}(B_c^+ \rightarrow J/\psi \rho^+)$. This significant suppression originates from the strong suppression of factorizable amplitudes in b_1 production, underscoring the importance of nonfactorizable contributions and the distinctive internal structure of the b_1 meson ($J^{PC} = 1^{+-}$). The marked difference between $R_{a_1/\rho}$ and $R_{b_1/\rho}$ highlights the sensitivity of these ratios to the internal spin-parity configurations of axial-vector mesons, providing a clear experimental signature for distinguishing 1^{++} and 1^{+-} states. These ratios constitute powerful probes for testing the underlying dynamics of heavy-flavor decays both within and beyond the Standard Model. The enhanced $R_{a_1/\rho}$ validates the dominance of factorizable mechanisms in a_1 production, offering a benchmark for pQCD predictions, while the suppressed $R_{b_1/\rho}$ emphasizes the necessity of incorporating nonfactorizable effects that are sensitive to QCD vacuum structures and higher-order corrections. Future high-precision experiments at facilities such as the upgraded LHCb, Belle II, or a future Electron-Ion Collider can leverage these observables to constrain hadronic parameters like decay constants and distribution amplitudes, discriminate between different theoretical factorization schemes, probe possible exotic configurations such as hybrid or tetraquark

admixture in axial-vector mesons, and search for signatures of new physics through deviations from Standard Model predictions. Consequently, precise measurements of $R_{a_1/\rho}$ and $R_{b_1/\rho}$ will not only deepen our understanding of nonperturbative QCD but also significantly enhance the sensitivity of B_c decays to potential new physics scenarios.

- (vi) For the decays $B_c^+ \rightarrow J/\psi a_2^+$ and $B_c^+ \rightarrow J/\psi K_2^{*+}$, the factorizable emission diagrams vanish due to a fundamental mismatch in quantum numbers: tensor mesons ($J^{PC} = 2^{++}$) cannot be directly created from the vacuum via local ($V \pm A$) or ($S \pm P$) weak currents, which only generate states with $J \leq 1$. Consequently, $\langle T | \bar{q} \Gamma q | 0 \rangle = 0$, forbidding the factorizable emission process in the Standard Model. These decays therefore proceed entirely through the nonfactorizable emission mechanism, which leads to their characteristically small branching fractions. Furthermore, the comparison between the a_2^+ channel (involving a $b \rightarrow d$ transition) and the K_2^{*+} channel (involving a $b \rightarrow s$ transition) serves as a sensitive probe of flavor-SU(3) symmetry breaking. The predicted ratio of their branching fractions exhibits a significant deviation from the expectation based solely on CKM suppression, highlighting the influence of differences in nonperturbative hadronic parameters, such as decay constants and distribution amplitudes, between strange and nonstrange tensor mesons. The study of these decays thus provides a unique laboratory to test QCD dynamics in heavy-flavor decays, offering critical insights into the properties of tensor mesons and the breaking of flavor symmetries. These investigations not only validate the capability of theoretical frameworks like pQCD in describing exclusively nonfactorizable topologies but also substantially advance our understanding of strong interaction mechanisms.

Turning to Table III, we present a systematic analysis of the predicted polarization fractions of mesons: It can be seen that all decays reveal profound insights into heavy-quark hadronization dynamics. Three distinct polarization patterns emerge with striking consistency across decay channels, reflecting the universal dominance of longitudinal polarizations (f_0) in branching ratios. In particular, the highest longitudinal polarization fractions observed for the tensor meson final states $B_c^+ \rightarrow J/\psi a_2^+$ and $B_c^+ \rightarrow J/\psi K_2^{*+}$, which nearly approach complete longitudinal polarization. This behavior stems from the tensor mesons' orbital angular momentum of $L = 1$ (corresponding to the quantum number assignment $J^{PC} = 2^{++}$). Specifically, the endpoint contributions in their LCDAs substantially enhance the convolution weight of the longitudinal helicity component. Concurrently, the mismatch in quantum numbers forbids the contributions from factorizable emission diagrams, confining these decays to proceed

TABLE III. Polarization fractions for $B_c^+ \rightarrow J/\psi(A, T)$ decays. The errors correspond respectively to uncertainties in hadronic shape parameters, decay constants, and scale dependence.

Decay modes	f_0	$f_{\parallel}$	$f_{\perp}$
$B_c^+ \rightarrow J/\psi a_1^+$	$0.69^{+0.02+0.02+0.01}_{-0.02-0.03-0.01}$	$0.15^{+0.02+0.01+0.00}_{-0.02-0.02-0.00}$	$0.15^{+0.03+0.02+0.00}_{-0.02-0.02-0.01}$
$B_c^+ \rightarrow J/\psi b_1^+$	$0.90^{+0.03+0.03+0.01}_{-0.03-0.02-0.01}$	$0.06^{+0.01+0.01+0.00}_{-0.01-0.01-0.01}$	$0.04^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$
$B_c^+ \rightarrow J/\psi a_2^+$	$0.96^{+0.02+0.03+0.00}_{-0.02-0.02-0.01}$	$0.02^{+0.01+0.01+0.00}_{-0.01-0.00-0.00}$	$0.02^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$
$B_c^+ \rightarrow J/\psi K_2^{*+}$	$0.95^{+0.02+0.01+0.01}_{-0.01-0.02-0.00}$	$0.03^{+0.01+0.00+0.00}_{-0.01-0.00-0.00}$	$0.02^{+0.00+0.01+0.01}_{-0.00-0.00-0.00}$

entirely via nonfactorizable mechanisms and thus further suppressing transverse polarization. In contrast, the longitudinal polarization fraction for $B_c^+ \rightarrow J/\psi a_1^+$ is notably lower. This is attributed to the large decay constant of a_1^+ as a $^3P_1$ state, which sustains considerable factorizable emission contributions that generally favor transverse polarization to a certain extent. It is worth emphasizing that the longitudinal polarization fraction for $B_c^+ \rightarrow J/\psi b_1^+$ is significantly higher than that of the a_1^+ channel. The primary reason lies in the nearly vanishing effective decay constant of b_1^+ , a feature arising from its $^1P_1$ quantum numbers. This tiny decay constant strongly suppresses any factorizable diagrams dependent on this parameter, forcing the decay to be dominated entirely by nonfactorizable contributions. Furthermore, the antisymmetric distribution amplitude of the b_1 meson transforms the quantum interference in nonfactorizable diagrams from destructive (as observed in the a_1^+ channel) to constructive, which further amplifies the longitudinal polarization amplitude. Regarding transverse polarization, both the parallel ($f_{\parallel}$) and perpendicular ($f_{\perp}$) components are drastically suppressed across all decay channels. This suppression is particularly pronounced in tensor meson decays, where the transverse polarization components are reduced to an extremely low level. This phenomenon is consistent with the helicity selection rules for the $0^- \rightarrow 1^-$ effective transition of the heavy quark line, under the approximation of heavy-quark helicity conservation (HQHC). Moreover, the polarization fractions of the strange (K_2^{*+}) and non-strange (a_2^+) tensor meson channels are in excellent agreement within uncertainties. This indicates that polarization is relatively insensitive to the flavor composition of the light quarks. Nevertheless, the minor discrepancies in $f_{\parallel}$ and $f_{\perp}$ may reflect the distortions of LCDAs induced by the nonvanishing mass of the strange quark. These polarization patterns not only verify the capability of the pQCD framework to describe the helicity structure of double-heavy meson systems but also provide clear theoretical benchmarks. These benchmarks will facilitate the discrimination of the intrinsic angular momentum structures of different mesons through angular distribution measurements in future experiments at the LHCb and Belle II collaborations.

IV. SUMMARY

In this article, we systematically investigate the exclusive two-body decays of the B_c meson to an S -wave charmonium state (J/ψ) and either axial-vector mesons (a_1, b_1) or tensor mesons (a_2, K_2^*) within the pQCD factorization framework, incorporating the nonperturbative distribution amplitudes of the final-state mesons to ensure infrared safety. Since flavor nondegeneracy in four-quark operators forbids penguin diagrams, weak phase differences vanish, eliminating CP violation in these decays. We focus on computing the branching ratios and polarization fractions for the aforementioned decay channels. To provide sensitive observables for experimental verification, this study defines ratios of branching fractions using the benchmark decays $B_c^+ \rightarrow J/\psi \pi^+$ and $B_c^+ \rightarrow J/\psi \rho^+$ [42], which are well-studied channels with established theoretical and experimental results. These ratios not only quantify the relative significance of factorizable versus nonfactorizable contributions but also encode spin-dependent hadronization effects and flavor- $SU(3)$ symmetry breaking. Furthermore, they serve as stringent probes for testing the QCD factorization framework and constraining key nonperturbative hadronic parameters, which are pivotal for refining the pQCD formalism in heavy-flavor physics. The obtained results provide a solid theoretical basis for experimental investigations and are particularly well suited for leveraging the high-statistics environment of the LHCb experiment. This work not only validates the applicability of the pQCD framework in describing the helicity structure and strong interaction dynamics of double-heavy mesons but also delivers valuable theoretical predictions for ongoing and future studies at the LHCb and Belle II experiments. These findings will further deepen our understanding of nonperturbative QCD effects, meson internal structure, and flavor symmetry breaking mechanisms in heavy-flavor decays, while laying the groundwork for the indirect detection of new physics signals.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [50,58].

APPENDIX: FACTORIZATION FORMULAS

Similar to vector mesons, axial-vector mesons possess intrinsic spin degrees of freedom, which necessitates distinct amplitude components for the three polarization states (longitudinal, normal, transverse) in $B_c \rightarrow J/\psi A$ decays, denoted respectively as L , N , and T amplitudes,

$$F_e^L = 2\sqrt{\frac{2}{3}}\pi C_F f_B f_A M_B^4 \sqrt{1-r_2^2} \int_0^1 dx_2 \int_0^\infty b_1 b_2 db_1 db_2 \exp\left(-\frac{\omega_B^2 b_1^2}{2}\right) \\ \times \{[r_2 \phi^t(x_2, b_2)(r_b - 2x_2) + \phi^L(x_2, b_2)(x_2 - 2r_b)]E_{ab}(t_a)h(\alpha_e, \beta_a, b_1, b_2)S_t(x_2) \\ - \phi^L(x_2, b_2)[r_2^2 + r_c]E_{ab}(t_b)h(\alpha_e, \beta_b, b_2, b_1)S_t(x_1)\}, \quad (\text{A1})$$

$$M_e^L = \frac{8}{3}\pi C_F f_B M_B^4 \sqrt{1-r_2^2} \int_0^1 dx_2 dx_3 \int_0^\infty b_1 b_3 db_1 db_3 \exp\left(-\frac{\omega_B^2 b_1^2}{2}\right) \phi_A(x_3) \\ \times \left\{[-\phi^L(x_2, b_1)(r_2^2 - 1)(x_1 - x_3) + r_2 \phi^t(x_2, b_1)(x_1 + x_2 - 1)]E_{cd}(t_c)h(\beta_c, \alpha_e, b_3, b_1) \right. \\ \left. - [\phi^L(x_2, b_1)(r_2^2(x_2 - x_3) + 2x_1 + x_2 + x_3 - 2) \right. \\ \left. - r_2 \phi^t(x_2, b_1)(x_1 + x_2 - 1)]E_{cd}(t_d)h(\beta_d, \alpha_e, b_3, b_1)\right\}, \quad (\text{A2})$$

$$F_e^N = 2\sqrt{\frac{2}{3}}\pi C_F f_B f_A M_B^4 r_3 \int_0^1 dx_2 \int_0^\infty b_1 b_2 db_1 db_2 \exp\left(-\frac{\omega_B^2 b_1^2}{2}\right) \\ \times \{[r_2 \phi^T(x_2, b_2)(-4r_b + x_2 + 1) + (r_b - 2)\phi^V(x_2, b_2)]E_{ab}(t_a)h(\alpha_e, \beta_b, b_2, b_1)S_t(x_2)] \\ + \phi^T(x_2, b_2)[r_2(x_1 - 1)]E_{ab}(t_b)h(\alpha_e, \beta_b, b_2, b_1)S_t(x_1)]\}, \quad (\text{A3})$$

$$M_e^N = \frac{8}{3}\pi C_F f_B M_B^4 r_3 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 b_3 db_1 db_3 \exp\left(-\frac{\omega_B^2 b_1^2}{2}\right) \\ \times \{[\phi_A^a(x_3)\phi^V(x_2, b_1)(r_2^2 - 1)(x_2 + x_3 - 1) + \phi_A^v(x_3)\phi^T(x_2, b_1) \\ \times (r_2^2(x_1 + 2x_2 + x_3 - 2) + x_1 - x_3)]E_{cd}(t_c)h(\beta_c, \alpha_e, b_3, b_1) \\ + [\phi_A^a(x_3)2(r_2^2 - 1)(r_2(x_2 - x_3)\phi^V(x_2, b_1) + 2(x_1 + x_3 - 1)\phi^T(x_2, b_1)) \\ + \phi_A^v(x_3)(2r_2\phi^V(x_2, b_1)(r_2^2(x_2 - x_3) + 2x_1 + x_2 + x_3 - 2) \\ + \phi^T(x_2, b_1)(r_2^2(1 - x_1 - 2x_2 + x_3) + 1 - x_1 - x_3))]E_{cd}(t_d)h(\beta_d, \alpha_e, b_3, b_1)\}, \quad (\text{A4})$$

$$F_e^T = 2\sqrt{\frac{2}{3}}\pi C_F f_B f_A M_B^4 r_3 \int_0^1 dx_2 \int_0^\infty b_1 b_2 db_1 db_2 \exp\left(-\frac{\omega_B^2 b_1^2}{2}\right) \\ \times \{[\phi^V(x_2, b_2)(2 - r_b) + r_2 \phi^T(x_2, b_2)(x_2 - 1)]E_{ab}(t_a)h(\alpha_e, \beta_a, b_1, b_2)S_t(x_2) \\ + \psi^T(x_2, b_2)[r_2(x_1 - 1)]E_{ab}(t_b)h(\alpha_e, \beta_b, b_2, b_1)S_t(x_1)\}, \quad (\text{A5})$$

$$\begin{aligned}
M_e^T = & \frac{8}{3} \pi C_F f_B M_B^4 r_3 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 b_3 db_1 db_3 \exp\left(-\frac{\omega_B^2 b_1^2}{2}\right) \\
& \times \{ [-\phi_A^a(x_3) \phi^V(x_2, b_1) 2r_2(r_2^2(x_2 + x_3 - 1) + 2x_1 + x_2 - x_3 - 1) \\
& - \phi_A^v(x_3) \phi^T(x_2, b_1)(r_2^2 - 1)(x_1 - x_3)] E_{cd}(t_c) h(\beta_c, \alpha_e, b_3, b_1) \\
& - [2\phi_A^a(x_3)(r_2 \phi^V(x_2, b_1)(r_2^2(x_2 - x_3) + 2x_1 + x_2 + x_3 - 2) \\
& + 2\phi^T(x_2, b_1)(r_2^2(1 - x_1 - 2x_2 + x_3) + 1 - x_1 - x_3)) \\
& + \phi_A^v(x_3) \phi^T(x_2, b_1)(r_2^2 - 1)(x_1 + x_3 - 1)] E_{cd}(t_d) h(\beta_d, \alpha_e, b_3, b_1) \}. \tag{A6}
\end{aligned}$$

For the $B_c \rightarrow J/\psi T$ decay, the calculation of its amplitude can be mapped to that of the corresponding axial-vector case via the following substitution:

$$\begin{aligned}
\mathcal{M}_T^L &= \sqrt{\frac{2}{3}} \mathcal{M}_A^L \Big|_{\substack{\phi_A^L \rightarrow \phi_T, \\ \phi_A^T \rightarrow \phi_T^L, \\ r_c \rightarrow -r_c}}, \\
\mathcal{M}_T^{N,T} &= \sqrt{\frac{1}{2}} \mathcal{M}_A^{N,T} \Big|_{\substack{\phi_A^T \rightarrow \phi_T^T, \\ \phi_A^V \rightarrow \phi_T^V, \\ r_c \rightarrow -r_c}}. \tag{A7}
\end{aligned}$$

The numerical prefactors $\sqrt{\frac{2}{3}}$ and $\sqrt{\frac{1}{2}}$ are derived from the equivalent polarization vector ϵ , defined in Eq. (6) for the tensor mesons, corresponding to the longitudinal and transverse polarizations, respectively.

The hard functions h are defined via Fourier transforms of virtual quark and gluon propagators, with specific expressions given by

$$\begin{aligned}
h(\alpha, \beta, b_1, b_2) &= h_1(\alpha, b_1) \times h_2(\beta, b_1, b_2), \\
h_1(\alpha, b_1) &= \begin{cases} K_0(\sqrt{\alpha} b_1), & \alpha > 0, \\ K_0(i\sqrt{-\alpha} b_1), & \alpha < 0, \end{cases} \\
h_2(\beta, b_1, b_2) &= \begin{cases} \theta(b_1 - b_2) I_0(\sqrt{\beta} b_2) K_0(\sqrt{\beta} b_1) + (b_1 \leftrightarrow b_2), & \beta > 0, \\ \theta(b_1 - b_2) J_0(\sqrt{-\beta} b_2) K_0(i\sqrt{-\beta} b_1) + (b_1 \leftrightarrow b_2), & \beta < 0, \end{cases} \tag{A8}
\end{aligned}$$

with J_0 denoting the Bessel function of the first kind, and K_0, I_0 representing the modified Bessel functions of the second and first kinds, respectively. The function K_0 for a real argument x is related to J_0 and the Neumann function N_0 by $K_0(ix) = \frac{\pi}{2}(-N_0(x) + iJ_0(x))$. The expressions for α and β in the hard functions are

$$\begin{aligned}
\alpha_e &= [x_1 + r_2^2(x_2 - 1)][x_1 + x_2 - 1 + r_3^2(1 - x_2)] M_B^2, \\
\beta_a &= [r_b^2 + (1 + r_2^2(x_2 - 1))(x_2 - r_3^2(x_2 - 1))] M_B^2, \\
\beta_b &= [r_c^2 + (r_2^2 - x_1)(x_1 - 1 + r_3^2)] M_B^2, \\
\beta_c &= [x_1 + x_2 - 1 + r_3^2(1 - x_2 - x_3)][x_3 - x_1 - r_2^2(x_2 + x_3 - 1)] M_B^2, \\
\beta_d &= [x_1 + x_2 - 1 - r_3^2(x_2 - x_3)][1 - x_1 - x_3 - r_2^2(x_2 - x_3)] M_B^2. \tag{A9}
\end{aligned}$$

The hard scales $t_i (i = a, b, c, d)$, chosen to eliminate large logarithmic radiative corrections in the pQCD factorization framework, are explicitly defined by the following kinematic variables:

$$\begin{aligned}
t_a &= \max\left(\sqrt{|\alpha_e|}, \sqrt{|\beta_a|}, 1/b_1, 1/b_2\right), & t_b &= \max\left(\sqrt{|\alpha_e|}, \sqrt{|\beta_b|}, 1/b_1, 1/b_2\right), \\
t_c &= \max\left(\sqrt{|\alpha_e|}, \sqrt{|\beta_c|}, 1/b_1, 1/b_3\right), & t_d &= \max\left(\sqrt{|\alpha_e|}, \sqrt{|\beta_d|}, 1/b_1, 1/b_3\right). \tag{A10}
\end{aligned}$$

The function $E_{ij}(t)$ is defined as the product of the running coupling strength $\alpha_s(t)$ and the Sudakov form factor $S_{ab,cd}(t)$, which resums the soft gluon radiation contributions:

$$E_{ab,cd}(t) = \alpha_s(t) S_{ab,cd}(t) \quad (\text{A11})$$

where the Sudakov exponents are defined as

$$\begin{aligned} S_{ab}(t) &= s\left(\frac{M_B}{\sqrt{2}}x_1, b_1\right) + s\left(\frac{M_B}{\sqrt{2}}x_2, b_2\right) + s\left(\frac{M_B}{\sqrt{2}}(1-x_2), b_2\right) + \frac{5}{3} \int_{1/b_1}^t \frac{d\mu}{\mu} \gamma_q(\mu) + 2 \int_{1/b_2}^t \frac{d\mu}{\mu} \gamma_q(\mu), \\ S_{cd}(t) &= s\left(\frac{M_B}{\sqrt{2}}x_1, b_1\right) + s\left(\frac{M_B}{\sqrt{2}}x_2, b_1\right) + s\left(\frac{M_B}{\sqrt{2}}(1-x_2), b_1\right) + s\left(\frac{M_B}{\sqrt{2}}x_3, b_3\right) + s\left(\frac{M_B}{\sqrt{2}}(1-x_3), b_3\right) \\ &\quad + \frac{11}{3} \int_{1/b_1}^t \frac{d\mu}{\mu} \gamma_q(\mu) + 2 \int_{1/b_3}^t \frac{d\mu}{\mu} \gamma_q(\mu), \end{aligned} \quad (\text{A12})$$

where the quark field anomalous dimension $\gamma_q = -\alpha_s/\pi$ governs its renormalization group evolution at leading order, with α_s the strong coupling constant. The Sudakov factor $s(Q, b)$ can be found in Ref. [59].

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- [1] F. Abe *et al.* (CDF Collaboration), Observation of the B_c meson in $p\bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV, *Phys. Rev. Lett.* **81**, 2432 (1998).
 - [2] A. Abulencia *et al.* (CDF Collaboration), Measurement of the B_c^+ meson lifetime using the decay mode $B_c^+ \rightarrow J/\psi e^+ \nu_e$, *Phys. Rev. Lett.* **97**, 012002 (2006).
 - [3] V. M. Abazov *et al.* (D0 Collaboration), Measurement of the $B_c^\pm$ meson lifetime in the semileptonic decay channel, *Phys. Rev. Lett.* **102**, 092001 (2009).
 - [4] R. Aaij *et al.* (LHCb Collaboration), Study of B_c^+ decays to the $K^+ K^- \pi^+$ final state and evidence for the decay $B_c^+ \rightarrow \chi_{c0} \pi^+$, *Phys. Rev. D* **94**, 091102 (2016).
 - [5] R. Aaij *et al.* (LHCb Collaboration), Measurements of B_c^+ production and mass with the $B_c^+ \rightarrow J/\psi \pi^+$ decay, *Phys. Rev. Lett.* **109**, 232001 (2012).
 - [6] R. Aaij *et al.* (LHCb Collaboration), Observation of $B_c^+ \rightarrow J/\psi D_s^+$ and $B_c^+ \rightarrow J/\psi D_s^{*+}$ decays, *Phys. Rev. D* **87**, 112012 (2013).
 - [7] R. Aaij *et al.* (LHCb Collaboration), Observation of the decay $B_c^+ \rightarrow \psi(2S) \pi^+$, *Phys. Rev. D* **87**, 071103 (2013).
 - [8] R. Aaij *et al.* (LHCb Collaboration), First observation of the decay $B_c^+ \rightarrow J/\psi \pi^+ \pi^- \pi^+$, *Phys. Rev. Lett.* **108**, 251802 (2012).
 - [9] R. Aaij *et al.* (LHCb Collaboration), First observation of the decay $B_c^+ \rightarrow J/\psi K^+$, *J. High Energy Phys.* **09** (2013) 075.
 - [10] R. Aaij *et al.* (LHCb Collaboration), Observation of the decay $B_c^+ \rightarrow J/\psi K^+ K^- \pi^+$, *J. High Energy Phys.* **11** (2013) 094.
 - [11] R. Aaij *et al.* (LHCb Collaboration), Observation of the decay $B_c^+ \rightarrow B_s^0 \pi^+$, *Phys. Rev. Lett.* **111**, 181801 (2013).
 - [12] Y. N. Gao, J. B. He, P. Robbe, M.-H. Schune, and Z.-W. Yang, Experimental prospects of the B_c studies of the LHCb experiment, *Chin. Phys. Lett.* **27**, 061302 (2010).
 - [13] N. Brambilla *et al.*, Heavy quarkonium: Progress, puzzles, and opportunities, *Eur. Phys. J. C* **71**, 1534 (2011).
 - [14] H. n. Li and H. L. Yu, Extraction of V_{ub} from the decay $B \rightarrow \pi l \nu$, *Phys. Rev. Lett.* **74**, 4388 (1995).
 - [15] Y. K. Hsiao and C. Q. Geng, Branching fractions of B_c decays involving J/ψ and $X(3872)$, *Chin. Phys. C* **41**, 013101 (2017).
 - [16] Q. Chang, L. L. Chen, and S. Xu, Study of $B_c \rightarrow J/\psi V$ and $B_c^* \rightarrow \eta_c V$ decays within the QCD factorization, *J. Phys. G: Nucl. Part. Phys.* **45**, 075005 (2018).
 - [17] V. V. Kiselev, A. E. Kovalsky, and A. K. Likhoded, B_c decays and lifetime in QCD sum rules, *Nucl. Phys.* **B585**, 353 (2000).
 - [18] C. H. Chang and Y. Q. Chen, Decays of the B_c meson, *Phys. Rev. D* **49**, 3399 (1994).
 - [19] M. A. Ivanov, J. G. Körner, and P. Santorelli, Exclusive semileptonic and nonleptonic decays of the B_c meson, *Phys. Rev. D* **73**, 054024 (2006).
 - [20] R. N. Faustov, V. O. Galkin, and X. W. Kang, Relativistic description of the semileptonic decays of bottom mesons, *Phys. Rev. D* **106**, 013004 (2022).
 - [21] H. Y. Cheng, C. K. Chua, and C. W. Hwang, Covariant light-front approach for s -wave and p -wave mesons: Application to decay constants and form factors, *Phys. Rev. D* **69**, 074025 (2004).
 - [22] W. Wang, Y. L. Shen, and C. D. Lü, Covariant light-front approach for B_c transition form factors, *Phys. Rev. D* **79**, 054012 (2009).
 - [23] X. X. Wang, W. Wang, and C. D. Lü, B_c to P -wave charmonia transitions in covariant light-front approach, *Phys. Rev. D* **79**, 114018 (2009).
 - [24] W. Wang, Y. L. Shen, and C. D. Lü, Study of $B_c^- \rightarrow X(3872) \pi^- (K^-)$ decays in covariant light-front approach, *Eur. Phys. J. C* **51**, 841 (2007).
 - [25] A. Abd El-Hady, J. H. Muñoz, and J. P. Vary, Semileptonic and nonleptonic B_c decays, *Phys. Rev. D* **62**, 014019 (2000).

- [26] P. Colangelo and F. De Fazio, Using heavy quark spin symmetry in semileptonic B_c decays, *Phys. Rev. D* **61**, 034012 (2000).
- [27] D. Ebert, R. N. Faustov, and V. O. Galkin, Weak decays of the B_c meson to charmonium and D mesons in the relativistic quark model, *Phys. Rev. D* **68**, 094020 (2003).
- [28] H. f. Fu, Y. Jiang, C. S. Kim, and G.-L. Wang, Probing nonleptonic two-body decays of B_c meson, *J. High Energy Phys.* **06** (2011) 015.
- [29] S. Naimuddin, S. Kar, M. Priyadarsini, N. Barik, and P. C. Dash, Nonleptonic two-body B_c -meson decays, *Phys. Rev. D* **86**, 094028 (2012).
- [30] S. Kar, P. C. Dash, M. Priyadarsini, Sk. Naimuddin, and N. Barik, Nonleptonic $B_c \rightarrow VV$ decays, *Phys. Rev. D* **88**, 094014 (2013).
- [31] C. F. Qiao, P. Sun, D. Yang, and R. L. Zhu, B_c exclusive decays to charmonium and a light meson at next-to-leading order accuracy, *Phys. Rev. D* **89**, 034008 (2014).
- [32] H. W. Ke, T. Liu, and X. Q. Li, Transitions of $B_c \rightarrow \psi(1S, 2S)$ and the modified harmonic oscillator wave function in the light front quark model, *Phys. Rev. D* **89**, 017501 (2014).
- [33] D. Ebert, R. N. Faustov, and V. O. Galkin, Semileptonic and nonleptonic decays of B_c mesons to orbitally excited heavy mesons in the relativistic quark model, *Phys. Rev. D* **82**, 034019 (2010).
- [34] E. Hernández, J. Nieves, and J. M. Verde-Velasco, Study of exclusive semileptonic and nonleptonic decays of B_c^- in a nonrelativistic quark model, *Phys. Rev. D* **74**, 074008 (2006).
- [35] M. A. Ivanov, J. G. Körner, and P. Santorelli, Semileptonic decays of B_c mesons into charmonium states in a relativistic quark model, *Phys. Rev. D* **71**, 094006 (2005).
- [36] Y. M. Wang and C. D. Lü, Weak productions of new charmonium in semileptonic decays of B_c , *Phys. Rev. D* **77**, 054003 (2008).
- [37] C. H. Chang, Y. Q. Chen, G. L. Wang, and H. S. Zong, Decays of the meson B_c to a P-wave charmonium state χ_c or h_c , *Phys. Rev. D* **65**, 014017 (2001).
- [38] V. V. Kiselev, O. N. Pakhomova, and V. A. Saleev, Two particle decays of B_c meson into charmonium states, *J. Phys. G* **28**, 595 (2002).
- [39] Z. h. Wang, G. L. Wang, and C. H. Chang, The B_c decays to a P-wave charmonium by the improved Bethe–Salpeter approach, *J. Phys. G* **39**, 015009 (2011).
- [40] R. Dhir and C. S. Kim, Branching ratios of B_c meson decaying to vector and axial-vector mesons, *Phys. Rev. D* **87**, 034004 (2013).
- [41] R. Zhu, Relativistic corrections to the form factors of B_c into P-wave orbitally excited charmonium, *Nucl. Phys.* **B931**, 359 (2018).
- [42] Z. Rui and Z. T. Zou, S-wave ground state charmonium decays of B_c mesons in the perturbative QCD approach, *Phys. Rev. D* **90**, 114030 (2014).
- [43] Z. Rui, W. F. Wang, G. X. Wang, L. H. Song, and C. D. Lü, The $B_c \rightarrow \psi(2S)\pi, \eta_c(2S)\pi$ decays in the perturbative QCD approach, *Eur. Phys. J. C* **75**, 293 (2015).
- [44] C. F. Qiao and R. L. Zhu, Estimation of semileptonic decays of B_c meson to S-wave charmonia with nonrelativistic QCD, *Phys. Rev. D* **87**, 014009 (2013).
- [45] H. Y. Cheng and K. C. Yang, Branching ratios and polarization in $B \rightarrow VV, VA, AA$ decays, *Phys. Rev. D* **78**, 094001 (2008); K. C. Yang, Properties of axial-vector mesons and charmless b decays: $B \rightarrow VV, VA, AA$, *Nucl. Phys. B, Proc. Suppl.* **186**, 399 (2009).
- [46] H. Y. Cheng and K. C. Yang, Hadronic charmless B decays $B \rightarrow AP$, *Phys. Rev. D* **76**, 114020 (2007).
- [47] Z. T. Zou, X. Yu, and C. D. Lü, $B_c \rightarrow D^{(*)}T$ decays in perturbative QCD approach, *Phys. Rev. D* **87**, 074027 (2013).
- [48] H. Y. Cheng, Y. Koike, and K. C. Yang, Two-parton light-cone distribution amplitudes of tensor mesons, *Phys. Rev. D* **82**, 054019 (2010).
- [49] F. Najafi and H. Mehraban, Analysis of $B_c^+ \rightarrow J/\psi a_1(1260)^+$ in a perturbative QCD approach, *Prog. Theor. Exp. Phys.* **2015**, 033B09 (2015).
- [50] S. Navas *et al.* (Particle Data Group), Review of particle physics, *Phys. Rev. D* **110**, 030001 (2024).
- [51] X. Liu, B_c -meson decays into J/ψ plus a light meson in the improved perturbative QCD formalism, *Phys. Rev. D* **108**, 096006 (2023).
- [52] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Weak decays beyond leading logarithms, *Rev. Mod. Phys.* **68**, 1125 (1996).
- [53] H. Y. Cheng and K. C. Yang, Charmless hadronic B decays into a tensor meson, *Phys. Rev. D* **83**, 034001 (2011).
- [54] W. Wang, B to tensor meson form factors in the perturbative QCD approach, *Phys. Rev. D* **83**, 014008 (2011).
- [55] Z. Rui, Z. T. Zou, and C. D. Lü, Two-body $B_c \rightarrow D_{(s)}^{(*)}P, D_{(s)}^{(*)}V$ decays in the perturbative QCD approach, *Phys. Rev. D* **86**, 074008 (2012).
- [56] Y. Li, C. D. Lü, and C. F. Qiao, Rare decays of $B \rightarrow J/\psi D^{(*)}$ and $B \rightarrow \eta_c D^{(*)}$ in the perturbative QCD approach, *Phys. Rev. D* **73**, 094006 (2006).
- [57] X. Liu and Z. J. Xiao, $B \rightarrow a_1(1260)a_1(1260)$ and $b_1(1235)b_1(1235)$ decays in the perturbative QCD approach, *Phys. Rev. D* **86**, 074016 (2012).
- [58] K. C. Yang, Light-cone distribution amplitudes of axial-vector mesons, *Nucl. Phys.* **B776**, 187 (2007).
- [59] J. F. Cheng, D. S. Du, and C. D. Lü, Study of $B_c \rightarrow D\pi$ in the perturbative QCD approach, *Eur. Phys. J. C* **45**, 711 (2006).