

Gravitational form factors of the nucleon in the Skyrme model based on scale-invariant chiral perturbation theory

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We investigate the role of the QCD scale anomaly in the gravitational form factors of the nucleon—particularly the $D(t)$ form factor—as well as the associated stress distribution and internal forces, using a Skyrme model based on the scale-invariant chiral perturbation theory. A distinctive feature of this model is the inclusion of both the pion and the scalar meson, which respectively capture the effects of the current quark mass and gluonic quantum contributions to the scale anomaly. By varying the mass of the scalar meson, we evaluate the sensitivity of the gluonic scale anomaly to the nucleon properties. We find that the gluonic scale anomaly plays a crucial role in satisfying the stability conditions of the nucleon and provides an internal confining force. Moreover, we also evaluate the momentum-transfer dependence of $D(t)$, which closely reproduces the lattice QCD results. With an appropriate choice of the anomalous dimension associated with the quark mass, its forward-limit value (i.e., the D -term) also matches the lattice data well.

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I. INTRODUCTION

The mechanisms that confine quarks and gluons within hadrons still remain one of the most fundamental problems in QCD. A promising route to understanding confinement lies in exploring QCD's nonperturbative phenomena—most notably the quark-antiquark (chiral) condensate, the gluon condensate, and the various patterns of symmetry breaking. Recently, the internal stress distribution within hadrons has attracted growing interest for investigating the connection between confinement and nonperturbative QCD phenomena [1–3].

The internal stress distributions—specifically, the pressure profile in 2018 [4] and the shear force distribution in 2021 [5,6]—was extracted from experiment for the first time. Furthermore, the upcoming Electron-Ion Collider (EIC) experiment is highly anticipated to provide more detailed measurements. Motivated by these experimental developments, theoretical investigations have rapidly

progressed [7–74]. The stress distribution can be obtained from the form factors that characterize the matrix elements of the hadronic energy momentum tensor (EMT), known as the gravitational form factors (GFFs) (for reviews, see Refs. [75,76]). These distributions would be connected to the forces that confine quarks and gluons within a hadron, which is expected to be closely related to the QCD confinement mechanism. Therefore, investigating the nonperturbative QCD properties connected to the internal stress distributions would be valuable for gaining a new insight into the QCD confinement mechanism.

From this perspective, a notable nonperturbative QCD quantity would be the scale anomaly, which is represented by the trace part of the EMT. While QCD is classically scale invariant in the chiral limit, this symmetry is broken by quantum effects and then becomes anomalous. This scale anomaly is actually reflected in the definite contribution to the finite nucleon mass. Indeed, in Ref. [77], Ji demonstrated that the scale anomaly accounts for a non-negligible contribution to the nucleon mass. Given this fact that the nucleon mass is associated with the time component of the EMT, its other components are also expected to be affected by the scale anomaly. However, the influence of the scale anomaly on spatial components such as the pressure and the shear force has not been intensively investigated.

To elucidate the role of the scale anomaly in the stress distribution from the low-energy perspective, we employed

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the Skyrme model based on scale-invariant chiral perturbation theory (sChPT) [78–101] in our earlier work [2]. In this model framework, a scalar meson field is incorporated into chiral perturbation theory (ChPT) to effectively describe the scale anomaly, while the Skyrme term is introduced to stabilize the soliton solution, which is identified with the nucleon (called a skyrmion [102,103]). In more detail, the scalar meson couples to the dilatation current J_D^μ to account for the anomalous contributions arising from the current quark mass and gluonic quantum corrections, enabling a systematic treatment of the scale anomaly via the partially conserved dilatation current (PCDC) relation. Within the sChPT framework, the scale anomaly is characterized by the pion mass, the dilaton mass, their decay constants, and the anomalous dimension parameter. By using the skyrmion approach based on sChPT, our earlier work demonstrated that the gluonic scale anomaly gives the dominant contribution to the negative pressure inside the nucleon and plays a leading role in the D -term, defined as a forward limit of the GFF $D(t)$.

As an extension of our earlier work [2], in this paper, we develop the analysis by further investigating the following details:

- (a) By varying the values of the meson mass, the decay constant, and the anomalous dimension parameter, we control the strength of the scale anomaly and investigate its correlation with the stress distribution inside the nucleon. Such insights to be gained in this study are particularly important for understanding how the pressure distribution may change under external conditions, where the strength of the scale anomaly is expected to vary, such as finite temperature or finite density.
- (b) We investigate the mechanical stability of the nucleon by conducting a more detailed analysis of its internal stress distribution, the D -term, and the balance of internal forces. Through this analysis, we also explore the connection between the (negative) pressure and the stability condition. We find that the gluonic scale anomaly makes a negative contribution to the internal pressure in the tangential direction, which is essential for satisfying the stability condition in terms of the D -term. Furthermore, we analyze how the gluonic scale anomaly is related to the internal force inside the nucleon.
- (c) Building on our previous result for the D -term, we show the momentum-transfer dependence of the $D(t)$ form factor in our model and compare it with recent lattice QCD results, demonstrating that the Skyrme model reproduces the momentum-transfer dependence of $D(t)$ observed in the lattice QCD simulations. This reproduction not only supports the quantitative reliability of our analysis but also provides a robust estimate of the D -term, whose precise value has yet to be firmly established.

The remainder of this paper is organized as follows: Section II provides a review of the GFFs for spin-1/2 hadrons, as well as their relation to the internal energy density and stress distributions. We also introduce the method of decomposing these distributions based on the scale anomaly, laying the groundwork for evaluating the anomalous contributions. In addition, we present the nucleon's stability conditions and define the internal force. In Sec. III, we present the Skyrme model based on sChPT, which enables the effective incorporation of the scale anomaly via the PCDC relation. Section IV presents our numerical results for the energy density and the stress distribution, along with a verification of the stability conditions. We also discuss the contribution of the scale anomaly to the internal force. Furthermore, we show the $D(t)$ form factor, which reproduces the recent lattice observations. Finally, Sec. V summarizes our main results and discusses their broader implications.

II. GRAVITATIONAL FORM FACTORS FOR SPIN-1/2 PARTICLES

In this section, we review the definition of the GFFs of the nucleon—a spin-1/2 particle—and present the corresponding expressions for the energy density and stress distributions. To elucidate the role of the scale anomaly, we decompose the EMT into its trace and traceless parts. This general decomposition is founded on Ji's [77] mass decomposition, which is formulated in a model-independent manner. In addition, we introduce the stability conditions of the nucleon derived from the energy density and stress distributions, and we define the internal force inside the nucleon based on the static EMT.

A. Matrix elements of the energy-momentum tensor

The matrix elements of the EMT for spin-1/2 particles, under Lorentz, CP, and CPT symmetries, can be generally expressed as

$$\begin{aligned} \langle p', s' | \Theta_{\mu\nu}(x) | p, s \rangle \\ = \bar{u}' \left[A(t) \frac{P_\mu P_\nu}{M_N} + J(t) \frac{i(P_\mu \sigma_{\nu\rho} + P_\nu \sigma_{\mu\rho}) \Delta^\rho}{2M_N} \right. \\ \left. + D(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu} \Delta^2}{4M_N} \right] u e^{i(p'-p)x}, \end{aligned} \quad (1)$$

where $\sigma_{\mu\nu}$ is defined as $\sigma_{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$ with the Dirac matrix γ^μ ; the momenta P^μ and Δ^μ are defined by $P^\mu = (p^\mu + p'^\mu)/2$ and $\Delta^\mu = p'^\mu - p^\mu$, and $t = \Delta^2$; M_N is a nucleon mass; $s, s' = \pm 1/2$ represents the spin state of a hadron; and $u(p, s)$ is the Dirac spinor. Here, we employ the Minkowski metric $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ and normalization condition as $\bar{u}(p') u(p) = 2E$, with the energy $E = \sqrt{M_N^2 + \vec{\Delta}^2}/4$ and $\langle p' | p \rangle = 2p^0 (2\pi)^3 \delta^{(3)}(\vec{p}' - \vec{p})$.

In the forward limit, $t = 0$, the GFFs take the following values:

$$A(0) = 1, \quad J(0) = \frac{1}{2}, \quad (2)$$

where these constraints reflect the Poincaré symmetry. We emphasize that, in contrast, the D -term, defined as $D = D(0)$, is subject to no general constraints. This difference arises from the fact that mass and angular momentum correspond to the conserved charges (generators) of space-time translations and rotations in the Poincaré group, whereas the D -term, encoding internal stress distributions, does not.

B. Energy density and stress distributions

To investigate the spatial structure of the EMT following the physical interpretation proposed in Ref. [104], we adopt the Breit frame, where the time component of the momentum transfer vanishes ($\Delta^0 = 0$). In this frame, the average momentum and momentum transfer are given by $P^\mu = (E, 0, 0, 0)$ and $\Delta^\mu = (0, \vec{\Delta})$, respectively, with $t = -|\vec{\Delta}|^2$. We define the static EMT as the Fourier transform of the matrix elements of the EMT with respect to the spatial momentum transfer $\vec{\Delta}$:

$$\Theta_{\mu\nu}^{\text{static}}(\vec{r}, \vec{s}) = \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{r}\cdot\vec{\Delta}} \frac{\langle p', s' | \Theta_{\mu\nu}(x=0) | p, s \rangle}{\bar{u}(p')u(p)}, \quad (3)$$

where, in the Breit frame, the matrix element is written as follows:

$$\begin{aligned} & \langle p', s' | \Theta_{00}(0) | p, s \rangle \\ &= 2M_N E \left[A(t) - \frac{t}{4M_N^2} (A(t) - 2J(t) + D(t)) \right] \delta_{ss'}, \end{aligned} \quad (4)$$

$$\langle p', s' | \Theta_{0i}(0) | p, s \rangle = 2M_N E \left[J(t) \frac{(-i\vec{\Delta} \times \vec{\sigma}_{s's})_i}{2M_N} \right], \quad (5)$$

$$\langle p', s' | \Theta_{ij}(0) | p, s \rangle = 2M_N E \left[D(t) \frac{\Delta_i \Delta_j - \delta_{ij} \vec{\Delta}^2}{4M_N^2} \right] \delta_{ss'}, \quad (6)$$

where $\vec{\sigma}_{s's} = \chi_{s'}^\dagger \vec{\sigma} \chi_s$ with the nucleon Pauli spinors χ_s normalized as $\chi_{s'}^\dagger \chi_s = \delta_{ss'}$ and the Pauli matrix $\vec{\sigma}$.

In the following, we restrict the analysis to the case $s = s'$ and neglect spin distributions while focusing on the stress distributions within the nucleon. For a spherically symmetric system, the energy density $\epsilon(r)$, pressure $p(r)$, and shear force $s(r)$ are defined in terms of the static EMT as follows:

$$\epsilon(r) = \Theta_{00}^{\text{static}}(r), \quad (7)$$

$$p(r) = \frac{1}{3} \delta^{ij} \Theta_{ij}^{\text{static}}(r), \quad (8)$$

$$s(r) = \frac{3}{2} \left(\frac{r^i r^j}{r^2} - \frac{1}{3} \delta^{ij} \right) \Theta_{ij}^{\text{static}}(r). \quad (9)$$

From Eq. (6), the GFF $D(t)$, which encodes information about the internal forces and mechanical properties of the nucleon, can be expressed as

$$D(t) = -6M_N \int d^3x \left(r^i r^j - \frac{1}{3} \delta^{ij} r^2 \right) \frac{j_2(\Delta r)}{(\Delta r)^2} \Theta_{ij}^{\text{static}}(r), \quad (10)$$

where $j_2(\Delta r)$ is the spherical Bessel function of order 2, and $\Delta = |\vec{\Delta}|$ denotes the magnitude of the three-momentum transfer. The nucleon mass and the D -term can also be related to the energy density and the pressure or shear force distributions as follows:

$$M_N = \int d^3x \epsilon(r), \quad (11)$$

$$D = M_N \int d^3x r^2 p(r) = -\frac{4}{15} M_N \int d^3x r^2 s(r). \quad (12)$$

So far, the stress tensor has been expressed in terms of a three-dimensional orthogonal Cartesian coordinate. However, assuming a spherical nucleon shape may be not well suited to this coordinate. Hence, one can rewrite the stress tensor to better reflect spherical symmetry. By diagonalizing the stress tensor along its eigenvector directions, all off-diagonal (shear) components become zero, leaving only the eigenvalues as the principal (normal) stresses. This diagonalization corresponds to expressing the spatial components of EMT in the spherical coordinate system as $\Theta_{\alpha\beta} = \text{diag}(p_r, p_\theta, p_\phi)$, with $\alpha, \beta = r, \phi, \theta$. Note again that p_r , p_θ , and p_ϕ are the eigenvalues of Θ , and their corresponding eigenvectors are denoted by $\vec{e}_r$, $\vec{e}_\theta$, and $\vec{e}_\phi$, respectively. In the case where the nucleon has a spherical shape, we have $p_\theta = p_\phi$. Accordingly, the pressures in the radial and tangential directions are given by

$$p_r(r) = \frac{2}{3} s(r) + p(r), \quad (13)$$

$$p_{\theta,\phi}(r) = -\frac{1}{3} s(r) + p(r). \quad (14)$$

Based on these expressions in Eqs. (13) and (14), we will illustrate the spatial distribution of the pressures projected onto the transverse plane of the nucleon in Sec. IV C.

C. The decomposition based on the scale anomaly

The trace part of the EMT encodes the scale anomaly. At the classical level, QCD is scale-invariant in the chiral limit. However, once the current quark mass and the quantum

correction are taken into account, the conservation law of the dilatation current becomes anomalous [105,106]:

$$\Theta^\mu{}_\mu = \partial_\mu J_D^\mu = (1 + \gamma_m) \sum_f m_f \bar{q}_f q_f + \frac{\beta(g_s)}{2g_s} \text{Tr}(G_{\mu\nu} G^{\mu\nu}), \quad (15)$$

where J_D^μ is the dilatation current, γ_m the quark-mass anomalous dimension, $\beta(g_s)$ the QCD beta function, $G_{\mu\nu}$ the $SU(N_c)$ field strength, and m_f and q_f ($\bar{q}_f$) are the current quark masses and quark (antiquark) fields with flavor index f .

In order to express the scale anomaly explicitly in energy density and pressure distributions, we decompose the EMT into trace and the traceless parts:

$$\Theta_{\mu\nu}^{\text{static}}(\vec{r}) = \bar{\Theta}_{\mu\nu}^{\text{static}} + \hat{\Theta}_{\mu\nu}^{\text{static}}, \quad (16)$$

$$\bar{\Theta}_{\mu\nu}^{\text{static}}(\vec{r}) = \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{r}\cdot\vec{\Delta}} \frac{\langle p' | \bar{\Theta}_{\mu\nu}(0) | p \rangle}{\bar{u}(p') u(p)}, \quad (17)$$

$$\hat{\Theta}_{\mu\nu}^{\text{static}}(\vec{r}) = \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{r}\cdot\vec{\Delta}} \frac{\langle p' | \hat{\Theta}_{\mu\nu}(0) | p \rangle}{\bar{u}(p') u(p)}. \quad (18)$$

Here, $\bar{\Theta}_{\mu\nu} \equiv \Theta_{\mu\nu} - \frac{1}{4} g_{\mu\nu} \Theta^\rho{}_\rho$ represents the traceless part of the EMT, which can be interpreted as the dynamical energy of quarks and gluons, while $\hat{\Theta}_{\mu\nu} \equiv \frac{1}{4} g_{\mu\nu} \Theta^\rho{}_\rho$ represents the trace part, which is interpreted as the energy arising from the scale anomaly. In accordance with the definition in Eqs. (7)–(9), we similarly decompose the energy density $\epsilon = \bar{\epsilon} + \hat{\epsilon}$, pressure $p = \bar{p} + \hat{p}$, and shear force $s = \bar{s}$:

$$\begin{aligned} \bar{\epsilon}(r) &= \bar{\Theta}_{00}^{\text{static}}(r), & \hat{\epsilon}(r) &= \hat{\Theta}_{00}^{\text{static}}(r), \\ \bar{p}(r) &= \frac{1}{3} \delta^{ij} \bar{\Theta}_{ij}^{\text{static}}(r), & \hat{p}(r) &= \frac{1}{3} \delta^{ij} \hat{\Theta}_{ij}^{\text{static}}(r), \\ \bar{s}(r) &= \frac{3}{2} \left(\frac{r^i r^j}{r^2} - \frac{1}{3} \delta^{ij} \right) \bar{\Theta}_{ij}^{\text{static}}(r), \end{aligned} \quad (19)$$

where the trace part of the shear force is absent due to the tensorial structure of the EMT.

D. Model-independent relations associated with the nucleon's stability

In this subsection, we present model-independent relations associated with the nucleon's stability, which are expressed through the energy density, the stress distribution, and their decomposed components.

The static EMT conservation law, $\partial^i \Theta_{ij}^{\text{static}} = 0$, yields the differential equation

$$\frac{2}{3} s'(r) + \frac{2}{r} s(r) + p'(r) = 0, \quad (20)$$

where the prime denotes a derivative with respect to r . This relation connects the pressure $p(r)$ and shear force $s(r)$, demonstrating that they are not independent. Consequently, although $s(r)$ does not directly appear in the diagonal part of the EMT, it is related to $p(r)$ through this differential equation, implying that the shear force is indirectly connected to the scale anomaly.

Moreover, the static EMT conservation also implies the von Laue condition [107]:

$$\int_0^\infty dr r^2 p(r) = 0. \quad (21)$$

This implies that, to ensure mechanical stability, the pressure must exhibit both positive and negative regions in its radial dependence, and their contributions must cancel out upon integration over space. In addition to the information on the pressure, the following constraint on its local behavior is required in any region inside the nucleon (see Ref. [108]):

$$p_r(r) = \frac{2}{3} s(r) + p(r) > 0. \quad (22)$$

This local constraint also ensures that the nucleon does not collapse. Using this constraint together with Eqs. (12) and (22), it follows that the D -term takes the negative value,

$$D < 0. \quad (23)$$

The negativity of the D -term reflects the nucleon's mechanical stability.

Next, we present the relations obtained from the decomposed components in Eq. (19). Considering the spatial integral of the anomalous part $\hat{\epsilon}(r) = \Theta_{\mu\mu}^{\text{static}}/4$, one can obtain the anomalous mass $\hat{M}_N$ originating from the scale anomaly as

$$\hat{M}_N = 4\pi \int_0^\infty dr r^2 \hat{\epsilon}(r) = \frac{1}{4} M_N. \quad (24)$$

This result implies that the scale anomaly contributes 25% of the nucleon mass. The remaining 75% originates from the dynamical part,

$$\bar{M}_N = 4\pi \int_0^\infty dr r^2 \bar{\epsilon}(r) = \frac{3}{4} M_N. \quad (25)$$

Furthermore, using $\Theta_{\mu\mu}^{\text{static}} = \hat{\epsilon}(r) - 3\hat{p}(r)$, the spatial integral of the anomalous part $\hat{p}(r)$ reads [34,109]

$$4\pi \int_0^\infty dr r^2 \hat{p}(r) = -\frac{1}{4} M_N. \quad (26)$$

Applying the von Laue condition to the dynamical part $\bar{p}(r)$ leads to

$$4\pi \int_0^\infty dr r^2 \bar{p}(r) = \frac{1}{4} M_N. \quad (27)$$

E. Internal force density inside the nucleon

In Ref. [4], the negative pressure inside the nucleon, defined in three-dimensional Cartesian coordinates, was referred to as confining pressure, yet it is unclear whether the negative pressure can definitively be regarded as the confining pressure that binds the quarks and gluons inside the nucleon. A natural approach to addressing this ambiguity would be to define a force density based on the static EMT:

$$\mathcal{F}^j(\vec{r}) \equiv \partial^i \Theta_{ij}^{\text{static}}(\vec{r}). \quad (28)$$

For a spherically symmetric nucleon, this force density is expressed as follows:

$$\begin{aligned} \vec{\mathcal{F}}(r) &= \mathcal{F}^r(r) \vec{e}_r, \\ \mathcal{F}^r(r) &= p'_r(r) + \frac{2}{r}(p_r(r) - p_{\theta,\phi}(r)). \end{aligned} \quad (29)$$

Although the force density is directed along the radial direction, it consists of the contributions from both the radial and tangential pressures. Furthermore, this radial force can be decomposed into the dynamical and anomalous parts, which must cancel each other out, as required by the static EMT conservation law,

$$\mathcal{F}^r(r) = \tilde{\mathcal{F}}^r(r) + \hat{\mathcal{F}}^r(r) = 0. \quad (30)$$

This indicates the force balance inside the nucleon, which is equivalent to Eq. (20) and represents one of the stability conditions.

III. SKYRME MODEL BASED ON THE SCALE-INVARIANT CHIRAL PERTURBATION THEORY

In this section, we present the analytical expressions for the nucleon's internal distributions—energy density, pressure, and shear force—by following the definitions given in Sec. II and using the Skyrme model based on sChPT. In this model framework, the lightest scalar meson field is incorporated through the low-energy theorem associated with the dilatation current, which serves as a central ingredient in decomposing the nucleon's internal distributions in terms of the scale anomaly.

A. Model Lagrangian and PCDC relation

To reflect the scale anomaly in the Skyrme model, we incorporate the scalar field into the ChPT Lagrangian through the low-energy theorem. This theorem, known as the PCDC relation, connects the divergence of the dilatation current to the overlap amplitude between the broken current associated with the scale symmetry and the lightest isosinglet scalar meson ϕ :

$$\langle 0 | \partial_\mu J_D^\mu(x) | \phi(p) \rangle = -f_\phi m_\phi^2 e^{-ip \cdot x}, \quad (31)$$

where f_ϕ and m_ϕ are the decay constant and mass of the scalar meson. An effective model that satisfies this PCDC relation is given by the following Lagrangian [78–101]:

$$\begin{aligned} \mathcal{L} &= \frac{f_\pi^2}{4} \left(\frac{\chi}{f_\phi} \right)^2 \text{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2} \text{Tr}([U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]) \\ &\quad + \frac{f_\pi^2 m_\pi^2}{4} \left(\frac{\chi}{f_\phi} \right)^{3-\gamma_m} \text{Tr}(U + U^\dagger) \\ &\quad + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{1}{4} m_{\phi 0}^2 f_\phi^2 \left(\frac{\chi}{f_\phi} \right)^4 \left[\ln \left(\frac{\chi}{f_\phi} \right) - \frac{1}{4} \right], \end{aligned} \quad (32)$$

where f_π and m_π are the pion decay constant and mass, respectively; e is the Skyrme parameter; γ_m represents the anomalous dimension parameter within the effective model approach; and $m_{\phi 0}$ corresponds to the scalar meson mass in the chiral limit. The Lagrangian in Eq. (32) is formulated as an extension of the ChPT Lagrangian typically used in the conventional Skyrme model. The fundamental building block of ChPT is the $SU(2)$ chiral field in the nonlinear representation, defined as $U = \exp(i\vec{\pi} \cdot \vec{\tau}/f_\pi)$, where $\vec{\pi}$ denotes the pion fields and $\vec{\tau}$ are the Pauli matrices in the isospin space. The chiral field is scale invariant and carries the scale dimension of zero. The scalar field χ , introduced as the extension, is the conformal compensator and is nonlinearly parametrized as $\chi = f_\phi e^{\phi/f_\phi}$. This χ field has the scale dimension of 1. By considering the scale transformation based on the corresponding scale dimensions, one can easily find the scale anomaly in the effective model,

$$\begin{aligned} \partial_\mu J_D^\mu &= -(1 + \gamma_m) \frac{m_\pi^2 f_\pi^2}{4} \left(\frac{\chi}{f_\phi} \right)^{3-\gamma_m} \text{tr}[U + U^\dagger] \\ &\quad - \frac{m_{\phi 0}^2 f_\phi^2}{4} \left(\frac{\chi}{f_\phi} \right)^4. \end{aligned} \quad (33)$$

This result clearly satisfies the PCDC relation in Eq. (31).

To gain further insight into the effective model result for the scale anomaly, we consider taking the vacuum expectation value of the relevant operators. Suppose that the vacuum expectation values of U and χ are, respectively, given by

$$\langle 0 | U | 0 \rangle = 1, \quad \langle 0 | \chi | 0 \rangle = f_\phi. \quad (34)$$

From this, the vacuum expectation value of the scale anomaly is evaluated as

$$\langle 0 | \partial_\mu J_D^\mu(x) | 0 \rangle = -(1 + \gamma_m) f_\pi^2 m_\pi^2 - \frac{f_\phi^2 m_{\phi 0}^2}{4}. \quad (35)$$

Since the pion mass term is interpreted as representing the contribution from the current quark masses of QCD via the Gell-Mann-Oakes-Renner relation, $f_\pi^2 m_\pi^2 = -m_f \langle 0 | \bar{q}q | 0 \rangle$,

we find that the first term in Eq. (35) corresponds to the quark mass contribution to the scale anomaly in Eq. (15). In contrast, the remaining second term in Eq. (35) can be interpreted as corresponding to the gluonic contribution in Eq. (15). In other words, the current quark mass part of the scale anomaly is described by the pion mass and its decay constant, while the gluonic anomalous part is mimicked by the scalar meson mass and its decay constant. Hence, within the effective model, we can control the strength of each scale-anomalous part by varying the values of meson masses and decay constants. We emphasize that this separate treatment of these anomalous contributions could not be achieved within the framework of the conventional Skyrme model.

The contribution from each component of the scale anomaly is also reflected in the scalar meson mass. By taking into account the matching between the effective model and QCD considered above, the scalar meson mass derived from the effective model can be expressed as

$$\begin{aligned} m_\phi^2 &= m_{\phi 0}^2 - (3 - \gamma_m)^2 \frac{f_\pi^2 m_\pi^2}{f_\phi^2} \\ &= -\frac{4}{f_\phi^2} \langle 0 | \frac{\beta(g_s)}{2g_s} \text{Tr}(G_{\mu\nu} G^{\mu\nu}) | 0 \rangle \\ &\quad + (3 - \gamma_m)^2 \frac{m_f \langle 0 | \bar{q}q | 0 \rangle}{f_\phi^2}. \end{aligned} \quad (36)$$

This shows that, in the chiral limit, the scalar meson mass is governed by the gluonic scale anomaly. When including the current quark mass, the quark condensate contributes negatively to the scalar meson mass, resulting in its reduction.

B. Classical solutions

To describe the nucleon based on the sChPT, we employ the skyrmion approach [102,103], in which the baryon is represented as a soliton solution known as the skyrmion. This skyrmion has a finite size and carries a nonzero baryon number. Hence, it can be identified as the nucleon with the baryon number 1.

To obtain the soliton solution, we impose the hedgehog ansatz for the chiral field U and the spherical symmetric configuration for the conformal compensator χ as

$$U(\vec{x}) = \exp(i\vec{\tau} \cdot \vec{x}F(r)), \quad (37)$$

$$\chi(\vec{x}) = f_\phi C(r), \quad (38)$$

where $F(r)$ and $C(r)$ are dimensionless functions. Substituting the ansatz into the sChPT Lagrangian and performing the spatial integration, we obtain the static energy functional as follows:

$$\begin{aligned} E[F, C] &= 4\pi \int_0^\infty dr r^2 \left[\frac{f_\pi^2}{2} C^2 \left(F'^2 + 2 \frac{\sin^2 F}{r^2} \right) \right. \\ &\quad + \frac{\sin^2 F}{2e^2 r^2} \left(\frac{\sin^2 F}{r^2} + 2F'^2 \right) - f_\pi^2 m_\pi^2 C^{3-\gamma_m} \cos F \\ &\quad \left. + \frac{f_\phi^2}{2} \left\{ C'^2 + \frac{m_{\phi 0}^2}{2} \left(C^4 \left(\ln C - \frac{1}{4} \right) \right) \right\} - \epsilon^{\text{sub}} \right], \end{aligned} \quad (39)$$

where ϵ^{sub} is introduced to subtract constant contributions from the energy density. We see that setting $C(r) = 1$, the static energy E reduces to the conventional skyrmion energy.

By minimizing the energy functional $E[F, C]$, the equations of motion (EOMs) for $F(r)$ and $C(r)$ are given by

$$\begin{aligned} (r^2 C^2 + 2\sin^2 F) F'' + 2(r C^2 + r^2 C C') F' \\ + 2F'^2 \sin F \cos F - 2C^2 \sin F \cos F - 2 \frac{\sin^3 F \cos F}{r^2} \\ - \left(\frac{m_\pi}{ef_\pi} \right)^2 r^2 C^{3-\gamma_m} \sin F = 0, \end{aligned} \quad (40)$$

$$\begin{aligned} C'' + \frac{2}{r} C' - \left(\frac{f_\pi}{f_\phi} \right)^2 C \left(F'^2 + 2 \frac{\sin^2 F}{r^2} \right) \\ + (3 - \gamma_m) \left(\frac{m_\pi}{ef_\phi} \right)^2 C^2 \cos F - \left(\frac{m_{\phi 0}}{ef_\pi} \right)^2 C^3 \ln C = 0. \end{aligned} \quad (41)$$

To construct a soliton with baryon number 1, we impose the boundary condition on the profile function $F(r)$ as follows:

$$F(0) = \pi, \quad F(r \rightarrow \infty) = 0. \quad (42)$$

The boundary conditions for $C(r)$ are chosen so that the soliton solution is regular at $r = 0$ and its energy finite:

$$\left. \frac{dC}{dr} \right|_{r=0} = 0, \quad C(r \rightarrow \infty) = C_\infty, \quad (43)$$

where C_∞ is determined by the stationary condition in the vacuum,

$$\begin{aligned} 0 &= \left. \frac{dV(F, C)}{dC} \right|_{r \rightarrow \infty} \\ &= (3 - \gamma_m) f_\pi^2 m_\pi^2 - f_\phi^2 m_{\phi 0}^2 C_\infty^4 \ln C_\infty. \end{aligned} \quad (44)$$

In the following, we briefly provide the practical techniques used in the numerical calculation. To accurately evaluate the GFFs [Eq. (10)] and to numerically satisfy global relations—such as the von Laue condition [Eq. (21)] and other relations [Eqs. (24)–(27)]—it is necessary to integrate over r up to infinity. However, in the numerical

calculations, it is not possible to solve Eqs. (40) and (41) up to spatial infinity, and the computation must be truncated at a finite value of r . In addition, even with truncation, it is still difficult to capture the behavior of the profile functions $F(r)$ and $C(r)$ in the asymptotic region far from $r = 0$. To analyze their asymptotic behaviors, it is useful to employ the Padé-like approximate solutions [110], given as follows:

$$F(r) = \frac{\pi + \sum_{k=1}^n a_k r^k}{1 + \sum_{k=1}^n b_k r^k + b_{n+1} r^{n+1} \exp(m_\pi r)}, \quad (45)$$

$$C(r) = C_\infty - \frac{\sum_{k=0}^n c_k r^k}{1 + \sum_{k=1}^n d_k r^k + d_{n+1} r^{n+1} \exp(m_{\phi 0} r)}, \quad (46)$$

where a_k , b_k , c_k , and d_k are determined by fitting the above ansatz to the numerical solutions for $F(r)$ and $C(r)$. These approximate analytical solutions exactly satisfy the boundary conditions in Eqs. (42) and (43). We find that $n = 3$ is already sufficient to reproduce all global quantities with high accuracy.

C. The static EMT from the Skyrme model

As we have seen in the previous subsection, a baryon is described as a soliton solution of the U and χ fields. These solutions do not carry spin and isospin quantum numbers, which are referred to as the classical solutions, $U_{\text{cl}}(\vec{x})$ and $\chi_{\text{cl}}(\vec{x})$. This may raise concerns about whether the static EMT can be appropriately described using classical solutions. Nevertheless, within the large- N_c limit, the static EMT can still be expressed at the classical level, and the GFF $D(t)$ can also be evaluated accordingly [111,112]. In this subsection, we begin with the quantization procedure for the skyrmion and demonstrate that the static EMT can be reduced to a form that is sufficiently described at the classical level.

To assign spin and isospin to the skyrmion, one performs collective quantization of the classical solitons by introducing translation and rotation:

$$U(t, \vec{x}) = A(t) U_{\text{cl}}[R(B(t))(\vec{x} - \vec{X}(t))] A^\dagger(t), \quad (47)$$

$$\chi(t, \vec{x}) = \chi_{\text{cl}}(R(B(t))(\vec{x} - \vec{X}(t))), \quad (48)$$

where the translational, isospin, and rotational zero mode are parametrized by the vector $\vec{X}$ and the $SU(2)$ matrices $A(t)$ and $B(t)$, respectively, with $R_{ij}(B(t)) = \text{Tr}[\tau^i B \tau^j B^\dagger] / 2 \in SO(3)$. Note that here, t denotes the time component. By treating the collective coordinates as the canonical variables, the quantum numbers are assigned to the skyrmion. Within the Skyrme model, the matrix element of Θ_{ij} is then expressed in the Breit frame as follows:

$$\begin{aligned} & \langle \vec{q}/2 | \Theta_{ij} [U(t, \vec{x}), \chi(t, \vec{x})] | -\vec{q}/2 \rangle \\ &= \exp[-i\vec{q} \cdot \vec{x}] R(B)_{ia}^T R(B)_{jb}^T \int d^3x \exp[i\vec{q} \cdot R(B)^T \vec{x}] \\ & \quad \times \Theta_{ab} [U(t, \vec{x}), \chi(t, \vec{x})]. \end{aligned} \quad (49)$$

Since the contribution of collective rotations generally appears at subleading order in the large- N_c expansion, we approximate the EMT with collective rotations as

$$\Theta_{ij} [U(t, \vec{x}), \chi(t, \vec{x})] \simeq \Theta_{ij}^{\text{cl}} [U_{\text{cl}}(\vec{x}), \chi_{\text{cl}}(\vec{x})]. \quad (50)$$

Under this approximation, the GFF $D(t)$ obtained from the matrix elements in Eq. (49) reduce to the expression

$$D(t) \simeq -6M_N \int d^3x \left(r^i r^j - \frac{1}{3} \delta^{ij} r^2 \right) \frac{j_2(\Delta r)}{(\Delta r)^2} \Theta_{ij}^{\text{cl}}, \quad (51)$$

where the nonrelativistic limit $E \rightarrow M_N$ is taken, and comparing with the general expression for the GFF $D(t)$ in Eq. (10), the static EMT tensor within the skyrmion approach reads

$$\Theta_{ij}^{\text{static}}(r) \simeq \Theta_{ij}^{\text{cl}} [U_{\text{cl}}(\vec{x}), \chi_{\text{cl}}(\vec{x})]. \quad (52)$$

The time component of the static EMT is also expressed as $\Theta_{00}^{\text{static}}(r) \simeq \Theta_{00}^{\text{cl}} [U_{\text{cl}}(\vec{x}), \chi_{\text{cl}}(\vec{x})]$ within the leading-order approximation in the large- N_c expansion.

In general, the EMT is obtained as $T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\mathcal{L}\sqrt{-g})}{\delta g^{\mu\nu}}$. However, this EMT in sChPT does not reproduce the scale anomaly in Eq. (33). To reconcile this discrepancy, an improved term should be added to the EMT, which is given by $\theta_{\mu\nu} = -(\partial_\mu \partial_\nu - g_{\mu\nu} \partial_\rho \partial^\rho) \chi^2 / 6$. Accordingly, the improved EMT in sChPT is evaluated as

$$\Theta_{\mu\nu} = T_{\mu\nu} + \theta_{\mu\nu} = \bar{\Theta}_{\mu\nu} + \hat{\Theta}_{\mu\nu}, \quad (53)$$

$$\begin{aligned} \bar{\Theta}_{\mu\nu} = & \frac{f_\pi^2}{2} \left(\frac{\chi}{f_\phi} \right)^2 \text{Tr}(\partial_\mu U^\dagger \partial_\nu U) + \frac{1}{8e^2} \text{Tr}([U^\dagger \partial_\mu U, U^\dagger \partial_\rho U][U^\dagger \partial_\nu U, U^\dagger \partial^\rho U]) + \frac{2}{3} \partial_\mu \chi \partial_\nu \chi - \frac{1}{3} \chi \partial_\mu \partial_\nu \chi \\ & - \frac{1}{4} g_{\mu\nu} \left\{ \frac{f_\pi^2}{2} \left(\frac{\chi}{f_\phi} \right)^2 \text{Tr}(\partial_\rho U^\dagger \partial^\rho U) + \frac{1}{8e^2} \text{Tr}[U^\dagger \partial_\rho U, U^\dagger \partial_\sigma U]^2 + \frac{2}{3} \partial_\rho \chi \partial^\rho \chi - \frac{1}{3} \chi \partial_\rho \partial^\rho \chi \right\}, \end{aligned} \quad (54)$$

$$\hat{\Theta}_{\mu\nu} = \hat{\Theta}_{\mu\nu}^q + \hat{\Theta}_{\mu\nu}^g, \quad (55)$$

$$\hat{\Theta}_{\mu\nu}^q = -\frac{1}{4}g_{\mu\nu}(1+\gamma_m)\frac{f_\pi^2 m_\pi^2}{4}\left(\frac{\chi}{f_\phi}\right)^{3-\gamma_m}\text{Tr}(U+U^\dagger), \quad (56)$$

$$\hat{\Theta}_{\mu\nu}^g = -\frac{1}{4}g_{\mu\nu}\frac{m_{\phi 0}^2 f_\phi^2}{4}\left(\frac{\chi}{f_\phi}\right)^4. \quad (57)$$

By substituting the classical solutions into the improved EMT, we obtain the static EMT in the skyrmion approach, $\Theta_{\mu\nu}^{\text{cl}}[F(r), C(r)]$. The decomposed components of the energy density and stress distribution are given by

$$\bar{\epsilon}(r) = 3\bar{p}(r) = \frac{f_\pi^2}{4}C^2\left(F'^2 + 2\frac{\sin^2 F}{r^2}\right) + \frac{\sin^2 F}{2e^2 r^2}\left(2F'^2 + \frac{\sin^2 F}{r^2}\right) + \frac{f_\phi^2}{6}C'^2 - \frac{f_\phi^2}{6}\frac{CC'}{r} - \frac{f_\phi^2}{12}CC'', \quad (58)$$

$$\hat{\epsilon}(r) = -\hat{p}(r) = \hat{\epsilon}^q(r) + \hat{\epsilon}^g(r) = -\hat{p}^q(r) - \hat{p}^g(r), \quad (59)$$

$$\hat{\epsilon}^q(r) = -\hat{p}^q(r) = -\frac{1}{4}(1+\gamma_m)f_\pi^2 m_\pi^2 C^{3-\gamma_m} \cos F - \hat{\epsilon}^{q,\text{sub}}, \quad (60)$$

$$\hat{\epsilon}^g(r) = -\hat{p}^g(r) = -\frac{1}{4}\left[\frac{1}{4}m_{\phi 0}^2 f_\phi^2 C^4\right] - \hat{\epsilon}^{g,\text{sub}}, \quad (61)$$

$$\bar{s}(r) = \left(f_\pi^2 C^2 + \frac{\sin^2 F}{e^2 r^2}\right)\left(F'^2 - \frac{\sin^2 F}{r^2}\right) + \frac{f_\phi^2}{3}\left(2C'^2 + \frac{CC'}{r} - CC''\right), \quad (62)$$

where $\hat{\epsilon}^{q(g),\text{sub}}$ denotes the subtraction term introduced to ensure that the EMT density is normalized to vanish at spatial infinity. These decomposed components show that the trace part of $\hat{\epsilon}$ ($\hat{p}$) is further decomposed into the current quark mass part of the scale anomaly $\hat{\epsilon}^q$ ($\hat{p}^q$) and the gluonic anomalous part $\hat{\epsilon}^g$ ($\hat{p}^g$). This further decomposition is thanks to the distinctive features of sChPT, where the scale anomaly can be separated into contributions from the pion mass and the scalar meson mass, as was discussed around Eq. (35).

In the skyrmion approach, the nucleon mass M_N is obtained by integrating the energy density over space, following the general expression in Eq. (11). The decomposed energies obtained in the skyrmion approach clearly indicate that the anomalous mass $\hat{M}_N$ indeed accounts for 25% of the nucleon mass. Interestingly, the anomalous mass consists of two parts: the current quark mass part and the gluonic anomalous part,

$$\begin{aligned} \hat{M}_N &= \hat{M}_N^q + \hat{M}_N^g, \\ \hat{M}_N^{q(g)} &= 4\pi \int_0^\infty dr r^2 \hat{\epsilon}^{q(g)}. \end{aligned} \quad (63)$$

The detailed amounts of the anomalous contributions will be evaluated in the next section.

IV. RESULTS

In this section, we numerically elucidate the influence of the scale anomaly on the energy density, the pressure, the

force density, and the GFF $D(t)$. Since our aim is to evaluate the contribution of the scale anomaly, we fix the pion mass to its physical value and vary the scalar meson mass, considering three different values: $m_{\phi 0} = 550, 720$, and 1000 MeV. (The strength of the gluonic scale anomaly can be controlled by the scalar meson mass.) As for the decay constant of the scalar meson, it is fixed at $f_\phi = 240$ MeV, which provides a reasonable description of the skyrmion properties within sChPT with $m_{\phi 0} = 720$ MeV [85].

Additionally, since the anomalous dimension also affects the strength of the scale anomaly, we also vary its value. In Ref. [89], the value of γ_m in the effective model is discussed to lie within the range $-1 < \gamma_m \leq 2$. Following this range, we consider $\gamma_m = -0.9, 0, 2$.

In the skyrmion approach, the parameters f_π and e are highly sensitive to the nucleon mass and other properties. Therefore, we adopt two parameter sets. One set, $f_\pi = 68$ MeV and $e = 5.45$, is chosen to reproduce the nucleon and Δ masses and obtain a realistic soliton size [103], though the pion decay constant deviates from the physical value. The other set, $f_\pi = 93$ MeV and $e = 4.75$, is fixed to the physical value of the pion decay constant, with the value of e determined by referring to the axial coupling g_A [85,113].

A. Energy density and mass decomposition

In this subsection, we present the spatial distribution of the energy density and its decomposition based on Eqs. (58)–(61).

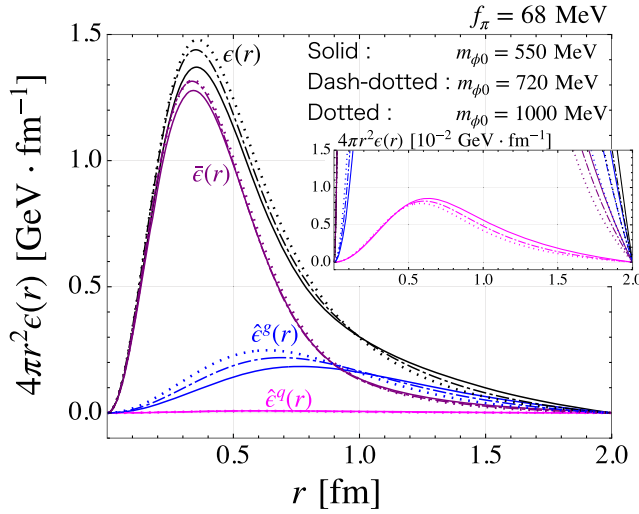


FIG. 1. Spatial distribution of the energy density inside the nucleon for the case of $f_\pi = 68$ MeV and $\gamma_m = 0$, and its decomposition. The dynamical part $\bar{\epsilon}(r)$, the current quark mass part of the scale anomaly $\hat{\epsilon}^q(r)$, and that of the gluonic anomaly part $\hat{\epsilon}^g(r)$ are depicted in purple, magenta, and blue, respectively. Their sum $\epsilon(r)$ is represented by the black line. The solid, dash-dotted, and dotted lines indicate the results for $m_{\phi 0} = 550$ MeV, 720 MeV, and 1000 MeV, respectively. An inset shows a close-up view of $\hat{\epsilon}^q(r)$.

First, we display the result for $f_\pi = 68$ MeV and $\gamma_m = 0$ in Fig. 1. This figure shows that the energy density is dominated by the dynamical part, while the contributions from the scale anomaly are subleading. We also investigate the effect of the scalar meson mass on the energy density, noting that changes in this mass effectively modulate the strength of the gluonic scale anomaly. In the region near the origin ($r = 0$), increasing the scalar meson mass leads to a larger energy density. However, in the asymptotic region far from $r = 0$, a larger scalar meson mass causes the energy density to fall off more rapidly toward zero. This trend stems from the behavior of the dynamical energy density $\bar{\epsilon}$ and the gluonic anomalous part $\hat{\epsilon}^g$. Although the

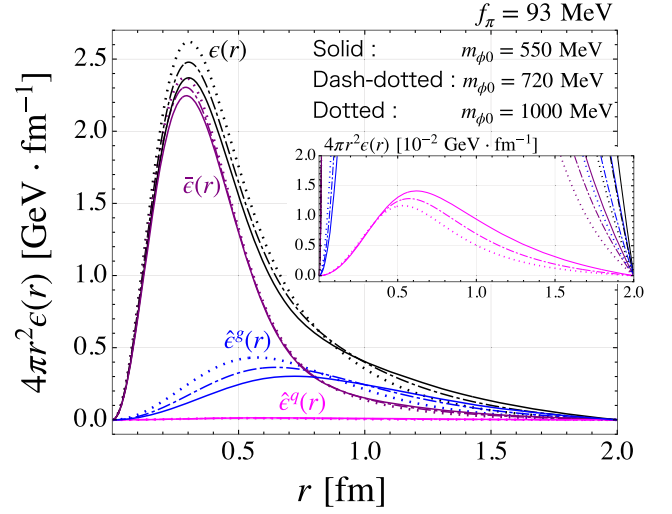


FIG. 2. Same as Fig. 1, but for $f_\pi = 93$ MeV and $\gamma_m = 0$.

current-quark-mass part of the scale anomaly $\hat{\epsilon}^q$ behaves oppositely, its magnitude is negligible.

Using the evaluated energy density, we estimate the nucleon mass. In the current setup with $f_\pi = 68$ MeV and $\gamma_m = 0$, the values of M_N corresponding to each scalar meson mass are listed in Table I. The values of M_N for all scalar meson masses are close to the empirical value. Regarding the anomalous part $\hat{M}_N$, which should account for 25% of M_N , the gluonic anomalous mass $\hat{M}_N^g$ is dominant, comprising about 24% of M_N .

We now turn to the alternative setup with $f_\pi = 93$ MeV and $\gamma_m = 0$, and present the result in Fig. 2. Qualitatively, the behavior is similar to that observed in the previous setup with $f_\pi = 68$ MeV and $\gamma_m = 0$, shown in Fig. 1. In contrast, the spatial distribution of the energy density shows enhancement near the origin in the current setup, resulting in a larger nucleon mass. Nevertheless, the gluonic anomalous mass $\hat{M}_N^g$ remains nearly unchanged, accounting for approximately 24% of M_N , as shown in Table I.

TABLE I. Summary of calculated nucleon observables for various parameter sets $(f_\pi, m_{\phi 0})$ and anomalous dimension γ_m . Listed are the nucleon mass M_N , the gluonic anomaly contribution to the mass (in percent), the D -term value $D(0)$, the mass radius $\sqrt{\langle r^2 \rangle^{\text{mass}}}$, and the mechanical radius $\sqrt{\langle r^2 \rangle^{\text{mech}}}$.

$(f_\pi, m_{\phi 0})/\text{MeV}$	Anomalous dim. γ_m	M_N/MeV	Gluonic anomaly (%)	D -term: $D = D(0)$	$\sqrt{\langle r^2 \rangle^{\text{mass}}}/\text{fm}$	$\sqrt{\langle r^2 \rangle^{\text{mech}}}/\text{fm}$
(68, 550)	0	923.6	24.1	-5.69	0.81	0.82
(68, 720)	0	927.6	24.2	-4.12	0.74	0.74
(68, 1000)	0	932.5	24.2	-3.03	0.68	0.72
(93, 550)	0	1397.4	24.0	-12.14	0.75	0.76
(93, 720)	0	1409.1	24.2	-8.64	0.69	0.66
(93, 1000)	0	1425.4	24.3	-5.69	0.61	0.62
(68, 720)	-0.9	928.8	24.9	-4.19	0.75	0.75
(68, 720)	2	925.3	22.8	-3.98	0.76	0.71

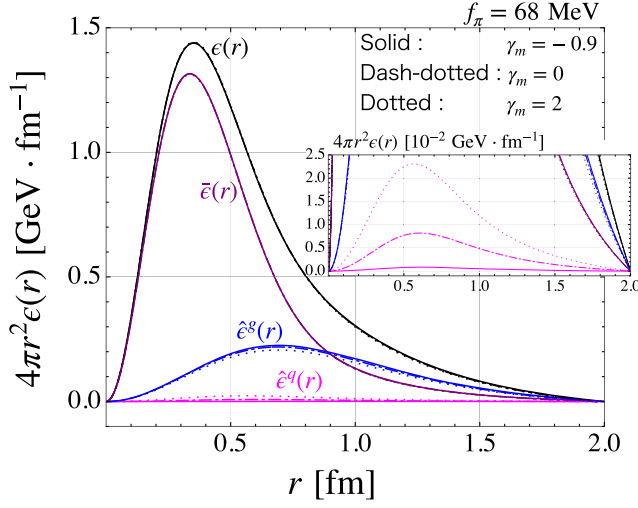


FIG. 3. Impact of γ_m on the energy density for $f_\pi = 68$ MeV. The scalar meson mass is fixed at $m_{\phi 0} = 720$ MeV.

We also consider the impact of γ_m on the spatial distribution of the energy density. Figure 3 shows the results for varying $\gamma_m = -0.9, 0, 2$, while f_π and m_ϕ are fixed at $f_\pi = 68$ MeV and $m_{\phi 0} = 720$ MeV. Although varying γ_m does not significantly affect the overall energy density, an increase in γ_m slightly enhances $\hat{\epsilon}^q(r)$ while reducing $\hat{\epsilon}^g(r)$. This is because the current-quark-mass part of the scale anomaly $\hat{\epsilon}^q(r)$ scales with the anomalous dimension γ_m , while the gluonic anomalous part $\hat{\epsilon}^g(r)$ correspondingly diminishes to preserve the total anomalous mass proportion of 25%.

Incidentally, using the obtained energy density, we also numerically evaluate the mass radius, which is defined as

$$\sqrt{\langle r^2 \rangle^{\text{mass}}} = \left(\frac{\int d^3x r^2 \epsilon(r)}{\int d^3x \epsilon(r)} \right)^{1/2}. \quad (64)$$

For the case of $f_\pi = 68$ MeV with $m_{\phi 0} = 720$ MeV and $\gamma_m = 0$, the mass radius is estimated to be $\sqrt{\langle r^2 \rangle^{\text{mass}}} = 0.74$ fm. This evaluation is qualitatively consistent with lattice results [53]. Results for other parameter sets are listed in Table I.

B. Pressure and its decomposition

Having analyzed the spatial distribution of the energy density in the previous subsection, we now focus on the pressure and its decomposition.

Figure 4 presents the spatial distribution of the pressure distribution for $f_\pi = 68$ MeV and $\gamma_m = 0$. The dynamical part is positive throughout space and is localized near the origin ($r = 0$). On the other hand, the anomalous part takes negative values and is dominated by the gluonic anomaly contribution, which retains substantial values even at locations far from $r = 0$. As a result, the total pressure, obtained by summing the dynamical and anomalous

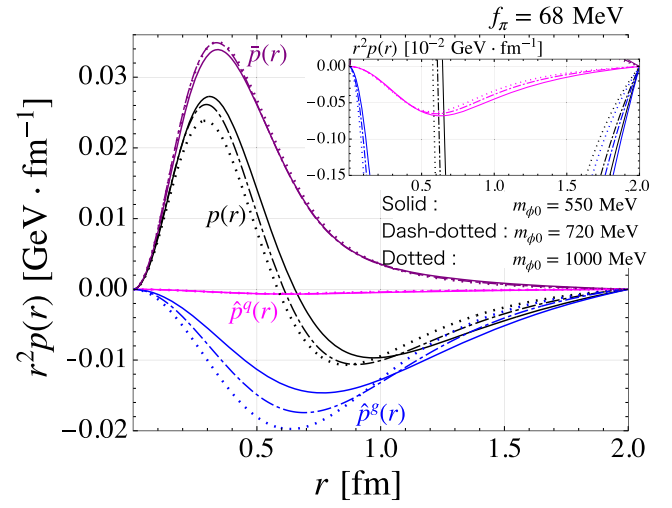


FIG. 4. Same as Fig. 1, but showing the pressure instead of the energy density.

contributions, is positive near $r = 0$, decreases with distance, and eventually flips to negative values. At larger distances, it asymptotically approaches zero from below. Regarding the effect of the scalar meson mass, the total pressure is suppressed in the region near the origin. This behavior is in contrast to that observed in the energy density. This implies that the anomalous part near $r = 0$ is more sensitive to the scalar meson mass than the dynamical part, which can be interpreted as the suppression in pressure induced by the increased strength of the gluonic scale anomaly. Looking at the asymptotic region far from the origin, we find that the asymptotic behavior is similar to that of the energy density.

By performing the spatial integral of the obtained pressure, we confirm that the present Skyrme model satisfies the von Laue condition in Eq. (21). Furthermore, the integrated contributions of the anomalous part $\hat{p}$ and the dynamical part $\bar{p}$ also fulfill the relations in Eqs. (26) and (27), respectively. These results ensure that the Skyrme model based on sChPT reliably describes a stable object.

We then plot the result for the alternative setup with $f_\pi = 93$ MeV and $\gamma_m = 0$ in Fig. 5. As shown earlier for the energy density in the previous subsection, the qualitative behavior of the pressure remains unchanged compared to the previous setup with $f_\pi = 68$ MeV and $\gamma_m = 0$. Only the overall magnitude of the pressure becomes larger.

The impact of γ_m on the pressure is shown in Fig. 6. Consistent with the analysis of the energy density, the total pressure remains largely insensitive to variations in γ_m . Regarding the behavior of the anomalous part, the magnitude of $\hat{p}^q$ increases with γ_m , while $\hat{p}^g$ is correspondingly suppressed to satisfy the relation in Eq. (26).

C. The radial and tangential pressure in a 2D plane

In this subsection, we present the spatial distribution of the pressure projected onto the transverse x - y plane of the

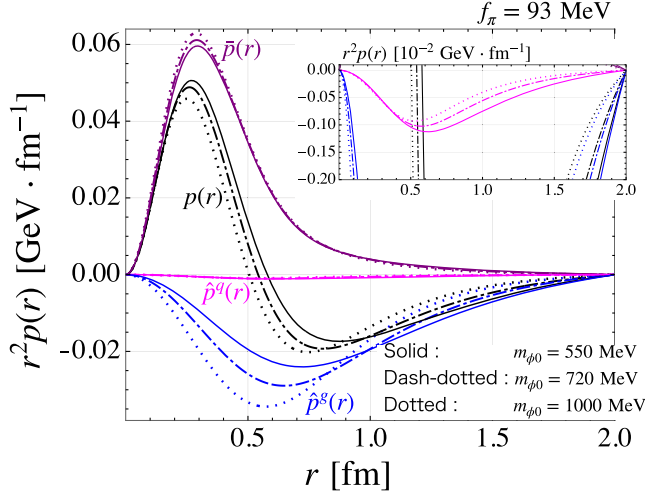


FIG. 5. Spatial distribution of the pressure in the alternative setup with $f_\pi = 93$ MeV and $\gamma_m = 0$.

nucleon, and we discuss the stability based on the spherical coordinates in Eqs. (13) and (14).

First, we show the radial component of the pressure together with its corresponding eigenvector, $p_r \vec{e}_r$, in Fig. 7. The arrows in this figure indicate the direction of the eigenvector: when an arrow points outward from the center, it represents a positive radial pressure in that direction; when it points inward, the radial pressure is negative. In other words, a positive/negative pressure corresponds to a stretching/squeezing effect on the nucleon. The color of each arrow indicates the magnitude of the pressure (its absolute value). The figure clearly shows that the radial pressure points outward at every spatial point, indicating that the nucleon undergoes the stretching effect along the radial direction throughout the entire region. Such behavior is consistent with the stability condition in Eq. (22).

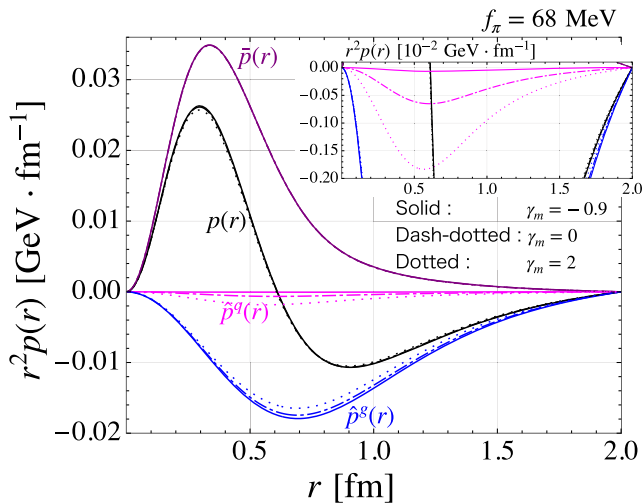


FIG. 6. Same as in Fig. 3, but showing the pressure instead of the energy density.

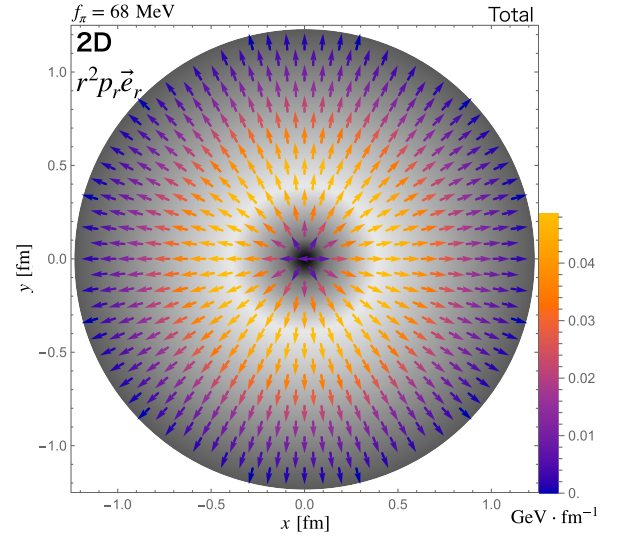


FIG. 7. Spatial distribution of the radial pressure together with its corresponding eigenvector, $p_r \vec{e}_r$, on the transverse x - y plane for the case of $f_\pi = 68$ MeV with $m_{\phi 0} = 720$ MeV and $\gamma_m = 0$.

It is worth noting that using the radial pressure, we numerically evaluate the mechanical radius, defined as

$$\sqrt{\langle r^2 \rangle^{\text{mech}}} = \left(\frac{\int d^3x r^2 p_r(r)}{\int d^3x p_r(r)} \right)^{1/2}. \quad (65)$$

For the case of $f_\pi = 68$ MeV with $m_{\phi 0} = 720$ MeV and $\gamma_m = 0$, the mechanical radius is estimated to be $\sqrt{\langle r^2 \rangle^{\text{mech}}} = 0.74$ fm, which is also qualitatively consistent with the lattice result [53].

We also show the tangential component of the pressure together with its corresponding eigenvector, $p_\phi \vec{e}_\phi$, in Fig. 8. Similarly to the radial case, counterclockwise rotation indicates positive tangential pressure, while clockwise rotation indicates negative tangential pressure. Furthermore, the color represents its magnitude. Figure 8(a) shows that the tangential pressure is positive (stretching) near the nucleon core, but its sign flips around $r \simeq 0.4$ fm, beyond which the tangential pressure becomes negative (squeezing). As will be shown below, this negative behavior plays a crucial role in ensuring the stability of the nucleon, as it is closely related to the negative D -term discussed in Eq. (23). The D -term can be rewritten in terms of the radial coordinate as

$$D = M_N \int d^3x r^2 \frac{p_r(r) + 2p_\phi(r)}{3}. \quad (66)$$

As shown above, the radial pressure p_r is positive throughout the spatial region. Hence, a sufficiently negative tangential pressure p_ϕ is essential to yield the negative D -term, which is required by the stability condition in Eq. (23). To further examine the origin of this negative

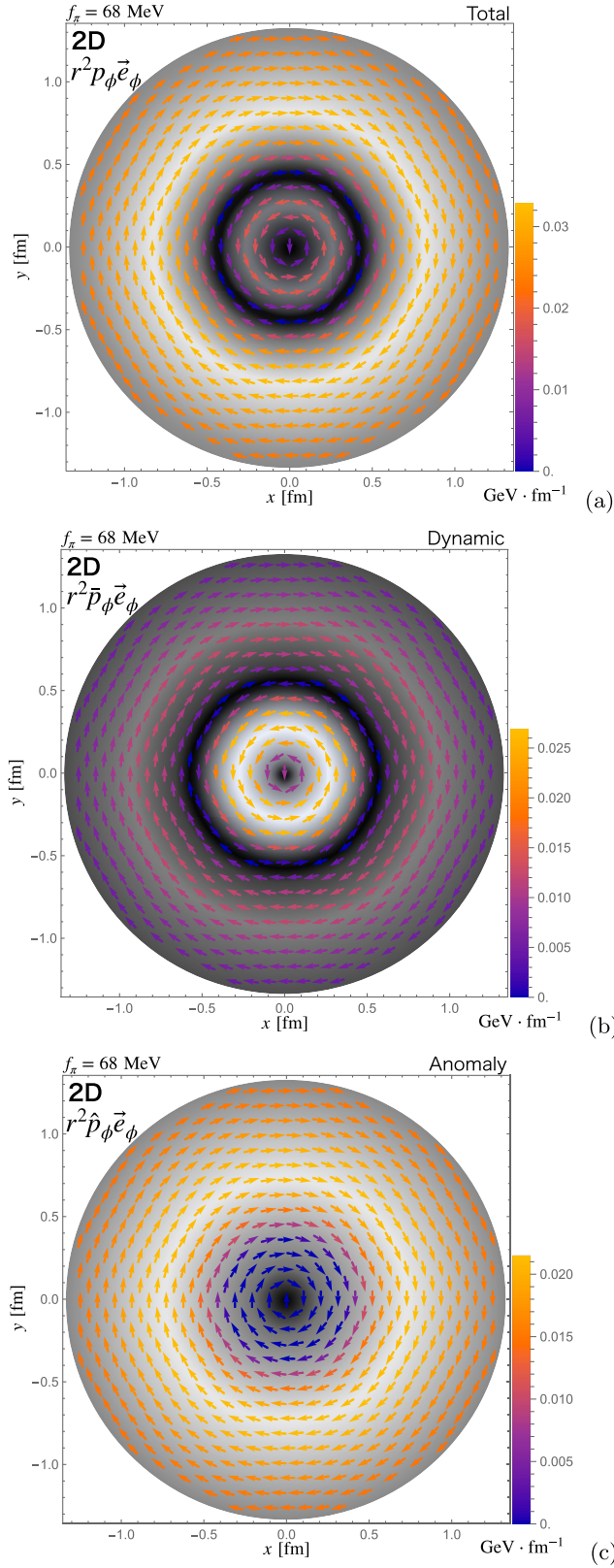


FIG. 8. (a) Spatial distribution of the tangential pressure together with its corresponding eigenvector, $p_\phi \vec{e}_\phi$, on the transverse x - y plane for the case of $f_\pi = 68$ MeV with $m_{\phi 0} = 720$ MeV and $\gamma_m = 0$. The other two panels show its decomposition into (b) the dynamical part ($\bar{p}_\phi$) and (c) the anomalous part ($\hat{p}_\phi$).

contribution, we consider the decomposition of the tangential pressure in the following.

Panels (b) and (c) in Fig. 8 show the decomposition of the tangential pressure into the dynamical part ($\bar{p}_\phi$) and the anomalous part ($\hat{p}_\phi$). Both parts contribute negatively in the outer region of the nucleon. However, the anomalous part is dominant, while the dynamical part is negligibly small. As discussed in the previous section, the anomalous part is primarily governed by the gluonic scale anomaly. These findings emphasize the crucial role of the gluonic scale anomaly in generating the negative tangential pressure that yields the negative D -term.

D. Internal confining force

In this subsection, we present numerical results for the internal force inside the nucleon based on Eq. (30), and we discuss its role in the confining mechanism of the nucleon.

To examine the force, we focus on the anomalous part, $\hat{\mathcal{F}}_r$, and present its spatial distribution in Fig. 9. Note that the dynamical force exhibits behavior opposite to the anomalous part in order to ensure the force balance inside the nucleon, as shown in Eq. (30). Regardless of the setup, the anomalous force takes small positive values only in the very vicinity of the nucleon core. However, it becomes negative throughout most of the interior of the nucleon.

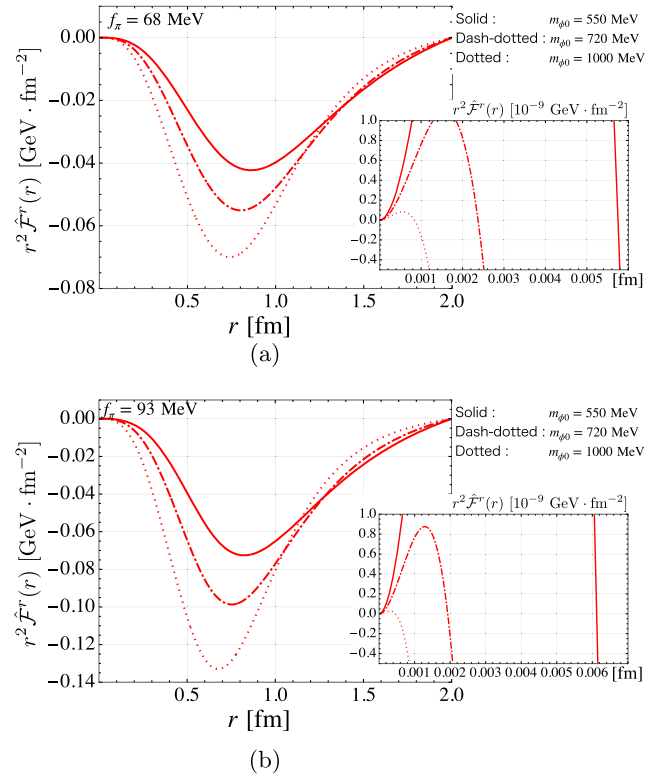


FIG. 9. The anomalous part of the force density for (a) $f_\pi = 68$ MeV and (b) $f_\pi = 98$ MeV, with the anomalous dimension fixed at $\gamma_m = 0$.

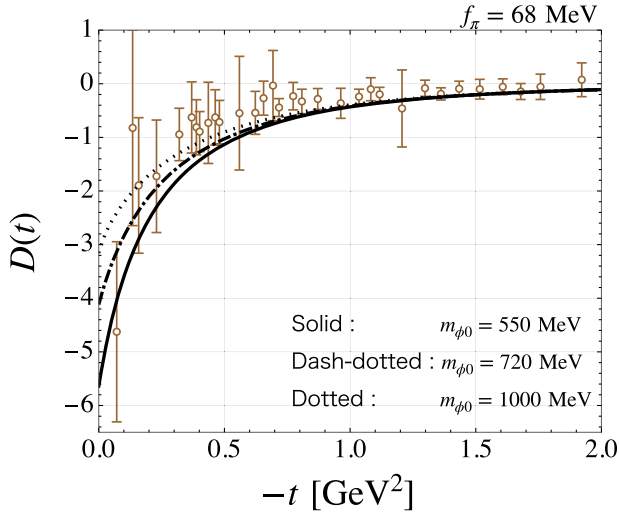


FIG. 10. Momentum-transfer dependence of the $D(t)$ form factor for $f_\pi = 63$ MeV (black curve), compared with the lattice results from Ref. [53] (points with error bars).

Since the negative force is directed inward from the outside, it represents a confining force. This implies that the scale anomaly—especially the gluonic scale anomaly—plays a crucial role in the confinement mechanism of the nucleon.

We also investigate the effect of the scalar meson mass on the force. As the scalar meson mass increases, the strength of the anomalous force becomes intense, implying that a larger scale anomaly results in a more pronounced confining effect.

E. $D(t)$ form factor

Based on the pressure results obtained above, we evaluate the momentum-transfer dependence of the $D(t)$ form factor and compare it with the recent lattice data [53], as shown in Fig. 10. The figure indicates that the momentum-transfer dependence in the Skyrme model qualitatively agrees with that in the lattice result. In particular, for $f_\pi = 68$ MeV and $m_{\phi} = 720$ MeV, the model successfully reproduces the lattice data. In this case, the D -term is evaluated as

$$D = D(0) = -4.12. \quad (67)$$

This result is in qualitative agreement with recent lattice QCD findings obtained via dipole fits, within the quoted systematic uncertainties [53]. However, their analysis using the z -expansion method yields a somewhat larger D -term compared to our prediction. In addition, recent model-independent estimates based on dispersion relations have also extracted the D -term [56]. Compared to these previous observations, our result is somewhat smaller. To reconcile this discrepancy, we take into account the finite value of the anomalous dimension. As shown in Table I, in particular for $\gamma_m = 2$, the D -term reaches $D = -3.98$, which is close

to the values obtained from both the lattice z -expansion analysis and the model-independent estimate.¹

We also comment on the impact of the scalar meson mass on the D -term. Unlike the nucleon mass, the D -term is significantly affected by changes in the scalar meson mass, as shown in Table I. As discussed in Sec. IV B, this is likely because the anomalous part $\hat{p}$ is more sensitive to changes in the mass than the dynamical part $\bar{p}$, which leads to a substantial variation in the D -term.

V. SUMMARY AND OUTLOOK

In this study, we extend our previous analysis further. As a brief reminder, in our earlier work using a Skyrme model based on sChPT, we revealed that the gluonic scale anomaly gives a negative contribution to the pressure in Cartesian coordinates and constitutes the dominant part of the D -term [2]. Building on this previous result, the main results obtained in the present study are as follows:

- (a) By varying the scalar meson mass, which controls the strength of the gluonic scale anomaly, we investigated the response of the gluonic scale anomaly in the energy density and the pressure. While the energy density somewhat changes with the scalar meson mass, the nucleon mass remains largely unaffected. In contrast, the anomalous part of the pressure exhibits a strong sensitivity to the scalar meson mass, leading to a significant variation in the D -term. Since the magnitude of the scale anomaly is expected to change in high-temperature and high-density environments, the insights obtained in this work may be useful for studying the properties of nucleons under such extreme conditions.
- (b) To better understand the internal structure of the nucleon, we analyzed the pressure distribution in spherical coordinates. We confirmed that the radial pressure satisfies the stability condition of the nucleon. For the tangential pressure, we found that it is positive in the inner region and becomes negative toward the outer region. This negative tangential pressure plays a crucial role in satisfying the stability condition associated with a negative D -term. We then identified that the main contribution to this negative tangential pressure originates from the gluonic scale anomaly. Furthermore, by examining the internal force density, we found that the forces are balanced inside the nucleon, and the confining force is predominantly generated by the gluonic scale anomaly.
- (c) We also investigated the momentum-transfer dependence of the GFF $D(t)$. The result obtained from the Skyrme model successfully reproduces the

¹In our present setup, the model parameters were adjusted to reproduce the lattice results for the $D(t)$ form factor, but this parameter setup causes the $A(t)$ form factor to deviate from the lattice results outside the error bars.

momentum-transfer dependence observed in recent lattice QCD calculations [53]. Based on the best-fit agreement with the lattice results, the D -term is found to be $D = -4.12$. However, this estimate is somewhat smaller than those reported in some previous studies [53,56]. To reconcile this deviation, we find that this value can be brought closer to earlier results by adjusting the anomalous dimension. In addition, we also make a comparison of the mass radius and the mechanical radius with the lattice QCD results [53]. This agreement with lattice QCD reinforces the quantitative reliability of our model analysis and, at the same time, provides a robust estimate of the D -term, whose precise value has not yet been firmly established.

In closing, we comment on the remaining issues and the implications of our analysis below.

First, in this study, our analysis is based on classical soliton solutions. In order to yield more realistic results, one needs to quantize the soliton, which corresponds to the next-to-leading order in the large N_c expansion. However, as pointed out by the previous studies, the quantized soliton would fail to satisfy the stability condition for the nucleon [23,114]. Resolving this issue remains an important direction for future work.

We also offer a few remarks on the current model framework. In the present analysis, the sChPT Lagrangian includes only the contributions from the pion and the scalar meson. Extending the model to incorporate higher excited states, such as vector mesons, is expected to yield more refined and comprehensive results. Further investigation based on an extended framework would be a valuable direction for future research.

Although this study has addressed the internal force structure, the skyrmion approach cannot describe the forces acting on the quark constituents of the nucleon. Nevertheless, recent studies have proposed an approach to characterize the forces acting on quarks through the static EMT [115]. In light of this development, it may be possible to infer the forces on quarks even within the framework of the Skyrme model. Alternatively, one could investigate the quark-level forces more directly by employing models that incorporate explicit quark degrees of freedom, such as the chiral quark soliton model [116–118] or the quark-meson coupling model [119,120].

There are also ongoing discussions regarding the relation between GFFs and spatial distributions. Our analysis follows the interpretation of hadrons as a continuum

medium proposed in Ref. [104], wherein the static EMT in the Breit frame is interpreted as three-dimensional spatial distributions such as pressure and shear forces. Based on this interpretation, in this study, we evaluated the density distribution in the Breit frame on the equal-time surface $t = 0$ in the instant form. However, it has been pointed out that, for light hadrons such as the nucleon, relativistic effects can lead to ambiguities in the definition of such densities [121,122]. To avoid that problem, two-dimensional transverse densities in the light-front form have been widely discussed [3,123]. In the present work, however, we leave them for future work. Furthermore, alternative interpretations, distinct from the continuum medium viewpoint of Ref. [104], have been proposed [115,124], and discussions based on those frameworks could be of interest; however, they lie beyond the scope of the present work.

One of the key features of the present study is the decomposition of the static EMT into dynamical and anomalous parts. Based on this decomposition, we analyzed the pressure distribution and force balance inside the nucleon. This method is universally applicable to any type of hadron. Recently, some of the present authors have applied this decomposition analysis to the pion and found that the scale anomaly plays a significant role even in the pion's internal structure [3]. Since this decomposition can also be applied to other hadrons, such as excited states of the nucleon and heavy hadrons, this approach may contribute to a deeper understanding of the confinement mechanism in hadronic systems.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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