

Extended AdS spacetime without boundaries and entanglement without holography

Cesar Arias ^{*}

*Departamento de Matemática, Pontificia Universidad Católica de Chile
Avenida Vicuña Mackenna 4860, Santiago, Chile*



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We glue together two copies of pure anti-de Sitter (AdS) spacetime along their conformal boundaries creating a manifold without boundaries. The resulting space, which in dimension $d + 2$ we denote by $\text{AdS}_{\pm}^{d+2}$, has the topology of $S^2 \times \Sigma^d$, where Σ^d is a d manifold without boundary. Acting with $\mathbb{Z}_n$ on the S^2 factor amounts to coupling a pair of membranes at the north and south poles of the two-sphere. Moreover, extending the domain of the two-sphere polar coordinate from $[0, \pi]$ to the interval $[0, (2N - 1)\pi]$, where $N > 1$, enables the coupling of one stack of N coincident membranes at each pole of the two-sphere ($2N$ membranes in total). Assuming the existence of a quantum gravity theory on the glued spacetime, we compute the classical approximation of the entanglement entropy across an entangling surface consisting of the two antipodal stacks of membranes. We find that the resulting entropy exhibits a boundary cutoff divergence that can be canceled by taking the limit of an infinite number of membranes. This large- N cancelation—possible only in the doubled, extended geometry without boundaries—yields a finite, universal quarter-area law. The calculation does not require details of the quantum theory other than its infrared limit, which we assume to be Einstein gravity.

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I. INTRODUCTION

One of the most salient and universal features that arises from the interplay between quantum theory and gravity is the appearance of area laws for entropy. A prime example is the Bekenstein-Hawking formula [1,2] for the entropy of a black hole, whose origins and implications have driven an important part of the recent research in string theory and gravity, as it arguably provides a semiclassical window into quantum aspects of spacetime.

Originally derived in the context of black hole thermodynamics, the area law has since revealed itself in a broader array of settings. Within the AdS/CFT correspondence [3], the Ryu-Takayanagi formula [4] generalizes the relation between entropy and area to quantum field theories with gravitational duals. The formula equates the entanglement entropy of a region in the boundary dual theory to one-quarter of the area of a minimal surface in the bulk. This geometrization of entanglement has further fueled the idea that spacetime itself may be a manifestation of quantum

entanglement [5], and the recurrence of the one-quarter prefactor suggests a universality that extends beyond the black hole thermal case.

Building on this framework, Lewkowycz and Maldacena [6] provided a derivation of the Ryu-Takayanagi formula from first principles within the gravitational path integral. By extending the replica trick [7] to the bulk and analyzing replica-symmetric geometries, they demonstrated that the dominant saddle-point contribution to the path integral yields an area term from a codimension two surface fixed under the replica symmetry. In the semiclassical limit, this surface is precisely the extremal surface of the Ryu-Takayanagi prescription. This work grounded the relation between entropy and area in a more rigorous formulation of quantum gravity, reinforcing the ubiquity of the one-quarter of the area law in holographic settings.

Despite the central role of holography in crystallizing these ideas, many of the phenomena it captures appear to reflect more general principles. In particular, the appearance of area laws in vacuum entanglement entropy across a wide variety of quantum systems [8–13] suggests that the geometric nature of entanglement entropy may not require a dual boundary theory to be realized. Similarly, black hole entropy, with its universal structure, emerges from semiclassical gravity without invoking any particular holographic argument.

This raises the following question: to what extent can the relation between entanglement entropy and the area of

^{*}Contact author: cesar.arias@uc.cl

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minimal surfaces be understood and derived in nonholographic settings? In particular, can area laws of the Ryu-Takayanagi type be reproduced or reinterpreted intrinsically within the bulk spacetime without appealing to a dual boundary description?

The goal of this note is to argue that Ryu-Takayanagi-like formulas can arise from more direct gravitational and field-theoretic considerations in anti-de Sitter (AdS) spacetime, with no need of holography.

To this end, we construct a geometry consisting of two copies of AdS glued along their conformal boundaries. Unlike a single AdS copy, this doubled geometry allows for the coupling of an arbitrary number of membranes. These are minimal surfaces that appear as the fixed points of a bulk $\mathbb{Z}_n$ symmetry, and are organized into two antipodal stacks of N coincident membranes each. We argue that, in a semiclassical approximation, the entanglement entropy between two spacetime subregions separated by the two stacks of membranes leads to a novel area law.

For a finite number N of membranes, the entropy depends on a boundary cutoff and diverges as this cutoff is taken to zero. However, this divergence can be suppressed by taking the limit of an infinite number of membranes. In this limit, the resulting entanglement entropy becomes cutoff independent and obeys a Ryu-Takayanagi-like formula.

The article is organized as follows. In Sec. II we construct our extended geometry by gluing along the boundary two copies of AdS space. In Sec. III we show that the extended geometry naturally admits the coupling of an arbitrary number of membranes, organized into two antipodal stacks of N coincident membranes each. In Sec. IV we devise a spacetime subregion separated by the two stacks of membranes, and compute the associated entanglement entropy. We conclude in Sec. V with some final comments and remarks.

II. EXTENDED ADS SPACE WITHOUT BOUNDARIES

In this section we assemble two copies of AdS space by fully overlapping their conformal boundaries. This construction results in a negatively curved manifold without boundaries, which hereafter we refer to as extended AdS space without boundaries; in $d + 2$ dimensions we denote this manifold by $\text{AdS}_{\pm}^{d+2}$.

We first focus on the lower dimensional cases and devise the two and three-dimensional extended spaces $\text{AdS}_{\pm}^2$ and $\text{AdS}_{\pm}^3$. We then use these as a seed to recursively define

$$\text{AdS}_{\pm}^{d+2} = S^2 \times \text{AdS}_{\pm}^d, \quad d \geq 2. \quad (2.1)$$

As we will elaborate on further below, Eq. (2.1) follows from the fact that a single copy of pure AdS^{d+2} space can be realized as the product $D^2 \times \Sigma^d$, where D^2 is a two-

dimensional disk and Σ^d a negatively curved d manifold without boundary that we take to be $\Sigma^d = \text{AdS}_{\pm}^d$, and where thus the boundary of AdS^{d+2} has the topology of $S^1 \times \Sigma^d$. Hence, in this context, gluing two copies of AdS means taking the corresponding copies of the disks D^2 and gluing them along their boundary circle.

To begin with, let us recall some elementary facts. Anti-de Sitter spacetime of dimension $d + 2$ can be defined as a hyperboloid embedded in flat ambient space $\mathbb{R}^{2,d+1}$. Taking the ambient space coordinates to be $X^A = (T^1, T^2, X^a)$, where $a = 1, \dots, d + 1$, the ambient space metric

$$\eta = -(dT^1)^2 - (dT^2)^2 + \sum_{a=1}^{d+1} (dX^a)^2. \quad (2.2)$$

The embedding $\text{AdS}^{d+2} \hookrightarrow \mathbb{R}^{2,d+1}$ is thus defined by means of the hypersurface equation

$$X^2 = \eta_{AB} X^A X^B = -(T^1)^2 - (T^2)^2 + \sum_{a=1}^{d+1} (X^a)^2 = -L^2, \quad (2.3)$$

where L denotes the AdS^{d+2} radius. Different parametrization of the coordinates $X^A \in \mathbb{R}^{2,d+1}$ gives rise to different induced geometries on AdS^{d+2} , which are given by the pull-back of the flat metric (2.2) onto the hypersurface defined by (2.3), namely

$$g_{\text{AdS}} = \eta|_{X^2 = -L^2}. \quad (2.4)$$

The induced geometry (2.4) in turn induces the topology of AdS space.

A. Extended AdS^2

Let us first consider the lowest dimensional case and construct the extended version of AdS^2 . To this end, we take the coordinates $(T^1, T^2, X) \in \mathbb{R}^{2,1}$ to be parametrized as

$$\begin{aligned} T^1 &= L \sec \theta \cos(t/L), & T^2 &= L \sec \theta \sin(t/L), \\ X &= L \tan \theta, \end{aligned} \quad (2.5)$$

where $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ and $t \in \mathbb{R}$. It is direct to check that the choice above satisfies the hypersurface constraint (2.3). The corresponding induced metric on (a single copy of) AdS^2 is obtained by restricting the flat embedding space metric η to the surface $X^2 = -L^2$ by means of the parametrization (2.5). This gives

$$g_{\text{AdS}} = \frac{L^2 d\theta^2 - dt^2}{\cos^2 \theta}. \quad (2.6)$$

The advantage of these coordinates is that we can now glue two copies of AdS^2 simply by extending the range of

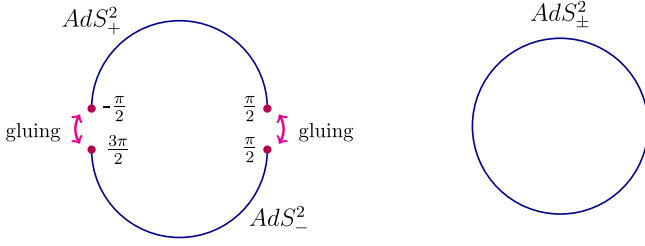


FIG. 1. Construction of extended AdS^2 at fixed time. On the left, the two copies are glued along the boundaries. Each of these copies, denoted by AdS^2_+ and AdS^2_- , are coordinatized by θ , which according to (2.7) runs over an interval of length π . On the right, the resulting space AdS^2_+ has no boundary and the fixed time topology of a circle, with the coordinate θ running over an interval of length 2π .

the polar coordinate from $[-\frac{\pi}{2}, \frac{\pi}{2}]$ to the interval $[-\frac{\pi}{2}, \frac{3\pi}{2}]$. Indeed, taking these two copies to be defined as

$$\begin{aligned} \text{AdS}^2_+ &:= \left\{ (T^1, T^2, X) \in \mathbb{R}^{2,1} \mid -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \right\}, \\ \text{AdS}^2_- &:= \left\{ (T^1, T^2, X) \in \mathbb{R}^{2,1} \mid \frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2} \right\}, \end{aligned} \quad (2.7)$$

then each of them has a boundary given by a pair of points (a 0-sphere) times $\mathbb{R}$; the boundary of AdS^2_+ is defined by the pair $\{-\frac{\pi}{2}, \frac{\pi}{2}\}$, while the boundary of AdS^2_- is determined by $\{\frac{\pi}{2}, \frac{3\pi}{2}\}$. Thus, the space

$$\text{AdS}^2_{\pm} = \text{AdS}^2_+ \cup \text{AdS}^2_- \quad (2.8)$$

can be defined as the result of overlapping the two boundaries by pairwise identifying the four points that compose them, that is, identifying the pair of points located at $\theta = \frac{\pi}{2}$ and the pair of points located at $\theta = -\frac{\pi}{2}$ and $\theta = \frac{3\pi}{2}$. Note that the range of the polar coordinate is chosen such that, in the resulting glued space, the north and south poles of the spatial circle are located at $\theta = 0$ and π , respectively. The construction is depicted in Fig. 1.

B. Extended AdS^3

For the three-dimensional extension, we solve the hyper-surface equation (2.3) by parametrizing the embedding space coordinates $(T^1, T^2, X^1, X^2) \in \mathbb{R}^{2,2}$ as

$$\begin{aligned} T^1 &= L \sec \theta \cos(t/L), & T^2 &= L \sec \theta \sin(t/L), \\ X^1 &= L \tan \theta \cos \phi, & X^2 &= L \tan \theta \sin \phi, \end{aligned} \quad (2.9)$$

where

$$0 \leq \theta \leq \frac{\pi}{2}, \quad 0 \leq \phi \leq 2\pi, \quad -\infty < t < \infty. \quad (2.10)$$

Note that in this case the polar coordinate θ ranges over an interval of length $\pi/2$, in contrast to the two-dimensional case where it spans an interval of length π .

From (2.4) and (2.9), it follows that the induced AdS geometry is

$$g_{\text{AdS}} = \frac{L^2(d\theta^2 + \sin^2 \theta d\phi^2) - dt^2}{\cos^2 \theta}, \quad (2.11)$$

which in turn induces the topology of $D^2 \times \mathbb{R}$, where D^2 is a two dimensional disk with coordinates (θ, ϕ) . The conformal boundary of AdS^3 is located at the boundary of disk $\theta = \frac{\pi}{2}$ and has the topology of $S^1 \times \mathbb{R}$.

The metric (2.11) is invariant under the shift $\theta \rightarrow \pi - \theta$. This symmetry amounts to taking

$$\begin{aligned} \text{AdS}^3_+ &:= \left\{ (T^1, T^2, X^1, X^2) \in \mathbb{R}^{2,2} \mid 0 \leq \theta \leq \frac{\pi}{2} \right\}, \\ \text{AdS}^3_- &:= \left\{ (T^1, T^2, X^1, X^2) \in \mathbb{R}^{2,2} \mid \frac{\pi}{2} \leq \theta \leq \pi \right\}, \end{aligned} \quad (2.12)$$

as two different copies of AdS^3 . These can be glued together along the boundary simply extending the range of the polar coordinate:

$$\begin{aligned} \text{AdS}^3_{\pm} &= \text{AdS}^3_+ \cup \text{AdS}^3_- \\ &= \{(T^1, T^2, X^1, X^2) \in \mathbb{R}^{2,2} \mid 0 \leq \theta \leq \pi\}. \end{aligned} \quad (2.13)$$

It is important to point out that, strictly speaking, when gluing two manifolds M and N along their boundaries, one needs to specify the way the gluing is done via an invertible map $\varphi: \partial M \rightarrow \partial N$ such that the resulting space $M \cup_{\varphi} N$ is uniquely defined [14]. In our case, since we are considering two copies of the same manifold, it suffices to take φ as the identity map; for simplicity we omit decorations regarding this point.

C. Extended AdS^{d+2}

We now formulate the general case of extended AdS space in dimension $d+2$, with $d \geq 2$. As we will see, the construction is a recursion that uses as a seed the lower-dimensional cases introduced above. To this end, we first parametrize

$$\begin{aligned} T^1 &= L \sec \theta t^1, & T^2 &= L \sec \theta t^2, \\ X^i &= L \sec \theta x^i (i = 1, \dots, d-1), \\ X^d &= L \tan \theta \cos \phi, & X^{d+1} &= L \tan \theta \sin \phi, \end{aligned} \quad (2.14)$$

where

$$0 \leq \theta \leq \frac{\pi}{2}, \quad 0 \leq \phi \leq 2\pi, \quad (2.15)$$

and where $-\infty < (t^1, t^2, x^i) < \infty$ are coordinates in the embedding subspace $\mathbb{R}^{2,d+1} \subset \mathbb{R}^{2,d+1}$. These are given by the restriction

$$(t^1, t^2, x^i) = \frac{1}{L}(T^1, T^2, X^i, X^d, X^{d+1})|_{\theta=0}. \quad (2.16)$$

Next, we observe that the parametrization (2.14) solves the hypersurface equation (2.3) if and only if the coordinates (t^1, t^2, x^i) satisfy the constraint

$$-(t^1)^2 - (t^2)^2 + \sum_{i=1}^{d-1} (x^i)^2 = -1. \quad (2.17)$$

This constraint, which under (2.14) is the $\theta = 0$ restriction of (2.3), defines a hypersurface of negative curvature of codimension three with respect to the embedding space $\mathbb{R}^{2,d+1}$, and a submanifold of codimension embedded in AdS^{d+2} ; we denote this submanifold by Σ^d .

To elucidate the significance of Σ^d it is useful to compute the induced metric on the AdS^{d+2} hyperboloid (2.3). Making use of the parametrization (2.14) and imposing (2.17), a direct calculation yields

$$g_{\text{AdS}} = \frac{L^2(d\theta^2 + \sin^2 \theta d\phi^2) + h}{\cos^2 \theta}, \quad (2.18)$$

where

$$h = -(dt^1)^2 - (dt^2)^2 + \sum_{i=1}^{d-1} (dx^i)^2 \quad (2.19)$$

is the induced metric on Σ^d . Note that the explicit form of h depends on the parametrization of the coordinates (t^1, t^2, x^i) , but for our purposes we do not need to make any particular choice of it. Global Einstein geometries of this type were first studied in [15,16].

From (2.18) it follows that the induced topology on a single copy of AdS is

$$\text{AdS}^{d+2} \cong D^2 \times \Sigma^d, \quad (2.20)$$

where D^2 is the two-dimensional disk with coordinates (θ, ϕ) , and Σ^d is the submanifold defined by Eq. (2.17). Requiring AdS^{d+2} to have a single, connected boundary we take Σ^d to have no boundary so that

$$\partial(\text{AdS}^{d+2}) \cong S^1 \times \Sigma^d. \quad (2.21)$$

Since the constraint (2.17) enforces Σ^d to have negative curvature, we further take

$$\Sigma^d = \text{AdS}_{\pm}^d, \quad (2.22)$$

where we recall that the extended space $\text{AdS}_{\pm}^d$ has no boundary.

The gluing of two copies of AdS^{d+2} along their boundaries is almost identical to the three-dimensional case. Since we are dealing with a manifold of topology D^2 times a manifold without boundaries (the latter is given by $\mathbb{R}$ in the case of dimension 3, and by $\text{AdS}_{\pm}^d$ in the case of dimension $d+2$), then all we need to do is to glue together the two corresponding copies of D^2 along their S^1 boundary. Indeed, due to the symmetry $\theta \rightarrow \pi - \theta$ exhibited by (2.18) we can define

$$\begin{aligned} \text{AdS}_{+}^{d+2} &:= \left\{ (T^1, T^2, X^i, X^d, X^{d+1}) \in \mathbb{R}^{2,d+1} \mid 0 \leq \theta \leq \frac{\pi}{2} \right\}, \\ \text{AdS}_{-}^{d+2} &:= \left\{ (T^1, T^2, X^i, X^d, X^{d+1}) \in \mathbb{R}^{2,d+1} \mid \frac{\pi}{2} \leq \theta \leq \pi \right\}, \end{aligned} \quad (2.23)$$

where both copies have the boundary located at $\theta = \pi/2$ (the point where the defining function $u = \cos \theta$ vanishes). Then, the extended space is directly obtained by extending the range of the polar coordinate as

$$\begin{aligned} \text{AdS}_{\pm}^{d+2} &= \text{AdS}_{+}^{d+2} \cup \text{AdS}_{-}^{d+2} \\ &= \{(T^1, T^2, X^i, X^d, X^{d+1}) \in \mathbb{R}^{2,d+1} \mid 0 \leq \theta \leq \pi\}. \end{aligned} \quad (2.24)$$

Consequently and due to (2.18)–(2.24), we observe that the glued, extended space can then be thought of as the recursion

$$\text{AdS}_{\pm}^{d+2} = S^2 \times \text{AdS}_{\pm}^d, \quad d \geq 2, \quad (2.25)$$

with seeds the lower dimensional cases $\text{AdS}_{\pm}^2$ and $\text{AdS}_{\pm}^3$ spelled out in Secs. II A and II B, respectively. A sketch of the construction is displayed in Fig. 2.

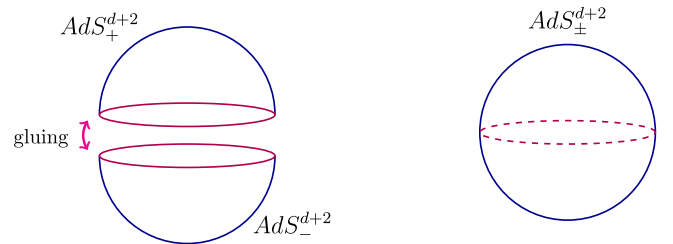


FIG. 2. Gluing of two copies of $\text{AdS}^{d+2} \cong D^2 \times \Sigma^d$. On the left, the corresponding disk factor of each copy glued along their S^1 boundary, located at $\theta = \pi/2$. The resulting manifold has the topology of a two-sphere, depicted on the right.

III. COUPLING MEMBRANES

We now examine how the geometry of extended AdS space naturally admits the coupling of membranes. In what follows we consider $d \geq 2$ and denote the metric on the extended space $\text{AdS}_{\pm}^{d+2}$ by $g_{\pm}$. This is given by (2.18) but with an extended polar coordinate, that is

$$g_{\pm} = \frac{L^2(d\theta^2 + \sin^2\theta d\phi^2) + h}{\cos^2\theta}, \quad 0 \leq \theta \leq \pi. \quad (3.1)$$

It is important to recall that the line element above exhibits the shift symmetry $\theta \rightarrow \pi - \theta$ and induces the topology of $S^2 \times \Sigma^d$ on $\text{AdS}_{\pm}^{d+2}$, where Σ^d is a manifold of negative curvature without boundary that we take to be $\text{AdS}_{\pm}^d$; this choice yields the recursive definition (2.25) presented earlier.

A. Antipodal defects

The backreaction of codimension two cosmic strings and membranes generates conical defects [17]. These are characterized by a deficit angle, which is in turn related to the tension of the string or membrane.

In our case, due to the induced topology of the extended space $\text{AdS}_{\pm}^{d+2} \cong S^2 \times \Sigma^d$, one can straightforwardly induce a pair of conical defects by means of the quotient $S^2/\mathbb{Z}_n$; its fixed points are located at the north and south poles of the two-sphere, and each of them supports a singular surface that we will denote by Σ_+^d and Σ_-^d , respectively.

Indeed, the action of $\mathbb{Z}_n$ can be defined, upon analytic continuation of the orbifold parameter $n \geq 1$, by the azimuthal identification

$$\phi \sim \phi + \frac{2\pi}{n}, \quad (3.2)$$

under which, locally about $\theta \rightarrow 0, \pi$ and in terms of the variable $\rho = L\theta$, the metric (3.1) behaves to leading order in ρ as

$$g_{\pm} \approx d\rho^2 + \frac{\rho^2}{n^2} d\phi^2 + h. \quad (3.3)$$

The latter manifestly exhibits the presence of conical defects at the north and south poles of the two-sphere, both having a deficit angle given by $\Delta\phi = 2\pi(1 - \frac{1}{n})$. This maneuver amounts to thinking of the extended space $\text{AdS}_{\pm}^{d+2}$ as the smooth, $n \rightarrow 1$ limit of the product $S^2/\mathbb{Z}_n \times \Sigma^d$. As anticipated, for $n > 1$ we denote the two singular surfaces supported by the antipodal defects by

$$\Sigma_+^d = (S^2/\mathbb{Z}_n \times \Sigma^d)|_{\theta=0} \quad \text{and} \quad \Sigma_-^d = (S^2/\mathbb{Z}_n \times \Sigma^d)|_{\theta=\pi}. \quad (3.4)$$

Since the extrinsic curvatures of Σ_+^d and Σ_-^d vanish, these surfaces are minimal.

B. Coupling an arbitrary number of membranes

Invariants of metrics on manifolds with conical defects, locally of the type $\mathcal{M} = C \times \Sigma$ where C is a cone, have been studied in [18]. The nontrivial holonomy about the conical singularity yields a Riemann tensor containing a delta function with support on the codimension two singular surface Σ . This translates into a Ricci scalar that decomposes as

$$R^{(n)} = R - 4\pi \left(1 - \frac{1}{n}\right) \delta_{\Sigma} \quad (3.5)$$

where $R = R^{(1)}$ is the curvature scalar of the smooth manifold and $\int_{\mathcal{M}} f \delta_{\Sigma} = \int_{\Sigma} f$. If the manifold $\mathcal{M}$ has globally a number of conical defects supporting several singular surfaces the last term in (3.5) should be replaced by a sum over all such surfaces.

Regarding the geometry of the extended space $\text{AdS}_{\pm}^{d+2}$, since we have the two singular surfaces (3.4) supported by antipodal conical defects of the same deficit angle, decomposition (3.5) reads

$$R^{(n)} = R - 4\pi \left(1 - \frac{1}{n}\right) (\delta_{\Sigma_+} + \delta_{\Sigma_-}). \quad (3.6)$$

The relation above has an immediate consequence when considering the gravitational action on the extended manifold with defects, which we recall has the induced topology of $S^2/\mathbb{Z}_n \times \Sigma^d$. Indeed, when integrating (3.6) one obtains the usual Einstein–Hilbert terms plus two extra Nambu–Goto terms with support on the singular surfaces (3.4), that is

$$\begin{aligned} I[\text{AdS}_{\pm}/\mathbb{Z}_n] &= \frac{1}{16\pi G} \int_{\text{AdS}_{\pm}} d^{d+2}x \sqrt{|\det g_{\pm}|} (R - 2\Lambda) \\ &\quad - \frac{1}{4G} \left(1 - \frac{1}{n}\right) \int_{\Sigma_+} d^d y \sqrt{|\det h|} \\ &\quad - \frac{1}{4G} \left(1 - \frac{1}{n}\right) \int_{\Sigma_-} d^d y \sqrt{|\det h|}, \end{aligned} \quad (3.7)$$

where y denotes the coordinates on Σ_+ and Σ_- . Note that since the extended manifold $\text{AdS}_{\pm}^{d+2}$ is the result of gluing two copies of AdS space along their boundaries, the corresponding boundary terms in (3.7) cancel each other due to their opposite orientations. Also note that, as Riemannian submanifolds, Σ_+ and Σ_- are both endowed with the same induced metric h , and the last two terms in (3.7) are simply the area functional of the singular surfaces coupled through the tension

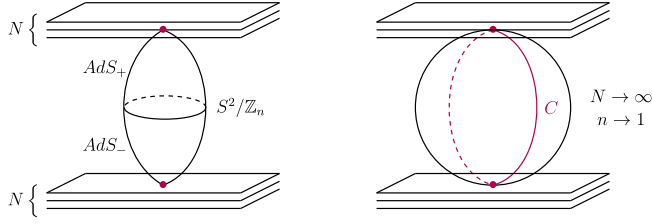


FIG. 3. Extended AdS space without boundaries and with $2N$ coupled membranes. To the left, the two fixed points of the $S^2/\mathbb{Z}_n$ quotient support a stack of N tensionful, coincident membranes, with the polar coordinates defined on the interval (3.9). To the right, the tensionless limit $n \rightarrow 1$ of an infinite number of membranes on a smooth geometry.

$$\tau = \frac{1}{4G} \left(1 - \frac{1}{n} \right). \quad (3.8)$$

The tensionless limit $n \rightarrow 1$ corresponds to a smooth geometry without defects.

It is crucial for our purposes to point out that, due to the isometries of extended AdS space, one can further couple an arbitrary number of coincident membranes to the action (3.7). Indeed, instead of considering only the two singular surfaces (3.4), one can extend the domain of the 2-sphere polar coordinate from $0 \leq \theta \leq \pi$ to the interval

$$0 \leq \theta \leq (2N - 1)\pi, \quad N \geq 1. \quad (3.9)$$

As a result, the geometry $(\text{AdS}_{\pm}^{d+2}, g_{\pm})$ exhibits N coincident defects at the north pole of the 2-sphere, and N coincident defects at the south pole, all of them supporting a total of $2N$ singular membranes; for finite N , the action (3.7) becomes

$$I[\text{AdS}_{\pm}/\mathbb{Z}_n] = \frac{1}{16\pi G} \int_{\text{AdS}_{\pm}} d^{d+2}x \sqrt{|\det g_{\pm}|} (R - 2\Lambda) - \frac{N}{4G} \left(1 - \frac{1}{n} \right) [A(\Sigma_+) + A(\Sigma_-)], \quad (3.10)$$

where $A(S)$ denotes the area of the hypersurface S . Evidently, θ can be extended as to run over the entire semipositive real line $\theta \in [0, \infty]$, which translates into an infinite number of coupled membranes; this limit will be essential in regularizing the entropy divergence in even dimensions. A cartoon of the geometry with membranes is given in Fig. 3.

IV. ENTANGLEMENT ENTROPY

The coupling of an arbitrary number of membranes to the gravity action (3.10) on the glued, extended space $\text{AdS}_{\pm}$ would not have been possible on a single copy of AdS. The appearance of a new parameter, N , counting the number of membranes, amounts to taking the tensionless limit of these

without neglecting their contribution to the action of the system. Indeed, it is direct to see that the Nambu-Goto terms in (3.10) remain present in the tensionless limit $n \rightarrow 1$ when $N \rightarrow \infty$, and vanish for finite N .

As we will next see, the limit of a large number of membranes will play an important role when computing the entanglement entropy for an entangling surface consisting of the two antipodal stacks of membranes. To begin with, we assume there exists some UV-complete gravity theory on the extended space $(\text{AdS}_{\pm}, g_{\pm})$, whose low-energy limit is well approximated by Einstein theory. The classical approximation of the partition function of this theory is then given by the Euclidean gravity action

$$\log Z[\text{AdS}_{\pm}] \approx -I[\text{AdS}_{\pm}]. \quad (4.1)$$

Note that under this assumption and in this approximation, no details regarding the full quantum theory are required.¹

We next consider a meridian circle C at constant azimuthal angle, illustrated on the right of Fig. 3, and take the bipartition $C = C_A \cup C_B$, where C_A and C_B are the two semicircles with boundary points the north and south poles of the original two-sphere. Consequently, the product

$$C \times \Sigma = (C_A \times \Sigma) \cup (C_B \times \Sigma) \quad (4.2)$$

defines a bipartition into two regions that we assume to be entangled, say $A = C_A \times \Sigma$ and $B = C_B \times \Sigma$, where the entanglement surface coincides with the two antipodal sets of membranes, that is

$$\partial A = \partial B = \Sigma_+ \cup \Sigma_-. \quad (4.3)$$

A similar entangling region, with an entangling surface given by two antipodal points on a holographic boundary two-sphere, was considered in [20].

We further attach to $C \times \Sigma$ a Hilbert space H that factorizes² as $H = H_A \otimes H_B$, and consider a state $|\Psi\rangle \in H$ with reduced density matrix $\rho = \text{Tr}_{H_B} |\Psi\rangle\langle\Psi|$, which we take to be normalized.

We are interested in computing the $n \rightarrow 1$ limit of the Rényi entropies

¹While the full quantum dynamics of gravity on manifolds without boundary remains largely unexplored, this assumption is common in studies of gravitational entropy and is consistent with the effective field theory approach to quantum gravity, where one expects universal IR behavior irrespective of UV completion [19]. The assumption has also proven to be useful in extracting reliable physical predictions at energy scales far below the Planck scale.

²The assumption that the Hilbert space factorizes is known to be subtle in gauge theories and gravity, where local degrees of freedom cannot always be sharply localized. However, in semi-classical settings and for entanglement across geometrically separated regions, this assumption is often made as an effective approximation [21–23].

$$S^{(n)} = \frac{1}{1-n} \log \text{Tr} \rho^n. \quad (4.4)$$

To this end we employ the replica method [7], which in general gives

$$\text{Tr} \rho^n = \frac{Z[B_n]}{Z[B]^n}, \quad (4.5)$$

where B_n denotes the n -fold branched cover of (4.2). In our case, since C is a circle partitioned into two semicircles, the branched cover of $C \times \Sigma$ is built simply by gluing n copies of $C \cong S^1$ along their north and south poles. Crucially, the resulting manifold has the topology of $S^2 \times \Sigma$, which is exactly the topology of the AdS extension (2.25), that is

$$B_n = \text{AdS}_\pm. \quad (4.6)$$

Note that the branched cover B_n is not singular but it admits the action of $\mathbb{Z}_n$, just as $\text{AdS}_\pm$ does. Of course, once this action is implemented the resulting manifold $B_n/\mathbb{Z}_n = \text{AdS}_\pm/\mathbb{Z}_n$ is conically singular.

From (4.5) and (4.6), it follows that the entropies (4.4) are given, in the semiclassical approximation (4.1), by

$$\begin{aligned} S^{(n)} &= \frac{1}{n-1} (I[B_n] - nI[B]) \\ &= \frac{n}{n-1} (I[\text{AdS}_\pm/\mathbb{Z}_n] - I[\text{AdS}_\pm]), \end{aligned} \quad (4.7)$$

where the last equality made use of the relation $I[B_n] = nI[B_n/\mathbb{Z}_n]$; it follows that the entanglement entropy

$$S_E = \lim_{n \rightarrow 1} S^{(n)} = n^2 \partial_n I[\text{AdS}_\pm/\mathbb{Z}_n]|_{n=1}. \quad (4.8)$$

Consequently, the calculation of the entropy (4.8) reduces to the calculation of the on-shell, Euclidean action $I[\text{AdS}_\pm/\mathbb{Z}_n]$ on the singular manifold $\text{AdS}_\pm/\mathbb{Z}_n$. As spelled out in Sec. III B and by means of the extension (3.9), this action includes the coupling of $2N$ membranes. Using the metric (3.1) and the values

$$R = -\frac{(d+1)(d+2)}{L^2}, \quad \Lambda = -\frac{d(d+1)}{2L^2}, \quad (4.9)$$

for the Ricci scalar R and cosmological constant Λ , both in dimension $d+2$, the Euclidean³ on-shell value of (3.10) is

³The Euclidean action carries an overall minus sign compared to the Lorentzian functional (3.10).

$$\begin{aligned} I[\text{AdS}_\pm/\mathbb{Z}_n] &\approx \frac{d+1}{4nG} \left[\int_0^\pi \frac{\sin \theta}{|\cos \theta|^{d+2}} d\theta \right] A(\Sigma) \\ &\quad + \frac{N}{4G} \left(1 - \frac{1}{n} \right) [A(\Sigma_+) + A(\Sigma_-)]. \end{aligned} \quad (4.10)$$

In the previous expression we have denoted by $A(\Sigma) = \int dy^d \sqrt{|\det h|}$, which, due to the form of the metric (3.1) equals the areas $A(\Sigma_+)$ and $A(\Sigma_-)$.

Note that the integral in θ in the first term of (4.10) blows up along the overlapped boundaries located at $\theta = \pi/2$; in terms of the defining function $u = \cos \theta$, the (principal value of the) singular integral is given by

$$\begin{aligned} \int_0^\pi \frac{\sin \theta}{|\cos \theta|^{d+2}} d\theta &= \int_{-1}^{-\varepsilon} \frac{du}{|u|^{d+2}} + \int_\varepsilon^1 \frac{du}{|u|^{d+2}} \\ &= \frac{2}{d+1} \left(\frac{1}{\varepsilon^{d+1}} - 1 \right) \end{aligned} \quad (4.11)$$

where $\varepsilon > 0$ is a boundary cutoff.

By virtue of the integral (4.11), the on-shell action (4.10) becomes

$$\begin{aligned} I[\text{AdS}_\pm/\mathbb{Z}_n] &\approx \left[\frac{1}{n} \left(\frac{1}{\varepsilon^{d+1}} - 1 \right) + N \left(1 - \frac{1}{n} \right) \right] \\ &\quad \times \frac{A(\Sigma_+) + A(\Sigma_-)}{4G}. \end{aligned} \quad (4.12)$$

It follows that the entropy (4.8) in this case is given by

$$S_E = \left(1 + N - \frac{1}{\varepsilon^{d+1}} \right) \frac{A(\Sigma_+ \cup \Sigma_-)}{4G}. \quad (4.13)$$

We observe that the entropy above depends on the boundary cutoff ε in a way that resembles the usual leading, short-distance divergence of any entanglement entropy in dimension $d+2$, namely $1/\varepsilon^{d+1}$. However, in the present case, the freedom to couple an arbitrary number of membranes to the system amounts to take the double limit $N \rightarrow \infty$ and $\varepsilon \rightarrow 0$ in a controlled way such that the divergences $N - 1/\varepsilon^{d+1}$ cancel each other. To this end, it suffices to take $N = 10^k$ and $\varepsilon = 10^{-k/(d+1)}$, with $k \rightarrow \infty$.

It is important to point out that, according to (3.9), the $N \rightarrow \infty$ limit requires one to extend the interval of the polar coordinate θ to the whole semipositive real line. It is also crucial to observe that this cancelation of the divergences is only consistent in the tensionless limit $n \rightarrow 1$, in which the coupling constant $N(1 - 1/n)$ in the action (3.10) remains finite; when this is the case the entanglement entropy

$$S_E = \frac{A(\Sigma_+ \cup \Sigma_-)}{4G}, \quad (4.14)$$

which is exactly the area law for the set of fixed points $\Sigma_+ \cup \Sigma_-$.

Finally, we would like to emphasize that although the above mechanism for canceling divergences may appear fine-tuned, it reflects a physically motivated balance. In the tensionless limit $n \rightarrow 1$, the membranes become nondynamical, yet their collective backreaction survives in the large- N limit. This suggests that the extended geometry continues to encode a nontrivial contribution to the entanglement entropy, while avoiding the short-distance singularities that typically arise in local quantum field theory.

V. CONCLUSIONS

In this work, we introduced a novel geometric framework for studying entanglement entropy in gravitational systems that do not rely on holography. By gluing together two copies of pure AdS spacetime along their conformal boundaries we constructed a manifold without boundary, which we denoted by $\text{AdS}_{\pm}^{d+2}$, and which exhibits the topology of $S^2 \times \Sigma^d$, where Σ^d is a negatively curved manifold without boundary. This extended space admits a natural coupling to codimension-two membranes, which appear as fixed-point minimal surfaces under the action of a discrete rotational symmetry acting on the two-sphere.

Within this setup, we studied the semiclassical entanglement entropy across an entangling surface defined by two antipodal stacks of N coincident membranes. Our main result is that the entropy obeys a universal area law, analogous to the Ryu–Takayanagi formula, yet derived entirely from bulk gravitational dynamics, without invoking any boundary dual description. Although a UV divergence inevitably arises from the near-boundary region, we showed that this divergence can be canceled in a controlled manner by taking the limit of an infinite number of membranes, thereby regularizing the entropy and restoring the area law. This large- N mechanism for canceling short-distance divergences is the main insight of this work.

This construction provides a demonstration that area laws can arise from the intrinsic geometry of the bulk spacetime alone, independent of the existence of black

holes or a holographic correspondence. The possibility of capturing Ryu–Takayanagi-type entropy purely from within the gravitational path integral on a manifold without boundary suggests a more universal origin for the relation between entanglement entropy and minimal surfaces, one that goes beyond holography.

There are several directions that would be worth exploring further. In particular, it remains an open question whether the large- N mechanism responsible for the cancellation of divergences admits a deeper interpretation—arguably in terms of coarse-graining over microscopic degrees of freedom localized at the defects. Another important question is whether this setup can be embedded in, or derived from, a UV-complete theory of gravity. It would be interesting to investigate whether the extended AdS geometry introduced here can be realized, for instance, as the near-horizon limit of a brane configuration in string theory or M theory.

Ultimately, the results presented here support the idea that entanglement entropy and area laws may be deeply rooted in the geometric and topological structure of spacetime, regardless of whether a boundary theory exists. Our doubled AdS construction offers a concrete and tractable model in which these ideas can be further tested and developed.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

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