Letter

Celestial symmetries of black hole horizons

Romain Ruzziconi[1,*] and Céline Zwikel[2,†]

[1]*Mathematical Institute, University of Oxford, Andrew Wiles Building, Radcliffe Observatory Quarter, Woodstock Road, Oxford, OX2 6GG, United Kingdom*

[2]*Perimeter Institute for Theoretical Physics, 31 Caroline Street North, Waterloo, Ontario N2L 2Y5, Canada*

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We establish a correspondence between the gravitational phase space at null infinity and the subleading phase space near a finite-distance null hypersurface, such as a black hole horizon. Within this framework, we identify the celestial $Lw_{1+\infty}$ symmetries in the subleading phase space at the horizon by constructing their canonical generators and imposing self-duality conditions. This leads to an infinite tower of conserved charges in the absence of radiation, revealing new gravitational observables relevant to black hole physics.

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Introduction. Symmetries play a central role in physics, offering a powerful organizing principle. In four-dimensional (4D) gravity with asymptotically flat boundary conditions—relevant for modeling gravitational waves and astrophysical black holes—the symmetry algebra is the Bondi-Metzner-Sachs (BMS) algebra [1,2]. This infinite-dimensional extension of Poincaré symmetry includes supertranslations and superrotations [3–7], which place constraints on the infrared structure of scattering amplitudes [8]. The associated charges, constructed via a Noether procedure [9–11], encode important dynamical information through flux-balance laws at null infinity, such as the Bondi mass loss formula [1,12].

It was early understood that self-dual asymptotically flat spacetimes possess even more symmetries, as revealed by the Penrose nonlinear graviton construction in twistor theory [13,14] (see also [15–17]). In this framework, deformations of the complex structure on twistor space exhibit an $Lw_{1+\infty}$ symmetry, a kind of "higher-spin generalization" of BMS symmetry. These symmetries are powerful enough to render the self-dual sector of gravity integrable.

The $Lw_{1+\infty}$ algebra has also emerged in the context of celestial holography, a proposed framework for flat space holography where 4D asymptotically flat gravity is conjectured to be dual to a 2D conformal field theory living on the celestial sphere. Within this setting, the celestial operator product expansions [18–20], derived from the collinear limit of graviton amplitudes, were shown to organize into the $Lw_{1+\infty}$ algebra [21,22]. This structure now plays a central role in the formulation of flat space holograms [23,24].

As in the case of BMS symmetries, it is natural to seek canonical generators for the $Lw_{1+\infty}$ symmetries via a Noether-like construction. Recent progress has achieved this both from a spacetime perspective [25,26], using the Ashtekar-Streubel radiative phase space at null infinity [27], and from a twistor theory approach [28]. These symmetries organize the radial expansion of the metric and are closely related to gravitational multipole moments [29].

A central motivation for studying gravity in asymptotically flat spacetimes, and for developing flat space holography, lies in the desire to understand black holes in the sky [30], which may carry essential clues toward a quantum theory of gravity. Black hole symmetries have been shown to account for the Bekenstein-Hawking entropy formula in 3D gravity [31], and this perspective has been extended to 4D Kerr black holes [32,33]. Analogs of BMS symmetries at black hole horizons [34–40]—often referred to as "soft hair"—have been proposed to play a key role in the microscopic understanding of black hole entropy. However the action of $Lw_{1+\infty}$ symmetries is yet to be unveiled at the horizon and is expected to play a major part in this discussion.

An important challenge in this endeavor is that the geometry of a null hypersurface at finite distance is much more complicated that the one at null infinity: the induced metric has to be time dependent to encode radiative degrees of freedom and the dynamics is encoded in Raychaudhuri [41] and Damour [42,43] equations. This drastically complicates the phase space discussion and does not allow

*Contact author: romain.ruzziconi@maths.ox.ac.uk

†Contact author: czwikel@perimeterinstitute.ca

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one to directly import the results from null infinity to the horizon. Relations between null infinity and bulk null hypersurfaces have recently been discussed in [44–46].

The goal of this Letter is to identify the celestial $Lw_{1+\infty}$ symmetries of black hole horizons.[1] More precisely, we establish two main results: (i) We demonstrate a precise correspondence between the Ashtekar-Streubel phase space at null infinity and the subleading phase space near a null hypersurface, such as a black hole horizon. This is achieved using Penrose's conformal compactification, the behavior of the Newman-Penrose (NP) formalism under Weyl rescalings, and covariant phase space techniques. (ii) Leveraging this correspondence, we identify the $Lw_{1+\infty}$ charges at the horizon and show that, upon imposing self-duality conditions, their fluxes integrated over the horizon yield the canonical generators of the celestial symmetries. In particular, in the absence of radiation through the horizon, these charges are conserved, giving rise to an infinite tower of observables relevant for black hole physics.

This Letter is complemented by a companion paper [47], which provides a comprehensive discussion of the null hypersurface phase space and detailed derivations of some of the results presented here.

NP formalism at null infinity. In this section, we review salient features of the phase space at null infinity using NP formalism [48] (we follow the notations and conventions of [26,49]) and list the key ingredients to identify the $Lw_{1+\infty}$ symmetries in that setup.

The NP formalism is a powerful framework based on the use of a null tetrad $(\ell, n, m, \bar{m})$ in the first-order Cartan formulation of general relativity,

$$g_{\mu\nu} = -\ell_\mu n_\nu - \ell_\nu n_\mu + m_\mu \bar{m}_\nu + m_\nu \bar{m}_\mu. \tag{1}$$

Notably, this formalism has been applied in asymptotically flat spacetime at null infinity [50] to derive the solution space and discuss the characteristic initial value problem. The Newman-Unti tetrad has the following expansion near $\mathscr{I} \equiv \{r_{\mathscr{I}} \to \infty\}$:

$$\begin{aligned} &\ell = -\partial_{r_{\mathscr{I}}}, \\ &n = (-1 + \mathcal{O}(r_{\mathscr{I}}^{-1}))\partial_v + \mathcal{O}(r_{\mathscr{I}})\partial_{r_{\mathscr{I}}} + \mathcal{O}(r_{\mathscr{I}}^{-1})^A \partial_A, \\ &m = \frac{1}{r_{\mathscr{I}}} m_0^A \partial_A + \mathcal{O}(r_{\mathscr{I}}^{-2}), \qquad \bar{m} = \frac{1}{r_{\mathscr{I}}} \bar{m}_0^A \partial_A + \mathcal{O}(r_{\mathscr{I}}^{-2}) \end{aligned} \tag{2}$$

where v is the null time along $\mathscr{I}$ and the two-dimensional boundary metric $q_{AB} = m_A^0 \bar{m}_B^0 + \bar{m}_A^0 m_B^0$ is usually fixed on the phase space. The components of the Weyl tensor expressed in this tetrad are required to obey the Peeling theorem

$$\Psi_n = \frac{\Psi_n^0}{r_{\mathscr{I}}^{5-n}} + \mathcal{O}(r_{\mathscr{I}}^{n-6}). \tag{3}$$

In particular the leading component Ψ_4^0 is purely radiative. Furthermore, the spin coefficients σ and λ, which play a crucial role in the radiative phase space at $\mathscr{I}$, have the following expansions:

$$\sigma = \frac{\sigma_0}{r_{\mathscr{I}}^2} + \mathcal{O}(r_{\mathscr{I}}^{-3}), \qquad \lambda = \lambda_0 + \frac{\lambda_1}{r_{\mathscr{I}}} + \mathcal{O}(r_{\mathscr{I}}^{-2}). \tag{4}$$

On shell, we have $\lambda_0 = \partial_v q_{AB} \bar{m}_0^A \bar{m}_0^B = 0$, which is the well-known statement that $\mathscr{I}$ has no intrinsic shear. Additionally, we have $\lambda_1 = \partial_v \bar{\sigma}_0$, which corresponds to the Bondi news [1].

Two key ingredients to identify the celestial symmetries at $\mathscr{I}$ from a spacetime perspective [25,26,28] are the following:

(i) The recursion relations encoding the hierarchies of the integrable self-dual sector of gravity are the starting point of the analysis,

$$\begin{aligned} -\partial_v Q_s &= \eth Q_{s-1} - (s+1)\sigma_0 Q_{s-2} \\ s &= -1, 0, 1, 2, 3, \ldots \end{aligned} \tag{5}$$

where $\eth$ encodes angular derivative on the sphere [51]. Each Q_s for $s > -2$ corresponds to a charge aspect of spin weight s and $Q_{-2} \equiv \Psi_4^0 = \partial_v \lambda_1$ is the radiation responsible for the nonconservation. These recursion relations correspond to an infinite tower of flux-balance laws at null infinity. For $s = -2, \ldots, 2$, we have $Q_s = \Psi_{2-s}^0$ and Eq. (5) follows from the Bianchi identities. For $s > 2$, the Q_s's appear in the $1/r_{\mathscr{I}}$ expansion of Ψ_0. Self-dual gravity and full gravity exhibit the same pattern only at leading orders near null infinity. In [26], deviations with respect to the self-dual sector were observed for $s \geq 4$, and obtaining (5) from the full gravity theory requires a truncation of the Bianchi identities.

(ii) The Ashtekar-Streubel symplectic structure on the radiative phase space [27] can be obtained from standard covariant phase space methods [9] by pushing a Cauchy slice to $\mathscr{I}$,

$$\boldsymbol{\Omega}_{\mathscr{I}} = \frac{1}{8\pi G} \int_{\mathscr{I}} dv d^2x \sqrt{q} \delta\lambda_1 \wedge \delta\sigma_0 + \text{c.c.} \tag{6}$$

where c.c. denotes the complex conjugate terms. The corresponding Poisson bracket is used to compute the $Lw_{1+\infty}$ charge algebra.

[1] Most of the discussion in this Letter applies to generic null hypersurfaces, not only to black hole horizons. For conciseness, we will slightly abuse the terminology of "horizons" throughout the text, as our ultimate goal is to apply this analysis to black hole horizons.

In the next two sections we find the analog of these two statements at the horizon.

From null infinity to the horizon. The derivation of the solution space in the NP formalism around a null hypersurface at finite distance will be addressed in full details in [47] (see also [52] for preliminary results in the nonradiative sector). As mentioned in the Introduction, one of the important technical issues is that the intrinsic geometry of the null hypersurface contains genuine gravitational degrees of freedom, by contrast with null infinity. However, as we shall now explain, the use of the NP formalism allows us to drastically simplify the analysis and identify the part of the solution space that is connected to null infinity via Penrose's conformal compactification [53–56].

One of the advantages of the NP formalism [48] is to trade tensor fields on spacetime for simpler spin and boost weighted scalars. Additionally, consider the rescaling of the tetrad [57]

$$\ell^\mu \to \Omega^{-2}\ell^\mu, \qquad n^\mu \to n^\mu,$$
$$m^\mu \to \Omega^{-1}m^\mu, \qquad \bar{m}^\mu \to \Omega^{-1}\bar{m}^\mu \tag{7}$$

inducing a Weyl rescaling of the metric $g_{\mu\nu} \to \Omega^2 g_{\mu\nu}$, $\Omega > 0$. A weighted scalar η has well-defined Weyl weight W if it transforms as $\eta \to \Omega^W \eta$ under (7). Examples of weighted scalars are given by the components of the Weyl tensors and the spin coefficients λ and σ,

$$\Psi_n : W = -(5-n), \qquad \sigma : W = -2, \qquad \lambda : W = 0. \tag{8}$$

Acting with the bare derivative operators ∂_v and $\eth$ on weighted scalars does not produce well-defined Weyl weighted scalars. To remedy this issue, one can introduce the conformal Geroch-Held-Penrose (GHP) derivative operators [57,58],

$$-\partial_v \to \text{Þ}'_{\mathscr{C}}, \qquad \eth \to \eth_{\mathscr{C}} \tag{9}$$

constructed from the bare operators by correcting them with spin coefficients, playing the role of a Weyl connection, so that they have well-defined weights. In [47], we display the explicit expression of these correction terms.

We now have all the tools in hand to map the relevant part of the solution space at null infinity to the one at the horizon. We implement the Penrose conformal compactification [53–56] on the (partially) "off-shell" radial expansions presented in the previous section, as the Einstein equations themselves are not Weyl covariant. Starting from the radial expansion of the tetrad (2) and performing the compactification with conformal factor Ω, see (7), we find

$$\ell \to \Omega^{-2}\partial_{r_{\mathscr{I}}} = \partial_r, \qquad n \to \Omega^0 n = -\partial_v + \mathcal{O}(r)$$
$$m \to \Omega^{-1}\left[\frac{1}{r_{\mathscr{I}}} m_0^A \partial_A + \mathcal{O}(r_{\mathscr{I}}^{-1})\right] = m_0^A \partial_A + \mathcal{O}(r), \tag{10}$$

and idem for $\bar{m}$. We identified the conformal factor $\Omega \sim r_{\mathscr{I}}^{-1} \sim r$ with the radial coordinate $r \geq 0$ ($r = 0$ is the locus of a finite-distance null hypersurface $\mathcal{H}$), and $v \in \mathbb{R}$ is now the null time along $\mathcal{H}$. By contrast with null infinity, the leading terms m_0^A and $\bar{m}_0^A$ characterizing the intrinsic geometry of $\mathcal{H}$ are now part of the phase space and encode genuine radiative degrees of freedom. Applying the conformal transformation on the Peeling theorem (3), we find the Taylor expansion

$$\Psi_n \to \Psi'_n = \Omega^{n-5}\left[\frac{\Psi_n^0}{r_{\mathscr{I}}^{5-n}} + \mathcal{O}(r_{\mathscr{I}}^{n-6}))\right]$$
$$= \Psi_n^0 + \mathcal{O}(r) \tag{11}$$

where we used the Weyl weights of Ψ_n in (8). By analogy with null infinity, we will show that Ψ_4^0 contains information on the radiation going through the horizon. Finally, the spin-coefficient expansions (4) are mapped onto the following expansions at finite distance:

$$\sigma \to \sigma' = \sigma_0 + \mathcal{O}(r), \quad \lambda \to \lambda' = \lambda_0 + r\lambda_1 + \mathcal{O}(r^2). \tag{12}$$

A key difference compared to null infinity is that, on shell, λ_0 does not generically vanish at finite distance—it coincides with the intrinsic shear of the null hypersurface and we have $\Psi_4^0 = -\text{Þ}'_{\mathscr{C}}\lambda_0$.

In the NP formalism, the tetrad and spin-coefficient equations are not Weyl covariant, but the recursion relations are. Indeed, using the conformal GHP operators (9), one can "covariantize" the recursion relations (5) to obtain

$$\text{Þ}'_{\mathscr{C}} Q_s = \eth_{\mathscr{C}} Q_{s-1} - (s+1)\sigma_0 Q_{s-2}. \tag{13}$$

This covariantization is harmless at null infinity, as all the correction terms involved in this procedure vanish at $\mathscr{I}$. Exploiting the Weyl covariance of these expressions, we can perform the conformal rescaling and reinterpret these equations at the horizon. However, at finite distance, the correction terms in the GHP derivative operators with respect to the bare operators are crucial as the intrinsic geometry at the horizon is nontrivial. For example, one of these corrections involves the surface gravity of the black hole. Hence, (13) constitutes the recursion relations at the horizon and, in particular, $Q_s = \Psi_{2-s}^0$, $s = -2, \ldots, 2$ are now the components of the Weyl tensor in the Taylor expansion (11), and the Q_s's for $s > 2$ appear in the expansion of Ψ_0. Notice that, for spin $s = -1, 0, 1, 2$, (13) are the Bianchi identities at finite distance.

Subleading phase space. As mentioned in the section on NP formalism at null infinity, a key ingredient to identify the $Lw_{1+\infty}$ symmetries from a spacetime perspective at null infinity are self-duality conditions. These are necessary only at subleading order in the radial expansion of the spin coefficients and recursion relations, where the deviation between self-dual and full gravity starts to occur [25,26]. At finite distance, self-duality conditions are already visible at leading order in r since the intrinsic geometry on the null hypersurface contains genuine degrees of freedom involving both helicities. We propose conditions on the spin coefficients,

$$\mu_0 = 0, \qquad \bar{\lambda}_0 = 0, \qquad \mathrm{Re}(\gamma_0) = \text{constant} \tag{14}$$

where μ_0 is the expansion of $\mathcal{H}$ and $\mathrm{Re}(\gamma_0) = -2\kappa$ with κ the surface gravity. Here we work in the usual complexified setup where the functions and their barred versions are complex and independent (e.g. λ_0 and $\bar{\lambda}_0$ are not complex conjugate). The conditions (14) ensure that $[P'_{\mathscr{C}}, \eth_{\mathscr{C}}] = 0$. Furthermore, we will require the following constraints on the variations:

$$\delta m_A^0 = 0, \quad \delta\mathrm{Re}(\gamma_0) = 0, \quad \delta\sqrt{q} = 0, \quad \delta\tau_0 = 0 \tag{15}$$

where τ_0 is related to the twist of $\mathcal{H}$. One can check that the above conditions (14) and (15) are consistent with each other through the NP equations for the spin coefficients [48] and imply self-duality conditions eliminating one of the two graviton helicities. Generically, $\Psi_4^0 = -P'_{\mathscr{C}}\lambda_0 \neq 0$, so that there is still one helicity radiating through the horizon.

The Einstein-Hilbert symplectic structure $\boldsymbol{\Omega}$ can be obtained from the standard covariant phase space methods [9]. For the radial expansion near a null hypersurface obtained in the previous section, we have

$$\boldsymbol{\Omega} = \boldsymbol{\Omega}^{(0)} + r\boldsymbol{\Omega}^{(1)} + \mathcal{O}(r^2) \tag{16}$$

where $\boldsymbol{\Omega}^{(0)}$ is the usual leading symplectic structure discussed in previous literature, see e.g. [40], and $\boldsymbol{\Omega}^{(1)}$ is the subleading symplectic structure. Remarkably, under the conditions (14) and (15), the leading symplectic structure vanishes, $\boldsymbol{\Omega}^{(0)} = 0$, and we find

$$\boldsymbol{\Omega} = \frac{r}{8\pi G}\oint\int dv\,\delta\sigma_0 \wedge \delta\lambda_0 + \delta\bar{p}^A \wedge \delta\bar{m}_A^0 + \mathcal{O}(r^2) \tag{17}$$

where $\oint = \int d^2x\sqrt{q}$. The first term is the analog of the Ashtekar-Streubel symplectic structure (6) at infinity for one helicity sector (i.e. there is no c.c. term). An important difference compared to the canonical pair at $\mathscr{I}$ is that, on shell, $\lambda_0 = -\bar{m}_0^A P'_{\mathscr{C}}\bar{m}_A^0$ is the intrinsic shear of $\mathcal{H}$ and is not related to the extrinsic shear σ_0. The bracket between these symplectic variables can be deduced by inverting the symplectic structure

$$\{\sigma_0(v_1, x_1^A), \lambda_0(v_2, x_2^A)\} = \frac{8\pi G}{r\sqrt{q}}\delta^2(x_1^A - x_2^A)\delta(v_1 - v_2). \tag{18}$$

The second symplectic pair in (17) involves $\bar{p}^A$, whose explicit expression in terms of the spin coefficients will be provided in [47], and $\bar{m}_A^0$. As we will see in the next section, the latter canonical variable does not appear in the $Lw_{1+\infty}$ symmetry generators. Hence, it will not play any role in the computation of the $Lw_{1+\infty}$ charge algebra.

This concludes the identification between the leading Ashtekar-Streubel phase space at $\mathscr{I}$ and the subleading phase space at the horizon, which is the first main result of this Letter. We will exploit this direct connection in the next section to identify the celestial symmetries of the horizon.

Canonical $Lw_{1+\infty}$ symmetries. The construction of the $Lw_{1+\infty}$ surface charges H_s at the horizon follows a similar heuristic procedure than the one used at null infinity [25,26], and we refer to [47] for details. We require the following two criteria: (i) the charges are conserved in the absence of radiation characterized by $\Psi_4^0 = 0$, and (ii) their associated integrated fluxes are the canonical generators of $Lw_{1+\infty}$ symmetries in the subleading phase space at the horizon. The general form of these $Lw_{1+\infty}$ surface charges, defined on a cut $v =$ constant of a $r =$ constant hypersurface close to the horizon is

$$H_s = \frac{r}{8\pi G}\oint T_s Q_s + \text{correction terms}. \tag{19}$$

The charges are labeled by $s = -1, 0, 1, \ldots$, the spin weight of their associated charge aspect Q_s. The T_s's are the associated symmetry parameters. These are weighted scalars satisfying $P'_{\mathscr{C}}T_s = 0$, generalizing the time-independence condition of the parameters at null infinity. The global factor r indicates that these charges are associated with the subleading phase space at the horizon (17). The correction terms in (19) are determined by requiring the criterium of conservation (i). The explicit form of these correction terms can be found in [47] up to spin 2. Using the recursion relations (13), the commutation properties of the GHP operators (9) (see [57,58]), and the self-duality conditions (14), we show that

$$\Psi_4^0 \equiv Q_{-2} = 0 \quad \Rightarrow \quad \frac{dH_s}{dv} = 0 \tag{20}$$

for all $s \geq -1$. Ψ_4^0 is responsible for the nonconservation of the charges and can thus be identified with the radiation through the horizon. In the absence of it, we have an infinite tower of conserved charges.

Analogous to the situation at null infinity, we can show that the symmetries $s = 0$ and $s = 1$ are related to the supertranslation and superrotations symmetries at the horizon [37–39] and refer to [47] for details. However,

our charges are associated with the subleading phase space and are thus of different nature than those considered in previous literature. Furthermore, the higher-spin charges provide an infinite tower of new symmetries and observables that do not have a direct diffeomorphism interpretation. For the Kerr solution, we have $H_{-1}^{\mathrm{Kerr}}=0$, and H_0^{Kerr} encodes the mass parameter m while H_1^{Kerr} probes its angular momentum parameter a. H_2^{Kerr} is nonvanishing and involves a combination of m and a. We expect these charges to be interesting observables for an observer sitting just outside of the horizon.

The integrated fluxes are defined by

$$\mathcal{F}_s=\int dv\partial_v H_s=H_s|_{v=+\infty}-H_s|_{v=-\infty} \tag{21}$$

and their explicit expressions appear in [47] up to spin 2. The integral over v converges for all s if, as in [25,26,28,59], we impose Schwarzian falloffs: $\lim_{v\to\pm\infty}\lambda_0\sim e^{-|v|^2}$. This is consistent with the action of the $Lw_{1+\infty}$ symmetries on the phase space if the parameters satisfy the wedge condition: $\eth_{\mathscr{C}}^{s+2}T_s=0$. Under this assumption, the soft terms in the fluxes vanish and, in particular, $\mathcal{F}_{-1}=0$.

Finally, using (18) and imposing the self-duality conditions (14) and (15), the computation of the algebra follows similar steps as those performed at null infinity [25,26] (see also [28,60,61] for nonperturbative results). We find

$$\begin{aligned}&\{\mathcal{F}_{T_{s_1}},\mathcal{F}_{T_{s_2}}\}=\mathcal{F}_{T_{s_1+s_2-1}},\\&T_{s_1+s_2-1}=(s_2+1)T_{s_2}\eth_{\mathscr{C}}T_{s_1}-(s_1+1)T_{s_1}\eth_{\mathscr{C}}T_{s_2}\end{aligned} \tag{22}$$

where we noted $\mathcal{F}_s\equiv\mathcal{F}_{T_s}$. This corresponds to a representation of the $Lw_{1+\infty}$ algebra in terms of weighted scalars, hence satisfying the requirement (ii). Therefore, we have shown that $\mathcal{F}_s$ are the canonical generators for the $Lw_{1+\infty}$ symmetries at the horizon.

Discussion. In this Letter, we have established a map between the Ashtekar-Streubel phase space at null infinity and the subleading phase space at the horizon. Using this correspondence, we have imported the celestial symmetries from null infinity to the horizon. This work contributes to relating physics at null infinity and at finite distance.

Indeed, our work provides a clear setup to apply the ideas of celestial holography to black hole physics. For instance, it would be rewarding to relate our analysis with [62] where a holographic state for a self-dual black hole is provided, and understand the precise role of the subleading phase space in this context.

The celestial $Lw_{1+\infty}$ symmetries at null infinity possess a natural interpretation on twistor space. It would be interesting to derive the surface charge expressions obtained here from first principles using similar method as in [28]. This would require defining an analog of asymptotic twistor space adapted to black hole horizons.

As mentioned in the Introduction, from the point of view of the horizon, symmetries corresponding to soft hair could account for black hole entropy. It would be interesting to study whether the $Lw_{1+\infty}$ symmetries play a role in this discussion.

Finally, our construction provides meaningful observables for an observer outside of the black hole, but close enough to probe the first subleading order of the metric. It would be interesting to understand the significance of the $Lw_{1+\infty}$ charges in the recent developments of black hole physics, such as the analysis of the photon ring [30,63–65] or the use of multipole moments at the horizon in numerical relativity [66–69].

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