

Decays $f_1 \rightarrow a_0\pi, \pi\pi\pi(\eta)$ and $f_1 \rightarrow KK\pi$ in the chiral $U(3) \times U(3)$ quark NJL model

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The branching fractions of the axial vector meson decays $f_1 \rightarrow a_0\pi, \pi\pi\eta, f_1 \rightarrow \rho\pi, \pi\pi\pi$, and $f_1 \rightarrow KK\pi$ are calculated in the standard $U(3) \times U(3)$ quark Nambu–Jona-Lasinio model. The intermediate channels with the states $a_0(980)\pi$ and $f_0(500)\eta$ are taken into account in the decay $f_1 \rightarrow \pi\pi\eta$. In the case of the scalar mesons, the $\bar{q}q$ representation as a chiral symmetric partners of the pseudoscalar mesons is used. It is shown that the decays $f_1 \rightarrow \rho\pi$ and $f_1 \rightarrow 3\pi$ occur due to the mass difference of the u and d quarks. All the results are obtained without using any additional arbitrary parameters and are in satisfactory agreement with the known experimental data.

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I. INTRODUCTION

Axial vector mesons play an important role in the processes of hadron interactions at low energies. The theoretical analysis of various mesonic τ lepton decays with strange and nonstrange mesons in the final state showed that axial vector channels are dominant in the calculation of decay widths [1–5]. In particular, in the τ lepton decay into three π mesons, the main contribution is given by the axial vector channel with the intermediate meson $a_1(1260)$ [6–12]. At the same time, the τ decays with the production of the axial vector meson $\tau \rightarrow f_1(1285)\pi\nu_\tau$ are intensively investigated from both theoretical and experimental points of view [13–16]. The properties and decays of the axial vector meson $f_1(1285)$ were studied based on the π meson scattering on nucleons $\pi^-N \rightarrow N\pi^-(\pi^+\pi^-\pi^0, \pi^+\pi^-\eta)$ [17]. Based on the analysis of experimental data, the estimations for the decay widths ratio $\Gamma(f_1 \rightarrow \pi^+\pi^-\pi^0)/\Gamma(f_1 \rightarrow \pi^+\pi^-\eta)$ and the restrictions for the branching fraction $\text{Br}(f_1 \rightarrow \pi\rho) < 0.31\%$ were obtained. The recent investigations of the f_1 meson production in the pp collisions have been carried out by the LHCb and ALICE collaborations at CERN [18,19].

For theoretical study of low energy processes of meson interactions, various phenomenological models are applied,

which are usually based on the approximate chiral symmetry of strong interactions. One of the successful models of this type is the Nambu–Jona-Lasinio (NJL) model. The NJL model [20–32] is a phenomenological quark model based on the chiral group $U(3) \times U(3)$ that has successfully proved itself in describing the processes of meson interactions at low energies.

Significant progress has been achieved in the study of the anomalous decays of the axial vector mesons $a_1(1260)$ and $f_1(1285)$ [33–38]. In particular, within the NJL model the effective Lagrangians for the vertices AVV, AAAV, and the widths of the anomalous decays $f_1 \rightarrow \rho(\omega)\gamma, f_1 \rightarrow \pi\pi\gamma, f_1 \rightarrow \rho\pi\pi$, and $a_1 \rightarrow \omega\pi\pi$ were obtained in agreement with the known experimental data [35–37]. The theoretical prediction for the decay $f_1 \rightarrow \rho\gamma$ is compatible with the recent data from the CLAS collaboration [39] which deviate significantly from the average PDG data [13].

In the present work, the decays of the axial vector meson $f_1(1285)$ are considered in the framework of the NJL model. The branching fraction of the decay $f_1 \rightarrow a_0\pi$ is calculated, and the dominant role of the contribution of the scalar mesons in the description of the process $f_1 \rightarrow \pi\pi\eta$ is demonstrated. The decays $f_1 \rightarrow \rho\pi$ and $f_1 \rightarrow 3\pi$ that occur due to isospin breaking are also considered. It is shown that the branching fraction is mostly determined by the triangle quark diagram $f_1 \rightarrow \rho^\pm\pi^\mp$ occurring due to the difference of the mass of the u and d quarks. Besides, the decay of the f_1 meson with the strange particles $f_1 \rightarrow KK\pi$ is considered.

The paper is organized in the following way. The Lagrangian of the quark-meson interactions in the NJL model is presented in Sec. II. In Sec. III the decays $f_1 \rightarrow a_0\pi$ and $f_1 \rightarrow \pi\pi\eta$ are considered. In Sec. IV the suppressed decays $f_1 \rightarrow \rho\pi$ and $f_1 \rightarrow \pi\pi\pi$ are described.

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In Sec. V the amplitude and branching fractions of the decays with the strange mesons $f_1 \rightarrow KK\pi$ are given. The discussion of the results is given in the Conclusion.

II. LAGRANGIAN OF THE NJL MODEL

The NJL model is formulated based on the quantum chromodynamics (QCD) motivated local 4-quark chiral

interaction in the minimal order in $1/N_c$ (the mean field approximation) [23,24,28,32]. The Lagrangian has been obtained after the bosonization and renormalization in the one-loop quark approximation (see e.g. [32], Secs. 2.1 and 2.2). The fragment of the NJL model Lagrangian containing the quark-meson vertices needed for our calculations has the following form:

$$\begin{aligned} \Delta L_{\text{int}} = & \bar{q} \left\{ \sum_{i=0,\pm} \left[ig_\pi \gamma^5 \lambda_i^\pi \pi^i + ig_K \gamma^5 \lambda_i^K K^i + \frac{g_\rho}{2} \gamma^\mu \lambda_i^\rho \rho_\mu^i + \frac{g_{a_1}}{2} \gamma^\mu \gamma^5 \lambda_i^{a_1} a_{1\mu}^i + \frac{g_{K^*}}{2} \gamma^\mu \lambda_i^K K_\mu^{*i} + g_\sigma \lambda_i^{a_0} a_0^i + g_{K_0^*} \lambda_i^K K_0^{*i} \right] \right. \\ & + ig_K \gamma^5 \lambda_0^K \bar{K}^0 + \frac{g_{K^*}}{2} \gamma^\mu \lambda_0^K \bar{K}_\mu^{*0} + g_{K_0^*} \lambda_0^K \bar{K}_0^{*0} + \frac{1}{2} \gamma^\mu \gamma^5 (g_{f_1}^\mu \lambda_u^{f_1} \cos \phi - g_{f_1}^s \lambda_s^{f_1} \sin \phi) f_{1\mu} \\ & + \frac{1}{2} \gamma^\mu \gamma^5 (g_{f_1}^\mu \lambda_u^{f_1} \sin \phi + g_{f_1}^s \lambda_s^{f_1} \cos \phi) f'_{1\mu} + (g_\sigma \lambda_u^0 \cos \bar{\theta}_\sigma - g_\sigma^s \lambda_s^0 \sin \bar{\theta}_\sigma) f_0 \\ & \left. + i\gamma^5 (g_\eta^\mu \lambda_u^\eta \sin \bar{\theta}_\eta + g_\eta^s \lambda_s^\eta \cos \bar{\theta}_\eta) \eta \right\} q, \end{aligned} \quad (1)$$

where $q = (u, d, s)$ is a quark triplet, λ are the linear combinations of the Gell-Mann matrices, the isoscalar axial vector mesons $f_1 = f_1(1285)$ and $f'_1 = f_1(1420)$ are considered as mixed states, $\phi = 24^\circ$ is the mixing angle of the f_1 mesons [18], $\bar{\theta}_\sigma = \theta_0 - \theta_\sigma$ is the mixing angle of the f_0 mesons [40] and $\bar{\theta}_\eta = \theta_0 - \theta_\eta$ is the mixing angle of the η mesons [40], where $\theta_0 = 35.3^\circ$ is the ideal mixing angle and $\theta_\sigma = 24^\circ$, $\theta_\eta = -19^\circ$.

The scalar mesons are considered here as quark-antiquark states, being the chiral partners of the pseudoscalar mesons.

The coupling constants of the mesons with quarks are the result of the renormalization of the Lagrangian in the one-loop quark approximation

$$\begin{aligned} g_\sigma &= \sqrt{\frac{1}{4I_{20}}}, & g_\eta^\mu &\approx g_\pi = \sqrt{\frac{Z_\pi}{4I_{20}}}, & g_\rho &= g_{a_1} = g_{f_1}^\mu = \sqrt{\frac{3}{2I_{20}}}, & g_K &= \sqrt{\frac{Z_K}{4I_{11}}}, \\ g_\sigma^s &= \sqrt{\frac{1}{4I_{02}}}, & g_\eta^s &= \sqrt{\frac{Z_s}{4I_{02}}}, & g_{f_1}^s &= \sqrt{\frac{3}{2I_{02}}}, & g_{K^*} &= \sqrt{\frac{3}{2I_{11}}}, & g_{K_0^*} &= \sqrt{\frac{1}{4I_{11}}}, \end{aligned} \quad (2)$$

where Z_π , Z_s , and Z_K are the additional renormalization constants appearing as a result of taking into account transitions between the pseudoscalar and axial vector states

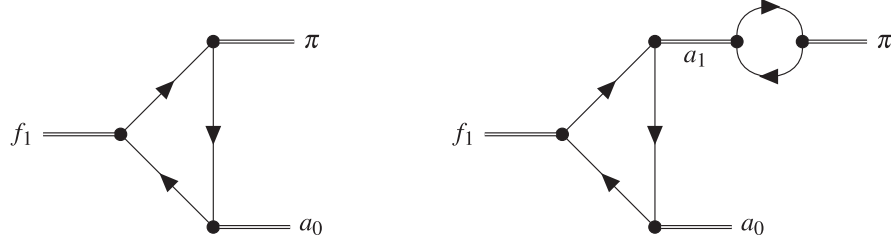
$$\begin{aligned} Z_\pi &= \left(1 - 6 \frac{m_u^2}{M_{a_1(1260)}^2} \right)^{-1}, & Z_K &= \left(1 - \frac{3(m_u + m_s)^2}{2M_{K_{1A}}^2} \right)^{-1}, & Z_s &= \left(1 - 6 \frac{m_s^2}{M_{f_1(1420)}^2} \right)^{-1}, \\ M_{K_{1A}}^2 &= \left(\frac{\sin^2 \alpha}{M_{K_1(1270)}^2} + \frac{\cos^2 \alpha}{M_{K_1(1400)}^2} \right)^{-1}. \end{aligned} \quad (3)$$

Here $m_u = 270$ MeV and $m_s = 420$ MeV are the constituent masses of the u and s quarks. In most cases, the difference between the masses of the u and d quarks may be neglected. In the cases where it is impossible, the value $m_d - m_u = 4$ MeV is used. This difference was obtained by analyzing the process $\omega \rightarrow \pi^+ \pi^-$ [24]. The mixing angle of the meson states $K_1(1270)$ and $K_1(1400)$ is $\alpha = 57^\circ$.

The integrals I_{nm} appear in quark loops during the renormalization of the Lagrangian and take the following form:

$$I_{nm} = -i \frac{N_c}{(2\pi)^4} \int \frac{\theta(\Lambda^2 + k^2)}{(m_u^2 - k^2)^n (m_s^2 - k^2)^m} d^4 k, \quad (4)$$

where $\Lambda = 1265$ MeV is the cutoff parameter. It has been obtained along with the constituent quark mass m_u by using the experimentally known processes $\rho \rightarrow \pi\pi$ and $\pi \rightarrow \mu\nu$ [32].

FIG. 1. Quark diagrams of the decay $f_1 \rightarrow a_0\pi$.

III. DECAYS $f_1 \rightarrow a_0\pi$ AND $f_1 \rightarrow \pi\pi\eta$

The decay $f_1 \rightarrow a_0\pi$ is described with the quark diagrams presented in Fig. 1. The amplitude takes the form

$$\mathcal{M}(f_1 \rightarrow a_0\pi) = -ig_{f_1}^\mu \sqrt{Z_\pi} \cos \phi (p_{a_0} - A_\pi p_\pi)^\mu e_\mu^*(p_{f_1}), \quad (5)$$

where p_{a_0} , p_π are the momenta of the mesons, $e_\mu(p_{f_1})$ is the polarization vector of the meson f_1 . The factor $A_\pi = 1 - 12m_u^2/M_{a_1}^2$ takes into account the nondiagonal $a_1 - \pi$ transition in the final state. The width of the decay $f_1 \rightarrow a_0\pi$ can be calculated by using the formula

$$\Gamma(f_1 \rightarrow a_0\pi) = \frac{1}{2J_{f_1} + 1} \frac{\sqrt{E_\pi^2 - M_\pi^2}}{8\pi M_{f_1}^2} \cdot |\mathcal{M}(f_1 \rightarrow a_0\pi)|^2, \quad (6)$$

where $J_{f_1} = 1$, $E_\pi = (M_{f_1}^2 + M_\pi^2 - M_{a_0}^2)/2M_{f_1}^2$ is the energy of the π meson in the rest system of the f_1 meson.

As a result, the following estimation can be obtained for the branching fraction of the decay $f_1 \rightarrow a_0\pi$

$$\text{Br}(f_1 \rightarrow a_0\pi) = (39.4 \pm 5.91)\%. \quad (7)$$

Calculation uncertainties in the framework of the NJL model can be estimated at a level of 15%. The main source is partial conservation of axial current (PCAC). There are a number of other sources of uncertainties, such as the experimental errors of the meson parameters. The value 15% is based on a comparison of numerous calculations carried out within the NJL model earlier with the known experimental results. This error obtained from the real calculation covers all possible sources including PCAC.

The obtained estimation of the branching fraction is in agreement with the experimental data given in the PDG [13]

$$\text{Br}(f_1 \rightarrow a_0\pi)_{\text{PDG}} = 38.0 \pm 4.0\%. \quad (8)$$

Similar calculations can be carried out for the decay $f_1 \rightarrow f_0\eta$ where $f_0 = f_0(500)$. In this decay, the transition $\eta - f_1$ takes place instead of the $\pi - a_1$ transition in the amplitude (5). The decay is described by the following amplitude:

$$\mathcal{M}(f_1 \rightarrow f_0\eta) = (g_{f_1}^\mu \sqrt{Z_\pi} \cos \phi \cos \bar{\theta}_\sigma \sin \bar{\theta}_\eta - \sqrt{2}g_{f_1}^s \sqrt{Z_s} \sin \phi \sin \bar{\theta}_\sigma \cos \bar{\theta}_\eta) (p_{f_0} - A_\eta p_\eta)^\mu e_\mu^*(p_{f_1}), \quad (9)$$

where the constant describing the $\eta - f_1$ transition takes the form

$$A_\eta = 1 - 3\sqrt{6} \frac{(m_u g_\sigma \cos \phi^2 \cos \bar{\theta}_\sigma + \sqrt{2}m_s g_\sigma^s \sin \phi^2 \sin \bar{\theta}_\sigma)(m_u \sqrt{Z_\pi} \cos \phi \sin \bar{\theta}_\eta - m_s \sqrt{Z_s} \sin \phi \cos \bar{\theta}_\eta)}{M_{f_1(1285)}^2 \cdot (g_{f_1}^\mu \sqrt{Z_\pi} \cos \phi \cos \bar{\theta}_\sigma \sin \bar{\theta}_\eta - \sqrt{2}g_{f_1}^s \sqrt{Z_s} \sin \phi \sin \bar{\theta}_\sigma \cos \bar{\theta}_\eta)} - 3\sqrt{6} \frac{(m_u g_\sigma \cos \phi \sin \phi \cos \bar{\theta}_\sigma - \sqrt{2}m_s g_\sigma^s \sin \phi \cos \phi \sin \bar{\theta}_\sigma)(m_u \sqrt{Z_\pi} \sin \phi \sin \bar{\theta}_\eta + m_s \sqrt{Z_s} \cos \phi \cos \bar{\theta}_\eta)}{M_{f_1(1420)}^2 \cdot (g_{f_1}^\mu \sqrt{Z_\pi} \cos \phi \cos \bar{\theta}_\sigma \sin \bar{\theta}_\eta - \sqrt{2}g_{f_1}^s \sqrt{Z_s} \sin \phi \sin \bar{\theta}_\sigma \cos \bar{\theta}_\eta)}. \quad (10)$$

As a result, the following theoretical estimation of the branching fraction of the decay $f_1 \rightarrow f_0\eta$ can be obtained in the quark NJL model.

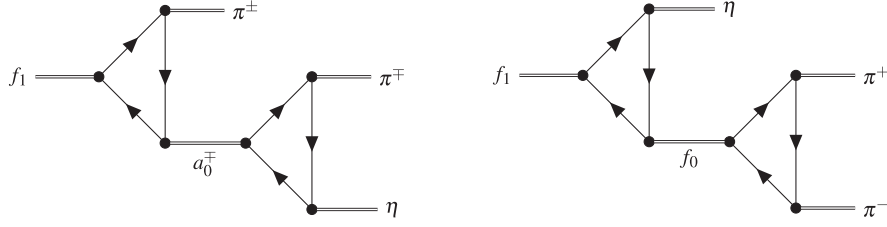
$$\text{Br}(f_1 \rightarrow f_0\eta) = (4.2 \pm 0.63)\%. \quad (11)$$

The decay $f_1 \rightarrow \pi\pi\eta$ is described by the contributions of two channels with the intermediate mesons $a_0\pi$ and $f_0\eta$. The quark diagrams describing the decay $f_1 \rightarrow \pi\pi\eta$ are

shown in Fig. 2. The total amplitude of the considered decay can be represented in the following form:

$$\mathcal{M}(f_1 \rightarrow \pi^- \pi^+ \eta) = i(\mathcal{M}_{a_0^+} + \mathcal{M}_{a_0^-} + \mathcal{M}_{f_0})^\mu e_\mu^*(p_{f_1}), \quad (12)$$

where the contributions of the channels with the intermediate mesons a_0^+ , a_0^- , and f_0 are presented in the brackets. The expressions for these channels take the form

FIG. 2. Quark diagrams of the decay $f_1 \rightarrow \pi^+ \pi^- \eta$.

$$\mathcal{M}_{a_0^+} = g_{f_1}^\mu \sqrt{Z_\pi} \cos \phi \text{BW}_{a_0^+} g_{a_0^+ \pi \eta} (p_{a_0^+}^2) (p_{a_0^+} - A_\pi p_{\pi^-})_\mu, \quad (13)$$

$$\mathcal{M}_{a_0^-} = g_{f_1}^\mu \sqrt{Z_\pi} \cos \phi \text{BW}_{a_0^-} g_{a_0^- \pi \eta} (p_{a_0^-}^2) (p_{a_0^-} - A_\pi p_{\pi^+})_\mu, \quad (14)$$

$$\mathcal{M}_{f_0} = (g_{f_1}^\mu \sqrt{Z_\pi} \cos \phi \cos \bar{\theta}_\sigma \sin \bar{\theta}_\eta - \sqrt{2} g_{f_1}^s \sin \phi \sqrt{Z_s} \sin \bar{\theta}_\sigma \cos \bar{\theta}_\eta) \text{BW}_{f_0} g_{f_0 \pi \pi} (p_{f_0}^2) (p_{f_0} - A_\eta p_\eta)_\mu, \quad (15)$$

where the factors A_π and A_η are shown in (5) and (9), the momenta of the intermediate mesons are defined as $p_{a_0^+} = p_{\pi^+} + p_\eta$, $p_{a_0^-} = p_{\pi^-} + p_\eta$, $p_{f_0} = p_{\pi^+} + p_{\pi^-}$. The functions $g_{a_0^\pm \pi \eta}$ and $g_{f_0 \pi \pi}$ describe the decays $a_0 \rightarrow \pi \eta$ and $f_0 \rightarrow \pi \pi$

$$g_{a_0^\pm \pi \eta} = 4m_u \sqrt{Z_\pi} g_\eta^\mu \sin \bar{\theta}_\eta \left[1 - \frac{3}{2} \frac{p_{a_0^\pm}^2 - M_\eta^2}{M_{a_1}^2} - \frac{3}{2} \cos \phi^2 \left(1 - \frac{m_s g_{f_1}^\mu g_\eta^s}{m_u g_{f_1}^\mu g_\eta^\mu} \text{tg} \phi \text{ctg} \bar{\theta}_\eta \right) \frac{p_{a_0^\pm}^2 - M_\pi^2}{M_{f_1(1285)}^2} - \frac{3}{2} \sin \phi^2 \left(1 + \frac{m_s g_{f_1}^\mu g_\eta^s}{m_u g_{f_1}^\mu g_\eta^\mu} \text{ctg} \phi \text{ctg} \bar{\theta}_\eta \right) \frac{p_{a_0^\pm}^2 - M_\pi^2}{M_{f_1(1420)}^2} \right], \quad (16)$$

$$g_{f_0 \pi \pi} = 4m_u \sqrt{Z_\pi} g_\pi \cos \bar{\theta}_\sigma \left(1 - 3 \frac{p_{f_0}^2 - M_\pi^2}{M_{a_1}^2} \right). \quad (17)$$

The intermediate mesons are described with the Breit-Wigner propagator

$$\text{BW}_M = \frac{1}{M_M^2 - p^2 - i\sqrt{p^2} \Gamma_M}, \quad (18)$$

where the widths and masses of the intermediate mesons $M_{a_0} = 980$ MeV, $M_{f_0} = 600$ MeV, $\Gamma_{a_0} = 75$ MeV, and $\Gamma_{f_0} = 450$ MeV [13].

In the calculated amplitude, besides the u and d quark loops, the s quark loops have been taken into account when considering the transitions between the axial vector f_1 meson and pseudoscalar η meson.

As a result, the following estimations of the branching fraction of the decay $f_1 \rightarrow \eta \pi \pi$ have been obtained by using the amplitude (12) in the NJL model

$$\text{Br}(f_1 \rightarrow \pi^+ \pi^- \eta)_{\text{NJL}} = (27.5 \pm 4.13)\%. \quad (19)$$

In the case of taking into account only up and down quark loops in the $f_1 - \eta$ transitions, the following value of the branching fraction is obtained

$$\text{Br}(f_1 \rightarrow \pi^+ \pi^- \eta)_{\text{NJL}} = (36.5 \pm 5.48)\%. \quad (20)$$

The obtained theoretical estimations are in agreement with the experimental data [13]

$$\text{Br}(f_1 \rightarrow \pi^+ \pi^- \eta)_{\text{PDG}} = (35 \pm 15)\%. \quad (21)$$

IV. DECAYS $f_1 \rightarrow \rho \pi$ AND $f_1 \rightarrow \pi^- \pi^+ \pi^0$

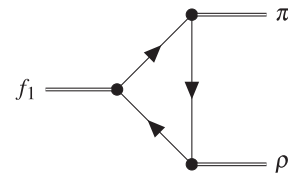
The processes $f_1 \rightarrow \rho \pi$ and $f_1 \rightarrow \pi^- \pi^+ \pi^0$ occur due to the difference between the u and d quark masses.

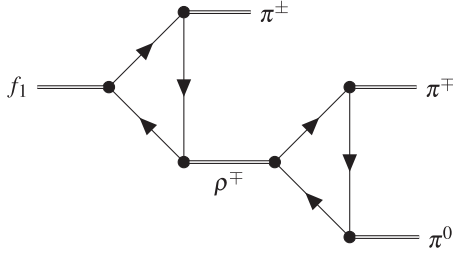
The diagram of the process $f_1 \rightarrow \rho \pi$ is shown in Fig. 3.

The amplitude of the process in the NJL model has the following form:

$$\mathcal{M}(f_1 \rightarrow \rho \pi) = -i6(m_d - m_u)g_\pi \cos \phi e_\mu^*(p_{f_1}) e^\mu(p_\rho), \quad (22)$$

where $e^\mu(p_\rho)$ is the polarization vector of the ρ meson.

FIG. 3. The diagram of the decay $f_1 \rightarrow \rho \pi$.

FIG. 4. The diagram of the decay $f_1 \rightarrow \pi^- \pi^+ \pi^0$.

The branching fraction obtained by using this amplitude is

$$\text{Br}(f_1 \rightarrow \rho\pi) = (1.9 \pm 0.3) \times 10^{-3}. \quad (23)$$

This theoretical estimation for the branching fraction does not contradict the experimental restrictions [13]

$$\text{Br}(f_1 \rightarrow \rho\pi)_{\text{PDG}} < 3.1 \times 10^{-3}. \quad (24)$$

The main channel determining the width of the process $f_1 \rightarrow \pi^- \pi^+ \pi^0$ is the vector channel. The diagram of this process is shown in Fig. 4.

The appropriate amplitude takes the following form:

$$\begin{aligned} \mathcal{M}(f_1 \rightarrow \pi^- \pi^+ \pi^0) = & -i6(m_d - m_u)g_\rho g_\pi \cos \phi \\ & \times e_\mu^*(p_{f_1})[(p_{\pi^0} - p_{\pi^-})^\mu \text{BW}_{\rho^+}^{p_{\pi^0} + p_{\pi^-}} \\ & + (p_{\pi^0} - p_{\pi^+})^\mu \text{BW}_{\rho^+}^{p_{\pi^0} + p_{\pi^+}}], \end{aligned} \quad (25)$$

where p_{π^0} , p_{π^+} , and p_{π^-} are the meson momenta in the final states.

The branching fraction obtained by using this amplitude is

$$\text{Br}(f_1 \rightarrow \pi^- \pi^+ \pi^0) = (3.2 \pm 0.5) \times 10^{-3}, \quad (26)$$

and is in agreement with the experimental data [13]

$$\text{Br}(f_1 \rightarrow \pi^- \pi^+ \pi^0)_{\text{PDG}} = (3.0 \pm 0.9) \times 10^{-3}. \quad (27)$$

V. DECAYS $f_1 \rightarrow KK\pi$

The decay $f_1 \rightarrow K^+ K^0 \pi^-$ contains the contributions of the channels with the intermediate vector and scalar mesons K^{*0} , K^{*-} , a_0 , K_0^{*0} , and K_0^{*-} . The total amplitude of the decay takes the form

$$\mathcal{M}(f_1 \rightarrow K^+ K^0 \pi^-) = -i2\sqrt{2}g_\pi Z_K (\mathcal{M}_{K^{*0}} + \mathcal{M}_{K^{*-}} + \mathcal{M}_{a_0} + \mathcal{M}_{K_0^{*0}} + \mathcal{M}_{K_0^{*-}}) e_\mu(p_{f_1}), \quad (28)$$

where the contributions from the channels are given by

$$\mathcal{M}_{K^{*0}} = \frac{3}{8}((3m_u - m_s)g_{f_1}^\mu \cos \phi + \sqrt{2}(3m_s - m_u)g_{f_1}^s \sin \phi) \text{BW}_{K^{*0}}(a_\pi p_{\pi^-} - a_K p_{K^+})_\mu, \quad (29)$$

$$\mathcal{M}_{K^{*-}} = \frac{3}{8}((3m_u - m_s)g_{f_1}^\mu \cos \phi + \sqrt{2}(3m_s - m_u)g_{f_1}^s \sin \phi) \text{BW}_{K^{*-}}(a_\pi p_{\pi^-} - a_K p_{K^0})_\mu, \quad (30)$$

$$\mathcal{M}_{a_0} = -(2m_u - m_s)g_{f_1}^\mu \cos \phi \text{BW}_{a_0}(p_{a_0} - A_\pi p_{\pi^-})_\mu, \quad (31)$$

$$\mathcal{M}_{K_0^{*0}} = \frac{1}{2}m_s(g_{f_1}^\mu \cos \phi + \sqrt{2}g_{f_1}^s \sin \phi) \text{BW}_{K_0^{*0}}(p_{K_0^{*0}} - p_{K^0})_\mu, \quad (32)$$

$$\mathcal{M}_{K_0^{*-}} = \frac{1}{2}m_s(g_{f_1}^\mu \cos \phi + \sqrt{2}g_{f_1}^s \sin \phi) \text{BW}_{K_0^{*-}}(p_{K_0^{*-}} - p_{K^+})_\mu, \quad (33)$$

where the constants a_π and a_K describe the transitions in the decay $K^{*0} \rightarrow K^+ \pi^-$

$$a_\pi = 1 - \frac{3m_u(3m_u - m_s)}{M_{a_1}^2}, \quad a_K = 1 - \frac{3m_s(m_u + m_s)}{M_{K_{1A}}^2}. \quad (34)$$

By using this amplitude, one can obtain the estimation of the branching fraction of the decay $\text{Br}(f_1 \rightarrow K^+ K^0 \pi^-) = (3.3 \pm 0.5)\%$. The total width of the decay $f_1 \rightarrow KK\pi$ includes the combinations of the mesons in the final state $f_1 \rightarrow K^+ K^0 \pi^-$, $f_1 \rightarrow K^- K^0 \pi^+$, $f_1 \rightarrow K^+ K^- \pi^0$ and $f_1 \rightarrow K^0 \bar{K}^0 \pi^0$. As a result, in the framework of the NJL model, for the decay $f_1 \rightarrow KK\pi$ one can obtain the estimation $\text{Br}(f_1 \rightarrow KK\pi)_{\text{NJL}} = (9.9 \pm 1.5)\%$. The experimental value provided for the branching fraction of this process is [13]

$$\text{Br}(f_1 \rightarrow KK\pi)_{\text{PDG}} = (9.0 \pm 0.4)\%. \quad (35)$$

VI. CONCLUSION

In this paper, we have considered the main strong decays of the axial-vector meson $f_1(1285)$. All calculations were performed in the one-loop quark approximation

corresponding to the leading order of expansions in $1/N_c$. For the branching fraction of the decay width with the production of the scalar meson $f_1 \rightarrow a_0\pi$, the value $(39.4 \pm 5.91)\%$ was obtained, which is in agreement with the experimental data. In this case, the scalar meson a_0 was considered as a chiral-symmetric quark-antiquark partner of the pseudoscalar π -meson. The decay $f_1 \rightarrow \eta\pi^+\pi^-$ is calculated taking into account the intermediate channels with the scalar mesons $a_0^\pm$ and f_0 . The contribution of the channels with the mesons $a_0^\pm$ to the branching fraction is 15.6%. The contribution of the channel with the scalar meson f_0 turns out to be comparable with the channels $\text{Br}(f_1 \rightarrow a_0^\pm\pi^\mp \rightarrow \eta\pi^+\pi^-) = 7.0\%$. The total branching fraction partial width taking into account the positive interference of channels a_0 and f_0 is $\text{Br}(f_1 \rightarrow \eta\pi^+\pi^-) = 27.5\%$.

From a theoretical point of view, the decay $f_1 \rightarrow a_0\pi$ has recently been considered in [41] within the mechanism of the triangular singularity, and the estimated value $\text{Br}(f_1 \rightarrow \eta\pi^+\pi^-) = 31\%$ is in agreement with the

experiment and the results of the present work. It should be noted that in our recent work [12] the $\tau \rightarrow 3\pi\nu_\tau$ decay was considered taking into account the intermediate channel with the scalar meson f_0 . In this work, good results were obtained for the spectral function and decay width in agreement with the CLEO data [42]. At the same time, an important role of the $f_0\pi$ channel contribution to the $\tau \rightarrow 3\pi\nu_\tau$ decay width was shown, which improved the agreement with the experiment.

The suppression of the decays $f_1 \rightarrow \rho^\pm\pi^\mp$ and $f_1 \rightarrow \pi^+\pi^-\pi^0$ was also investigated. We showed that these decays occur due to the difference in the masses of the light u and d quarks. In the decay $f_1 \rightarrow \pi^+\pi^-\pi^0$ the charged vector meson channel including $\rho^\pm$ completely determines the decay width because it is the only possible intermediate state in this process. One can obtain the estimation for the branching fraction ratio $\text{Br}(f_1 \rightarrow \pi^+\pi^-\pi^0)/\text{Br}(f_1 \rightarrow \pi^+\pi^-\eta) = 1.16\%(0.87\%)$. This estimation within the NJL model agrees with the data of the VES and BES III experiments [17,43]

$$\frac{\text{Br}(f_1 \rightarrow \pi^+\pi^-\pi^0)}{\text{Br}(f_1 \rightarrow \pi^+\pi^-\eta)} = (0.86 \pm 0.16 \pm 0.20), \quad \frac{\text{Br}(f_1 \rightarrow \pi^+\pi^-\pi^0)}{\text{Br}(f_1 \rightarrow \pi^+\pi^-\eta)} = (1.23 \pm 0.55).$$

The obtained results for the decay involving the strange mesons $f_1 \rightarrow KK\pi$ are presented in Sec. V. Channels with vector and scalar mesons K^* , a_0 and K_0^* are determined for the decay. An analysis of the decay width $f_1 \rightarrow K^+K^0\pi^-$ shows that the contribution of the vector channel is 0.23%. The scalar contributions give 1.9% and the total contribution taking into account the interference is $\text{Br}(f_1 \rightarrow K^+K^0\pi^-) = 3.3\%$. As a result, for the decay $f_1 \rightarrow KK\pi$ taking into account the combination of meson charges in the final state, the resulting partial width $\text{Br}(f_1 \rightarrow KK\pi) = 9.9\%$ is obtained. This result is in qualitative agreement with the PDG data [13].

The calculations carried out in the present work show that along with various representations of the structure of scalar mesons (see review papers, for example, [44–46]) in the case under consideration the $\bar{q}q$ representation works quite satisfactorily at low energies where the scalar meson is considered as a chirally symmetric partner of the pseudoscalar meson. In the framework of this approach,

strong decays of strange scalar mesons $K_0^*(700)$ and $K_0^*(1430)$ [47] have also been recently investigated. In addition, the processes of strange scalar meson production in tau lepton decays [48] were considered. As a future development of this work, the problem of inclusion of tetraquark scalar states in consistency with the chiral symmetry could be considered. The influence of the radially excited mesons in the intermediate states of these processes can also be investigated.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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