

Electric flux tube solutions in SU(3) gauge theory

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We consider electric flux tube solutions in SU(3) gauge theory with scalar fields in the fundamental representation. Such solutions can possibly be constructed in two classes, corresponding to the two maximally commuting generators λ_3 and λ_8 of SU(3). We successfully construct the 3-type solution with two scalar fields and show that it is immune to decay through Schwinger pair production of gauge bosons, but we have not been able to construct an 8-type solution.

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I. INTRODUCTION

Magnetic type solutions in non-Abelian gauge theories, including magnetic monopoles and cosmic strings, have been of interest for many decades. Electric-type solutions, however, have only recently been discovered and their existence and properties remain largely unexplored. The key to the new electric string solutions is the discovery by Brown and Weisberger (BW) [1] that there are an infinite number of gauge inequivalent non-Abelian gauge fields, in addition to embedded Maxwellian gauge fields, all of which lead to identical electric fields. Electric fields resulting from Maxwellian gauge fields are vacuum solutions of the equations of motion but such electric fields are unstable to rapid Schwinger pair production of gauge particles and will eventually dissipate [2–4]. On the other hand, electric fields generated from the BW gauge fields need volume-filling sources but are classically stable for a wide range of parameters [5,6] and also quantumly stable (“unexciting”) to Schwinger pair production of gauge particles [7].

Here we are interested in BW gauge fields that result in electric-type solutions. Since such solutions require non-trivial sources, it is necessary to include “matter” fields that are charged under the non-Abelian symmetry. In Refs. [8,9] electric-type solutions were constructed in an SU(2) non-Abelian gauge theory with the matter field chosen to be a scalar field transforming in the fundamental representation of SU(2). The “electric string” solution, equivalently an “electric flux tube”, was derived using some guesswork,

and it seemed somewhat of a miracle that all the equations of motion could be solved for this highly nonlinear system. The question we address in this paper is if the same miracle can be extended to other gauge groups, and SU(3) in particular because of its relevance to quantum chromodynamics (QCD).

At first sight, it appears that the SU(2) solution can simply be embedded into SU(3). However, this turns out not to be the case for the reason that the SU(2) Pauli spin matrices, σ^a , ($a = 1, 2, 3$), satisfy

$$\{\sigma^a, \sigma^b\} = 2\delta^{ab}, \quad (1)$$

whereas the SU(3) Gell-Mann matrices, λ^a , ($a = 1, \dots, 8$) satisfy

$$\{\lambda^a, \lambda^b\} = \frac{4}{3}\delta^{ab} + 2d^{abc}\lambda^c, \quad (2)$$

where d^{abc} are nonvanishing in general. Further, SU(3) has two elements in its Cartan subalgebra, and there are two different “electric fields”, one in the λ^3 direction and the other in the λ^8 direction. Thus there are two types of electric solutions that one can consider in SU(3). We shall call these “3-type” and “8-type” solutions.

Construction of the 3-type electric field largely follows the procedures developed for the SU(2) case [8,9] except that we find that a single scalar field cannot source the BW gauge fields. Instead we need to introduce two scalar fields that transform in the fundamental representation of SU(3). With such matter fields we can source a uniform electric field in the λ^3 direction as well as an electric string solution (see Sec. III). Construction of the 8-type electric field turns out to be more complicated since $d^{abc} \neq 0$ and will be reserved for future work.

One motivation for investigating the BW gauge fields is that they are quantumly stable. This is shown explicitly for the 3-type electric field in Sec. IV.

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We start out in Sec. II by describing 3-type and 8-type BW gauge fields for SU(3) and their field strengths. In Sec. III we analyze 3-type electric solutions with two scalar fields in the fundamental representation and summarize the solution in Sec. III E. We discuss questions for future work in the concluding Sec. V. We also show in the Appendix that a single scalar field in the adjoint representation of SU(3) cannot provide suitable sources.

II. GAUGE FIELDS

SU(3) gauge fields, W_μ^a , of the Maxwellian type that give rise to an electric field in the z -direction in space and in the λ^b direction in group space may be written as

$$W_\mu^a = -Et\partial_\mu z \delta^{ab}. \quad (3)$$

In contrast, a general set of BW gauge fields that result in uniform electric fields along the z -direction and in the $a = 3$ and $a = 8$ directions in group space can be written as

$$\begin{aligned} W_\mu^1 &= \frac{\Omega_1}{g} \partial_\mu t, & W_\mu^2 &= -\frac{\Omega_1 E}{\Omega^2} \partial_\mu z, & W_\mu^4 &= \frac{\Omega_2}{g} \partial_\mu t, \\ W_\mu^5 &= \frac{\Omega_2 E}{\Omega^2} \partial_\mu z, & W_\mu^6 &= \frac{\Omega_3}{g} \partial_\mu t, & W_\mu^7 &= -\frac{\Omega_3 E}{\Omega^2} \partial_\mu z. \end{aligned} \quad (4)$$

Using the expression for the field strength tensor,

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + gf^{abc} W_\mu^b W_\nu^c, \quad (5)$$

where f^{abc} are the SU(3) structure constants defined using the commutator of the Gell-Mann matrices [10],

$$[\lambda_a, \lambda_b] = 2if^{abc} \lambda_c, \quad (6)$$

we obtain

$$W_{\mu\nu}^3 = \frac{E}{2\Omega^2} (\Omega_2^2 + \Omega_3^2 - 2\Omega_1^2) (\partial_\mu t \partial_\nu z - \partial_\mu z \partial_\nu t) \quad (7)$$

$$W_{\mu\nu}^8 = \frac{\sqrt{3}E}{2\Omega^2} (\Omega_2^2 - \Omega_3^2) (\partial_\mu t \partial_\nu z - \partial_\mu z \partial_\nu t). \quad (8)$$

Setting $\Omega_2^2 = \Omega_3^2$ results in a constant electric field in the $a = 3$ direction (3-type) whereas setting $\Omega_1^2 = (\Omega_2^2 + \Omega_3^2)/2$ results in a constant electric field in the $a = 8$ direction (8-type).

The gauge four-currents are defined as

$$j^{\mu a} \equiv D_\nu W^{\mu\nu a} = \partial_\nu W^{\mu\nu a} + gf^{abc} W_\nu^b W^{\mu\nu c}, \quad (9)$$

which can be computed for the above BW gauge fields as follows:

$$j_\mu^1 = \frac{gE^2}{2\Omega^4} \Omega_1 (\Omega_2^2 + \Omega_3^2 - 2\Omega_1^2) \partial_\mu t \quad (10)$$

$$j_\mu^2 = \frac{E}{2\Omega^2} \Omega_1 (\Omega_2^2 + \Omega_3^2 - 2\Omega_1^2) \partial_\mu z \quad (11)$$

$$j_\mu^4 = \frac{gE^2}{2\Omega^4} \Omega_2 (\Omega_1^2 + \Omega_3^2 - 2\Omega_2^2) \partial_\mu t \quad (12)$$

$$j_\mu^5 = -\frac{E}{2\Omega^2} \Omega_2 (\Omega_1^2 + \Omega_3^2 - 2\Omega_2^2) \partial_\mu z \quad (13)$$

$$j_\mu^6 = \frac{gE^2}{2\Omega^4} \Omega_3 (\Omega_1^2 + \Omega_2^2 - 2\Omega_3^2) \partial_\mu t \quad (14)$$

$$j_\mu^7 = \frac{E}{2\Omega^2} \Omega_3 (\Omega_1^2 + \Omega_2^2 - 2\Omega_3^2) \partial_\mu z, \quad (15)$$

with other components vanishing.

In what follows, we will investigate only 3-type electric fields and attempt to construct electric string solutions following [8,9].

III. ELECTRIC FIELD IN λ_3 DIRECTION

A. Homogeneous electric fields

Electric fields in the λ_3 direction may be obtained by setting $\Omega_2^2 = \Omega_3^2 = 0$. We also denote $\Omega_1 = \Omega$. This results in the following gauge fields:

$$W_\mu^1 = \frac{\Omega}{g} \partial_\mu t, \quad W_\mu^2 = -\frac{E}{\Omega} \partial_\mu z, \quad (16)$$

with all other $W_\mu^b = 0$. The only nonvanishing field strength component is

$$W_{\mu\nu}^3 = -E(\partial_\mu t \partial_\nu z - \partial_\mu z \partial_\nu t). \quad (17)$$

These gauge fields can be written in the temporal gauge $W_t^a = 0$ by acting with a gauge transformation U ,

$$W_\mu \rightarrow W'_\mu = UW_\mu U^\dagger + \frac{2i}{g} U \partial_\mu U^\dagger, \quad (18)$$

where $W_\mu = W_\mu^a \lambda_a$. Following [9], it can be checked that

$$U = e^{i\lambda_2 \pi/4} e^{i\lambda_1 \pi/4} e^{-i\lambda_1 \Omega t/2} \quad (19)$$

results in

$$W_\mu^1 = -\frac{\epsilon}{g} \cos \Omega t \partial_\mu z, \quad W_\mu^2 = -\frac{\epsilon}{g} \sin \Omega t \partial_\mu z, \quad (20)$$

along with $W_\mu^b = 0$ for $b = 3, \dots, 8$ where we have defined

$$\epsilon = \frac{gE}{\Omega}. \quad (21)$$

The nonvanishing field strength tensor for the gauge fields in the temporal gauge can be shown to be

$$W_{\mu\nu}^1 = \frac{\epsilon\Omega}{g} \sin(\Omega t) (\partial_\mu t \partial_\nu z - \partial_\nu t \partial_\mu z) \quad (22)$$

$$W_{\mu\nu}^2 = -\frac{\epsilon\Omega}{g} \cos(\Omega t) (\partial_\mu t \partial_\nu z - \partial_\nu t \partial_\mu z). \quad (23)$$

Note that the electric field W_{0i}^a is time-dependent in the temporal gauge. However, the gauge-invariant energy density in the electric fields,

$$\mathcal{E}_{\text{elec}} = \frac{1}{2} (W_{0i}^a)^2 = \frac{\epsilon^2 \Omega^2}{2g^2} = \frac{1}{2} E^2, \quad (24)$$

is time-independent.

Using (9), the four-currents for the gauge fields in the temporal gauge can be computed as follows:

$$j_\mu^1 = -\frac{\epsilon\Omega^2}{g} \cos(\Omega t) \partial_\mu z \quad (25)$$

$$j_\mu^2 = -\frac{\epsilon\Omega^2}{g} \sin(\Omega t) \partial_\mu z \quad (26)$$

$$j_\mu^3 = -\frac{\epsilon^2 \Omega}{g} \partial_\mu t, \quad (27)$$

while other components vanish.

B. Scalar fields

Next, we would like to obtain scalar fields that can consistently source the gauge fields in (20). We consider the following Lagrangian for the $SU(3)$ gauge fields coupled to two¹ scalar fields in the fundamental representation [11],

$$L = -\frac{1}{4} W_{\mu\nu}^a W^{\mu\nu a} + |D_\mu \Phi_1|^2 + |D_\mu \Phi_2|^2 - V(\Phi_1, \Phi_2), \quad (28)$$

where

$$D_\mu \Phi_i = \partial_\mu \Phi_i - i \frac{g}{2} W_\mu^a \lambda^a \Phi_i, \quad (29)$$

for $i = 1, 2$ and

$$V(\Phi_1, \Phi_2) = m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 + \kappa_1 |\Phi_1|^4 + \kappa_2 |\Phi_2|^4 + \kappa_{12} [(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2]. \quad (30)$$

¹We will see later that a single scalar field cannot source the gauge fields.

The equation of motion for the scalar field Φ_i is

$$D_\mu D^\mu \Phi_i + \delta_i V(\Phi_1, \Phi_2) = 0, \quad i = 1, 2, \quad (31)$$

where δ_i denotes differentiation with respect to Φ_i . We evaluate

$$\begin{aligned} D_\mu D^\mu \Phi_i &= \partial_t^2 \Phi_i + \frac{g^2}{4} W_z^a W_z^b \lambda^a \lambda^b \Phi_i \\ &= \partial_t^2 \Phi_i + \frac{g^2}{6} (W_z^a)^2 \Phi_i + \frac{g^2}{4} W_z^a W_z^b d^{abc} \lambda^c \Phi_i, \end{aligned} \quad (32)$$

where we have used the temporal gauge $W_t^a = 0$ and $\Phi_i = \Phi_i(t)$ to cancel out and simplify terms in the first line, and used the $SU(3)$ identity,

$$\lambda^a \lambda^b = \frac{2}{3} \delta^{ab} + d^{abc} \lambda^c + i f^{abc} \lambda^c, \quad (33)$$

along with the total antisymmetry of the structure constants, f^{abc} , to get the second line. We also have

$$\delta_i V(\Phi_1, \Phi_2) = m_i^2 \Phi_i + 2\kappa_i |\Phi_i|^2 \Phi_i + 2\kappa_{ij} (\Phi_i^\dagger \Phi_j) \Phi_j, \quad (34)$$

where $j \neq i$.

Let us denote $M \equiv g^2 W_z^a W_z^b d^{abc} \lambda^c / 4$ where repeated indices $a, b, c = 1, \dots, 8$ are summed over so that M is a 3×3 matrix. We can use the eigenvectors of M to construct an orthonormal basis for the set of scalar fields.

For the gauge fields in (20), we have

$$M = \frac{g^2}{4} ((W_z^1)^2 d^{118} \lambda^8 + (W_z^2)^2 d^{228} \lambda^8) = \frac{\epsilon^2}{2\sqrt{3}} \lambda^8, \quad (35)$$

which has the orthonormal eigenvectors,

$$\psi_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \psi_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \psi_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad (36)$$

with eigenvalues $\epsilon^2/12, \epsilon^2/12$ and $-\epsilon^2/6$ respectively. As we will show below, it turns out that a solution can be obtained simply by writing the two scalar fields as a linear combination of ψ_1 and ψ_2 .

We write down an ansatz for the two scalar fields using the Hopf parametrization as follows:

$$\begin{aligned} \Phi_1 &= \eta_1 (\cos \alpha_1 e^{i\beta_1} \psi_1 + \sin \alpha_1 e^{i\gamma_1} \psi_2) \\ &= \eta_1 \begin{pmatrix} \cos \alpha_1 e^{i\beta_1} \\ \sin \alpha_1 e^{i\gamma_1} \\ 0 \end{pmatrix}, \end{aligned} \quad (37)$$

$$\begin{aligned}\Phi_2 &= \eta_2 (\cos \alpha_2 e^{i\beta_2} \psi_1 + \sin \alpha_2 e^{i\gamma_2} \psi_2) \\ &= \eta_2 \begin{pmatrix} \cos \alpha_2 e^{i\beta_2} \\ \sin \alpha_2 e^{i\gamma_2} \\ 0 \end{pmatrix},\end{aligned}\quad (38)$$

where $\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_1, \gamma_2$ are functions of t while η_1, η_2 are constants.

We will further require the two scalar fields to be orthogonal. This ensures that their corresponding interaction term in the potential (30) vanishes. Demanding $\Phi_1^\dagger \Phi_2 = 0$ gives

$$\cos \alpha_1 \cos \alpha_2 e^{i(\beta_2 - \beta_1)} + \sin \alpha_1 \sin \alpha_2 e^{i(\gamma_2 - \gamma_1)} = 0, \quad (39)$$

which can be further simplified as follows:

$$\cot \alpha_1 \cot \alpha_2 e^{i(\beta_2 - \beta_1 - \gamma_2 + \gamma_1)} = -1. \quad (40)$$

Since the rhs is real-valued, we require that

$$\beta_2 - \beta_1 - \gamma_2 + \gamma_1 = n\pi, \quad n \in \mathbb{Z}. \quad (41)$$

This results in

$$\cot \alpha_1 \cot \alpha_2 = (-1)^{n+1}, \quad (42)$$

which can be solved by

$$\alpha_1 + (-1)^{n+1} \alpha_2 = (2m - 1)\pi/2, \quad m \in \mathbb{Z}. \quad (43)$$

For the sake of simplicity, we set $m = 0$ and $n = 0$ to obtain

$$\beta_2 - \beta_1 = \gamma_2 - \gamma_1 \equiv \theta \quad (44)$$

$$\alpha_2 = \alpha_1 + \pi/2, \quad (45)$$

where it is sufficient to consider θ to be a constant.

Thus, the final set of orthogonal scalar fields Φ_1 and Φ_2 ,

$$\Phi_1 = \eta_1 \begin{pmatrix} \cos \alpha e^{i\beta} \\ \sin \alpha e^{i\gamma} \\ 0 \end{pmatrix}, \quad \Phi_2 = \eta_2 e^{i\theta} \begin{pmatrix} -\sin \alpha e^{i\beta} \\ \cos \alpha e^{i\gamma} \\ 0 \end{pmatrix}, \quad (46)$$

where we have denoted $\alpha_1 \equiv \alpha, \beta_1 \equiv \beta$ and $\gamma_1 \equiv \gamma$.

The four-current for the two scalar fields is

$$J_\mu^a = \frac{ig}{2} \sum_{i=1,2} [\Phi_i^\dagger \lambda^a D_\mu \Phi_i - \text{H.c.}]. \quad (47)$$

Using the definition of the gauge covariant derivative (29) and the $SU(3)$ identity (33), we can expand the terms in the four-current as follows:

$$\begin{aligned}\Phi_i^\dagger \lambda^a D_\mu \Phi_i &= \Phi_i^\dagger \lambda^a \partial_\mu \Phi_i - \frac{ig}{2} W_\mu^b \Phi_i^\dagger \lambda^a \lambda^b \Phi_i \\ &= \Phi_i^\dagger \lambda^a \partial_\mu \Phi_i - \frac{ig}{3} W_\mu^a |\Phi_i|^2 + \frac{g}{2} f^{abc} W_\mu^b (\Phi_i^\dagger \lambda^c \Phi_i) \\ &\quad - \frac{ig}{2} d^{abc} W_\mu^b (\Phi_i^\dagger \lambda^c \Phi_i).\end{aligned}\quad (48)$$

Hence, the final expression for the four-current becomes

$$\begin{aligned}J_\mu^a &= \sum_{i=1,2} \left\{ \frac{ig}{2} [\Phi_i^\dagger \lambda^a \partial_\mu \Phi_i - \text{H.c.}] \right. \\ &\quad \left. + \frac{g^2}{3} W_\mu^a |\Phi_i|^2 + \frac{g^2}{2} d^{abc} W_\mu^b (\Phi_i^\dagger \lambda^c \Phi_i) \right\}.\end{aligned}\quad (49)$$

The nonvanishing components of the four-current for the set of scalar fields in (46) can now be computed. Setting $\mu = z$ in (49) we obtain

$$J_z^a = \sum_{i=1,2} \left\{ \frac{g^2}{3} W_z^a |\Phi_i|^2 + \frac{g^2}{2} d^{abc} W_z^b (\Phi_i^\dagger \lambda^c \Phi_i) \right\}. \quad (50)$$

Substituting (46) in (50), we obtain the $a = 8$ component,

$$J_z^8 = -\frac{g\epsilon}{2\sqrt{3}} (\eta_1^2 - \eta_2^2) \cos(\Omega t + \beta - \gamma) \sin 2\alpha. \quad (51)$$

This must vanish to match the corresponding gauge field four-current components that are given in (25)–(27). Then we can either set $\eta_1 = \eta_2$ or set $\sin(2\alpha) = 0$. The latter choice turns out to be inconsistent with the condition $j_t^8 = 0$, and so we are forced to set $\eta_1 = \eta_2 \equiv \eta$. Note that both η_1 and η_2 have to be nonvanishing, which means that our construction would fail if we only searched for a solution with a single scalar field.

The nontrivial z-component of the scalar field four-current can be written as

$$J_z^a = g^2 \eta^2 W_z^a. \quad (52)$$

Comparing with (25)–(26), we obtain

$$\eta^2 = \frac{\Omega^2}{g^2} \Rightarrow \eta = \frac{\Omega}{g}. \quad (53)$$

Setting $\mu = t$, we get

$$J_t^a = \frac{ig}{2} \sum_{i=1,2} [\Phi_i^\dagger \lambda^a \dot{\Phi}_i - \dot{\Phi}_i^\dagger \lambda^a \Phi_i], \quad (54)$$

where overdots denote time derivatives. Substituting (46) in (54) and setting $\eta_1 = \eta_2 = \eta$, the nontrivial t-component of the scalar field four-current is

$$\begin{aligned}
J_t^1 &= 2g\eta^2 \sin(\beta - \gamma)\dot{\alpha} \\
J_t^2 &= 2g\eta^2 \cos(\beta - \gamma)\dot{\alpha} \\
J_t^3 &= -g\eta^2(\dot{\beta} - \dot{\gamma}) \\
J_t^8 &= -\frac{g\eta^2}{\sqrt{3}}(\dot{\beta} + \dot{\gamma}).
\end{aligned} \tag{55}$$

Comparing with (27), we see that J_t^1 and J_t^2 can be made to vanish by setting $\dot{\alpha} = 0$, whereas J_t^8 can be shown to vanish by demanding $\dot{\gamma} = -\dot{\beta}$. Matching the remaining nontrivial component J_t^3 yields

$$2g^2\eta^2\dot{\beta} = \epsilon^2\Omega \Rightarrow \dot{\beta} = \frac{\epsilon^2}{2\Omega}, \tag{56}$$

where we have substituted $\eta^2 = \Omega^2/g^2$. Thus, the final set of orthogonal scalar field solutions can be written as

$$\Phi_1 = \eta \begin{pmatrix} z_1 e^{i\omega t} \\ z_2 e^{-i\omega t} \\ 0 \end{pmatrix}, \quad \Phi_2 = \eta \begin{pmatrix} w_1 e^{i\omega t} \\ w_2 e^{-i\omega t} \\ 0 \end{pmatrix}, \tag{57}$$

where $z_1, z_2, w_1, w_2 \in \mathbb{C}$ are constants satisfying the following constraints:

$$\begin{aligned}
|z_1|^2 + |z_2|^2 &= 1 \\
|w_1|^2 + |w_2|^2 &= 1 \\
z_1 w_1^* + z_2 w_2^* &= 0,
\end{aligned} \tag{58}$$

and we have defined

$$\omega = \frac{\epsilon^2}{2\Omega}. \tag{59}$$

Note that even though we made the simplifying assumption $m = n = 0$ [see (41) and (43)], the final solution, as written above, is valid generally requiring only that the two scalar fields be orthogonal.

Thus far we have demanded that the scalar field four-current, J_μ^a , matches the gauge field four-current, j_μ^a , and in doing so we have satisfied the gauge field equations of motion. Next, we need to demand that both Φ_1 and Φ_2 satisfy their own equations of motion,

$$D_\mu D^\mu \Phi_i + V'(\Phi_i) = 0, \tag{60}$$

where now

$$V(\Phi_i) = m_i^2 |\Phi_i|^2 + \kappa_i |\Phi_i|^4, \tag{61}$$

as there are no interaction terms since the two scalar fields are orthogonal $\Phi_1^\dagger \Phi_2 = \Phi_2^\dagger \Phi_1 = 0$. Substituting (57) in (32), we obtain

$$D_\mu D^\mu \Phi_i = -(\omega^2 - \epsilon^2/4)\Phi_i. \tag{62}$$

We also have

$$\begin{aligned}
V'(\Phi_i) &= m_i^2 \Phi_i + 2\kappa_i |\Phi_i|^2 \Phi_i \\
&= (m_i^2 + 2\kappa_i \eta^2) \Phi_i.
\end{aligned} \tag{63}$$

Putting everything together, (60) gives

$$-\omega^2 + \frac{\epsilon^2}{4} + m_i^2 + 2\kappa_i \eta^2 = 0. \tag{64}$$

Substituting $\epsilon^2 = 2\omega\Omega$ from (59) and solving the resulting quadratic equation, we obtain

$$\omega = \frac{1}{2} \left[\frac{\Omega}{2} \pm \sqrt{\frac{\Omega^2}{4} + 4 \left(m_i^2 + \frac{2\kappa_i}{g^2} \Omega^2 \right)} \right]. \tag{65}$$

Since a single value of ω must satisfy the equation of motion for both scalar fields, this implies additional constraints on the model parameters. In particular, we must have

$$m_1^2 + \frac{2\kappa_1}{g^2} \Omega^2 = m_2^2 + \frac{2\kappa_2}{g^2} \Omega^2. \tag{66}$$

The 3-type solution we have found in Eqs. (20), (57) for a uniform electric field in SU(3) is very similar to the solution in SU(2). The main difference is that the solution in SU(2) only required one scalar field, while in SU(3) we need at least two scalar fields.

C. Electric string solution

In the previous subsection, we derived the uniform BW electric field solution for all equations of motion. We now build on the uniform solution to construct electric string (flux tube) solutions by attaching a profile function $f(r)$ to the gauge field configuration (20) where r is the cylindrical radial coordinate. Accordingly, we write

$$W_\mu^1 = -\frac{\epsilon}{g} \cos \Omega t f(r) \partial_\mu z, \quad W_\mu^2 = -\frac{\epsilon}{g} \sin \Omega t f(r) \partial_\mu z. \tag{67}$$

The nonvanishing field strength tensor components are

$$\begin{aligned}
W_{\mu\nu}^1 &= \frac{\epsilon}{g} \Omega \sin \Omega t f(r) (\partial_\mu t \partial_\nu z - \partial_\nu t \partial_\mu z) \\
&\quad - \frac{\epsilon}{g} \cos \Omega t f'(r) (\partial_\mu r \partial_\nu z - \partial_\nu r \partial_\mu z)
\end{aligned} \tag{68}$$

$$\begin{aligned}
W_{\mu\nu}^2 &= -\frac{\epsilon}{g} \Omega \cos \Omega t f(r) (\partial_\mu t \partial_\nu z - \partial_\nu t \partial_\mu z) \\
&\quad - \frac{\epsilon}{g} \sin \Omega t f'(r) (\partial_\mu r \partial_\nu z - \partial_\nu r \partial_\mu z),
\end{aligned} \tag{69}$$

and the nonvanishing gauge four-currents are

$$j_\mu^1 = -\frac{\epsilon}{g} \cos \Omega t \left[f'' + \frac{f'}{r} + \Omega^2 f \right] \partial_\mu z \quad (70)$$

$$j_\mu^2 = -\frac{\epsilon}{g} \sin \Omega t \left[f'' + \frac{f'}{r} + \Omega^2 f \right] \partial_\mu z \quad (71)$$

$$j_\mu^3 = -\frac{\epsilon^2 \Omega}{g} f^2 \partial_\mu t. \quad (72)$$

We include separate profile functions in each of the orthogonal scalar fields (57),

$$\Phi_1 = \eta h_1(r) \begin{pmatrix} z_1 e^{i\omega t} \\ z_2 e^{-i\omega t} \\ 0 \end{pmatrix}, \quad \Phi_2 = \eta h_2(r) \begin{pmatrix} w_1 e^{i\omega t} \\ w_2 e^{-i\omega t} \\ 0 \end{pmatrix}. \quad (73)$$

Note that while z_1, z_2, w_1, w_2 satisfy the normalization and orthogonality constraints in (58), η and ω are arbitrary and will be fixed by the equations of motion.

The strategy now is to use the gauge field equations of motion to determine the gauge and scalar field profile functions. We will then use the scalar field equations of motion to obtain constraints on the parameters.

We would like to solve the following set of equations:

$$j_\mu^a = \sum_{i=1,2} \left\{ \frac{ig}{2} [\Phi_i^\dagger \lambda^a \partial_\mu \Phi_i - \text{H.c.}] + \frac{g^2}{3} W_\mu^a |\Phi_i|^2 + \frac{g^2}{2} d^{abc} W_\mu^b (\Phi_i^\dagger \lambda^c \Phi_i) \right\}, \quad (74)$$

where j_μ^a is defined in (70)–(72) and Φ_1 and Φ_2 are defined in (73). Substituting $\mu = t$ and $a = 3$ gives

$$-g\omega\eta^2(h_1^2 + h_2^2) = -\frac{\epsilon^2}{g}\Omega f^2, \quad (75)$$

and substituting $\mu = t$ and $a = 8$ results in $h_1 = h_2 \equiv h$. Substituting back in (75), we obtain

$$2g\omega\eta^2 h^2 = \frac{\epsilon^2}{g}\Omega f^2 \Rightarrow h(r) = \frac{\epsilon}{g\eta} \sqrt{\frac{\Omega}{2\omega}} f(r). \quad (76)$$

We must demand $\omega/\Omega > 0$ since h is a real-valued function. For $\mu = z$ in (74), we obtain

$$f'' + \frac{f'}{r} + (\Omega^2 - \eta^2 g^2 h^2) f = 0. \quad (77)$$

Further on, substituting the definition of $h(r)$ from (76), we can write

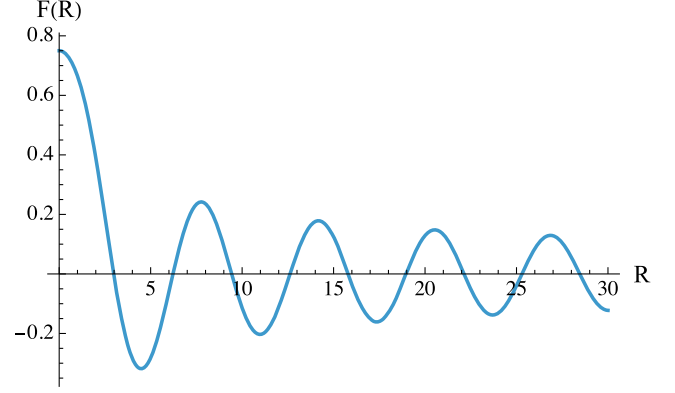


FIG. 1. $F(R)$ vs R for $F_0 = 0.75$.

$$f'' + \frac{f'}{r} + \Omega^2 \left(1 - \frac{\epsilon^2}{2\omega\Omega} f^2 \right) f = 0. \quad (78)$$

This is the equation for the gauge profile function resulting from the gauge field equation of motion. We can plot this numerically by defining rescaled variables,

$$R = \Omega r, \quad F = \frac{\epsilon}{\sqrt{2\omega\Omega}} f, \quad (79)$$

to write

$$F'' + \frac{F'}{R} + (1 - F^2) F = 0. \quad (80)$$

Following [9], we choose boundary conditions such that $F(0) = F_0 \neq 0$ and $F(\infty) \rightarrow 0$ along with $F'(0) = 0$. If $F_0 > 1$, then the F^3 term dominates and there are no solutions that fall off asymptotically as $R \rightarrow \infty$. Hence, we must restrict to $F_0 \leq 1$. A numerical plot of $F(R)$ for $F_0 = 0.75$ is shown in Fig. 1.

Now we consider the scalar field equation of motion, (60), to obtain

$$h'' + \frac{h'}{r} + (\omega^2 - m^2) h - \frac{\epsilon^2}{4} f^2 h - 2\kappa\eta^2 h^3 = 0, \quad (81)$$

where we have set $m_1 = m_2 \equiv m$ and $\kappa_1 = \kappa_2 \equiv \kappa$. Substituting (76) and simplifying, we obtain

$$f'' + \frac{f'}{r} + \left[(\omega^2 - m^2) - \left(\frac{\epsilon^2}{4} + \frac{\kappa\epsilon^2\Omega}{g^2\omega} \right) f^2 \right] f = 0. \quad (82)$$

Consistency with (78) implies

$$\Omega^2 = \omega^2 - m^2 \quad (83)$$

$$\frac{\Omega}{2\omega} = \frac{1}{4} + \frac{\kappa\Omega}{g^2\omega}. \quad (84)$$

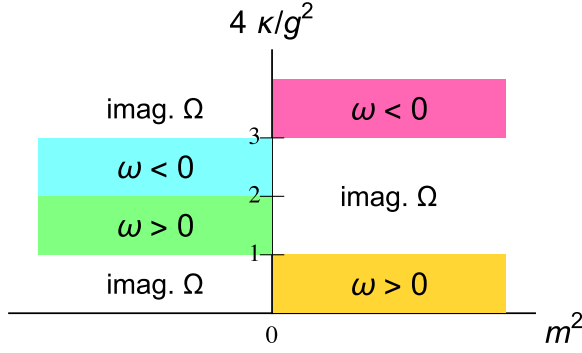


FIG. 2. Allowed regions in parameter space spanned by different values of m^2 and $4\kappa/g^2$ are shaded. The solution is valid in regions with $\omega > 0$.

We can solve (84) to obtain

$$\omega = 2 \left(1 - \frac{2\kappa}{g^2} \right) \Omega. \quad (85)$$

As given below Eq. (76), we demand $\omega/\Omega > 0$ which implies $0 \leq \kappa < g^2/2$. We can also solve (83) and (84) simultaneously to write

$$\Omega^2 = \frac{m^2}{4(1 - 2\kappa/g^2)^2 - 1} \quad (86)$$

$$\omega^2 = \frac{4(1 - 2\kappa/g^2)^2 m^2}{4(1 - 2\kappa/g^2)^2 - 1}. \quad (87)$$

Since Ω and ω are real, for $m^2 > 0$ we have $0 \leq \kappa < g^2/4$, while for $m^2 < 0$ we have $g^2/4 < \kappa < g^2/2$ since κ is bounded from above as discussed earlier. These allowed regions in parameter space are illustrated in Fig. 2.

While the gauge and scalar field equations admit solutions in both shaded regions of Fig. 2, the physical interpretation differs. For $m^2 > 0$, the true vacuum is at $\Phi_{1,2} = 0$, consistent with our boundary condition $h(r) \rightarrow 0$ as $r \rightarrow \infty$. For $m^2 < 0$, however, the scalar potential has a true vacuum at nonzero $|\Phi|$; in that case our boundary condition $\Phi_{1,2} \rightarrow 0$ at large r places the configuration asymptotically in the *false* vacuum. Accordingly, the $m^2 < 0$ branch should be viewed as a classical field configuration embedded in a false-vacuum background (a cylindrical “bubble” interpretation) rather than as an isolated, stable string in the true vacuum.

D. Asymptotic falloff and string tension

The numerical solution shown in Fig. 1 exhibits oscillations with a slowly decaying profile. To make the large- r falloff explicit, we linearize the profile equation at large radius. In terms of the rescaled variables $R = \Omega r$ and $F = \frac{\epsilon}{\sqrt{2\omega\Omega}} f$, the profile equation is

$$F'' + \frac{F'}{R} + (1 - F^2)F = 0. \quad (88)$$

For $R \gg 1$ we have $|F| \ll 1$ and the equation linearizes to

$$F'' + \frac{F'}{R} + F = 0, \quad (89)$$

whose solutions are Bessel functions. Imposing regularity at $R = 0$ selects $F(R) \simeq F_\infty J_0(R)$ at large R . Using the asymptotic expansion of J_0 , we obtain

$$F(R) \sim F_\infty \sqrt{\frac{2}{\pi R}} \cos\left(R - \frac{\pi}{4}\right), \quad (R \rightarrow \infty), \quad (90)$$

and hence $f(r) \propto r^{-1/2}$ with an oscillatory tail. Equivalently, $\sqrt{r}f(r)$ approaches an oscillatory function with approximately constant amplitude at large r .

This slow falloff implies that the tension which is the energy per unit length is not finite. The dominant contributions to the energy density arise from the gauge-field terms containing f^2 and f'^2 (and similarly from the scalar sector since $h \propto f$). Using $f \sim r^{-1/2} \cos(\Omega r)$, we have $f'^2 \sim \Omega^2/r$ and $f^2 \sim 1/r$, so the energy density behaves as $\mathcal{E}(r) \propto 1/r$ at large r . Therefore, the tension with a radial cutoff R_c ,

$$\mu(R_c) = 2\pi \int_0^{R_c} dr r \mathcal{E}(r), \quad (91)$$

diverges linearly with R_c (with possible subleading logarithmic contributions depending on how the derivative terms are grouped). Thus, in the classical Yang-Mills-scalar theory with massless gauge fields, the electric flux tube is axisymmetric and concentrated near the z -axis but is not exponentially localized in the Nielsen-Olesen sense; instead, it has an oscillatory long-range tail leading to a classically divergent tension similar to global monopole solutions [12].

Finally, for $m^2 < 0$ the interpretation is stronger: the true vacuum lies at nonzero scalar vacuum expectation value while our solution approaches $\Phi_{1,2} \rightarrow 0$ as $r \rightarrow \infty$, i.e., it asymptotes to the false vacuum. After shifting the potential so that the true vacuum has zero energy density, there remains a nonzero vacuum-energy density difference between the false and true vacua, and consequently the energy per unit length diverges quadratically, $\mu(R_c) \propto R_c^2$. For this reason, the $m^2 < 0$ branch should be regarded as a false-vacuum configuration rather than an isolated string in the true vacuum.

E. Summary of electric string solution

Finally, we summarize the electric string solution,

$$\Phi_1 = \frac{\epsilon}{g} \sqrt{\frac{\Omega}{2\omega}} f(r) \begin{pmatrix} z_1 e^{i\omega t} \\ z_2 e^{-i\omega t} \\ 0 \end{pmatrix}, \quad (92)$$

$$\Phi_2 = \frac{\epsilon}{g} \sqrt{\frac{\Omega}{2\omega}} f(r) \begin{pmatrix} w_1 e^{i\omega t} \\ w_2 e^{-i\omega t} \\ 0 \end{pmatrix} \quad (93)$$

$$W_\mu^1 = -\frac{\epsilon}{g} \cos \Omega t f(r) \partial_\mu z, \quad (94)$$

$$W_\mu^2 = -\frac{\epsilon}{g} \sin \Omega t f(r) \partial_\mu z \quad (95)$$

$$W_\mu^b = 0, \quad b = 3, \dots, 8, \quad (96)$$

where

$$\begin{aligned} |z_1|^2 + |z_2|^2 &= 1 \\ |w_1|^2 + |w_2|^2 &= 1 \\ z_1 w_1^* + z_2 w_2^* &= 0, \end{aligned} \quad (97)$$

along with

$$\Omega^2 = \frac{m^2}{4(1 - 2\kappa/g^2)^2 - 1} \quad (98)$$

$$\omega^2 = \frac{4(1 - 2\kappa/g^2)^2 m^2}{4(1 - 2\kappa/g^2)^2 - 1}. \quad (99)$$

The solution is valid for $0 \leq \kappa < g^2/4$ for $m^2 > 0$ and $g^2/4 < \kappa < g^2/2$ for $m^2 < 0$. The profile function satisfies (78) along with $f(0) \leq \sqrt{2\omega\Omega}/\epsilon$ and $f'(0) = 0$. In addition, the large- r asymptotics of the profile is oscillatory with $f(r) \propto r^{-1/2}$, which implies that the classical string tension diverges linearly with a radial cutoff for $m^2 > 0$, while for $m^2 < 0$ the configuration asymptotes to the false vacuum and the energy per unit length diverges quadratically.

IV. SCHWINGER PAIR PRODUCTION

In this section, we consider quantum fluctuations Q_μ^a about the background gauge fields (denoted A_μ^a) that are responsible for the electric field. The motivation is to show that the electric field is stable to Schwinger particle production which, in temporal gauge, is equivalent to particle production in a time-dependent background [7].

Charged excitations in our SU(3) model include gauge particles and scalar particles, and an electric field can

Schwinger produce both kinds of particles. However, the mass parameter of the scalar fields can be taken to be arbitrarily large, thus suppressing Schwinger production of the scalar excitations. Hence, we will only consider Schwinger pair production of gauge excitations, some of which can be massless. Since the Schwinger process can be viewed as particle production in a time-dependent background, we only need to show that our electric field background is independent of time for the dynamics of the gauge particles.

Consider perturbations about the gauge field background which give rise to electric fields in the λ_3 direction. The treatment is similar to that of [7]; however, we now need to consider perturbations in the $a = 3, \dots, 8$ components as well. Consider

$$W_\mu^a = A_\mu^a + Q_\mu^a. \quad (100)$$

We can combine the fields to write them as follows:

$$w_\mu^\pm = a_\mu^\pm + e^{\mp i\Omega t} q_\mu^\pm \quad (101)$$

$$y_\mu^\pm = b_\mu^\pm + e^{\mp i\Omega t/2} s_\mu^\pm \quad (102)$$

$$z_\mu^\pm = c_\mu^\pm + e^{\pm i\Omega t/2} t_\mu^\pm, \quad (103)$$

where we have defined

$$w_\mu^\pm = W_\mu^2 \pm iW_\mu^1 \quad (104)$$

$$y_\mu^\pm = W_\mu^5 \pm iW_\mu^4 \quad (105)$$

$$z_\mu^\pm = W_\mu^7 \pm iW_\mu^6, \quad (106)$$

and the background fields now take the following form:

$$a_\mu^\pm = \mp \frac{i\epsilon}{g} e^{\mp i\Omega t} f(r) \partial_\mu z \quad (107)$$

$$b_\mu^\pm = 0, \quad c_\mu^\pm = 0 \quad (108)$$

$$A_\mu^3 = 0, \quad A_\mu^8 = 0. \quad (109)$$

Substituting these backgrounds in (101)–(103) gives

$$w_\mu^\pm = e^{\mp i\Omega t} \left[\mp \frac{i\epsilon}{g} f(r) \partial_\mu z + q_\mu^\pm \right] \quad (110)$$

$$y_\mu^\pm = e^{\mp i\Omega t/2} s_\mu^\pm \quad (111)$$

$$z_\mu^\pm = e^{\pm i\Omega t/2} t_\mu^\pm \quad (112)$$

$$W_\mu^3 = Q_\mu^3, \quad W_\mu^8 = Q_\mu^8. \quad (113)$$

Next, we can compute the field strength tensor using (5),

$$\begin{aligned}
w_{\mu\nu}^{\pm} &= W_{\mu\nu}^2 \pm iW_{\mu\nu}^1 \\
&= \partial_{\mu}w_{\nu}^{\pm} - \partial_{\nu}w_{\mu}^{\pm} \pm igw_{\mu}^{\pm}W_{\nu}^3 \mp igW_{\mu}^3w_{\nu}^{\pm} \\
&\quad + \frac{g}{2}y_{\mu}^{\pm}z_{\nu}^{\mp} - \frac{g}{2}z_{\mu}^{\mp}y_{\nu}^{\pm}.
\end{aligned} \tag{114}$$

Substituting (110)–(113) in (114) gives

$$\begin{aligned}
w_{\mu\nu}^{\pm} &= e^{\mp i\Omega t} \left[a_{\mu\nu}^{\pm} + (\mathcal{D}_{\mu}q_{\nu}^{\pm} - \mathcal{D}_{\nu}q_{\mu}^{\pm}) \right. \\
&\quad \left. \pm ig(q_{\mu}^{\pm}Q_{\nu}^3 - q_{\nu}^{\pm}Q_{\mu}^3) + \frac{g}{2}(s_{\mu}^{\pm}t_{\nu}^{\mp} - s_{\nu}^{\pm}t_{\mu}^{\mp}) \right]
\end{aligned} \tag{115}$$

with

$$\begin{aligned}
a_{\mu\nu}^{\pm} &= -\frac{\epsilon\Omega}{g}f(r)(\partial_{\mu}t\partial_{\nu}z - \partial_{\nu}t\partial_{\mu}z) \\
&\quad \mp \frac{i\epsilon}{g}f'(r)(\partial_{\mu}r\partial_{\nu}z - \partial_{\nu}r\partial_{\mu}z)
\end{aligned} \tag{116}$$

$$\mathcal{D}_{\mu}q_{\nu}^{\pm} = \partial_{\mu}q_{\nu}^{\pm} \mp i\Omega\partial_{\mu}tq_{\nu}^{\pm} + \epsilon f(r)\partial_{\mu}zQ_{\nu}^3. \tag{117}$$

Similarly, we have

$$\begin{aligned}
y_{\mu\nu}^{\pm} &= W_{\mu\nu}^5 \pm iW_{\mu\nu}^4 \\
&= \partial_{\mu}y_{\nu}^{\pm} - \partial_{\nu}y_{\mu}^{\pm} - \frac{g}{2}w_{\mu}^{\pm}z_{\nu}^{\pm} + \frac{g}{2}z_{\mu}^{\pm}w_{\nu}^{\pm} \\
&\quad \pm ig(y_{\mu}^{\pm}x_{\nu}^+ - x_{\mu}^+y_{\nu}^{\pm}),
\end{aligned} \tag{118}$$

where we have defined $x_{\mu}^+ = (W_{\mu}^3 + \sqrt{3}W_{\mu}^8)/2$. Substituting (110)–(113) in (118) gives

$$\begin{aligned}
y_{\mu\nu}^{\pm} &= e^{\mp i\Omega t/2} \left[(\mathcal{D}_{\mu}s_{\nu}^{\pm} - \mathcal{D}_{\nu}s_{\mu}^{\pm}) \right. \\
&\quad \left. - \frac{g}{2}(q_{\mu}^{\pm}t_{\nu}^{\pm} - t_{\mu}^{\pm}q_{\nu}^{\pm}) \pm ig(s_{\mu}^{\pm}x_{\nu}^+ - x_{\mu}^+s_{\nu}^{\pm}) \right],
\end{aligned} \tag{119}$$

where

$$\mathcal{D}_{\mu}s_{\nu}^{\pm} = \partial_{\mu}s_{\nu}^{\pm} \mp \frac{i\Omega}{2}\partial_{\mu}ts_{\nu}^{\pm} \pm \frac{i\epsilon}{2}f(r)\partial_{\mu}z t_{\nu}^{\pm}. \tag{120}$$

Similarly,

$$\begin{aligned}
z_{\mu\nu}^{\pm} &= W_{\mu\nu}^7 \pm iW_{\mu\nu}^6 \\
&= \partial_{\mu}z_{\nu}^{\pm} - \partial_{\nu}z_{\mu}^{\pm} + \frac{g}{2}w_{\mu}^{\mp}y_{\nu}^{\pm} - \frac{g}{2}y_{\mu}^{\pm}w_{\nu}^{\mp} \\
&\quad \pm igx_{\mu}^{-}z_{\nu}^{\pm} \mp igz_{\mu}^{\pm}x_{\nu}^{-},
\end{aligned} \tag{121}$$

where $x_{\mu}^{-} = (W_{\mu}^3 - \sqrt{3}W_{\mu}^8)/2$. Once again, substituting (110)–(113) in (121) yields

$$\begin{aligned}
z_{\mu\nu}^{\pm} &= e^{\pm i\Omega t/2} \left[(\mathcal{D}_{\mu}t_{\nu}^{\pm} - \mathcal{D}_{\nu}t_{\mu}^{\pm}) \right. \\
&\quad \left. + \frac{g}{2}(q_{\mu}^{\mp}s_{\nu}^{\pm} - q_{\nu}^{\mp}s_{\mu}^{\pm}) \pm ig(x_{\mu}^{-}t_{\nu}^{\pm} - t_{\mu}^{\pm}x_{\nu}^{-}) \right],
\end{aligned} \tag{122}$$

where we have defined

$$\mathcal{D}_{\mu}t_{\nu}^{\pm} = \partial_{\mu}t_{\nu}^{\pm} \pm \frac{i\Omega}{2}\partial_{\mu}tt_{\nu}^{\pm} \pm \frac{i\epsilon}{2}f(r)\partial_{\mu}zs_{\nu}^{\pm}. \tag{123}$$

Following the same procedure, we can write

$$\begin{aligned}
W_{\mu\nu}^3 &= \partial_{\mu}W_{\nu}^3 - \partial_{\nu}W_{\mu}^3 - \frac{ig}{2}(w_{\mu}^{+}w_{\nu}^{-} - w_{\mu}^{-}w_{\nu}^{+}) \\
&\quad - \frac{ig}{4}(y_{\mu}^{+}y_{\nu}^{-} - y_{\mu}^{-}y_{\nu}^{+}) + \frac{ig}{4}(z_{\mu}^{+}z_{\nu}^{-} - z_{\mu}^{-}z_{\nu}^{+})
\end{aligned} \tag{124}$$

$$\begin{aligned}
W_{\mu\nu}^8 &= \partial_{\mu}W_{\nu}^8 - \partial_{\nu}W_{\mu}^8 - \frac{i\sqrt{3}g}{4}(y_{\mu}^{+}y_{\nu}^{-} - y_{\mu}^{-}y_{\nu}^{+}) \\
&\quad - \frac{i\sqrt{3}g}{4}(z_{\mu}^{+}z_{\nu}^{-} - z_{\mu}^{-}z_{\nu}^{+}),
\end{aligned} \tag{125}$$

where substituting (110)–(113) in (124) and (125) gives

$$\begin{aligned}
W_{\mu\nu}^3 &= \mathcal{D}_{\mu}Q_{\nu}^3 - \mathcal{D}_{\nu}Q_{\mu}^3 - \frac{ig}{2}(q_{\mu}^{+}q_{\nu}^{-} - q_{\mu}^{-}q_{\nu}^{+}) \\
&\quad - \frac{ig}{4}(s_{\mu}^{+}s_{\nu}^{-} - s_{\mu}^{-}s_{\nu}^{+}) + \frac{ig}{4}(t_{\mu}^{+}t_{\nu}^{-} - t_{\mu}^{-}t_{\nu}^{+})
\end{aligned} \tag{126}$$

$$\begin{aligned}
W_{\mu\nu}^8 &= \mathcal{D}_{\mu}Q_{\nu}^8 - \mathcal{D}_{\nu}Q_{\mu}^8 - \frac{i\sqrt{3}g}{4}(s_{\mu}^{+}s_{\nu}^{-} - s_{\mu}^{-}s_{\nu}^{+}) \\
&\quad - \frac{i\sqrt{3}g}{4}(t_{\mu}^{+}t_{\nu}^{-} - t_{\mu}^{-}t_{\nu}^{+}),
\end{aligned} \tag{127}$$

respectively, where we have defined

$$\mathcal{D}_{\mu}Q_{\nu}^3 = \partial_{\mu}Q_{\nu}^3 - \epsilon f(r)\partial_{\mu}z q_{\nu}^2, \tag{128}$$

along with $\mathcal{D}_{\mu}Q_{\nu}^8 = \partial_{\mu}Q_{\nu}^8$.

The gauge field Lagrangian density can then be written as

$$\begin{aligned}
\mathcal{L}_g &= -\frac{1}{4}W_{\mu\nu}^a W^{\mu\nu a} \\
&= -\frac{1}{4}[w_{\mu\nu}^{+}w^{\mu\nu -} + y_{\mu\nu}^{+}y^{\mu\nu -} + z_{\mu\nu}^{+}z^{\mu\nu -} \\
&\quad + W_{\mu\nu}^3 W^{\mu\nu 3} + W_{\mu\nu}^8 W^{\mu\nu 8}].
\end{aligned} \tag{129}$$

Substituting all the above field strength tensors in the gauge field Lagrangian density (129), we can check that the explicit time dependence completely cancels out.

This means that the dynamics of the gauge perturbations denoted by $q_{\mu}^{\pm}$, $s_{\mu}^{\pm}$, $t_{\mu}^{\pm}$, Q_{μ}^3 , and Q_{μ}^8 occur in a time-independent background, and they will not be excited. Equivalently, their quanta will not be produced.

V. CONCLUSIONS

We have constructed 3-type electric string solutions in $SU(3)$ non-Abelian gauge theory, as summarized in Sec. III E. The construction follows the procedure in Refs. [8,9] for $SU(2)$ theory but requires two scalar fields in the fundamental representation instead of just one. The construction of 8-type electric strings remains an open question.

Such electric solutions may potentially be relevant to non-Abelian gauge theories such as QCD, though the fundamental scalar fields would have to arise as effective degrees of freedom, perhaps due to backreaction of quantum excitations. A novelty of the 3-type electric string solutions, and also of the uniform 3-type electric field, is that they are stable to the quantum production of gauge bosons due to the Schwinger process.

We emphasize that the construction in this paper is classical: we establish the existence of stationary solutions of the Yang-Mills-scalar equations and, in Sec. IV, show that the explicit time dependence of the background cancels out of the gauge-fluctuation dynamics relevant for Schwinger production of gauge bosons. At the same time, in an asymptotically free non-Abelian gauge theory, the coupling grows in the infrared, and genuinely nonperturbative physics is expected at sufficiently large distances. Correspondingly, the oscillatory $r^{-1/2}$ tail and the associated classical divergence of the tension should be interpreted within the classical effective description.

There are a number of follow-up questions that we plan to address in the future. Are the 3-type solutions stable as in the $SU(2)$ case [5]? Can such solutions be studied using lattice methods? Can fundamental scalar fields arise as emergent degrees of freedom in pure non-Abelian gauge theory?

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX: NO SOLUTION WITH SCALAR FIELD IN THE ADJOINT REPRESENTATION

In this appendix, we show that there exists no solution with the scalar fields in the adjoint representation. Instead of providing a general proof for this, we will choose a scalar field that satisfies certain consistency conditions and then show that the corresponding scalar field four-current does not satisfy the gauge field equations.

Consider the homogeneous electric field in the λ_3 direction. The nonvanishing $SU(3)$ gauge field components are [see (20)]

$$W_\mu^1 = -\frac{\epsilon}{g} \cos \Omega t \partial_\mu z, \quad W_\mu^2 = -\frac{\epsilon}{g} \sin \Omega t \partial_\mu z, \quad (\text{A1})$$

and the corresponding non-vanishing four-current components are [see (25)–(27)]

$$j_\mu^1 = -\frac{\epsilon \Omega^2}{g} \cos(\Omega t) \partial_\mu z \quad (\text{A2})$$

$$j_\mu^2 = -\frac{\epsilon \Omega^2}{g} \sin(\Omega t) \partial_\mu z \quad (\text{A3})$$

$$j_\mu^3 = -\frac{\epsilon^2 \Omega}{g} \partial_\mu t. \quad (\text{A4})$$

The four-current for the adjoint scalar ϕ^a is

$$J_\mu^a = g f^{abc} \phi^b D_\mu \phi^c, \quad (\text{A5})$$

where the covariant derivative on ϕ^a is now defined as

$$D_\mu \phi^a = \partial_\mu \phi^a + g f^{abc} W_\mu^b \phi^c. \quad (\text{A6})$$

Since f^{abc} is totally antisymmetric, one straightforward constraint on ϕ^a is

$$\phi^a J_\mu^a = 0, \quad (\text{A7})$$

and repeated indices $a, b, c = 1, \dots, 8$ are summed over. Setting $\mu = t$ gives $\phi^3 = 0$ while setting $\mu = z$ gives

$$\cos(\Omega t) \phi^1 + \sin(\Omega t) \phi^2 = 0. \quad (\text{A8})$$

Hence, we can choose the nonvanishing components of ϕ^a as

$$\phi^1 = -\eta \sin(\Omega t), \quad \phi^2 = \eta \cos(\Omega t). \quad (\text{A9})$$

Using (A5), the nonvanishing components of the scalar field four-current are

$$J_\mu^1 = -\epsilon g \eta^2 \cos(\Omega t) \partial_\mu z \quad (\text{A10})$$

$$J_\mu^2 = -\epsilon g \eta^2 \sin(\Omega t) \partial_\mu z \quad (\text{A11})$$

$$J_\mu^3 = g \eta^2 \Omega \partial_\mu t. \quad (\text{A12})$$

Comparing with (A2)–(A4) gives

$$\eta = \frac{\Omega}{g} \quad (\text{A13})$$

and

$$\epsilon^2 = -\Omega^2. \quad (\text{A14})$$

Since both ϵ and Ω are real, there is no consistent nontrivial solution with scalar fields in the adjoint representation.

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