

Investigating the two-pion exchange of the double charm DD^* chiral interactions and T_{cc}

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Under chiral effective field theory, we study the S -wave DD^* interactions up to second chiral order at one-loop level, which contain full contact, one-pion-exchange (OPE) and two-pion-exchange (TPE) contributions. Here, we adopt a new subtraction scheme of the two-particle-reducible contributions and attempt different regularization schemes uniquely for the TPE contributions since they are highly divergent on the momentum transfer. Under these schemes, we conclude that, in the $I = 0$ channel, the TPE contribution is repulsive, then the competition between this powerful repulsing TPE and the other two (contact and OPE) results in a quite weak attraction. This explains why T_{cc} has an extremely small binding energy if treated as the $I = 0$ DD^* bound state. This feature resembles that of hidden charm $D\bar{D}^*$ as we investigated in previous work [Phys. Rev. D **105**, 034013 (2022)], which also interpreted the extremely near-threshold phenomenon of $X(3872)$. In addition, we also solve the Bethe-Salpeter equation with the chiral interactions for a consistency check.

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I. INTRODUCTION

The elaborate study of the heavy hadron interactions is a crucial aspect in nonperturbative QCD physics. It helps us to uncover the interacting mechanism of low energy QCD where the fundamental quarks and gluons have been confined and can also be directly utilized to understand the heavy exotic states and near-threshold phenomena, even serving as a nonperturbative QCD input to other areas. The advantage in the heavy hadron sector is that the emergent symmetry, i.e., the heavy quark symmetry, can be applied to benefit our studies.

There have been many kinds of methods to deal with the heavy hadron interactions and related exotic states. For example, the one-boson-exchange (OBE) model is one of the appropriate dynamical models. This model constructs effective meson interactions by taking into account various meson exchanges (π , σ , ρ , ω mesons, etc.). For example, Ref. [1] successfully applied the OBE model to the $D\bar{D}^*$ and $X(3872)$ study. See Ref. [2] for details about the OBE model. Other hadron-level approaches have also been widely used, such as the contact-range method, local hidden gauge approach, nonrelativistic effective field

theory, and other effective Lagrangian approaches [3–7]. There also exists quark-level approaches, where the interactions and concerned multiquark systems are evaluated based on the quark model [8–11]. In addition, lattice studies play an indispensable role in the area [12,13]. We also refer to Refs. [14–21] for other approaches not mentioned. Among them, chiral field effective theory (ChEFT) is one of the promising frameworks dealing with the heavy hadron interactions, which will be adopted in this work.

ChEFT was originally proposed to deal with the nucleon-nucleon interaction [22,23], it has been fully developed in the last decades, and it gives us a deep understanding of the nuclear force [24–30]. Similar to chiral perturbation theory [31], the ChEFT framework respects the chiral symmetry and arranges the low energy nucleon-nucleon interaction in a systematic way, so that the interaction is classified as direct contact and multi-Goldstone exchange contributions. Here unique schemes such as the Weinberg scheme have to be applied to keep a correct power counting. In the Weinberg scheme [22,23], with the standard power-counting rule, one calculates the two-particle-irreducible (2PI) diagrams, i.e., the effective potential, then iterates it into the equation (e.g., the Schrödinger or Lippmann-Schwinger equation) to retrieve the rest of the two-particle-reducible (2PR) contributions.

In a word, two advantages of ChEFT are prominent: the interaction is dealt with in a systematic way, and it manifests as the multi-Goldstone exchanges; so it is promising to apply ChEFT to the heavy hadron interactions. In the heavy hadron sector, ChEFT adopting the Weinberg scheme has indeed been developed in recent

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years. In Ref. [32], the authors introduced the ChEFT framework to study the doubly bottomed system $\bar{B}\bar{B}$ up to next-to-leading order (NLO); especially, they analyzed the two-pion-exchange contributions in detail. Based on this pioneer work [32], the authors in Ref. [33] developed the framework and applied the ChEFT to the doubly charmed DD^* system. A molecular state in the $I = 0$ channel was predicted then, which just matched the T_{cc}^+ state discovered later. The author in Ref. [34] extended to the hidden charm $D\bar{D}^*$ system by considering the proper charge conjugation properties of the heavy hadrons. The authors in Ref. [35] systematically studied the $D^{(*)}\bar{B}^{(*)}$ systems and their coupled-channel effects. Following this framework, other two-heavy-hadron systems were also studied [30,36–45].

Follow our previous work [33], we continue to investigate the DD^* interactions. DD^* is a typical two-heavy-hadron system and similar to NN , due to their same isospin configurations and similar interactions with the Goldstone bosons, so we can draw lessons from the research in the NN sector. Furthermore, because of the additional symmetry, i.e., heavy quark symmetry, the interactions in the heavy hadron sector can be simplified to a good approximation. Specifically, in this work the heavy hadron formalism will be adopted. Therefore, the study of the DD^* interactions can surely help us to reveal the nonperturbative QCD dynamics. On the other hand, due to the observation of T_{cc} in recent years [46,47], the related DD^* interactions are now surely under the spotlight.

In 2021, T_{cc}^+ was observed by the LHCb Collaboration in the distribution of the $D^0 D^0 \pi^+$ invariant mass [46,47]; its mass difference referring to the $D^0 D^{*+}$ threshold is only around 300 keV. Its extremely near-threshold nature as well as the exotic $cc\bar{u}\bar{d}$ content implicate T_{cc}^+ as a possible DD^* molecular candidate. In theory, T_{cc} and the DD^* system have been also studied in various models, such as the molecular approaches [6,48–57], quark-level approaches [58–64], and lattice simulations [65–75], as well as the production mechanisms [76–79].

So in this work we will study the DD^* interactions within ChEFT. In addition to our earlier work mentioned above [33], there have been other works discussing the DD^* interactions and T_{cc}^+ under ChEFT recently. The authors in Ref. [80] revisited the DD^* chiral interactions of Ref. [33] by introducing the local momentum-space regularization. The authors in Ref. [81] investigated the two-pion-exchange contributions in a general formalism (without the heavy hadron approach). Despite not being DD^* , authors in Ref. [82] studied the two-pion exchange of the $B^{(*)}B^{(*)}$ and $B^{(*)}\bar{B}^{(*)}$ systems, where the coupled channels were taken into account.

Notice that, in the investigation of the chiral interaction, to properly study the heavy multi-quark states and exotic phenomena, an unitarization procedure is necessary. The unitarization methods have been widely applied to different areas, e.g., meson-meson interactions, NN interactions,

physics at LHC energies, and quantum gravity. Especially, it achieves successes in the studies of hadronic bound states and resonances such as $f_0(500)$ and $\Lambda(1405)$; see Refs. [83–86] for details. In this work, due to the lack of the unitarization, we preliminarily discuss the relation between our calculated interaction and observed T_{cc}^+ state.

In this research, we study the DD^* chiral interactions under ChEFT within heavy hadron formalism, up to NLO at one-loop level, where the Weinberg scheme is adopted. Confronting previous work [33], here we will further correct the defect of the previous 2PR subtraction method and elaborately explore the regularization schemes uniquely for the two-pion exchange because of the intrinsic difficulty it faces. We also solve the Bethe-Salpeter (BS) equation to explore the consistency between different iterative equations. Actually, main conclusions do not change with our new results, but new insights about the DD^* binding force and the T_{cc} state appear.

This paper is organized as follows. In Sec. II, we briefly review the chiral interactions of the concerned DD^* system, express the calculated potentials of the individual contact, one-pion-exchange (OPE) and two-pion-exchange (TPE) contributions, then discuss the 2PR reduction scheme and the regularization of the potentials especially for the TPE. In Sec. III, we present the numerical results along with an elaborate discussion of the TPE contributions under different schemes. In Sec. IV, we iterate the DD^* chiral interactions into the BS equation for comparing. The last section is the summary.

II. THE CHIRAL INTERACTIONS OF THE DD^* SYSTEM

Following Refs. [32–34], we continue to investigate the DD^* interactions under ChEFT. This framework arranges the amplitudes with an expansion parameter $\epsilon = p/\Lambda_\chi$ (p stands for a small momentum, pion mass, or $D - D^*$ mass splitting). Specifically, we will construct the Lagrangians and interaction potentials by the order parameter ϵ with Weinberg's power-counting scheme [22,23]. Note that in this work the flavor $SU(2)$ symmetry is adopted.

A. The chiral Lagrangians

As in Ref. [33], we will calculate the amplitudes or interaction potentials up to second order $O(\epsilon^2)$ at one-loop level, so the $D^{(*)}D^{(*)}\pi$ Lagrangian and 4 $D^{(*)}$ contact Lagrangian at leading order are needed.

Under the flavor $SU(2)$ symmetry, the $DD^*\pi$ Lagrangian at $O(\epsilon^1)$ reads [87–89]

$$\begin{aligned} \mathcal{L}_{H\phi}^{(1)} = & -\langle (iv \cdot \partial H) \bar{H} \rangle + \langle H v \cdot \Gamma \bar{H} \rangle + g \langle H \not{v} \gamma_5 \bar{H} \rangle \\ & - \frac{1}{8} \delta \langle H \sigma^{\mu\nu} \bar{H} \sigma_{\mu\nu} \rangle, \end{aligned} \quad (1)$$

with the axial vector field $u_\mu = \frac{i}{2} \{\xi^\dagger, \partial_\mu \xi\}$ and chiral connection $\Gamma_\mu = \frac{i}{2} [\xi^\dagger, \partial_\mu \xi]$, where $\xi = \exp(i\phi/2f)$ with

$$\phi = \sqrt{2} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} & \pi^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} \end{pmatrix}. \quad (2)$$

The (D, D^*) doublet H is

$$\begin{aligned} H &= \frac{1 + \not{P}}{2} (P_\mu^* \gamma^\mu + i P \gamma_5), \\ \bar{H} &= \gamma^0 H^\dagger \gamma^0 = (P_\mu^\dagger \gamma^\mu + i P^\dagger \gamma_5) \frac{1 + \not{P}}{2}, \\ P &= (D^0, D^+), \quad P_\mu^* = (D^{*0}, D^{*+})_\mu. \end{aligned} \quad (3)$$

The contact Lagrangian at $O(\epsilon^0)$ is also needed [3,32,90],

$$\begin{aligned} \mathcal{L}_{4H}^{(0)} &= D_a \text{Tr}[H \gamma_\mu \bar{H}] \text{Tr}[H \gamma^\mu \bar{H}] \\ &+ D_b \text{Tr}[H \gamma_\mu \gamma_5 \bar{H}] \text{Tr}[H \gamma^\mu \gamma_5 \bar{H}] \\ &+ E_a \text{Tr}[H \gamma_\mu \tau^a \bar{H}] \text{Tr}[H \gamma^\mu \tau_a \bar{H}] \\ &+ E_b \text{Tr}[H \gamma_\mu \gamma_5 \tau^a \bar{H}] \text{Tr}[H \gamma^\mu \gamma_5 \tau_a \bar{H}], \end{aligned} \quad (4)$$

where the low energy constants (LECs) D_a, D_b, E_a, E_b should be determined by other methods. The Lagrangians at $O(\epsilon^2)$, which are needed to renormalize the $O(\epsilon^2)$ loop amplitudes, can be found in Refs. [32,33].

The adopted Weinberg power-counting scheme [22,23] tells us that, with the chiral power counting, we calculate the sum of the 2PI diagrams first, then iterate it into the Schrödinger, Lippmann-Schwinger, or BS equation, where the 2PR contributions can be restored. Here we will calculate the interaction potentials, then solve the Schrödinger equation.

B. The diagrams of the DD^* scattering and the DD^* interaction potentials

The flavor wave functions $|I; K\rangle$ of the DD^* system with isospin $I = 0, 1$ and momentum K can be written as

$$\begin{aligned} |0, 0; K\rangle &= \frac{1}{\sqrt{2}} (|D^0 D^{*+}\rangle - |D^+ D^{*0}\rangle), \\ |1, 0; K\rangle &= \frac{1}{\sqrt{2}} (|D^0 D^{*+}\rangle + |D^+ D^{*0}\rangle), \\ |1, 1; K\rangle &= |D^+ D^{*+}\rangle, \\ |1, -1; K\rangle &= |D^0 D^{*0}\rangle. \end{aligned} \quad (5)$$

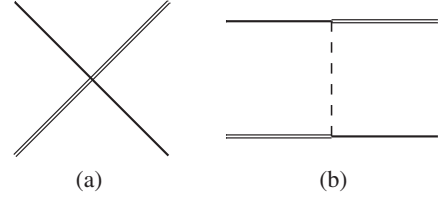


FIG. 1. Tree-level diagrams of the DD^* system at $O(\epsilon^0)$: (a) the contact diagram; (b) the OPE diagram. The solid, double solid, and dashed lines stand for D, D^* , and π , respectively.

Considering the above flavor wave functions, we can calculate interaction potentials $\mathcal{V}^{I=0,1}$ up to the chiral order $O(\epsilon^2)$ at one-loop level (NLO). Just at this order, the nontrivial TPE contribution emerges.

We now show the calculated potentials containing all the 2PI contributions. At $O(\epsilon^0)$ (tree level), there exists a OPE and a contact 2PI diagram, which are shown in Fig. 1. With the help of the Lagrangians (1) and (4), we obtain their contributions to $\mathcal{V}^{I=0,1}$,

$$\mathcal{V}_{\text{contact}(0)}^{I=0} = 2(D_a + D_b - 3E_a - 3E_b) \epsilon \cdot \epsilon^*, \quad (6)$$

$$\mathcal{V}_{\text{contact}(0)}^{I=1} = 2(D_a - D_b + E_a - E_b) \epsilon \cdot \epsilon^*, \quad (7)$$

$$\mathcal{V}_{1\pi(0)}^{I=0} = \frac{3}{4} \frac{g^2}{f^2} \frac{1}{q^2 - m^2} q \cdot \epsilon q \cdot \epsilon^*, \quad (8)$$

$$\mathcal{V}_{1\pi(0)}^{I=1} = \frac{1}{4} \frac{g^2}{f^2} \frac{1}{q^2 - m^2} q \cdot \epsilon q \cdot \epsilon^*. \quad (9)$$

In the above expressions, the subscript (0) of $\mathcal{V}$ stands for the order $O(\epsilon^0)$, q denotes the momentum transfer between initial and final states, and m, m_1 , and m_2 are the masses of π, D , and D^* , respectively. In addition, ϵ and ϵ^* are the polarization vectors.

For the diagrams at $O(\epsilon^2)$, there appear one-loop TPE diagrams, as well as one-loop corrections to the $O(\epsilon^0)$ contact and OPE diagrams, which are shown in Figs. 2 and 3. As in Ref. [33], we ignore the finite parts of the $O(\epsilon^2)$ tree-level amplitudes here due to the lack of their LEC information; of course, their divergent parts are used to renormalize the $O(\epsilon^2)$ one-loop diagrams [33]. According to Figs. 2 and 3, we now show the calculated $\mathcal{V}$ at $O(\epsilon^2)$ as individual contact, OPE and TPE contributions,

$$\begin{aligned} \mathcal{V}_{\text{contact}(2)}^{I=0} &= \frac{3g^2}{2f^2} \{ -C_1(d-1) \partial_\omega J_{22}^b(-\delta) - C_1(d-2) \partial_\omega J_{22}^b(0) - C_1 \partial_\omega J_{22}^b(\delta) - C_1 J_{22}^h(-\delta, \delta) - C_1 J_{22}^h(\delta, -\delta) \\ &- (dC_2 + 2C_4) J_{22}^g(-\delta, -\delta) - C_2(d-3)(d-2) J_{22}^g(0, 0) - 2C_3 J_{22}^g(\delta, \delta) - 2C_4 J_{22}^g(-\delta, \delta) - 2C_4 J_{22}^g(\delta, -\delta) \\ &- 2C_5(d-3)(d-2) J_{22}^h(0, -\delta) - 2C_5(d-3)(d-2) J_{22}^h(-\delta, 0) \} \epsilon \cdot \epsilon^*, \end{aligned} \quad (10)$$

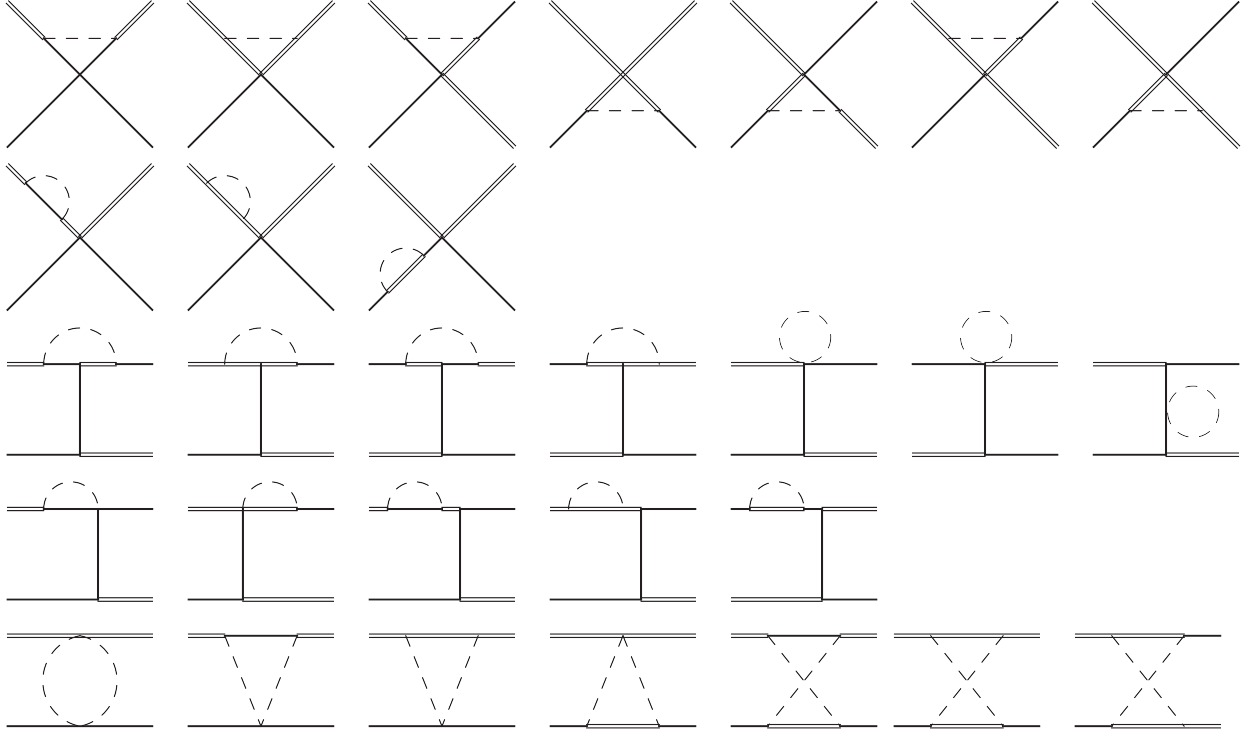


FIG. 2. One-loop 2PI diagrams of the DD^* system at $O(\epsilon^2)$. The solid, double solid, and dashed lines stand for D , D^* , and π , respectively.

$$\begin{aligned} \mathcal{V}_{\text{contact}(2)}^{I=1} = & \frac{g^2}{2f^2} \{ -3C_2(d-1)\partial_\omega J_{22}^b(-\delta) - 3C_2(d-2)\partial_\omega J_{22}^b(0) + 3(C_4 - C_3)\partial_\omega J_{22}^b(\delta) + (C_4 - C_3)J_{22}^b(-\delta, \delta) \\ & + (C_4 - C_3)J_{22}^b(\delta, -\delta) - 4C_3J_{22}^g(\delta, \delta) + 4C_4J_{22}^g(-\delta, \delta) + 4C_4J_{22}^g(\delta, -\delta) - 2C_5(d-3)(d-2)J_{22}^g(0, -\delta) \\ & - 2C_5(d-3)(d-2)J_{22}^g(-\delta, 0) + [(2-3d)D_a + (d-2)D_b + (d-6)E_a + 5(d-2)E_b]J_{22}^g(-\delta, -\delta) \\ & - (d-3)(d-2)(3D_a - D_b - E_a - 5E_b)J_{22}^g(0, 0) \} \epsilon \cdot \epsilon^*, \end{aligned} \quad (11)$$

$$\begin{aligned} \mathcal{V}_{1\pi(2)}^{I=0} = & \frac{-3}{16f^4} \frac{1}{q^2 - m^2} \{ g^4[3(d-1)\partial_\omega J_{22}^b(-\delta) + 3(d-2)\partial_\omega J_{22}^b(0) + 3\partial_\omega J_{22}^b(\delta) + (d-3)(d-2)J_{22}^g(0, -\delta) \\ & + (d-3)(d-2)J_{22}^g(-\delta, 0) - J_{22}^g(-\delta, \delta) - J_{22}^g(\delta, -\delta)] - 8g^2J_0^c \} q \cdot \epsilon q \cdot \epsilon^*, \end{aligned} \quad (12)$$

$$\begin{aligned} \mathcal{V}_{1\pi(2)}^{I=1} = & \frac{-1}{16f^4} \frac{1}{q^2 - m^2} \{ g^4[3(d-1)\partial_\omega J_{22}^b(-\delta) + 3(d-2)\partial_\omega J_{22}^b(0) + 3\partial_\omega J_{22}^b(\delta) + (d-3)(d-2)J_{22}^g(0, -\delta) \\ & + (d-3)(d-2)J_{22}^g(-\delta, 0) - J_{22}^g(-\delta, \delta) - J_{22}^g(\delta, -\delta)] - 8g^2J_0^c \} q \cdot \epsilon q \cdot \epsilon^*, \end{aligned} \quad (13)$$

$$\begin{aligned} \mathcal{V}_{2\pi(2)}^{I=0} = & \frac{3}{16f^4} \{ [-3(d-3)J_{22}^B(-\delta, 0)\bar{q}^4 + (d-3)J_{22}^R(-\delta, 0)\bar{q}^4 - 6(d-3)J_{32}^B(-\delta, 0)\bar{q}^4 + 2(d-3)J_{32}^R(-\delta, 0)\bar{q}^4 \\ & - 3(d-3)J_{43}^B(-\delta, 0)\bar{q}^4 + (d-3)J_{43}^R(-\delta, 0)\bar{q}^4 + 6(d-3)J_{21}^B(-\delta, 0)\bar{q}^2 - (d-3)J_{21}^R(-\delta, 0)\bar{q}^2 + (d-3)J_{21}^R(-\delta, -\delta)\bar{q}^2 \\ & + 3(d-3)(2d+1)J_{31}^B(-\delta, 0)\bar{q}^2 + 3J_{31}^B(-\delta, \delta)\bar{q}^2 - (d-3)(2d+1)J_{31}^R(-\delta, 0)\bar{q}^2 - J_{31}^R(-\delta, \delta)\bar{q}^2 \\ & + 3(d-3)(2d+1)J_{42}^B(-\delta, 0)\bar{q}^2 + 3J_{42}^B(-\delta, \delta)\bar{q}^2 - (d-3)(2d+1)J_{42}^R(-\delta, 0)\bar{q}^2 - J_{42}^R(-\delta, \delta)\bar{q}^2 \\ & - 3(d-3)(d-2)(d+1)J_{41}^B(-\delta, 0) - 3(d+1)J_{41}^B(-\delta, \delta) + (d-3)(d-2)(d+1)J_{41}^R(-\delta, 0) + (d+1)J_{41}^R(-\delta, \delta)]g^4 \\ & + 2[(d-3)q_0J_{11}^S(0)\bar{q}^2 - q_0J_{11}^T(-\delta)\bar{q}^2 + 3(d-3)q_0J_{22}^S(0)\bar{q}^2 - 3q_0J_{22}^T(-\delta)\bar{q}^2 + 2(d-3)J_{24}^S(0)\bar{q}^2 - 2J_{24}^T(-\delta)\bar{q}^2 \\ & + 2(d-3)q_0J_{32}^S(0)\bar{q}^2 - 2q_0J_{32}^T(-\delta)\bar{q}^2 + 2(d-3)J_{33}^S(0)\bar{q}^2 - 2J_{33}^T(-\delta)\bar{q}^2 - (d-3)(d-2)q_0J_{21}^S(0) - q_0J_{21}^S(\delta) \} \end{aligned}$$

$$\begin{aligned}
 & + (d-1)q_0J_{21}^T(-\delta) - 2(d-3)(d-2)q_0J_{31}^S(0) - 2q_0J_{31}^S(\delta) + 2(d-1)q_0J_{31}^T(-\delta) - 2(d-3)(d-2)J_{34}^S(0) - 2J_{34}^S(\delta) \\
 & + 2(d-1)J_{34}^T(-\delta)]g^2 + q_0^2J_0^F + 4q_0^2J_{11}^F + 4q_0^2J_{21}^F + 4J_{22}^F\} \varepsilon \cdot \varepsilon^* + \frac{2g^2}{16f^4} \left\{ \frac{3}{2} [-3(d-3)J_{22}^B(-\delta, 0)\vec{q}^2 + 3J_{22}^B(-\delta, \delta)\vec{q}^2 \right. \\
 & + (d-3)J_{22}^R(-\delta, 0)\vec{q}^2 - J_{22}^R(-\delta, \delta)\vec{q}^2 - 6(d-3)J_{32}^B(-\delta, 0)\vec{q}^2 + 6J_{32}^B(-\delta, \delta)\vec{q}^2 + 2(d-3)J_{32}^R(-\delta, 0)\vec{q}^2 \\
 & - 2J_{32}^R(-\delta, \delta)\vec{q}^2 - 3(d-3)J_{43}^B(-\delta, 0)\vec{q}^2 + 3J_{43}^B(-\delta, \delta)\vec{q}^2 + (d-3)J_{43}^R(-\delta, 0)\vec{q}^2 - J_{43}^R(-\delta, \delta)\vec{q}^2 + 6(d-3)J_{21}^B(-\delta, 0) \\
 & - 3J_{21}^B(-\delta, \delta) - (d-3)J_{21}^R(-\delta, 0) + (d-3)J_{21}^R(-\delta, -\delta) + J_{21}^R(-\delta, \delta) + 3(d^2-9)J_{31}^B(-\delta, 0) - 3(d+3)J_{31}^B(-\delta, \delta) \\
 & - (d^2-9)J_{31}^R(-\delta, 0) + (d+3)J_{31}^R(-\delta, \delta) + 3(d^2-9)J_{42}^B(-\delta, 0) - 3(d+3)J_{42}^B(-\delta, \delta) - (d^2-9)J_{42}^R(-\delta, 0) \\
 & + (d+3)J_{42}^R(-\delta, \delta)]g^2 + 3(d-3)q_0J_{11}^S(0) - 3q_0J_{11}^S(\delta) + 9(d-3)q_0J_{22}^S(0) - 9q_0J_{22}^S(\delta) + 6(d-3)J_{24}^S(0) - 6J_{24}^S(\delta) \\
 & \left. + 6(d-3)q_0J_{32}^S(0) - 6q_0J_{32}^S(\delta) + 6(d-3)J_{33}^S(0) - 6J_{33}^S(\delta) \right\} q \cdot \varepsilon q \cdot \varepsilon^*, \quad (14)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{V}_{2\pi(2)}^{\prime=1} = & \frac{-1}{16f^4} \{ [(d-3)J_{22}^B(-\delta, 0)\vec{q}^4 + 5(d-3)J_{22}^R(-\delta, 0)\vec{q}^4 + 2(d-3)J_{32}^B(-\delta, 0)\vec{q}^4 + 10(d-3)J_{32}^R(-\delta, 0)\vec{q}^4 \\
 & + (d-3)J_{43}^B(-\delta, 0)\vec{q}^4 + 5(d-3)J_{43}^R(-\delta, 0)\vec{q}^4 - 5(d-3)J_{21}^R(-\delta, 0)\vec{q}^2 - 5(d-3)J_{21}^R(-\delta, -\delta)\vec{q}^2 \\
 & + ((5-2d)d+3)J_{31}^B(-\delta, 0)\vec{q}^2 - J_{31}^B(-\delta, \delta)\vec{q}^2 - 5(d-3)(2d+1)J_{31}^R(-\delta, 0)\vec{q}^2 - 5J_{31}^R(-\delta, \delta)\vec{q}^2 \\
 & + ((5-2d)d+3)J_{42}^B(-\delta, 0)\vec{q}^2 - J_{42}^B(-\delta, \delta)\vec{q}^2 - 5(d-3)(2d+1)J_{42}^R(-\delta, 0)\vec{q}^2 - 5J_{42}^R(-\delta, \delta)\vec{q}^2 \\
 & + (d-3)(d-2)(d+1)J_{41}^B(-\delta, 0) + (d+1)J_{41}^B(-\delta, \delta) + 5(d-3)(d-2)(d+1)J_{41}^R(-\delta, 0) + 5(d+1)J_{41}^R(-\delta, \delta)]g^4 \\
 & + 2[(d-3)q_0J_{11}^S(0)\vec{q}^2 - q_0J_{11}^T(-\delta)\vec{q}^2 + 3(d-3)q_0J_{22}^S(0)\vec{q}^2 - 3q_0J_{22}^T(-\delta)\vec{q}^2 + 2(d-3)J_{24}^S(0)\vec{q}^2 - 2J_{24}^T(-\delta)\vec{q}^2 \\
 & + 2(d-3)q_0J_{32}^S(0)\vec{q}^2 - 2q_0J_{32}^T(-\delta)\vec{q}^2 + 2(d-3)J_{33}^S(0)\vec{q}^2 - 2J_{33}^T(-\delta)\vec{q}^2 - (d-3)(d-2)q_0J_{21}^S(0) - q_0J_{21}^S(\delta) \\
 & + (d-1)q_0J_{21}^T(-\delta) - 2(d-3)(d-2)q_0J_{31}^S(0) - 2q_0J_{31}^S(\delta) + 2(d-1)q_0J_{31}^T(-\delta) - 2(d-3)(d-2)J_{34}^S(0) - 2J_{34}^S(\delta) \\
 & + 2(d-1)J_{34}^T(-\delta)]g^2 + q_0^2J_0^F + 4q_0^2J_{11}^F + 4q_0^2J_{21}^F + 4J_{22}^F\} \varepsilon \cdot \varepsilon^* + \frac{-2g^2}{16f^4} \left\{ \frac{1}{2} [(d-3)J_{22}^B(-\delta, 0)\vec{q}^2 - J_{22}^B(-\delta, \delta)\vec{q}^2 \right. \\
 & + 5(d-3)J_{22}^R(-\delta, 0)\vec{q}^2 - 5J_{22}^R(-\delta, \delta)\vec{q}^2 + 2(d-3)J_{32}^B(-\delta, 0)\vec{q}^2 - 2J_{32}^B(-\delta, \delta)\vec{q}^2 + 10(d-3)J_{32}^R(-\delta, 0)\vec{q}^2 \\
 & - 10J_{32}^R(-\delta, \delta)\vec{q}^2 + (d-3)J_{43}^B(-\delta, 0)\vec{q}^2 - J_{43}^B(-\delta, \delta)\vec{q}^2 + 5(d-3)J_{43}^R(-\delta, 0)\vec{q}^2 - 5J_{43}^R(-\delta, \delta)\vec{q}^2 + J_{21}^B(-\delta, \delta) \\
 & - 5(d-3)J_{21}^R(-\delta, 0) - 5(d-3)J_{21}^R(-\delta, -\delta) + 5J_{21}^R(-\delta, \delta) - (d^2-9)J_{31}^B(-\delta, 0) + (d+3)J_{31}^B(-\delta, \delta) \\
 & - 5(d^2-9)J_{31}^R(-\delta, 0) + 5(d+3)J_{31}^R(-\delta, \delta) - (d^2-9)J_{42}^B(-\delta, 0) + (d+3)J_{42}^B(-\delta, \delta) - 5(d^2-9)J_{42}^R(-\delta, 0) \\
 & + 5(d+3)J_{42}^R(-\delta, \delta)]g^2 + (d-3)q_0J_{11}^S(0) - q_0J_{11}^S(\delta) + 3(d-3)q_0J_{22}^S(0) - 3q_0J_{22}^S(\delta) + 2(d-3)J_{24}^S(0) - 2J_{24}^S(\delta) \\
 & \left. + 2(d-3)q_0J_{32}^S(0) - 2q_0J_{32}^S(\delta) + 2(d-3)J_{33}^S(0) - 2J_{33}^S(\delta) \right\} q \cdot \varepsilon q \cdot \varepsilon^*, \quad (15)
 \end{aligned}$$

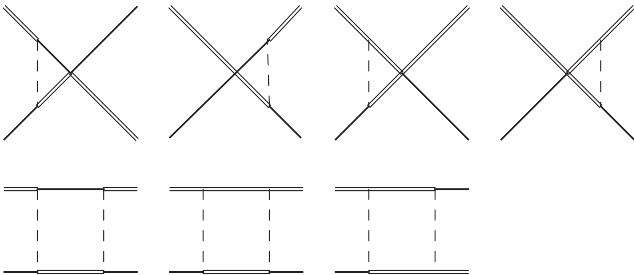


FIG. 3. One-loop 2PR diagrams of the DD^* system at $O(\epsilon^2)$. The solid, double solid, and dashed lines stand for D , D^* , and π , respectively.

where $C_1 = D_a + D_b - 3(E_a + E_b)$, $C_2 = D_a - D_b + E_a - E_b$, $C_3 = D_a + E_a$, $C_4 = D_b + E_b$, and $C_5 = D_b - 3E_b$. In the above, d is the space-time dimension originated from the dimensional regularization, δ is the $D - D^*$ mass splitting, and $\mathcal{V}$ relates to the scattering amplitude $\mathcal{M}$ via $\mathcal{V} = \mathcal{M}/(-\sqrt{\prod 2M_i} \prod 2M_f)$. In addition, considering the S -wave interactions, we have $\vec{\varepsilon} \cdot \vec{\varepsilon}^* \rightarrow 1$ and $\vec{\varepsilon} \cdot \vec{p} \vec{\varepsilon}^* \cdot \vec{p} \rightarrow \vec{p}^2/(d-1)$.

The definitions of the loop function J are shown in Eqs. (A1)–(A6) (see also Refs. [32,33]). Also we list the chiral order of each J in Table II, so one can conveniently count the orders of Eqs. (10)–(15). They are all collected in Appendix A.

Under Fourier transformation, we can get the potential $\mathcal{V}(\mathbf{r})$,

$$\mathcal{V}(\mathbf{r}) = \int \frac{d\mathbf{p}}{(2\pi)^3} \mathcal{V}(\mathbf{p}) e^{i\mathbf{p}\cdot\mathbf{r}}, \quad (16)$$

where $\mathcal{V}(\mathbf{p})$ is a polynomial of $\mathbf{p}$, so the integral will be highly divergent with increasing of the order.

C. The subtraction of the 2PR contributions and the regularization of the potentials

To obtain a 2PI effective potential and further substitute it into the equations, we have to subtract the 2PR contributions appearing in all the 2PR diagrams, because they violate the power-counting rule and cause pinch singularities in our heavy hadron formalism.

Now take a closer look at a 2PR diagram, e.g., the box diagram (TPE contribution) in Fig. 2. When closing the energy contour integral, we are able to collect two poles in the loop function: the pion pole and the heavy meson pole. The pion pole belongs to the 2PI part and satisfies the power counting, whereas the heavy meson pole belongs to the iterated OPE contribution (i.e., 2PR part) and has an enhanced chiral order that violates the power counting. Furthermore, the heavy meson poles located in upper and lower planes come too close, which produces a pinch singularity in the heavy hadron formalism. Therefore, we have to preserve only the pion pole.

To collect the pion pole, in our previous work [33] we applied the substitution

$$\begin{aligned} & \int d^4l \frac{1}{v \cdot l + a + i\epsilon} \frac{1}{-v \cdot l - a + i\epsilon} \times \dots \\ & \rightarrow \int d^4l \frac{1}{v \cdot l + a + i\epsilon} (-1) \frac{1}{v \cdot l + a + i\epsilon} \times \dots \end{aligned} \quad (17)$$

when calculating the loop functions. However, such substitution only subtracts the heavy meson pole in the single channel diagrams (e.g., $DD^* \rightarrow D^*D \rightarrow DD^*$); the coupled-channel diagrams (e.g., $DD^* \rightarrow D^*D^* \rightarrow DD^*$) are intact. So we extend the subtraction to contain all the types of the 2PR diagrams,

$$\begin{aligned} & \int d^4l \frac{1}{v \cdot l + a + i\epsilon} \frac{1}{-v \cdot l - b + i\epsilon} \times \dots \\ & \rightarrow \int d^4l \frac{1}{v \cdot l + a + i\epsilon} (-1) \frac{1}{v \cdot l + b + i\epsilon} \times \dots \end{aligned} \quad (18)$$

Note that, although the 2PR parts of the coupled-channel 2PR diagrams avoid the pinch singularities due to a tiny separation of the two heavy meson poles (originated from the mass splitting δ), their contributions are still abnormally enhanced ($\sim 1/\delta$). Therefore, the subtractions of these contributions are surely needed.

After discussing the subtraction of the 2PR contributions, we now turn to the regularization of the calculated 2PI potential. In previous work [33,34], we use a simple Gaussian cutoff $\exp(-\vec{q}^{2n}/\Lambda^{2n})$ to regularize the potential. However, simple use of this regulator leads to two drawbacks in the TPE calculation: an overestimation of the TPE, as well as a highly sensitive dependence on the cutoff Λ .

The reason is that the calculated TPE potential is a polynomial of momentum $\vec{q}$ (especially contains the terms of high $\vec{q}$ power), this NLO contribution just shows a good convergence within a low $\vec{q}$ region (empirically, say, up to 2π mass, around 300 MeV). When we apply $\exp(-\vec{q}^{2n}/\Lambda^{2n})$, the main contribution of the potential will come from the region $|\vec{q}| \lesssim \Lambda$. So if Λ is below 300 MeV, the convergence would be quite good, i.e., the TPE contribution (NLO) would be relatively small. However, in practice, Λ is generally chosen to be 500–1000 MeV, which means that the contributions in the region $|\vec{q}| \lesssim 500$ (~ 1000) MeV are all included, breaking up the convergence and causing TPE to be abnormally large. Actually, with the increase of Λ , the contributions from the high $\vec{q}$ region will grow rapidly and become dominant; note that these contributions mainly attribute to the terms of high $\vec{q}$ powers in the potential.

As we discussed above, with a relatively high Λ , the main contribution of the potential comes from the high $\vec{q}$ region and the terms of high $\vec{q}$ powers, which means the TPE is dominated by short distance contribution. In the coordinate space, this TPE potential would be a short distance potential, the range it reaches can be shorter than a contact potential. Therefore, if insisting on using the practically determined Λ , we need to suppress the contribution of the TPE in the region $m_{\text{ch}} \lesssim |\vec{q}| \lesssim \Lambda$ (where m_{ch} is typically around 2π mass, about 300 MeV) in addition to the regulator $\exp(-\vec{q}^{2n}/\Lambda^{2n})$.

So, here we could adopt the following schemes to deal with the above problems in the TPE potential:

Scheme I: As we explained above, the chiral expansion is reasonable, while the high $\vec{q}$ contribution is excluded, only when cutoff Λ is low, so we just constrain Λ to 0–450 MeV here. In this case, we do not need to introduce additional strategies to suppress the TPE. However, due to the low chosen Λ , the potential may become less attractive with the already fixed LECs; therefore, in this scheme, we refit the LECs.

Scheme II: In this scheme, we do not constrain the Λ ; instead, we introduce a regulator $\mathcal{F}_{2\pi}(\vec{q})$ to the TPE contribution in addition to the overall regulator $\exp(-\vec{q}^{2n}/\Lambda^{2n})$. The purpose of $\mathcal{F}_{2\pi}(\vec{q})$ is to constrain the highly divergent contribution beyond $|\vec{q}| = m_{\text{ch}}$. For $\mathcal{F}_{2\pi}(\vec{q})$, we use

$$\mathcal{F}_{2\pi}(\vec{q}) = \left(\frac{\mu_{2\pi}^2}{\vec{q}^2 + \mu_{2\pi}^2} \right)^{n_{2\pi}} \quad (19)$$

with $n_{2\pi} = 4$. We adjust $\mu_{2\pi}$ so that the strength of the TPE outside the perturbation region ($|\vec{q}| \gtrsim m_{\text{ch}}$) does not surpass the strength in $|\vec{q}| \lesssim m_{\text{ch}}$.

Scheme III: In this scheme, we also do not constrain the Λ , however, different from scheme II, we adopt another strategy to suppress the TPE. We preserve the contribution in the region $|\vec{q}| \lesssim m_{\text{ch}}$, then, we set the momentum potential $\mathcal{V}_{2\pi}^{\text{III}}(\vec{q})$ (superscript III stands for scheme III) to a constant starting from $|\vec{q}| = m_{\text{ch}}$, i.e.,

$$\mathcal{V}_{2\pi}^{\text{III}}(\vec{q}) = \begin{cases} \mathcal{V}_{2\pi}(\vec{q}) & |\vec{q}| \leq m_{\text{ch}}, \\ \mathcal{V}_{2\pi}(m_{\text{ch}}) & |\vec{q}| \geq m_{\text{ch}}, \end{cases} \quad (20)$$

where $\mathcal{V}_{2\pi}(\vec{q})$ is the unmodified TPE contribution. As we discussed above, we choose $m_{\text{ch}} = 300$ MeV. This scheme has advantages over scheme II: it not only suppresses the TPE in the high $\vec{q}$ region, but lets the TPE have the same ultraviolet behavior as the OPE and contact (they are both constants in the large $|\vec{q}|$ region), as we will discuss below.

Here we give some comments on schemes II and III. When we look into the individual contributions of contact, OPE, and TPE in momentum space later, in Fig. 4, we can find that, with the increase of $\vec{q}$, the contact and OPE tend to constants because of the contact terms in them, which means that their sensitivities on Λ are same. It further indicates that the two contributions can adopt a same regularization procedure, i.e., they may be regularized with a same regulator as well as the same Λ . However, the TPE is not the case. The TPE contribution goes to infinity because of the terms of high $\vec{q}$ power, so its ultraviolet

behavior is different from that of the contact or OPE. Consequently, the TPE is very sensitive on the cutoff parameter Λ , so a different regularization or Λ may be applied. This is the reason we introduce schemes II and III.

Especially in scheme III, we set the TPE to a constant beyond m_{ch} within which the expansion is valid, so that all three contributions have consistent ultraviolet behaviors. Based on this, we are ready to apply an unified regularization scheme to them. Therefore, scheme III is most promising among the three schemes.

At last, note that, in all the schemes including I–III, we apply the following overall regulator $\mathcal{F}(\vec{q})$ to the contact, OPE, and TPE contributions,

$$\mathcal{F}(\vec{q}) = \exp(-\vec{q}^{2n}/\Lambda^{2n}). \quad (21)$$

Of course, we emphasize again that, besides $\mathcal{F}(\vec{q})$, the TPE is further dealt with using Eq. (19) (scheme II) or Eq. (20) (scheme III).

After an elaborate numerical analysis, we find that the above three schemes give similar potential behaviors and bound state conclusions; so in the following section we adopt our best choice, i.e., scheme III, to present the numerical results and discussions. Later, we just briefly mention the results of other two schemes.

III. NUMERICAL RESULTS OF THE DD^* POTENTIALS

With the calculated expressions of $\mathcal{V}(q)$ and the 2PR subtraction scheme, as well as the proposed regularization schemes in Sec. II C, we now present the numerical results and discuss the behaviors of the DD^* potentials with different schemes in detail. In this work, we use the input parameters following Refs. [33,34], especially the low energy constants $D_a^{\text{RSM}} = -6.62$, $E_a^{\text{RSM}} = -5.74$, $D_b^{\text{RSM}} = 0$, and $E_b^{\text{RSM}} = 0$, where RSM means they are obtained using the resonance saturation model. In addition, we use $q^0 = \delta$.

A. The original DD^* potentials in momentum space

In this subsection, we discuss the potentials in momentum space without any scheme applied, i.e., with only overall regulator $\mathcal{F}(\vec{q})$ (21) attached. We list the results in Fig. 4. Here we consider the $I = 0$ channel. Note that $q = |\vec{q}|$ is the 3-momentum in Fig. 4.

From Fig. 4 we can see that, for the results without the regulator ($\Lambda = \infty$), when we restrict ourselves to the low q region (e.g., $q \leq 0.3$ GeV) the $O(e^2)$ TPE is small, which means that the convergence of our chiral expansion series is good. Move sights to the high q region, the contact and OPE contributions both tend to finite constants with the increase of q , which is reasonable because they have zeroth power of q . However, the TPE contribution rapidly goes to divergence with the increase of q because of the terms of high q power in TPE, so its convergence surely breaks up.

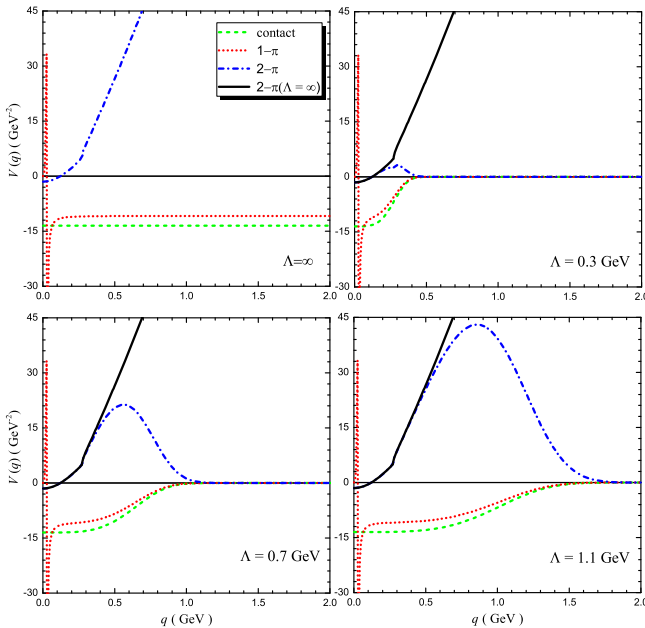


FIG. 4. The original $I = 0$ DD^* potentials $\mathcal{V}(q)$ in momentum space attached with only overall regulator (21), varying with cutoff Λ . Note that $\Lambda = \infty$ just stands for the totally unregulated case. $q = |\vec{q}|$ is the 3-momentum in units of GeV.

As we discussed before, it may indicate that the contact and OPE can be applied with a same regularization scheme, while the TPE cannot be. In conclusion, the high q behavior of the TPE is totally different from that of the contact or OPE. So we, indeed, need to adopt a distinctive regularization strategy for the TPE contribution.

We now focus on the results with the overall regulator $\mathcal{F}(\vec{q})$ turning on. At $\Lambda = 0.3$ GeV, when looking at the general line shape, we observe that the TPE contribution is small compared to the contact and OPE contributions, which shows a good convergence. As we explained in the previous section, a low Λ means only the contribution within the low q region is preserved. When moving the cutoff to a higher value ($\Lambda = 0.7, 1.1$ GeV) in Fig. 4, we see the suppressed higher region contributions are gradually recovered. Among them, the TPE is much more sensitive to the Λ than the contact and OPE, and it consequently becomes abnormally large at a high Λ , so that its relative magnitude is far beyond the contact and OPE contributions. As we dictated in the previous section, such TPE will become a short distance interaction which possesses a shorter range than the contact interaction. In a word, the TPE should be treated differently in the regularization procedure.

We also mention that, in Fig. 4, the regularized TPE preserves the unregularized TPE potential (black line) in the low q region, while in the high q region they are suppressed to zero by $\mathcal{F}(\vec{q})$. So the use of the regulator (21) is reasonable because the low momentum contributions are well preserved.

At last, we discuss the general behaviors of all the contributions. From Fig. 4, we see the contact and OPE are attractive while the TPE is repulsive. The judgment of the repulsive TPE is one of our most important conclusions. Note that it is consistent with the results of the $I = 0$ $\bar{B}\bar{B}^*$ system, where the $\delta = 0$ limit is considered [36], and also consistent with the $D^{(*)}\bar{B}^{(*)}$ calculation in Ref. [35].

B. $I = 0$ DD^* potentials under proposed schemes

1. Scheme III

Through the above investigations, we speculate that a careful treatment of the TPE is needed. So in this subsection we show the $I = 0$ results applying scheme III, which is the most promising choice among the three schemes as discussed in Sec. II C. As we said, in this scheme we constrain the TPE using Eq. (20). This scheme has an advantage over scheme II: the TPE and other two contributions will have a same high momentum behavior, so that we can confidently use the same regularization scheme (21) on them.

We depict the potentials in momentum space varying with Λ in Fig. 5. From the $\Lambda = \infty$ case, we see in this scheme that the TPE goes to a constant just as other contributions. As we discussed in Sec. II C, in this situation

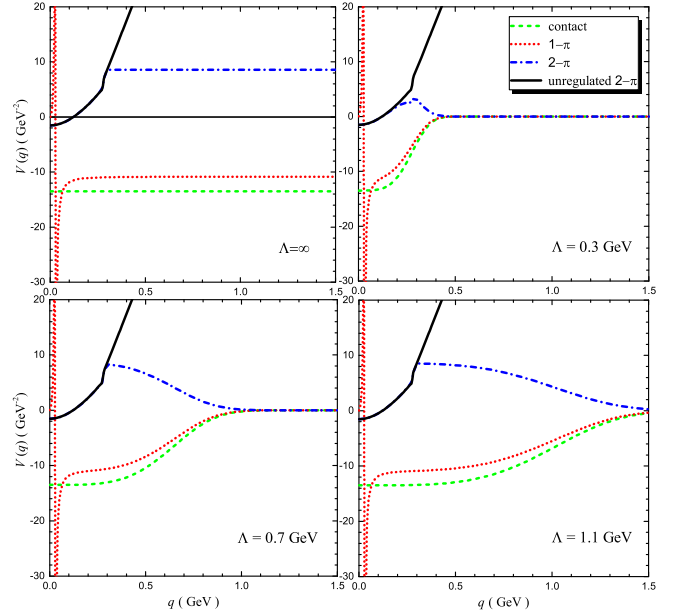


FIG. 5. The $I = 0$ DD^* potentials $\mathcal{V}(q)$ under scheme III, varying with cutoff Λ . $q = |\vec{q}|$ is the 3-momentum in units of GeV. The “unregulated $2 - \pi$ ” stands for the totally unmodified TPE contribution [$\Lambda = \infty$, and no application of Eq. (20)].

all contributions have a same high q (ultraviolet) behavior, so we are able to use the same regularization procedure [Eq. (21)] on them. It brings a profitable consequence that all three contributions have highly consistent dependence on the Λ , as we show in Fig. 5 with Λ varying from 0.3 to 1.1 GeV. So the TPE is no longer dominant over the other two at high Λ , and the expansion shows a good convergence. In other words, it solves the problem discussed in Sec. II C.

In Fig. 6 we present the potentials $\mathcal{V}(r)$. As we expect, the behaviors of the TPE contributions are consistent with the $\mathcal{V}(q)$ shown in Fig. 5: the TPE rises synchronously with the contact and OPE on Λ and grows smoothly when Λ varies from 0.3 to 1.1 GeV; also, it does not get overwhelmingly large at high Λ . In Fig. 6 at $\Lambda = 0.94$ GeV, the potential $\mathcal{V}(r)$ can produce the mass of the T_{cc} state when substituted in the Schrödinger equation.

From Fig. 6, we can see that the competition between the powerful repulsing TPE and the other two (contact and OPE) leads to a quite weak attraction, which explains why T_{cc} has an extremely small binding energy if treated as the $I = 0$ DD^* bound state. Therefore, we believe the repulsive nature of the TPE in our framework is in accord with the experimental observation. On the other hand, if the TPE is attractive, we expect all three attractive contributions should have caused a much deeper $I = 0$ DD^* bound state.

The above feature of T_{cc} ($I = 0$ DD^*) is also very similar to that of $X(3872)$ ($I = 0$ $D\bar{D}^*$) presented in our previous work [34]. In Ref. [34], we find that, for $X(3872)$ treated as a 0^+1^{++} $D\bar{D}^*$ bound state, its TPE is repulsive while the

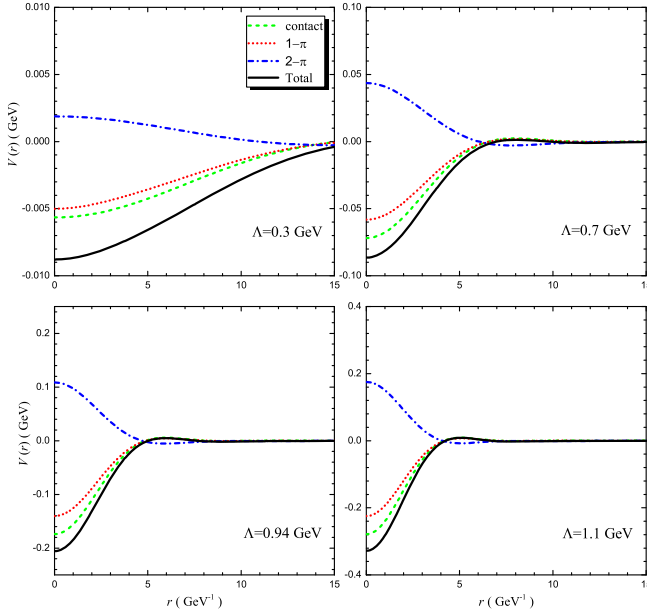


FIG. 6. The $I = 0$ DD^* potentials $\mathcal{V}(r)$ under scheme III, varying with cutoff Λ . Note that the potentials at $\Lambda = 0.94$ GeV correspond to the observed T_{cc} .

contact and OPE are attractive, which leads to a small binding energy. In a word, this common feature shared by $X(3872)$ and T_{cc} , especially the repulsive TPE, is responsible for their extremely near-threshold phenomena.

2. Brief discussion of schemes I and II

As we dictated at the end of Sec. II C, we will not present the details of the scheme I and II results here, but give a short discussion.

In scheme I, other than using an additional regularization on the TPE, we actually restrict cutoff Λ to a low value to maintain the convergence, as well as suppress the high q contribution. Here we collect the results in Figs. 9 and 10 in Appendix B. In this scheme, we observe that, because of the adopted low Λ , the potentials span across large distances, while the magnitudes of the potentials are relatively small. This means that, at these low Λ , they are very long-range potentials.

In scheme II, we apply an additional factor $\mathcal{F}_{2\pi}(\vec{q})$ (19) to constrain the TPE outside the perturbation region ($|\vec{q}| \gtrsim m_{ch}$). Numerical results show that the TPE contribution in the high q region is largely suppressed compared to the original highly divergent unmodified TPE just as in scheme III, though under a different strategy.

In a word, in all three schemes the potential behaviors are similar, and their conclusion about the bound state is also the same: the near-threshold nature of the T_{cc} state may originate from the competition between the TPE and the other two.

There is a comment about schemes III and II. They all preserve the TPE contribution in the low q region,

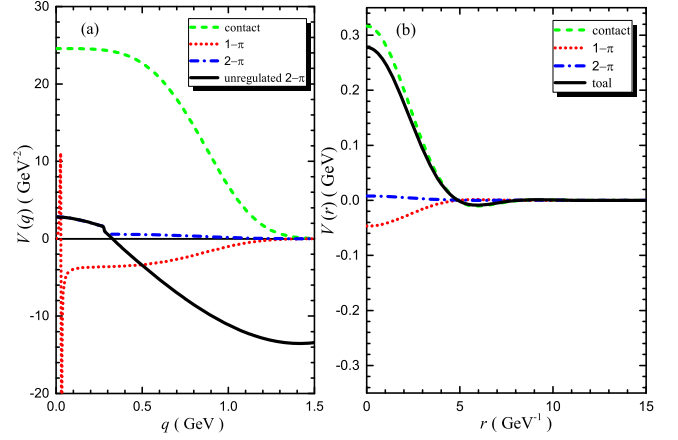


FIG. 7. The $I = 1$ DD^* potentials under scheme III: (a) the potentials $\mathcal{V}(q)$; (b) the potentials $\mathcal{V}(r)$. $q = |\vec{q}|$ is the 3-momentum in units of GeV. The unregulated $2 - \pi$ stands for the totally unmodified TPE contribution [$\Lambda = \infty$, and no application of Eq. (20)].

while they modify that in the high q region. We can find scheme II completely cuts off the high q contribution, whereas scheme III retains it to let the TPE coincide with the contact and OPE at high q . Obviously, scheme III is better. However, notice that the TPE at high q are largely cut by the regulator (21), so their final potentials present not much discrepancy.

C. $I = 1$ DD^* potentials

Having discussed the $I = 0$ DD^* potentials in the different schemes, we now show the $I = 1$ potentials. Here we adopt scheme III.

We first depict the $I = 1$ DD^* potentials in momentum space in Fig. 7(a). We can find that the contact contribution is dominant over the small OPE and TPE; it is also repulsive.

The $I = 1$ DD^* potentials in coordinate space are shown in Fig. 7(b). Consistent with the $\mathcal{V}(q)$ in Fig. 7(a), the contact potential is dominant and repulsive; so the total potential $\mathcal{V}(r)$ behaves as a strong repulsive force. Indeed, when solving the Schrödinger equation, we cannot find a bound state solution within a wide range of Λ .

IV. SOLVING THE BS EQUATION WITH THE DD^* CHIRAL INTERACTIONS

In this section, we alternatively solve the BS equation to see the consistency between the results of the two equations.

A. The DD^* BS equation and interaction kernels

In this subsection, we first reduce the BS equation, then give the expressions of the DD^* interaction kernels.

For the DD^* system carrying momentum K and mass M , after the center of mass motion is removed, the BS wave

function is defined as

$$\begin{aligned}\chi_{ij}^{I,\mu}(x, K) &= \langle 0 | T[\mathcal{D}_i(\lambda_2 x) \mathcal{D}_j^{*\mu}(-\lambda_1 x)] | I; K \rangle \\ &= C_{I,I_3}^{ij} \chi^{I,\mu}(x, K).\end{aligned}\quad (22)$$

Here $\mathcal{D}(\mathcal{D}^{*\mu})$ is the field operator of the $D(D^*)$ meson, x is the relative coordinate between D and D^* , and we assume $D(D^*)$ possesses the momentum $k_1(k_2)$ and mass $m_1(m_2)$. The subscript $i(j) = 1, 2$ is the flavor index, I and I_3 are isospin and third component, respectively, and $\lambda_{1(2)} = m_{1(2)}/(m_1 + m_2)$. Note that, in the above, the isospin coefficients C_{I,I_3}^{ij} are determined according to the isospin wave functions (5),

$$\begin{aligned}C_{0,0}^{12} &= \frac{1}{\sqrt{2}}, & C_{0,0}^{21} &= -\frac{1}{\sqrt{2}}, \\ C_{1,0}^{12} &= \frac{1}{\sqrt{2}}, & C_{1,0}^{21} &= \frac{1}{\sqrt{2}}, \\ C_{1,1}^{22} &= 1, & C_{1,-1}^{11} &= 1.\end{aligned}\quad (23)$$

The BS wave function in momentum space $\chi_{ij}^{I,\mu}(k, K)$ is defined via

$$\chi_{ij}^{I,\mu}(x, K) = \int \frac{d^4 k}{(2\pi)^4} \chi_{ij}^{I,\mu}(k, K) e^{-ikx}, \quad (24)$$

where k is the relative momentum between D and D^* . Using k we also have $k_1 = \lambda_1 K + k$ and $k_2 = \lambda_2 K - k$. $\chi_{ij}^{I,\mu}(k, K)$ satisfies the following homogeneous integral equation:

$$\begin{aligned}\chi_{ij}^{I,\mu}(k, K) &= s_D(k_1) s_{D^*}^{\nu\mu}(k_2) \int \frac{d^4 l}{(2\pi)^4} \chi_{mn}^{I,\alpha}(l, K) \mathcal{K}_{mnij,\alpha\nu}(l, k, K),\end{aligned}\quad (25)$$

where $s_{D^{(*)}}$ is the propagator for $D^{(*)}$, and $\mathcal{K}$ is the interaction kernel. Notice that l is a relative momentum too, so that $l_1 = \lambda_1 K + l$ and $l_2 = \lambda_2 K - l$. Taking into account $\chi_{ij}^{I,\mu} = C_{I,I_3}^{ij} \chi^{I,\mu}$ and C_{I,I_3}^{ij} values (23), we reduce Eq. (25) to the equation of $\chi^{I,\mu}$,

$$\chi^\mu(k) = s_D(k_1) s_{D^*}^{\nu\mu}(k_2) \int \frac{d^4 l}{(2\pi)^4} \mathcal{K}_{\alpha\nu}^I(l, k, K) \chi^\alpha(l), \quad (26)$$

where χ^μ is the abbreviation of $\chi^{I,\mu}$. The specific $I = 0, 1$ kernels $\mathcal{K}$ read

$$\begin{aligned}\mathcal{K}_{\alpha\nu}^{I=0}(l, k, K) &= \mathcal{K}^{D^0 D^{*+} \rightarrow D^0 D^{*+}}(l, k, K) - \mathcal{K}^{D^+ D^{*0} \rightarrow D^0 D^{*+}}(l, k, K), \\ \mathcal{K}_{\alpha\nu}^{I=1}(l, k, K) &= \mathcal{K}^{D^0 D^{*+} \rightarrow D^0 D^{*+}}(l, k, K) + \mathcal{K}^{D^+ D^{*0} \rightarrow D^0 D^{*+}}(l, k, K).\end{aligned}\quad (27)$$

Note that, in general, a kernel is expressed as a function of the momentum transfer q and p : $\mathcal{K}(l, k, K) = \mathcal{K}(p, q, K)$ with relations $q = l_1 - k_1$ and $p = l_1 - k_2$.

For solving the equations later, we decompose the relative momentum k into the longitudinal and transverse parts: $k = k_\ell v + k_t$, $v \cdot k_t = 0$, with $v = K/M$ being the four velocity of the DD^* system. Here we consider the instantaneous approximation: $\mathcal{K}(p, q, K) \simeq \mathcal{K}(p_t, q_t, K)$. In this approach, we need to integrate out the longitudinal part of Eq. (26) to arrive at a three-dimensional integral equation. Defining $\chi^\mu(k_t) = \int dk_\ell / 2\pi \chi^\mu(k)$, we perform the contour integrals $\int dk_\ell / 2\pi$ on both sides of Eq. (26) and obtain

$$\begin{aligned}\chi^\mu(k_t) &= \int \frac{d^3 l_t}{(2\pi)^3} \left[\frac{-i(-g^{\nu\mu} + k_2^\nu k_2^\mu / m_2^2)|_{k_\ell = -(\lambda_1 M + \omega_1)}}{(-2\omega_1)(M + \omega_1 + \omega_2)(M + \omega_1 - \omega_2)} \right. \\ &\quad \left. + \frac{i(-g^{\nu\mu} + k_2^\nu k_2^\mu / m_2^2)|_{k_\ell = \lambda_2 M - \omega_2}}{(M - \omega_2 + \omega_1)(M - \omega_2 - \omega_1)(2\omega_2)} \right] \\ &\quad \times \mathcal{K}_{\alpha\nu}^I(p_t, q_t, K) \chi^\alpha(l_t),\end{aligned}\quad (28)$$

with $\omega_i = \sqrt{m_i^2 - k_i^2}$.

We consider the $J^P = 1^+$ state only, so the BS wave function $\chi^\mu(k_t)$ can be constructed as

$$\chi^\mu(k_t) = \varepsilon^\mu(K) \varphi(k_t). \quad (29)$$

In addition, we normalize the obtained BS wave function with

$$i \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 l}{(2\pi)^4} \bar{\chi}^\mu(k) \left[\frac{\partial}{\partial K^0} I_{\mu\nu}(k, l, K) \right] \chi^\nu(l) = 2K^0. \quad (30)$$

According to the 2PI diagrams in Figs. 1 and 2, we now calculate $\mathcal{K}^{I=0,1}$ [defined in Eq. (27)] up to the order $O(\epsilon^2)$. Here we show the $\mathcal{K}$ at order $O(\epsilon^0)$,

$$\mathcal{K}_{\text{contact}(0)}^{I=0,\mu\nu} = -8im_1 m_2 (D_a + D_b - 3E_a - 3E_b) g^{\mu\nu}, \quad (31)$$

$$\mathcal{K}_{\text{contact}(0)}^{I=1,\mu\nu} = -8im_1 m_2 (D_a - D_b + E_a - E_b) g^{\mu\nu}, \quad (32)$$

$$\mathcal{K}_{1\pi(0)}^{I=0,\mu\nu} = -3im_1 m_2 \frac{g^2}{f^2} \frac{1}{p^2 - m^2} p^\mu p^\nu, \quad (33)$$

TABLE I. The bound state solutions of the DD^* BS equations in the $I = 0, 1$ channels. The cutoff parameter Λ , calculated mass M , and binding energy E relative to the $D^0 D^{*+}$ threshold are in units of GeV, MeV, and MeV, respectively. Note that the solution at $\Lambda = 0.79$ GeV corresponds to the observed T_{cc} state.

$I = 0$			$I = 1$	
Λ	M	E	M	E
0.5	...	...	...	...
0.6	...	...	...	...
0.79	3874.7	0.36	...	...
0.8	3874.2	0.89	...	...
0.9	3869.9	5.19	...	...
1.0	3860.8	14.29	...	...
1.1	3845.8	29.29	...	...
1.3	3796.8	78.90	...	...
1.5	3705.8	169.29	...	...

$$\mathcal{K}_{1\pi(0)}^{I=1,\mu\nu} = -im_1 m_2 \frac{g^2}{f^2} \frac{1}{p^2 - m^2} p^\mu p^\nu, \quad (34)$$

while $\mathcal{K}$ at order $O(\epsilon^2)$ are collected in Appendix C.

B. The numerical results

Here we solve the obtained BS equation above numerically. For the regularization of the TPE, we adopt scheme III (20). Note that, due to the distinct formalism compared to the Schrödinger equation, the application of scheme III here is slightly different. For the terms with the structure $g^{\mu\nu}$ in the kernels $\mathcal{K}$, we just apply Eq. (20), while for the terms with the structure $q^\mu q^\nu$ (or $p^\mu p^\nu$), we replace $q^\mu q^\nu$ with $q^\mu q^\nu m_{\text{ch}}^2 / \vec{q}^2$ beyond $|\vec{q}| = m_{\text{ch}}$ for consistency.

After solving the BS equation, we obtain bound state solutions at some typical Λ values, which are listed in Table I. We can see that the $I = 0$ channel has a bound state

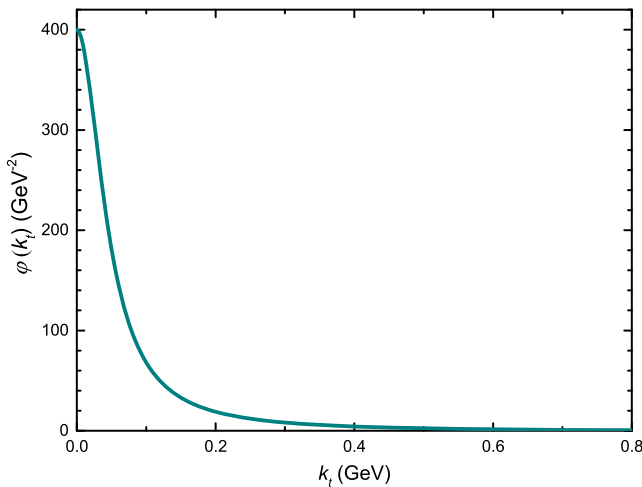


FIG. 8. The obtained $I = 0$ DD^* BS wave function $\varphi(k_t)$ which corresponds to the solution at $\Lambda = 0.79$ GeV, i.e., the T_{cc} state. k_t denotes the 3-momentum $|\vec{k}_t|$ in units of GeV.

solution emerging at around $\Lambda = 0.79$ GeV, and the binding energy increases with the variation of Λ . Also, the solution at $\Lambda = 0.79$ corresponds to the observed T_{cc} state, while for the $I = 1$ channel there does not exist any solution in all the Λ range. These conclusions are surely consistent with those obtained by solving the Schrödinger equation in Sec. III.

Here we also depict the normalized scalar BS wave function $\varphi(k_t)$ of Eq. (29) in Fig. 8. From Fig. 8, we can find that the behavior of $\varphi(k_t)$ accords with that of the typical bound state wave function.

V. SUMMARY

In this work, we studied the DD^* interactions under chiral effective field theory. Within heavy hadron formalism, we calculate the DD^* potentials up to order $O(\epsilon^2)$ (NLO) at one-loop level. Comparing to our previous work [33], we adopt a new 2PR subtraction scheme for the 2PR diagrams and discuss multiple regularization strategies unique for the TPE. We elaborately investigate the potential behaviors under the proposed regularizations of the TPE. At last, we solve the Schrödinger equations with the obtained potentials and search for the bound state solutions as well as their relations to the experimental findings. In addition, we also solve the BS equations of the DD^* interactions for a consistency check.

Specifically, among the investigation, our improvements of the framework are as follows. First, due to the intermediate $D^* D^*$ states as the coupled channel appearing in some 2PR diagrams, we apply a proper 2PR subtraction method to remove the additional poles brought by the coupled channels, so that all the heavy meson poles are just subtracted. Second, because the TPE contribution is highly divergent on the momentum transfer q far more than the contact and OPE, we believe a distinctive regularization may be imposed on the TPE. So, in this work, we introduce schemes to additionally deal with the TPE regularization. The proposed schemes can properly tackle the TPE in the high q region, so that the TPE and the other two (contact and OPE) all exhibit consistent and unified behaviors.

Here we list our main conclusions and key findings. We find that there exists a bound state solution in the $I = 0$ channel, while no solution can be found in the $I = 1$ channel, so the observed T_{cc} state indeed can be treated as the $I = 0$ DD^* bound state. However, the interaction detail is most interesting: For the $I = 0$ T_{cc} state, the TPE contribution is repulsive, then the competition between the repulsing TPE and other two contributions leads to a quite weak attraction. This just explains why the binding energy of T_{cc} is extremely small.

The above feature of T_{cc} just resembles $X(3872)$. As we indicated in Ref. [34], small binding energy of $X(3872)$ also comes from the balance between the repulsing TPE and the other two if it is treated as a $0^+ 1^{++} D\bar{D}^*$ bound state. In conclusion, the repulsive nature of the TPE, which

is shared by both T_{cc} and $X(3872)$, is responsible for their extremely near-threshold phenomena.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [91].

APPENDIX A: DEFINITIONS OF THE LOOP FUNCTIONS

Following Refs. [32,33], we define the loop function J as

$$i \int \frac{d^D l \mu^{4-D}}{(2\pi)^D} \frac{1}{(l^2 - m^2 + i\epsilon)} \equiv J_0^c(m), \quad (A1)$$

$$i \int \frac{d^D l \mu^{4-D}}{(2\pi)^D} \frac{\{1, l^\alpha, l^\alpha l^\beta, l^\alpha l^\beta l^\gamma\}}{[(+/-)v \cdot l + \omega + i\epsilon](l^2 - m^2 + i\epsilon)} \\ \equiv \{J_0^{a/b}, v^\alpha J_{11}^{a/b}, v^\alpha v^\beta J_{21}^{a/b} + g^{\alpha\beta} J_{22}^{a/b}, (g \vee v) J_{31}^{a/b} + v^\alpha v^\beta v^\gamma J_{32}^{a/b}\}(m, \omega), \quad (A2)$$

$$i \int \frac{d^D l \mu^{4-D}}{(2\pi)^D} \frac{\{1, l^\alpha, l^\alpha l^\beta, l^\alpha l^\beta l^\gamma\}}{(v \cdot l + \omega_1 + i\epsilon)[(+/-)v \cdot l + \omega_2 + i\epsilon](l^2 - m^2 + i\epsilon)} \\ \equiv \{J_0^{g/h}, v^\alpha J_{11}^{g/h}, v^\alpha v^\beta J_{21}^{g/h} + g^{\alpha\beta} J_{22}^{g/h}, (g \vee v) J_{31}^{g/h} + v^\alpha v^\beta v^\gamma J_{32}^{g/h}\}(m, \omega_1, \omega_2), \quad (A3)$$

$$i \int \frac{d^D l \mu^{4-D}}{(2\pi)^D} \frac{\{1, l^\alpha, l^\alpha l^\beta, l^\alpha l^\beta l^\gamma\}}{(l^2 - m_1^2 + i\epsilon)[(q + l)^2 - m_2^2 + i\epsilon]} \\ \equiv \{J_0^F, q^\alpha J_{11}^F, q^\alpha q^\beta J_{21}^F + g^{\alpha\beta} J_{22}^F, (g \vee q) J_{31}^F + q^\alpha q^\beta q^\gamma J_{32}^F\}(m_1, m_2, q), \quad (A4)$$

$$i \int \frac{d^D l \mu^{4-D}}{(2\pi)^D} \frac{\{1, l^\alpha, l^\alpha l^\beta, l^\alpha l^\beta l^\gamma, l^\alpha l^\beta l^\gamma l^\delta\}}{[(+/-)v \cdot l + \omega + i\epsilon](l^2 - m_1^2 + i\epsilon)[(q + l)^2 - m_2^2 + i\epsilon]} \\ \equiv \{J_0^{T/S}, q^\alpha J_{11}^{T/S} + v^\alpha J_{12}^{T/S}, g^{\alpha\beta} J_{21}^{T/S} + q^\alpha q^\beta J_{22}^{T/S} + v^\alpha v^\beta J_{23}^{T/S} + (q \vee v) J_{24}^{T/S}, (g \vee q) J_{31}^{T/S} + q^\alpha q^\beta q^\gamma J_{32}^{T/S} + (q^2 \vee v) J_{33}^{T/S} \\ + (g \vee v) J_{34}^{T/S} + (q \vee v^2) J_{35}^{T/S} + v^\alpha v^\beta v^\gamma J_{36}^{T/S}, (g \vee g) J_{41}^{T/S} + (g \vee q^2) J_{42}^{T/S} + q^\alpha q^\beta q^\gamma q^\delta J_{43}^{T/S} + (g \vee v^2) J_{44}^{T/S} + v^\alpha v^\beta v^\gamma v^\delta J_{45}^{T/S} \\ + (q^3 \vee v) J_{46}^{T/S} + (q^2 \vee v^2) J_{47}^{T/S} + (q \vee v^3) J_{48}^{T/S} + (g \vee q \vee v) J_{49}^{T/S}\}(m_1, m_2, \omega, q), \quad (A5)$$

$$i \int \frac{d^D l \mu^{4-D}}{(2\pi)^D} \frac{\{1, l^\alpha, l^\alpha l^\beta, l^\alpha l^\beta l^\gamma, l^\alpha l^\beta l^\gamma l^\delta\}}{(v \cdot l + \omega_1 + i\epsilon)[(+/-)v \cdot l + \omega_2 + i\epsilon](l^2 - m_1^2 + i\epsilon)[(q + l)^2 - m_2^2 + i\epsilon]} \\ \equiv \{J_0^{R/B}, q^\alpha J_{11}^{R/B} + v^\alpha J_{12}^{R/B}, g^{\alpha\beta} J_{21}^{R/B} + q^\alpha q^\beta J_{22}^{R/B} + v^\alpha v^\beta J_{23}^{R/B} + (q \vee v) J_{24}^{R/B}, (g \vee q) J_{31}^{R/B} + q^\alpha q^\beta q^\gamma J_{32}^{R/B} + (q^2 \vee v) J_{33}^{R/B} \\ + (g \vee v) J_{34}^{R/B} + (q \vee v^2) J_{35}^{R/B} + v^\alpha v^\beta v^\gamma J_{36}^{R/B}, (g \vee g) J_{41}^{R/B} + (g \vee q^2) J_{42}^{R/B} + q^\alpha q^\beta q^\gamma q^\delta J_{43}^{R/B} + (g \vee v^2) J_{44}^{R/B} + v^\alpha v^\beta v^\gamma v^\delta J_{45}^{R/B} \\ + (q^3 \vee v) J_{46}^{R/B} + (q^2 \vee v^2) J_{47}^{R/B} + (q \vee v^3) J_{48}^{R/B} + (g \vee q \vee v) J_{49}^{R/B}\}(m_1, m_2, \omega_1, \omega_2, q), \quad (A6)$$

with

$$q \vee v \equiv q^\alpha v^\beta + q^\beta v^\alpha, \quad g \vee q \equiv g^{\alpha\beta} q^\gamma + g^{\alpha\gamma} q^\beta + g^{\beta\gamma} q^\alpha, \quad g \vee v \equiv g^{\alpha\beta} v^\gamma + g^{\alpha\gamma} v^\beta + g^{\beta\gamma} v^\alpha, \\ q^2 \vee v \equiv q^\beta q^\gamma v^\alpha + q^\alpha q^\gamma v^\beta + q^\alpha q^\beta v^\gamma, \quad q \vee v^2 \equiv q^\gamma v^\alpha v^\beta + q^\beta v^\alpha v^\gamma + q^\alpha v^\beta v^\gamma, \\ g \vee g \equiv g^{\alpha\beta} g^{\gamma\delta} + g^{\alpha\delta} g^{\beta\gamma} + g^{\alpha\gamma} g^{\beta\delta}, \\ g \vee q^2 \equiv q^\alpha q^\beta g^{\gamma\delta} + q^\alpha q^\delta g^{\beta\gamma} + q^\alpha q^\gamma g^{\beta\delta} + q^\gamma q^\delta g^{\alpha\beta} + q^\beta q^\delta g^{\alpha\gamma} + q^\beta q^\gamma g^{\alpha\delta}, \\ g \vee v^2 \equiv v^\alpha v^\beta g^{\gamma\delta} + v^\alpha v^\delta g^{\beta\gamma} + v^\alpha v^\gamma g^{\beta\delta} + v^\gamma v^\delta g^{\alpha\beta} + v^\beta v^\delta g^{\alpha\gamma} + v^\beta v^\gamma g^{\alpha\delta},$$

TABLE II. The chiral order of each loop function J . Note that n stands for the order $O(\epsilon^n)$ of a J function.

J	$J_0^{a/b}$	$J_{11}^{a/b}$	$J_{21}^{a/b}$	$J_{22}^{a/b}$	$J_{31}^{a/b}$	$J_{32}^{a/b}$	$J_0^{g/h}$	$J_{11}^{g/h}$	$J_{21}^{g/h}$	$J_{22}^{g/h}$	$J_{31}^{g/h}$	$J_{32}^{g/h}$	J_0^F	J_{11}^F	J_{21}^F	J_{22}^F	J_{31}^F	J_{32}^F
n	1	2	3	3	4	4	0	1	2	2	3	3	0	0	0	2	2	0
J	$J_0^{T/S}$	$J_{11}^{T/S}$	$J_{12}^{T/S}$	$J_{21}^{T/S}$	$J_{22}^{T/S}$	$J_{23}^{T/S}$	$J_{24}^{T/S}$	$J_{31}^{T/S}$	$J_{32}^{T/S}$	$J_{33}^{T/S}$	$J_{34}^{T/S}$	$J_{35}^{T/S}$	$J_{36}^{T/S}$	J_0^c				
n	-1	-1	0	1	-1	1	0	1	-1	0	2	1	2	2				
J	$J_{41}^{T/S}$	$J_{42}^{T/S}$	$J_{43}^{T/S}$	$J_{44}^{T/S}$	$J_{45}^{T/S}$	$J_{46}^{T/S}$	$J_{47}^{T/S}$	$J_{48}^{T/S}$	$J_{49}^{T/S}$									
n	3	1	-1	3	3	0	1	2	2									
J	$J_0^{R/B}$	$J_{11}^{R/B}$	$J_{12}^{R/B}$	$J_{21}^{R/B}$	$J_{22}^{R/B}$	$J_{23}^{R/B}$	$J_{24}^{R/B}$	$J_{31}^{R/B}$	$J_{32}^{R/B}$	$J_{33}^{R/B}$	$J_{34}^{R/B}$	$J_{35}^{R/B}$	$J_{36}^{R/B}$					
n	-2	-2	-1	0	-2	0	-1	0	-2	-1	1	0	1					
J	$J_{41}^{R/B}$	$J_{42}^{R/B}$	$J_{43}^{R/B}$	$J_{44}^{R/B}$	$J_{45}^{R/B}$	$J_{46}^{R/B}$	$J_{47}^{R/B}$	$J_{48}^{R/B}$	$J_{49}^{R/B}$									
n	2	0	-2	2	2	-1	0	1	1									

$$\begin{aligned}
 q^3 \vee v &\equiv q^\beta q^\gamma q^\delta v^\alpha + q^\alpha q^\gamma q^\delta v^\beta + q^\alpha q^\beta q^\delta v^\gamma + q^\alpha q^\beta q^\gamma v^\delta, & q \vee v^3 &\equiv q^\delta v^\alpha v^\beta v^\gamma + q^\gamma v^\alpha v^\beta v^\delta + q^\beta v^\alpha v^\gamma v^\delta + q^\alpha v^\beta v^\gamma v^\delta, \\
 q^2 \vee v^2 &\equiv q^\gamma q^\delta v^\alpha v^\beta + q^\beta q^\delta v^\alpha v^\gamma + q^\alpha q^\delta v^\beta v^\gamma + q^\beta q^\gamma v^\alpha v^\delta + q^\alpha q^\gamma v^\beta v^\delta + q^\alpha q^\beta v^\gamma v^\delta, \\
 g \vee q \vee v &\equiv q^\beta v^\alpha g^{\gamma\delta} + q^\alpha v^\beta g^{\gamma\delta} + q^\delta v^\alpha g^{\beta\gamma} + q^\gamma v^\alpha g^{\beta\delta} + q^\alpha v^\delta g^{\beta\gamma} + q^\alpha v^\gamma g^{\beta\delta} + q^\delta v^\gamma g^{\alpha\beta} + q^\delta v^\beta g^{\alpha\gamma} + q^\gamma v^\delta g^{\alpha\beta} \\
 &\quad + q^\gamma v^\beta g^{\alpha\delta} + q^\beta v^\delta g^{\alpha\gamma} + q^\beta v^\gamma g^{\alpha\delta}.
 \end{aligned} \tag{A7}$$

In the above, J^b is related to J^a via

$$\begin{aligned}
 J_0^b &= J_0^a, & J_{11}^b &= -J_{11}^a, & J_{21}^b &= J_{21}^a, & J_{22}^b &= J_{22}^a, \\
 J_{31}^b &= -J_{31}^a, & J_{32}^b &= -J_{32}^a.
 \end{aligned} \tag{A8}$$

J^g and J^h can be reduced to

$$J^g(\omega_1, \omega_2) = \frac{1}{\omega_2 - \omega_1} [J^a(\omega_1) - J^a(\omega_2)], \tag{A9}$$

$$J^h(\omega_1, \omega_2) = \frac{1}{\omega_2 + \omega_1} [J^a(\omega_1) + J^b(\omega_2)]. \tag{A10}$$

J^S is related to J^T via

$$\begin{aligned}
 J_0^S(v \cdot q) &= J_0^T(-v \cdot q), & J_{11}^S(v \cdot q) &= J_{11}^T(-v \cdot q), \\
 J_{12}^S(v \cdot q) &= -J_{12}^T(-v \cdot q), & J_{21}^S &= J_{21}^T(-v \cdot q), \\
 J_{22}^S(v \cdot q) &= J_{22}^T(-v \cdot q), & J_{23}^S(v \cdot q) &= J_{23}^T(-v \cdot q), \\
 J_{24}^S(v \cdot q) &= -J_{24}^T(-v \cdot q), & J_{31}^S(v \cdot q) &= J_{31}^T(-v \cdot q), \\
 J_{32}^S(v \cdot q) &= J_{32}^T(-v \cdot q), & J_{33}^S(v \cdot q) &= -J_{33}^T(-v \cdot q), \\
 J_{34}^S(v \cdot q) &= -J_{34}^T(-v \cdot q), & J_{35}^S(v \cdot q) &= J_{35}^T(-v \cdot q), \\
 J_{36}^S(v \cdot q) &= -J_{36}^T(-v \cdot q), & J_{41}^S(v \cdot q) &= J_{41}^T(-v \cdot q), \\
 J_{42}^S(v \cdot q) &= J_{42}^T(-v \cdot q), & J_{43}^S(v \cdot q) &= J_{43}^T(-v \cdot q), \\
 J_{44}^S(v \cdot q) &= J_{44}^T(-v \cdot q), & J_{45}^S(v \cdot q) &= J_{45}^T(-v \cdot q), \\
 J_{46}^S(v \cdot q) &= -J_{46}^T(-v \cdot q), & J_{47}^S(v \cdot q) &= J_{47}^T(-v \cdot q), \\
 J_{48}^S(v \cdot q) &= -J_{48}^T(-v \cdot q), & J_{49}^S(v \cdot q) &= -J_{49}^T(-v \cdot q).
 \end{aligned} \tag{A11}$$

J^R and J^B can be reduced to

$$J^R(\omega_1, \omega_2) = \frac{1}{\omega_2 - \omega_1} [J^T(\omega_1) - J^T(\omega_2)], \tag{A12}$$

$$J^B(\omega_1, \omega_2) = \frac{1}{\omega_2 + \omega_1} [J^T(\omega_1) + J^S(\omega_2)]. \tag{A13}$$

Notice that in our main text the loop functions $J_{ij}^{a/b}(m, \omega)$, $J_{ij}^{g/h}(m, \omega_1, \omega_2)$, $J_{ij}^F(m_1, m_2, q)$, $J_{ij}^{T/S}(m_1, m_2, \omega, q)$, and $J_{ij}^{R/B}(m_1, m_2, \omega_1, \omega_2, q)$ in Eqs. (A1)–(A6) are abbreviated as $J_{ij}^{a/b}(\omega)$, $J_{ij}^{g/h}(\omega_1, \omega_2)$, J_{ij}^F , $J_{ij}^{T/S}(\omega)$, and $J_{ij}^{R/B}(\omega_1, \omega_2)$, respectively.

Here we also list the chiral order of each J in Table II based on the definitions (A1)–(A6).

APPENDIX B: THE RESULTS OF THE POTENTIALS IN SCHEME I

Here we show the $I = 0$ results under scheme I in Figs. 9 and 10. We choose three typical Λ values: 0.25, 0.35, and 0.45 GeV.

Because of the insufficient strengths of the potentials at such low Λ , we cannot get the bound state solution. Therefore, by solving the Schrödinger equation with the $I = 0$ $\mathcal{V}(r)$ at $\Lambda = 0.45$ GeV, we refit the binding energy to the location of the observed T_{cc} state by justifying the LEC D_a and E_a . Note that, for an illustration, we do not search for the parameter region of the LECs, but vary them around previous D_a^{RSM} and E_a^{RSM} with a same multiplier: $D_a = cD_a^{\text{RSM}}$, $E_a = cE_a^{\text{RSM}}$. At last, we get $c = 2.7$ at $\Lambda = 0.45$ GeV, and we show the potential $\mathcal{V}(r)$ with the refitted LECs in the last diagram of Fig. 10.

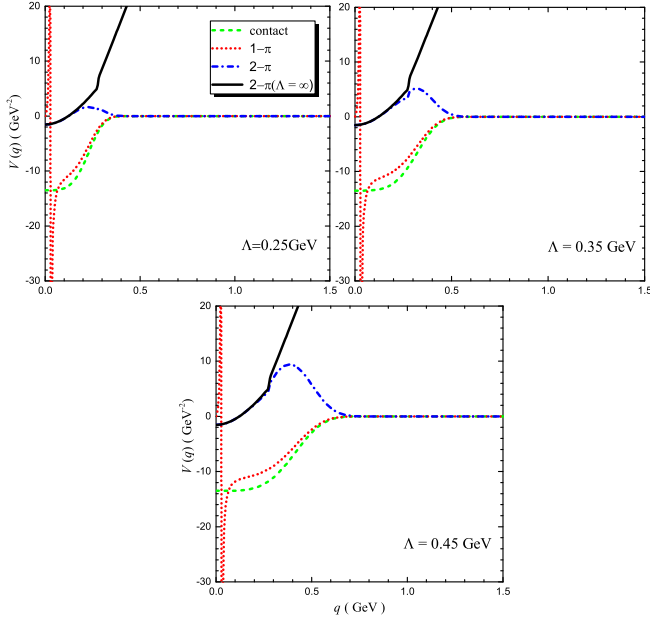


FIG. 9. The $I=0$ DD^* potentials $\mathcal{V}(q)$ under scheme I, varying with cutoff Λ . $q = |\vec{q}|$ is the 3-momentum in units of GeV.

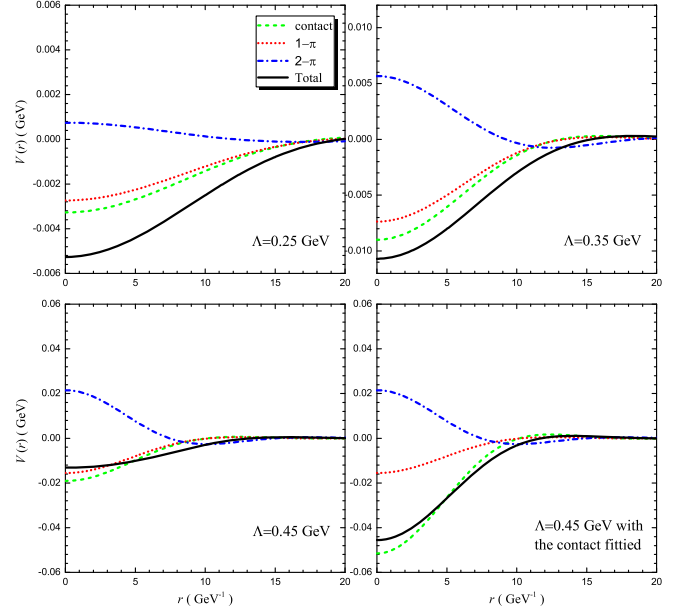


FIG. 10. The $I=0$ DD^* potentials $\mathcal{V}(r)$ under scheme I, varying with cutoff Λ . Note that the LECs of the contact contribution in the last diagram are fitted to T_{cc} mass.

APPENDIX C: THE INTERACTION KERNELS OF THE DD^* SYSTEMS AT $O(\epsilon^2)$

Here we list the kernels $\mathcal{K}^{I=0,1}$ at the second order $O(\epsilon^2)$,

$$\begin{aligned} \mathcal{K}_{\text{contact}(2)}^{I=0,\mu\nu} = & im_1 m_2 \frac{g^2}{f^2} \{ 6(D_a + D_b - 3E_a - 3E_b) [(d-1)\partial_\omega J_{22}^b(-\delta) + (d-2)\partial_\omega J_{22}^b(0) + \partial_\omega J_{22}^b(\delta)] + 6(d-2)(d-3) \\ & \times (D_a - D_b + E_a - E_b) J_{22}^g(0,0) + 6[d(D_a - D_b + E_a - E_b) + 2(D_b + E_b)] J_{22}^g(-\delta, -\delta) + 12(D_a + E_a) J_{22}^g(\delta, \delta) \\ & + 12(D_b + E_b) J_{22}^g(-\delta, \delta) + 12(D_b + E_b) J_{22}^g(\delta, -\delta) \} g^{\mu\nu}, \end{aligned} \quad (C1)$$

$$\begin{aligned} \mathcal{K}_{\text{contact}(2)}^{I=1,\mu\nu} = & im_1 m_2 \frac{g^2}{f^2} \{ 6(D_a - D_b + E_a - E_b) [(d-1)\partial_\omega J_{22}^b(-\delta) + (d-2)\partial_\omega J_{22}^b(0) + \partial_\omega J_{22}^b(\delta)] + 2(d-2)(d-3) \\ & \times (3D_a - D_b - E_a - 5E_b) J_{22}^g(0,0) + 4(d-2)(d-3)(D_b - 3E_b) J_{22}^g(0, -\delta) + 4(d-2)(d-3)(D_b - 3E_b) J_{22}^g(-\delta, 0) \\ & + 2[(3d-2)D_a - (d-2)D_b + (6-d)E_a + (10-5d)E_b] J_{22}^g(-\delta, -\delta) + 8(D_a + E_a) J_{22}^g(\delta, \delta) \\ & - 8(D_b + E_b) J_{22}^g(-\delta, \delta) - 8(D_b + E_b) J_{22}^g(\delta, -\delta) \} g^{\mu\nu}, \end{aligned} \quad (C2)$$

$$\begin{aligned} \mathcal{K}_{1\pi(2)}^{I=0,\mu\nu} = & im_1 m_2 \frac{1}{p^2 - m^2} \left\{ \frac{g^4}{f^4} \left[\frac{9}{4} ((d-1)\partial_\omega J_{22}^b(-\delta) + (d-2)\partial_\omega J_{22}^b(0) + \partial_\omega J_{22}^b(\delta)) + \frac{3}{4} (d-2)(d-3) J_{22}^g(0, -\delta) \right. \right. \\ & \left. \left. + \frac{3}{4} (d-2)(d-3) J_{22}^g(-\delta, 0) - \frac{3}{4} J_{22}^g(-\delta, \delta) - \frac{3}{4} J_{22}^g(\delta, -\delta) \right] - 6 \frac{g^2}{f^4} J_0^c \right\} p^\mu p^\nu, \end{aligned} \quad (C3)$$

$$\begin{aligned} \mathcal{K}_{1\pi(2)}^{I=1,\mu\nu} = & im_1 m_2 \frac{1}{p^2 - m^2} \left\{ \frac{g^4}{f^4} \left[\frac{3}{4} ((d-1)\partial_\omega J_{22}^b(-\delta) + (d-2)\partial_\omega J_{22}^b(0) + \partial_\omega J_{22}^b(\delta)) + \frac{1}{4} (d-2)(d-3) J_{22}^g(0, -\delta) \right. \right. \\ & \left. \left. + \frac{1}{4} (d-2)(d-3) J_{22}^g(-\delta, 0) - \frac{1}{4} J_{22}^g(-\delta, \delta) - \frac{1}{4} J_{22}^g(\delta, -\delta) \right] - 2 \frac{g^2}{f^4} J_0^c \right\} p^\mu p^\nu, \end{aligned} \quad (C4)$$

$$\mathcal{K}_{2\pi(2)}^{I=0,\mu\nu} = \mathcal{K}_{2\pi(2)}^{I=0,\mu\nu}(q) + \mathcal{K}_{2\pi(2)}^{I=0,\mu\nu}(p), \quad (C5)$$

with

$$\begin{aligned}
 \mathcal{K}_{2\pi(2)}^{I=0,\mu\nu}(q) = & im_1 m_2 \frac{-3}{4f^4} \{ [(d-3)J_{22}^R(-\delta, 0)\vec{q}^4 + 2(d-3)J_{32}^R(-\delta, 0)\vec{q}^4 + (d-3)J_{43}^R(-\delta, 0)\vec{q}^4 - (d-3)J_{21}^R(-\delta, 0)\vec{q}^2 \\
 & - (2d+1)(d-3)J_{31}^R(-\delta, 0)\vec{q}^2 - J_{31}^R(-\delta, \delta)\vec{q}^2 - (2d+1)(d-3)J_{42}^R(-\delta, 0)\vec{q}^2 - J_{42}^R(-\delta, \delta)\vec{q}^2 \\
 & + (d-3)(d-2)(d+1)J_{41}^R(-\delta, 0) + (d+1)J_{41}^R(-\delta, \delta)]g^4 + 2[(d-3)q_0J_{11}^S(0)\vec{q}^2 - q_0J_{11}^T(-\delta)\vec{q}^2 \\
 & + 3(d-3)q_0J_{22}^S(0)\vec{q}^2 - 3q_0J_{22}^T(-\delta)\vec{q}^2 + 2(d-3)J_{24}^S(0)\vec{q}^2 - 2J_{24}^T(-\delta)\vec{q}^2 + 2(d-3)q_0J_{32}^S(0)\vec{q}^2 - 2q_0J_{32}^T(-\delta)\vec{q}^2 \\
 & + 2(d-3)J_{33}^S(0)\vec{q}^2 - 2J_{33}^T(-\delta)\vec{q}^2 - (d-3)(d-2)q_0J_{21}^S(0) - q_0J_{21}^S(\delta) + (d-1)q_0J_{21}^T(-\delta) \\
 & - 2(d-3)(d-2)q_0J_{31}^S(0) - 2q_0J_{31}^S(\delta) + 2(d-1)q_0J_{31}^T(-\delta) - 2(d-3)(d-2)J_{34}^S(0) - 2J_{34}^S(\delta) \\
 & + 2(d-1)J_{34}^T(-\delta)]g^2 + q_0^2J_0^F + 4q_0^2J_{11}^F + 4q_0^2J_{21}^F + 4J_{22}^F\}g^{\mu\nu} \\
 & + im_1 m_2 \frac{-3g^2}{2f^4} \left\{ \frac{-1}{2} [-(d-3)J_{22}^R(-\delta, 0)\vec{q}^2 + J_{22}^R(-\delta, \delta)\vec{q}^2 \right. \\
 & - 2(d-3)J_{32}^R(-\delta, 0)\vec{q}^2 + 2J_{32}^R(-\delta, \delta)\vec{q}^2 - (d-3)J_{43}^R(-\delta, 0)\vec{q}^2 + J_{43}^R(-\delta, \delta)\vec{q}^2 + (d-3)J_{21}^R(-\delta, 0) - J_{21}^R(-\delta, \delta) \\
 & + (d^2-9)J_{31}^R(-\delta, 0) - (d+3)J_{31}^R(-\delta, \delta) + (d^2-9)J_{42}^R(-\delta, 0) - (d+3)J_{42}^R(-\delta, \delta)]g^2 + (d-3)q_0J_{11}^S(0) - q_0J_{11}^S(\delta) \\
 & + 3(d-3)q_0J_{22}^S(0) - 3q_0J_{22}^S(\delta) + 2(d-3)J_{24}^S(0) - 2J_{24}^S(\delta) + 2(d-3)q_0J_{32}^S(0) - 2q_0J_{32}^S(\delta) + 2(d-3)J_{33}^S(0) \\
 & \left. - 2J_{33}^S(\delta) \right\} q^\mu q^\nu, \tag{C6}
 \end{aligned}$$

$$\mathcal{K}_{2\pi(2)}^{I=0,\mu\nu}(p) = im_1 m_2 \frac{-3(d-3)g^4}{4f^4} J_{21}^R(-\delta, -\delta)[\vec{p}^2 g^{\mu\nu} + p^\mu p^\nu], \tag{C7}$$

$$\mathcal{K}_{2\pi(2)}^{I=1,\mu\nu} = \mathcal{K}_{2\pi(2)}^{I=1,\mu\nu}(q) + \mathcal{K}_{2\pi(2)}^{I=1,\mu\nu}(p), \tag{C8}$$

with

$$\begin{aligned}
 \mathcal{K}_{2\pi(2)}^{I=1,\mu\nu}(q) = & im_1 m_2 \frac{1}{4f^4} \{ (-5)[-(d-3)J_{22}^R(-\delta, 0)\vec{q}^4 - 2(d-3)J_{32}^R(-\delta, 0)\vec{q}^4 - (d-3)J_{43}^R(-\delta, 0)\vec{q}^4 + (d-3)J_{21}^R(-\delta, 0)\vec{q}^2 \\
 & + (d-3)(2d+1)J_{31}^R(-\delta, 0)\vec{q}^2 + J_{31}^R(-\delta, \delta)\vec{q}^2 + (d-3)(2d+1)J_{42}^R(-\delta, 0)\vec{q}^2 + J_{42}^R(-\delta, \delta)\vec{q}^2 \\
 & - (d-3)(d-2)(d+1)J_{41}^R(-\delta, 0) - (d+1)J_{41}^R(-\delta, \delta)]g^4 + 2[(d-3)q_0J_{11}^S(0)\vec{q}^2 - q_0J_{11}^T(-\delta)\vec{q}^2 \\
 & + 3(d-3)q_0J_{22}^S(0)\vec{q}^2 - 3q_0J_{22}^T(-\delta)\vec{q}^2 + 2(d-3)J_{24}^S(0)\vec{q}^2 - 2J_{24}^T(-\delta)\vec{q}^2 + 2(d-3)q_0J_{32}^S(0)\vec{q}^2 - 2q_0J_{32}^T(-\delta)\vec{q}^2 \\
 & + 2(d-3)J_{33}^S(0)\vec{q}^2 - 2J_{33}^T(-\delta)\vec{q}^2 - (d-3)(d-2)q_0J_{21}^S(0) - q_0J_{21}^S(\delta) + (d-1)q_0J_{21}^T(-\delta) \\
 & - 2(d-3)(d-2)q_0J_{31}^S(0) - 2q_0J_{31}^S(\delta) + 2(d-1)q_0J_{31}^T(-\delta) - 2(d-3)(d-2)J_{34}^S(0) - 2J_{34}^S(\delta) \\
 & + 2(d-1)J_{34}^T(-\delta)]g^2 + q_0^2J_0^F + 4q_0^2J_{11}^F + 4q_0^2J_{21}^F + 4J_{22}^F\}g^{\mu\nu} \\
 & + im_1 m_2 \frac{g^2}{2f^4} \left\{ \frac{-5}{2} [-(d-3)J_{22}^R(-\delta, 0)\vec{q}^2 + J_{22}^R(-\delta, \delta)\vec{q}^2 - 2(d-3)J_{32}^R(-\delta, 0)\vec{q}^2 + 2J_{32}^R(-\delta, \delta)\vec{q}^2 \right. \\
 & - (d-3)J_{43}^R(-\delta, 0)\vec{q}^2 + J_{43}^R(-\delta, \delta)\vec{q}^2 + (d-3)J_{21}^R(-\delta, 0) - J_{21}^R(-\delta, \delta) + (d^2-9)J_{31}^R(-\delta, 0) - (d-3)J_{31}^R(-\delta, \delta) \\
 & + (d^2-9)J_{42}^R(-\delta, 0) - (d+3)J_{42}^R(-\delta, \delta)]g^2 + (d-3)q_0J_{11}^S(0) - q_0J_{11}^S(\delta) + 3(d-3)q_0J_{22}^S(0) - 3q_0J_{22}^S(\delta) \\
 & \left. + 2(d-3)J_{24}^S(0) - 2J_{24}^S(\delta) + 2(d-3)q_0J_{32}^S(0) - 2q_0J_{32}^S(\delta) + 2(d-3)J_{33}^S(0) - 2J_{33}^S(\delta) \right\} q^\mu q^\nu, \tag{C9}
 \end{aligned}$$

$$\mathcal{K}_{2\pi(2)}^{I=1,\mu\nu}(p) = im_1 m_2 \frac{-5(d-3)g^4}{4f^4} J_{21}^R(-\delta, -\delta)[\vec{p}^2 g^{\mu\nu} + p^\mu p^\nu]. \tag{C10}$$

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