

Constraining quintessence models with integrated Sachs-Wolfe thermal Sunyaev-Zel'dovich cross-correlations: A comparative analysis of thawing, tracker, and scaling-freezing dynamics

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(Received 27 August 2025; accepted 12 February 2026; published 3 March 2026)

We present constraints on quintessence dark energy models using the observational detection of the integrated Sachs-Wolfe (ISW)–thermal Sunyaev-Zeldovich (tSZ) cross-correlation dataset. Our analysis compares three classes of quintessence dynamics: thawing, tracker, and scaling-freezing with the standard Λ cold dark matter (Λ CDM) cosmology. Through a comprehensive likelihood analysis, we derive best-fit values and 68% confidence intervals for key cosmological parameters, finding $\Omega_m = 0.322^{+0.027}_{-0.030}$ and $\sigma_8 = 0.735^{+0.045}_{-0.035}$ for Λ CDM, with deviations in alternative models consistent within 1σ . For the thawing model, we consider an exponential potential with slope $\lambda = 0.736^{+0.270}_{-0.227}$, while for the tracker and scaling-freezing models, we use inverse axionlike and double exponential potentials, respectively. Observationally, the tracker model yields $n = 5.651^{+1.625}_{-1.604}$ and $f = 0.258^{+0.149}_{-0.096}$, and the scaling-freezing model gives $\lambda_1 = 0.405^{+0.293}_{-0.322}$ and $\lambda_2 = 23.226^{+7.975}_{-7.258}$. The dimensionless tSZ amplitude ($\tilde{W}^{\text{SZ}}$) and cosmic infrared background (CIB) parameters are tightly constrained across all models, providing additional insights into astrophysical foregrounds. Our results demonstrate the effectiveness of ISW-tSZ cross-correlations as a probe of dark energy dynamics, with the thawing quintessence model yielding the lowest χ^2_{min} among the tested scenarios, and highlight the need for future high-precision measurements to distinguish between quintessence models and Λ CDM.

DOI: [10.1103/grt1-234m](https://doi.org/10.1103/grt1-234m)

I. INTRODUCTION

The nature of dark energy, which constitutes approximately 70% of the energy density of the universe, remains one of the most profound mysteries in cosmology. Since its discovery through observations of distant type Ia supernovae [1,2], dark energy has been primarily characterized by its equation of state parameter w , which relates the pressure p to the energy density ρ via $w = p/\rho$. The simplest model, the Λ cold dark matter (Λ CDM) model,

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corresponds to $w = -1$, where the cosmological constant (CC), Λ , is the dark energy component. However, observations allow for the possibility of a dynamical dark energy (DDE) component, where w evolves with time [3–9].

Among the various proposals for dynamical dark energy (DDE) [5,10], *quintessence* [11–13], a minimally coupled canonical scalar field rolling slowly at late times, has emerged as a compelling alternative to the CC. We restrict our analysis to the nonphantom regime $w \geq -1$, a fundamental constraint imposed by the positive kinetic energy of canonical scalar fields [14,15]. This choice is further motivated by DESI Year 1 results, which reports phantom-crossing behavior at high redshifts ($z > 0.55$), but also find $w(z) \geq -1$ at low redshifts ($z < 0.55$) [16] precisely the regime where the ISW-tSZ cross-correlation has peak sensitivity ($z \approx 0.45$ – 0.6) [17]. The evolution of quintessence models is broadly classified into three categories [18]. In *tracker* models, the scalar field energy density decays more slowly than the background energy density, eventually dominating and driving late-time acceleration [14,19,20]. *Scaling-freezing* models exhibit an initial scaling behavior, where the scalar field energy density scales with the background [21], followed by a transition to dominance due to a shallow potential or nonminimal coupling [22–24]. In contrast, *thawing* models involve a field that remains frozen in the early Universe, mimicking a CC, and begins evolving only recently [25]. Tracker and scaling-freezing models exhibit attractor behavior that helps to mitigate the cosmic coincidence problem [19,20], whereas thawing dynamics is more sensitive to initial conditions.

In this work, we focus on constraining the quintessence dark energy models using the cross-correlation of large-scale structure observables, specifically the integrated Sachs-Wolfe (ISW) effect and the thermal Sunyaev-Zel’dovich (tSZ) effect. The ISW effect [26] provides a direct signature of dark energy at low redshifts by capturing the change in the energy of cosmic microwave background (CMB) photons as they traverse time-evolving gravitational potentials [27–33]. In [17], the underlying ISW effect was successfully measured through its cross-correlation with tSZ data, yielding a 3.6σ detection. The thermal Sunyaev-Zel’dovich (tSZ) effect, which arises from the inverse Compton scattering of CMB photons off hot electrons in galaxy clusters, serves as a complementary probe of large-scale structure [34]. By combining the ISW and tSZ maps from the *Planck* satellite, we can extract valuable information on the growth of cosmic structures and the evolution of dark energy [35,36]. Modeling the cross-correlation dataset with a quintessence model provides an innovative method to estimate the baryon-to-matter density ratio, i.e., Ω_b/Ω_m , where Ω_b denotes the normalized baryonic energy density and Ω_m the normalized total matter density. Quintessence fields, characterized by a dynamical equation of state, affect the growth of cosmic structures and the relative distribution of matter components [37,38].

This model’s flexibility in describing the evolution of dark energy allows for more accurate predictions of the matter power spectrum and its components, including the baryon density fraction. By leveraging the cross-correlation of ISW and tSZ effects, we can derive constraints on Ω_b/Ω_m , offering insights into the baryonic matter’s role in cosmic evolution and structure formation. This approach provides a novel and independent method to constrain Ω_b , complementing existing measurements from the CMB and baryon acoustic oscillations (BAO) [39,40]. By providing a robust test of quintessence models, this work contributes to the broader effort of refining cosmological parameters and advancing our knowledge of the Universe’s fundamental components.

In this study, we utilize our robust detection of the ISW-tSZ cross-correlation from [17] as the primary dataset to test the quintessence model and derive constraints on the baryon density fraction alongside other cosmological and model parameters. The paper is organized as follows: Sec. II describes the models used. Section III discusses the matter power spectrum. Section IV presents the ISW-tSZ power spectrum. Section V details the likelihood method and results. Section VI summarizes our conclusions. Throughout, we assume a spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) cosmology, fixing $n_s = 0.965$, $\tau = 0.0540$, and $\ln(10^{10}A_s) = 3.043$ [39].

II. THE MODELS

We consider the minimally coupled canonical scalar field (ϕ) governed by the following action:

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi) \right] + \mathcal{S}_m + \mathcal{S}_r \quad (1)$$

Here, $M_{\text{Pl}} = (1/\sqrt{8\pi G})$ is the reduced Planck mass and $V(\phi)$ represents the scalar field potential. $\mathcal{S}_m$ and $\mathcal{S}_r$ represent the action for matter and radiation, respectively. Varying the action (1) with respect to the metric $g_{\mu\nu}$ gives Einstein’s field equation

$$M_{\text{Pl}}^2 G_{\mu\nu} = T_{\mu\nu}^{(m)} + T_{\mu\nu}^{(r)} + T_{\mu\nu}^{(\phi)}, \quad (2)$$

where

$$T_{\mu\nu}^{(\phi)} = \phi_{;\mu} \phi_{;\nu} - \frac{1}{2} g_{\mu\nu} (\nabla \phi)^2 - g_{\mu\nu} V(\phi). \quad (3)$$

The equation of motion of the scalar field can be calculated by varying the action (1) with respect to the scalar field ϕ , and it is given by

$$\square \phi - V_\phi(\phi) = 0, \quad (4)$$

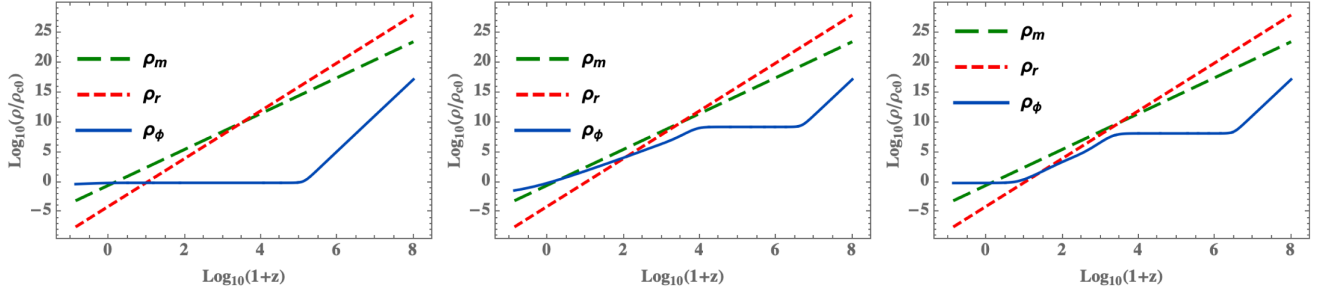


FIG. 1. Different scalar field dynamics has been shown. Left figure shows the thawing dynamics while middle and the right figures show the tracker and scaling-freezing dynamics, respectively.

where the subscript ϕ denotes the derivative with respect to ϕ .

In the flat FLRW metric, the line element is given by

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j. \quad (5)$$

where $a(t)$ is the scale factor. The Friedmann equations are given by

$$3H^2 M_{\text{Pl}}^2 = \rho_m + \rho_r + \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad (6)$$

$$(2\dot{H} + 3H^2)M_{\text{Pl}}^2 = -\frac{1}{3}\rho_r - \frac{1}{2}\dot{\phi}^2 + V(\phi). \quad (7)$$

The equation of motion of the scalar field is given by

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0. \quad (8)$$

The effective equation of state (EoS) is given by

$$w_{\text{eff}} = -\left(1 + \frac{2}{3}\frac{\dot{H}}{H^2}\right). \quad (9)$$

The scalar field energy density ρ_ϕ and pressure p_ϕ are given by

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad (10)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \quad (11)$$

The EoS for the scalar field is therefore

$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}. \quad (12)$$

For a quintessence field, the EoS (w_ϕ) can vary from 1 to -1 depending upon the relative values of the kinetic and potential terms. Hence, the scalar field energy density varies as $\rho_\phi \sim a^{-n}$, where $0 \leq n \leq 6$. When $n = 0$, the scalar field

behaves like a CC *i.e.*, the potential completely dominates the kinetic term; and for $n = 6$, the kinetic term completely dominates over the potential term. This wide variation in the evolution of ρ_ϕ can be classified into three classes: thawing, tracker, and scaling-freezing. Figure 1 shows the different dynamics of a scalar field in a flat FLRW background. The left figure depicts thawing dynamics, where the scalar field remains frozen for most of the universe's history and starts evolving in the recent past, giving rise to dynamics distinct from a CC. The middle figure represents tracker dynamics, in which the scalar field has a frozen period in the past and, once it exits this frozen period, almost—but not completely—follows the background. In fact, ρ_ϕ decays slightly slower than the background, which eventually makes it the dominant component. The right figure shows scaling-freezing dynamics. For an exponential (EXP) potential with a steep slope, ρ_ϕ completely scales with the background after the past frozen period. This is known as scaling behavior [21]. This is an attractor solution that makes the scalar field follow the background indefinitely. This dynamics cannot give rise to late-time acceleration. However, if we adopt a potential *e.g.*, a double exponential (DEXP) potential, where the potential behaves like a steep exponential in the past and becomes shallow in the recent epoch, this change in slope can give rise to dynamics that feature scaling behavior in the past followed by slow-roll evolution in recent times. This is known as scaling-freezing dynamics.

Here, we consider three distinct potentials representing the three different classes of dynamics mentioned above: an EXP potential for thawing dynamics [18,21,25], an inverse axionlike (IAX) potential [41] for tracker dynamics, and a DEXP potential [42,43] for scaling-freezing dynamics

$$V(\phi) = \begin{cases} V_0 e^{-\lambda\phi} & (\text{EXP}) \\ V_0 \left(1 - \cos\left(\frac{\phi}{f}\right)\right)^{-n} & (\text{IAX}) \\ V_1 e^{-\lambda_1\phi} + V_2 e^{-\lambda_2\phi} & (\text{DEXP}) \end{cases} \quad (13)$$

In all cases, V_0 , V_1 , V_2 set the energy scales of the potentials. In the EXP potential, λ determines the slope of the potential and controls how quickly the field thaws at late times. For tracker dynamics, f and n control the tracker

behavior and are treated as free parameters. For the scaling-freezing potential, λ_1 and λ_2 represent the slopes of the two exponential terms, with the first term dominating during the scaling regime and the second term becoming important at late times when the field rolls slowly.

III. MATTER POWER SPECTRUM

In the subhorizon ($k^2 \gg a^2 H^2$) and quasi-static ($|\dot{\phi}| \lesssim H|\dot{\phi}| \ll k^2|\phi|$) approximations, the evolution of the matter density contrast δ follows a second-order differential equation. Specifically, under these assumptions, the perturbations evolve according to

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\bar{\rho}_m\delta = 0, \quad (14)$$

where $\bar{\rho}_m$ is the background matter density and G is Newton's gravitational constant. The solution of Eq. (14) can be written as the superposition of growing (D_+) and decaying (D_-) modes.

Using the growing mode D_+ , we define the growth factor f as

$$f = \frac{d \ln D_+}{d \ln a}. \quad (15)$$

The evolution of σ_8 , the root mean square amplitude of mass fluctuations within a sphere of radius $8 h^{-1}$ Mpc, with redshift is given by

$$\sigma_8(z) = \sigma_8(0) \frac{D_+(z)}{D_+(0)}. \quad (16)$$

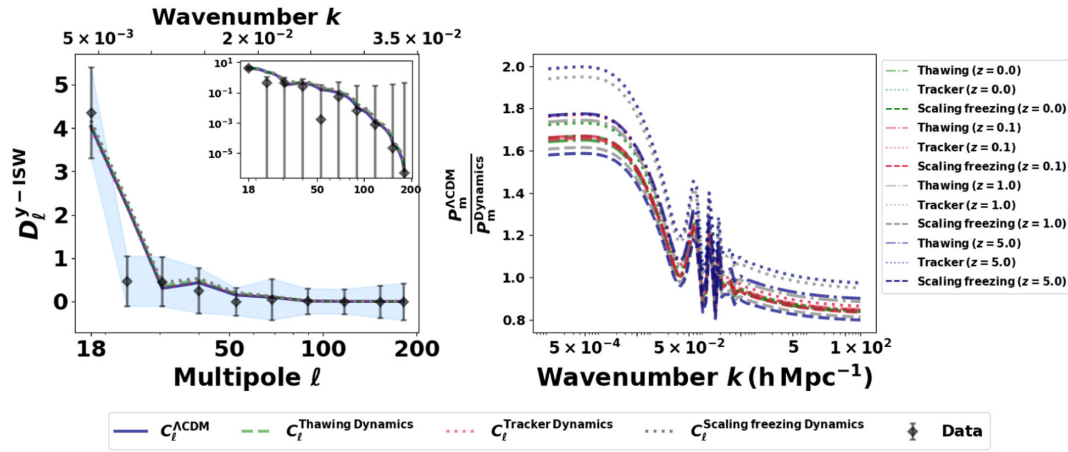


FIG. 2. Left panel: angular power spectrum of the integrated Sachs-Wolfe (ISW)-thermal Sunyaev-Zeldovich (tSZ) effect, with uncertainties shown as an error band (blue) and error bars (gray). The data is presented in Table 1 of [17]. The theoretical angular power spectra for various cosmological models are plotted alongside the observed data. An inset log-log plot (top right) highlights the detailed fit of each model to the data, demonstrating that the Λ CDM model provides a better statistical fit compared to the thawing, tracker, and scaling-freezing dynamics models. $D_\ell^{y-ISW} = 10^{14} \ell(\ell+1) C_\ell^{y-ISW} / 2\pi$. Right panel: Ratios of the Λ CDM matter power spectrum ($P_m^{\Lambda\text{CDM}}$) to the matter power spectra computed for quintessence dynamics at redshifts $z = 0.0$, $z = 0.1$, $z = 1.0$, and $z = 5.0$. Different line styles and colors distinguish the thawing, tracker, and scaling-freezing dynamics models.

To set the normalization, we fix the initial value of $D_+(z)$ during the matter-dominated era, which is essentially the same as in the Λ CDM model.

The power spectrum can be written as [44,45]

$$\mathcal{P}(t, k) \propto A_H^2 |D_+(a)|^2 T^2(k) \left(\frac{k}{H_0} \right)^{n_s}. \quad (17)$$

Here, A_H is a normalization factor, and n_s denotes the spectral index of scalar perturbations generated during inflation. The function $T(k)$ represents the transfer function, which encodes the evolution of perturbations from the primordial epoch to the late-time matter-dominated universe [46]. Specifically, $T(k)$ relates the primordial curvature perturbation to the comoving matter perturbation. In this work, we adopt the Eisenstein-Hu fitting formula for $T(k)$ [46].

In Fig. 2, we present the ratios of the matter power spectra

$$\frac{P_m^{\Lambda\text{CDM}}(k, z^i)}{P_m^{\text{Dynamics}}(k, z^i)}, \quad (18)$$

evaluated at redshifts $z^i = 0.0, 0.1, 1.0$, and 5.0 . These results highlight the deviations of the quintessence models from the standard Λ CDM scenario across different scales and redshifts. For Λ CDM the matter power spectrum $P(k)$ is computed using the CAMB code [47].

IV. ISW-TSZ POWER SPECTRUM

A. Theoretical framework

The observed temperature anisotropies on the sky can be decomposed into spherical harmonics as follows:

$$\Delta T(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi), \quad (19)$$

where $a_{\ell m}$ are the spherical harmonic coefficients, and $Y_{\ell m}(\theta, \phi)$ are the spherical harmonic basis functions.

The angular power spectrum is then estimated from these coefficients as

$$C_{\ell} = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2. \quad (20)$$

Theoretically, to compute the angular power spectrum for the ISW-tSZ cross-correlation, we employ the Limber approximation, which is well suited for small-angle analyses. While this approximation becomes less accurate at very low multipoles (ℓ), it remains robust for $\ell \gtrsim 10$, consistent with the multipole range probed by our cross-correlation dataset. Under the Limber approximation, the angular power spectrum for the cross-correlation between two observables A and B (e.g., ISW and tSZ) can be expressed as

$$C_{\ell}^{A,B} = \frac{2}{\pi} \int k^2 dk P(k) W_{\ell}^A(k) W_{\ell}^B(k), \quad (21)$$

where $P(k)$ is the matter power spectrum, and $W_{\ell}^A(k)$ and $W_{\ell}^B(k)$ are the window functions associated with observables A and B , respectively.

For the ISW-tSZ cross-correlation, the combined window function is given by

$$W_{\ell}^{\text{ISW-tSZ}}(k) = W_{\ell}^{\text{ISW}}(k) W_{\ell}^{\text{tSZ}}(k), \quad (22)$$

with $W_{\ell}^{\text{ISW}}(k)$ and $W_{\ell}^{\text{tSZ}}(k)$ being the individual window functions for the ISW and tSZ effects, respectively. These window functions are directly related to the kernels described in Sec. 4.3 of [17].

B. Observational data

The observed angular power spectra are computed by decomposing the full-sky ISW and Compton y -parameter maps into spherical harmonics. Beam effects and masking are accounted for during this process. The Limber approximation, discussed in Sec. IV.1, is valid for the multipole range $\ell \gtrsim 10$, ensuring consistency between the theoretical predictions and the observed data.

The observed maps are processed using the publicly available pseudo- C_{ℓ} MASTER algorithm implemented in the NaMaster package, which efficiently handles complex masks and high-resolution pixelization while mitigating mode-coupling effects introduced by the survey mask. The observed maps are decomposed into spherical harmonics to compute the angular power spectrum C_{ℓ} , with corrections applied for beam effects and masking.

The covariance matrix is computed using a combination of Gaussian and non-Gaussian contributions: the Gaussian covariance is estimated from simulations using NaMaster, while the non-Gaussian contribution is modeled using the PYCCL code to account for trispectrum effects, ensuring accurate treatment of uncertainties. To validate the results, they are compared with outputs from the ANAFast subroutine in the HEALPix software package and tested against Monte Carlo (MC) simulations.

For the detection, 1000 simulated ISW sky maps are generated, with 900 used to compute the covariance matrix and 100 reserved to quantify the statistical significance. The true ISW-tSZ cross-correlation is detected at 3.6σ confidence level, rejecting the no-correlation hypothesis. This comprehensive approach ensures robust characterization of the ISW-tSZ signal and its statistical significance. The autocorrelation of the Planck tSZ data is measured at a 25σ CL, while the detection of the ISW autocorrelation is still challenging due to its dominance by cosmic variance on large scales.

V. LIKELIHOOD METHOD AND RESULTS

We constrain quintessence models by comparing theoretical predictions of the ISW-tSZ cross-correlation [$C_{\ell}^{\text{th}}(\Theta)$] with observed angular power spectra (C_{ℓ}^{obs}). The observed angular power spectra, and their associated uncertainties, as reported in Table 2 of [48], are binned into 10 multipole (ℓ) bins spanning the range $\ell_{\text{eff}}^{\text{min}} = 18.0$ to $\ell_{\text{eff}}^{\text{max}} = 181.0$. The binning process involves averaging the power spectrum values within each bin to obtain representative measurements. To perform this analysis, we define the parameter vector Θ , which encompasses various categories of parameters

$$\Theta = \left\{ \begin{array}{ll} \text{Cosmological Parameters:} & \Omega_b, \Omega_m, h, \sigma_8, \\ \text{Foreground Parameters:} & B_{\text{CIB}}, A_{\text{CIB}}, \\ \text{tSZ Amplitude Parameter:} & \tilde{W}^{\text{SZ}}, \\ \text{Model Parameters:} & \lambda, V_0, n, f, \\ & \lambda_1, \lambda_2, V_1, \log V_2 \end{array} \right\}. \quad (23)$$

The likelihood function is constructed using the full covariance matrix Cov_{tot} described in Sec. 2.4 of [17], with the χ^2 statistic defined as

$$\chi^2(\Theta) = [C_{\ell}^{\text{obs}} - C_{\ell}^{\text{th}}(\Theta)] \text{Cov}_{\text{tot}}^{-1} [C_{\ell}^{\text{obs}} - C_{\ell}^{\text{th}}(\Theta)]^T. \quad (24)$$

The likelihood is then expressed as $\mathcal{L}(\Theta) \sim e^{-\chi^2/2}$ while the posterior probabilities are obtained via Bayes' theorem

$$P(\Theta|d) \propto P(\Theta) \mathcal{L}(d|\Theta), \quad (25)$$

with flat priors on all parameters within physically motivated ranges, as shown in Table I. For the EXP potential, accelerated expansion can be obtained for $\lambda < \sqrt{3}$ and V_0

TABLE I. Prior distributions for the parameters of three quintessence potentials: EXP, IAX, and DEXP. The priors are defined as uniform distributions ($\mathcal{U}$) over specified ranges, reflecting physically motivated constraints on the model parameters. These priors ensure broad exploration of parameter space while maintaining computational feasibility.

Potential type	Parameter	Prior range
EXP	λ	$\mathcal{U}[0, 1.5]$
	V_0	$\mathcal{U}[0, 2]$
IAX	n	$\mathcal{U}[0, 10]$
	f	$\mathcal{U}[0.01, 2]$
	V_0	$\mathcal{U}[0, 2]$
DEXP	λ_1	$\mathcal{U}[-1, 1]$
	λ_2	$\mathcal{U}[0, 50]$
	V_1	$\mathcal{U}[0.1, 2]$
	$\log V_2$	$\mathcal{U}[0, 14]$

should be of similar order to the current dark energy density, ρ_{DE0} . $\lambda = 0$ can mimic the CC. For the IAX potential, $n = 0$ and $V_0 = \rho_{\text{DE0}}$ gives a CC-like behavior. $n > 0$ gives tracker behavior. Since $n < 0$ makes the potential oscillatory, we cannot get tracker behavior and hence we do not consider $n < 0$. f should be of the order of or less than the Planck mass, M_{pl} . Larger values of f can give rise to larger scalar field EoS at $z = 0$ for a fixed n , which can make the models less favored by the data. For the DEXP potential, the parameters λ_1 and V_0 play the same role as the parameters of the EXP potential. But the parameters λ_2 and V_2 determine the dynamics during the early time, more specifically the scaling behavior during the intermediate stages, which requires larger values of λ_2 so that the potential behaves sufficiently steep during the intermediate stages. V_2 determines the energy scale of the early frozen period of the scalar field and the duration of the scaling regime. In general, $V_2 \geq \rho_{\text{DE0}}$. The CC-like behavior for the DEXP potential requires $\lambda_1 = \lambda_2 = 0$, $V_1 \sim \rho_{\text{DE0}}$ and V_2 can be either zero or of similar order

to V_1 . For estimating parameters, we employ nested sampling using the `UltraNest` package [49,50], which is well-suited for exploring high-dimensional parameter spaces and estimating Bayesian evidence. The algorithm initializes live points randomly within the prior ranges and iteratively replaces the point with the lowest likelihood until convergence criteria are met. Convergence is monitored by ensuring the uncertainty on the log-evidence falls below a specified threshold (typically < 0.1), and that the effective sample size (ESS) exceeds 1000, ensuring robust posterior sampling. The parameter space is explored with `UltraNest`'s default settings, including adaptive refinement and dynamic updates to the proposal distribution for efficient sampling. Posterior distributions are then analyzed using `GetDist` [51], with marginalized constraints summarized in Table II. Best-fit model predictions are shown in Fig. 2, demonstrating excellent agreement between the cosmological models and observations.

A. Impact on cosmological parameters

We analyze how the best-fit cosmological parameters vary across different dark energy models, focusing on their consistency with the Λ CDM. In Table II, we present the best-fit values and 68% confidence limits on all parameters. Figure 3 shows the graphical representation of the best-fit value and corresponding 1σ bound on the parameter for each scenario. Figure 4 shows the 1σ and 2σ contours of the seven free parameters, cosmological, foreground and tSZ amplitude parameters listed in Eq. (23). According to Table II while most parameters remain broadly consistent across models, some show mild but notable deviations, which we quantify in terms of their statistical significance (σ). For the baryon density parameter, Ω_b , the differences are relatively small: the thawing model is consistent with Λ CDM at 0.236σ , while the tracker and scaling-freezing models show consistencies of 0.512σ and 0.000σ , respectively. These minor variations, all well within 1σ , indicate a high degree of consistency in Ω_b across the models. This agreement suggests that Ω_b is a robust parameter, largely

TABLE II. Best-fit values and 68% confidence limits for cosmological, astrophysical, foreground, and model parameters.

Model	Ω_b	Ω_m	h	σ_8	$\tilde{W}^{\text{SZ}}$	$B_{\text{CIB}} [\times 10^{-4}]$	A_{CIB}	λ
Thawing	$0.049^{+0.003}_{-0.003}$	$0.277^{+0.032}_{-0.027}$	$0.694^{+0.023}_{-0.018}$	$0.746^{+0.042}_{-0.041}$	$3.522^{+0.336}_{-0.284}$	$5.986^{+1.806}_{-1.658}$	$0.863^{+0.046}_{-0.041}$	$0.736^{+0.270}_{-0.227}$
Tracker	$0.046^{+0.003}_{-0.002}$	$0.352^{+0.015}_{-0.015}$	$0.671^{+0.010}_{-0.008}$	$0.712^{+0.039}_{-0.028}$	$3.861^{+0.216}_{-0.222}$	$3.184^{+1.236}_{-1.065}$	$0.815^{+0.042}_{-0.045}$	...
Scaling-Freezing	$0.048^{+0.002}_{-0.003}$	$0.301^{+0.029}_{-0.026}$	$0.691^{+0.021}_{-0.019}$	$0.729^{+0.045}_{-0.037}$	$3.713^{+0.391}_{-0.338}$	$4.540^{+1.706}_{-1.546}$	$0.854^{+0.044}_{-0.043}$	...
Λ CDM	$0.048^{+0.003}_{-0.003}$	$0.322^{+0.027}_{-0.030}$	$0.676^{+0.015}_{-0.012}$	$0.735^{+0.045}_{-0.035}$	$2.934^{+0.377}_{-0.370}$	$2.815^{+1.088}_{-1.059}$	$0.900^{+0.080}_{-0.069}$	...
Model	V_0	n	f	λ_1	λ_2	V_1	$\log V_2$	
Thawing	$0.170^{+0.155}_{-0.121}$	...	...	...	...	...	...	
Tracker	...	$5.651^{+1.625}_{-1.604}$	$0.258^{+0.149}_{-0.096}$	...	...	...	...	
Scaling-freezing	...	...	...	$0.405^{+0.293}_{-0.322}$	$23.226^{+7.975}_{-7.258}$	$0.467^{+0.283}_{-0.203}$	$7.746^{+2.385}_{-2.056}$	
Λ CDM	...	...	...	...	...	...	...	

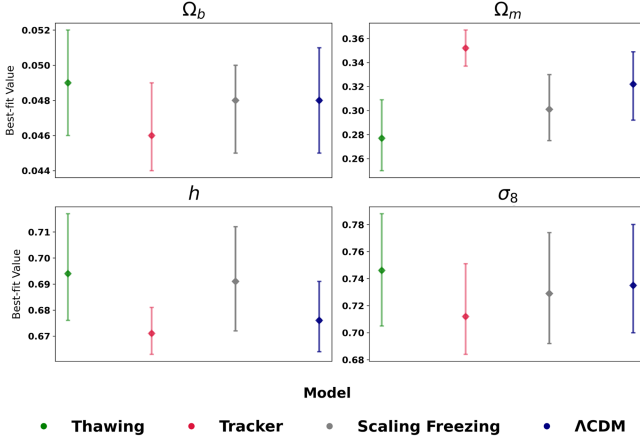


FIG. 3. Final estimates of model parameters with uncertainties. Final estimates of our model parameters are computed as the marginalized 1D (M1D) values from their respective posterior distributions. Each panel corresponds to an individual cosmological parameter (Ω_b , Ω_m , h , and σ_8), displaying the best-fit values grouped by the models, while error bars depict the lower and upper uncertainties associated with each estimate.

insensitive to the choice of different dark energy models considered in this paper.

In contrast, the matter density parameter, Ω_m , exhibits some variation across the models. The thawing model deviates from Λ CDM by 1.10σ , while the tracker and scaling-freezing models differ by 0.93σ and 0.53σ , respectively. These deviations remain within the range of statistical insignificance ($< 2\sigma$), indicating no strong evidence for tension between the models. However, the tracker model predicts a slightly higher Ω_m compared to the other models, which could reflect differences in its treatment of dark energy or the relative contributions of matter components. While these variations are not statistically significant, they highlight the sensitivity of Ω_m to the underlying assumptions of each model and emphasize the need for further investigation into these differences with more precise data.

The baryon density fraction relative to the total matter density, Ω_b/Ω_m , provides additional insight into the distribution of baryonic and dark matter across different cosmological models. The estimated values for Ω_b/Ω_m are as follows:

$$\Omega_b/\Omega_m = \begin{cases} \text{Thawing:} & 0.177^{+0.017}_{-0.019}, \\ \text{Tracker:} & 0.131^{+0.006}_{-0.005}, \\ \text{Scaling-Freezing:} & 0.160^{+0.014}_{-0.016}, \\ \Lambda\text{CDM:} & 0.149^{+0.012}_{-0.011}. \end{cases}$$

These results highlight variations in Ω_b/Ω_m across the models, reflecting differences in their underlying assumptions about the interplay between baryonic and dark matter components. Notably, the thawing model predicts a higher baryon fraction compared to Λ CDM, while the tracker

model yields the lowest value. This underscores the sensitivity of the baryon-to-matter ratio to the treatment of dark energy and matter dynamics.

We now turn to the constraints on the Hubble constant, h across the different models. The thawing model is consistent with Λ CDM at 0.73σ , while the tracker and scaling-freezing models show consistencies of 0.31σ and 0.62σ , respectively. These small deviations, all within 1σ , suggest that the expansion rate of the universe remains largely unaffected by the choice of the model. This agreement reinforces the robustness of h as a cosmological parameter and supports its reliability in cosmological studies.

For the amplitude of matter fluctuations, σ_8 , the differences between the models are small but noticeable. The thawing model is consistent with Λ CDM at 0.20σ , while the tracker and scaling-freezing models show consistencies of 0.40σ and 0.11σ , respectively. These deviations remain well within the range of statistical insignificance ($< 2\sigma$), indicating no strong evidence for tension between the models. The slightly higher σ_8 values in the thawing model could indicate differences in the models' predictions for the linear growth of perturbations, which may arise from variations in the effective dark energy equation of state or the interaction between dark energy and matter.

The dimensionless tSZ amplitude parameter ($\tilde{W}^{\text{SZ}}$) exhibits notable sensitivity across the different dark energy models, reflecting its dependence on the underlying cosmological dynamics. For the Λ CDM model, $\tilde{W}^{\text{SZ}}$ is constrained to $2.934^{+0.377}_{-0.370}$, which serves as a baseline for comparison. In contrast, the thawing, tracker, and scaling-freezing models yield higher values of $3.522^{+0.336}_{-0.284}$, $3.861^{+0.216}_{-0.222}$, and $3.713^{+0.391}_{-0.338}$, respectively. These variations are closely tied to differences in the matter density (Ω_m) and the amplitude of matter fluctuations (σ_8), as models with higher Ω_m or σ_8 tend to produce stronger tSZ signals due to enhanced clustering of matter and hotter gas reservoirs in galaxy clusters.

B. Implications for dark energy dynamics

We now discuss extensively the constraints on model parameters in the quintessence class, which reveal insights into the dynamics of dark energy. For the thawing model (Fig. 5), the slope of the potential λ is constrained to $0.74^{+0.14}_{-0.16}$, indicating a dynamics explicitly different than the Λ CDM at the 1σ level as $\lambda = 0$ is outside 1σ . This result is particularly interesting as the recent BAO observation by DESI points toward a dynamical behavior of dark energy [6,7,9,16,52,53]. It is worth noting that the ISW-tSZ cross-correlation signal is dominated by low multipoles ($\ell \lesssim 50$), as shown in Fig. 2, and is primarily sensitive to low redshifts in this regime (see bottom panel of Fig. 6 in our detection analysis [17]). For these scales, the redshift kernel peaks at $z \approx 0.45\text{--}0.6$, with the majority of its weight concentrated at $z < 1.0$. This range overlaps with the low-redshift regime

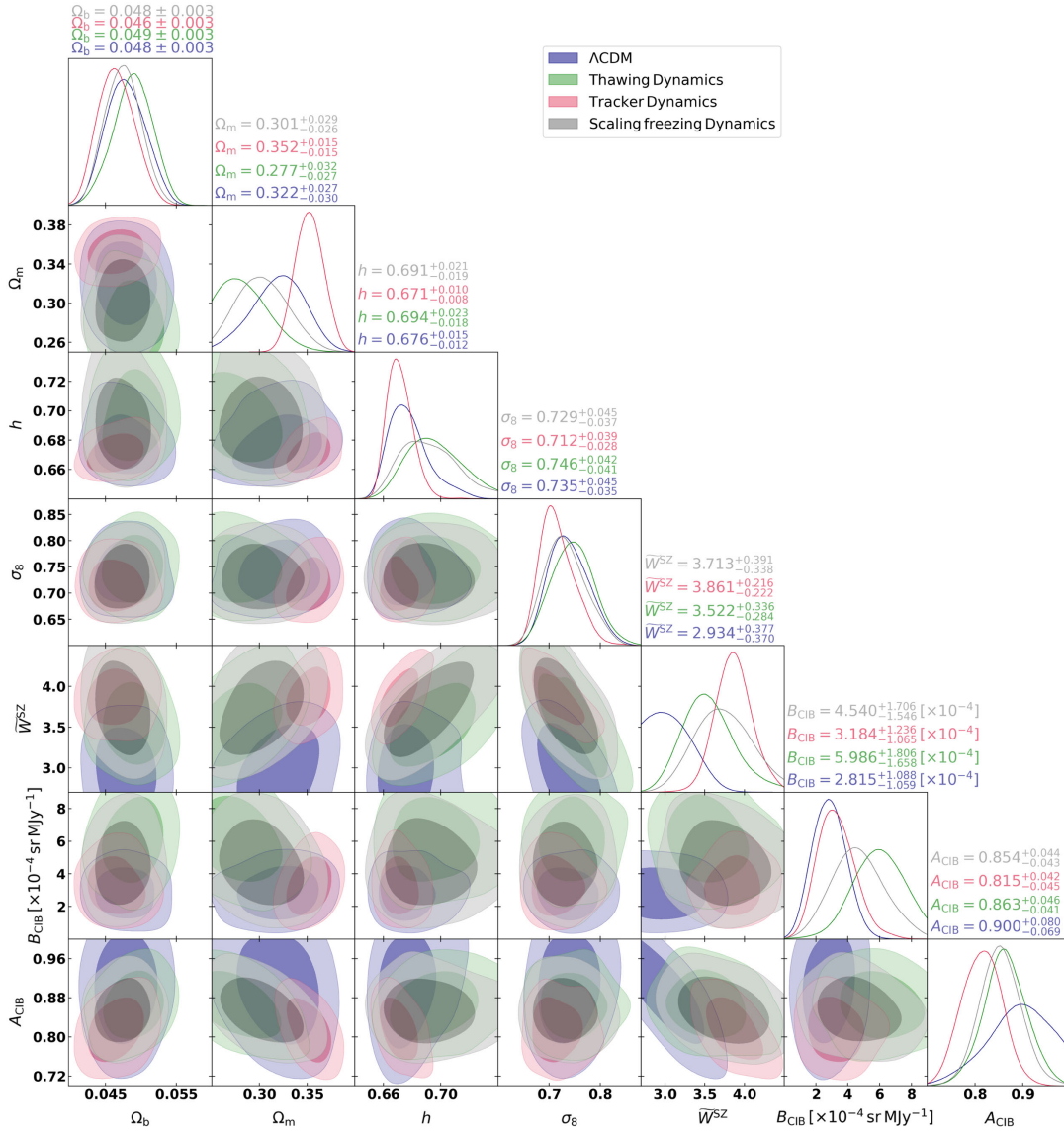


FIG. 4. Posterior distributions from MCMC with the full covariance matrix for the seven free parameters in our quintessence model. The figures show the joint constraints for all parameter pairs and the marginalized distributions for each parameter along the table diagonal.

($z < 0.55$) where DESI Year 1 finds $w(z) \geq -1$ [16]. Consequently, our preference for thawing quintessence ($w > -1$) is consistent with DESI's findings in this overlapping epoch. Rather than being in tension, the two probes are complementary: ISW-tSZ provides an independent evidence of nonphantom dark energy dynamics where DESI also supports $w \geq -1$. Canonical quintessence models are restricted to $w \geq -1$ and cannot reproduce the phantom-crossing behavior ($w < -1$) reported in some DESI analyses. Our constraints therefore apply only to dynamical dark energy within the nonphantom regime. The other parameter, V_0 , is consistent with low-energy scales ($V_0 = 0.17 \pm 0.10$). In the tracker model (Fig. 6), we find $n = 5.57^{+1.37}_{-1.42}$ and $f = 0.27^{+0.15}_{-0.09}$, also suggesting a significant departure from Λ CDM dynamics. However, given the modest significance (3.6σ) of the ISW-tSZ detection, the

preference for $n \approx 5.6$ in the tracker model should be interpreted as indicative rather than conclusive, pending validation from higher-precision data. For the scaling-freezing model, the scaling-freezing dynamics with DEXP potential parameters ($\lambda_1, \lambda_2, V_1, \log V_2$) are tightly constrained (Fig. 7), with $\lambda_2 = 23.3^{+6.23}_{-5.61}$ and $\log V_2 = 7.93 \pm 1.80$. These values highlight the role of early time scaling behavior in shaping the late-time evolution of the field. Having discussed the constraints on model parameters, we show in Figs. 5, 6, and 7, the complete posterior distributions of the parameter spaces and their correlations.

To further explore the implications of these constraints, we analyze the integrated Sachs-Wolfe (ISW) effect. We find that the scaling freezing model exhibits the strongest ISW signal with an ISW strength of 0.270, followed by the thawing model (0.224), Λ CDM (0.205), and the tracker

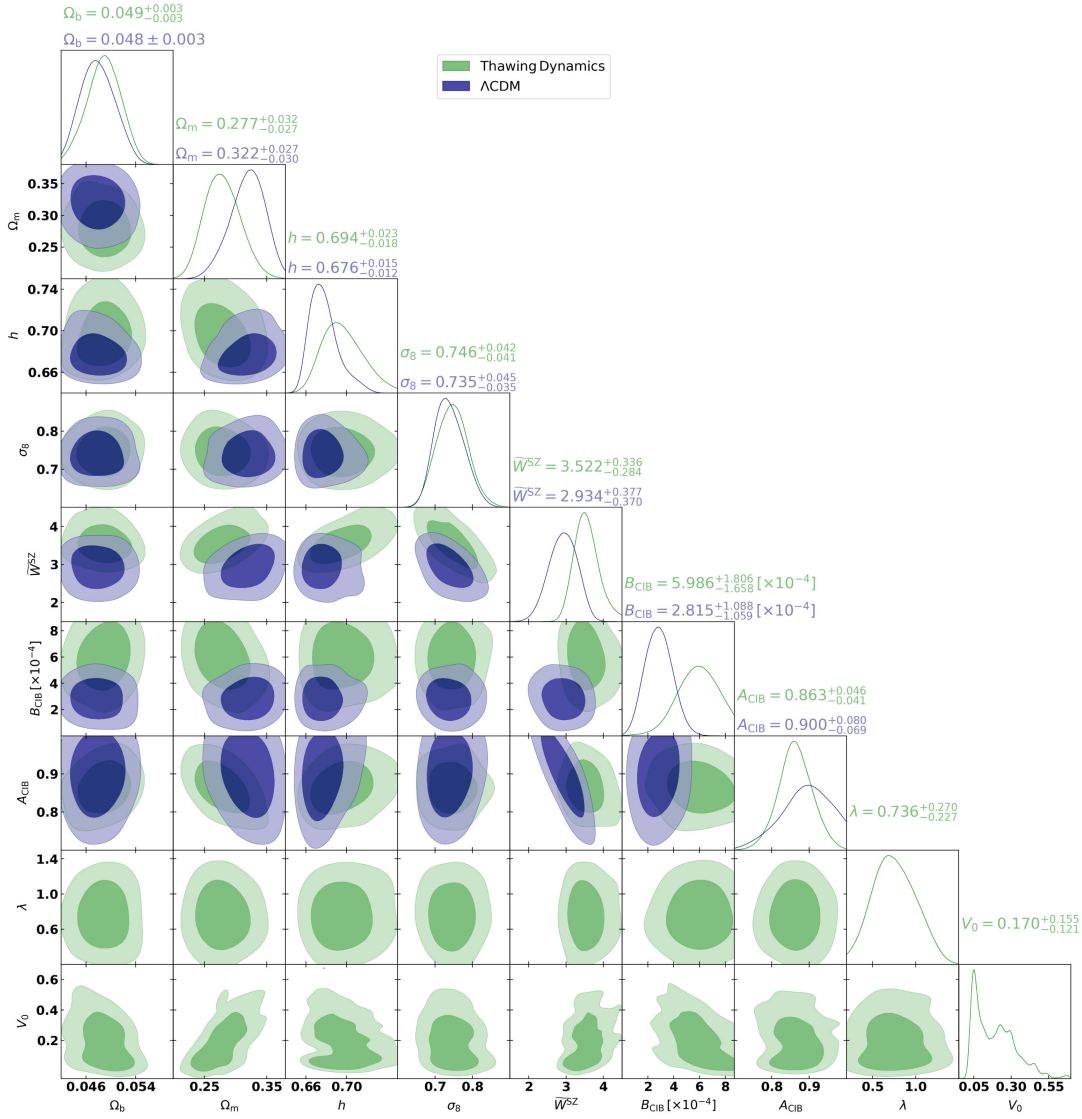


FIG. 5. The posterior distributions and correlations of key cosmological parameters from thawing dynamics of quintessence models compared to the Λ CDM framework. The constraints includes the thawing dynamics sensitive parameters. The filled contours represent the 68% and 95% confidence regions for the thawing dynamics (green) and Λ CDM (navy) models, illustrating the differences in parameter estimates and their uncertainties, as well as the impact of different dynamical behaviors on cosmological constraints.

model (0.197). The higher ISW strength in the scaling freezing model can be attributed to its more pronounced evolution of the gravitational potential during the matter-to-dark-energy transition, reflecting a more rapid change in the underlying dark energy dynamics compared to the other models. This result suggests that dark energy models characterized by significant deviations from Λ CDM behavior, particularly those with steep transitions in their equation of state, are likely to produce stronger ISW effects. Such findings have important implications for understanding the nature of dark energy, as they highlight the sensitivity of the ISW effect to the dynamical properties of dark energy.

The ISW-tSZ cross-correlation probes the large-scale, quasilinear decay of gravitational potentials, statistically

averaged over the full sky. In contrast, the so-called ISW “tension” arises from local, nonlinear measurements using stacked cosmic superstructures [54–56]. While these approaches are complementary, they probe different aspects of structure growth and are subject to distinct systematics. Among our models, scaling-freezing yields the strongest ISW signal (0.270), followed by thawing (0.224), Λ CDM (0.205), and tracker (0.197). The scaling-freezing model is statistically disfavored by our ISW-tSZ data ($\Delta\text{BIC} = +3.36$), while the preferred thawing model produces a modest enhancement relative to Λ CDM.

Future observations aimed at improving the precision of ISW measurements, such as deeper surveys of large-scale structure and high-resolution cosmic microwave background

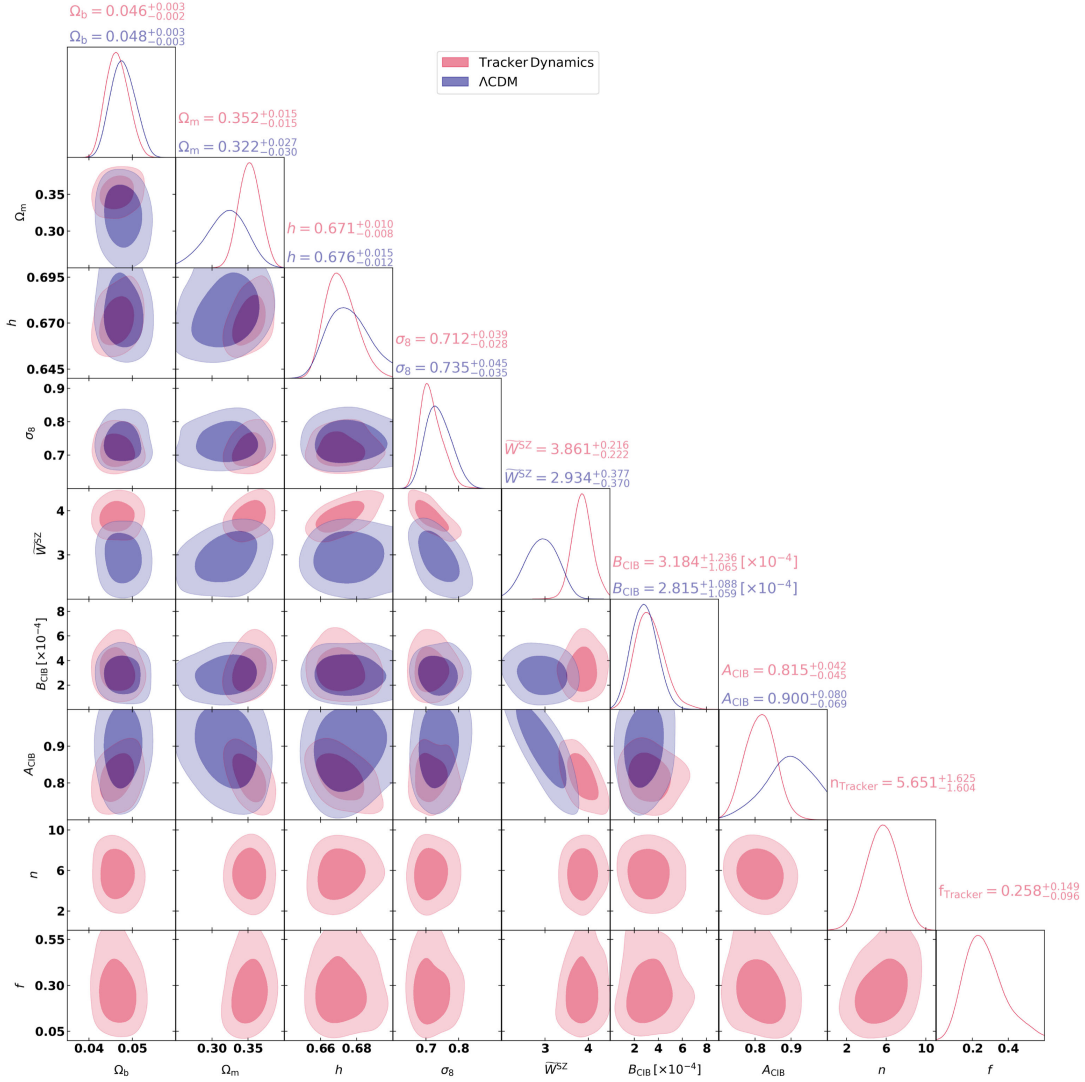


FIG. 6. Exploring the parameter space of tracker dynamics in quintessence models compared to the standard cosmological scenario. This analysis presents the 68% and 95% confidence regions for tracker dynamics (crimson) and Λ CDM (navy), along with the one-dimensional distributions along the diagonal. The results reveal distinct differences in parameter estimates and uncertainties. Notably, the tracker model indicates a higher baryon density and exhibits different behaviors in matter density and the Hubble parameter, suggesting that the dynamics of dark energy can significantly influence cosmological constraints. This study highlights the importance of considering alternative dark energy models to enhance our understanding of the Universe's evolution.

data, could provide critical tests for distinguishing between competing dark energy models and further constraining their parameter spaces. In addition to this, recent work by [57] reports the second model-dependent measurement of the gravitational potential decay rate (DR) using DESI DR9 galaxy samples. Their analysis achieves a detection significance of 3.1σ across the redshift range $0.2 \leq z < 1.4$, highlighting the growing importance of DR as a cosmological observable. Future work could incorporate DR measurements alongside ISW analyses to further refine constraints on both constant and time-varying dark energy models. These efforts would not only enhance our understanding of the late-time expansion history of the universe but also shed light on the fundamental physics governing dark energy.

C. Comparison of cosmological models

To evaluate the viability of the cosmological models under consideration, we compute the minimum chi-square ($\chi^2_{\min}$), the Akaike information criterion (AIC), and the Bayesian information criterion (BIC), along with their differences relative to the standard Λ CDM model. These metrics jointly assess goodness-of-fit and model complexity, with lower values indicating stronger statistical support. The AIC and BIC are defined as

$$\begin{aligned} \text{AIC} &= 2N + \chi^2_{\min}, \\ \text{BIC} &= N \ln(K) + \chi^2_{\min}, \end{aligned} \quad (26)$$

where N is the number of free parameters, $K = 10$ is the number of effective multipole bins in the ISW-tSZ data, and

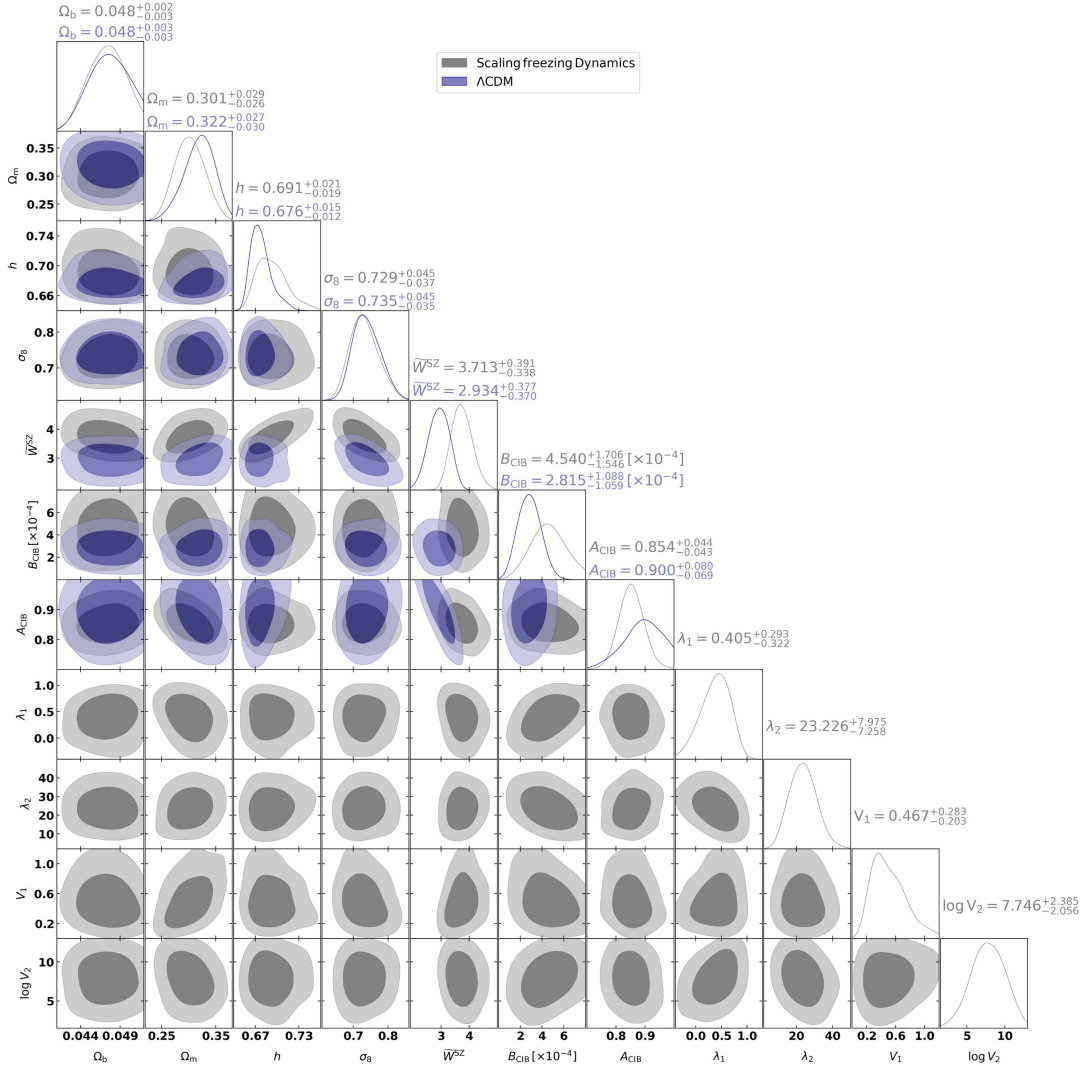


FIG. 7. Comparison of parameter constraints from scaling-freezing dynamics of quintessence models and the Λ CDM framework. The triangle plot illustrates the posterior distributions and correlations of key cosmological parameters, including baryon density (Ω_b), matter density (Ω_m), Hubble parameter (h), and the amplitude of density fluctuations (σ_8). The filled contours represent the 68% and 95% confidence regions for the scaling-freezing dynamics (gray) and Λ CDM (navy) models, highlighting the differences in parameter estimates and their uncertainties.

$\chi^2_{\min}$ quantifies the fit quality. The relative differences $\Delta AIC = AIC_{\text{model}} - AIC_{\Lambda\text{CDM}}$ and $\Delta BIC = BIC_{\text{model}} - BIC_{\Lambda\text{CDM}}$ measure the evidence for or against each model compared to Λ CDM. Following standard interpretation [58], $|\Delta| < 2$ implies substantial support, while $|\Delta| > 10$ indicates decisive disfavor.

Critically, all $\chi^2_{\min}$ values are evaluated at the *maximum likelihood point* of the posterior distribution, ensuring statistically valid comparison across nested model families. As theoretically required, both the thawing and scaling-freezing models extensions of Λ CDM that recover it in the limit of vanishing field dynamics yield $\chi^2_{\min} \leq \chi^2_{\min}(\Lambda\text{CDM})$, showing proper sampling of the Λ CDM boundary and the validity of our inference.

Table III shows that the thawing model achieves the lowest $\chi^2_{\min} = 10.16$, compared to Λ CDM ($\chi^2_{\min} = 16.39$). It also has the lowest AIC (28.16) and BIC (30.89), giving

TABLE III. Goodness-of-fit and information criteria for cosmological models, evaluated at maximum likelihood (ML) parameters. All metrics satisfy nested model consistency [$\chi^2_{\min} \leq \chi^2_{\min}(\Lambda\text{CDM})$].

Model	$\chi^2_{\min}$ (ML)	AIC	BIC	ΔAIC	ΔBIC
Λ CDM	16.39	30.39	32.51	0.00	0.00
Thawing	10.16	28.16	30.89	-2.23	-1.62
Tracker	12.29	30.29	33.02	-0.10	0.51
Scaling-freezing	10.54	32.54	35.87	2.15	3.36

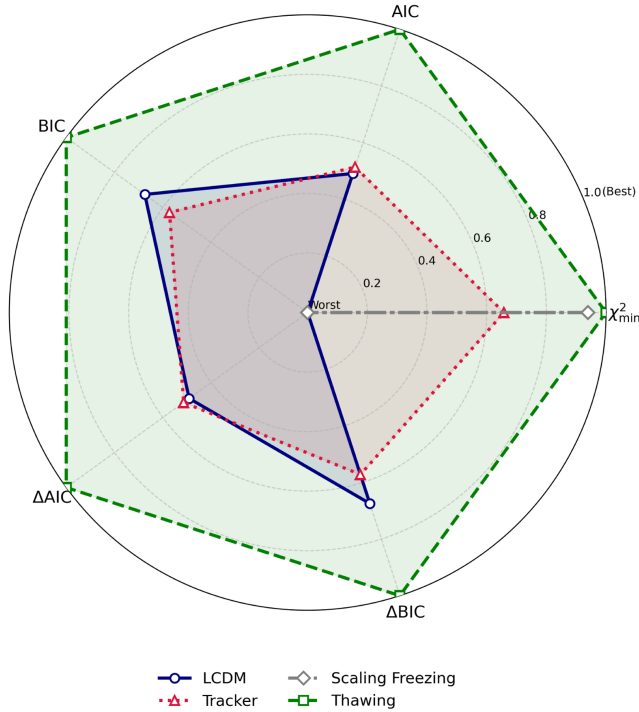


FIG. 8. Radar chart comparing the statistical performance of four cosmological models: Λ CDM, thawing, tracker, and scaling-freezing. Performance is evaluated using five metrics: $\chi^2_{\min}$, AIC/BIC (information criteria balancing fit and parsimony), and their relative differences to Λ CDM (ΔAIC , ΔBIC). All metrics follow a “lower-is-better” paradigm and are independently normalized to [0,1], where 1 = best and 0 = worst model for each metric. This ensures larger polygon areas correspond to superior overall performance. The thawing model (green) dominates with the largest polygon, reflecting an optimal balance of fit and parsimony. Scaling-freezing (dark gray) follows, with strong $\chi^2_{\min}$ performance offset by lower parsimony (weak AIC/BIC scores). Tracker (crimson) and Λ CDM (navy) have progressively smaller polygons, indicating weaker overall performance. Better-performing models are plotted on top for visibility.

$\Delta BIC = -1.62$ relative to Λ CDM, which indicates substantial support under standard criteria [58]. The scaling-freezing model has a comparable $\chi^2_{\min} = 10.54$ but higher BIC (35.87) due to its larger parameter count ($N = 11$), yielding $\Delta BIC = +3.36$. The tracker model gives $\chi^2_{\min} = 12.29$ and $\Delta BIC = +0.51$. The best-fit (maximum-likelihood) thawing slope is $\lambda = 0.974$, distinct from the Λ CDM limit ($\lambda = 0$); the Bayesian posterior median is $\lambda = 0.736^{+0.270}_{-0.227}$. All extended models satisfy $\chi^2_{\min} \leq \chi^2_{\min}(\Lambda\text{CDM})$, as required for nested models. The relative performance of the models is visualized in Fig. 8.

VI. CONCLUSION

We have presented constraints on canonical quintessence dark energy models: thawing, tracker, and scaling-freezing

using the 3.6σ ISW-tSZ cross-correlation detection from [17]. This probe is sensitive to late-time gravitational potential decay ($z \lesssim 1$) and operates independently of early Universe physics, avoiding degeneracies present in joint CMB + BAO analyses.

Within this dataset, the thawing model achieves the lowest $\chi^2_{\min} = 10.16$ compared to Λ CDM ($\chi^2_{\min} = 16.39$), with $\Delta BIC = -1.62$ and $\Delta AIC = -2.23$, indicating substantial statistical support under standard criteria [58]. The best-fit thawing slope is $\lambda = 0.974$, distinct from the Λ CDM limit ($\lambda = 0$). All extended models satisfy the theoretical requirement $\chi^2_{\min} \leq \chi^2_{\min}(\Lambda\text{CDM})$ for nested scenarios.

The analysis reveals a mild but persistent $\sim 1.7\sigma$ tension in σ_8 , with ISW-tSZ-inferred values peaking near 0.74, compared to the *Planck* 2018 value of 0.811 ± 0.012 . This discrepancy remains after applying Gaussian *Planck* priors on $(\Omega_b, \Omega_m, h, \sigma_8)$, suggesting it is not an artifact of weak external constraints, but likely reflects a characteristic of the ISW-tSZ data itself.

Importantly, the preference for the thawing model persists when incorporating *Planck* 2018 constraints on $(\Omega_b, \Omega_m, h, \sigma_8)$: the statistical support shifts from $\Delta BIC = -1.62$ (without priors) to $\Delta BIC = -3.83$ (with priors). Model-specific parameters show minimal shifts (Appendixes A and B), indicating that the preference is not solely a consequence of weak constraints on background cosmology. However, since external priors can affect model comparison through parameter volume effects, this preference should be viewed as internal consistency within our framework, not as definitive evidence for dynamical dark energy.

Future CMB experiments (CMB-S4, Simons Observatory) [59,60] and large-scale structure surveys (DESI, Euclid, Rubin LSST) [61–64] will dramatically improve the precision of ISW cross-correlations. When combined with galaxy clustering, weak lensing, or supernova data, these datasets will enable robust tests of dark energy dynamics and determine whether the thawing preference persists in a full cosmological context. While joint analyses with BAO, SNe Ia, and CMB are essential for definitive constraints, they lie beyond the scope of this probe-focused study. We refer the reader to companion works by our coauthor and collaborators: [14] for tracker models, and [65] for thawing and scaling-freezing dynamics, where these models are constrained using the full cosmological concordance dataset. For now, the ISW-tSZ cross-correlation serves as a complementary probe of late-time cosmic acceleration, with our analysis showing consistency with nonphantom dynamical dark energy.

Software: This work made use of the following software packages: HEALPix [66], MASTER [67], NaMaster [68], UltraNest [49,50], and GetDist [51].

ACKNOWLEDGMENTS

The postdoctoral funds of the Guangdong Technion-Israel Institute of Technology, China support this project. A. I. acknowledges the partial support of the Alliance of International Science Organizations, Grant No. ANSO-VF-2024-01. X. C. acknowledges support from the National Natural Science Foundation of China, Grant No. NSFC 12361141814.

DATA AVAILABILITY

The data that support the findings of this article are openly available [17,69].

APPENDIX A: STABILITY OF COSMOLOGICAL PARAMETERS UNDER EXTERNAL CONSTRAINTS

To test the stability of our inference under external constraints, we repeat the analysis applying Gaussian priors on base cosmological parameters from *Planck* 2018 TT,

TE, EE + lowE + lensing results [39]

$$\Omega_b h^2 = 0.02237 \pm 0.00015$$

$$\Omega_c h^2 = 0.1200 \pm 0.0012$$

$$H_0 = 67.36 \pm 0.54 \text{ km/s/Mpc}$$

$$\sigma_8 = 0.8110 \pm 0.0120$$

The resulting posterior distributions for Ω_b , Ω_m , h , and σ_8 are compared in Fig. 9 between runs with no prior (light blue) and with Planck Gaussian priors (navy). The black dashed line in each panel indicates the mean of the Planck prior.

As shown in Fig. 9, the inclusion of Planck priors has a clear and differential effect across parameters. For Ω_b and h , the posteriors are pulled toward the Planck values, with the 1σ credible intervals narrowing substantially by factors of 4 to 9. This indicates that while ISW-tSZ data alone provides only weak constraints on these parameters, it remains broadly consistent with Planck when combined with

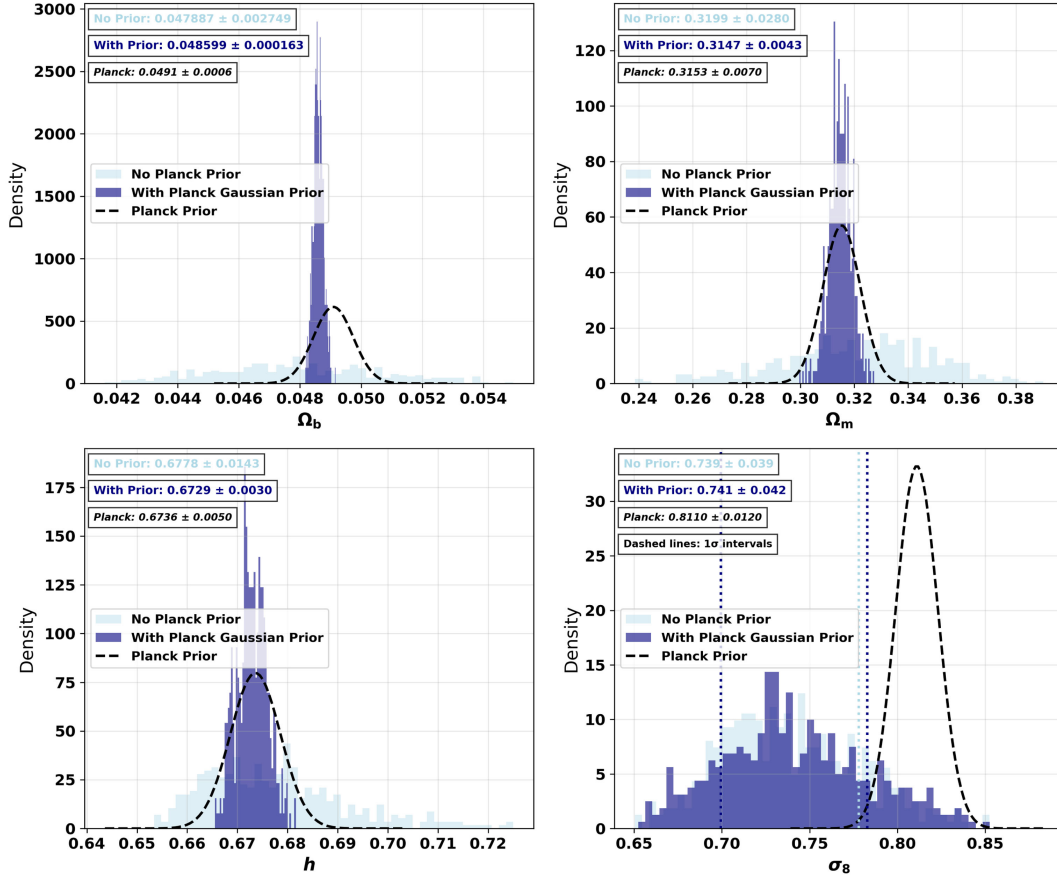


FIG. 9. One-dimensional marginal posterior distributions for Λ CDM parameters derived from ISW-tSZ data alone, comparing chains run without Planck priors (light blue) and with Planck Gaussian priors (navy). The black dashed line indicates the Planck prior mean for each parameter. For Ω_b , Ω_m , and h , the Planck prior significantly tightens the posterior, bringing estimates within 0.35σ of the Planck mean, with credible intervals reduced by factors of 4 to 9 compared to the no-prior case. In contrast, the posterior for σ_8 remains largely unaffected by the prior: the mean shifts only slightly ($\Delta\mu = +0.0023$), and the distribution is inconsistent with the Planck value at approximately 1.7σ . Dashed light blue and navy lines in the σ_8 panel denote the 1σ credible intervals for the respective posteriors.

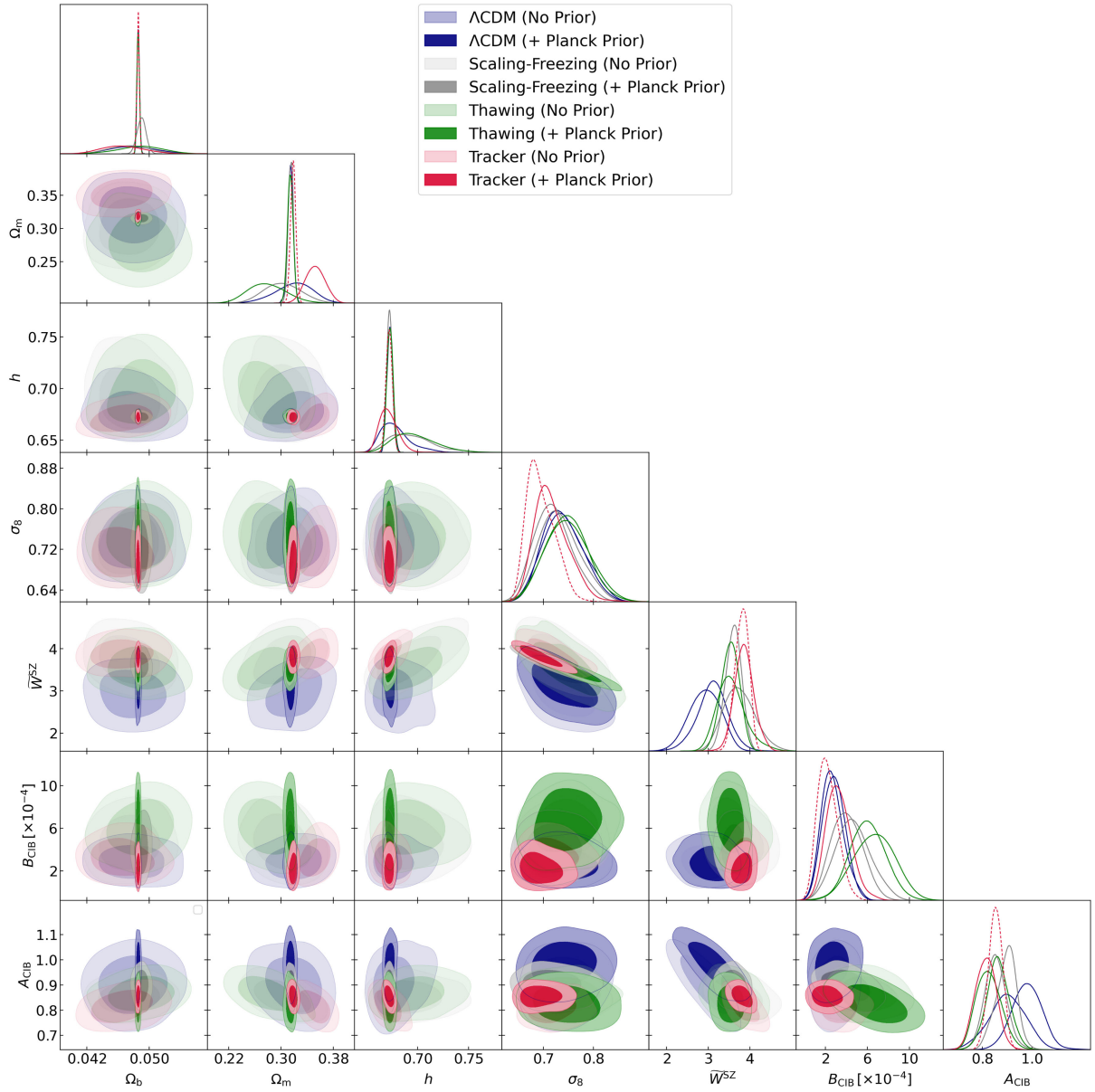


FIG. 10. Triangle plot showing 68% and 95% credible regions for cosmological parameters in four models (Λ CDM, scaling freezing, thawing, tracker) using ISW-tSZ data alone (lighter contours) and with Planck Gaussian priors (darker contours).

external information. The posterior for Ω_m also tightens and shifts toward the Planck value, though less dramatically than for Ω_b and h , reflecting residual degeneracies with other parameters in the ISW-tSZ likelihood.

Critically, the posterior for σ_8 shows minimal response to the Planck prior: the peak remains near 0.74, while Planck's value is 0.811 ± 0.012 , corresponding to a $\sim 1.7\sigma$ tension that persists even after incorporating the prior. The persistence of the σ_8 offset unchanged by external priors reflects a difference in probe sensitivity: the ISW-tSZ signal is primarily sensitive to the integrated gravitational potential along the line of sight, not to the present-day amplitude of matter fluctuations.

These findings highlight two features of the ISW-tSZ probe. First, it is not a direct probe of σ_8 ; its insensitivity to small-scale structure leads to weaker constraints and potential biases in this parameter when used in isolation. Second, the persistent σ_8 tension underscores the complementary nature of ISW-tSZ: it may probe aspects of late-time cosmology that are not fully captured by standard Λ CDM fits to CMB data. Resolving this tension will require joint analyses with probes sensitive to small-scale structure, such as galaxy clustering, weak lensing, or cluster counts.

To further evaluate the impact of Planck priors on our main conclusion, we performed a full comparison of all four models (Λ CDM, thawing, tracker, scaling freezing)

under Planck Gaussian priors (Fig. 10). This plot demonstrates that the thawing model continues to show the strongest preference over Λ CDM, with tighter constraints and higher density in regions of parameter space favored by the data. All other models exhibit broader posteriors and less pronounced peaks compared to thawing, consistent with their higher complexity and lower statistical support.

Critically, the σ_8 tension persists across all models: the posterior remains centered near 0.74, regardless of whether they are canonical scalar field models or extended versions. This indicates that the tension is not an artifact of the prior, but likely reflects a characteristic of the ISW-tSZ signal. The persistent σ_8 offset further suggests that the ISW-tSZ signal encodes late-time physics that is not fully aligned with early universe CMB constraints.

The thawing model remains preferred over Λ CDM under Planck priors ($\Delta\text{BIC} = -3.83$), and the model-specific parameters show minimal shifts (Appendix B). This indicates that the preference is not merely an artifact of the weak constraints on background cosmological parameters in the baseline analysis. To assess whether these priors impact the model-specific dynamical parameters of our dark energy frameworks (rather than just shared cosmological parameters), we turn to Appendix B.

APPENDIX B: INTERNAL CONSISTENCY CHECK: STABILITY OF DYNAMICAL DARK ENERGY PARAMETERS UNDER EXTERNAL PRIORS

Building on the cosmological parameter stability shown in Appendix A, we now examine whether the model-specific parameters of our quintessence frameworks—thawing (λ),

tracker (n), and scaling-freezing (λ_1) are affected by external constraints.

As shown in Fig. 11, the application of Planck Gaussian priors induces negligible shifts in the posterior distributions of model-specific parameters. For scaling-freezing λ_1 , the 1σ interval changes from $[0.08, 0.70]$ (no prior) to $[-0.10, 0.60]$ (with prior), a width variation of 13.6%. For thawing λ , the interval shifts from $[0.51, 1.01]$ to $[0.46, 0.99]$, a 6.4% change. For tracker n , the shift is even smaller: from $[4.05, 7.28]$ to $[3.97, 7.22]$, a mere 0.9% change.

This contrasts sharply with cosmological parameters such as Ω_b , whose uncertainties shrink by factors of 4–9 under the same priors. The difference arises because Planck priors constrain early universe physics, while model-specific parameters govern late-time dark energy behavior, precisely the regime probed by ISW-tSZ.

The minimal response of these parameters to external priors shows that the relative ranking of models is not driven by volume effects in the background cosmology. However, it does not imply that the ISW-tSZ data provides definitive evidence for dynamical dark energy. The observed preference for thawing should be interpreted as internal consistency within our modeling framework, not as confirmation of new physics.

Future joint analyses with probes that directly trace the expansion history (e.g., SNe Ia, BAO, weak lensing) will be needed to test whether this preference persists in a broader cosmological context. For this work, which isolates the ISW-tSZ signal as a stand-alone probe, the stability of model-specific parameters under Planck priors serves as a necessary internal consistency check, ensuring that our conclusions are not artifacts of unconstrained base cosmology.

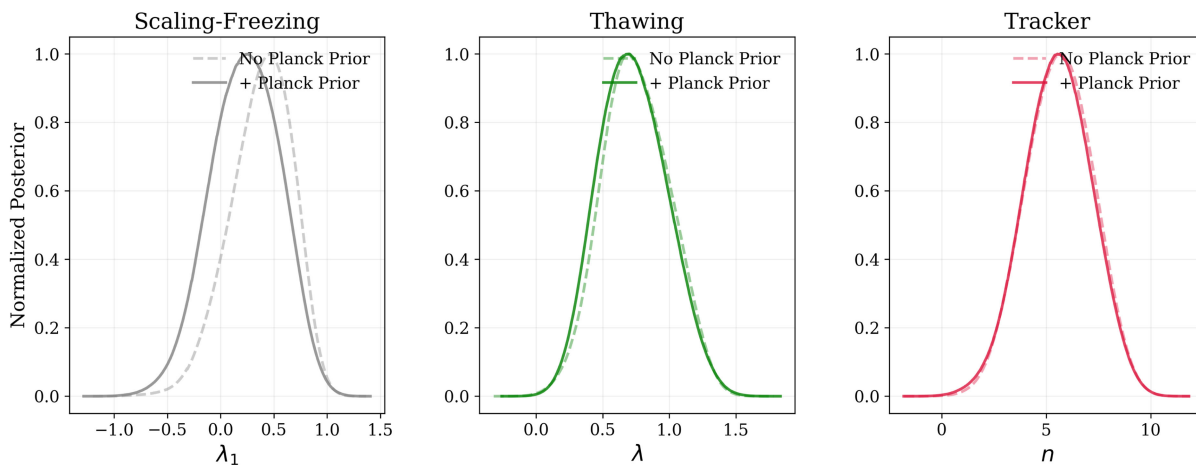


FIG. 11. One-dimensional marginal posterior distributions for model-specific dark energy parameters: scaling-freezing λ_1 (left), thawing λ (middle), and tracker n (right). Light dashed lines show posteriors from ISW-tSZ data alone (no Planck prior), while dark solid lines show posteriors with Planck Gaussian priors applied. The Planck prior has minimal impact on these parameters: 1σ credible interval widths change by only 13.6% (λ_1), 6.4% (λ), and 0.9% (n)—a stark contrast to the 300%–800% narrowing observed for cosmological parameters like Ω_b . This stability confirms that the inferred dark energy dynamics are weakly degenerate with background cosmology.

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