

Improved constraints on pion fragmentation functions from simulated electron-ion collider data

Maryam Soleymaninia^{1,2,*} Hamzeh Khanpour^{3,4,1,†} Majid Azizi^{1,‡} and Hadi Hashamipour^{5,§}

¹*School of Particles and Accelerators, Institute for Research in Fundamental Sciences (IPM),
P.O.Box 19395-5531, Tehran, Iran*

²*Department of Computer Engineering, UAE.C., Islamic Azad University, Dubai, United Arab Emirates*

³*AGH University, Faculty of Physics and Applied Computer Science,
Al. Mickiewicza 30, 30-055 Krakow, Poland*

⁴*Department of Physics, University of Science and Technology of Mazandaran,
P.O.Box 48518-78195, Behshahr, Iran*

⁵*Istituto Nazionale di Fisica Nucleare, Gruppo collegato di Cosenza,
I-87036 Arcavacata di Rende, Cosenza, Italy*



(Received 27 March 2025; accepted 3 September 2025; published 22 September 2025)

We present a quantitative assessment of the anticipated impact of future Electron-Ion Collider (EIC) measurements on the extraction of parton-to-pion fragmentation functions (FFs). Our analysis combines simulated semi-inclusive deep-inelastic scattering (SIDIS) pseudo-data at EIC energies of 45 GeV and 140 GeV with existing single-inclusive electron-positron annihilation (SIA) and SIDIS experimental data. The pion fragmentation functions are determined at next-to-leading-order (NLO) accuracy using a perturbative quantum chromodynamics framework and a neural network parametrization. Uncertainties are rigorously estimated through Monte Carlo sampling, accounting for both experimental errors and variations in input parton distribution functions. Our results demonstrate that incorporating EIC pseudo-data reduces their uncertainties, especially at medium- to-large momentum fractions (z). This improvement is particularly pronounced for the gluon and selected quark FFs, highlighting the substantial role that EIC measurements will play in achieving high-precision extractions of FFs and informing future experimental and theoretical developments in collider physics.

DOI: [10.1103/gm92-w63j](https://doi.org/10.1103/gm92-w63j)

I. INTRODUCTION

Fragmentation functions (FFs) are fundamental components of quantum chromodynamics (QCD), describing the non-perturbative transition of partons into observed hadrons. These functions serve as critical inputs for theoretical predictions across various high-energy collision processes, including single-inclusive electron-positron annihilation (SIA), semi-inclusive deep-inelastic scattering (SIDIS), and hadron-hadron collisions. Accurate determination of FFs is essential for testing QCD factorization, deepening our understanding of hadronization mechanisms and refining global fits aimed at extracting parton

distribution functions (PDFs). Traditionally, SIA data have provided the primary constraints on FFs due to their straightforward theoretical interpretation. However, SIDIS measurements are indispensable for distinguishing quark and antiquark fragmentation and achieving comprehensive flavor separation [1–3].

The electron-ion collider (EIC), scheduled to commence operations around 2030 at Brookhaven National Laboratory, will be a state-of-the-art, high-luminosity facility designed to systematically investigate the internal structure of nucleons and nuclei with unprecedented precision [4–8]. This new capability promises to significantly advance our comprehension of QCD dynamics, particularly illuminating the interplay between sea quarks and gluons [8,9].

One of the main scientific objectives of the EIC will be high-precision DIS and SIDIS measurements, providing detailed and differential constraints on both PDFs and FFs [10–13]. In addition to improving the determination of sea quark and gluon distributions, simulated DIS data from the EIC will also enable enhanced constraints on valence quark distributions. Recent studies indicate that the inclusion of these projected datasets significantly reduces uncertainties

*Contact author: Maryam_Soleymaninia@ipm.ir

†Contact author: Hamzeh.Khanpour@cern.ch

‡Contact author: Ma.Azizi@ipm.ir

§Contact author: Hadi.Hashamipour@lnf.infn.it

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](https://creativecommons.org/licenses/by/4.0/). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

in the strong coupling constant extraction, driven by the substantial extension of the accessible kinematic ranges in both Bjorken x and Q^2 [14–16]. Furthermore, the high-statistics SIDIS data expected from the EIC will substantially decrease uncertainties in FF extractions, especially for charge-separated quark distributions and gluon fragmentation. Thus, EIC measurements will complement and enhance existing constraints from SIA and other high-energy processes, facilitating significant refinements in global determinations of FFs.

The impact of projected SIDIS EIC data on proton PDFs as well as pion and kaon fragmentation functions was previously investigated in Ref. [17], employing a reweighting technique to incorporate realistic pseudodata uncertainties into established global extractions. Their analysis utilized cross section data unfolded for detector effects, highlighting the substantial improvements expected for charged pion and kaon FFs.

In this study, we perform a comprehensive global QCD analysis integrating charged pion SIDIS pseudodata from the EIC with all available SIA and real SIDIS datasets. Our analysis represents the first instance of directly incorporating EIC SIDIS pseudodata into a global fit to quantify their precise impact on FF extractions. A significant feature of our approach is the adoption of a neural network-based methodology, which provides enhanced flexibility and accuracy compared to traditional parametrization techniques.

Our findings demonstrate that the inclusion of EIC pseudodata substantially improves the precision of pion FFs, particularly at the previously poorly constrained large-momentum fraction (z) region. In fact, uncertainty bands of FFs are notably reduced, particularly for gluon and up-quark FFs, when EIC pseudodata are included. This underscores the critical role future EIC data will play in addressing the persistent challenge of precisely constraining gluon FFs.

The paper is structured as follows. Section II outlines the theoretical framework for inclusive hadron production in electron-positron annihilation and semi-inclusive deep-inelastic scattering. It describes our neural network-based parametrization of FFs and details the methodology for uncertainty estimation. Section III introduces the experimental datasets used, including a discussion of the generation of EIC pseudodata and applied kinematic cuts. Section IV presents our main results, focusing on the quality of the global fit and the specific impacts of incorporating EIC data on pion FFs, along with a detailed comparison with previous results from the MAPFF and NNFF1.0 analyses. Finally, Sec. V summarizes our conclusions and outlines potential future directions for extending this analysis.

II. THEORETICAL AND METHODOLOGICAL SETUP

In this analysis, we investigate charged pion FFs utilizing experimental data from two complementary processes:

the SIA and SIDIS. Each of these processes offers distinct and essential constraints on the flavor-dependent FFs. The cross-section for the SIA process, describing hadron production in electron-positron annihilation, is expressed as a convolution of a perturbatively calculable hard-scattering coefficient function and nonperturbative FFs [3,18],

$$\frac{d\sigma^{\text{SIA}}}{dQ^2 dz_h} = \sum_j C_j^{\text{SIA}}(z_h, Q^2) \otimes D_j(z_h, \mu), \quad (1)$$

where z_h represents the fraction of momentum carried by the observed hadron relative to the parent parton, and $\otimes$ denotes the convolution integral.

Similarly, the SIDIS process involves an electron scattering off a nucleon with the subsequent detection of a hadron in the final state. The SIDIS cross section is given by a double convolution involving the PDFs, FFs, and the corresponding hard-scattering coefficient functions,

$$\frac{d\sigma^{\text{SIDIS}}}{dQ^2 dx_B dz_h} = \sum_{i,j} C_{ij}^{\text{SIDIS}}(x_B, z_h, Q^2) \otimes f_i(x_B, \mu) \otimes D_j(z_h, \mu). \quad (2)$$

In this expression, x_B is the Bjorken scaling variable, and again, z_h is the hadronic momentum fraction. Additional theoretical details and derivations can be found in Refs. [3,18].

Our global analysis of SIA and SIDIS data is carried out within a perturbative QCD framework at next-to-leading-order (NLO) accuracy. To parametrize the FFs, we adopt a flexible neural network (NN) approach, enabling robust and unbiased uncertainty propagation from both experimental data and input PDFs. Specifically, the NNPDF4.0 PDF sets [19] are employed in calculating the SIDIS cross sections. To perform this global QCD analysis, we utilize the publicly available MontBlanc package [1,20], accessible from [21].

As heavy-quark mass corrections for SIDIS processes are not yet well established, we perform our analysis within the zero-mass variable flavor number scheme (ZM-VFNS). In this approximation, all active partons are treated as massless; however, we incorporate partial heavy-quark mass dependence by matching different active-flavor number subschemes at heavy-quark thresholds. The charm and bottom thresholds are set at $m_c = 1.51$ GeV and $m_b = 4.92$ GeV, respectively, consistent with the NNPDF4.0 sets [19,22]. Below these thresholds, heavy-quark FFs remain constant and do not evolve.

In this analysis, we utilize a framework that combines a neural network parametrization of FFs with a Monte Carlo representation of FF uncertainties. This approach, which has been extensively used by the NNPDF Collaboration to determine PDFs of the proton and of the nuclei, allows us to reduce model bias in FF parametrization as much as

possible and to faithfully propagate experimental and PDF uncertainties into FFs [1,20]. These features are essential to achieve the methodological accuracy of FFs that are utilized to analyze, e.g., high-precision hadron production measurements at the Large Hadron Collider (LHC) and, in the future, at the EIC.

The neural network is optimized by means of a gradient descent algorithm that makes use of the knowledge of the analytic derivatives of the neural network itself with respect to its free parameters [1].

As discussed in our previous work [23], the inclusive SIA data only allow for the determination of the total summed quark and antiquark FFs by including the total inclusive cross sections for light, charm, and bottom quarks. By incorporating SIDIS datasets into the analysis, we gain the ability to directly constrain the individual quark and antiquark FFs for light quarks. For the parametrization of π^+ , we adopt the following basis:

$$\begin{aligned} D_u^{\pi^+}, D_d^{\pi^+}, D_s^{\pi^+} &= D_u^{\pi^+}, \\ D_{s^+}^{\pi^+}, D_{c^+}^{\pi^+}, D_{b^+}^{\pi^+}, D_g^{\pi^+}. \end{aligned} \quad (3)$$

The parameterization is introduced at the initial scale $\mu_0 = 5$ GeV. To reduce model bias in the parametrization of FFs, we employ a feed-forward neural network (NN) architecture at the initial scale. The FFs for each parton flavor are defined through the NN model as

$$zD_i^{h^+}(z, Q_0) = (N_i(z; \theta) - N_i(1; \theta))^2, \quad (4)$$

where $N_i(z; \theta)$ is the NN output, and θ stands for the (internal) parameters of the NN. The subtraction at $z = 1$ ensures that the FFs vanish at the endpoint, and squaring the result guarantees positivity. We deliberately avoid introducing prefactors such as $z^\alpha(1-z)^\beta$ to regulate the endpoint behavior, thereby minimizing theoretical bias and allowing the data to drive the shape of the FFs.

The neural network is a multilayer feed-forward perceptron [24]. The output $\xi_i^{(l)}$ of the i th node in the l th layer is given by

$$\xi_i^{(l)} = g\left(\sum_j \omega_{ij}^{(l)} \xi_j^{(l-1)} + \theta_i^{(l)}\right), \quad (5)$$

where g is the activation function (sigmoid in hidden layers, linear in the output layer), and $\omega_{ij}^{(l)}$ and $\theta_i^{(l)}$ are the weights and biases, respectively. These parameters are determined by minimizing the χ^2 function.

We use a (1-20-7) architecture: one input node (for z), one hidden layer with 20 nodes, and seven output nodes for the FF combinations. While the use of NNs can mitigate model bias, we acknowledge that design choices such as activation functions and constraints can affect uncertainty

estimates. This effect is discussed, for example, in NNPDF4.0 [19]. Similar strategies to ours are also adopted in recent global FF fits such as Refs. [1,25], where the flexibility of the NN allows for unbiased extraction of FFs even in the underconstrained small- and large- z regions.

We choose the initial scale of parametrization for the fragmentation functions to be $Q_0 = 5$ GeV. This scale lies above the bottom-quark mass threshold ($m_b = 4.92$ GeV). This choice is advantageous for two reasons: first, it avoids any heavy-quark threshold crossings during evolution, keeping the number of active flavors fixed at $n_f = 5$, and thereby eliminates the need for time-like matching conditions across thresholds, which are currently only known to NLO accuracy [26]. Second, in the variable flavor number scheme (VFNS), this allows us to parametrize charm and bottom FFs on the same footing as light-quark and gluon FFs, accommodating their significant nonperturbative components. Although some SIDIS data lie below Q_0 , we perform backward evolution using numerically stable time-like Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution routines, which have been validated down to $Q \sim 1.4$ GeV. This approach is consistent with previous global analyses such as MAPFF [1,20], where the same initial scale is adopted and reliable backward evolution is achieved. We confirm that the numerical stability of our evolution routines is preserved throughout the full Q^2 range of our analysis.

By including the SIDIS data in the QCD fit, the combinations of quark FFs, $D_{u^+}^{\pi^+}$, and $D_{d^+}^{\pi^+}$ are considered to be decomposed. The strange quark and heavy quark distributions are assumed to be symmetric, as expressed by

$$D_q^{\pi^+} = D_{\bar{q}}^{\pi^+}, \quad q = s, c, b. \quad (6)$$

Thus, by adding the SIDIS data, the number of independent distributions increases to seven in comparison to five in [27]. We find that these flavor combinations for quark FFs provide the best fit quality and accuracy in hadron production.

Uncertainties on the FFs are determined by means of the Monte Carlo sampling method properly accounting for all sources of experimental uncertainties, including that of parton distribution functions. This method allows us to incorporate the uncertainty information coming from datasets and input PDFs using Monte Carlo, avoiding a full time-consuming refit but preserving the statistical rigor for the uncertainty estimates [20].

In the present work, hadron mass and target mass corrections (TMCs) have not been included in our analysis. We intend to address these aspects in future studies. Additionally, forthcoming analyses will integrate recent experimental data, such as the BESIII measurements for $\pi^\pm$ production in electron-positron annihilation [28], as well as newly reported charged pion multiplicities from SIDIS

measurements on proton and deuteron targets provided by the JLab collaboration [29].

III. EXPERIMENTAL DATA

This section details the experimental datasets analyzed in our work, including the SIA and SIDIS, highlighting their roles in constraining FFs and specifying their kinematic coverage. It also introduces and describes the simulated EIC pseudodata for SIDIS measurements, emphasizing their complementary role and discussing the covered kinematic ranges. Finally, this section outlines the criteria used for data selection and describes the specific kinematic cuts applied in our analysis to ensure theoretical consistency and reliability.

A. SIA and SIDIS datasets

SIA measurements directly probe the FFs integrated over quark and antiquark flavors. Due to their inclusive nature, these measurements primarily constrain the charge-weighted sum of the FFs, limiting sensitivity to individual flavor distinctions. However, the availability of SIA data sets from different experiments at multiple center-of-mass energies allow for constraints on the gluon FF through DGLAP evolution [30–32]. In contrast, SIDIS provides direct sensitivity to individual quark and antiquark FFs due to the flavor-dependent nature of the PDFs. SIDIS measurements typically cover lower energy scales ($1 \leq Q^2 \leq 8 \text{ GeV}^2$), significantly aiding the determination of gluon FF via QCD evolution. Moreover, experiments using isoscalar targets such as deuterium and lithium further enhance flavor separation, particularly distinguishing valence from sea quark contributions. Therefore, the combination of both SIA and SIDIS datasets provides a powerful, complementary approach to FF extraction. This complementarity, reinforced by carefully chosen kinematic cuts, ensures theoretical consistency and robustness in the global extraction of FFs. Details on the specific experimental datasets used from SIA and SIDIS processes are provided in the following subsections.

1. The SIA datasets

SIA provides a clean experimental environment for studying fragmentation as it does not involve the complexities of initial-state parton distributions. In this process, an electron and a positron annihilate into a virtual photon (or Z boson), which subsequently produces a quark-antiquark pair that fragments into final-state hadrons,

$$e^+ + e^- \rightarrow h + X. \quad (7)$$

The experimental SIA data used in this analysis are collected from various high-energy experiments covering a wide range of center-of-mass energies $\sqrt{s}$,

- (i) *CERN*: ALEPH [33], DELPHI [34], OPAL [35],

- (ii) *DESY*: TASSO [36–38],
- (iii) *KEK*: BELLE [39], TOPAZ [40], and
- (iv) *SLAC*: BABAR [41], TPC [42], SLD [43].

The measurements span a kinematic range from $\sqrt{s} \sim 10 \text{ GeV}$ (B-factory experiments such as BABAR and BELLE) to the $\sqrt{s} = M_Z$ (LEP experiments such as ALEPH, DELPHI, OPAL, and SLD).

2. The SIDIS datasets

In the SIDIS process, a charged lepton scatters off a nucleon, exchanging a virtual photon that probes the nucleon's parton structure. A hadron is detected in the final state, leading to the reaction

$$\ell + N \rightarrow \ell' + h + X. \quad (8)$$

The cross-section depends on both the partonic structure of the nucleon (PDFs) and the hadronization of the struck quark.

The SIDIS data used in this study are collected from

- (i) *COMPASS* (CERN): Muon beam ($E_\mu = 160 \text{ GeV}$) on a ${}^6\text{LiD}$ target [44], and
- (ii) *HERMES* (DESY): Electron/positron beams ($E_e = 27.6 \text{ GeV}$) on hydrogen and deuterium targets [45].

B. Simulated EIC SIDIS data

In addition to the SIA measurements from LEP, DESY, SLAC, and KEK, and SIDIS data from HERMES and COMPASS, this analysis incorporates the EIC pseudodata for the semi-inclusive deep-inelastic scattering process.

We show that the projected EIC measurements for SIDIS at $\sqrt{s} = 140 \text{ GeV}$ and $\sqrt{s} = 45 \text{ GeV}$ provide crucial constraints on FFs. These datasets are generated using Monte Carlo simulations with realistic experimental uncertainties, accounting for detector effects and kinematic acceptance criteria. The EIC is designed as a high-energy, high-luminosity polarized collider ($\mathcal{L} \sim 10^{33} - 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$) and has evolved through two preconceptual designs.

To span the full kinematic coverage of the EIC, studies are performed for lepton beam energies of 5 GeV and 20 GeV, in combination with proton beam energies of 100 GeV and 250 GeV, respectively. The pseudodata are generated using the Monte Carlo event generator PYTHIA-6 to simulate DIS events, assuming a statistical uncertainty corresponding to an integrated luminosity of 10 fb^{-1} . Only events satisfying $Q^2 > 1 \text{ GeV}^2$, a squared invariant photon-nucleon mass of $W^2 > 10 \text{ GeV}^2$, and an inelasticity cut of $0.01 < y < 0.95$ are considered. For further details about EIC SIDIS data, we refer the reader to Ref. [17].

The two energy configurations complement each other in kinematic coverage: the $\sqrt{s} = 140 \text{ GeV}$ data extend the reach to lower x_B values, probing the sea-quark region, whereas the $\sqrt{s} = 45 \text{ GeV}$ data provide complementary

constraints at higher x_B , aiding in the extraction of valence-quark distributions. The inclusion of these pseudodata in global fits is expected to significantly reduce uncertainties, particularly for the gluon FF at medium to large z and for the charge-separated light-quark FFs. These improvements highlight the EIC transformative role in refining our understanding of fragmentation processes and hadronization mechanisms in QCD.

We emphasize that the pseudo-data included in our analysis are not generated using general-purpose event generators such as PYTHIA6, which are based on leading-order (LO) matrix elements, parton showers, and phenomenological hadronization models. Instead, we adopt a fully perturbative approach based on QCD collinear factorization at NLO. Specifically, the central values of the SIDIS cross sections are computed using the MontBlanc program, which performs NLO calculations via fast convolutions of stored grids of hard coefficient functions with input fragmentation functions MAPFF [1]. This ensures consistency with the theoretical framework used in our global QCD analysis, which employs two-loop time-like DGLAP splitting kernels for FF evolution.

The pseudodata generation follows a controlled procedure: each bin's central value σ_i^{exp} is derived from the theoretical prediction σ_i^{theory} as

$$\sigma_i^{\text{exp}} = \sigma_i^{\text{theory}}(1 + r_i \delta_i^{\text{uncorr}} + s \delta^{\text{corr}}), \quad (9)$$

where r_i and s are independent Gaussian random variables, δ_i^{uncorr} is the uncorrelated uncertainty in bin i , and δ^{corr} is the correlated systematic uncertainty, set to 3.5% across all bins. The uncorrelated uncertainty is defined as

$$\delta_i^{\text{uncorr}} = \sqrt{(\delta_i^{\text{stat}})^2 + (\delta_i^{\text{sys}})^2}. \quad (10)$$

Subsequently, the central values are subjected to random fluctuations according to the above formula. By construction, this ensures that the calculated χ^2/N_{data} for the pseudodata is expected to be close to unity, thereby validating the internal consistency between the generated data and the theoretical model.

With this procedure, the pseudodata reflect the expected statistical and systematic precision of future lepton colliders, while being fully compatible with the NLO theoretical framework used in our analysis. Since the pseudodata are not generated using LO parton shower models or hadronization schemes, no theoretical mismatch is introduced, and their inclusion does not create tension with existing SIDIS or SIA measurements. This justifies their role in evaluating the constraining power of future EIC measurements on fragmentation functions.

To further clarify the relative precision of the simulated EIC pseudodata compared to existing measurements, in Figs. 1 and 2, we present the relative total uncertainties

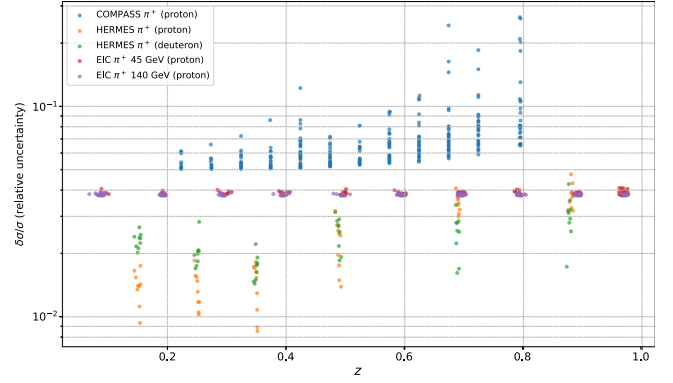


FIG. 1. Relative total uncertainties $\delta\sigma/\sigma$ versus z for the π^+ SIDIS datasets: HERMES, COMPASS, and EIC ($\sqrt{s} = 45$ and 140 GeV).

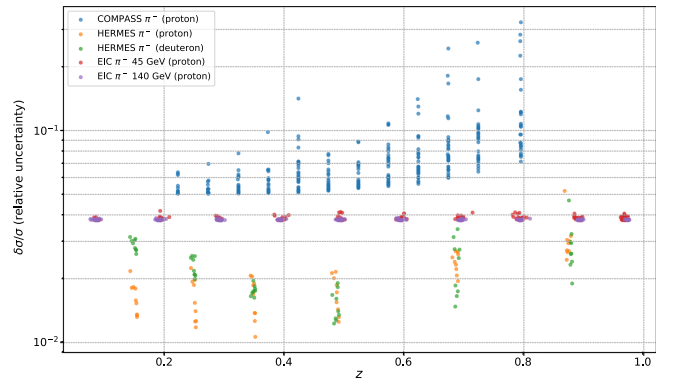


FIG. 2. Same as Fig. 1 but for the π^- SIDIS datasets.

$\delta\sigma/\sigma$ as a function of the fragmentation variable z for the π^+ and π^- multiplicity datasets, respectively. These figures include the SIDIS data from HERMES (proton and deuteron targets), COMPASS, and the projected EIC pseudodata at $\sqrt{s} = 45$ GeV and 140 GeV. It is clearly observed that the EIC pseudodata exhibit significantly smaller uncertainties than COMPASS data at all ranges of z for both π^+ and π^- . However, HERMES measurements typically have smaller uncertainties across a broad z range in comparison to EIC pseudodata, particularly at low values of z . This highlights the enhanced constraining power of the EIC data and supports the conclusion that their inclusion can lead to substantially improved determinations of fragmentation functions in global QCD analyses.

C. Data selection and kinematical cuts

In this analysis, we incorporate all available experimental datasets on pion production from SIA and SIDIS experiments, along with the simulated SIDIS pseudodata from the EIC, to extract pion FFs. Below, we discuss the kinematic coverage of these datasets and detail the specific cuts on the fragmentation variable z and the energy scale Q applied throughout this study. Figure 3 shows the coverage of the SIDIS, SIA, and EIC datasets in the (z, Q) plane.

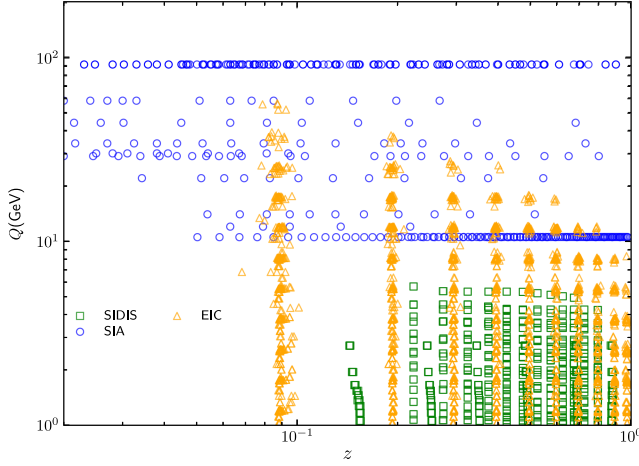


FIG. 3. Kinematic coverage in the (z, Q) plane for the SIDIS, SIA, and EIC simulated SIDIS pseudodata included in this analysis.

The SIA data predominantly cover higher Q regions with an extended range in z . In contrast, the SIDIS data span a wide range of z , covering low-to-intermediate values and partially extending into the high- z region. The moderate range of Q covered by SIDIS provides valuable flavor-sensitive information on fragmentation across different energy scales. The projected EIC measurements substantially enhance the existing kinematic coverage by extending into higher Q regions while maintaining a comprehensive span in z . This extended coverage is especially beneficial for the high- z region, where current FF extractions suffer from larger uncertainties. By bridging SIDIS and SIA data, the EIC pseudodata significantly improve constraints on both quark and gluon FFs, enhance flavor separation, and refine our understanding of hadronization mechanisms across diverse kinematic domains.

To ensure the validity of perturbative QCD and the applicability of QCD evolution, we impose specific kinematic cuts on the data. Particularly for the SIA data, we exclude very low values of z , as this region is dominated by nonperturbative effects. Following established analyses [1,23], we restrict the SIA data to the kinematic range $0.02 \leq z \leq 0.9$ for center-of-mass energies near the Z -boson mass (M_Z) and to $0.075 \leq z \leq 0.9$ for other center-of-mass energies. For the SIDIS data from HERMES and COMPASS, we retain data points within $0.2 \leq z \leq 0.8$. The broader and more precise kinematic reach of the EIC pseudodata allows us to extend the z coverage to $0.1 \leq z \leq 0.98$, enabling a more comprehensive extraction of FFs across an extended phase space.

At low energy scales (Q), higher-order perturbative corrections become increasingly significant, making it necessary to account for them to maintain reliable theoretical predictions. However, at NLO accuracy, perturbative corrections become less reliable at very low Q . Therefore, we exclude data at very small Q values from the SIDIS

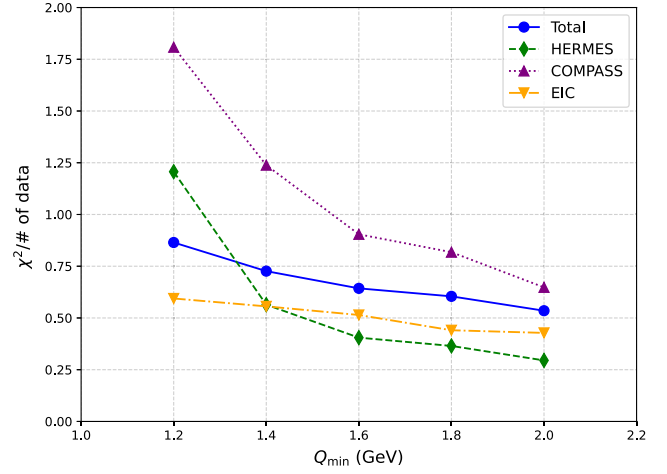


FIG. 4. Variation of χ^2 per data point as a function of the applied $Q_{\min}$ cut in the SIDIS datasets. The curves show χ^2/N_{data} for the total dataset (blue), HERMES (green), COMPASS (purple), and EIC pseudodata (yellow) at different $Q_{\min}$ thresholds.

datasets to ensure the robustness and reliability of our extractions.

To determine an appropriate lower bound on Q , we studied how the fit quality depends on the choice of $Q_{\min}$. Figure 4 shows the variation of χ^2/N_{data} as a function of the minimum energy scale $Q_{\min}$ (in GeV) for different SIDIS datasets used in the extraction of pion FFs at NLO accuracy. The trend of the total χ^2/N_{data} reflects the interplay among datasets and the constraints they impose for different choices of $Q_{\min}$.

The SIDIS datasets from HERMES and COMPASS show increasing values of χ^2/N_{data} at lower $Q_{\min}$, indicating rising theoretical tensions and uncertainties at these lower energy scales. This suggests that SIDIS data at low Q introduce greater variability in the fit, potentially due to nonperturbative contributions or limitations inherent in the NLO pQCD framework. In contrast, the inclusion of EIC pseudodata has a stabilizing effect, particularly noticeable at small $Q_{\min}$, mitigating fluctuations and significantly improving overall fit consistency. This highlights the crucial role of the EIC pseudodata in providing additional constraints and yielding a more stable and reliable determination of FFs at lower energy scales.

Based on the observed behavior of χ^2/N_{data} , we choose $Q_{\min} = 1.4$ GeV as the optimal lower cutoff. This selection ensures an appropriate balance between retaining sufficient data at low energy scales and minimizing the tensions indicated by higher χ^2/N_{data} values observed at lower $Q_{\min}$. The stability of the fit quality around this chosen value indicates that it represents the optimal compromise between precision and consistency in the global fit. After applying these kinematic cuts, our final dataset includes a total of $N_{\text{data}} = 2434$ data points, comprising $N_{\text{data}} = 498$ from SIDIS, $N_{\text{data}} = 1559$ from EIC pseudodata, and $N_{\text{data}} = 377$ from SIA measurements.

In summary, our results confirm the complementary nature of the different datasets. The HERMES and COMPASS SIDIS data provide essential information at moderate energy scales, while EIC pseudodata significantly reduce uncertainties and enhance precision, particularly in lower energy regions. Our results show that the inclusion of EIC pseudodata leads to a more constrained, stable, and consistent extraction of FFs, emphasizing its importance for future high-precision QCD analyses.

IV. RESULTS

In this section, we present the main results and key findings of our global QCD analysis. We conducted two distinct QCD fits: the first fit, labeled “Global Fit,” incorporates the SIA data along with SIDIS measurements from the HERMES and COMPASS experiments, while the second fit, labeled “Global Fit + EIC,” additionally includes simulated SIDIS EIC data. We begin by evaluating the quality of these fits by examining the χ^2/N_{data} values obtained for each experimental dataset considered. Next, we present the extracted pion FFs resulting from the combined analysis of SIA, SIDIS, and EIC datasets. We then assess the specific impact of incorporating projected EIC measurements on the precision of the FFs for different parton species. Finally, we compare our extracted FFs to other existing sets available in the literature.

A. Assessment of fit quality and dataset consistency

We discuss the quality of these fits by examining the χ^2/N_{data} value obtained for each experimental dataset for both QCD fits. Figure 5 shows the values of χ^2/N_{data} for the various experimental datasets included in our analysis, categorized into SIDIS, SIA, and EIC pseudodata. The SIDIS measurements are from the HERMES and COMPASS experiments, while the SIA datasets include results from multiple electron-positron annihilation experiments, specifically BELLE, BABAR, TASSO, TPC, TOPAZ, ALEPH, DELPHI, OPAL, and SLD. As previously discussed, the EIC pseudodata incorporated into this analysis correspond to projected measurements at center-of-mass energies of 45 GeV and 140 GeV.

The distribution of χ^2/N_{data} values provides insights into the consistency of each dataset within the global fit. For the SIDIS datasets (HERMES and COMPASS), the observed moderate χ^2 values reflect the complexity associated with modeling hadronic final states in SIDIS processes. Among the SIA datasets, lower χ^2 values for experiments such as BELLE, TOPAZ, and TPC highlight their robust and stable constraints derived from clean e^+e^- annihilation data. In contrast, certain datasets, notably the DELPHI uds measurements, exhibit higher χ^2/N_{data} values, likely due to greater systematic uncertainties or mild internal tensions. This observation is consistent with previous results from the MAPFF analysis [1].

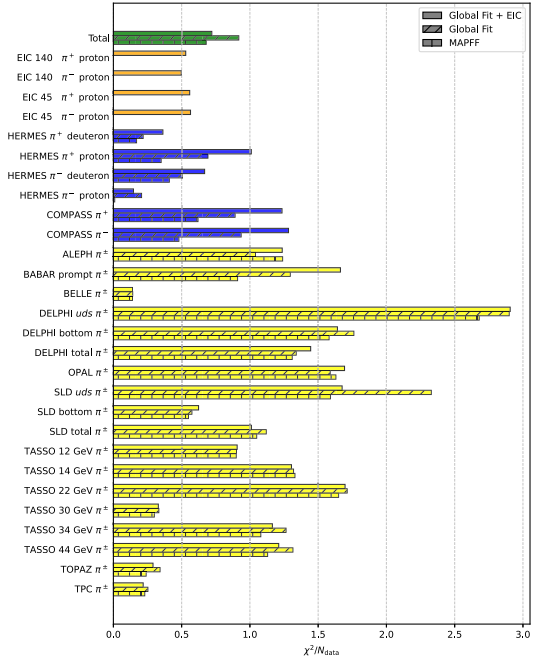


FIG. 5. Comparison of the χ^2/N_{data} values for three fits: Global Fit, Global Fit + EIC, and MAPFF. The yellow bars represent the SIA datasets, the blue bars denote the SIDIS datasets, and the orange bars show the EIC datasets. The topmost green bar summarizes the total χ^2/N_{data} across all datasets combined.

The EIC data particularly improve precision in kinematic regions where current SIDIS measurements provide limited accuracy. The total χ^2/N_{data} , represented by the green bars in Fig. 5, summarizes the overall quality of the global fit across all datasets. The χ^2/N_{data} for the Global Fit is calculated as 0.919, and for the Global Fit + EIC, it is 0.721. These values suggest that our global analysis successfully balances the constraints from diverse experimental sources, effectively accommodating systematic uncertainties and variations inherent in the individual datasets.

In Fig. 5, the χ^2/N_{data} values are also compared with the MAPFF results [1] for all individual data sets. Both our “Global Fit” analysis and the MAPFF analysis utilize the same datasets, with minor differences in the kinematic cuts applied to the HERMES and COMPASS SIDIS data. Specifically, MAPFF applies a virtuality cut of $Q_{\text{cut}} = 2$ GeV for the HERMES and COMPASS SIDIS datasets, whereas our analysis uses a looser cut of $Q_{\text{cut}} = 1.4$ GeV for all SIDIS data. In addition, MAPFF employs the NNPDF3.1 [46] PDF sets for computing SIDIS cross sections, while we use the more recent NNPDF4.0 [47] sets. Although both analyses are based on the MontBlanc framework, some differences in the results can be observed. These differences primarily arise from the variation in the number of data points included from each dataset, as well as from the choice of PDF sets used in the two analyses.

B. Impact of EIC pseudodata on the precision of Pion FFs

The anticipated SIDIS measurements from the EIC are expected to significantly affect the global extraction of FFs. These measurements are crucial for global QCD analyses, as SIDIS data primarily facilitate the separation of quark and antiquark FFs, substantially enhancing flavor differentiation. To quantitatively assess the impact of future EIC SIDIS measurements, we performed two sets of QCD fits with and without EIC SIDIS pseudodata.

Figure 6 presents a comparison of the extracted pion FFs at NLO accuracy for various parton species at the initial scale $Q = 5$ GeV from both the Global Fit and the Global Fit + EIC. The upper panel of each plot shows the absolute distributions, while the lower panel displays the ratios relative to the central values of the corresponding fits. The results show FFs for the up quark ($D_{\pi^+}^u$), antidown quark ($D_{\pi^+}^{\bar{d}}$), antiup quark ($D_{\pi^+}^{\bar{u}}$), and strange quark ($D_{\pi^+}^s$). The figure also displays the gluon ($D_{\pi^+}^g$) and heavy-quark FFs.

From Fig. 6, several key observations can be made. We focus only on the reduction of uncertainties in the extracted FFs when EIC pseudo-data are included in the fit. The shaded bands show that these uncertainties become much smaller with the addition of the EIC data, as expected from their projected precision. This reduction is particularly evident for the up quark ($D_{\pi^+}^u$), antiup quark ($D_{\pi^+}^{\bar{u}}$) and antidown quark ($D_{\pi^+}^{\bar{d}}$) FFs, emphasizing that future EIC measurements substantially enhance the precision of favored and unfavored quark FFs, particularly in the currently poorly constrained high- z region.

The gluon FF ($D_{\pi^+}^g$) also shows improvements in its uncertainty band, although this reduction is more modest compared to the quark FFs, reflecting ongoing challenges in accurately constraining gluon fragmentation. Overall, these comparisons illustrate that the inclusion of EIC pseudodata notably enhances the precision of FFs. This improvement underscores the critical role future EIC measurements will play in refining global QCD analyses of FFs.

C. Comparison with other FF sets

Now, we compare our extracted pion FFs with two other existing sets available in the literature. Our global QCD analyses, with and without incorporating EIC pseudodata (labeled “Global Fit + EIC” and “Global Fit”), are compared with the MAPFF extraction [1] and results from the NNFF1.0 collaboration [27]. Figure 7 presents a detailed comparison among these four sets of pion FFs at NLO accuracy and at the initial scale $Q = 5$ GeV. The upper panel of each plot shows the absolute distributions of the set, while the lower panel displays the ratios relative to the central values of the corresponding sets.

Our Global Fit + EIC and Global Fit analyses in comparison with the MAPFF extraction, utilize similar

datasets, with the primary differences being the inclusion of EIC pseudodata in our Global Fit + EIC analysis and slight variations in the kinematic cuts applied to the HERMES and COMPASS SIDIS datasets. The kinematic cut on the virtuality Q in MAPFF is $Q_{\text{cut}} = 2$ GeV to the HERMES and COMPASS SIDIS datasets, while it is $Q_{\text{cut}} = 1.4$ GeV in our analyses for all SIDIS datasets. Moreover, MAPFF has used the NNPDF3.1 [46] PDF sets as input to the computation of SIDIS cross sections. However, we use the NNPDF4.0 [47] PDF sets. In contrast, the NNFF1.0 QCD fit exclusively uses SIA datasets. The upper plots in Fig. 7 show the FFs for the up-type quark combination ($D_{\pi^+}^{u+}$) and down-type quark combination ($D_{\pi^+}^{d+}$), while the lower plots display the FFs for the strange quark combination ($D_{\pi^+}^{s+}$) and gluon ($D_{\pi^+}^g$).

Comparison of our Global Fit results with the MAPFF analysis shows similar behavior in both the central values and the error bands for all types of light partons. However, for the gluon fragmentation function, significant differences are observed between our baseline analysis (Global Fit) and the MAPFF results across the entire range of z , particularly at low z , which can be related to difference of kinematic cuts for Q of SIDIS datasets in these two analyses. Comparing our Global Fit + EIC results with those of the MAPFF analysis, we observe largely consistent central values across all parton species, owing to similarities in flavor decomposition methods. The uncertainty bands, indicated by shaded regions, differ noticeably between these two extractions. For the quark FFs $D_{\pi^+}^{u+}$ and $D_{\pi^+}^{d+}$, already constrained well by existing data, uncertainties are relatively small and similar between both analyses. Nevertheless, incorporating EIC pseudo-data leads to additional uncertainty reduction at high- z , highlighting improved constraints in this kinematic region. More substantial improvements are observed in the gluon FF ($D_{\pi^+}^g$), especially at intermediate and high z . The inclusion of EIC pseudodata significantly narrows the gluon uncertainty bands, underscoring the critical impact of future EIC measurements in precisely constraining gluon fragmentation.

Comparisons between our Global Fit + EIC results and the NNFF1.0 extraction reveal substantial differences in both central values and uncertainties, especially at medium and lower z . These discrepancies primarily result from NNFF1.0’s exclusive reliance on SIA datasets, limiting its ability to constrain FFs effectively. Consequently, NNFF1.0 uncertainties are considerably broader compared to our analysis, which benefits from additional constraints provided by SIDIS and projected EIC measurements.

Overall, this comparative analysis emphasizes the significant impact anticipated from incorporating EIC pseudodata in global FFs extractions. It particularly highlights expected enhancements for gluon, u-quark, and anti-u-quark FFs compared to other FFs sets.

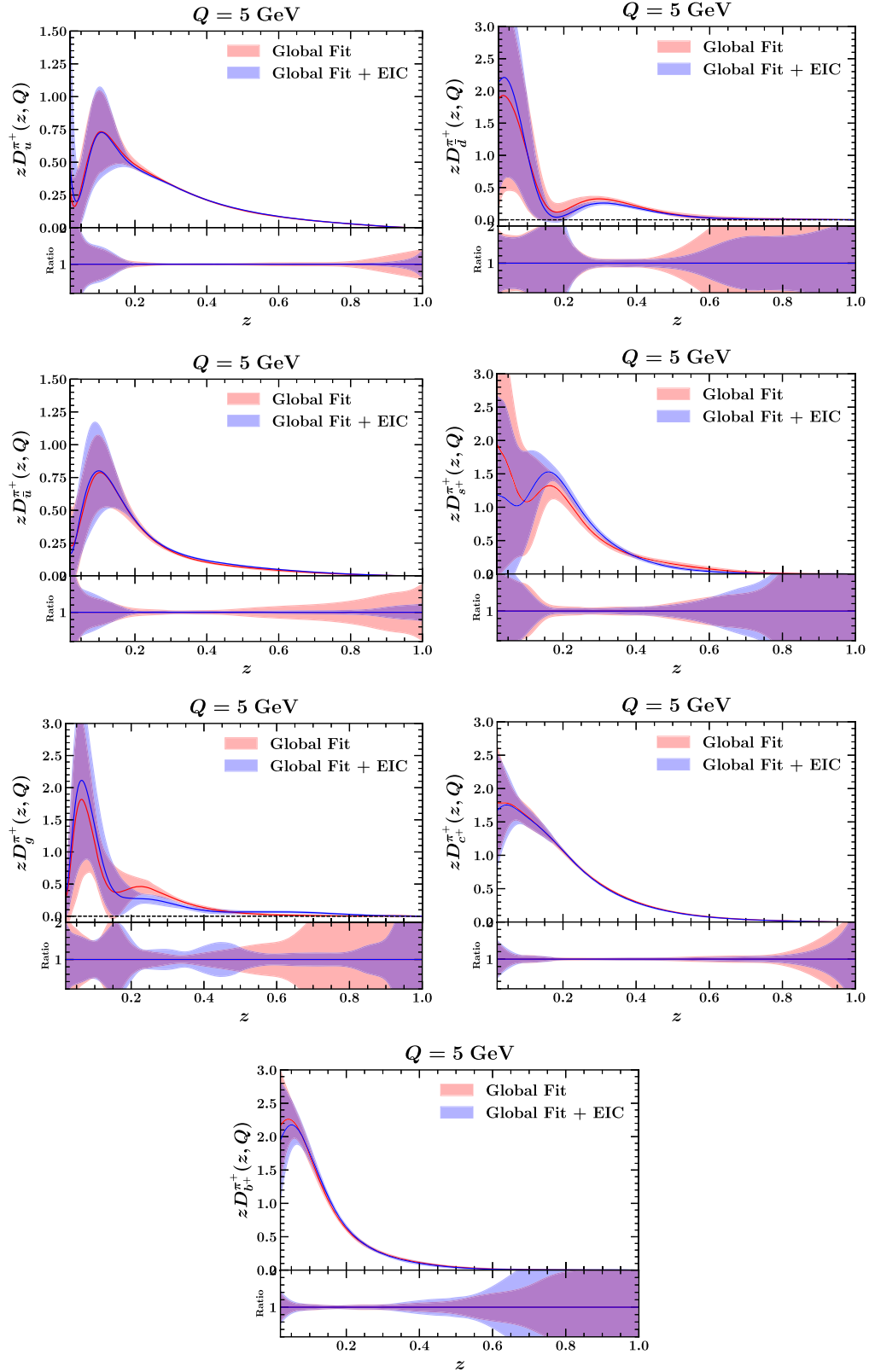


FIG. 6. Comparison of the π^+ FFs at NLO accuracy for various parton species at the initial scale $Q = 5$ GeV, extracted from a global fit without EIC data (red bands) and with EIC pseudodata included (blue bands). The results show FFs in the upper panels for the up quark ($D_{\pi^+}^u$), the antidown quark ($D_{\pi^+}^{\bar{d}}$), the antiup quark ($D_{\pi^+}^{\bar{u}}$), the strange quark ($D_{\pi^+}^s$), the gluon ($D_{\pi^+}^g$), and the heavy-quark FFs. Lower panels represent the ratios of all the sets to the central values of the corresponding sets.

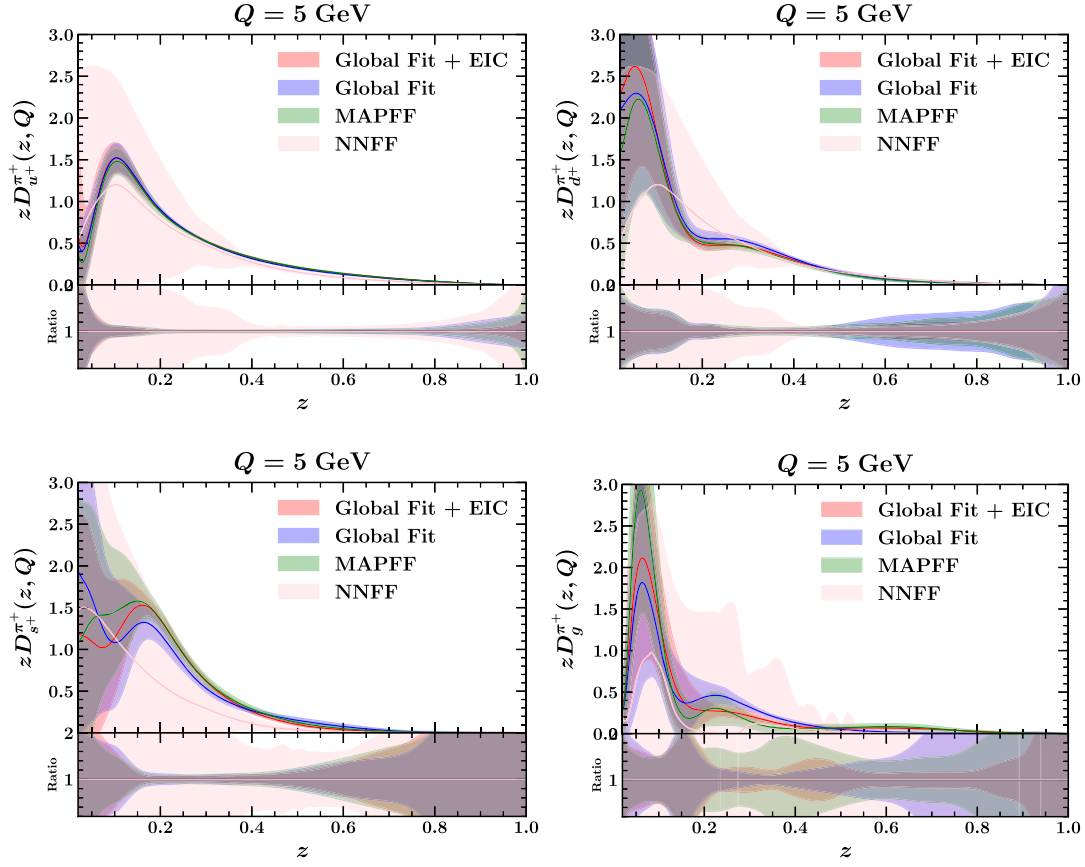


FIG. 7. Comparison of pion (π^+) FFs at NLO accuracy for various parton species at the initial scale $Q = 5$ GeV, obtained from our global fit with and without, including EIC pseudodata (red and blue bands respectively), the MAPFF extraction [1] (green bands), and the NNFF1.0 analysis [27] (pink bands). The upper plots show FFs for the up-type quark combination ($D_{\pi^+}^{u^+}$) and down-type quark combination ($D_{\pi^+}^{d^+}$), while the lower plots display the FFs for the strange quark combination ($D_{\pi^+}^{s^+}$) and gluon ($D_{\pi^+}^g$). Lower panels represent the ratios of all the sets to the central values of the corresponding sets. The shaded bands represent the 1σ uncertainty intervals, computed using the Monte Carlo sampling method.

V. SUMMARY AND OUTLOOK

Semi-inclusive deep-inelastic scattering offers a powerful and versatile means of probing both the flavor structure of the nucleon and the mechanisms through which partons hadronize into observed final-state hadrons. Utilizing the framework of QCD factorization, the SIDIS cross-sections can be accurately described in terms of nonperturbative fragmentation functions, thereby providing stringent constraints on their extraction from experimental measurements.

In this study, we performed a detailed analysis to quantify the anticipated impact of future SIDIS measurements from the electron-ion collider on the determination of pion FFs. By conducting a global QCD fit, including existing SIA and SIDIS data, complemented by simulated pseudo-data expected from the EIC SIDIS, we have explicitly demonstrated how forthcoming EIC measurements will enhance the precision of FF extractions.

Our analysis reveals that the inclusion of EIC pseudodata leads to substantial improvements in the precision of FFs, especially in the poorly constrained regions of large

momentum fractions (z). We found that the uncertainty bands, particularly for the gluon and up quark FFs, show significant reductions upon inclusion of EIC pseudodata. This highlights the crucial role of future EIC SIDIS measurements in constraining quark and gluon FFs, addressing a long-standing challenge in fragmentation phenomenology.

Comparisons with the SIA and SIDIS-based extraction indicate that the EIC pseudodata significantly tighten constraints on gluon FFs, underscoring the potential of the EIC to advance the current state-of-the-art in FF precision. This comparative analysis clearly emphasizes that SIDIS measurements at the EIC will complement existing data by probing a wider kinematic range, thereby improving our knowledge of hadronization mechanisms.

In the near future, we plan to extend this study to kaon FFs. Given the increased sensitivity of kaon production to strange quark distributions, this analysis will enhance our understanding of flavor-dependent fragmentation processes.

Additionally, we will explore the impact of new data from the BESIII experiment on the production of charged pions ($\pi^\pm$) in electron-positron annihilation [28]. We will also examine the recently reported multiplicities for charged pion production in SIDIS on proton and deuteron targets from the JLab collaboration [29].

ACKNOWLEDGMENTS

The authors gratefully acknowledge the helpful discussions provided by Charlotte Van Hulse. We also thank the

School of Particles and Accelerators at the Institute for Research in Fundamental Sciences (IPM) for their financial support of this project. Hamzeh Khanpour appreciates the financial support from NAWA under Grant No. BPN/ULM/2023/1/00160, as well as from the IDUB program at AGH University.

DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

-
- [1] R. Abdul Khalek, V. Bertone, A. Khoukli, and E. R. Nocera (MAP (Multi-dimensional Analyses of Partonic distributions) Collaboration), Pion and kaon fragmentation functions at next-to-next-to-leading order, *Phys. Lett. B* **834**, 137456 (2022).
 - [2] J. Gao, C. Liu, X. Shen, H. Xing, and Y. Zhao, Global analysis of fragmentation functions to charged hadrons with high-precision data from the LHC, *Phys. Rev. D* **110**, 114019 (2024).
 - [3] M. Soleymaninia, H. Hashamipour, M. Salajegheh, H. Khanpour, H. Spiesberger, and U. G. Meißner, Determination of KS0 fragmentation functions including BESIII measurements and using neural networks, *Phys. Rev. D* **110**, 014019 (2024).
 - [4] R. Abdul Khalek, J. J. Ethier, and J. Rojo (NNPDF Collaboration), Nuclear parton distributions from lepton-nucleus scattering and the impact of an electron-ion collider, *Eur. Phys. J. C* **79**, 471 (2019).
 - [5] M. Klasen and K. Kovarik, Nuclear parton density functions from dijet photoproduction at the EIC, *Phys. Rev. D* **97**, 114013 (2018).
 - [6] M. Klasen, K. Kovarik, and J. Potthoff, Nuclear parton density functions from jet production in DIS at an EIC, *Phys. Rev. D* **95**, 094013 (2017).
 - [7] R. Abdul Khalek, U. D'Alesio, M. Arratia, A. Bacchetta, M. Battaglieri, M. Began, M. Boglione, R. Boughezal, R. Boussarie, G. Bozzi *et al.*, Snowmass 2021 White Paper: Electron ion collider for high energy physics, [arXiv:2203.13199](https://arxiv.org/abs/2203.13199).
 - [8] R. Abdul Khalek, A. Accardi, J. Adam, D. Adamiak, W. Akers, M. Albaladejo, A. Al-bataineh, M. G. Alexeev, F. Ameli, P. Antonioli *et al.*, Science requirements and detector concepts for the electron-ion collider: EIC Yellow Report, *Nucl. Phys. A* **1026**, 122447 (2022).
 - [9] M. Radici *et al.* (Electron-Ion Collider User Group Collaboration), The Electron-Ion Collider—A U.S. facility for the European community to explore the mysteries of the building blocks of matter, [arXiv:2211.02785](https://arxiv.org/abs/2211.02785).
 - [10] Y. Zhou, C. Cocuzza, F. Delcarro, W. Melnitchouk, A. Metz, and N. Sato (Jefferson Lab Angular Momentum (JAM) Collaboration), Revisiting quark and gluon polarization in the proton at the EIC, *Phys. Rev. D* **104**, 034028 (2021).
 - [11] R. A. Khalek, J. J. Ethier, E. R. Nocera, and J. Rojo, Self-consistent determination of proton and nuclear PDFs at the Electron Ion Collider, *Phys. Rev. D* **103**, 096005 (2021).
 - [12] N. Armesto, T. Cridge, F. Giuli, L. Harland-Lang, P. Newman, B. Schmookler, R. Thorne, and K. Wichmann, Impact of inclusive electron ion collider data on collinear parton distributions, *Phys. Rev. D* **109**, 054019 (2024).
 - [13] J. Jiménez-López, P. R. Newman, and K. Wichmann, Prospects for measurements of the longitudinal proton structure function FL at the Electron Ion Collider, *Phys. Rev. D* **111**, 056014 (2025).
 - [14] T. Kutz, J. R. Pybus, D. W. Upton, C. Cotton, A. Deshpande, A. Deur, W. B. Li, D. Nguyen, M. Nycz, and X. Zheng, High precision measurements of α_s at the future EIC, *Phys. Rev. D* **110**, 074004 (2024).
 - [15] M. Azizi, M. Soleymaninia, H. Hashamipour, M. Salajegheh, H. Khanpour, and U. G. Meißner, Revisiting constraints on proton PDFs from HERA DIS, Drell-Yan, W and Z boson production, and projected EIC measurements, *Phys. Rev. D* **111**, 034023 (2025).
 - [16] S. Cerci, Z. S. Demiroglu, A. Deshpande, P. R. Newman, B. Schmookler, D. Sunar Cerci, and K. Wichmann, Extraction of the strong coupling with HERA and EIC inclusive data, *Eur. Phys. J. C* **83**, 1011 (2023).
 - [17] E. C. Aschenauer, I. Borsa, R. Sassot, and C. Van Hulse, Semi-inclusive deep-inelastic scattering, parton distributions and fragmentation functions at a future electron-ion collider, *Phys. Rev. D* **99**, 094004 (2019).
 - [18] E. Moffat, W. Melnitchouk, T. C. Rogers, and N. Sato (Jefferson Lab Angular Momentum (JAM) Collaboration), Simultaneous Monte Carlo analysis of parton densities and fragmentation functions, *Phys. Rev. D* **104**, 016015 (2021).
 - [19] R. D. Ball *et al.* (NNPDF Collaboration), The path to proton structure at 1% accuracy, *Eur. Phys. J. C* **82**, 428 (2022).
 - [20] R. A. Khalek, V. Bertone, and E. R. Nocera (MAP (Multi-dimensional Analyses of Partonic distributions) Collaboration), Determination of unpolarized pion fragmentation functions using semi-inclusive deep-inelastic-scattering data, *Phys. Rev. D* **104**, 034007 (2021).
 - [21] <https://github.com/MapCollaboration/MontBlanc>

- [22] R. D. Ball *et al.* (NNPDF Collaboration), The path to N^3LO parton distributions, *Eur. Phys. J. C* **84**, 659 (2024).
- [23] M. Soleymaninia, H. Hashamipour, and H. Khanpour, Neural network QCD analysis of charged hadron fragmentation functions in the presence of SIDIS data, *Phys. Rev. D* **105**, 114018 (2022).
- [24] S. Forte, L. Garrido, J. I. Latorre, and A. Piccione, Neural network parametrization of deep inelastic structure functions, *J. High Energy Phys.* **05** (2002) 062.
- [25] M. Soleymaninia, H. Hashamipour, H. Khanpour, S. Shoeibi, and A. Mohamaditabar, Study of nuclear corrections on the charged hadron fragmentation functions in a neural network global QCD analysis, *Eur. Phys. J. Plus* **139**, 794 (2024).
- [26] M. Cacciari, P. Nason, and C. Oleari, Crossing heavy-flavor thresholds in fragmentation functions, *J. High Energy Phys.* **10** (2005) 034.
- [27] V. Bertone, S. Carrazza, N. P. Hartland, E. R. Nocera, and J. Rojo (NNPDF Collaboration), A determination of the fragmentation functions of pions, kaons, and protons with faithful uncertainties, *Eur. Phys. J. C* **77**, 516 (2017).
- [28] M. Ablikim *et al.* (BESIII Collaboration), Single inclusive $\pi^\pm$ and $K^\pm$ production in e^+e^- annihilation at center-of-mass energies from 2.000 to 3.671 GeV, *arXiv:2502.16084*.
- [29] H. Bhatt, P. Bosted, S. Jia, W. Armstrong, D. Dutta, R. Ent, D. Gaskell, E. Kinney, H. Mkrtchyan, S. Ali *et al.*, Flavor dependence of charged pion fragmentation functions, *Phys. Lett. B* **865**, 139485 (2025).
- [30] Y. L. Dokshitzer, Calculation of the structure functions for deep inelastic scattering and e^+e^- annihilation by perturbation theory in quantum chromodynamics., *Sov. Phys. JETP* **46**, 641 (1977).
- [31] V. N. Gribov and L. N. Lipatov, Deep inelastic e p scattering in perturbation theory, *Sov. J. Nucl. Phys.* **15**, 438 (1972).
- [32] G. Altarelli and G. Parisi, Asymptotic freedom in parton language, *Nucl. Phys.* **B126**, 298 (1977).
- [33] D. Buskulic *et al.* (ALEPH Collaboration), Inclusive $\pi^\pm$, $K^\pm$ and $(p\bar{p})$ differential cross-sections at the Z resonance, *Z. Phys. C* **66**, 355 (1995).
- [34] P. Abreu *et al.* (DELPHI Collaboration), $\pi^\pm$, $K^\pm$, p and $\bar{p}$ production in $Z^0 \rightarrow q\bar{q}$, $Z^0 \rightarrow b\bar{b}$, $Z^0 \rightarrow u\bar{u}$, $d\bar{d}$, $s\bar{s}$, *Eur. Phys. J. C* **5**, 585 (1998).
- [35] R. Akers *et al.* (OPAL Collaboration), Measurement of the production rates of charged hadrons in e^+e^- annihilation at the Z^0 , *Z. Phys. C* **63**, 181 (1994).
- [36] R. Brandelik *et al.* (TASSO Collaboration), Charged pion, kaon, proton and anti-proton production in high-energy e^+e^- annihilation, *Phys. Lett.* **94B**, 444 (1980).
- [37] M. Althoff *et al.* (TASSO Collaboration), Charged hadron composition of the final state in e^+e^- annihilation at high-energies, *Z. Phys. C* **17**, 5 (1983).
- [38] W. Braunschweig *et al.* (TASSO Collaboration), Pion, kaon and proton cross-sections in e^+e^- annihilation at 34-GeV and 44-GeV center-of-mass energy, *Z. Phys. C* **42**, 189 (1989).
- [39] M. Leitgab *et al.* (Belle Collaboration), Precision measurement of charged pion and kaon differential cross sections in e^+e^- annihilation at $s=10.52$ GeV, *Phys. Rev. Lett.* **111**, 062002 (2013).
- [40] R. Itoh *et al.* (TOPAZ Collaboration), Measurement of inclusive particle spectra and test of MLLA prediction in e^+e^- annihilation at $s^{*}(1/2) = 58$ -GeV, *Phys. Lett. B* **345**, 335 (1995).
- [41] J. P. Lees *et al.* (BABAR Collaboration), Production of charged pions, kaons, and protons in e^+e^- annihilations into hadrons at $\sqrt{s} = 10.54$ GeV, *Phys. Rev. D* **88**, 032011 (2013).
- [42] H. Aihara *et al.* (TPC/Two Gamma Collaboration), Charged hadron inclusive cross-sections and fractions in e^+e^- annihilation $\sqrt{s} = 29$ GeV, *Phys. Rev. Lett.* **61**, 1263 (1988).
- [43] K. Abe *et al.* (SLD Collaboration), Production of π^+ , π^- , K^+ , K^- , p and $\bar{p}$ in light (uds), c and b jets from Z^0 decays, *Phys. Rev. D* **69**, 072003 (2004).
- [44] C. Adolph *et al.* (COMPASS Collaboration), Multiplicities of charged pions and charged hadrons from deep-inelastic scattering of muons off an isoscalar target, *Phys. Lett. B* **764**, 1 (2017).
- [45] A. Airapetian *et al.* (HERMES Collaboration), Multiplicities of charged pions and kaons from semi-inclusive deep-inelastic scattering by the proton and the deuteron, *Phys. Rev. D* **87**, 074029 (2013).
- [46] R. D. Ball *et al.* (NNPDF Collaboration), Parton distributions from high-precision collider data, *Eur. Phys. J. C* **77**, 663 (2017).
- [47] R. D. Ball *et al.* (NNPDF Collaboration), Determination of the theory uncertainties from missing higher orders on NNLO parton distributions with percent accuracy, *Eur. Phys. J. C* **84**, 517 (2024).