

Analysis of the semileptonic decays of Ξ_{cc} and Ω_{cc} baryons in QCD sum rules

Guo-Liang Yu,^{1,3,*} Zhi-Gang Wang,^{1,3,†} Jie Lu^{2,‡}, Bin Wu,² Peng Yang,¹ and Ze Zhou¹

¹*Department of Mathematics and Physics, North China Electric Power University, Baoding 071003, People's Republic of China*

²*School of Physics, Southeast University, Nanjing 210094, People's Republic of China*

³*Hebei Key Laboratory of Physics and Energy Technology, North China Electric Power University, Baoding 071000, China*



(Received 10 January 2026; accepted 24 March 2026; published 13 April 2026)

We firstly carry out a systematic analysis on the spin $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$ weak transition process in the framework of three-point QCD sum rules, where the initial and final states are doubly and singly charmed baryons. On the phenomenological side, all possible couplings of interpolating current to hadronic states are considered. In doing operator production expansion on the QCD side, the contributions of the perturbative part and vacuum condensate terms of $\langle \bar{q}q \rangle$, $\langle g_s^2 GG \rangle$, $\langle \bar{q}g_s \sigma G q \rangle$, and $g_s^2 \langle \bar{q}q \rangle^2$ are all considered. After the form factors in the spacelike region ($Q^2 > 0$) are obtained, the numerical results are extrapolated into the timelike region ($Q^2 < 0$) by a fitting function. Using the predicted form factors, we finally analyze the semileptonic decays of $\Xi_{cc}^{++} \rightarrow \Sigma_c^{*+} l^+ \nu_l$, $\Xi_{cc}^{++} \rightarrow \Xi_c^{*+} l^+ \nu_l$, $\Omega_{cc}^+ \rightarrow \Xi_c^{*0} l^+ \nu_l$, and $\Omega_{cc}^+ \rightarrow \Omega_c^{*0} l^+ \nu_l$ with $l = e, \mu$. The predictions in this work can deepen our understanding of the dynamics in the decay processes of doubly heavy baryons and provide useful information to explore the possibility of new physics in heavy baryonic decay channels.

DOI: [10.1103/ghm9-2lmd](https://doi.org/10.1103/ghm9-2lmd)

I. INTRODUCTION

In recent years, many single heavy baryons have been discovered by Belle, BABAR, CLEO, and LHCb collaborations [1]. However, searching for doubly heavy baryons in experiments ended with no progress for a long time. The breakthrough came in 2017 with the discovery of the doubly charmed baryon Ξ_{cc}^{++} by LHCb collaboration [2]. This state was first observed in the decay channel $\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+$ with a measured mass $3621.40 \pm 0.72 \pm 0.14 \pm 0.27$ MeV and later was confirmed in another decay mode $\Xi_{cc}^{++} \rightarrow \Lambda_c^+ \pi^+$ [3,4]. We believe that more and more doubly heavy baryons will be discovered with the efforts of experimental and theoretical physicists. In experiments, the firstly discovered new type of baryons are usually the ground states, which can only be reconstructed via weak decay final states. Thus, studying the decay properties of doubly heavy baryons, especially the weak decaying processes, is of great interest to

theoretical physicists, which can provide important information to experimentalists for searching for new doubly heavy baryons.

As for doubly charmed baryons, key transitions of interest include the semileptonic decays, such as $\Xi_{cc}^{++} \rightarrow \Xi_c^{*+} l^+ \nu_l$, where form factors parametrize the hadronic matrix elements of the weak current. Thus, a central aspect of understanding the structure and dynamics of the doubly charmed baryons lies in the calculation of their form factors. In addition, precise knowledge of these form factors is essential for extracting fundamental parameters, notably the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements like V_{cs} or V_{cd} that are not only important parameters to deepen our understanding of the dynamics of the weak transition, but also are crucial to search for new physics beyond the standard model.

Although QCD has achieved remarkable success in the high-energy regions, it has nonperturbative properties in low-energy regions, which makes it difficult to understand the interactions at the hadronic level, especially in systems involving heavy quarks. In calculating the form factors and studying weak decays of heavy baryons, physicists mainly employed some phenomenological methods, such as the relativistic quark model [5–11], nonrelativistic quark model [12,13], the light-front quark model [14–23], MIT bag model [24], and the flavor symmetry method [25], to carry out this research. In the quark model, the momentum distribution of the constituent quarks in a baryon is

* Contact author: yuguoliang2011@163.com

† Contact author: zgwang@aliyun.com

‡ Contact author: 117693567997@163.com

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

approximately modeled by a wave function or a vertex function [5–23]. In addition, most of these above research works concentrate on the spin $\frac{1}{2} \rightarrow \frac{1}{2}$ transition process, and few works focus on the $\frac{1}{2} \rightarrow \frac{3}{2}$ process with the initial and final states being a doubly heavy baryon and a singly heavy one, respectively. Thus, it is an interesting and important work to perform a systematic analysis about this weak decay process. In this field, QCD sum rules (QCDSRs) is a very effective nonperturbative method, and it has been widely used to investigate the properties of hadrons, including the mass spectra, pole residues, strong coupling constants [26–33], and the form factors [34–47].

In the framework of three-point QCDSRs, it will be seen in the following section (Sec. II) that there are many different structures both on the phenomenological and QCD sides. The form factors F_i and G_i ($i = 1 \sim 4$) for the transition process of $\frac{1}{2} \rightarrow \frac{3}{2}$ are not independent of each other and each of them has relations with several structures. That is to say, each form factor cannot be determined only according to one structure. In our previous work [48], we successfully analyzed the form factors of $\Lambda_b \rightarrow \Lambda_c$ and $\Xi_b \rightarrow \Xi_c$ with the spin-parity $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$ by solving 12 linear equations about the form factors, where all the Dirac structures are considered and the vacuum condensates up to dimension 8 are included on the QCD side. In another work [49], we systematically analyzed the mass spectra and root mean square radius of singly heavy baryons Λ_Q , Σ_Q , and Ω_Q using the relativized quark model, which provided reliable input parameters for studying the weak decay process. In the present work, we will analyze the form factors of transition processes $\Xi_{cc}^{++} \rightarrow \Sigma_c^{*+}$, $\Xi_{cc}^{++} \rightarrow \Xi_c'^{*+}$, $\Omega_{cc}^+ \rightarrow \Xi_c'^{*0}$, and $\Omega_{cc}^+ \rightarrow \Omega_c'^{*0}$. As an application of the form

factors, the corresponding semileptonic decay processes will also be analyzed.

This article is organized as follows. In Sec. II, we explain how the form factors of transition processes $\Xi_{cc}^{++} \rightarrow \Sigma_c^{*+}$, $\Xi_{cc}^{++} \rightarrow \Xi_c'^{*+}$, $\Omega_{cc}^+ \rightarrow \Xi_c'^{*0}$, and $\Omega_{cc}^+ \rightarrow \Omega_c'^{*0}$ are analyzed in the framework of QCDSRs. In Sec. III, we explain in detail how the parameters of QCDSRs in the present work are determined, and compare our numerical results with those of other collaborations. With these form factors, the semileptonic decays with spin-parity $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$ are analyzed in Sec. IV. Section V is reserved for the conclusion. Some complicated formulas are shown in Appendices A and B.

II. QCD SUM RULES FOR THE FORM FACTORS

A. Transition form factors of $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$

It is known that the transition form factors are key parameters in analyzing the semileptonic decay processes. The Feynman diagram of this process can be explicitly illustrated in Fig. 1, where it includes the transition $c \rightarrow q' \bar{l} \nu_l$ at the quark level with $q' = d, s$. In this transition, the leptonic amplitude can be calculated using electroweak perturbation theory, while the hadronic transition matrix element cannot be calculated perturbatively because perturbative QCD is not valid for low energies. Commonly, it can be parametrized into several transition form factors, as in Eq. (1). In this equation, $J_\nu^{V-A} = \bar{q}' \gamma_\nu (1 - \gamma_5) c$ is the electroweak transition current, $u(p, s)$ and $u_\alpha(p', s')$ denote the spinor wave functions of initial ($\mathcal{B}_1$) and final ($\mathcal{B}_2^*$) baryons, while $F_i(q^2)$ and $G_i(q^2)$ ($i = 1 \sim 4$) are the vector and axial vector form factors with $q = p - p'$,

$$\begin{aligned} \langle \mathcal{B}_2^*(p') | J_\nu^{V-A} | \mathcal{B}_1(p) \rangle = & \bar{u}_{\alpha}^{\mathcal{B}_2^*}(p', s') \gamma_5 \left[\gamma_\nu \frac{p_\alpha}{m_{\mathcal{B}_1}} F_1(q^2) + \frac{p_\alpha p_\nu}{m_{\mathcal{B}_1}^2} F_2(q^2) + \frac{p_\alpha p'_\nu}{m_{\mathcal{B}_1} m_{\mathcal{B}_2^*}} F_3(q^2) + g_{av} F_4(q^2) \right] u^{\mathcal{B}_1}(p, s) \\ & - \bar{u}_{\alpha}^{\mathcal{B}_2^*}(p', s') \left[\gamma_\nu \frac{p_\alpha}{m_{\mathcal{B}_1}} G_1(q^2) + \frac{p_\alpha p_\nu}{m_{\mathcal{B}_1}^2} G_2(q^2) + \frac{p_\alpha p'_\nu}{m_{\mathcal{B}_1} m_{\mathcal{B}_2^*}} G_3(q^2) + g_{av} G_4(q^2) \right] u^{\mathcal{B}_1}(p, s). \end{aligned} \quad (1)$$

B. QCD sum rules

To study the form factors of $\Xi_{cc} \rightarrow \Sigma_c^*$, $\Xi_{cc} \rightarrow \Xi_c'^*$, $\Omega_{cc} \rightarrow \Xi_c'^*$, and $\Omega_{cc} \rightarrow \Omega_c'^*$, we firstly write the following three-point correlation function:

$$\begin{aligned} \Pi_{\mu\nu}^{V/A}(p, p') = & i^2 \int d^4x d^4y e^{ip'x} e^{i(p-p')y} \\ & \times \langle 0 | T \{ J_\mu^{\mathcal{B}_2^*}(x) J_\nu^{V-A}(y) \bar{J}^{\mathcal{B}_1}(0) \} | 0 \rangle, \end{aligned} \quad (2)$$

where T denotes the time-ordered product, $J^{\mathcal{B}_1}$ and $J^{\mathcal{B}_2^*}$ are the interpolating currents of initial and final baryons, and

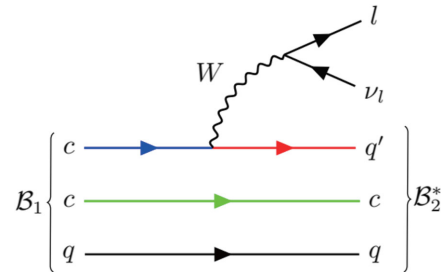


FIG. 1. The Feynman diagram of semileptonic decay processes of doubly charmed baryons.

$\bar{J} = J^\dagger \gamma_0$. These baryonic currents are taken in the following forms:

$$\begin{aligned} J^{\Xi_{cc}} &= \varepsilon_{nml} (c^{nT} C \gamma_\mu c^m) \gamma_\mu \gamma_5 q^l, \\ J^{\Omega_{cc}} &= \varepsilon_{nml} (c^{nT} C \gamma_\mu c^m) \gamma_\mu \gamma_5 s^l, \\ J_\mu^{\Sigma_c^*} &= \frac{1}{\sqrt{2}} \varepsilon_{nml} (u^{nT} C \gamma_\mu d^m + d^{nT} C \gamma_\mu u^m) c^l, \\ J_\mu^{\Xi_c^*} &= \frac{1}{\sqrt{2}} \varepsilon_{nml} (u^{nT} C \gamma_\mu s^m + s^{nT} C \gamma_\mu u^m) c^l, \\ J_\mu^{\Omega_c^*} &= \varepsilon_{nml} (s^{nT} C \gamma_\mu s^m) c^l, \end{aligned} \quad (3)$$

where ε_{nml} is the three-dimension Levi-Civita tensor, n, m , and l represent the color indices, and C is the charge conjugation matrix. In the framework of QCDSRs, the above three-point correlation function, Eq. (2), will be calculated at both hadron and quark levels, which are called the phenomenological side and the QCD side, respectively.

On the phenomenological side, a complete set of hadron states that can couple to the corresponding interpolating currents are inserted into the correlation function. Finishing the integral of coordinate space and using the double dispersion relation [26,27], we write the three-point correlation function as

$$\begin{aligned} \Pi_{\mu\nu}^{V/A}(p, p') &= \frac{\langle 0 | J_\mu^{\mathcal{B}_2^*} | \mathcal{B}_2^{*+}(p') \rangle \langle \mathcal{B}_2^{*+}(p') | J_\nu^{V-A} | \mathcal{B}_1^+(p) \rangle \langle \mathcal{B}_1^+(p) | \bar{J}^{\mathcal{B}_1} | 0 \rangle}{(p'^2 - m_{\mathcal{B}_2^{*+}}^2)(p^2 - m_{\mathcal{B}_1^+}^2)} \\ &+ \frac{\langle 0 | J_\mu^{\mathcal{B}_2^*} | \mathcal{B}_2^{*-}(p') \rangle \langle \mathcal{B}_2^{*-}(p') | J_\nu^{V-A} | \mathcal{B}_1^+(p) \rangle \langle \mathcal{B}_1^+(p) | \bar{J}^{\mathcal{B}_1} | 0 \rangle}{(p'^2 - m_{\mathcal{B}_2^{*-}}^2)(p^2 - m_{\mathcal{B}_1^+}^2)} \\ &+ \frac{\langle 0 | J_\mu^{\mathcal{B}_2^*} | \mathcal{B}_2^+(p') \rangle \langle \mathcal{B}_2^+(p') | J_\nu^{V-A} | \mathcal{B}_1^+(p) \rangle \langle \mathcal{B}_1^+(p) | \bar{J}^{\mathcal{B}_1} | 0 \rangle}{(p'^2 - m_{\mathcal{B}_2^+}^2)(p^2 - m_{\mathcal{B}_1^+}^2)} \\ &+ \frac{\langle 0 | J_\mu^{\mathcal{B}_2^*} | \mathcal{B}_2^-(p') \rangle \langle \mathcal{B}_2^-(p') | J_\nu^{V-A} | \mathcal{B}_1^+(p) \rangle \langle \mathcal{B}_1^+(p) | \bar{J}^{\mathcal{B}_1} | 0 \rangle}{(p'^2 - m_{\mathcal{B}_2^-}^2)(p^2 - m_{\mathcal{B}_1^+}^2)} \\ &+ \frac{\langle 0 | J_\mu^{\mathcal{B}_2^*} | \mathcal{B}_2^{*+}(p') \rangle \langle \mathcal{B}_2^{*+}(p') | J_\nu^{V-A} | \mathcal{B}_1^-(p) \rangle \langle \mathcal{B}_1^-(p) | \bar{J}^{\mathcal{B}_1} | 0 \rangle}{(p'^2 - m_{\mathcal{B}_2^{*+}}^2)(p^2 - m_{\mathcal{B}_1^-}^2)} \\ &+ \frac{\langle 0 | J_\mu^{\mathcal{B}_2^*} | \mathcal{B}_2^{*-}(p') \rangle \langle \mathcal{B}_2^{*-}(p') | J_\nu^{V-A} | \mathcal{B}_1^-(p) \rangle \langle \mathcal{B}_1^-(p) | \bar{J}^{\mathcal{B}_1} | 0 \rangle}{(p'^2 - m_{\mathcal{B}_2^{*-}}^2)(p^2 - m_{\mathcal{B}_1^-}^2)} \\ &+ \frac{\langle 0 | J_\mu^{\mathcal{B}_2^*} | \mathcal{B}_2^+(p') \rangle \langle \mathcal{B}_2^+(p') | J_\nu^{V-A} | \mathcal{B}_1^-(p) \rangle \langle \mathcal{B}_1^-(p) | \bar{J}^{\mathcal{B}_1} | 0 \rangle}{(p'^2 - m_{\mathcal{B}_2^+}^2)(p^2 - m_{\mathcal{B}_1^-}^2)} \\ &+ \frac{\langle 0 | J_\mu^{\mathcal{B}_2^*} | \mathcal{B}_2^-(p') \rangle \langle \mathcal{B}_2^-(p') | J_\nu^{V-A} | \mathcal{B}_1^-(p) \rangle \langle \mathcal{B}_1^-(p) | \bar{J}^{\mathcal{B}_1} | 0 \rangle}{(p'^2 - m_{\mathcal{B}_2^-}^2)(p^2 - m_{\mathcal{B}_1^-}^2)} + \dots, \end{aligned} \quad (4)$$

where the ellipses denote the contributions of excited and continuum states, while $\mathcal{B}_1^{+(-)}$ and $\mathcal{B}_2^{*+(-)}$ represent the initial and final positive (negative) parity baryons. The hadron vacuum matrix elements for initial and final baryons are defined as

$$\begin{aligned} \langle 0 | J_\mu^{\mathcal{B}_2^*}(0) | \mathcal{B}_2^{*+}(p', s') \rangle &= \lambda_{\mathcal{B}_2^{*+}} u_\mu(p', s'), \\ \langle 0 | J_\mu^{\mathcal{B}_2^*}(0) | \mathcal{B}_2^{*-}(p', s') \rangle &= \lambda_{\mathcal{B}_2^{*-}} i \gamma_5 u_\mu(p', s'), \\ \langle 0 | J_\mu^{\mathcal{B}_2^*}(0) | \mathcal{B}_2^+(p', s') \rangle &= \lambda_{\mathcal{B}_2^+} i \gamma_5 \left(\alpha \gamma_\mu - \frac{4\alpha}{m} p'_\mu \right) u(p', s'), \\ \langle 0 | J_\mu^{\mathcal{B}_2^*}(0) | \mathcal{B}_2^-(p', s') \rangle &= \lambda_{\mathcal{B}_2^-} \left(\alpha \gamma_\mu - \frac{4\alpha}{m} p'_\mu \right) u(p', s'), \\ \langle \mathcal{B}_1^+(p, s) | \bar{J}^{\mathcal{B}_1} | 0 \rangle &= \lambda_{\mathcal{B}_1^+} \bar{u}(p, s), \\ \langle \mathcal{B}_1^-(p, s) | \bar{J}^{\mathcal{B}_1} | 0 \rangle &= \lambda_{\mathcal{B}_1^-} \bar{u}(p, s) i \gamma_5. \end{aligned} \quad (5)$$

Here, $u(p, s)$ and $u_\mu(p', s')$ are the spinor wave functions of $J^P = \frac{1}{2}^\pm$ and $\frac{3}{2}^\pm$ baryons, which satisfy the Dirac and Rarita-Schwinger completeness relations,

$$\sum_s u(p, s) \bar{u}(p, s) = \not{p} + m_{B_1}, \quad (6)$$

$$\begin{aligned} & \sum_{s'} u_\mu^{B_2^*}(p', s') \bar{u}_\alpha^{B_2^*}(p', s') \\ &= -(\not{p}' + m_{B_2^*}) \left(g_{\mu\alpha} - \frac{\gamma_\mu \gamma_\alpha}{3} - \frac{2 p'_\mu p'_\alpha}{3 m_{B_2^*}^2} + \frac{1 p'_\mu \gamma_\alpha - p'_\alpha \gamma_\mu}{3 m_{B_2^*}} \right). \end{aligned} \quad (7)$$

As for the hadronic transition elements $\langle \mathcal{B}_2^{(*)\pm} | J_\nu^{V-A} | \mathcal{B}_1^\pm \rangle$, they can be parametrized as a series of form factors, which are illustrated in Eq. (A1) in Appendix A. By replacing the matrix elements in Eq. (4) with Eqs. (5) and (A1), the correlation function on the phenomenological side can be decomposed into different Dirac structures. From Eq. (4), we can see that the current J^{B_1} can couple to the baryons with $J^P = \frac{1}{2}^\pm$, and the current $J^{B_2^*}$ can couple to both baryons with $J^P = \frac{3}{2}^\pm$ and $J^P = \frac{1}{2}^\pm$. To extract the form factors of the decay process $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$, we must eliminate the interferences of the baryons with spin-parities $\frac{1}{2}^\pm$ and $\frac{3}{2}^-$ in the final states, and $\frac{1}{2}^-$ in the initial states. It is shown by Eq. (5) that the current $J^{B_2^*}$ coupling to baryons with $J^P = \frac{1}{2}^\pm$ is related to the structures of γ_μ and p'_μ . Hence, the structures proportional to γ_μ and p'_μ must be eliminated first. After these interferences are eliminated, the correlation function can be written as Eq. (A2) in Appendix A, and can be decomposed into the following different structures:

$$\Pi_{\mu\nu}^{\text{phy-V/A}} = \sum_1^{16} \Pi_i^{\text{phy-V/A}} e_{i\mu\nu}, \quad (8)$$

where

$$\begin{aligned} e_{i\mu\nu} &= \gamma_\nu \times \{I, \not{p}, \not{p}'\} \times \gamma_5 p_\mu \quad i = 1 \sim 4, \\ e_{i\mu\nu} &= \{I, \not{p}, \not{p}'\} \times \gamma_5 g_{\mu\nu} \quad i = 5 \sim 8, \\ e_{i\mu\nu} &= \{I, \not{p}, \not{p}'\} \times \gamma_5 p_\mu p_\nu \quad i = 9 \sim 12, \\ e_{i\mu\nu} &= \{I, \not{p}, \not{p}'\} \times \gamma_5 p_\mu p'_\nu \quad i = 13 \sim 16 \end{aligned} \quad (9)$$

are corresponding to vector part $\Pi_i^{\text{phy-V}}$, and

$$\begin{aligned} e_{i\mu\nu} &= \gamma_\nu \times \{I, \not{p}, \not{p}'\} \times p_\mu \quad i = 1 \sim 4, \\ e_{i\mu\nu} &= \{I, \not{p}, \not{p}'\} \times g_{\mu\nu} \quad i = 5 \sim 8, \\ e_{i\mu\nu} &= \{I, \not{p}, \not{p}'\} \times p_\mu p_\nu \quad i = 9 \sim 12, \\ e_{i\mu\nu} &= \{I, \not{p}, \not{p}'\} \times p_\mu p'_\nu \quad i = 13 \sim 16 \end{aligned} \quad (10)$$

are matched with the axial vector part $\Pi_i^{\text{phy-A}}$. In these above equations, a simplified expression $\{I, \not{p}, \not{p}'\}$ represents four different choices, I , $\not{p}$, $\not{p}'$, and $\not{p}\not{p}'$. The form factors are included in these expansion coefficients $\Pi_i^{\text{phy-V/A}}$, which are commonly named as scalar invariant amplitudes. In order to eliminate the contributions of baryons with negative parity, all these 32 structures will be used to determine the value of form factors for the process $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$.

On the side of QCD, we replace the interpolating currents in the three-point correlation function in Eq. (2) with their specific form in Eq. (3). Then, the correlation function on the side of QCD can be expressed as the following form after doing the operator production expansion (OPE):

$$\begin{aligned} \Pi_{\mu\nu}^{\text{QCD-V/A}}(p, p') &= 2\sqrt{2}i^2 \varepsilon_{nml} \varepsilon_{abc} \int d^4x d^4y e^{ip'x} e^{iqy} \{ Q^{la}(x) \\ &\quad \times \gamma_\alpha C Q^{ibT}(y) C \gamma_\nu (1 - \gamma_5) C q^{miT}(x - y) C \gamma_\mu q^{nc}(x) \gamma_5 \gamma_\alpha \}, \end{aligned} \quad (11)$$

where $Q^{ij}(x)$ is the full propagator of c quark and can be written as the following form in momentum space [50,51]:

$$\begin{aligned} Q^{ij}(x) &= \frac{i}{(2\pi)^4} \int d^4k e^{-ikx} \left\{ \frac{\delta^{ij}}{k - m_Q} - \frac{g_s G_{\alpha\beta}^n t_{ab}^n}{4} \frac{\sigma^{\alpha\beta}(k + m_Q) + (k + m_Q)\sigma^{\alpha\beta}}{(k^2 - m_Q^2)^2} \right. \\ &\quad \left. - \frac{g_s^2 (t^m t^n)_{ab} G_{\alpha\beta}^m G_{\mu\nu}^n (f^{\alpha\beta\mu\nu} + f^{\alpha\mu\beta\nu} + f^{\alpha\mu\nu\beta})}{4(k^2 - m_Q^2)^5} + \dots \right\}, \end{aligned} \quad (12)$$

with

$$f^{\alpha\beta\mu\nu} = (k + m_Q)\gamma^\alpha(k + m_Q)\gamma^\beta(k + m_Q) \\ \times \gamma^\mu(k + m_Q)\gamma^\nu(k + m_Q).$$

In Eq. (11), $q^{ij}(x)$ and $q^{ij}(x-y)$ are light quark full propagators of u , d , or s and they can be uniformly expressed as the following form in coordinate space [50,51]:

$$q^{ij}(x) = \frac{i\delta^{ij}\not{x}}{2\pi^2 x^4} - \frac{\delta^{ij}\langle\bar{q}q\rangle}{12} - \frac{\delta^{ij}x^2\langle\bar{q}g_s\sigma Gq\rangle}{192} \\ - \frac{\langle\bar{q}^j\sigma^{\mu\nu}q^i\rangle\sigma_{\mu\nu}}{8} - \frac{ig_s G_{\alpha\beta}^n t_{ij}^n (\not{x}\sigma^{\alpha\beta} + \sigma^{\alpha\beta}\not{x})}{32\pi^2 x^2} \\ - \frac{i\delta^{ij}x^2\kappa g_s^2\langle\bar{q}q\rangle^2}{7776} + \dots \quad (13)$$

Here, $\langle\bar{q}g_s\sigma Gq\rangle = \langle\bar{q}g_s\sigma_{\mu\nu}t^\alpha G_{\mu\nu}^a q\rangle$ and $t^\alpha = \frac{\lambda^\alpha}{2}$, λ^α ($\alpha = 1, \dots, 8$) are the Gell-Mann matrices, and $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$. By using the double dispersion relation, the correlation function on the QCD side can be written as

$$\Pi_{\mu\nu}^{\text{QCD-V/A}}(p, p') = \int_{s_{\min}}^{\infty} ds \int_{u_{\min}}^{\infty} du \frac{\rho_{\mu\nu}^{\text{QCD-V/A}}(s, u, q^2)}{(s - p^2)(u - p'^2)}, \quad (14)$$

where $\rho_{\mu\nu}^{\text{QCD}}(s, u, q^2)$ is the QCD spectral density with $s = p^2$ and $u = p'^2$. $s_{\min}$ and $u_{\min}$ are create thresholds for initial and final baryons, which are taken as $4m_c^2$ and m_c^2 ,

respectively. Eliminating the structures proportional to γ_μ and p'_μ , the correlation function on the QCD side can also be decomposed into the same structures as those on the phenomenological side,

$$\Pi_{\mu\nu}^{\text{QCD-V/A}} = \sum_{i=1}^{16} \Pi_i^{\text{QCD-V/A}} e_{i\mu\nu}, \quad (15)$$

where $e_{i\mu\nu}$ has the same form as Eqs. (9) and (10) for $\Pi_{\mu\nu}^{\text{QCD-V}}$ and $\Pi_{\mu\nu}^{\text{QCD-A}}$, respectively. In Eq. (15), $\Pi_i^{\text{QCD-V}}$ and $\Pi_i^{\text{QCD-A}}$ ($i = 1, 2, \dots, 16$) are scalar invariant amplitudes on the QCD side. These scalar invariant amplitudes can be represented as the summation of the perturbative part and vacuum condensate terms. The Feynman diagrams for the perturbative part and vacuum condensate terms of $\langle\bar{q}q\rangle$, $\langle g_s^2 GG \rangle$, $\langle\bar{q}g_s\sigma Gq\rangle$, and $g_s^2\langle\bar{q}q\rangle^2$ are explicitly shown in Fig. 2.

C. The QCD spectral density on the QCD side

For the correlation function that is related to the vector transition current $\bar{q}'\gamma_\nu c$ as an example, we now introduce how its spectral density $\rho_{\mu\nu}^{\text{QCD-V}}$ in Eq. (14) is calculated. Firstly, the contribution of the perturbative term is derived by substituting the free propagators in momentum space of both light and heavy quarks in Eq. (11). The corresponding Feynman diagram is shown as Fig. 2(a). After performing integrals on coordinates x and y , we write the perturbative part as

$$\Pi_{\mu\nu-(a)}^{\text{QCD-V0}}(p, p') = -\frac{12\sqrt{2}}{(2\pi)^8} \int d^4 k_1 d^4 k_2 d^4 k_3 d^4 k_4 \int d^4 q' \frac{\delta^4(q' - k_1 - k_4)\delta^4(p' - q' - k_3)\delta^4(q - k_2 + k_3)}{(k_1^2 - m_1^2)(k_2^2 - m_2^2)(k_3^2 - m_3^2)(k_4^2 - m_4^2)} N_{\mu\nu}, \quad (16)$$

where $N_{\mu\nu} = (k_1 + m_1)\gamma_\alpha(k_2 - m_2)\gamma_\nu(k_3 - m_3)\gamma_\mu(k_4 + m_4)\gamma_5\gamma_\alpha$. The superscript 0 represents that the dimension of the perturbative term is zero, and the subscript (a) denotes the contribution of Fig. 2(a). By setting all quark lines on-shell with Cutkosky's rule [52], the QCD spectral density function of this contribution can be derived as

$$\rho_{\mu\nu-(a)}^{\text{QCD-V0}}(s, u, q^2) = -\frac{12\sqrt{2}}{(2\pi)^8} \frac{(-2\pi i)^5}{(2\pi i)^3} \int_{(m_1+m_4)^2}^{(\sqrt{u}-m_3)^2} dr' \int d^4 k_1 \delta(k_1^2 - m_1^2) \delta[(q' - k_1)^2 - m_4^2] \\ \times \int d^4 k_3 \delta[(p' - k_3)^2 - r'] \delta(k_3^2 - m_3^2) \delta[(k_3 + p - p')^2 - m_2^2] N_{\mu\nu}, \quad (17)$$

where $q' = k_1 + k_4$ and $r' = q'^2$. The integral formulas for two and three Dirac delta functions in this equation are shown in Appendix B.

The quark condensate $\langle\bar{q}q\rangle$ has dimension 3 and it comes from the second term of the light quark full propagator in Eq. (13). Because heavy quarks do not contribute to this condensation, there is only one Feynman diagram originating from the light quark condensate in the present work. The corresponding Feynman diagram is shown in Fig. 2(b), which can be expressed as

of the full light quark propagator in Eq. (13). To finish the integration of the correlation function related to this contribution [Fig. 2(c)] in coordinate space, the coordinate x_μ is firstly replaced by $x_\mu \rightarrow i\partial/\partial p'_\mu$. Then, after performing the derivative on p'_μ , there will appear a high power factor $(k^2 - m^2)^{-n}$ in the Feynman integral. To obtain the spectral density with Cutkosky's rule, we adopt the following formula to lower the power of the denominator:

$$\frac{1}{(k^2 - m^2)^n} = \frac{1}{(n-1)!} \frac{\partial^{n-1}}{(\partial d)^{n-1}} \frac{1}{k^2 - d} \Big|_{d \rightarrow m^2}. \quad (20)$$

With Eq. (20), this kind of quark-gluon mixed condensate can be written as

$$\begin{aligned} \Pi_{\mu\nu-(c)}^{\text{QCD-V5}}(p, p') = & -\frac{i\sqrt{2}\langle\bar{q}g_s\sigma Gq\rangle}{16(2\pi)^4} \left\{ \frac{\partial}{\partial d} \int d^4k_3 \frac{N_{\mu\nu 1}^{V5}}{[(p' - k_3)^2 - d][(p - p' + k_3)^2 - m_2^2][k_3^2 - m_3^2]} \right. \\ & \left. + \frac{\partial^2}{2\partial d^2} \int d^4k_3 \frac{N_{\mu\nu 2}^{V5}}{[(p' - k_3)^2 - d][(p - p' + k_3)^2 - m_2^2][k_3^2 - m_3^2]} \right\}_{d \rightarrow m_1^2}. \end{aligned} \quad (21)$$

The second kind of mixed condensate originates from two quark lines, which can be seen in Figs. 2(d)–2(f). It is shown that one gluon emitted from a light or heavy quark line together with a quark-antiquark pair contributed by another light quark line build up this condensate. This contribution comes from the second term in Eq. (12) and the fourth term in Eq. (13). For Fig. 2(d) as an example, this term can finally be expressed as

$$\Pi_{\mu\nu-(d)}^{\text{QCD-V5}}(p, p') = \frac{i\sqrt{2}\langle\bar{q}g_s\sigma Gq\rangle}{192(2\pi)^4} \frac{\partial}{\partial d} \left\{ \int d^4k_3 \frac{N_{\mu\nu 3}^{V5}}{[(p' - k_3)^2 - d][(p - p' + k_3)^2 - m_2^2][k_3^2 - m_3^2]} \right\}_{d \rightarrow m_1^2}. \quad (22)$$

Similar to the quark condensate, the QCD spectral density of these mixed condensate terms can also be derived as

$$\begin{aligned} \rho_{\mu\nu-(c)}^{\text{QCD-V5}}(s, u, q^2) = & \frac{i\sqrt{2}\langle\bar{q}g_s\sigma Gq\rangle}{16(2\pi)^4} \frac{(-2\pi i)^3}{(2\pi i)^2} \left\{ \frac{\partial}{\partial d} \int d^4k_3 \delta[(p' - k_3)^2 - d] \delta[(p - p' + k_3)^2 - m_2^2] \delta[k_3^2 - m_3^2] N_{\mu\nu 1}^{V5} \right. \\ & \left. + \frac{\partial^2}{2\partial d^2} \int d^4k_3 \delta[(p' - k_3)^2 - d] \delta[(p - p' + k_3)^2 - m_2^2] \delta[k_3^2 - m_3^2] N_{\mu\nu 2}^{V5} \right\}_{d \rightarrow m_1^2} \end{aligned} \quad (23)$$

and

$$\begin{aligned} \rho_{\mu\nu-(d)}^{\text{QCD-V5}}(s, u, q^2) = & \frac{i\sqrt{2}\langle\bar{q}g_s\sigma Gq\rangle}{192(2\pi)^4} \frac{(-2\pi i)^3}{(2\pi i)^2} \frac{\partial}{\partial d} \left\{ \int d^4k_3 N_{\mu\nu 3}^{V5} \right. \\ & \left. \times \delta[(p' - k_3)^2 - d] \delta[(p - p' + k_3)^2 - m_2^2] \delta[k_3^2 - m_3^2] \right\}_{d \rightarrow m_1^2}, \end{aligned} \quad (24)$$

with

$$\begin{aligned} N_{\mu\nu 1}^{V5} = & -4[\not{p}' - k_3]\gamma_\alpha[\not{p} - \not{p}' + k_3 - m_2]\gamma_\nu[k_3 - m_3]\gamma_\mu\gamma_5\gamma_\alpha, \\ N_{\mu\nu 2}^{V5} = & 8[(\not{p}' - k_3 + m_1)m_1^2]\gamma_\alpha[\not{p} - \not{p}' + k_3 - m_2]\gamma_\nu[k_3 - m_3]\gamma_\mu\gamma_5\gamma_\alpha, \\ N_{\mu\nu 3}^{V5} = & (g_{\lambda\beta}g_{\rho\tau} - g_{\lambda\tau}g_{\rho\beta})[\sigma^{\beta\tau}(\not{p}' - k_3 + m_1) + (\not{p}' - k_3 + m_1)\sigma^{\beta\tau}] \\ & \times \gamma_\alpha[\not{p} - \not{p}' + k_3 - m_2]\gamma_\nu(k_3 - m_3)\gamma_\mu\sigma^{\lambda\rho}\gamma_5\gamma_\alpha. \end{aligned}$$

Based on the generation mechanism of the gluon condensate $\langle g_s^2 GG \rangle$, they can also be divided into two categories, which are shown in Figs. 2(g)–2(j) and 2(k)–2(p), respectively. For the contributions of Figs. 2(g) and 2(m) as examples, they finally can be expressed as

$$\begin{aligned} \Pi_{\mu\nu-(g)}^{\text{QCD-V4}}(p, p') &= \frac{\sqrt{2}\langle g_s^2 GG \rangle}{24(2\pi)^8} \int_{(m_1+m_4)^2}^{(\sqrt{u}-m_3^2)^2} dr' \left\{ \int d^4 k_3 \frac{1}{[k_3^2 - m_3^2][(p' - k_3)^2 - r'][(p - p' + k_3)^2 - m_2^2]} \right. \\ &\quad \times \left. \frac{\partial^4}{4! d^4} \int d^4 k_1 \frac{N_{\mu\nu 1}^{V4}}{[k_1^2 - d][(q' - k_1)^2 - m_4^2]} \right\}_{d \rightarrow m_1^2} \end{aligned} \quad (25)$$

and

$$\begin{aligned} \Pi_{\mu\nu-(m)}^{\text{QCD-V4}}(p, p') &= \frac{\sqrt{2}\langle g_s^2 GG \rangle}{192(2\pi)^8} \int_{(m_1+m_4)^2}^{(\sqrt{u}-m_3^2)^2} dr' \left\{ \frac{\partial}{\partial B} \int d^4 k_3 \frac{1}{[k_3^2 - m_3^2][(p' - k_3)^2 - r'][(p - p' + k_3)^2 - B]} \right. \\ &\quad \times \left. \frac{\partial}{\partial A} \int d^4 k_1 \frac{N_{\mu\nu 2}^{V4}}{[k_1^2 - A][(q' - k_1)^2 - m_4^2]} \right\}_{A \rightarrow m_1^2, B \rightarrow m_2^2}, \end{aligned} \quad (26)$$

with

$$\begin{aligned} N_{\mu\nu 1}^{V4} &= (g_{\lambda\beta}g_{\rho\tau} - g_{\lambda\tau}g_{\rho\beta})(f^{\lambda\rho\beta\tau} + f^{\lambda\beta\rho\tau} + f^{\lambda\beta\tau\rho})\gamma_\alpha[\not{p} - \not{p}' + \not{k}_3 - m_2]\gamma_\nu[\not{k}_3 - m_3]\gamma_\mu\gamma_5\gamma_\alpha, \\ N_{\mu\nu 2}^{V4} &= (g_{\lambda\beta}g_{\rho\tau} - g_{\lambda\tau}g_{\rho\beta})[\sigma^{\lambda\rho}(\not{k}_1 + m_1) + (\not{k}_1 + m_1)\sigma^{\lambda\rho}]\gamma_\alpha[(\not{p} - \not{p}' + \not{k}_3 - m_2)\sigma^{\tau\beta} + \sigma^{\tau\beta}(\not{p} - \not{p}' + \not{k}_3 - m_2)] \\ &\quad \times \gamma_\nu[\not{k}_3 - m_3]\gamma_\mu(\not{p}' - \not{k}_3 - \not{k}_1 + m_4)\gamma_5\gamma_\alpha. \end{aligned}$$

The spectral density of these above gluon condensates can also be derived by Cutkosky's rule. Their expressions are similar to that of the perturbative term [Eq. (17)]. The finally considered condensate term in the present work is the four-quark condensation, which is shown in Fig. 2(q). This figure describes that a light quark line emits a quark-antiquark pair and a gluon, and then the gluon contributes a quark-antiquark pair to produce the four-quark condensation. This contribution comes from the sixth term in Eq. (13) and can be calculated by doing the replacement $x_\mu \rightarrow i\partial/\partial p'_\mu$. This contribution can be expressed in the following form:

$$\Pi_{\mu\nu-(q)}^{\text{QCD-V6}}(p, p') = \frac{i\sqrt{2}g_s^2\langle \bar{q}q \rangle^2}{648(2\pi)^4} \left\{ \frac{\partial}{\partial d} \int \frac{N_{\mu\nu 1}^{V6}}{M(d, k_3)} d^4 k_3 + \frac{\partial^2}{2\partial d^2} \int \frac{N_{\mu\nu 2}^{V6}}{M(d, k_3)} d^4 k_3 + \frac{\partial^3}{6\partial d^3} \int \frac{N_{\mu\nu 3}^{V6}}{M(d, k_3)} d^4 k_3 \right\}_{d \rightarrow m_1^2} \quad (27)$$

with

$$\begin{aligned} M(d, k_3) &= [k_3^2 - m_3^2][(p' - k_3)^2 - d][(p - p' + k_3)^2 - m_2^2], \\ N_{\mu\nu 1}^{V6} &= -12\gamma^\beta\gamma^\alpha(\not{p} - \not{p}' + \not{k}_3 - m_2)\gamma_\nu(\not{k}_3 - m_3)\gamma_\mu\gamma_\beta\gamma_5\gamma_\alpha, \\ N_{\mu\nu 2}^{V6} &= (64k_3k_3^\beta - 48m_1k_3^\beta - 16(k_3 \cdot p')\gamma^\beta - 64k_3p'^\beta - 64p'k_3^\beta + 8m_3^2\gamma^\beta + 48m_1p'^\beta + 8p'^2\gamma^\beta + 64p'p'^\beta) \\ &\quad \times (\not{p}' - \not{k}_3 + m_1)\gamma^\alpha(\not{p} - \not{p}' + \not{k}_3 - m_2)\gamma_\nu(\not{k}_3 - m_3)\gamma_\mu\gamma_\beta\gamma_5\gamma_\alpha, \\ N_{\mu\nu 3}^{V6} &= (48m_3^2(m_1 - k_3)k_3^\beta + 96(m_1 - k_3)(k_3 \cdot p')p'^\beta - 96(m_1 - k_3)(k_3 \cdot p')k_3^\beta - 48m_3^2(m_1 - k_3)p'^\beta \\ &\quad + 48p'^2(m_1 - k_3)k_3^\beta - 48p'^2(m_1 - k_3)p'^\beta + 48m_3^2p'k_3^\beta + 96(k_3 \cdot p')p'p'^\beta - 96(k_3 \cdot p')p'k_3^\beta + 48p'^2p'k_3^\beta \\ &\quad - 48m_3^2p'p'^\beta - 48p'^2p'p'^\beta)(\not{p}' - \not{k}_3 + m_1)\gamma^\alpha(\not{p} - \not{p}' + \not{k}_3 - m_2)\gamma_\nu(\not{k}_3 - m_3)\gamma_\mu\gamma_\beta\gamma_5\gamma_\alpha. \end{aligned}$$

Its spectral density function can also be obtained by Cutkosky's rule and has a similar form to that of the quark condensate term.

Finally, we take the change of variables $p^2 \rightarrow -P^2$, $p'^2 \rightarrow -P'^2$, and $q^2 \rightarrow -Q^2$ in Eqs. (A2) and (14), and perform double Borel transformation for variables P^2 and P'^2 to both phenomenological and QCD sides. The variables P^2 and P'^2 are then replaced by Borel parameters M_1^2 and M_2^2 . It is shown by Eqs. (8) and (15) that there are 16 Dirac structures on both the phenomenological and QCD sides. Using the quark-hadron duality condition, we can establish 16 linear equations for the form factors. By solving these linear equations, we obtain the momentum-dependent form factors $F_i(Q^2)$ and $G_i(Q^2)$, which are shown in Eq. (A3) in Appendix A. In Eq. (A3), s_0 and u_0 are threshold parameters for initial and final state baryons, respectively. The spectral density functions $\rho_i^{\text{QCD-V/A}}(s, u, Q^2)$ include contributions of the perturbative part and different condensate terms, and $i = 1 \sim 16$ represent 16 different Dirac structures on the QCD side. The full expressions of these functions are too complex to be shown here for simplicity.

III. NUMERICAL RESULTS

From Eq. (A3), one can find that the final results of QCD SRs depend on some parameters, such as the Borel parameters M_1^2 and M_2^2 , the threshold parameters s_0 and u_0 , the vacuum condensate terms, the pole residues, and the masses of quarks and hadrons. Most of the input parameters in the present work are listed in Table I. We know that the mass of the heavy quark and the values of the vacuum condensates are energy dependent according to the renormalization group equation, and these dependencies can be expressed as follows:

$$\begin{aligned}
 m_c(\mu) &= m_c(m_c) \left[\frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{12}{33-2n_f}}, \\
 m_s(\mu) &= m_s(2 \text{ GeV}) \left[\frac{\alpha_s(\mu)}{\alpha_s(2 \text{ GeV})} \right]^{\frac{12}{33-2n_f}}, \\
 \langle \bar{q}q \rangle(\mu) &= \langle \bar{q}q \rangle(1 \text{ GeV}) \left[\frac{\alpha_s(1 \text{ GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{33-2n_f}}, \\
 \langle \bar{q}g_s\sigma Gq \rangle(\mu) &= \langle \bar{q}g_s\sigma Gq \rangle(1 \text{ GeV}) \left[\frac{\alpha_s(1 \text{ GeV})}{\alpha_s(\mu)} \right]^{\frac{2}{33-2n_f}}, \\
 \langle \bar{s}g_s\sigma Gs \rangle(\mu) &= \langle \bar{s}g_s\sigma Gs \rangle(1 \text{ GeV}) \left[\frac{\alpha_s(1 \text{ GeV})}{\alpha_s(\mu)} \right]^{\frac{2}{33-2n_f}}, \\
 \alpha_s(\mu) &= \frac{1}{b_0 t} \left[1 - \frac{b_1 \log t}{b_0^2} + \frac{b_1^2(\log^2 t - \log t - 1) + b_0 b_2}{b_0^4 t^2} \right], \quad (28)
 \end{aligned}$$

where $t = \log(\mu^2/\Lambda_{\text{QCD}}^2)$, $b_0 = (33 - 2n_f)/12\pi$, $b_1 = (153 - 19n_f)/24\pi^2$, $b_2 = (2857 - \frac{5033}{9}n_f + \frac{325}{27}n_f^2)/128\pi^3$, and Λ_{QCD} is taken to be 292 MeV for the quark flavor $n_f = 4$ in the present work [1]. The minimum subtraction masses of c and s quarks are taken to be the standard values

from the PDG [1], which are $m_c(m_c) = 1.275 \pm 0.025 \text{ GeV}$ and $m_s(\mu = 2 \text{ GeV}) = 0.095 \pm 0.005 \text{ GeV}$. As for the energy scale, a typical value of $\mu = 1 \text{ GeV}$ is adopted for charmed baryon [53].

To obtain reliable results, an appropriate working region for the Borel parameters should be selected. In this region, the values of form factors should have a weak dependence on the Borel parameters. This working region is commonly named the ‘‘Borel platform.’’ At the same time, two conditions should also be satisfied, which are the pole dominance and the convergence of OPE. The pole contribution is defined as

$$\text{Pole} = \frac{\int_{s_{\min}}^{s_0} ds \int_{u_{\min}}^{u_0} du \rho^{\text{QCD}}(s, u, q^2) \exp\left(-\frac{s}{M_1^2} - \frac{u}{M_2^2}\right)}{\int_{s_{\min}}^{\infty} ds \int_{u_{\min}}^{\infty} du \rho^{\text{QCD}}(s, u, q^2) \exp\left(-\frac{s}{M_1^2} - \frac{u}{M_2^2}\right)}. \quad (29)$$

The pole dominance requires that the pole contribution should be larger than 40%, and the convergence of OPE requires that the contribution of high-dimension condensate terms should be as small as possible. For the form factors F_1 and F_4 of the transition $\Xi_{cc}^{++} \rightarrow \Sigma_c^{*++}$ as examples, we introduce how the Borel platform is determined. Fixing $Q^2 = 1 \text{ GeV}^2$, we plot the pole contribution and the contributions of the perturbative part and different vacuum condensate terms in Fig. 3, which explicitly illustrates the dependence of the results on the Borel parameters. From this figure, we can see that the pole contribution (Pole) decreases with the increase of Borel parameters. In addition, we can also see that main contributions come from the quark condensate (D3) and perturbative part (D0), and the larger the values of the Borel parameters taken, the smaller the contributions of vacuum condensate terms become. That is to say, the condition of convergence of OPE can be well satisfied when the larger Borel parameters are taken. After repeated trial and error, the Borel platform for M_2^2 is

TABLE I. 1 Input parameters (IPs) in this work. The values of vacuum condensate are at the energy scale $\mu = 1 \text{ GeV}$.

IP	$m_{\Xi_{cc}(\frac{1}{2}^+)}$	$m_{\Xi_{cc}(\frac{1}{2}^-)}$	$m_{\Omega_{cc}(\frac{1}{2}^+)}$	$m_{\Omega_{cc}(\frac{1}{2}^-)}$	$m_{\Sigma_c^*(\frac{3}{2}^+)}$
Values (GeV)	3.621 [54]	3.932 [54]	3.750 [54]	4.049 [54]	2.518 [49]
IP	$m_{\Sigma_c^*(\frac{3}{2}^-)}$	$m_{\Xi_{cc}'(\frac{3}{2}^+)}$	$m_{\Xi_{cc}'(\frac{3}{2}^-)}$	$m_{\Omega_{cc}'(\frac{3}{2}^+)}$	$m_{\Omega_{cc}'(\frac{3}{2}^-)}$
Values (GeV)	2.829 [49]	2.645 [49]	2.958 [49]	2.762 [55]	3.062 [55]
IP	$\lambda_{\Xi_{cc}(\frac{1}{2}^+)}$	$\lambda_{\Omega_{cc}(\frac{1}{2}^+)}$	$\lambda_{\Sigma_c^*(\frac{3}{2}^+)}$	$\lambda_{\Xi_{cc}'(\frac{3}{2}^+)}$	$\lambda_{\Omega_{cc}'(\frac{3}{2}^+)}$
Values (GeV^3)	0.109 ± 0.020 [38]	0.129 ± 0.024 [38]	$\sqrt{2} \times (0.027 \pm 0.008)$ [53]	$\sqrt{2} \times (0.033 \pm 0.008)$ [53]	0.056 ± 0.012 [53]
IP	m_e	m_μ	$\langle \bar{q}q \rangle$	$\langle \bar{s}s \rangle$	$\langle \bar{q}g_s\sigma Gq \rangle$
Values	$0.511 \times 10^{-3} \text{ GeV}$ [1]	$106 \times 10^{-3} \text{ GeV}$ [1]	$-(0.23 \pm 0.01)^3 \text{ GeV}^3$ [27,51]	$(0.8 \pm 0.1) \langle \bar{q}q \rangle$ [27,51]	$m_0^2 \langle \bar{q}q \rangle$ [27,51]
IP	$\langle \bar{s}g_s\sigma Gs \rangle$	m_0^2	$\langle g_s^2 GG \rangle$		
Values	$m_0^2 \langle \bar{s}s \rangle$ [27,51]	$0.8 \pm 0.1 \text{ GeV}^2$ [27,51]	$0.47 \pm 0.15 \text{ GeV}^4$ [56–58]		

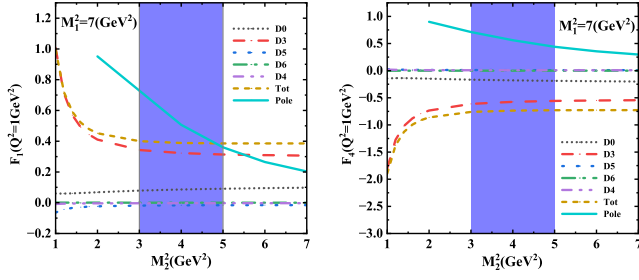


FIG. 3. Variations of the pole contributions and the contributions of the perturbative term and different vacuum condensates with respect to the Borel parameter M_2^2 . Results are shown for the form factors F_1 and F_4 of the transition $\Xi_{cc}^{++} \rightarrow \Sigma_c^{*+}$. The blue area denotes the Borel platform.

taken to be 3–5 GeV^2 , marked as the blue area in Fig. 3. In this region, the average value of the pole contribution is about 50% and, at the same time, the contributions of different dimensions satisfy the relation $D3 > D0 > D5 \gg D4 > D6$. The contributions originating from the high-dimension condensates $\langle g_s^2 GG \rangle$ and $g_s^2 \langle \bar{q}q \rangle^2$ are both smaller than 2%.

Another two parameters to be selected are the threshold parameters s_0 and u_0 , which are used to eliminate the contributions of excited and continuum states. Their values are usually taken to be $s_0 = (m_{B_1} + \Delta_1)^2$ and $u_0 = (m_{B_2} + \Delta_2)^2$. Commonly, the energy gap between the ground state and the first excited state of heavy baryons is about 500 MeV, thus the value of $\Delta_1 = \Delta_2 = 0.5 \text{ GeV}$ is taken in the present work. For the transition process $\Xi_{cc}^{++} \rightarrow \Sigma_c^{*+}$ as an example, we plot variations of the form factors with respect to the Borel parameters M_1^2 and M_2^2 in Fig. 4. From this figure, we can see that the results show good stability and weak dependence on the Borel parameters in the Borel platform.

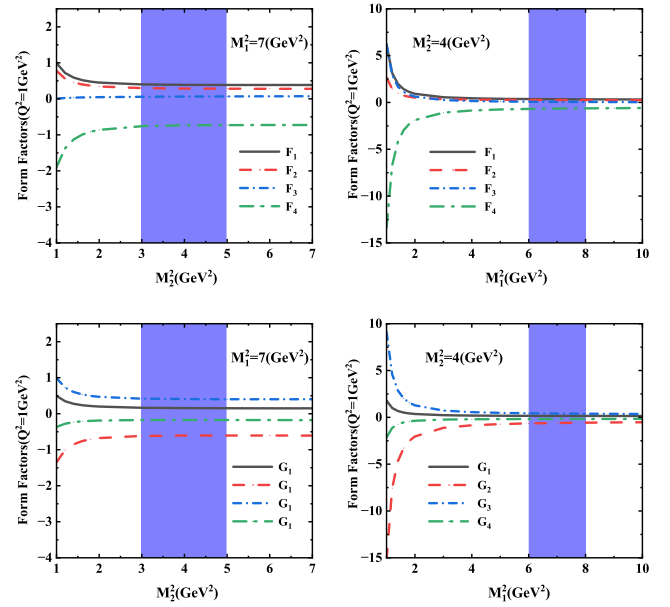


FIG. 4. Variations of the form factors ($\Xi_{cc}^{++} \rightarrow \Sigma_c^{*+}$ transition process) with respect to the Borel parameters M_1^2 and M_2^2 , where the blue area denotes the Borel platform.

After all the conditions of QCDSRs are satisfied, we can obtain the momentum-dependent form factors $F_i(Q^2)$ and $G_i(Q^2)$ by taking different values of Q^2 in the spacelike region ($Q^2 > 0$). The results are explicitly shown in Figs. 5 and 6, where Q^2 are in the range of 1–8 GeV^2 . To obtain the values of the form factors in the timelike region ($Q^2 < 0$), it is necessary to fit the numerical results with the appropriate analytical function and then extrapolate them into the timelike region. The z series expansion approach is commonly employed to fit the form factors [59]. With this method, the form factors can be expressed as

$$F_i(Q^2) = \frac{F(0)}{1 + Q^2/m_{\text{pole}}^2} \times \left\{ 1 + a \left[z(Q^2) - z(0) - \frac{1}{3} [z(Q^2)^3 - z(0)^3] \right] + b \left[z(Q^2)^2 - z(0)^2 - \frac{2}{3} [z(Q^2)^3 - z(0)^3] \right] \right\},$$

$$G_i(Q^2) = \frac{G(0)}{1 + Q^2/m_{\text{pole}}^2} \times \left\{ 1 + c \left[z(Q^2) - z(0) - \frac{1}{3} [z(Q^2)^3 - z(0)^3] \right] + d \left[z(Q^2)^2 - z(0)^2 - \frac{2}{3} [z(Q^2)^3 - z(0)^3] \right] \right\}, \quad (30)$$

with

$$z(Q^2) = \frac{\sqrt{t_+^2 + Q^2} - \sqrt{t_+^2 - t_-^2}}{\sqrt{t_+^2 + Q^2} + \sqrt{t_+^2 - t_-^2}},$$

$$t_{\pm} = (m_{B_1} \pm m_{B_2})^2.$$

In Eq. (30), $a \sim d$ are fitting parameters, and their values are all listed in Table II. The fitting curves of the form factors are explicitly shown in Figs. 5 and 6. From these

figures, we can see that the form factors are fitted well by the z series expansion approach. Thus, it is reliable for us to obtain the values of form factors in the timelike region, which can be used to analyze the semileptonic decays of Ξ_{cc} and Ω_{cc} baryons.

A similar analysis about the form factors was also carried out in the framework of the light-front quark model in Ref. [21]. For comparison, the results of the present work and those obtained with the quark model [21] are all listed in Table II. Note that if a factor of $\sqrt{2}$ is multiplied to F_i/G_i

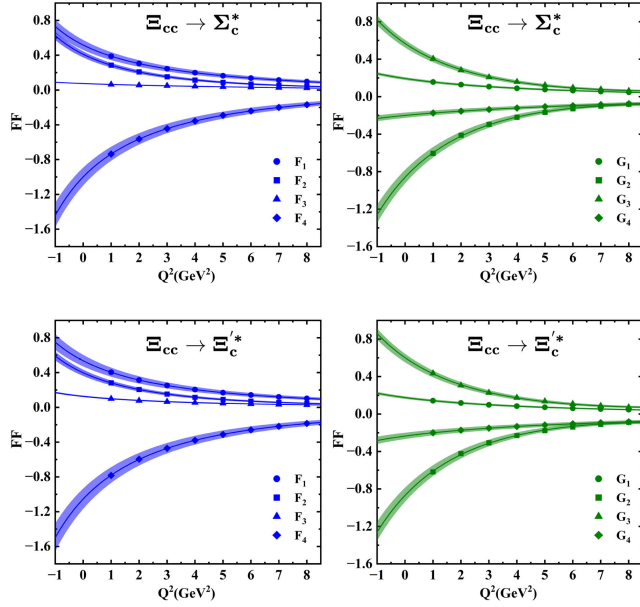


FIG. 5. The fitting results of the form factors for the transition processes $\Xi_{cc}^{++} \rightarrow \Sigma_c^{*+}$ and $\Xi_{cc}^{++} \rightarrow \Xi_c^{*+}$.

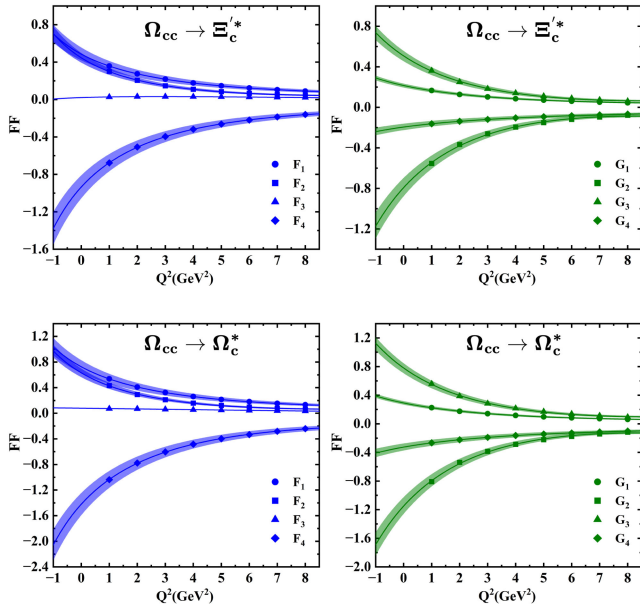


FIG. 6. The fitting results of the form factors for the transition processes $\Omega_{cc}^{+} \rightarrow \Xi_c^{*0}$ and $\Omega_{cc}^{+} \rightarrow \Omega_c^{*0}$.

for $\Xi_{cc}^{++} \rightarrow \Sigma_c^{*+}$, we can obtain the results for the transition $\Xi_{cc}^{++} \rightarrow \Sigma_c^{*0}$. Thus, the results of the latter are not listed in the table. It is shown that our results are apparently lower than the values of the quark model. Compared with the present work, another difference in Ref. [21] is that they employ the analytical function with a two-pole structure to fit the numerical results for the form factors. Their fitting curves illustrate that the absolute values of the form factors

become lower after being extrapolated into the timelike region. In the present work, the situation is exactly the opposite, where the absolute values increase after being fitted into the timelike region (see Figs. 5 and 6). From Table II, we can see that the on-shell values of $F(q_{\max}^2)/G(q_{\max}^2)$ and $q_{\max}^2 = (m_{B_1} - m_{B_2})^2$ are larger than the values at zero. In Ref. [23], the form factors of $\Xi_{cc}^{++} \rightarrow \Sigma_c^{*+}$ were also analyzed by the light-front quark model. The results of $F_1(0) \sim F_4(0)$ and $G_1(0) \sim G_4(0)$ are $(-0.0179, 0.374, -0.381, 0.429)$ and $(-0.453, 0, 0.320, -1.14)$, which are inconsistent with those of the present work and Ref. [21].

IV. SEMILEPTONIC DECAYS OF Ξ_{cc} AND Ω_{cc} BARYONS

As a phenomenological application, the results for the form factors are applied to analyze the decay widths and branching ratios of the semileptonic decays $\Xi_{cc}^{++} \rightarrow \Sigma_c^{*+} l^+ \nu_l$, $\Xi_{cc}^{++} \rightarrow \Xi_c^{*+} l^+ \nu_l$, $\Omega_{cc}^{+} \rightarrow \Xi_c^{*0} l^+ \nu_l$, and $\Omega_{cc}^{+} \rightarrow \Omega_c^{*0} l^+ \nu_l$. The effective Hamiltonian of these processes can be expressed as

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} (V_{cs}^* \bar{s} \gamma_\mu (1 - \gamma_5) c \bar{\nu}_l \gamma_\mu (1 - \gamma_5) l + V_{cd}^* \bar{d} \gamma_\mu (1 - \gamma_5) c \bar{\nu}_l \gamma_\mu (1 - \gamma_5) l), \quad (31)$$

where G_F is the Fermi constant and V_{cs}^* and V_{cd}^* are the CKM matrix elements. Their values are taken from the PDG [1],

$$G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}, \quad |V_{cd}| = 0.225, \quad |V_{cs}| = 0.974.$$

With this Hamiltonian, the transition matrix element of the semileptonic decay processes $\Xi_{cc}^{++} \rightarrow \Sigma_c^{*+}$, $\Xi_{cc}^{++} \rightarrow \Xi_c^{*+}$, $\Omega_{cc}^{+} \rightarrow \Xi_c^{*0}$, and $\Omega_{cc}^{+} \rightarrow \Omega_c^{*0}$ can be expressed as

$$\begin{aligned} T &= \langle \mathcal{B}_2 | H_{\text{eff}} | \mathcal{B}_1 \rangle \\ &= \frac{G_F}{\sqrt{2}} V_{cq}^* \langle \mathcal{B}_2 | \bar{q} \gamma_\mu (1 - \gamma_5) c | \mathcal{B}_1 \rangle \\ &\quad \times \langle l \bar{\nu}_l | \bar{\nu}_l \gamma_\mu (1 - \gamma_5) l | 0 \rangle. \end{aligned} \quad (32)$$

The leptonic matrix element can be formulated as the following form by electroweak perturbation theory:

$$\langle l \bar{\nu}_l | \bar{\nu}_l \gamma_\mu (1 - \gamma_5) l | 0 \rangle = \bar{u}_{\nu, s} \gamma_\mu (1 - \gamma_5) u_{l, s'}. \quad (33)$$

Here, $u_{\nu, s}$ and $u_{l, s'}$ are the spinor wave functions of $\bar{\nu}_l$ and l , and the subscripts $v(l)$ and $s(s')$ represent the four-momentum and spin, respectively. The helicity amplitudes will be used in the calculation and for the vector current and the axial vector current. They are written as

TABLE II. The values of form factors (FFs) at $Q^2 = 0$ and fitting parameters of different form factors by the z series expansion approach.

Mode	FF	$F_i(0)$	a	b	$F(q_{\max}^2)$	$F_i(0)$ [21]	FF	$G_i(0)$	c	d	$G(q_{\max}^2)$	$G_i(0)$ [21]
$\Xi_{cc}^{++} \rightarrow \Sigma_c^{*+}$	F_1	$0.52^{+0.06}_{-0.06}$	$-9.45^{+0.97}_{-1.19}$	$2.39^{+5.16}_{-4.12}$	$0.75^{+0.09}_{-0.09}$	-0.979	G_1	$0.19^{+0.01}_{-0.02}$	$-8.42^{+2.09}_{-2.28}$	$-5.77^{+16.11}_{-14.26}$	$0.24^{+0.01}_{-0.03}$	-5.792
	F_2	$0.41^{+0.03}_{-0.04}$	$-23.25^{+1.50}_{-1.52}$	$134.38^{+14.11}_{-14.71}$	$0.59^{+0.04}_{-0.06}$	0.66	G_2	$-0.86^{+0.08}_{-0.07}$	$-35.05^{+1.33}_{-1.49}$	$286.90^{+16.15}_{-14.34}$	$-1.09^{+0.10}_{-0.09}$	3.84
	F_3	$0.07^{+0.004}_{-0.004}$	$17.64^{+4.35}_{-4.35}$	$-285.62^{+33.18}_{-36.18}$	$0.10^{+0.006}_{-0.006}$	0.42	G_3	$0.57^{+0.04}_{-0.04}$	$-31.94^{+1.57}_{-1.73}$	$251.65^{+18.25}_{-16.57}$	$0.72^{+0.05}_{-0.05}$	2.28
	F_4	$-1.00^{+0.10}_{-0.10}$	$-13.50^{+0.98}_{-1.08}$	$47.38^{+4.95}_{-5.32}$	$-1.44^{+0.14}_{-0.14}$	-1.969	G_4	$-0.20^{+0.03}_{-0.03}$	$6.25^{+0.86}_{-0.47}$	$-129.65^{+1.37}_{-8.55}$	$-0.25^{+0.04}_{-0.04}$	0.393
$\Xi_{cc}^{++} \rightarrow \Xi_{cc}^{*+}$	F_1	$0.54^{+0.07}_{-0.07}$	$-13.53^{+0.57}_{-0.77}$	$57.73^{+2.26}_{-1.00}$	$0.69^{+0.09}_{-0.09}$	-1.148	G_1	$0.18^{+0.01}_{-0.02}$	$-9.13^{+2.32}_{-2.17}$	$13.49^{+16.19}_{-19.60}$	$0.21^{+0.01}_{-0.02}$	-10.350
	F_2	$0.40^{+0.04}_{-0.04}$	$-24.51^{+1.08}_{-1.30}$	$164.35^{+13.01}_{-10.26}$	$0.51^{+0.05}_{-0.05}$	0.72	G_2	$-0.87^{+0.08}_{-0.07}$	$-34.78^{+1.26}_{-1.28}$	$307.93^{+14.57}_{-14.61}$	$-1.04^{+0.10}_{-0.08}$	6.43
	F_3	$0.13^{+0.009}_{-0.009}$	$-6.92^{+2.41}_{-2.90}$	$-11.15^{+27.43}_{-21.35}$	$0.17^{+0.01}_{-0.01}$	0.52	G_3	$0.60^{+0.04}_{-0.05}$	$-31.53^{+1.42}_{-1.44}$	$267.88^{+15.77}_{-15.78}$	$0.72^{+0.05}_{-0.06}$	4.70
	F_4	$-1.05^{+0.12}_{-0.12}$	$-16.58^{+0.79}_{-0.84}$	$93.65^{+4.10}_{-4.49}$	$-1.34^{+0.15}_{-0.15}$	-2.297	G_4	$-0.24^{+0.03}_{-0.03}$	$-0.20^{+0.82}_{-0.68}$	$-62.14^{+2.57}_{-0.76}$	$-0.28^{+0.04}_{-0.04}$	0.404
$\Omega_{cc}^{+} \rightarrow \Xi_c^{*0}$	F_1	$0.49^{+0.07}_{-0.07}$	$-15.26^{+0.96}_{-0.91}$	$85.80^{+7.64}_{-9.14}$	$0.70^{+0.10}_{-0.10}$	-1.017	G_1	$0.22^{+0.02}_{-0.02}$	$-20.59^{+2.09}_{-2.18}$	$152.50^{+20.95}_{-19.94}$	$0.28^{+0.03}_{-0.03}$	-6.244
	F_2	$0.45^{+0.05}_{-0.05}$	$-33.23^{+0.96}_{-1.00}$	$285.01^{+10.65}_{-10.51}$	$0.65^{+0.07}_{-0.07}$	0.66	G_2	$-0.79^{+0.09}_{-0.09}$	$-40.54^{+1.22}_{-1.19}$	$386.80^{+15.42}_{-16.16}$	$-1.01^{+0.11}_{-0.11}$	3.86
	F_3	$0.02^{+0.001}_{-0.001}$	$144.87^{+21.10}_{-17.63}$	$-1612.00^{+0.20}_{-0.25}$	$0.03^{+0.001}_{-0.001}$	0.47	G_3	$0.51^{+0.05}_{-0.05}$	$-36.18^{+1.39}_{-1.40}$	$331.61^{+17.39}_{-17.45}$	$0.65^{+0.06}_{-0.06}$	2.73
	F_4	$-0.93^{+0.12}_{-0.12}$	$-19.10^{+1.01}_{-0.92}$	$133.04^{+8.86}_{-10.71}$	$-1.34^{+0.17}_{-0.17}$	-2.045	G_4	$-0.19^{+0.03}_{-0.03}$	$-4.04^{+1.51}_{-1.18}$	$0.03^{+11.02}_{-15.49}$	$-0.24^{+0.04}_{-0.04}$	0.389
$\Omega_{cc}^{+} \rightarrow \Omega_{cc}^{*0}$	F_1	$0.73^{+0.10}_{-0.10}$	$-18.73^{+1.25}_{-1.06}$	$132.11^{+12.21}_{-17.60}$	$0.94^{+0.13}_{-0.13}$	-1.178	G_1	$0.29^{+0.03}_{-0.02}$	$-20.49^{+1.77}_{-2.66}$	$160.42^{+30.94}_{-17.50}$	$0.35^{+0.04}_{-0.02}$	-10.670
	F_2	$0.63^{+0.07}_{-0.07}$	$-34.84^{+0.86}_{-0.96}$	$328.09^{+11.61}_{-10.20}$	$0.81^{+0.09}_{-0.09}$	0.73	G_2	$-1.15^{+0.12}_{-0.12}$	$-40.55^{+1.20}_{-1.30}$	$413.34^{+18.99}_{-17.28}$	$-1.38^{+0.14}_{-0.14}$	6.55
	F_3	$0.08^{+0.002}_{-0.004}$	$32.05^{+11.25}_{-8.90}$	$-469.48^{+121.27}_{-152.33}$	$0.10^{+0.002}_{-0.005}$	0.54	G_3	$0.78^{+0.07}_{-0.07}$	$-36.54^{+1.41}_{-1.43}$	$358.31^{+19.82}_{-19.52}$	$0.93^{+0.08}_{-0.08}$	4.85
	F_4	$-1.42^{+0.17}_{-0.18}$	$-21.62^{+0.94}_{-1.19}$	$170.48^{+14.91}_{-10.99}$	$-1.82^{+0.22}_{-0.23}$	-2.359	G_4	$-0.33^{+0.05}_{-0.05}$	$-8.89^{+1.50}_{-1.64}$	$46.77^{+20.28}_{-17.36}$	$-0.39^{+0.06}_{-0.06}$	0.415

$$\begin{aligned}
H_{\frac{1}{2}}^V &= -\sqrt{\frac{2}{3}}\alpha_{\frac{1}{2}}^V(\omega-1)\left[f_1^V(q^2)m_{B_1}-f_2^V(q^2)m_+ \right. \\
&\quad \left.+ f_3^V(q^2)\frac{m_{B_2^*}}{m_{B_1}}(m_{B_1}\omega-m_{B_2^*})+f_4^V(q^2)\frac{q^2}{m_{B_1}}\right], \\
H_{\frac{3}{2}0}^V &= -\sqrt{\frac{2}{3}}\alpha_{\frac{3}{2}0}^V[f_1^V(q^2)(m_{B_1}\omega-m_{B_2^*})-f_2^V(q^2)(\omega+1)m_- \\
&\quad + f_3^V(q^2)(\omega^2-1)m_{B_2^*}], \\
H_{\frac{1}{2}1}^V &= \sqrt{\frac{1}{6}}\alpha_{\frac{1}{2}1}^V[f_1^V(q^2)-2f_2^V(q^2)(\omega+1)], \\
H_{\frac{3}{2}1}^V &= -\frac{1}{\sqrt{2}}\alpha_{\frac{3}{2}1}^V f_1^V(q^2), \\
H_{\frac{1}{2}}^A &= \sqrt{\frac{2}{3}}\alpha_{\frac{1}{2}}^A(\omega+1)\left[f_1^A(q^2)m_{B_1}-f_2^A(q^2)m_- \right. \\
&\quad \left.+ f_3^A(q^2)\frac{m_{B_2^*}}{m_{B_1}}(m_{B_1}\omega-m_{B_2^*})+f_4^A(q^2)\frac{q^2}{m_{B_1}}\right], \\
H_{\frac{3}{2}0}^A &= \sqrt{\frac{2}{3}}\alpha_{\frac{3}{2}0}^A[f_1^A(q^2)(m_{B_1}\omega-m_{B_2^*})-f_2^A(q^2)(\omega-1)m_+ \\
&\quad + f_3^A(q^2)(\omega^2-1)m_{B_2^*}], \\
H_{\frac{1}{2}1}^A &= \sqrt{\frac{1}{6}}\alpha_{\frac{1}{2}1}^A[f_1^A(q^2)-2f_2^A(q^2)(\omega-1)], \\
H_{\frac{3}{2}1}^A &= \frac{1}{\sqrt{2}}\alpha_{\frac{3}{2}1}^A f_1^A(q^2), \tag{34}
\end{aligned}$$

where

$$\alpha_{\frac{1}{2}}^V = \alpha_{\frac{3}{2}0}^A = \sqrt{\frac{2m_{B_1}m_{B_2^*}(\omega+1)}{q^2}},$$

$$\alpha_{\frac{3}{2}0}^V = \alpha_{\frac{1}{2}1}^A = \sqrt{\frac{2m_{B_1}m_{B_2^*}(\omega-1)}{q^2}},$$

$$\alpha_{\frac{1}{2}1}^V = 2\sqrt{m_{B_1}m_{B_2^*}(\omega-1)},$$

$$\alpha_{\frac{3}{2}1}^A = 2\sqrt{m_{B_1}m_{B_2^*}(\omega+1)},$$

and

$$\omega = \frac{m_{B_1}^2 + m_{B_2^*}^2 - q^2}{2m_{B_1}m_{B_2^*}},$$

$$m_{\pm} = m_{B_1} \pm m_{B_2^*},$$

with m_{B_1} and $m_{B_2^*}$ being the masses of initial and final baryons. The parametrization of the transition matrix element in the present work [see Eq. (1)] is different from Eq. (34), and the relations between these form factors are as follows:

$$f_1^V(q^2) = F_4^V(q^2), \quad g_1^V(q^2) = G_4^V(q^2),$$

$$f_2^V(q^2) = F_1^V(q^2), \quad g_2^V(q^2) = G_1^V(q^2),$$

$$f_3^V(q^2) = \frac{m_{B_1}}{m_{B_2^*}}F_3^V(q^2) + F_2^V(q^2),$$

$$g_3^V(q^2) = \frac{m_{B_1}}{m_{B_2^*}}G_3^V(q^2) + G_2^V(q^2),$$

$$f_4^V(q^2) = F_2^V(q^2), \quad g_4^V(q^2) = G_2^V(q^2).$$

The amplitudes for negative helicity are given by

$$H_{-\lambda_2, -\lambda_W}^V = -H_{\lambda_2, \lambda_W}^V H_{-\lambda_2, -\lambda_W}^A = H_{\lambda_2, \lambda_W}^A,$$

where λ_2 and λ_W represent the polarizations of the final baryon and the intermediate W boson, respectively. The helicity amplitude for the J_{ν}^{V-A} current can be expressed as

$$H_{\lambda_2, \lambda_W} = H_{\lambda_2, \lambda_W}^V - H_{\lambda_2, \lambda_W}^A. \quad (35)$$

Finally, the total amplitude about the decay process $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$ can be expressed as the combinations of the helicity amplitudes,

$$\begin{aligned} \mathcal{H}_{\frac{1}{2} \rightarrow \frac{3}{2}} &= |H_{\frac{1}{2}, 1}|^2 + |H_{-\frac{1}{2}, -1}|^2 + |H_{\frac{3}{2}, 1}|^2 + |H_{-\frac{3}{2}, -1}|^2 + |H_{\frac{1}{2}, 0}|^2 \\ &+ |H_{-\frac{1}{2}, 0}|^2 + \frac{m_l^2}{2q^2} (3|H_{\frac{1}{2}, 1}|^2 + 3|H_{-\frac{1}{2}, -1}|^2 + |H_{\frac{1}{2}, 1}|^2 \\ &+ |H_{-\frac{1}{2}, -1}|^2 + |H_{\frac{3}{2}, 1}|^2 + |H_{-\frac{3}{2}, -1}|^2 + |H_{\frac{1}{2}, 1}|^2 \\ &+ |H_{-\frac{1}{2}, 0}|^2). \end{aligned} \quad (36)$$

According to the helicity amplitudes, differential decay widths can be decomposed into the following longitudinally and transversely polarized components:

$$\begin{aligned} \frac{d\Gamma_L}{dq^2} &= \frac{G_F^2 |V_{CKM}|^2 m_{B_2} (q^2 - m_l^2)^2}{192\pi^3 m_{B_1}^2 q^2} \sqrt{\omega^2 - 1} \left[|H_{\frac{1}{2}, 0}|^2 + |H_{-\frac{1}{2}, 0}|^2 \right. \\ &\quad \left. + \frac{m_l^2}{2q^2} (3|H_{\frac{1}{2}, 1}|^2 + 3|H_{-\frac{1}{2}, -1}|^2 + |H_{\frac{1}{2}, 0}|^2 + |H_{-\frac{1}{2}, 0}|^2) \right], \\ \frac{d\Gamma_T}{dq^2} &= \frac{G_F^2 |V_{CKM}|^2 m_{B_2} (q^2 - m_l^2)^2}{192\pi^3 m_{B_1}^2 q^2} \sqrt{\omega^2 - 1} \left[|H_{\frac{1}{2}, 1}|^2 \right. \\ &\quad \left. + |H_{-\frac{1}{2}, -1}|^2 + |H_{\frac{3}{2}, 1}|^2 + |H_{-\frac{3}{2}, -1}|^2 \right. \\ &\quad \left. + \frac{m_l^2}{2q^2} (|H_{\frac{1}{2}, 1}|^2 + |H_{-\frac{1}{2}, -1}|^2 + |H_{\frac{3}{2}, 1}|^2 + |H_{-\frac{3}{2}, -1}|^2) \right]. \end{aligned} \quad (37)$$

The dependences of the differential decay widths on q^2 are explicitly shown in Figs. 7 and 8. By integrating out the square momentum q^2 , the decay widths can be written as

$$\Gamma = \int_{m_l^2}^{(m_{B_1} - m_{B_2}^*)^2} \left\{ \frac{d\Gamma_L}{dq^2} + \frac{d\Gamma_T}{dq^2} \right\} dq^2. \quad (38)$$

The integrated partial decay widths, ratios of Γ_L/Γ_T , and the corresponding branching fractions are calculated, and the results are all listed in Table III. For comparison, the results in Ref. [21] are listed in the last two columns in the table. In Ref. [9], the authors used the covariant confined quark model to analyze the decay processes of $\Xi_{cc}^{++} \rightarrow \Xi_c^{*+} l^+ \nu_l$ and $\Omega_{cc}^+ \rightarrow \Omega_c^{*0} l^+ \nu_l$, where the decay widths are predicted to be 2.2×10^{-14} and 4.0×10^{-14} GeV. It can be

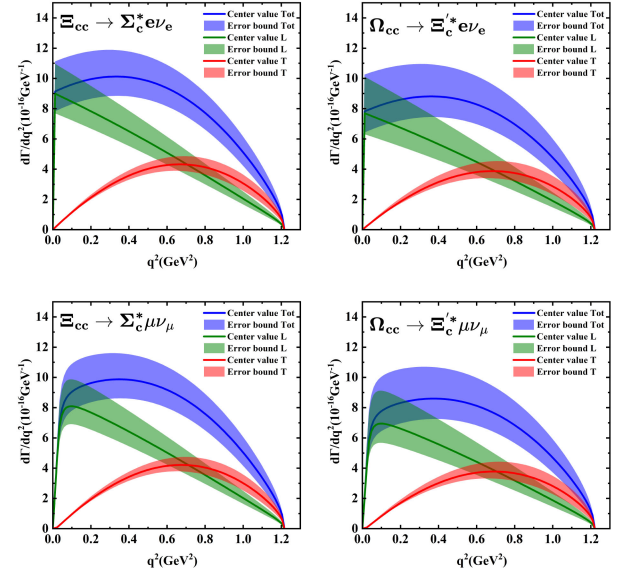


FIG. 7. Variations of the differential decay widths with respect to q^2 for the transition processes $\Xi_{cc}^{++} \rightarrow \Xi_c^{*+} l^+ \nu_l$ and $\Omega_{cc}^+ \rightarrow \Omega_c^{*0} l^+ \nu_l$.

seen that the predicted widths in the present work are slightly lower than those obtained by the quark model in Refs. [9,21]. In addition, the predicted results in the present work show that the decay channels $\Xi_{cc}^{++} \rightarrow \Xi_c^{*+} l^+ \nu_l$ and $\Omega_{cc}^+ \rightarrow \Omega_c^{*0} l^+ \nu_l$ have a much larger branching fraction than the other decay modes.

In the flavor SU(3) symmetry, there exist the following relations for the semileptonic decay processes:

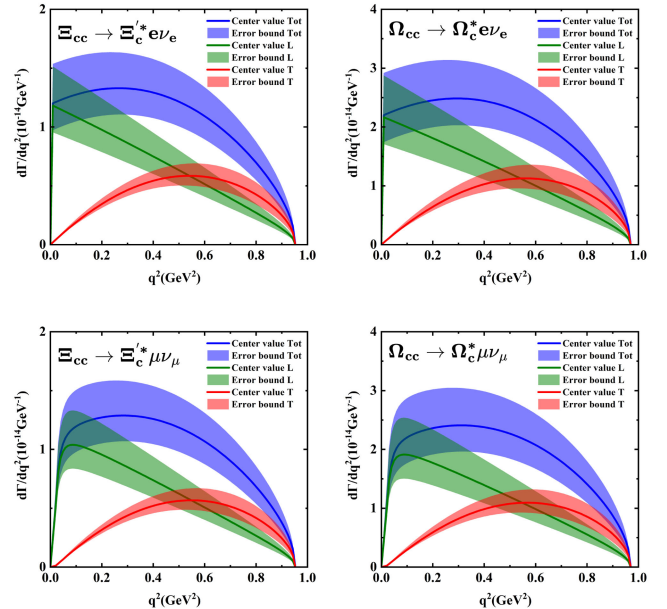


FIG. 8. Variations of the differential decay widths with respect to q^2 for the transition processes $\Omega_{cc}^+ \rightarrow \Xi_c^{*+} l^+ \nu_l$ and $\Omega_{cc}^+ \rightarrow \Omega_c^{*0} l^+ \nu_l$.

TABLE III. Results of semileptonic decay, where Γ is in units of 10^{-14} GeV and the lifetimes of Ξ_{cc} and Ω_{cc} are taken to be 256 and 206 fs, respectively.

Modes	This work			Ref. [21]	
	Γ	$\mathcal{B}(\%)$	Γ_L/Γ_T	Γ	Γ_L/Γ_T
$\Xi_{cc}^{++} \rightarrow \Sigma_c^{*+} e^+ \nu_e$	$0.096_{-0.01}^{+0.02}$	$0.37_{-0.05}^{+0.07}$	$1.65_{-0.08}^{+0.12}$	0.14	0.92
$\Xi_{cc}^{++} \rightarrow \Sigma_c^{*+} \mu^+ \nu_\mu$	$0.091_{-0.01}^{+0.02}$	$0.35_{-0.05}^{+0.06}$	$1.62_{-0.08}^{+0.12}$	0.14	0.92
$\Xi_{cc}^{++} \rightarrow \Xi_c^{*+} e^+ \nu_e$	$1.01_{-0.18}^{+0.24}$	$3.93_{-0.72}^{+0.93}$	$1.59_{-0.10}^{+0.13}$	1.74	1.08
$\Xi_{cc}^{++} \rightarrow \Xi_c^{*+} \mu^+ \nu_\mu$	$0.95_{-0.17}^{+0.23}$	$3.68_{-0.68}^{+0.88}$	$1.55_{-0.10}^{+0.12}$	1.74	1.08
$\Omega_{cc}^+ \rightarrow \Xi_c^{*0} e^+ \nu_e$	$0.085_{-0.01}^{+0.02}$	$0.27_{-0.05}^{+0.07}$	$1.60_{-0.11}^{+0.17}$	0.14	0.93
$\Omega_{cc}^+ \rightarrow \Xi_c^{*0} \mu^+ \nu_\mu$	$0.081_{-0.01}^{+0.02}$	$0.25_{-0.04}^{+0.06}$	$1.57_{-0.11}^{+0.17}$	0.14	0.93
$\Omega_{cc}^+ \rightarrow \Omega_c^{*0} e^+ \nu_e$	$1.95_{-0.38}^{+0.53}$	$4.82_{-0.35}^{+0.45}$	$1.54_{-0.11}^{+0.14}$	3.45	1.07
$\Omega_{cc}^+ \rightarrow \Omega_c^{*0} \mu^+ \nu_\mu$	$1.83_{-0.36}^{+0.50}$	$4.69_{-0.35}^{+0.44}$	$1.50_{-0.11}^{+0.14}$	3.45	1.07

$$\begin{aligned} \Gamma(\Xi_{cc}^{++} \rightarrow \Sigma_c^{*+} l^+ \nu) &= \Gamma(\Omega_{cc}^+ \rightarrow \Xi_c^{*0} l^+ \nu), \\ \Gamma(\Xi_{cc}^{++} \rightarrow \Xi_c^{*+} l^+ \nu) &= \frac{1}{2} \Gamma(\Omega_{cc}^+ \rightarrow \Omega_c^{*0} l^+ \nu). \end{aligned} \quad (39)$$

The predictions in the present work show that these relations are not strictly satisfied (see Table III), which reflects the SU(3) symmetry breaking effect. The reason can be ascribed to the fact that the masses of u and d quarks are taken to be zero in this work, however, the mass of the strange quark is not neglected.

V. CONCLUSIONS

In the present work, we firstly analyze the form factors of the transition processes $\Xi_{cc}^{++} \rightarrow \Sigma_c^{*+}$, $\Xi_{cc}^{++} \rightarrow \Xi_c^{*+}$, $\Omega_{cc}^+ \rightarrow \Xi_c^{*0}$, and $\Omega_{cc}^+ \rightarrow \Omega_c^{*0}$ in the framework of three-point QCDSRs. Considering the spin-parity of the initial and final baryons, these transition processes can be expressed as $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$. On the phenomenological side, if all couplings of interpolating currents to hadronic states are considered, there are many redundant contributions to the results. For example, the states with $J^P = \frac{1}{2}^-$ will lead to contaminations of the initial baryons, and the states $J^P = \frac{1}{2}^\pm$, $\frac{3}{2}^-$ will interfere with the final baryons. By eliminating the Dirac structures γ_μ and p'_μ and solving 16 linear equations, we successfully extracted the form factors for the process $\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$. As for the OPE on the QCD side, the perturbative part and vacuum condensate

terms of $\langle \bar{q}q \rangle$, $\langle g_s^2 GG \rangle$, $\langle \bar{q}g_s \sigma G q \rangle$, and $g_s^2 \langle \bar{q}q \rangle^2$ are considered. After the Q^2 dependence of the form factors is calculated, the numerical results are fitted to an analytical function and then extrapolated into the timelike region ($Q^2 < 0$). Using the predicted form factors, we analyze the decay properties of $\Xi_{cc}^{++} \rightarrow \Sigma_c^{*+} l^+ \nu_l$, $\Xi_{cc}^{++} \rightarrow \Xi_c^{*+} l^+ \nu_l$, $\Omega_{cc}^+ \rightarrow \Xi_c^{*0} l^+ \nu_l$, and $\Omega_{cc}^+ \rightarrow \Omega_c^{*0} l^+ \nu_l$. Although our predicted values for the form factors at zero are not consistent with those of other works, the difference becomes smaller after the numerical results are extrapolated into the timelike region. Thus, the final decay widths predicted in the present work are comparable with those of other collaborations.

More theoretical efforts in analyzing the weak decays of singly and doubly heavy baryons are still needed. Because the form factors that we have calculated in the present work are related to heavy-to-light transitions at the quark level, the heavy-to-heavy transition will be studied in the future. In addition, the form factors studied now are defined by vector and axial vector currents, while the tensor form factors are also important parameters in some other decay processes, such as the radiative and the dilepton decay modes.

ACKNOWLEDGMENTS

This project is supported by the National Natural Science Foundation under the Grant No. 12575083, and the Natural Science Foundation of the Hebei Province under the Grant No. A2024202008.

DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: FORM FACTOR OF $\mathcal{B}_1 \rightarrow \mathcal{B}_2^*$

In this appendix we first write down the matrix elements of the vector and axial vector currents J_ν^{V-A} between the baryon states with spin $\frac{1}{2}$ and $\frac{3}{2}$ [Eq. (A1)]. In these equations, $F_i^{\lambda_1 \lambda_2} / G_i^{\lambda_1 \lambda_2}$ ($i = 1 \sim 4$) and $F_i^{\prime \lambda_1 \lambda_2} / G_i^{\prime \lambda_1 \lambda_2}$ ($i = 1 \sim 3$) are the form factors, where λ_1 and λ_2 denote the parity of the initial and final states, respectively. After the interferences of the baryons with spin-parities $\frac{1}{2}^\pm$ and $\frac{3}{2}^-$ in the final states, and $\frac{1}{2}^-$ in the initial states, are eliminated, the correlation function can be written as Eq. (A2),

$$\begin{aligned}
& \langle \mathcal{B}_2^{*+}(p') | J_\nu^{V-A} | \mathcal{B}_1^+(p) \rangle \\
&= \bar{u}_a^{\mathcal{B}_2^{*+}}(p') \gamma_5 \left[\gamma_\nu \frac{P_\alpha}{m_{\mathcal{B}_1^+}} F_1^{++}(q^2) + \frac{P_\alpha P_\nu}{m_{\mathcal{B}_1^+}^2} F_2^{++}(q^2) + \frac{P_\alpha P'_\nu}{m_{\mathcal{B}_1^+} m_{\mathcal{B}_2^{*+}}} F_3^{++}(q^2) + g_{av} F_4^{++}(q^2) \right] u^{\mathcal{B}_1^+}(p) \\
&\quad - \bar{u}_a^{\mathcal{B}_2^{*+}}(p') \left[\gamma_\nu \frac{P_\alpha}{m_{\mathcal{B}_1^+}} G_1^{++}(q^2) + \frac{P_\alpha P_\nu}{m_{\mathcal{B}_1^+}^2} G_2^{++}(q^2) + \frac{P_\alpha P'_\nu}{m_{\mathcal{B}_1^+} m_{\mathcal{B}_2^{*+}}} G_3^{++}(q^2) + g_{av} G_4^{++}(q^2) \right] u^{\mathcal{B}_1^+}(p), \\
& \langle \mathcal{B}_2^{*-}(p') | J_\nu^{V-A} | \mathcal{B}_1^+(p) \rangle \\
&= \bar{u}_a^{\mathcal{B}_2^{*-}}(p') \left[\gamma_\nu \frac{P_\alpha}{m_{\mathcal{B}_1^+}} F_1^{+-}(q^2) + \frac{P_\alpha P_\nu}{m_{\mathcal{B}_1^+}^2} F_2^{+-}(q^2) + \frac{P_\alpha P'_\nu}{m_{\mathcal{B}_1^+} m_{\mathcal{B}_2^{*-}}} F_3^{+-}(q^2) + g_{av} F_4^{+-}(q^2) \right] u^{\mathcal{B}_1^+}(p) \\
&\quad - \bar{u}_a^{\mathcal{B}_2^{*-}}(p') \gamma_5 \left[\gamma_\nu \frac{P_\alpha}{m_{\mathcal{B}_1^+}} G_1^{+-}(q^2) + \frac{P_\alpha P_\nu}{m_{\mathcal{B}_1^+}^2} G_2^{+-}(q^2) + \frac{P_\alpha P'_\nu}{m_{\mathcal{B}_1^+} m_{\mathcal{B}_2^{*-}}} G_3^{+-}(q^2) + g_{av} G_4^{+-}(q^2) \right] u^{\mathcal{B}_1^+}(p), \\
& \langle \mathcal{B}_2^{*+}(p') | J_\nu^{V-A} | \mathcal{B}_1^-(p) \rangle \\
&= \bar{u}_a^{\mathcal{B}_2^{*+}}(p') \left[\gamma_\nu \frac{P_\alpha}{m_{\mathcal{B}_1^-}} F_1^{-+}(q^2) + \frac{P_\alpha P_\nu}{m_{\mathcal{B}_1^-}^2} F_2^{-+}(q^2) + \frac{P_\alpha P'_\nu}{m_{\mathcal{B}_1^-} m_{\mathcal{B}_2^{*+}}} F_3^{-+}(q^2) + g_{av} F_4^{-+}(q^2) \right] u^{\mathcal{B}_1^-}(p) \\
&\quad - \bar{u}_a^{\mathcal{B}_2^{*+}}(p') \gamma_5 \left[\gamma_\nu \frac{P_\alpha}{m_{\mathcal{B}_1^-}} G_1^{-+}(q^2) + \frac{P_\alpha P_\nu}{m_{\mathcal{B}_1^-}^2} G_2^{-+}(q^2) + \frac{P_\alpha P'_\nu}{m_{\mathcal{B}_1^-} m_{\mathcal{B}_2^{*+}}} G_3^{-+}(q^2) + g_{av} G_4^{-+}(q^2) \right] u^{\mathcal{B}_1^-}(p), \\
& \langle \mathcal{B}_2^{*-}(p') | J_\nu^{V-A} | \mathcal{B}_1^-(p) \rangle \\
&= \bar{u}_a^{\mathcal{B}_2^{*-}}(p') \gamma_5 \left[\gamma_\nu \frac{P_\alpha}{m_{\mathcal{B}_1^-}} F_1^{--}(q^2) + \frac{P_\alpha P_\nu}{m_{\mathcal{B}_1^-}^2} F_2^{--}(q^2) + \frac{P_\alpha P'_\nu}{m_{\mathcal{B}_1^-} m_{\mathcal{B}_2^{*-}}} F_3^{--}(q^2) + g_{av} F_4^{--}(q^2) \right] u^{\mathcal{B}_1^-}(p) \\
&\quad - \bar{u}_a^{\mathcal{B}_2^{*-}}(p') \left[\gamma_\nu \frac{P_\alpha}{m_{\mathcal{B}_1^-}} G_1^{--}(q^2) + \frac{P_\alpha P_\nu}{m_{\mathcal{B}_1^-}^2} G_2^{--}(q^2) + \frac{P_\alpha P'_\nu}{m_{\mathcal{B}_1^-} m_{\mathcal{B}_2^{*-}}} G_3^{--}(q^2) + g_{av} G_4^{--}(q^2) \right] u^{\mathcal{B}_1^-}(p), \\
& \langle \mathcal{B}_2^+(p') | J_\nu^{V-A} | \mathcal{B}_1^+(p) \rangle \\
&= \bar{u}^{\mathcal{B}_2^+}(p') \left[\frac{P_\nu}{m_{\mathcal{B}_1^+}} F_1'^{++}(q^2) + \frac{P'_\nu}{m_{\mathcal{B}_2^+}} F_2'^{++}(q^2) + \gamma_\nu F_3'^{++}(q^2) \right] u^{\mathcal{B}_1^+}(p) \\
&\quad - \bar{u}^{\mathcal{B}_2^+}(p') \left[\frac{P_\nu}{m_{\mathcal{B}_1^+}} G_1'^{++}(q^2) + \frac{P'_\nu}{m_{\mathcal{B}_2^+}} G_2'^{++}(q^2) + \gamma_\nu G_3'^{++}(q^2) \right] \gamma_5 u^{\mathcal{B}_1^+}(p), \\
& \langle \mathcal{B}_2^-(p') | J_\nu^{V-A} | \mathcal{B}_1^+(p) \rangle \\
&= \bar{u}^{\mathcal{B}_2^-}(p') \gamma_5 \left[\frac{P_\nu}{m_{\mathcal{B}_1^+}} F_1'^{+-}(q^2) + \frac{P'_\nu}{m_{\mathcal{B}_2^-}} F_2'^{+-}(q^2) + \gamma_\nu F_3'^{+-}(q^2) \right] u^{\mathcal{B}_1^+}(p) \\
&\quad - \bar{u}^{\mathcal{B}_2^-}(p') \gamma_5 \left[\frac{P_\nu}{m_{\mathcal{B}_1^+}} G_1'^{+-}(q^2) + \frac{P'_\nu}{m_{\mathcal{B}_2^-}} G_2'^{+-}(q^2) + \gamma_\nu G_3'^{+-}(q^2) \right] \gamma_5 u^{\mathcal{B}_1^+}(p), \\
& \langle \mathcal{B}_2^+(p') | J_\nu^{V-A} | \mathcal{B}_1^-(p) \rangle \\
&= \bar{u}^{\mathcal{B}_2^+}(p') \left[\frac{P_\nu}{m_{\mathcal{B}_1^-}} F_1'^{-+}(q^2) + \frac{P'_\nu}{m_{\mathcal{B}_2^+}} F_2'^{-+}(q^2) + \gamma_\nu F_3'^{-+}(q^2) \right] \gamma_5 u^{\mathcal{B}_1^-}(p) \\
&\quad - \bar{u}^{\mathcal{B}_2^+}(p') \left[\frac{P_\nu}{m_{\mathcal{B}_1^-}} G_1'^{-+}(q^2) + \frac{P'_\nu}{m_{\mathcal{B}_2^+}} G_2'^{-+}(q^2) + \gamma_\nu G_3'^{-+}(q^2) \right] \gamma_5 \gamma_5 u^{\mathcal{B}_1^-}(p), \\
& \langle \mathcal{B}_2^-(p') | J_\nu^{V-A} | \mathcal{B}_1^-(p) \rangle = \bar{u}^{\mathcal{B}_2^-}(p') \gamma_5 \left[\frac{P_\nu}{m_{\mathcal{B}_1^-}} F_1'^{--}(q^2) + \frac{P'_\nu}{m_{\mathcal{B}_2^-}} F_2'^{--}(q^2) + \gamma_\nu F_3'^{--}(q^2) \right] \gamma_5 u^{\mathcal{B}_1^-}(p) \\
&\quad - \bar{u}^{\mathcal{B}_2^-}(p') \gamma_5 \left[\frac{P_\nu}{m_{\mathcal{B}_1^-}} G_1'^{--}(q^2) + \frac{P'_\nu}{m_{\mathcal{B}_2^-}} G_2'^{--}(q^2) + \gamma_\nu G_3'^{--}(q^2) \right] \gamma_5 \gamma_5 u^{\mathcal{B}_1^-}(p),
\end{aligned} \tag{A1}$$

$\Pi_{\mu\nu}^{\text{phy}}(p, p')$

$$\begin{aligned}
& \lambda_{B_2^{*+}} \lambda_{B_1^{*+}} u_{\mu}^{B_2^{*+}}(p', s') \bar{u}_{\alpha}^{B_2^{*+}}(p', s') \gamma_5 \left[\frac{p_{\alpha} \gamma_{\nu}}{m_{B_1^{*+}}} F_1^{++}(q^2) + \frac{p_{\alpha} p_{\nu}}{m_{B_1^{*+}}^2} F_2^{++}(q^2) + \frac{p_{\alpha} p'_{\nu}}{m_{B_1^{*+}} m_{B_2^{*+}}} F_3^{++}(q^2) + g_{av} F_4^{++}(q^2) \right] u^{B_1^{*+}}(p, s) \bar{u}^{B_1^{*+}}(p, s) \\
&= \frac{(p'^2 - m_{B_2^{*+}}^2)(p^2 - m_{B_1^{*+}}^2)}{(p'^2 - m_{B_2^{*+}}^2)(p^2 - m_{B_1^{*+}}^2)} \\
& - \frac{\lambda_{B_2^{*+}} \lambda_{B_1^{*+}} u_{\mu}^{B_2^{*+}}(p', s') \bar{u}_{\alpha}^{B_2^{*+}}(p', s') \left[\frac{p_{\alpha} \gamma_{\nu}}{m_{B_1^{*+}}} G_1^{++}(q^2) + \frac{p_{\alpha} p_{\nu}}{m_{B_1^{*+}}^2} G_2^{++}(q^2) + \frac{p_{\alpha} p'_{\nu}}{m_{B_1^{*+}} m_{B_2^{*+}}} G_3^{++}(q^2) + g_{av} G_4^{++}(q^2) \right] u^{B_1^{*+}}(p, s) \bar{u}^{B_1^{*+}}(p, s)}{(p'^2 - m_{B_2^{*+}}^2)(p^2 - m_{B_1^{*+}}^2)} \\
& + \frac{\lambda_{B_2^{*-}} \lambda_{B_1^{*-}} u_{\mu}^{B_2^{*-}}(p', s') \bar{u}_{\alpha}^{B_2^{*-}}(p', s') \left[\frac{p_{\alpha} \gamma_{\nu}}{m_{B_1^{*-}}} F_1^{+-}(q^2) + \frac{p_{\alpha} p_{\nu}}{m_{B_1^{*-}}^2} F_2^{+-}(q^2) + \frac{p_{\alpha} p'_{\nu}}{m_{B_1^{*-}} m_{B_2^{*-}}} F_3^{+-}(q^2) + g_{av} F_4^{+-}(q^2) \right] u^{B_1^{*-}}(p, s) \bar{u}^{B_1^{*-}}(p, s)}{(p'^2 - m_{B_2^{*-}}^2)(p^2 - m_{B_1^{*-}}^2)} \\
& - \frac{\lambda_{B_2^{*-}} \lambda_{B_1^{*-}} u_{\mu}^{B_2^{*-}}(p', s') \bar{u}_{\alpha}^{B_2^{*-}}(p', s') \gamma_5 \left[\frac{p_{\alpha} \gamma_{\nu}}{m_{B_1^{*-}}} G_1^{+-}(q^2) + \frac{p_{\alpha} p_{\nu}}{m_{B_1^{*-}}^2} G_2^{+-}(q^2) + \frac{p_{\alpha} p'_{\nu}}{m_{B_1^{*-}} m_{B_2^{*-}}} G_3^{+-}(q^2) + g_{av} G_4^{+-}(q^2) \right] u^{B_1^{*-}}(p, s) \bar{u}^{B_1^{*-}}(p, s)}{(p'^2 - m_{B_2^{*-}}^2)(p^2 - m_{B_1^{*-}}^2)} \\
& + \frac{\lambda_{B_2^{*+}} \lambda_{B_1^{*-}} u_{\mu}^{B_2^{*+}}(p', s') \bar{u}_{\alpha}^{B_2^{*+}}(p', s') \left[\frac{p_{\alpha} \gamma_{\nu}}{m_{B_1^{*-}}} F_1^{-+}(q^2) + \frac{p_{\alpha} p_{\nu}}{m_{B_1^{*-}}^2} F_2^{-+}(q^2) + \frac{p_{\alpha} p'_{\nu}}{m_{B_1^{*-}} m_{B_2^{*+}}} F_3^{-+}(q^2) + g_{av} F_4^{-+}(q^2) \right] u^{B_1^{*-}}(p, s) \bar{u}^{B_1^{*-}}(p, s)}{(p'^2 - m_{B_2^{*+}}^2)(p^2 - m_{B_1^{*-}}^2)} \\
& - \frac{\lambda_{B_2^{*+}} \lambda_{B_1^{*-}} u_{\mu}^{B_2^{*+}}(p', s') \bar{u}_{\alpha}^{B_2^{*+}}(p', s') \gamma_5 \left[\frac{p_{\alpha} \gamma_{\nu}}{m_{B_1^{*-}}} G_1^{-+}(q^2) + \frac{p_{\alpha} p_{\nu}}{m_{B_1^{*-}}^2} G_2^{-+}(q^2) + \frac{p_{\alpha} p'_{\nu}}{m_{B_1^{*-}} m_{B_2^{*+}}} G_3^{-+}(q^2) + g_{av} G_4^{-+}(q^2) \right] u^{B_1^{*-}}(p, s) \bar{u}^{B_1^{*-}}(p, s)}{(p'^2 - m_{B_2^{*+}}^2)(p^2 - m_{B_1^{*-}}^2)} \\
& + \frac{\lambda_{B_2^{*-}} \lambda_{B_1^{*+}} u_{\mu}^{B_2^{*-}}(p', s') \bar{u}_{\alpha}^{B_2^{*-}}(p', s') \gamma_5 \left[\frac{p_{\alpha} \gamma_{\nu}}{m_{B_1^{*+}}} F_1^{--}(q^2) + \frac{p_{\alpha} p_{\nu}}{m_{B_1^{*+}}^2} F_2^{--}(q^2) + \frac{p_{\alpha} p'_{\nu}}{m_{B_1^{*+}} m_{B_2^{*-}}} F_3^{--}(q^2) + g_{av} F_4^{--}(q^2) \right] u^{B_1^{*+}}(p, s) \bar{u}^{B_1^{*+}}(p, s)}{(p'^2 - m_{B_2^{*-}}^2)(p^2 - m_{B_1^{*+}}^2)} \\
& - \frac{\lambda_{B_2^{*-}} \lambda_{B_1^{*+}} u_{\mu}^{B_2^{*-}}(p', s') \bar{u}_{\alpha}^{B_2^{*-}}(p', s') \left[\frac{p_{\alpha} \gamma_{\nu}}{m_{B_1^{*+}}} G_1^{--}(q^2) + \frac{p_{\alpha} p_{\nu}}{m_{B_1^{*+}}^2} G_2^{--}(q^2) + \frac{p_{\alpha} p'_{\nu}}{m_{B_1^{*+}} m_{B_2^{*-}}} G_3^{--}(q^2) + g_{av} G_4^{--}(q^2) \right] u^{B_1^{*+}}(p, s) \bar{u}^{B_1^{*+}}(p, s)}{(p'^2 - m_{B_2^{*-}}^2)(p^2 - m_{B_1^{*+}}^2)} \\
& + \dots, \tag{A2}
\end{aligned}$$

$$\begin{aligned}
F_1^{++}(Q^2) &= \frac{m_{B_1^{*+}} e^{\frac{m_{B_1^{*+}}^2}{M_1^2} + \frac{m_{B_2^{*+}}^2}{M_2^2}}}{(m_{B_2^{*+}} + m_{B_2^{*-}})(m_{B_1^{*+}} + m_{B_1^{*-}}) \lambda_{B_2^{*+}} \lambda_{B_1^{*+}}} \int_{s_{\min}}^{s_0} ds \int_{u_{\min}}^{u_0} du \{ [Q^2 - m_{B_2^{*-}} m_{B_1^{*-}} + m_{B_1^{*+}} m_{B_1^{*-}} + m_{B_2^{*+}} m_{B_2^{*-}}] \rho_4^{\text{QCD-V}}(s, u, Q^2) \\
& + \rho_1^{\text{QCD-V}}(s, u, Q^2) + [m_{B_2^{*+}} - m_{B_2^{*-}} - m_{B_1^{*-}}] \rho_2^{\text{QCD-V}}(s, u, Q^2) + [m_{B_1^{*+}} - m_{B_1^{*-}} - m_{B_2^{*-}}] \rho_3^{\text{QCD-V}}(s, u, Q^2) \}, \\
F_2^{++}(Q^2) &= - \frac{m_{B_1^{*+}}^2 e^{\frac{m_{B_1^{*+}}^2}{M_1^2} + \frac{m_{B_2^{*+}}^2}{M_2^2}}}{(m_{B_2^{*+}} + m_{B_2^{*-}})(m_{B_1^{*+}} + m_{B_1^{*-}}) \lambda_{B_2^{*+}} \lambda_{B_1^{*+}}} \int_{s_{\min}}^{s_0} ds \int_{u_{\min}}^{u_0} du \{ [Q^2 + m_{B_2^{*-}} m_{B_1^{*-}} + m_{B_1^{*+}} m_{B_1^{*-}} + m_{B_2^{*+}} m_{B_2^{*-}}] \rho_{12}^{\text{QCD-V}}(s, u, Q^2) \\
& + \rho_9^{\text{QCD-V}}(s, u, Q^2) + [m_{B_2^{*-}} + m_{B_1^{*+}} - m_{B_1^{*-}}] \rho_{11}^{\text{QCD-V}}(s, u, Q^2) + [m_{B_2^{*-}} - m_{B_2^{*+}} - m_{B_1^{*-}}] \rho_{10}^{\text{QCD-V}}(s, u, Q^2) \}, \\
F_3^{++}(Q^2) &= - \frac{m_{B_1^{*+}} m_{B_2^{*+}} e^{\frac{m_{B_1^{*+}}^2}{M_1^2} + \frac{m_{B_2^{*+}}^2}{M_2^2}}}{(m_{B_2^{*+}} + m_{B_2^{*-}})(m_{B_1^{*+}} + m_{B_1^{*-}}) \lambda_{B_2^{*+}} \lambda_{B_1^{*+}}} \int_{s_{\min}}^{s_0} ds \int_{u_{\min}}^{u_0} du \{ [Q^2 + m_{B_2^{*-}} m_{B_1^{*-}} + m_{B_1^{*+}} m_{B_1^{*-}} + m_{B_2^{*+}} m_{B_2^{*-}}] \rho_{16}^{\text{QCD-V}}(s, u, Q^2) \\
& + \rho_{13}^{\text{QCD-V}}(s, u, Q^2) + [m_{B_2^{*-}} - m_{B_1^{*-}} - m_{B_2^{*+}}] \rho_{14}^{\text{QCD-V}}(s, u, Q^2) + [m_{B_2^{*-}} + m_{B_1^{*+}} - m_{B_1^{*-}}] \rho_{15}^{\text{QCD-V}}(s, u, Q^2) \\
& + 2\rho_3^{\text{QCD-V}}(s, u, Q^2) + 2(m_{B_1^{*-}} - m_{B_2^{*-}} + m_{B_2^{*+}}) \rho_4^{\text{QCD-V}} \},
\end{aligned}$$

$$\begin{aligned}
F_4^{++}(Q^2) &= -\frac{e^{\frac{m_{B_1^+}^2}{M_1^2} + \frac{m_{B_2^{*+}}^2}{M_2^2}}}{(m_{B_2^{*+}} + m_{B_2^-})(m_{B_1^+} + m_{B_1^-})\lambda_{B_2^+}\lambda_{B_1^+}} \int_{s_{\min}}^{s_0} ds \int_{u_{\min}}^{u_0} du \{ [Q^2 + m_{B_2^+}m_{B_2^-} + m_{B_2^-}m_{B_1^-} + m_{B_1^+}m_{B_1^-}] \rho_8^{\text{QCD-V}}(s, u, Q^2) \\
&\quad + \rho_5^{\text{QCD-V}}(s, u, Q^2) + [m_{B_2^-} - m_{B_1^-} - m_{B_2^{*+}}] \rho_6^{\text{QCD-V}}(s, u, Q^2) + [m_{B_2^-} + m_{B_1^+} - m_{B_1^-}] \rho_7^{\text{QCD-V}}(s, u, Q^2) \}, \\
G_1^{++}(Q^2) &= -\frac{m_{B_1^+} e^{\frac{m_{B_1^+}^2}{M_1^2} + \frac{m_{B_2^{*+}}^2}{M_2^2}}}{(m_{B_2^{*+}} + m_{B_2^-})(m_{B_1^+} + m_{B_1^-})\lambda_{B_2^+}\lambda_{B_1^+}} \int_{s_{\min}}^{s_0} ds \int_{u_{\min}}^{u_0} du \{ [Q^2 + m_{B_2^-}m_{B_1^-} + m_{B_1^+}m_{B_1^-} + m_{B_2^+}m_{B_2^-}] \rho_4^{\text{QCD-A}}(s, u, Q^2) \\
&\quad + \rho_1^{\text{QCD-A}}(s, u, Q^2) + [m_{B_2^+} - m_{B_2^-} + m_{B_1^-}] \rho_2^{\text{QCD-A}}(s, u, Q^2) + [m_{B_1^-} - m_{B_1^+} - m_{B_2^-}] \rho_3^{\text{QCD-A}}(s, u, Q^2) \}, \\
G_2^{++}(Q^2) &= -\frac{m_{B_1^+}^2 e^{\frac{m_{B_1^+}^2}{M_1^2} + \frac{m_{B_2^{*+}}^2}{M_2^2}}}{(m_{B_2^{*+}} + m_{B_2^-})(m_{B_1^+} + m_{B_1^-})\lambda_{B_2^+}\lambda_{B_1^+}} \int_{s_{\min}}^{s_0} ds \int_{u_{\min}}^{u_0} du \{ [Q^2 + m_{B_2^-}m_{B_2^{*+}} + m_{B_1^+}m_{B_1^-} - m_{B_2^-}m_{B_1^-}] \rho_9^{\text{QCD-A}}(s, u, Q^2) \\
&\quad + \rho_{12}^{\text{QCD-A}}(s, u, Q^2) + [m_{B_2^-} + m_{B_1^-} - m_{B_2^{*+}}] \rho_{11}^{\text{QCD-A}}(s, u, Q^2) + [m_{B_2^-} + m_{B_1^-} - m_{B_1^+}] \rho_{10}^{\text{QCD-A}}(s, u, Q^2) \}, \\
G_3^{++}(Q^2) &= -\frac{m_{B_1^+}m_{B_2^{*+}} e^{\frac{m_{B_1^+}^2}{M_1^2} + \frac{m_{B_2^{*+}}^2}{M_2^2}}}{(m_{B_2^{*+}} + m_{B_2^-})(m_{B_1^+} + m_{B_1^-})\lambda_{B_2^+}\lambda_{B_1^+}} \int_{s_{\min}}^{s_0} ds \int_{u_{\min}}^{u_0} du \{ [Q^2 - m_{B_2^-}m_{B_1^-} + m_{B_1^+}m_{B_1^-} + m_{B_2^+}m_{B_2^-}] \rho_{13}^{\text{QCD-A}}(s, u, Q^2) \\
&\quad + \rho_{16}^{\text{QCD-A}}(s, u, Q^2) + [m_{B_1^-} - m_{B_1^+} + m_{B_2^-}] \rho_{14}^{\text{QCD-A}}(s, u, Q^2) + [m_{B_2^-} - m_{B_2^{*+}} + m_{B_1^-}] \rho_{15}^{\text{QCD-A}}(s, u, Q^2) \\
&\quad + 2\rho_3^{\text{QCD-A}}(s, u, Q^2) + 2(m_{B_2^+} - m_{B_1^-} - m_{B_2^-}) \rho_4^{\text{QCD-A}} \}, \\
G_4^{++}(Q^2) &= -\frac{e^{\frac{m_{B_1^+}^2}{M_1^2} + \frac{m_{B_2^{*+}}^2}{M_2^2}}}{(m_{B_2^{*+}} + m_{B_2^-})(m_{B_1^+} + m_{B_1^-})\lambda_{B_2^+}\lambda_{B_1^+}} \int_{s_{\min}}^{s_0} ds \int_{u_{\min}}^{u_0} du \{ [Q^2 + m_{B_2^+}m_{B_2^-} - m_{B_2^-}m_{B_1^-} + m_{B_1^+}m_{B_1^-}] \rho_5^{\text{QCD-A}}(s, u, Q^2) \\
&\quad + \rho_8^{\text{QCD-A}}(s, u, Q^2) + [m_{B_1^-} - m_{B_1^+} + m_{B_2^-}] \rho_6^{\text{QCD-A}}(s, u, Q^2) + [m_{B_2^-} + m_{B_1^-} - m_{B_2^{*+}}] \rho_7^{\text{QCD-A}}(s, u, Q^2) \}. \quad (\text{A3})
\end{aligned}$$

APPENDIX B: THE INTEGRAL FORMULAS FOR TWO AND THREE DIRAC DELTA FUNCTIONS

The integral formulas of two Dirac delta functions can be expressed as

$$\begin{aligned}
\int d^4k \delta(k^2 - m_1^2) \delta[(q' - k)^2 - m_4^2] &= \frac{\pi \sqrt{\lambda(r, m_1^2, m_4^2)}}{2r}, \\
\int d^4k \delta(k^2 - m_1^2) \delta[(q' - k)^2 - m_4^2] k_\mu &= \frac{\pi \sqrt{\lambda(r, m_1^2, m_4^2)}}{2r} \frac{r + m_1^2 - m_4^2}{2r} q'_\mu, \\
\int d^4k \delta(k^2 - m_1^2) \delta[(q' - k)^2 - m_4^2] k_\mu k_\nu &= \frac{\pi \sqrt{\lambda(r, m_1^2, m_4^2)}}{2r} \left[-\frac{\lambda(r, m_1^2, m_4^2)}{12r} g_{\mu\nu} + \left(m_1^2 + \frac{\lambda(r, m_1^2, m_4^2)}{3r} \right) \frac{q'_\mu q'_\nu}{q'^2} \right], \quad (\text{B1})
\end{aligned}$$

where $r = q'^2$, and $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$ is the triangle function.

The integral formulas of three Dirac delta functions can be given as follows:

$$\begin{aligned}
& \int d^4k \delta(k^2 - m_3^2) \delta[(k + p - p')^2 - m_2^2] \delta[(p' - k)^2 - m^2] = \frac{\pi}{2\sqrt{\lambda(s, u, q^2)}}, \\
& \int d^4k \delta(k^2 - m_3^2) \delta[(k + p - p')^2 - m_2^2] \delta[(p' - k)^2 - m^2] k_\mu = \frac{\pi}{2\sqrt{\lambda(s, u, q^2)}} (\alpha_1 p_\mu + \beta_1 p'_\mu), \\
& \int d^4k \delta(k^2 - m_3^2) \delta[(k + p - p')^2 - m_2^2] \delta[(p' - k)^2 - m^2] k_\mu k_\nu \\
& \quad = \frac{\pi}{2\sqrt{\lambda(s, u, q^2)}} [\alpha_2 g_{\mu\nu} + \beta_2 p_\mu p_\nu + \gamma_2 (p'_\mu p_\nu + p_\mu p'_\nu) + \delta_2 p'_\mu p'_\nu], \\
& \int d^4k \delta(k^2 - m_3^2) \delta[(k + p - p')^2 - m_2^2] \delta[(p' - k)^2 - m^2] k_\mu k_\nu k_\lambda \\
& \quad = \frac{\pi}{2\sqrt{\lambda(s, u, q^2)}} [\alpha_3 (p_\mu g_{\nu\lambda} + p_\nu g_{\mu\lambda} + p_\lambda g_{\mu\nu}) + \lambda_3 (p'_\mu g_{\nu\lambda} + p'_\nu g_{\mu\lambda} + p'_\lambda g_{\mu\nu}) + \gamma_3 p_\mu p_\nu p_\lambda \\
& \quad \quad + \rho_3 (p'_\mu p_\nu p_\lambda + p'_\nu p_\mu p_\lambda + p'_\lambda p_\mu p_\nu) + \delta_3 (p_\mu p'_\nu p'_\lambda + p_\nu p'_\mu p'_\lambda + p_\lambda p'_\nu p'_\mu) + \kappa_3 p'_\mu p'_\nu p'_\lambda], \tag{B2}
\end{aligned}$$

where $s = p^2$, $u = p'^2$, $q = p - p'$,

$$\begin{aligned}
\alpha_1 &= \frac{m^2(q^2 - s + u) + u(-2m_2^2 + q^2 + s - u) + m_3^2(-q^2 + s + u)}{\lambda(s, u, q^2)}, \\
\beta_1 &= \frac{m^2(q^2 + s - u) + m_2^2(-q^2 + s + u) - 2m_3^2s + q^4 - q^2s - 2q^2u - su + u^2}{\lambda(s, u, q^2)}, \tag{B3}
\end{aligned}$$

$$\begin{aligned}
\alpha_2 &= \frac{1}{2\lambda(s, u, q^2)} \{m^4q^2 - m^2[m_2^2(q^2 - s + u) + m_3^2(q^2 + s - u) + q^2(-q^2 + s + u)] + m_2^4u \\
& \quad + m_2^2[m_3^2(q^2 - s - u) + u(-q^2 - s + u)] + s[m_3^4 - m_3^2(q^2 - s + u) + q^2u]\}, \\
\beta_2 &= \frac{1}{\lambda(s, u, q^2)^2} \{m^4[q^4 - 2q^2(s - 2u) + (s - u)^2] - 2m^2[u(3m_2^2(q^2 - s + u) - 2q^4 + q^2(s + u) + (s - u)^2) \\
& \quad + m_3^2(q^4 + q^2(u - 2s) + s^2 + su - 2u^2)] - 2m_3^2u[3m_2^2(-q^2 + s + u) + q^4 + q^2(s - 2u) - 2s^2 + su + u^2] \\
& \quad + u^2[6m_2^4 - 6m_2^2(q^2 + s - u) + q^4 + q^2(4s - 2u) + (s - u)^2] + m_3^4[q^4 - 2q^2(s + u) + s^2 + 4su + u^2]\}, \\
\gamma_2 &= -\frac{1}{\lambda(s, u, q^2)} 2\left\{(-m^2 + m_3^2 + u)(-m^2 + m_2^2 - q^2 + u) - \frac{1}{\lambda(s, u, q^2)} 3(q^2 - s - u)[m^4q^2 - m^2(m_2^2(q^2 - s + u) \right. \\
& \quad + m_3^2(q^2 + s - u) + q^2(-q^2 + s + u)) + m_2^4u + m_2^2(m_3^2(q^2 - s - u) + u(-q^2 - s + u)) \\
& \quad \left. + s(m_3^4 - m_3^2(q^2 - s + u) + q^2u)] - 2m_3^2(-q^2 + s + u)\right\}, \\
\delta_2 &= \frac{1}{\lambda(s, u, q^2)^2} \{m^4[q^4 + q^2(4s - 2u) + (s - u)^2] - 2m^2[m_2^2(q^4 + q^2(s - 2u) - 2s^2 + su + u^2) \\
& \quad + q^2(3m_3^2s + 2s^2 + 3su - 3u^2) + (s - u)(3m_3^2s + u(s - u)) - q^6 - q^4(s - 3u)] \\
& \quad + m_2^4(q^4 - 2q^2(s + u) + s^2 + 4su + u^2) - 2m_2^2(q^2(-3m_3^2s + s^2 + 3su + 3u^2) \\
& \quad + 3m_3^2s(s + u) + q^6 - q^4(2s + 3u) - u(-2s^2 + su + u^2)) + 6m_3^4s^2 - 4m_3^2q^4s + 2m_3^2q^2s^2 + 8m_3^2q^2su \\
& \quad + 2m_3^2s^3 + 2m_3^2s^2u - 4m_3^2su^2 + q^8 - 2q^6s - 4q^6u + q^4s^2 + 2q^4su + 6q^4u^2 + 4q^2s^2u + 2q^2su^2 - 4q^2u^3 \\
& \quad + s^2u^2 - 2su^3 + u^4\}. \tag{B4}
\end{aligned}$$

The expressions for the coefficients $\alpha_3 \sim \kappa_3$ are too complicated to write down explicitly. In addition, a geometric constraint will be introduced in the integration of three Dirac delta functions, which is actually a constraint on the phase space and it has the following form:

$$-1 \leq \cos \theta = \frac{(u - q^2 + m_2^2 - m^2)(s + u - q^2) + 2s(m^2 - u - m_3^2)}{\sqrt{(u - q^2 + m_2^2 - m^2)^2 - 4sm_3^2} \sqrt{\lambda(s, u, q^2)}} \leq 1. \quad (\text{B5})$$

-
- [1] S. Navas *et al.* (Particle Data Group), *Phys. Rev. D* **110**, 030001 (2024).
- [2] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **119**, 112001 (2017).
- [3] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **121**, 162002 (2018).
- [4] R. Aaij *et al.* (LHCb Collaboration), *J. High Energy Phys.* **02** (2020) 049.
- [5] R. N. Faustov and V. O. Galkin, *Phys. Rev. D* **98**, 093006 (2018).
- [6] D. Ebert, R. N. Faustov, and V. O. Galkin, *Phys. Rev. D* **73**, 094002 (2006).
- [7] A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Körner, and V. E. Lyubovitskij, *Phys. Rev. D* **80**, 034025 (2009).
- [8] T. Gutsche, M. A. Ivanov, J. G. Körner, and V. E. Lyubovitskij, *Phys. Rev. D* **98**, 074011 (2018).
- [9] T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, and Z. Tyulemisov, *Phys. Rev. D* **100**, 114037 (2019).
- [10] M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, and A. G. Rusetsky, *Mod. Phys. Lett. A* **13**, 181 (1998).
- [11] M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, and A. G. Rusetsky, *Phys. Rev. D* **57**, 5632 (1998).
- [12] C. Albertus, E. Hernandez, and J. Nieves, *Phys. Rev. D* **71**, 014012 (2005).
- [13] H. Y. Cheng, *Phys. Rev. D* **56**, 2799 (1997).
- [14] Z. X. Zhao, *Eur. Phys. J. C* **78**, 756 (2018).
- [15] Z. X. Zhao, *Chin. Phys. C* **42**, 093101 (2018).
- [16] C. K. Chua, *Phys. Rev. D* **99**, 014023 (2019).
- [17] C. K. Chua, *Phys. Rev. D* **100**, 034025 (2019).
- [18] H. W. Ke, N. Hao, and X. Q. Li, *Eur. Phys. J. C* **79**, 540 (2019).
- [19] J. Zhu, Z. T. Wei, and H. W. Ke, *Phys. Rev. D* **99**, 054020 (2019).
- [20] Y. S. Li, X. Liu, and F. S. Yu, *Phys. Rev. D* **104**, 013005 (2021).
- [21] X. H. Hu, R. H. Li, and Z. P. Xing, *Eur. Phys. J. C* **80**, 320 (2020).
- [22] Y. S. Li and X. Liu, *Phys. Rev. D* **105**, 013003 (2022).
- [23] F. Lu, H. W. Ke, X. H. Liu, and Y. L. Shi, *Eur. Phys. J. C* **83**, 412 (2023).
- [24] C. Q. Geng, C. W. Liu, and T. H. Tsai, *Phys. Rev. D* **102**, 034033 (2020).
- [25] W. Wang, Z. P. Xing, and J. Xu, *Eur. Phys. J. C* **77**, 800 (2017).
- [26] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, *Nucl. Phys. B* **147**, 385 (1979).
- [27] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, *Nucl. Phys. B* **147**, 448 (1979).
- [28] P. Colangelo and A. Khodjamirian, *arXiv:hep-ph/0010175*.
- [29] Z. G. Wang, *Front. Phys.* **21**, 016300 (2026).
- [30] Z. G. Wang, *Eur. Phys. J. C* **78**, 826 (2018).
- [31] Z. G. Wang, *Nucl. Phys. B* **926**, 467 (2018).
- [32] S. Q. Zhang and C. F. Qiao, *arXiv:2512.24706*.
- [33] J. Lu, D. Y. Chen, G. L. Yu, Z. G. Wang, and Z. Zhou, *Eur. Phys. J. C* **85**, 1061 (2025).
- [34] Y. m. Wang, Y. Li, and C. D. Lu, *Eur. Phys. J. C* **59**, 861 (2009).
- [35] Y. M. Wang and Y. L. Shen, *J. High Energy Phys.* **02** (2016) 179.
- [36] A. Khodjamirian, C. Klein, T. Mannel, and Y. M. Wang, *J. High Energy Phys.* **09** (2011) 106.
- [37] Z. X. Zhao, R. H. Li, Y. L. Shen, Y. J. Shi, and Y. S. Yang, *Eur. Phys. J. C* **80**, 1181 (2020).
- [38] Y. J. Shi, W. Wang, and Z. X. Zhao, *Eur. Phys. J. C* **80**, 568 (2020).
- [39] K. Azizi, Y. Sarac, and H. Sundu, *Eur. Phys. J. A* **48**, 2 (2012).
- [40] T. M. Aliev, S. Bilmis, and M. Savci, *Phys. Lett. B* **847**, 138287 (2023).
- [41] L. Khajouei and K. Azizi, *Phys. Rev. D* **111**, 074018 (2025).
- [42] Y. Miao, H. Deng, K. S. Huang, J. Gao, and Y. L. Shen, *Chin. Phys. C* **46**, 113107 (2022).
- [43] Z. P. Xing and Z. X. Zhao, *Eur. Phys. J. C* **81**, 1111 (2021).
- [44] Z. X. Zhao, X. Y. Sun, F. W. Zhang, Y. P. Xing, and Y. T. Yang, *Phys. Rev. D* **108**, 116008 (2023).
- [45] S. Q. Zhang and C. F. Qiao, *Phys. Rev. D* **108**, 074017 (2023).
- [46] Z. Neishabouri, K. Azizi, and H. R. Moshfegh, *Phys. Rev. D* **110**, 014010 (2024).
- [47] M. S. Tousi, K. Azizi, and H. R. Moshfegh, *Phys. Rev. D* **110**, 114001 (2024).
- [48] J. Lu, G. L. Yu, D. Y. Chen, Z. G. Wang, and B. Wu, *Eur. Phys. J. C* **85**, 1382 (2025).
- [49] G. L. Yu, Z. Y. Li, Z. G. Wang, J. Lu, and M. Yan, *Nucl. Phys. B* **990**, 116183 (2023).
- [50] P. Pascual and R. Tarrach, *Lect. Notes Phys.* **194** (1984).
- [51] L. J. Reinders, H. Rubinstein, and S. Yazaki, *Phys. Rep.* **127**, 1 (1985).
- [52] R. E. Cutkosky, *J. Math. Phys. (N.Y.)* **1**, 429 (1960).
- [53] Z. G. Wang, *Eur. Phys. J. C* **68**, 459 (2010).

-
- [54] G. L. Yu, Z. Y. Li, Z. G. Wang, J. Lu, and M. Yan, *Eur. Phys. J. A* **59**, 126 (2023).
- [55] Z. Y. Li, G. L. Yu, Z. G. Wang, J. Z. Gu, J. Lu, and H. T. Shen, *Chin. Phys. C* **47**, 073105 (2023).
- [56] S. Narison, *Phys. Lett. B* **693**, 559 (2010).
- [57] S. Narison, *Phys. Lett. B* **706**, 412 (2012).
- [58] S. Narison, *Phys. Lett. B* **707**, 259 (2012).
- [59] C. G. Boyd, B. Grinstein, and R. F. Lebed, *Phys. Rev. Lett.* **74**, 4603 (1995).