

Constraints on invisible $B^+ \rightarrow K^+ X$ decays from the Belle II $B^+ \rightarrow K^+ \nu \bar{\nu}$ measurement

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The Belle II measurement of the branching fraction for $B^+ \rightarrow K^+ \nu \bar{\nu}$ shows a 2.7σ excess over the Standard Model prediction and motivates new-physics explanations such as axionlike particles, Higgs-like scalars, or beyond Standard Model gauge bosons. A two-body decay $B^+ \rightarrow K^+ X$ with an invisible X provides a natural candidate explanation. This work provides a comprehensive test of this hypothesis using Belle II's public model-agnostic likelihood. Posterior distributions are derived for the resonance mass m_X and the branching fraction, and a modified frequentist upper-limit mass scan is performed. The data favor a resonance with mass $m_X = 2.1^{+0.2}_{-0.1}$ GeV and the product $\mathcal{B}(B^+ \rightarrow K^+ X) \cdot \mathcal{P}_{X,\text{inv}} = 9.2^{+1.8}_{-3.4} \times 10^{-6}$, where $\mathcal{P}_{X,\text{inv}}$ is the probability that X (and its decay products) are undetected. Bayes's factors indicate a very strong preference for the Standard Model plus resonance over the Standard Model-only hypothesis. A frequentist likelihood-ratio test favors the Standard Model plus resonance hypothesis by 3.0σ . A light invisible resonance plus the Standard Model therefore provides a compelling description of the Belle II data.

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I. INTRODUCTION

Flavor-changing neutral current processes are sensitive probes of physics beyond the Standard Model (SM) because they are suppressed in the SM. Rare meson decays with missing energy are particularly interesting because they are sensitive to new light particles that escape detection. One such process is $B^+ \rightarrow K^+ \nu \bar{\nu}$, for which the Belle II Collaboration has reported evidence [1] for a 2.7σ excess over the SM.

A two-body decay $B^+ \rightarrow K^+ X$ [2,3], involving a new light boson X , provides an attractive explanation for this observed excess.¹ This scenario is defined by the boson mass m_X and the bsX interaction strength. While previous

studies explored this model [2,3,10–14], they assumed a negligible width ($\Gamma_X \ll m_X$) and relied on approximations, as detailed experimental information was unavailable.

This work provides the first rigorous test of this hypothesis by exploiting the model-agnostic likelihood of the $B^+ \rightarrow K^+ \nu \bar{\nu}$ measurement published by the Belle II Collaboration [15,16]. This allows a quantification of the preferred parameter space for an invisible resonant contribution, described by a Breit-Wigner distribution with mass m_X and decay width Γ_X .

In Sec. II the theoretical background is presented. Section III describes the Belle II analysis, the reinterpretation method, and the resonance model. Bayesian credible intervals are derived in Sec. IV, and modified frequentist upper limits from a mass scan are derived in Sec. V. Finally, goodness-of-fit and model comparison are assessed in Secs. VI and VII, respectively, before concluding in Sec. VIII.

II. THEORETICAL BACKGROUND

Models explaining the excess with a two-body decay can be distinguished by the spin of the boson X , its decay width Γ_X , and the bsX coupling. Among spin-0 bosons, axionlike particles (ALPs) provide a well-motivated explanation for the excess [2,17–20]. The decay width is model dependent, and one must distinguish cases in which X decays to undetectable particles inside the detector from cases in

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¹A light resonance has also been proposed as an explanation for the deviation from the SM prediction in $b \rightarrow s \ell \bar{\ell}$; see, e.g., Refs. [4–9] for early references.

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which it decays outside the detector. The branching fraction $\mathcal{B}(B \rightarrow KX)$ is determined by the vector coupling bsX , while the axial-vector coupling determines its contribution to $B \rightarrow K^*X$. As the ALP-fermion coupling is proportional to the fermion mass, ALP decays to neutrinos are negligible. Flavor-violating axion couplings naturally occur in flavor models [21,22] and their phenomenology has been studied in detail in Ref. [23].

Higgs-like scalars [24–27] acquire the bsX coupling through their mixing with the SM Higgs boson, which also determines their decays to SM particles. Decays to SM neutrinos are thus negligible. This results in strong constraints from searches with displaced vertices, such as $B \rightarrow K^{(*)}X(\rightarrow \mu^+\mu^-)$ [28–30]. These constraints can be avoided by introducing an additional decay channel to SM-singlet fermions [24], which suppresses $\mathcal{B}(X \rightarrow \mu^+\mu^-)$. In multi-Higgs models, the scalar X can mix with different Higgs bosons, which removes the tight relationship between the couplings of X and avoids constraints from visible decays [13,31–33]. Similarly, no tight relationship is present in models that induce the bsX coupling through the mixing of SM quarks with a heavy vectorlike fermion [34].

The proposed models with vector bosons include τ -philic gauge bosons [2,35] and dark gauge bosons [18,36]. τ -philic gauge bosons predominantly decay into undetectable τ neutrinos, which escape the Belle II detector. Dark gauge bosons acquire a bsX coupling by mixing with the Z boson. Avoiding constraints from visible decays into SM fermions requires a large decay width into undetectable dark sector particles and thus decays to neutrinos are negligible. A detailed discussion of the flavor phenomenology of light dark vector bosons is provided in Ref. [37].

All discussed models have a small bsX coupling; for example, the dimensionless bsX gauge coupling of the $B_3 - L_3$ gauge boson is $\mathcal{O}(10^{-8})$ [2] and the effective decay constant in the axionlike particle scenario is $\mathcal{O}(10^8 \text{ GeV})$ [2,18]. Similarly, existing models have only considered a small decay width with $\Gamma_X \ll m_X$. For a strongly coupled dark sector boson, the decay width Γ_X could be comparable to the mass, $\Gamma_X \sim m_X$.

To illustrate this point, consider two scenarios: a Higgs-like scalar and a dark gauge boson mixing with the Z boson, where the X partial decay width into undetectable final states, denoted Γ_X^{undet} , is constrained by invisible Higgs and Z boson decays, respectively.² The Higgs-like scalar partial width Γ_X^{undet} is constrained to [24]

$$\Gamma_X^{\text{undet}} \lesssim 0.2m_X \left(\frac{4 \times 10^{-3}}{\sin \theta} \right)^2 \left(\frac{\mathcal{B}(h \rightarrow \text{inv})}{0.107} \right) \quad (1)$$

in terms of the scalar mixing angle θ and the invisible branching ratio of the SM Higgs, $\mathcal{B}(h \rightarrow \text{inv}) \leq 0.107$ at 95% CL [38]. Similarly, the dark-gauge-boson partial

width Γ_X^{undet} into undetectable final states is given by [18,39]

$$\Gamma_X^{\text{undet}} \simeq \frac{\alpha_X m_X}{6} \lesssim \frac{N_\nu \Gamma(Z \rightarrow \nu\bar{\nu}) m_X}{4m_Z \epsilon_Z^2}, \quad (2)$$

where α_X is the dark fine structure constant and $N_\nu = 2.9963 \pm 0.0074$ [40,41] is the number of neutrinos. Due to the small $X - Z$ mixing angle $\epsilon_Z \simeq 10^{-4.6}$ [18] required to explain the excess, the constraint from the number of neutrinos N_ν is weaker than the requirement of a perturbative dark gauge coupling.

In conclusion, the X partial width into undetectable final states, Γ_X^{undet} , can be as large as 0.4 GeV for a Higgs-like scalar and even comparable to the mass for a dark gauge boson mixing with the Z boson. X decays to neutrinos are negligible and hence the interference of the two-body decay with the SM can be neglected. Consequently, the analysis considers both narrow and broad decay widths and does not consider interference with the SM. Note that the large decay width has to arise predominantly from decays to undetectable particles, and hence contributes to the invisible $B^+ \rightarrow K^+X$ signature.

III. ANALYSIS AND REINTERPRETATION METHOD

A. Belle II $B^+ \rightarrow K^+\nu\bar{\nu}$ analysis

The Belle II $B^+ \rightarrow K^+\nu\bar{\nu}$ analysis [1] uses two complementary strategies on nearly orthogonal datasets from $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B^+B^-$: an inclusive tagging analysis (ITA) and a hadronic tagging analysis (HTA). The ITA reconstructs the K^+ and infers missing energy using two boosted decision trees (BDT₁, BDT₂) for background suppression, achieving high efficiency. The HTA fully reconstructs the companion B and applies one classifier (BDTh), yielding lower background at the price of a reduced efficiency.

The $B^+ \rightarrow K^+\nu\bar{\nu}$ signal is based on a SM kinematic prediction [42], scaled by a normalization factor μ_{SM} . A HISTFACTORY [43] likelihood implemented in PYHF [44,45] uses 4×3 bins in $\eta(\text{BDT}_2) \times q_{\text{rec}}^2$ (ITA)³ and 6 bins in $\eta(\text{BDTh})$ (HTA), with an off-resonance control region (ITA) constraining continuum backgrounds. The combined fit with 231 nuisance parameters⁴ yielded $\mu_{\text{SM}} = 4.6 \pm 1.3$, corresponding to $\mathcal{B}(B^+ \rightarrow K^+\nu\bar{\nu}) = (2.3 \pm 0.5(\text{stat})_{-0.4}^{+0.5}(\text{syst})) \times 10^{-5}$, with 3.5σ significance over background-only and 2.7σ over the SM [based on

³The reconstructed q^2 is defined as $q_{\text{rec}}^2 = \frac{s}{4} + M_K^2 - \sqrt{s}E_K^*$, where $\sqrt{s}$ is the center-of-mass collision energy, M_K is the nominal kaon mass, and E_K^* is the reconstructed energy of the kaon in the collision center-of-mass frame. The signal B meson is assumed to be at rest in the e^+e^- center-of-mass frame.

⁴A full list of systematic uncertainties and corresponding nuisance parameters can be found in Ref. [1].

²See Appendix A for details.

High Precision Quantum Chromodynamics (HPQCD) form factors [46]].

B. Model-agnostic likelihood

The measurement is reinterpreted using a model-agnostic likelihood that isolates reconstruction effects from kinematic variations [15,47]. The key ingredient is the joint number density $\nu_0(x, z)$, which represents the expected event distribution in reconstructed bins x for a kinematic variable z under a null hypothesis $\sigma_0(z)$. This density encodes everything from detector response to analysis procedure, effectively folding theoretical distributions into reconstructed ones.

For an alternative theory with kinematic prediction $\sigma_1(z)$, expected rates are obtained by reweighting: $\nu_1(x) = \int dz \nu_0(x, z) w(z)$, with $w(z) = \sigma_1(z)/\sigma_0(z)$. A discrete sum is used, $\nu_{1,x} = \sum_z \nu_{0,xz} w_z$, which is computationally fast and requires only the precomputed joint density and the ratio of predictions.

The variable $z = q^2$ represents the true dineutrino invariant mass squared, and x corresponds to the analysis bins: $\eta(\text{BDT}_2) \times q_{\text{rec}}^2$ (ITA) and $\eta(\text{BDTh})$ (HTA). The joint density uses 100 q^2 bins (plus one negative- q^2 bin for events outside this range due to issues with Monte Carlo truth matching) derived from SM signal simulation. The reweighting is implemented as a custom modifier in the PYHF/HISTFACTORY framework, combined with the published $B^+ \rightarrow K^+ \nu \bar{\nu}$ likelihood.

The method's main limitation arises when $\sigma_1(z)$ is substantially larger than $\sigma_0(z)$, inflating weights and reducing effective precision. In this analysis, all weights satisfy $w(z) < 5$, and hence show no sign of excessive inflation.

C. Signal model

To test a light invisible resonance contributing to the $B^+ \rightarrow K^+ \nu \bar{\nu}$ signal, the decay $B^+ \rightarrow K^+ X$ with X invisible is modeled. A two-body resonance produces a peak at $q^2 = m_X^2$, described by a relativistic Breit-Wigner density:

$$f_{\text{BW}}(q^2 | m_X, \Gamma_X) = \frac{k}{(q^2 - m_X^2)^2 + m_X^2 \Gamma_X^2}, \quad k = \frac{m_X \Gamma_X}{\pi/2 + \arctan(m_X/\Gamma_X)}, \quad (3)$$

where m_X is the resonance mass, Γ_X is the decay width, and k ensures normalization. Since both the B and K mesons are spinless, the decay kinematics do not depend on the spin of the boson X . Consequently, the analysis is insensitive to this quantity.

Adding the two-body model to the $B^+ \rightarrow K^+ \nu \bar{\nu}$ SM expectation,

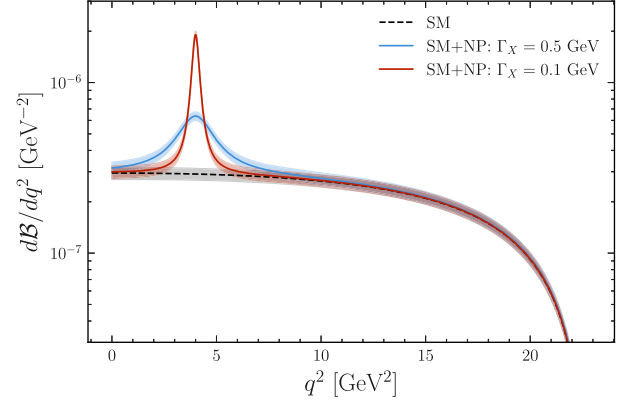


FIG. 1. Predicted differential branching fraction from Eq. (5) for two resonance widths at $\mu_X = 10^{-6}$ and $m_X = 2$ GeV. Bands include hadronic form factor and 5% SM normalization uncertainties. Form factor uncertainties on the resonance contribution are not considered.

$$\sigma_0(q^2) = \mu_{\text{SM}} \frac{d\mathcal{B}_{\text{SM}}}{dq^2}, \quad (4)$$

the total differential branching fraction is

$$\sigma_1(q^2) = \frac{d\mathcal{B}}{dq^2} = \mu_{\text{SM}} \frac{d\mathcal{B}_{\text{SM}}}{dq^2} + \mu_X f_{\text{BW}}(q^2 | m_X, \Gamma_X), \quad (5)$$

with the SM signal strength μ_{SM} . The new-physics signal strength is μ_X , such that $\mathcal{B}(B^+ \rightarrow K^+ X) \mathcal{P}_{X,\text{inv}} = \mu_X$, where $\mathcal{P}_{X,\text{inv}} = \mathcal{P}_{X,\text{out}} + (1 - \mathcal{P}_{X,\text{out}}) \cdot \mathcal{B}(X \rightarrow \text{undet})$ is the sum of the probability for the particle to decay outside the detector coverage, $\mathcal{P}_{X,\text{out}}$, and the probability to decay into undetectable final states inside the detector coverage (see, e.g., Ref. [48]). Here $\mathcal{B}(X \rightarrow \text{undet})$ denotes the branching fraction for X decays to undetectable final states. For the large X decay widths into undetectable final states considered in this analysis, $\mathcal{P}_{X,\text{inv}} \approx \mathcal{B}(X \rightarrow \text{undet})$.

The $B^+ \rightarrow K^+ \nu \bar{\nu}$ SM contribution is treated as background. Interference effects with the new-physics signal are neglected following the discussion in Sec. II. The SM prediction is based on the HPQCD form factors, and their uncertainties are included as systematic uncertainties [46]. In addition, μ_{SM} is constrained around unity by an extra 5% normalization uncertainty, applied on top of the other systematics. This accounts for the 4.4% uncertainty on $|V_{ts}^* V_{tb}|^2$ and the 2.3% uncertainty on the SM Wilson coefficient $|C_{\text{VL}}^{\text{SM}}|^2$ [42].

The parameters μ_X and m_X are inferred for two fixed widths, $\Gamma_X \in \{0.1, 0.5\}$ GeV. The coarse q_{rec}^2 binning used in the Belle II analysis [1] limits the sensitivity to narrow structures in the spectrum. As a result, widths below ~ 0.1 GeV produce bin-averaged distributions that are indistinguishable from the case $\Gamma_X = 0.1$ GeV. This value effectively represents the narrow-width limit and covers all narrow-width models discussed above. The broader width

TABLE I. Prior ranges for the new-physics signal strength μ_X and resonance mass m_X , for the two resonance widths.

$\Gamma_X =$	0.1 GeV	0.5 GeV
μ_X	[0.0, 24.0]	[0.0, 32.0]
m_X [GeV]	[1.5, 3.0]	[1.5, 3.2]

$\Gamma_X = 0.5$ GeV tests a strongly coupled boson which can be realized in scenarios such as Higgs-like scalars or dark gauge bosons. A future Belle II analysis with finer q_{rec}^2 binning would significantly improve the mass resolution. Figure 1 shows the predicted differential branching fraction for both widths, assuming $\mu_X = 10^{-6}$ and $m_X = 2$ GeV.

IV. POSTERIOR AND CREDIBLE INTERVALS

This section presents the Bayesian reinterpretation of the Belle II $B^+ \rightarrow K^+ \nu \bar{\nu}$ measurement using the signal model described in Eq. (5). Posterior distributions and credible intervals are derived for the signal strength μ_X and mass m_X , assuming fixed resonance widths of $\Gamma_X = 0.1$ GeV and 0.5 GeV.

The HISTFACTORY likelihood [43] is translated into a Bayesian posterior for inference,⁵

$$f(\boldsymbol{\eta}, \boldsymbol{\chi} | \mathbf{n}, \mathbf{a}) \propto f(\mathbf{n} | \boldsymbol{\nu}(\boldsymbol{\eta}, \boldsymbol{\chi})) f(\boldsymbol{\chi} | \mathbf{a}) f(\boldsymbol{\eta}). \quad (6)$$

Here, $f(\mathbf{n} | \boldsymbol{\nu}(\boldsymbol{\eta}, \boldsymbol{\chi}))$ is the likelihood, given observed and expected bin yields $\mathbf{n}$ and $\boldsymbol{\nu}(\boldsymbol{\eta}, \boldsymbol{\chi})$, respectively; $f(\boldsymbol{\chi} | \mathbf{a})$ encodes priors for nuisance parameters $\boldsymbol{\chi}$ using auxiliary data $\mathbf{a}$ (normally distributed), and $f(\boldsymbol{\eta})$ specifies priors for unconstrained parameters $\boldsymbol{\eta}$. Prior distributions for the parameters of interest are chosen to be uniform over the ranges specified in Table I. Ranges are chosen to cover the full posterior, as verified *a posteriori*. Appendix B examines the robustness of the results with respect to alternative prior choices. The posterior is implemented and sampled with BAYESIAN PYHF [49] using PYMC [50] as the back end.

Marginal posteriors for μ_X and m_X are obtained by sampling the joint posterior with Markov chain Monte Carlo and marginalizing over nuisance parameters. Figure 2 shows 1- and 2-dimensional marginal distributions for the two widths, with contours enclosing 68% and 95% credible regions.

From 1-dimensional marginal distributions, highest-density intervals (HDIs) are calculated at 68% and 95% probability. Table II summarizes posterior modes and credible intervals.

Direct comparison of the observed data with the predicted yields at the posterior mode for the unconstrained $B^+ \rightarrow K^+ \nu \bar{\nu}$ SM (μ_{SM} unconstrained, $\mu_X = 0$) and

⁵Throughout this work, f denotes the relevant probability density or likelihood, identified by its arguments.

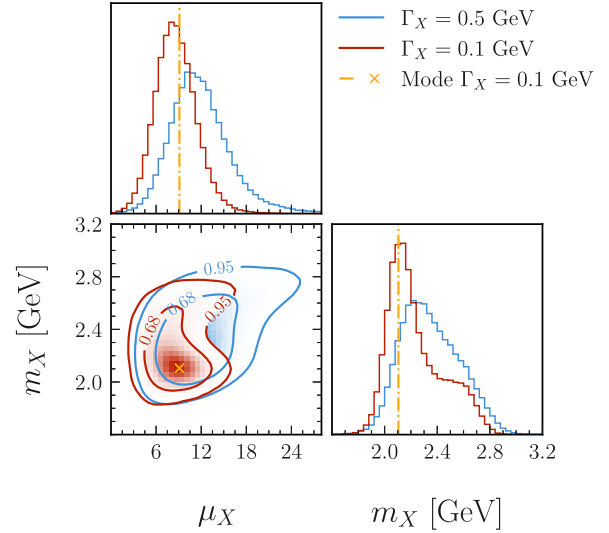


FIG. 2. Marginalized posterior distributions for $B^+ \rightarrow K^+ X$ model parameters from Eq. (5) for two resonance widths $\Gamma_X = 0.1$ and 0.5 GeV. Diagonal panels show 1-dimensional densities; the off-diagonal panel shows 2-dimensional densities (linear scale). Contours enclose 68% and 95% credible regions. The dash-dotted yellow lines and cross mark the posterior mode.

$B^+ \rightarrow K^+ X$ ($\Gamma_X = 0.1$ GeV) models is illustrated in Fig. 3, for the highest sensitivity region of the analysis (ITA, $\eta(\text{BDT}_2) > 0.98$). The $B^+ \rightarrow K^+ X$ model fits the data significantly better than the unconstrained SM prediction, as indicated by the smaller pull values.

For the considered widths, m_X peaks sharply at 2.1–2.2 GeV with an extended tail toward higher masses. The peak location is consistent across widths, indicating the data favor a resonance in this range regardless of the assumed width. This agrees with the excess observed in Ref. [1] and its interpretations presented in Refs. [2,3,11].

The new-physics signal strength μ_X deviates clearly from zero for both widths, indicating a preference for a nonzero $B^+ \rightarrow K^+ X$ contribution. Posterior modes increase with width: $\mu_X \times 10^6 = 9.2$ and 11.1 for $\Gamma_X = 0.1$ and 0.5 GeV, respectively. This is expected because broader resonances distribute the same total rate over a larger q^2 range, requiring higher normalization to match the observed excess.

TABLE II. Posterior mode and HDIs at 68% and 95% credible levels for $B^+ \rightarrow K^+ X$ model parameters from Eq. (5), derived from Fig. 2. The product of branching fractions is $\mathcal{B}(B^+ \rightarrow K^+ X) \cdot \mathcal{P}_{X,\text{inv}} = \mu_X$.

Parameter	Γ_X [GeV]	Mode	68% HDI	95% HDI
$\mu_X \times 10^6$	0.1	9.2	[5.8, 11.0]	[3.4, 14.0]
	0.5	11.1	[7.8, 15.0]	[4.2, 20.2]
m_X [GeV]	0.1	2.1	[2.0, 2.3]	[1.9, 2.7]
	0.5	2.2	[2.1, 2.5]	[2.0, 2.8]

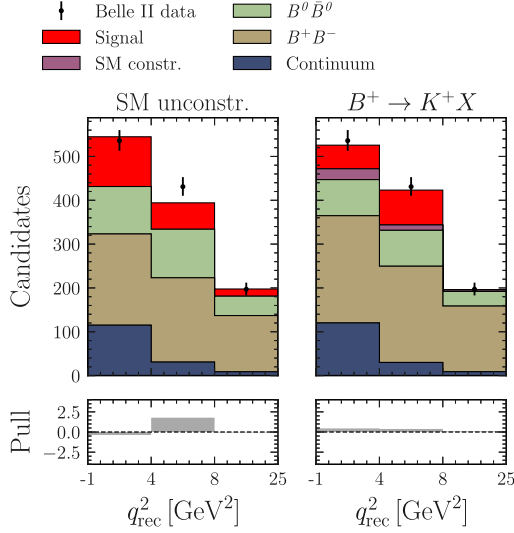


FIG. 3. Observed and predicted best-fit yields in the highest sensitivity region. Signal predictions are for the unconstrained $B^+ \rightarrow K^+ \nu \bar{\nu}$ SM (left) and $B^+ \rightarrow K^+ X$ with $\Gamma_X = 0.1$ GeV (right). For $B^+ \rightarrow K^+ X$, the constrained $B^+ \rightarrow K^+ \nu \bar{\nu}$ SM background is shown separately. Background yields include neutral and charged B -meson decays and summed continuum categories. Lower panels show pulls.

The narrow resonance ($\Gamma_X = 0.1$ GeV) yields

$$\mathcal{B}(B^+ \rightarrow K^+ X) \cdot \mathcal{P}_{X,\text{inv}} = 9.2_{-3.4}^{+1.8} \times 10^{-6}, \quad (7)$$

roughly twice the $B^+ \rightarrow K^+ \nu \bar{\nu}$ SM expectation $\mathcal{B}_{\text{SM}} = (4.97 \pm 0.37) \times 10^{-6}$ [42].

V. UPPER LIMIT MASS SCAN

A modified frequentist scan over the resonance mass m_X is performed to derive upper limits at the 95% confidence level on the product of branching fractions $\mathcal{B}(B^+ \rightarrow K^+ X) \cdot \mathcal{P}_{X,\text{inv}}$. The confidence limit is determined using the CL_s criterion [51],

$$CL_s = \frac{p(q_{\text{obs}} | B^+ \rightarrow K^+ X + \text{SM})}{p(q_{\text{obs}} | \text{SM})} = 0.05. \quad (8)$$

Here q_{obs} is the upper limit test statistic evaluated on data, and $p(q_{\text{obs}} | B^+ \rightarrow K^+ X + \text{SM})$ and $p(q_{\text{obs}} | \text{SM})$ are the corresponding p -values under the $B^+ \rightarrow K^+ X + \text{SM}$ and SM-only hypotheses, respectively.

Figure 4 shows the observed and expected limits. The observed limit exceeds the SM expectation by over 2 standard deviations in the mass range $m_X \in [1.8, 2.8]$ GeV for $\Gamma_X = 0.1$ GeV, and $m_X \in [1.7, 3.0]$ GeV for $\Gamma_X = 0.5$ GeV.

These numerical results agree well with the upper credible intervals from Sec. IV. The observed upper limits exhibit a similar shape to the contours of the 2-dimensional posterior in Fig. 2.

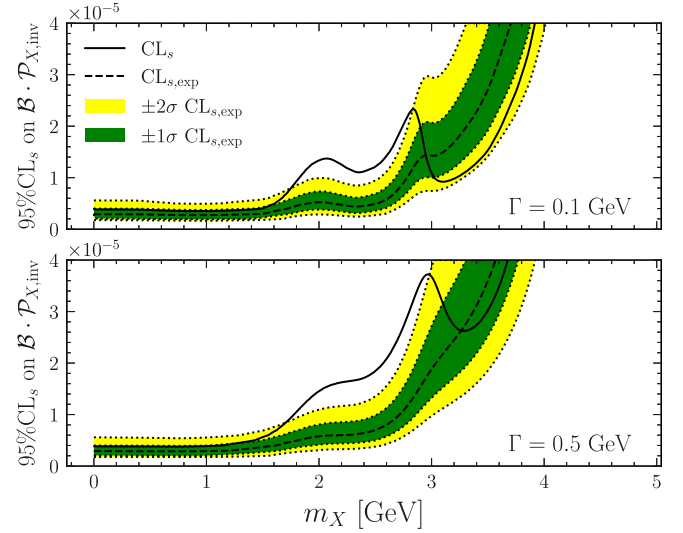


FIG. 4. Modified frequentist 95% CL_s upper limit on $\mathcal{B}(B^+ \rightarrow K^+ X) \cdot \mathcal{P}_{X,\text{inv}}$ as a function of m_X (solid black), compared to the expected SM limit (dashed black) with $\pm 1\sigma$ (green) and $\pm 2\sigma$ (yellow) bands.

At higher masses ($m_X > 3.5$ GeV), sensitivity degrades due to reduced experimental efficiency. Localized features in the expected limit near 2 and 3 GeV correspond to analysis bin boundaries.

These results are compared to the preliminary dedicated search for $B \rightarrow KX$ using 711 fb $^{-1}$ Belle data [52]. This reinterpretation yields stronger constraints in the low-mass region and uniquely retains sensitivity around $m_X \approx 2$ GeV, a region excluded in the dedicated search. Conversely, the dedicated analysis achieves superior sensitivity at high masses.

VI. GOODNESS OF FIT

The fit quality of the $B^+ \rightarrow K^+ X$ hypothesis is assessed with a saturated-likelihood statistic:

$$p_{\text{gof}} = \int_{t_{\text{obs}}}^{\infty} dt_{\text{gof}} f(t_{\text{gof}}), \quad t_{\text{gof}} = -2 \ln \frac{f(\mathbf{n}, \mathbf{a} | \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\chi}})}{f_{\text{sat}}(\mathbf{n}, \mathbf{a} | \hat{\boldsymbol{\chi}})}, \quad (9)$$

where $f(\mathbf{n}, \mathbf{a} | \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\chi}})$ is the likelihood at the global best fit, and f_{sat} is the saturated likelihood that represents a perfect fit to the observed data.

The sampling distribution $f(t_{\text{gof}})$ is obtained from fits to toy data generated at the best fit of each $B^+ \rightarrow K^+ X$ width [53], and p_{gof} is the fraction with $t > t_{\text{obs}}$.

The goodness-of-fit values, summarized in Table III, are high and nearly width independent: $p_{\text{gof}} = 0.83, 0.82$ for $\Gamma_X = 0.1$ and 0.5 GeV, respectively. Thus about 80% of toys yield worse fits than the data, indicating excellent fit quality. The weak dependence on Γ_X reflects the limited resolution due to the coarse q_{rec}^2 binning.

TABLE III. Bayes's factors relative to the background-only model (BKG) and the constrained SM, and goodness-of-fit p -values for $B^+ \rightarrow K^+ X$ at the two considered widths.

Model	Γ_X	$\log_{10} B_{\text{BKG}}$	$\log_{10} B_{\text{SM}}$	p_{gof}
$B^+ \rightarrow K^+ X$	0.1 GeV	2.82	1.71	0.83
$B^+ \rightarrow K^+ X$	0.5 GeV	2.93	1.83	0.82
unconst. SM	...	2.02	0.92	0.58

For comparison, the unconstrained SM yields $p_{\text{gof}} = 0.58$, which also indicates a good fit to the data.

VII. MODEL COMPARISON

Model comparison quantifies the *relative* support for competing hypotheses. The $B^+ \rightarrow K^+ X$ model is assessed against two references: background only ($\mu_{\text{SM}} = \mu_X = 0$) and the constrained SM (μ_{SM} constrained, $\mu_X = 0$), using Bayes's factors and frequentist hypothesis testing.

Bayesian comparison uses the Bayes's factor (ratio of marginal likelihoods) [54] shown in Table III. For $\Gamma_X = 0.1$ and 0.5 GeV the values are $\log_{10} B_{\text{SM}} = 1.71$ and 1.83 (*very strong* preference over the constrained SM on Jeffreys's scale [55]) and $\log_{10} B_{\text{BKG}} = 2.82$ and 2.93 (*decisive* preference over the background-only hypothesis). The $B^+ \rightarrow K^+ X$ model is preferred over the unconstrained SM, consistent with a resonance better capturing the excess near $q_{\text{rec}}^2 \sim 3\text{--}7 \text{ GeV}^2$.⁶ Because the marginal likelihood integrates over the full parameter space, these Bayes's factors already include the look-elsewhere effect [56] and thus represent global, not merely local, preference for $B^+ \rightarrow K^+ X$.

To complement the Bayesian comparison, $B^+ \rightarrow K^+ X$ is tested against the constrained SM with a likelihood-ratio statistic and toy-based p -values:

$$p = \int_{t_{\text{obs}}}^{\infty} dt f(t), \quad t = -2 \ln \frac{f(\mathbf{n}, \mathbf{a} | \boldsymbol{\eta} = \mathbf{0}, \hat{\boldsymbol{\chi}})}{f(\mathbf{n}, \mathbf{a} | \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\chi}})}, \quad (10)$$

where $f(\mathbf{n}, \mathbf{a} | \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\chi}})$ is the global best fit and $f(\mathbf{n}, \mathbf{a} | \boldsymbol{\eta} = \mathbf{0}, \hat{\boldsymbol{\chi}})$ fixes the parameters of interest to their null values while optimizing nuisances. The sampling distribution $f(t)$ is obtained from toys generated under the SM hypothesis, and p is the fraction with $t > t_{\text{obs}}$.

For $B^+ \rightarrow K^+ X$ with $\Gamma_X = 0.1$ GeV, the resulting global p -value is 1.4×10^{-3} , a global 3.0σ deviation from the SM. This exceeds the 2.7σ in Ref. [1] because the

⁶The preference of the $B^+ \rightarrow K^+ X$ model over the unconstrained SM can be quantified by the difference between the logarithms of the Bayes's factors given in Table III. This is $\log_{10} B(B^+ \rightarrow K^+ X) - \log_{10} B(\text{unconst. SM}) = 0.8$ and 0.9 for $\Gamma_X = 0.1$ and 0.5 GeV, respectively, indicating a *substantial* preference for $B^+ \rightarrow K^+ X$ over the unconstrained SM on Jeffreys's scale.

$B^+ \rightarrow K^+ X$ shape matches the observed excess better than an overall enhancement of the SM distribution.

The look-elsewhere effect in m_X is included, as toys are drawn under the constrained $B^+ \rightarrow K^+ \nu \bar{\nu}$ SM and each fit floats μ_X and m_X .

VIII. CONCLUSION

This work presents a reinterpretation of the Belle II measurement of $B^+ \rightarrow K^+ \nu \bar{\nu}$ using the published model-agnostic likelihood to test the hypothesis of a two-body decay $B^+ \rightarrow K^+ X$ into a light invisible boson X . The data strongly prefer the resonance hypothesis, with a posterior mode for the mass at $m_X = 2.1^{+0.2}_{-0.1}$ GeV and a product $\mathcal{B}(B^+ \rightarrow K^+ X) \cdot \mathcal{P}_{X,\text{inv}} = 9.2^{+1.8}_{-3.4} \times 10^{-6}$. A modified frequentist mass scan confirms the excess is localized around $m_X \approx 2.1$ GeV, consistent with the Bayesian results, and provides the most stringent upper limits for the $m_X < 2.1$ GeV region to date.

Bayesian model comparison indicates a very strong preference for the $B^+ \rightarrow K^+ X$ model over the constrained SM and a decisive preference over the background-only hypothesis. A complementary frequentist hypothesis test favors $B^+ \rightarrow K^+ X$ over the constrained SM with a global significance of 3.0σ . This significance exceeds the 2.7σ reported by the Belle II Collaboration [1] because the kinematic shape of the resonance better matches the observed excess than an overall enhancement of the SM distribution.

The central values for the mass and branching fraction are consistent with previous reinterpretations based on simplified likelihoods [2,3,11]. While uncertainty intervals vary across studies, this analysis uniquely incorporates full experimental uncertainties, providing the most robust estimates to date.

The analysis demonstrates that a light invisible resonance provides an excellent description of the Belle II data, significantly outperforming the SM. This motivates further measurements to clarify the presence of contributions beyond the SM.

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DATA AVAILABILITY

Posterior samples for the results presented in this work are publicly available on Zenodo [57]. A lightweight code repository for performing simple fits and reproducing the posterior plots is also available [58].

APPENDIX A: DETAILS FOR EXAMPLE MODELS WITH A LARGE DECAY WIDTH

For completeness, we provide the relevant parts of the Lagrangians for the two example models in Sec. II with a large decay width of the boson X .

The minimal model of the Higgs-like real scalar X requires the additional interactions [24]

$$\mathcal{L} \supset -\kappa H^\dagger H X - \frac{1}{2} \lambda_{HX} H^\dagger H X^2 - \left(\frac{1}{2} N^T C (\mu_N + y_X X) N + y_\nu \bar{L} H N + \text{H.c.} \right) \quad (\text{A1})$$

where H denotes the SM Higgs doublet and N is a SM-singlet fermion that allows X to decay into undetectable SM-singlet fermions N . The terms in the first line introduce mixing between the Higgs boson h and X , parametrized by $\sin \theta$, and contribute to invisible Higgs decays $h \rightarrow XX$. The mass of N is given by $y_X \langle X \rangle$. The SM-singlet fermion can further decay to neutrinos through the neutrino Yukawa interaction $\bar{L} H N$. For simplicity, we take the N mass to be negligible and set $\kappa = 0$ in Eq. (1). Nonzero κ relaxes the relationship between $\sin \theta$ and $h \rightarrow XX$. See Sec. V in Ref. [24] for further details.

An example model for a dark gauge boson X has been introduced in Ref. [18]. The model additionally introduces a dark fermion χ , which is a dark matter candidate and provides an efficient way for X to decay into undetectable dark fermions, a dark Higgs boson ϕ , which gives mass to the dark gauge boson X , and an electroweak doublet scalar η , which introduces mass mixing between the dark gauge boson X and the SM Z boson. The additional contribution to the mixing of X with the Z boson is important to enhance the coupling of X to the weak neutral current J_{NC}^μ relative to the coupling to the electromagnetic current J_{em}^μ in

$$\mathcal{L} \supset \left(e \epsilon J_{\text{em}}^\mu + \frac{g}{2 \cos \theta_w} \epsilon_Z J_{\text{NC}}^\mu \right) X_\mu, \quad (\text{A2})$$

TABLE IV. Posterior modes and 68%/95% HDIs for $B^+ \rightarrow K^+ X$ model [Eq. (5)] with $\Gamma_X = 0.1$ GeV based on alternative priors (compare Table II).

Priors	Parameter	Mode	68% HDI	95% HDI
Eq. (B1)	$\mu_X \times 10^6$	9.0	[5.7, 10.8]	[3.4, 13.8]
	m_X [GeV]	2.1	[2.0, 2.4]	[1.9, 2.7]
Eq. (B2)	$\mu_X \times 10^6$	9.0	[5.7, 11.0]	[3.0, 14.4]
	m_X [GeV]	2.1	[2.0, 2.4]	[1.9, 2.7]

$$J_{\text{NC}}^\mu = \sum_f (T_{3f} - 2Q_f \sin^2 \theta_w) \bar{f} \gamma^\mu f - T_{3f} \bar{f} \gamma^\mu f, \quad (\text{A3})$$

$$J_{\text{em}}^\mu = \sum_f Q_f \bar{f} \gamma^\mu f, \quad (\text{A4})$$

where ϵ denotes the kinetic mixing, ϵ_Z parametrizes the coupling of X to the weak neutral current, f labels SM fermions, Q_f is its electric charge, and T_3 is the third generator of $\text{SU}(2)_L$. See Sec. III in Ref. [18] for further details.

APPENDIX B: PRIOR SENSITIVITY STUDY

Bayesian inference depends on prior distributions. Robustness is assessed by comparing baseline uniform priors to two alternative specifications for the $B^+ \rightarrow K^+ X$ model [Eq. (5)] with $\Gamma_X = 0.1$ GeV.

The first alternative uses a truncated-normal prior for μ_X and a uniform prior for m_X :

$$f(\mu_X) = \begin{cases} \mathcal{N}(\mu_X | \mu = 0, \sigma = 20) & \mu_X \geq 0 \\ 0 & \mu_X < 0 \end{cases}, \quad f(m_X) = \mathcal{U}([1.5, 3.0] \text{ GeV}), \quad (\text{B1})$$

where $\mathcal{U}$ denotes a uniform distribution.

The second alternative adopts a uniform prior in squared mass m_X^2 , yielding a triangular prior in m_X :

$$f(\mu_X) = \mathcal{U}([0, 24]), \quad f(m_X) \propto \begin{cases} m_X & m_X \leq 4.8 \text{ GeV} \\ 0 & m_X > 4.8 \text{ GeV} \end{cases} \quad (\text{B2})$$

Table IV summarizes posterior modes and credible intervals for both alternatives. The mass m_X is robustly determined: the posterior mode ($m_X = 2.1$ GeV) and credible intervals show minimal prior sensitivity. The new-physics signal strength μ_X shifts slightly from 9.2 (uniform) to 9.0 (truncated-normal and triangular), with similarly robust credible intervals.

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