

QCD analysis of $B \rightarrow \pi\pi$ form factors and the $|V_{ub}|$ extraction from B_{l4} decays

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We present a comprehensive QCD analysis of the $B \rightarrow \pi\pi$ form factors within the framework of light-cone sum rules, employing two-pion light-cone distribution amplitudes (2π DAs). For the first time, we derive the twist-three 2π DAs explicitly by exploiting the conformal symmetry of massless QCD. This breakthrough enables the first systematic calculation of higher-power corrections to the $B \rightarrow \pi\pi$ -type transitions. Leveraging the recently measured partial decay fractions of $B^+ \rightarrow \pi^+\pi^-l^+\nu_l$ by the Belle detector, we obtain $|V_{ub}| = (4.27 \pm 0.49)_{\text{data}} \pm 0.55_{\text{LCSR}} \times 10^{-3}$ within the dominant region of the ρ resonance. We clarify that the relatively small value of $|V_{ub}|$ extracted from $B \rightarrow \rho l\nu$, when compared to the golden channel $B \rightarrow \pi l\nu$, arises due to the finite width and nonresonant effects. We anticipate future measurements of purely isovector and isoscalar B_{l4} decays, which will aid in reducing uncertainties by enhancing our understanding of the $\pi\pi$ phase shifts in the two-pion distribution amplitudes through heavy flavor decays.

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Introduction. The determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{ub} is a high priority in the era of precision flavor physics for uncovering new interactions beyond the Standard Model [1–3]. Semileptonic decays induced by the charged transition $b \rightarrow ul\nu$ offer two avenues to determine $|V_{ub}|$, namely through the processes with inclusive and exclusive final states. For a long time, the inclusive determination has exhibited approximately 2.5σ discrepancy compared to the exclusive determination [4,5]. This inconsistency, known as the $|V_{ub}|$ puzzle, remains unresolved to date.

Compared to the inclusive approach, the exclusive determination of $|V_{ub}|$ has significantly smaller uncertainty due to the superior signal-to-background ratio and the increasingly precise QCD predictions [6–11]. Recently, the Belle II collaboration conducted simultaneous measurements of the partial branching fractions for $B^0 \rightarrow \pi^-l^+\nu_l$ and $B^+ \rightarrow \rho^0l^+\nu_l$ decays [12]. They derived $|V_{ub}|$ from these two modes by leveraging constraints on

the form factors (FFs) from lattice QCD (LQCD) [5] and the light-cone sum rules (LCSRs) [13],

$$|V_{ub}|_{B \rightarrow \pi l\nu} = (3.73 \pm 0.10 \pm 0.16_{\text{LQCD+LCSRs}}) \times 10^{-3},$$

$$|V_{ub}|_{B \rightarrow \rho l\nu} = (3.19 \pm 0.22 \pm 0.26_{\text{LCSRs}}) \times 10^{-3}.$$

The result derived from the $B \rightarrow \rho$ transition aligns with prior analyses [14–17], yet it is notably small than that determined from the $B \rightarrow \pi l\nu_l$ decay. In contrast, the $|V_{ub}|$ extracted from the $B_s \rightarrow K^-\mu^+\nu$ decay is largely consistent with the golden channel [18,19]. This inconsistency has raised concerns regarding the uniformity of $|V_{ub}|$ determinations across different exclusive decays.

The signal state in a $B \rightarrow \rho l\nu_l$ -type decay is $\pi\pi l\nu_l$, as the ρ is an unstable resonance that decays almost exclusively into the $\pi\pi$ state. Consequently, a comprehensive analysis must account for the finite widths, the nonresonant backgrounds, and the contributions of different partial waves in the $\pi\pi$ invariant mass spectrum, rather than treating the ρ meson as a single particle at its pole mass [13] or approximating it with a simple Breit-Wigner line shape [12]. Recently, the $B^+ \rightarrow \pi^+\pi^-l^+\nu_l$ decay was measured by the Belle detector [20]. The branching fraction exhibits a significant deviation from the earlier analysis of $B^+ \rightarrow \rho^0l^+\nu_l$ decay by using the same dataset [17], revealing the importance of nonresonant contributions and S -wave effects which were previously underestimated.

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From a QCD perspective, the dynamics of four-body semileptonic decay (B_{l4}) [21,22] is governed by the $B \rightarrow \pi\pi$ FFs. They were pioneer studied from the light-cone sum rules (LCSRs) approaches. Compared to the B -meson LCSRs [23], the LCSRs with two-pion light-cone distribution amplitudes (2π DAs) [24–27] offer several unique advantages. First, the sum rule analysis of the correlation function is performed without resonant modeling assumptions. Second, it provides a unified framework to predict contributions from different partial waves. Last but not least, theoretical uncertainties are better controlled due to highly suppressed power corrections $\mathcal{O}(1/m_b)$ [28]. However, existing LCSRs' studies with 2π DAs have so far been limited to leading-twist accuracy. This limitation has hindered theoretical progress in B_{l4} decays.

In this work, we derive the subleading-twist 2π DAs. These findings represent the first comprehensive advance beyond the two-decades-old leading-twist paradigm [29], providing unprecedented insight into the structure of the $\pi\pi$ system and complementing recent studies of kinematical higher-twist generalized distribution amplitudes [30–32]. Our result enables the calculation of subleading power corrections to $B \rightarrow \pi\pi$ transition. As a key application, we determine $|V_{ub}|$ from the $B^+ \rightarrow \pi^+\pi^-l^+\nu_l$ channel.

Two-pion light-cone distribution amplitudes. The most general nonperturbative quantity in hard exclusive processes with two energetic pions in the final state is provided by the 2π DAs. The 2π DAs are defined by the vacuum to $\pi\pi$ matrix elements sandwiched with special nonlocal quark currents,

$$\langle \pi^+(k_1)\pi^-(k_2) | \bar{q}(0)\Gamma q(x) | 0 \rangle \propto \int du e^{i\bar{u}k \cdot x} \Phi^t(\bar{u}, \zeta, k^2). \quad (1)$$

Here $\Gamma \in \{\gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}, 1\}$, $k = k_1 + k_2$ is the invariant mass of $\pi\pi$ system. The superscript t denotes the twist that is defined by the difference between canonical dimension and conformal spin of the associated quark currents [33,34]. In addition to the longitudinal momentum fraction u carried by the current antiquark, hence $\bar{u} = 1 - u$ by the quark with respect to the total two-pion momentum, 2π DAs depend on two further kinematic variables: the one-pion momentum fraction along the light-cone direction $\zeta = k_1^+/k^+$, and the invariant mass squared k^2 .

Further details on the matrix element definition and twist clarification for 2π DAs are provided in the Supplemental Material [35]. Here, we focus on the double expansion formalism [36–38],

$$\Phi \propto \sum_n \sum_l^{n+1} B_{nl}(\mu, k^2) C_n^{3/2}(2u-1) C_l^{1/2}(2\zeta-1). \quad (2)$$

Here n is even (odd) and l is odd (even) for the isovector (isoscalar) $\pi\pi$ state. The expansion coefficient $B_{nl}(\mu, k^2)$

weights the universal distribution amplitudes at a given conformal power n and partial wave l . They encode both renormalization scale dependence and invariant mass dynamics via

$$B_{nl}^\parallel(\mu, k^2) = B_{nl}^\parallel(0) \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right]^{\frac{\gamma_n - \gamma_0}{2\beta_0}} \text{Exp} \left[k^2 \frac{d \ln B_{nl}^\parallel(0)}{dk^2} + \frac{k^4}{\pi} \int_{4m_\pi^2}^\infty ds \frac{\delta_l(s)}{s^2(s - k^2 - i0)} \right]. \quad (3)$$

The scale dependence of the coefficients, governed by the anomalous dimension γ_n that identifies to the single-meson case [39–41], reflects shrinking transverse parton separations at high momentum [42–45].

Near the threshold, the parameters $B_{nl}(0)$ and $d \ln B_{nl}^\parallel(0)/dk^2$ can be determined from the instanton model of the QCD vacuum. The instanton vacuum is modeled as a grand canonical ensemble, with the partition function expressed in terms of the collective coordinates. Within this framework, the mean values of these coordinates are determined by maximizing the partition function for a fixed number of pseudoparticles. A key element of this derivation is the introducing of a dimensionless parameter that is not uniquely fixed from first principles. As indicated in Table I of Ref. [46], the resulting uncertainties in the average instanton size $\bar{\rho}$ and the average instanton separation $\bar{R}$ are each approximately 10%. However, their ratio $\bar{\rho}/\bar{R} \sim 1/3$ is notably stable against variations in this parameter. In our analysis, we adopt the established values from Refs. [47,48], allowing the associated uncertainties to be safely neglected in the construction of the 2π distribution amplitudes.

For the resonant regions, Watson's theorem yields the k^2 dependence via Omnés solutions with the phase shifts δ_l in pion-pion scattering [49]. In practice, an excellent description of data up to $k^2 \sim 5 \text{ GeV}^2$ is achieved through an amplitude analysis that combines dispersion relations with unitarity constraints [50]. This analysis fits the phase shifts and inelasticity parameters to $\pi^- p \rightarrow \pi^- \pi^+ n$ hadronic data from threshold up to approximately 1.8 GeV. Final-state interactions between meson pairs, especially critical in the $K\bar{K}$ threshold region, are further constrained by $BABAR$ Dalitz plot analyses of $D_s^+ \rightarrow \pi^+\pi^-\pi^+$ and $K^+K^-\pi^+$ [51–53]. At higher energies, the full $\gamma\gamma \rightarrow \pi\pi$ amplitudes are described using a Regge parametrization [54–56].

In exclusive QCD processes, power-suppressed contributions arise from three sources: the “bad” component of the quark field with an incorrect spin projector, transverse momentum in the leading-twist components of the $q\bar{q}$ configuration, and higher Fock states such as $q\bar{q}g$ and $q\bar{q}q\bar{q}$. These contributions are encoded in subleading-twist LCDAs. For 2π DAs, the quark current operators match those of single-meson LCDAs (with the same quantum numbers), leading to three analogous terms in

the two-particle twist-three 2π DAs: a term connected to three-particle 2π DAs through equations of motion (EOMs), a quark mass-dependent term, and a part related to the leading-twist DAs. In this work, we focus exclusively on the latter contribution, referred to as the Wandzura-Wilczek (WW) contribution, for twist-three 2π DA. The remaining two terms are neglected since the three-particle 2π DAs, though unexplored, are expected to be power suppressed as $\mathcal{O}(1/m_b)$ in B decays, while the light-quark mass effects are negligible at $\mathcal{O}(m_q/m_\pi)$. Our definition of twist-three 2π DAs preserves the scale dependence and normalization of the single-meson parton momentum distribution, augmented by partial-wave polynomials at Gegenbauer order n with coupling coefficient $B_{nl}(\mu, k^2)$ [35].

$B \rightarrow \pi\pi$ form factors. The transition FFs between the B meson and the $\pi\pi$ system are defined through the matrix elements of the weak current,

$$\begin{aligned} & \langle \pi^+(k_1) \pi^-(k_2) | \bar{u} \gamma^\mu (1 - \gamma_5) b | \bar{B}^0(p_B) \rangle \\ &= i F_\perp \frac{\bar{q}_\perp^\mu}{\sqrt{k^2}} + F_t \frac{q^\mu}{\sqrt{q^2}} + F_0 \frac{2\sqrt{q^2} k_0^\mu}{\sqrt{\lambda}} + F_\parallel \frac{\bar{k}_\parallel^\mu}{\sqrt{k^2}}. \end{aligned} \quad (4)$$

The orthogonal basis of momentum vectors [21] are $\bar{q}_\perp^\mu = 2\varepsilon^{\mu\nu\rho\sigma} q_\nu k_\rho \bar{k}_\sigma / \sqrt{\lambda}$, $q^\mu = p_B^\mu - k^\mu$, $k_0^\mu = k^\mu - k \cdot q / q^2 q^\mu$, and $\bar{k}_\parallel^\mu = \bar{k}^\mu - 4(k \cdot q)(\bar{k} \cdot q) / \lambda k^\mu + 4k^2(\bar{k} \cdot q) / \lambda q^\mu$. The FFs F_i with $i = \{\perp, \parallel, t, 0\}$ depend on three kinematical variables: momentum transfer squared q^2 , dipion invariant

mass squared k^2 , and scalar produce $q \cdot \bar{k} = \beta_\pi \cos \theta_\pi \sqrt{\lambda} / 2$ that specifies the polar angle θ_π of the π^- meson in the $\pi\pi$ rest frame. Here $\bar{k} = k_1 - k_2$; the Källén function reads as $\lambda = m_B^4 - q^4 - k^4 - 2(m_B^2 q^2 + m_B^2 k^2 + q^2 k^2)$.

The derivation of $B \rightarrow \pi\pi$ FFs begins with the construction of approximate correlation functions [24–26],

$$\begin{aligned} C[\Gamma_\mu] &= i \int d^4x e^{-ip_B \cdot x} \langle \pi^+ \pi^- | T \{ j_\mu(0), j_{(B)}(x) \} | 0 \rangle, \\ C[\Gamma_5] &= i \int d^4x e^{-ip_B \cdot x} \langle \pi^+ \pi^- | T \{ j_5(0), j_{(B)}(x) \} | 0 \rangle, \end{aligned} \quad (5)$$

in which the weak current $j_\mu = \bar{q}_f \gamma_\mu (1 - \gamma_5) b$ (pseudoscalar current $j_5 = im_b \bar{q}_f \gamma_5 b$) and the B -meson interpolating current $j_{(B)} = im_b \bar{b} \gamma_5 q_f$, with f being the flavor index, are sandwiched between the vacuum and $\pi\pi$ state. The QCD calculation of the correlation functions is performed in the negative half-plane of q^2 , which requires the momentum transfer squared to satisfy $0 \leq |q^2| \leq q_{\max}^2 \sim 10 \text{ GeV}^2$. On the other hand, as q^2 shifts from large negative to positive values, the average distance between the two currents increases, leading to the formation of hadrons through long-distance quark-gluon interactions. In this context, the correlation functions can be expressed as the sum of contributions from all possible intermediate states. $B \rightarrow \pi\pi$ FFs are then extracted by applying the Borel transformation and subsequently performing the continuum subtraction:

$$\begin{aligned} F_\parallel(q^2, k^2, \theta_\pi) &= \frac{-im_b^2 \sqrt{k^2}}{2m_B^2 f_B} \int_{u_0}^1 du \left[\frac{(m_b^2 - q^2 + u^2 k^2) \Phi_\perp}{u^2 m_b f_{2\pi} \beta_\pi \cos \theta_\pi} + \frac{\Phi_\perp^{(v)}}{u} \right] e^{\frac{m_B^2 - s(u, q^2)}{M^2}}, \\ F_\perp(q^2, k^2, \theta_\pi) &= \frac{im_b^2 \sqrt{\lambda} k^2}{2m_B^2 f_B} \left\{ \int_{u_0}^1 du \left[\frac{\Phi_\perp}{u m_b f_{2\pi} \beta_\pi \cos \theta_\pi} + \frac{\Phi_\perp^{(a)}}{2u^2 M^2} \right] e^{\frac{m_B^2 - s(u, q^2)}{M^2}} + \frac{\Phi_\perp^{(a)}(u_0)}{2u_0(s_0 - q^2)} e^{\frac{m_B^2 - s_0}{M^2}} \right\}, \\ \sqrt{q^2} F_t(q^2, k^2, \theta_\pi) &= \frac{-im_b^2}{2m_B^2 f_B} \left\{ \int_{u_0}^1 du \left[\frac{(m_b^2 - q^2 + u^2 k^2) \Phi_\parallel}{u^2} + \frac{\sqrt{\lambda} \beta_\pi \cos \theta_\pi \Phi_\perp^{(v)}}{u} \right. \right. \\ &\quad \left. \left. - \frac{m_b k^2}{f_{2\pi}} \left(\frac{1}{u^2} - \frac{m_b^2 - q^2 + u^2 k^2}{u^3 M^2} \right) \Phi_\parallel^{(s)} \right] e^{\frac{m_B^2 - s(u, q^2)}{M^2}} + \frac{m_b k^2 (m_b^2 - q^2 + u_0^2 k^2)}{f_{2\pi} u_0 (s_0 - q^2)} \Phi_\parallel^{(s)}(u_0) e^{\frac{m_B^2 - s_0}{M^2}} \right\}, \\ \sqrt{q^2} F_0(q^2, k^2, \theta_\pi) &= \frac{\sqrt{\lambda} \sqrt{q^2}}{m_B^2 - q^2 - k^2} F_t + \frac{2q^2 \sqrt{k^2} \beta_\pi \cos \theta_\pi}{m_B^2 - q^2 - k^2} F_\parallel + \frac{\sqrt{\lambda} q^2}{im_B (m_B^2 - q^2 - k^2)} F_R, \\ F_R(q^2, k^2, \theta_\pi) &= \frac{m_b k^2}{2f_{2\pi} m_B f_B} \left\{ \int_{u_0}^1 du \left(\frac{1}{u^2} - \frac{m_b^2 - q^2 + u^2 k^2}{u^3 M^2} \right) \left[\frac{\Phi_\parallel^{(s)}}{2} + 2I_1[\Phi_\parallel^{(t)}] \right] e^{\frac{m_B^2 - s(u, q^2)}{M^2}} \right. \\ &\quad \left. - \frac{(m_b^2 - q^2 + u_0^2 k^2)}{u_0(s_0 - q^2)} \left[\frac{\Phi_\parallel^{(s)}(u_0)}{2} + 2I_1[\Phi_\parallel^{(t)}(u_0)] \right] e^{\frac{m_B^2 - s_0}{M^2}} \right\}. \end{aligned} \quad (6)$$

Equation (6) is the main result of this work, where the contributions from twist-three 2π DAs ($\Phi_\perp^{(a)}, \Phi_\perp^{(v)}, \Phi_\parallel^{(s)}, \Phi_\parallel^{(t)}$) are studied for the first time. Our leading-twist results ($\Phi_\perp, \Phi_\parallel$ terms) are consistent with the previous

works [24–26]. We consider the virtuality of the b quark at $s(u, q^2) = (m_b^2 - \bar{u}q^2 + u\bar{u}k^2)/u$, and u_0 is the solution of u by taking $s = s_0^B$. We take the sum rule parameters $s_0^B = 37.5 \pm 2.5$ and $M^2 = 16 \pm 4 \text{ GeV}^2$ as the same as in the previous leading-twist study [24–26]; other input parameters are all the same as in Ref. [26] and the references there. Additionally, we introduce an auxiliary function $I_1[\Phi_{\parallel}^{(l)}] = \int_0^u du' \Phi_{\parallel}^{(l)}(u', \zeta, k^2)$.

$|V_{ub}|$ extraction. The double-differential decay width can be expressed as:

$$\frac{d^2\Gamma}{dq^2 dk^2} = G_F^2 |V_{ub}|^2 \frac{\beta_\pi \sqrt{\lambda} q^2}{3(4\pi)^5 m_B^3} \left[\sum_{l=0,1,\dots} |F_0^{(l)}(q^2, k^2)|^2 + \beta_\pi^2 \sum_{l=1,3,\dots} (|F_{\parallel}^{(l)}(q^2, k^2)|^2 + |F_{\perp}^{(l)}(q^2, k^2)|^2) \right]. \quad (7)$$

The subsequent task is to derive the $B \rightarrow \pi\pi$ FFs for a specific partial wave $F_i^{(l)}(q^2, k^2)$ from the sum rules result given in Eq. (6). This is achieved by utilizing the orthogonality properties of the associated Legendre polynomials [25,26]; in detail, by multiplying $F_{\perp,\parallel}$ by $P_{\nu'}^{(1)}(\cos\theta_\pi) \sin\theta_\pi$ and F_t by $P_{\nu'}^{(0)}(\cos\theta_\pi)$, and subsequently integrating over θ_π . We find that the two-particle twist-three contributions vanish in the partial-wave FFs $F_{i=\parallel,\perp}^{(l=1,3,\dots)}$ due to our choice of partial-wave projectors. Specifically, for the transverse-helicity FFs $F_{\parallel,\perp}^{(l=1,3)}$, the projector $P_{\nu'}^{(1)}(\cos\theta_\pi) \sin\theta_\pi$ generates twist-3 integrands that are odd functions, causing their integrals over the polar angle to vanish. In contrast, these contributions persist for the timelike- and longitudinal-helicity FFs $F_{t,0}^{(l=0,1,\dots)}$, where the projector $P_{\nu'}^{(0)}(\cos\theta_\pi)$ preserves the even symmetry of the twist-three integrands. Notably, the $\Phi_{\parallel}^{(s)}$ terms provide a significant correction ($\sim 40\%$) to the leading-twist result in this channel.

While finite twist-three contributions are present in $B \rightarrow \rho$ FFs [13,57], we find no such contribution to the P -wave transverse-helicity form factors $F_{\parallel,\perp}^{(l=1)}$. This difference is attributed to our treatment of the twist-three 2π DAs, for which we consider only the WW term. In contrast, the LCSR calculations for $B \rightarrow \rho$ transition incorporate a substantial admixture of twist-two matrix elements into the twist-three distribution amplitudes (DAs) via the EOM. Although this admixture complicates a naive twist counting of the relative size of contributions of different DAs, the genuine twist-three contributions to the $B \rightarrow \rho$ FFs, primarily arising from these EOM terms, are indeed suppressed by $\mathcal{O}(m_\rho/m_b)$.

In Figs. 1 and 2, we present the P , F - and S , D -wave components of $B \rightarrow \pi^+\pi^-$ FFs, respectively. While the P -wave contribution dominates in the region of the ρ

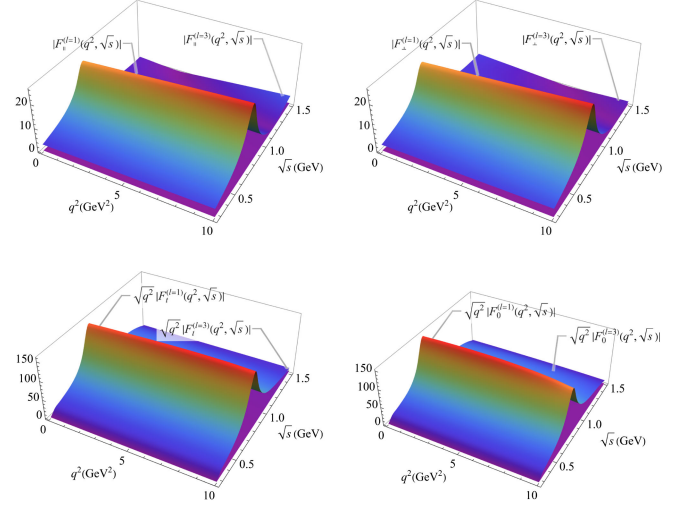


FIG. 1. P - and F -wave components to $B \rightarrow \pi^+\pi^-$ FFs.

resonance, we observe that the S -wave component provides a sizable contribution when the $\pi\pi$ invariant mass is not far away from the threshold value, and the D -wave component gives a significant contribution when the $\pi\pi$ invariant mass is large. In contrast, the F -wave component is relatively small in the considered kinematic regions.

Using the PDG value of $|V_{ub}| = (3.82 \pm 0.20) \times 10^{-3}$ [58], we calculate the partial branching fractions of $B^+ \rightarrow \pi^+\pi^-l^+\nu$. The results, presented in Table I, are compared with the Belle measurements [20]. The first and second uncertainties in our calculation arise from $|V_{ub}|$ and the LCSRs parameters, respectively. We estimate a $\sim 20\%$ uncertainty in the branching ratios from the LCSR approach, dominated by variations in M^2 . This translates to a $\sim 10\%$ uncertainty in the FFs themselves. Given that LCSRs are most reliable at small momentum transfer squared for heavy-to-light transitions, we consider seven bins with varying invariant mass ($\sqrt{s} = \sqrt{k^2}$) across different resonant regions, while restricting q^2 to the range $[0, 8] \text{ GeV}^2$.

We find good consistency in bins “2, 3” and “4, 5,” where the invariant mass lies within the ρ and f_0 resonances region, respectively. In this region, the 2π DAs is well understood, as the (predominant) asymptotic term is proportional to the well measured timelike FFs of the pion [59,60]. For bin “1,” where the invariant mass primarily resides in the region of light scalar resonance σ , our result is

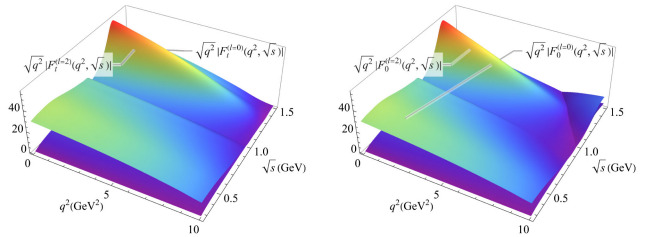


FIG. 2. S - and D -wave components of timelike- and longitudinal-helicity FFs.

TABLE I. Partial branching fractions $\Delta\mathcal{B}^i$ (in unit of 10^{-5}).

Bins	$\sqrt{s}$	q^2	$\Delta\mathcal{B}^i$	$\Delta\mathcal{B}^i$ [20]
1	$[4m_\pi^2, 0.6]$	[0, 8]	$0.27 \pm 0.03 \pm 0.06$	$0.84^{+0.39}_{-0.32} \pm 0.18$
2	(0.6, 0.9]	[0, 4]	$1.91 \pm 0.21 \pm 0.38$	$2.39^{+0.53}_{-0.47} \pm 0.32$
3	(0.6, 0.9]	(4, 8]	$1.54 \pm 0.17 \pm 0.27$	$2.16^{+0.47}_{-0.42} \pm 0.23$
4	(0.9, 1.2]	[0, 4]	$0.65 \pm 0.07 \pm 0.12$	$0.70^{+0.32}_{-0.25} \pm 0.20$
5	(0.9, 1.2]	(4, 8]	$0.41 \pm 0.04 \pm 0.08$	$0.64^{+0.28}_{-0.22} \pm 0.11$
6	(1.2, 1.5]	[0, 4]	$0.57 \pm 0.05 \pm 0.10$	$0.91^{+0.35}_{-0.28} \pm 0.12$
7	(1.2, 1.5]	(4, 8]	$0.16 \pm 0.02 \pm 0.02$	$0.64^{+0.32}_{-0.26} \pm 0.08$

approximately one-third of the experimental data. This discrepancy can be attributed to two main factors: (1) The nonasymptotic terms in the twist-three 2π DAs of the isoscalar $\pi\pi$ system have not yet been studied and are therefore not included in the current calculation, but these terms are known to play a significant role in $D \rightarrow S$ transitions, where S denotes a scalar meson [27,61]. (2) There is a need for a better understanding of the asymptotic term of the isoscalar 2π DAs in the σ resonance region, which is directly related to the $\pi\pi$ scattering phase shifts. In the larger invariant mass region $1.2 \leq \sqrt{s} \leq 1.5$ GeV, particularly in bin “7,” our results are significantly smaller than the experimental measurements. This discrepancy underscores the limited knowledge of 2π DAs in this region. Furthermore, since the intermediate resonances in this region are not ground states, the predictive power of LCSRs may be reduced.

The good consistency observed in bins “2” and “4” motivates us to extract $|V_{ub}|$ from the B_{14} decay in the regions of ρ and f_0 resonances. We obtain $|V_{ub}| = (4.27 \pm 0.49 \pm 0.55) \times 10^{-3}$ and $(3.96 \pm 0.47 \pm 0.52) \times 10^{-3}$ in these $\pi\pi$ invariant mass regions. The result is slightly larger but remain consistent with that obtained from the golden channel $B \rightarrow \pi l \nu$, compared to that obtained from the $B \rightarrow \rho l \nu$ channel. The first and second uncertainties originate from the experimental data and LCSR parameters; they are notably large at the current precision level. We anticipate that a complete LQCD simulation at large momentum transfer squared could reduce the theoretical uncertainty by approximately half. Recently, the $B \rightarrow \pi\pi$ transition FFs in the ρ resonance region have

been evaluated using LQCD with a simple Breit-Wigner model [62,63]. A novel parametrization of the $B \rightarrow \pi\pi$ form factors, based on partial-wave decompositions and series expansions, is proposed for the full phase space [64]. Consequently, the purely isovector decay $B^0 \rightarrow \pi^- \pi^0 l^+ \nu_l$ and the isoscalar decay $B^+ \rightarrow \pi^0 \pi^0 l^+ \nu_l$ are highly anticipated at the Belle II detector. These measurements will help distinguish between P - and S -wave contributions and improve our understanding of 2π DAs.

Conclusions. In this work, we perform the comprehensive calculation of the $B \rightarrow \pi\pi$ FFs using LCSR, incorporating an investigation of the twist-three 2π DAs. We demonstrate that these previously neglected contributions play a significant role, particularly in the timelike- and longitudinal-helicity FFs. Given the well-understood predominance of the P -wave contribution in the B_{14} decay $B^0 \rightarrow \pi^- \pi^+ l^+ \nu_l$, we extract the CKM matrix element $|V_{ub}|$. Our result, $|V_{ub}| = (4.27 \pm 0.49)_{\text{data}} \pm 0.55_{\text{LCSR}} \times 10^{-3}$, is consistent with that obtained from the golden channel $B \rightarrow \pi l \nu_l$. This consistency resolves the puzzle regarding the determination of $|V_{ub}|$ from different exclusive channels, particularly in light of finite width and nonresonant effects. The uncertainties in our determination can be further reduced through advancements in both theoretical frameworks and experimental precision. It is also feasible to perform a next-to-leading-order calculation for the $B \rightarrow \pi\pi$ form factors which would significantly enhance the accuracy of the QCD predictions [65–67]. Consequently, B_{14} decays offer a robust and independent method to address the $|V_{ub}|$ puzzle, providing a critical cross-check for existing determinations.

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Data availability. The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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