

New type of saddle in the Euclidean IKKT matrix model and its emergent geometry

Henry Liao^{1,*} and Reishi Maeta^{1,2,3,†}

¹*Department of Physics and Center for Theoretical Physics, National Taiwan University, Taipei 10617, Taiwan*

²*Graduate School of Advanced Science and Engineering, Hiroshima University, Higashi-Hiroshima 739-8526, Japan*

³*Physics Department, McGill University, Montreal, Quebec H3A 2T8, Canada*



(Received 29 December 2025; accepted 25 June 2026; published 16 July 2026)

We study the equation of motion of the Euclidean Ishibashi-Kawai-Kitazawa-Tsuchiya matrix model, and realize a new type of classical saddle that only exists in the $N \rightarrow \infty$ limit. Under the assumption that the matrices are the generators of $\mathfrak{so}(n, m)$, we identify a unique solution, that is, $\mathfrak{so}(1, 3)$. Even though it has 6 generators and thus 6 nonzero matrices, they are not independent due to the 2 Casimir constraints in $\mathfrak{so}(1, 3)$. Exploiting the Lie-algebraic structure and the Casimir constraints, we derive a four-dimensional space that a test scalar propagates on. The associated metric possesses $SU(2)$ isometry, which is closely related to the Taub-Newman-Unti-Tamburino/Bolt geometry and may, more broadly, be related to black hole physics.

DOI: [10.1103/gfcj-9vb3](https://doi.org/10.1103/gfcj-9vb3)

I. INTRODUCTION

As a promising proposal toward a nonperturbative formulation of type IIB superstring theory, the Ishibashi-Kawai-Kitazawa-Tsuchiya (IKKT) matrix model [1,2] is expected to describe nonperturbative objects such as a $D3$ -brane on which we live. In fact, this zero-dimensional matrix model suggests a picture that four-dimensional spacetime can emerge dynamically [3–6]. In addition, its low energy effective theory is closely related to general relativity and can be unitary [7].

There are other attempts to understand the IKKT matrix model. A holographic dictionary between type IIB supergravity and the IKKT matrix model has been established [8,9]. For the so-called polarized IKKT matrix model [10], in which special deformations are added while preserving maximal supersymmetry, there are similar discussions [11–15]. For other types of deformations, various studies have shown the emergence of four-dimensional near-realistic spacetime, which has an Friedmann–Lemaître–Robertson–Walker metric [16–18] or even chiral fermions [19,20].

Among these various developments, from the viewpoint of describing the real world in which we live, the original

IKKT matrix model without mass deformations is the most suitable for detailed analysis. This is because the IKKT matrix model is derived either (1) by the *large- N reduction* of ten-dimensional $\mathcal{N} = 1$ supersymmetric Yang-Mills theory [21–23], or (2) by *matrix regularization* of the Green-Schwarz action of type IIB superstring theory [1] (for type IIA superstring theory, see also [24,25]), and in both derivations no mass term is present.

However, for the massless theory, only a limited number of classical solutions are currently known [1,26–28], let alone solutions describing four-dimensional spacetime. Motivated by this situation, we try to find a new saddle of the original IKKT matrix model, which is defined in Euclidean signature. More concretely, we examined whether configurations of the form $A_\mu = M_{ab}$, constructed from the generators M_{ab} of $\mathfrak{so}(n, m)$, satisfy the equations of motion (2). As a result, we found that among these cases, $\mathfrak{so}(1, 3)$ is the only solution. Furthermore, we analyzed what geometry emerges from such a saddle. It turns out that the resulting geometry describes a four-dimensional Euclidean spacetime with $SU(2)$ isometry, which bears a strong resemblance to the Taub-Newman-Unti-Tamburino/Bolt geometry [29,30]. Since such a geometry may be related to black hole physics, the $\mathfrak{so}(1, 3)$ solution could provide the first example of this kind of connection in the IKKT matrix model.

At the beginning of Sec. II, we briefly mention our setup, and show the action and the equation of motion in the IKKT matrix model. Then, we present a theorem stating that, as long as the matrix size is finite, the solutions to the equations of motion are restricted to diagonal matrix configurations. This is not necessarily true for

*Contact author: henryliao.physics@gmail.com

†Contact author: maeta-reishi@hiroshima-u.ac.jp

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](https://creativecommons.org/licenses/by/4.0/). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

infinite-dimensional matrices, and several nontrivial non-commutative matrix configurations are in fact known to satisfy the equations of motion. This motivates us to consider, in Sec. II C, an ansatz in which the matrices are infinite-dimensional. More specifically, they are generators of $\mathfrak{so}(n, m)$ with $n, m \geq 1$, which have only infinite-dimensional unitary representations.¹ We find that, under our ansatz, $\mathfrak{so}(1, 3)$ is the only possibility. This forms a new class of solutions of the massless model; see Sec. II B. To analyse its emergent geometry, we consider a test scalar matrix to find the effective metric in the semiclassical limit. This procedure aligns with that in [31], and we provide the details in Sec. III. Then, we conclude in Sec. IV.

II. CLASSICAL SADDLES OF THE EUCLIDEAN BOSONIC IKKT MATRIX MODEL

In the Euclidean IKKT matrix model, the action of the bosonic sector reads

$$S_{\text{IKKT,B}} = -\frac{1}{4} \text{tr}(\delta_{\mu\rho} \delta_{\nu\sigma} [A^\mu, A^\nu] [A^\rho, A^\sigma]). \quad (1)$$

Here, the matrices A^μ are Hermitian. The equation of motion for A^μ is given by

$$\delta_{\mu\rho} [A^\mu, [A^\rho, A^\sigma]] = 0, \quad (2)$$

which can also be regarded as the equation of motion of the full IKKT matrix model under the assumption that the fermionic matrices are set to 0. Note that during the derivation, we have assumed that the cyclic property of the trace is always valid, and this is assumed throughout this paper.

A. Necessity of infinite-dimensional A^μ

If the matrices A_μ are finite-dimensional, the solutions to the equation of motion (2) are restricted to diagonal matrices. This can be shown as follows: Starting with (2), we multiply from the left by $\delta_{\nu\sigma} A^\nu$ and take the trace, which yields

$$\begin{aligned} 0 &= \text{tr}(\delta_{\nu\sigma} A^\nu \delta_{\mu\rho} [A^\mu, [A^\rho, A^\sigma]]) \\ &= -\text{tr}(\delta_{\mu\rho} \delta_{\nu\sigma} [A^\mu, A^\nu] [A^\rho, A^\sigma]) \\ &= \text{tr}([i[A_\mu, A_\nu]] [i[A_\mu, A_\nu]]). \end{aligned} \quad (3)$$

Since the A_μ are Hermitian matrices, the combination $i[A_\mu, A_\nu]$ is also Hermitian; for each pair of μ, ν , $i[A_\mu, A_\nu]$ has real eigenvalues and thus $\text{tr}(i[A_\mu, A_\nu])^2$ is non-negative. Hence, the last line of (3) can only vanish if all eigenvalues of $i[A_\mu, A_\nu]$ vanish for all μ, ν , meaning that

¹The algebra $\mathfrak{so}(n, m)$ is different from the spacetime Lorentz symmetry in general.

the A^μ commute. This is essentially the same as the proof given in Appendix A of [16].

It should be noted that this proof is valid only for finite-dimensional A_μ . In fact, it is clear that the Heisenberg algebra, which is defined by $[X, Y] = Z$, $[X, Z] = 0$ and $[Y, Z] = 0$, is a solution of (2). Moreover, some nilpotent and solvable Lie algebras are solutions as well [27]. This apparent contradiction originates from the fact that the trace can be ill-defined for infinite-dimensional matrices. In all of the examples above, the Hermitian A^μ realize the algebras as their unitary representations, which are infinite dimensional. Therefore, they do not fall within the scope of the proof. What we consider in this study is precisely such infinite-dimensional unitary representations.

B. Classification of the IKKT saddles

Before discussing our solution, we make a brief detour to discuss the classification of the saddles that can occur in (2).

By simple observation, (2) contains three operations: (a) the inner commutator, (b) the outer commutator and (c) the summation over spacetime indices. To satisfy the equation of motion (2), the expression can vanish at any one of the steps, leading to three different categories of solutions.

The solutions mentioned previously fall either into (a) or (b). In particular, the case where $[X, Y] = 0$ falls into (a), while the cases of Heisenberg, solvable and nilpotent Lie algebras are contained in (b). There is no known third class of solution so far. In particular, the solution we found below is the first known example of the class (c) in the IKKT matrix model.

C. $\mathfrak{so}(n, m)$ solution

In general, it is difficult to control matrices of infinite size. To make this possible, we consider a Lie algebra ansatz for A^μ , which allows us to manipulate commutators algebraically without specifying the representation *a priori*. The size of matrices is then determined by specifying the unitary representation of the chosen Lie algebra ansatz. In particular, we are seeking matrices of infinite size that solve (2). A convenient and physical choice is to consider the case of the Lorentz algebra with signature (n, m) . Since the corresponding unitary representations are infinite-dimensional when $n, m \geq 1$ [32], we expect that, if solutions exist, $n, m \geq 1$ is required.

The convention we use to define the Lorentz algebra is as follows

$$[M_{ab}, M_{cd}] = -i(h_{ad}M_{bc} + h_{bc}M_{ad} - h_{ac}M_{bd} - h_{bd}M_{ac}) \quad (4)$$

where $a, b = 1, \dots, d$ with $d = n + m$, and h_{ab} is the signature matrix with n entries equal to $+1$ and m entries equal to -1 . Next, we assume that the A_μ have a one-to-one correspondence with the generators M_{ab} without any

further linear combinations. That is, the dimension of spacetime is aligned with that of the algebra, which is $\frac{d(d-1)}{2}$, and the explicit ansatz is written as

$$(A_1, A_2, \dots, A_{\frac{d(d-1)}{2}}) = (M_{01}, M_{02}, \dots, M_{(d-1)(d)}). \quad (5)$$

In terms of the generators of the Lorentz algebra, the equation of motion (2) now reads

$$\sum_{a,b} [M_{ab}, [M_{ab}, M_{ij}]] = 0. \quad (6)$$

After applying (4) twice, we obtain

$$2(h_{ii} + h_{jj})(\text{tr}(h) - h_{ii} - h_{jj})M_{ij} = 0 \quad (7)$$

for all $i, j = 1, \dots, d$. The equality holds when one of the following two situations is true:

- (1) $h_{ii} = -h_{jj}$
- (2) $\text{tr}(h) = h_{ii} + h_{jj}$.

The generators of the Lorentz algebra correspond to either boosts or rotations.

For the case of boosts, we have one timelike index and one spacelike index for i, j , namely, $h_{ii} = -h_{jj}$.² Hence, the boost generators automatically satisfy the equation of motion.

On the other hand, the rotation generators do not satisfy the first case and are restricted by the second one. Since i, j can be both timelike or both spacelike for rotation generators, we have two equations

$$\text{tr}(h) = +2 \quad (8)$$

$$\text{tr}(h) = -2, \quad (9)$$

where the first one is the equation for rotations in timelike directions, while the second one is the equation for rotations in spacelike directions. Obviously, we cannot satisfy them simultaneously. Hence, the only possibility to remove the contradiction is to have only one timelike or one spacelike index, which means we only have to satisfy one of them. Below, we consider the case of a single timelike index, meaning we set $\mathfrak{so}(n, m)$ to $\mathfrak{so}(1, m)$. That means we only need to satisfy (9), which reads

$$1 - m = -2 \quad (10)$$

implying $m = 3$. Hence, we get $\mathfrak{so}(1, 3)$ generators as our solution. By a similar calculation for the case of a single spacelike index, we obtain $\mathfrak{so}(3, 1)$. Overall, within our ansatz with $\mathfrak{so}(n, m)$, the only solution is $\mathfrak{so}(1, 3)$.

²An index i is called timelike or spacelike, if $h_{ii} = +1$ or $h_{ii} = -1$, respectively. This terminology is not related to spacetime.

Unlike the case of $\mathfrak{so}(3)$, for which S^2 is naturally interpreted as the emergent geometry, it is not intuitive to argue for the emergent geometry of this $\mathfrak{so}(1, 3)$ solution. In fact, we cannot even argue that the emergent geometry possesses $SO(1, 3)$ isometry, since the ambient metric is Euclidean. Hence, in the following, we proceed to discuss how a metric can emerge from this solution in the Euclidean IKKT matrix model. We note that the use of Lie-algebra generators to construct classical solutions in IKKT-type matrix models was pioneered in [33, 34], where saddle points of the mass-deformed Lorentzian IKKT matrix model were studied; in particular, the solutions of [34] include those based on the $SO(n, m)$ group.

III. EMERGENT GEOMETRY FROM THE $\mathfrak{so}(1, 3)$ MATRIX BACKGROUND

To obtain the geometric properties of the $\mathfrak{so}(1, 3)$ background, we follow the approach of [31] and introduce a test scalar matrix whose semiclassical equation of motion allows us to read off the effective metric. In the following, we provide a detailed derivation for the effective metric to demonstrate how a nontrivial curved geometry emerges from the $\mathfrak{so}(1, 3)$ matrix configuration.

A. Poisson structure of $\mathfrak{so}(1, 3)$

In the semiclassical limit, we expect commutators to be approximated by Poisson brackets and the matrices A^μ to become position or embedding coordinates a^μ . Hence, it is crucial to find the Poisson structure that realizes the $\mathfrak{so}(1, 3)$ commutation relations, and the general form is analyzed by Kirillov, Kostant, and Souriau (KKS) [35].

Concretely, we demand that the embedding coordinates satisfy

$$\{a^\mu, a^\nu\} = f^{\mu\nu}{}_\rho a^\rho, \quad (11)$$

where $f^{\mu\nu}{}_\rho$ are the structure constants of $\mathfrak{so}(1, 3)$. The remaining task is to define a nondegenerate Poisson tensor so that the Poisson bracket in (11) is well defined.

Naively, we have a Poisson tensor $\omega^{\mu\nu} \equiv f^{\mu\nu}{}_\rho a^\rho$ defined on the six-dimensional embedding space. However, $\omega^{\mu\nu}$ is degenerate due to the presence of the two Casimirs in $\mathfrak{so}(1, 3)$. This degeneracy can be eliminated by fixing the values of the Casimirs, thereby restricting us to a four-dimensional subspace $\mathcal{M}_4$. Note that the detailed construction of $\mathcal{M}_4$ is explained in Sec. III B.

The Poisson bracket is thus understood as the induced Poisson bracket on $\mathcal{M}_4$, that is,

$$\{F(s), G(s)\}_{\mathcal{M}_4} = \omega^{ab}(s) \partial_{s^a} F(s) \partial_{s^b} G(s), \quad (12)$$

where s^a are coordinates on $\mathcal{M}_4$ ($a = 1, \dots, 4$), and ω^{ab} is the nondegenerate Poisson tensor, i.e., the inverse of the symplectic two-form ω_{ab} on $\mathcal{M}_4$. In particular, the

left-hand side of (11) is understood as

$$\{a^\mu(s), a^\nu(s)\}_{\mathcal{M}_4} = \omega^{ab}(s) \partial_{s^a} a^\mu(s) \partial_{s^b} a^\nu(s). \quad (13)$$

Equating it to the right-hand side of (11), we have

$$\omega^{ab}(s) \partial_{s^a} a^\mu(s) \partial_{s^b} a^\nu(s) = f^{\mu\nu}{}_\rho a^\rho(s). \quad (14)$$

Given the explicit embedding (see Sec. III B), (14) determines the nondegenerate Poisson tensor ω^{ab} on $\mathcal{M}_4$.

B. Construction of $\mathcal{M}_4$

In the $\mathfrak{so}(1,3)$ algebra, we have two Casimirs

$$C_1 = \sum_{i=1}^3 X_i^2 - Y_i^2, \quad (15)$$

$$C_2 = \sum_{i=1}^3 X_i Y_i, \quad (16)$$

where X_i and Y_i are boostlike and rotationlike generators of $SO(1,3)$. In the semiclassical limit, these matrices (as a unitary representation of $\mathfrak{so}(1,3)$) become embedding coordinates x_i and y_i , respectively; the Casimirs become constraints that define the four-dimensional surface $\mathcal{M}_4$ where the induced Poisson bracket lives. Concretely, $\mathcal{M}_4$ is obtained for fixed values of (C_1, C_2) by the constraints

$$\begin{aligned} C_1 &= \sum_{i=1}^3 x_i^2 - y_i^2, \\ C_2 &= \sum_{i=1}^3 x_i y_i. \end{aligned} \quad (17)$$

Note that these two quantities are the familiar invariants in electrodynamics, that is, $\mathbf{E}^2 - \mathbf{B}^2$ and $\mathbf{E} \cdot \mathbf{B}$.

The parametrization of $\mathcal{M}_4$ can be obtained straightforwardly by exploiting the symmetry of the constraints. First, since both constraints are invariant under simultaneous rotations of both x_i and y_i , we can separate two coordinates (out of four) that parametrize an S^2 . Next, there is an additional S^1 symmetry, that is, the rotation of y_i about x_i . Then, our chart takes the following explicit form.

First, we introduce the local coordinates s^a ($a = 1, \dots, 4$) on $\mathcal{M}_4$. Explicitly, we have

$$s^a = (r, \theta, \varphi, \chi), \quad (18)$$

where $r \in [r_*, \infty)$ with r_* being the lower bound that will be made explicit below, (θ, φ) parametrize an S^2 , and χ parametrizes an S^1 . The chart is then given by

$$\begin{aligned} \mathbf{x} &= \sqrt{C_1 + r^2} \hat{u}, \\ \mathbf{y} &= \frac{C_2}{\sqrt{C_1 + r^2}} \hat{u} + \sqrt{r^2 - \frac{C_2^2}{C_1 + r^2}} \hat{w}, \end{aligned} \quad (19)$$

where

$$\begin{aligned} \hat{u} &= (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta), \\ \hat{w} &= \cos \chi \hat{e}_1 + \sin \chi \hat{e}_2, \end{aligned} \quad (20)$$

and

$$\begin{aligned} \hat{e}_1 &= (\cos \theta \cos \varphi, \cos \theta \sin \varphi, -\sin \theta), \\ \hat{e}_2 &= (-\sin \varphi, \cos \varphi, 0). \end{aligned} \quad (21)$$

In this chart, the boundary is at $\sqrt{r^2 - \frac{C_2^2}{C_1 + r^2}} = 0$, which gives

$$r_* = \sqrt{\frac{-C_1 + \sqrt{C_1^2 + 4C_2^2}}{2}} > 0. \quad (22)$$

By construction, $\hat{u}, \hat{e}_1, \hat{e}_2$ form an orthonormal frame on S^2 , and $\hat{w}$ is a unit vector orthogonal to $\hat{u}$.

C. Test scalar and effective metric on $\mathcal{M}_4$

Following the strategy of [31], we extract the emergent geometry associated with the $\mathfrak{so}(1,3)$ background by introducing a test scalar matrix Φ in the adjoint representation. Concretely, we add

$$S_\Phi = \frac{1}{2} \text{tr}(\delta_{\mu\nu} [A^\mu, \Phi] [A^\nu, \Phi]) \quad (23)$$

to the bosonic action (1). The equation of motion for Φ is

$$\delta_{\mu\nu} [A^\mu, [A^\nu, \Phi]] = 0. \quad (24)$$

In the semiclassical limit, commutators are replaced by i times the Poisson brackets on $\mathcal{M}_4$, as defined in (13). Also, the matrix A^μ is mapped to the embedding function a^μ of $\mathcal{M}_4$ as determined by the semiclassical limit of (17); the matrix Φ corresponds to a scalar field ϕ that propagates on $\mathcal{M}_4$. Hence, the semiclassical limit of (24) is

$$-\delta_{\mu\nu} \{a^\mu(s), \{a^\nu(s), \phi(s)\}_{\mathcal{M}_4}\}_{\mathcal{M}_4} = 0. \quad (25)$$

Expanding the left-hand side in local coordinates yields a second-order differential equation on $\mathcal{M}_4$ of the form

$$G^{ab}(s) \partial_a \partial_b \phi(s) + \dots = 0, \quad (26)$$

where the ellipsis denotes terms with at most one derivative of ϕ . The tensor $G^{ab}(s)$ hence defines the effective inverse metric on which $\phi(s)$ propagates.

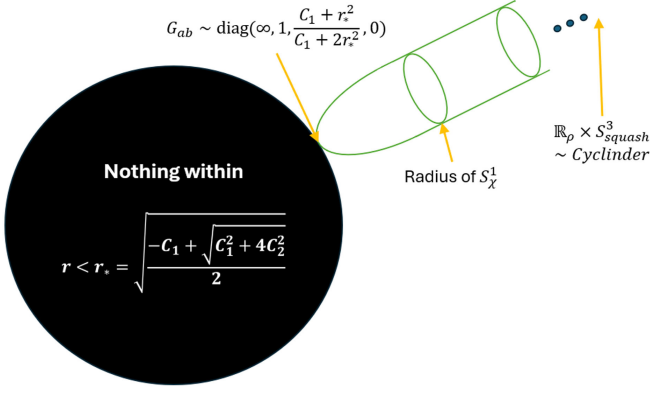


FIG. 1. Geometry of the effective metric (29). Here, we emphasize the relation between the radius in the χ direction and r , so each point in the plot represents an S_{squash}^2 parametrized by θ, φ . Note that one can easily check that the curvature is finite everywhere.

To make the structure of this emergent geometry explicit, it is convenient to use the $SU(2)$ left-invariant one-forms σ^i ($i = 1, 2, 3$) to express G_{ab} (the matrix inverse of G^{ab}). They are defined as

$$\begin{aligned}\sigma^1 &= \cos \chi d\theta + \sin \chi \sin \theta d\varphi, \\ \sigma^2 &= \sin \chi d\theta - \cos \chi \sin \theta d\varphi, \\ \sigma^3 &= d\chi + \cos \theta d\varphi,\end{aligned}\quad (27)$$

and we also define the shifted one-forms

$$\begin{aligned}\tilde{\sigma}^1 &= \sigma^1 + \frac{C_2 r}{(C_1 + r^2) \sqrt{r^4 + C_1 r^2 - C_2^2}} dr, \\ \tilde{\sigma}^3 &= \sigma^3 - \frac{C_2}{\sqrt{r^4 + C_1 r^2 - C_2^2}} \sigma^2.\end{aligned}\quad (28)$$

In the basis $(dr, \tilde{\sigma}^1, \tilde{\sigma}^2, \tilde{\sigma}^3)$, G_{ab} takes a diagonal form,

$$G_{ab} = \text{diag}\left(\frac{r^2}{A(r)}, 1, \frac{B(r)}{C(r)}, \frac{A(r)}{B(r)C(r)}\right), \quad (29)$$

where

$$A(r) = r^4 + C_1 r^2 - C_2^2, \quad (30)$$

$$B(r) = C_1 + r^2, \quad (31)$$

$$C(r) = C_1 + 2r^2. \quad (32)$$

Since the effective metric G_{ab} (29) only depends on r , G_{ab} has manifest $SU(2)$ isometry.

There are two asymptotic regions that are useful to understand the geometry. First, in the limit $r \rightarrow \infty$, the metric becomes

$$\lim_{r \rightarrow \infty} G_{ab} = \text{diag}\left(\frac{1}{r^2}, 1, \frac{1}{2}, \frac{1}{2}\right), \quad (33)$$

which, upon taking $\rho = \log r$, describes $\mathbb{R}_\rho \times S_{\text{squash}}^3$; hence, it is asymptotically cylindrical. Second, as we approach the boundary $r = r_*$, the radius of the $d\chi^2$ part of the metric shrinks to zero, which makes the metric singular. More specifically, the metric becomes

$$\lim_{r \rightarrow r_*} G_{ab} = \text{diag}\left(\infty, 1, \frac{C_1 + r_*^2}{C_1 + 2r_*^2}, 0\right) \quad (34)$$

with finite curvatures. We summarize these features in Fig. 1.

IV. CONCLUSION AND OUTLOOK

In this letter, we have discussed an unexpected and interesting solution of the Euclidean IKKT matrix model, namely the $\mathfrak{so}(1,3)$ solution. It forms a new class of solutions in the theory, since it satisfies the equation of motion (2) only after the summation is performed, see Sec. II B.

To read off the effective metric of the $\mathfrak{so}(1,3)$ background, we introduce a test scalar matrix as in [31] to identify how it propagates in the semiclassical limit. The resulting geometry is four-dimensional, since the Lie algebra $\mathfrak{so}(1,3)$ has six generators and two Casimir constraints. Furthermore, such a geometry, described in Fig. 1, has $SU(2)$ isometry, which is closely related to the Taub–Newman–Unti–Tamburino/Bolt geometry and thus could be related to black hole physics.

In the following, we discuss several unresolved issues in the present study, as well as possible directions for future research, from various perspectives.

(i) *Effective metric* We used the KKS form to read off the effective metric as in [31], but this is not the only possible approach. For example, there are approaches that attempt to extract geometry by interpreting the matrices as covariant-derivative operators [36,37], as well as an approach that defines a metric by utilizing block-diagonal configurations of matrices [38]. It is not yet clear how such differences in the extraction procedure affect the resulting effective metric, and further investigation will be required.

(ii) *Stability of the saddle* All analyses in this paper are based on tree-level calculations; however, the stability of the saddle should be discussed with quantum effects taken into account. According to [39], as long as the emergent spacetime has dimension at most four, the fermionic sector of the IKKT matrix model ensures that no one-loop divergences arise. Therefore, we expect that this saddle is stable at the one-loop level.

(iii) *Numerical computations* Beyond the one-loop analysis around the $\mathfrak{so}(1,3)$ saddle, understanding how dominant this configuration is within the full matrix

integral will require summing over the contributions of the infinitely many other saddles as well. Such calculations will in practice require numerical analysis, for which methods such as the generalized Lefschetz-thimble approach [40,41], the complex Langevin method [42,43] as in [4,44], or the bootstrap method [45,46] may be useful.

(iv) *Relation to holography* For the massless IKKT matrix model, the holographic dictionary was put forward in [8,9]. Since our $\mathfrak{so}(1,3)$ saddle is an exact solution of this model, it is expected that the corresponding supergravity solution can be constructed by invoking the holographic principle.

At the same time, the $\mathfrak{so}(1,3)$ generators cannot be a solution of the polarized IKKT matrix model, even in the large- N limit. In [11,14], the authors calculated the partition function of the polarized IKKT matrix model. Furthermore, a conjecture to define the IKKT matrix model through the massless limit of the polarized counterpart has been made in [14], where they proved it for $N = 2, 3$. However, as we have demonstrated in this work, there can be nontrivial saddles in the $N \rightarrow \infty$ limit, which may alter the calculation in their work.

(v) *Lorentzian vs. Euclidean* Throughout this study, we have consistently worked with the Euclidean IKKT matrix model and obtained emergent geometries with Euclidean signature. On the other hand, obtaining Lorentzian geometries may require starting from the Lorentzian IKKT matrix model. Although the definition of the Lorentzian IKKT matrix model had long remained ambiguous, a proposal for defining the theory was recently put forward [47]. Furthermore, the corresponding numerical studies

appear to suggest the emergence of $(1+3)$ -dimensional spacetime [48–50]. In particular, recent complex Langevin simulations of the Lorentzian model with a supersymmetry-inspired deformation, analogous to the one used to define the polarized IKKT matrix model in the Euclidean case, exhibit a phase in which a $(1+3)$ -dimensional expanding spacetime emerges with both space and time being smooth and real [50]. On the other hand, it is not yet understood whether these results persist in the undeformed massless limit, nor how the Euclidean and Lorentzian formulations are precisely related to each other—for instance, whether saddles of the Euclidean model, such as the one found in this work, have Lorentzian counterparts. Further investigation is necessary to clarify how realistic $(1+3)$ -dimensional spacetime can emerge.

ACKNOWLEDGMENTS

The authors are grateful for helpful discussions with Prof. Hikaru Kawai and Prof. Pei-Ming Ho. H.L. is supported in part by the Ministry of Science and Technology Grants No. 112-2112-M-002-024-MY3 and No. 112-2628-M-002-003-MY3. R.M. is supported by JST SPRING, Grant No. JPMJSP2132.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

-
- [1] N. Ishibashi, H. Kawai, Y. Kitazawa, and A. Tsuchiya, *Nucl. Phys.* **B498**, 467 (1997).
 - [2] H. Aoki, S. Iso, H. Kawai, Y. Kitazawa, A. Tsuchiya, and T. Tada, *Prog. Theor. Phys. Suppl.* **134**, 47 (1999).
 - [3] S.-W. Kim, J. Nishimura, and A. Tsuchiya, *Phys. Rev. Lett.* **108**, 011601 (2012).
 - [4] K. N. Anagnostopoulos, T. Azuma, K. Hatakeyama, M. Hirasawa, Y. Ito, J. Nishimura, S. K. Papadoudis, and A. Tsuchiya, *Eur. Phys. J. Spec. Top.* **232**, 3681 (2023).
 - [5] J. Nishimura and A. Tsuchiya, *J. High Energy Phys.* **06** (2019) 077.
 - [6] R. Brandenberger and J. Pasiicznik, *Phys. Rev. D* **112**, 026006 (2025).
 - [7] P.-M. Ho, H. Kawai, and H. C. Steinacker, *J. High Energy Phys.* **02** (2026) 070.
 - [8] F. Ciceri and H. Samtleben, *Phys. Rev. Lett.* **135**, 061601 (2025).
 - [9] F. Ciceri and H. Samtleben, *Phys. Rev. D* **113**, 046001 (2026).
 - [10] G. Bonelli, *J. High Energy Phys.* **08** (2002) 022.
 - [11] S. A. Hartnoll and J. Liu, *J. High Energy Phys.* **03** (2025) 060.
 - [12] S. A. Hartnoll and J. Liu, *SciPost Phys.* **19**, 099 (2025).
 - [13] S. Komatsu, A. Martina, J. Penedones, A. Vuignier, and X. Zhao, *J. High Energy Phys.* **12** (2025) 029.
 - [14] S. Komatsu, A. Martina, J. Penedones, A. Vuignier, and X. Zhao, *J. High Energy Phys.* **12** (2025) 030.
 - [15] C.-Y. Chou, J. Nishimura, and C.-T. Wang, *Phys. Rev. Lett.* **135**, 221601 (2025).
 - [16] H. C. Steinacker, *J. High Energy Phys.* **02** (2018) 033.
 - [17] M. Sperling and H. C. Steinacker, *J. High Energy Phys.* **07** (2019) 010.
 - [18] E. Battista and H. C. Steinacker, *Eur. Phys. J. C* **82**, 909 (2022).
 - [19] J. Nishimura and A. Tsuchiya, *J. High Energy Phys.* **12** (2013) 002.
 - [20] A. Chatzistavrakidis, H. Steinacker, and G. Zoupanos, *J. High Energy Phys.* **09** (2011) 115.
 - [21] T. Eguchi and H. Kawai, *Phys. Rev. Lett.* **48**, 1063 (1982).
 - [22] G. Parisi, *Phys. Lett.* **112B**, 463 (1982).

- [23] D.J. Gross and Y. Kitazawa, *Nucl. Phys.* **B206**, 440 (1982).
- [24] B. de Wit, J. Hoppe, and H. Nicolai, *Nucl. Phys.* **B305**, 545 (1988).
- [25] T. Banks, W. Fischler, S. H. Shenker, and L. Susskind, *Phys. Rev. D* **55**, 5112 (1997).
- [26] A. Manta and H. C. Steinacker, *J. High Energy Phys.* **02** (2026) 062.
- [27] A. Chatzistavarakidis, *Phys. Rev. D* **84**, 106010 (2011).
- [28] Y. Shibusa and T. Tada, *Phys. Lett. B* **579**, 211 (2004).
- [29] A. H. Taub, *Ann. Math.* **53**, 472 (1951).
- [30] E. Newman, L. Tamburino, and T. Unti, *J. Math. Phys.* (N.Y.) **4**, 915 (1963).
- [31] H. Steinacker, *J. High Energy Phys.* **12** (2007) 049.
- [32] W. Tung, *Group Theory in Physics*, G—Reference, Information and Interdisciplinary Subjects Series (World Scientific, Singapore, 1985), <https://books.google.co.jp/books?id=O89tgpOBO04C>.
- [33] S.-W. Kim, J. Nishimura, and A. Tsuchiya, *Phys. Rev. D* **86**, 027901 (2012).
- [34] S.-W. Kim, J. Nishimura, and A. Tsuchiya, *J. High Energy Phys.* **10** (2012) 147.
- [35] A. Kirillov, *Lectures on the Orbit Method*, Graduate Studies in Mathematics (American Mathematical Society, Providence, 2025).
- [36] M. Hanada, H. Kawai, and Y. Kimura, *Prog. Theor. Phys.* **114**, 1295 (2006).
- [37] K. Hattori, Y. Mizuno, and A. Tsuchiya, *Prog. Theor. Exp. Phys.* **2024**, 123B06 (2024).
- [38] S. Brahma, R. Brandenberger, and S. Laliberte, *J. High Energy Phys.* **09** (2022) 031.
- [39] D. N. Blaschke and H. Steinacker, *J. High Energy Phys.* **10** (2011) 120.
- [40] M. Cristoforetti, F. Di Renzo, and L. Scorzato (Aurora Science Collaboration), *Phys. Rev. D* **86**, 074506 (2012).
- [41] A. Alexandru, G. Basar, P. F. Bedaque, G. W. Ridgway, and N. C. Warrington, *J. High Energy Phys.* **05** (2016) 053.
- [42] G. Parisi, *Phys. Lett.* **131B**, 393 (1983).
- [43] J. R. Klauder, *Phys. Rev. A* **29**, 2036 (1984).
- [44] C.-Y. Chou, J. Nishimura, and A. Tripathi, *Phys. Rev. Lett.* **134**, 211601 (2025).
- [45] H. W. Lin, *J. High Energy Phys.* **06** (2020) 090.
- [46] V. Kazakov and Z. Zheng, *J. High Energy Phys.* **06** (2022) 030.
- [47] Y. Asano, J. Nishimura, W. Piensuk, and N. Yamamori, *Phys. Rev. Lett.* **134**, 041603 (2025).
- [48] M. Hirasawa, K. N. Anagnostopoulos, T. Azuma, K. Hatakeyama, J. Nishimura, S. Papadoudis, and A. Tsuchiya, *Proc. Sci., CORFU2023* (**2024**) 257 [arXiv:2407.03491].
- [49] K. N. Anagnostopoulos, T. Azuma, M. Hirasawa, J. Nishimura, A. Tsuchiya, and N. Yamamori, in *Proceedings of the 42th International Symposium on Lattice Field Theory* (2026), arXiv:2604.25564.
- [50] K. N. Anagnostopoulos, T. Azuma, M. Hirasawa, J. Nishimura, S. Papadoudis, and A. Tsuchiya, arXiv:2604.19836.