

Two-flavor color superconductivity in a Nambu-Jona-Lasinio model with color and charge neutrality

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Using a general Nambu–Jona-Lasinio (NJL) model including as many interaction channels as possible, and taking into account the constraints imposed by color and charge neutrality, we analyze how the different interaction channels contribute to charge-neutral two-flavor color-superconducting matter. After taking the Fierz transformation, the number of parameters in the NJL model is reduced and thereby quark-antiquark condensates and diquark condensates are related. Through a self-consistent solution of the gap equations, we find that in addition to the diquark and vector channels, the scalar-isovector, vector-isovector, and vector-isovector-color-octet channels are also important, while other channels can be neglected. Moreover, between the normal quark-matter phase and two-flavor color-superconducting phase, there is a gapless two-flavor color-superconducting phase. The fractions of quarks with different flavors and colors are calculated in both gapped and gapless two-flavor color-superconducting phases. In addition, we illustrate the phase diagrams in the diquark coupling and chemical potential plane, and discuss in detail how the dominant channels influence these phase diagrams.

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I. INTRODUCTION

The notion of color superconductivity (CSC) was introduced in the 1970s [1–3], in analogy to the Bardeen-Cooper-Schrieffer (BCS) theory in condensed matter physics [4–6]. In BCS theory, phonon exchange provides a weak attractive force between electrons, while in CSC, quarks experience an attractive interaction mediated by gluon exchange in the color antitriplet channel. This attraction enables quarks near the Fermi surface to form Cooper pairs.

Early studies of CSC claimed that the superconducting gap was only on the order of MeV [7], making it difficult to exhibit significant phenomenological effects. Consequently, this phenomenon initially received little attention. However, this situation changed in the 1990s after the subsequent research found that in the moderate-density regions, the gap of CSC could reach up to ≈ 100 MeV [8,9], substantially enhancing its physical significance. The CSC at high

densities was analyzed using a weak coupling approach in Refs. [10–12]. Since then, research in this field has flourished, with extensive investigations focusing on CSC in the moderate-density regimes and its implications for both phase structure of quantum chromodynamics (QCD) [13–17] and astrophysical observations [18–23]. For the deeper insights into the mechanisms and implications of CSC, we refer to some review articles, e.g., [24–27].

Color superconducting matter exhibits rich phase structures, a consequence of its diverse quark pairing patterns [27–30]. Among these pairing patterns, the color-flavor locked (CFL) phase is regarded as the ground state of QCD matter at asymptotic density [13,31,32]. In the moderate-density regime, due to the substantial mass difference between strange quark and up/down quarks, when the chemical potential is below the order of the strange quark mass, the CFL phase may be disfavored and replaced by a two-flavor color superconducting phase (2SC) [33–35]. When studying CSC in electrically neutral quark matter, such as the core of neutron stars, the phase diagram becomes even richer. The standard 2SC phase may evolve into a gapless 2SC (g2SC) phase with gapless quasiparticle excitations [36–38]. In the transition region between the 2SC and CFL phases, especially at finite temperatures, gapless CFL (gCFL) and superconductivity including up quark (uSC) phases are expected to emerge [15,16].

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The g2SC phase has recently attracted significant attention due to its stability. Studies have shown that the g2SC phase exhibits thermal stability under the constraint of local charge neutrality [36,37], but it also suffers from chromomagnetic instability associated with the Meissner effect [39,40]. The chromomagnetic instability arises not only in the g2SC phase but also in the standard 2SC phase when the mismatch between the chemical potentials of up and down quarks, $|\delta\mu|$, exceeds $\Delta/\sqrt{2}$ with Δ being the 2SC gap (its explicit definition will be given later) [39]. This instability suggests that the g2SC phase may not be a stable state but could transition to other phases, such as inhomogeneous phases like the crystalline phase [27,41–43], solitonic phase [44], gluonic phase [45,46], and so on. Consequently, further research is needed to clarify the exact nature and implications of these instabilities.

In the literature, the 2SC and g2SC phases have been studied within the framework of the Nambu–Jona-Lasinio (NJL) model [47,48], a powerful model used in a variety of phenomena of strong interaction at low energy [28,49–51]. The NJL model has many interaction channels, and their effects on the phase diagram and neutron star physics have been widely studied in, e.g., Refs. [15,52–55]. The influence of diquark channels on the CSC phase diagram was studied in Refs. [15,36], and the effects of the diquark channel combining with the vector and vector-isovector channels on the equations of state (EOS) of the neutron stars were investigated in Refs. [20,53,54]. The results show that in addition to the diquark channel, the vector and isovector channels also have significant effects on the EOS of neutron stars and the CSC phase diagram. In addition to the NJL models, the phase structure of CSC matter was also studied using the renormalization group approach or the Schwinger-Dyson equation [56–59].

In this work, we systematically investigate the effects of all the possible channels, particularly those often overlooked in the literature, in the most general NJL model on the QCD phase diagram and the EOS for neutron stars. The general NJL model has many interaction channels, especially those that could influence the charge-neutral 2SC phase. These channels are not independent but can be connected through a Fierz transformation. We further use the one-gluon-exchange (OGE) model to obtain the coupling constants of each channel. By imposing the electric-charge and color-charge neutrality, which were not seriously considered before [60,61], we analyze the contributions of each channel in more detail and identify the dominant interaction channels. Then we discuss their individual impacts on the phase diagram beyond the OGE model by varying their couplings.

We found that, apart from the diquark and vector channels which have been extensively discussed in the literature, the scalar-isovector, vector-isovector, and vector-isovector-color-octet channels also play significant roles, whereas the contributions from other channels are negligible. This arises from enforcing color and charge neutrality constraints, which enhance the importance of these channels in determining the phase structure. In addition, due to the color and charge neutrality conditions, we find that in the OGE model the system evolves from the chiral breaking phase to the g2SC phase, and then to the 2SC phase. Compared to the case without the charge neutrality, including charge neutrality introduces an additional intermediate g2SC phase, making the phase diagram more complex. Moreover, we examined how the ratio of the diquark-channel coupling constant H to the scalar-channel coupling constant $G_s^{(0)}$ influences the 2SC phase structure. We estimated the critical endpoint (CEP) at $H/G_s^{(0)} = 1.1$, $\mu = 312$ MeV. The g2SC phase is delineated by two phase boundaries, corresponding to $H/G_s^{(0)} \approx 0.67$ and $H/G_s^{(0)} \approx 0.81$. For the effects of distinct channels, we found that the vector channels shift the entire phase diagram toward larger chemical potential (μ), while the scalar-isovector channel extends the gapless phase region from $0.67 \lesssim H/G_s^{(0)} \lesssim 0.80$ to $0.67 \lesssim H/G_s^{(0)} \lesssim 0.85$ in the vicinity of the chiral phase transition. Moreover, the isovector channel lowers the values of $H/G_s^{(0)}$ corresponding to both the upper and lower boundaries of the g2SC phase.

The rest of this paper is organized as follows. In Sec. II, we derive the general NJL model and give the coupling relations by Fierz transformation. In Sec. III, we figure out the dominant channels in the OGE model. In Sec. IV, we discuss the effects of these channels on the 2SC phase structure separately at zero temperature. Finally, we make conclusions in Sec. V.

II. GENERAL FRAMEWORK

The most general Lorentz-invariant Lagrangian of the four-fermion interaction NJL model can be decomposed as [62]

$$\mathcal{L}_{\text{NJL}}^{(4)} = \mathcal{L}_{\text{sym}} + \mathcal{L}_{\text{det}}, \quad (1)$$

where $\mathcal{L}_{\text{sym}}$ is the Lagrangian preserving $SU(N_c) \times SU(N_f)_V \times SU(N_f)_A \times U(1)_V \times \{C, P, T\}$ symmetry and $\mathcal{L}_{\text{det}}$ breaks $U(1)_A$ symmetry. They can be generally expressed as

$$\begin{aligned}
\mathcal{L}_{\text{sym}} = & \bar{\psi}_a \psi_b \bar{\psi}_c \psi_d [C_1(SS + PP) \otimes (t_0 t_0 + tt) \otimes \delta\delta + C_2(VV + AA) \otimes (t_0 t_0 + tt) \otimes \delta\delta \\
& + C_3(VV + AA) \otimes t_0 t_0 \otimes \delta\delta + C_4(VV - AA) \otimes t_0 t_0 \otimes \delta\delta]_{ab,cd} \\
& + \bar{\psi}_a \psi_b \bar{\psi}_c \psi_d [C_5(SS + PP) \otimes (t_0 t_0 + tt) \otimes \lambda\lambda + C_6(VV + AA) \otimes (t_0 t_0 + tt) \otimes \lambda\lambda \\
& + C_7(VV + AA) \otimes t_0 t_0 \otimes \lambda\lambda + C_8(VV - AA) \otimes t_0 t_0 \otimes \lambda\lambda]_{ab,cd},
\end{aligned} \tag{2a}$$

$$\mathcal{L}_{\text{det}} = G(\det[\bar{\psi}(1 + \gamma^5)\psi] + \det[\bar{\psi}(1 - \gamma^5)\psi]), \tag{2b}$$

where $\{S, P, V, A\} = \{1, i\gamma_5, \gamma^\mu, \gamma^\mu \gamma_5\}$ are the Dirac matrices, $\{t_0, t_i\}$ are generators $U(N_f)$ with $\text{Tr}[t_a t_b] = 2\delta_{ab}$, and $\{\delta, \lambda_l\}$ are the identity and Gell-Mann matrices in color space. A simplified notation is often adopted, for example

$$\bar{\psi}_a \psi_b \bar{\psi}_c \psi_d [VV \otimes tt \otimes \lambda\lambda]_{ab,cd} = \sum_{i=1, l=1}^{i=N_f^2-1, l=8} \bar{\psi}_a (\gamma^\mu t_i \lambda_l)_{ab} \psi_b \bar{\psi}_c (\gamma_\mu t_i \lambda_l)_{cd} \psi_d = (\bar{\psi} \gamma^\mu t_i \lambda_l \psi)^2. \tag{3}$$

Some of the interaction terms in Lagrangian (1) are not independent but relate to each other by Fierz transformation through exchanging ψ_a and ψ_c or ψ_b and ψ_d . The interactions related by Fierz transformation are in fact dynamically equivalent. Therefore, independent interaction terms form a Fierz complete basis, i.e., the minimal set of interactions which can generate all other interactions by Fierz transformation [62,63]. Thus, we choose a Fierz complete interaction set

$$\begin{aligned}
\mathcal{L}_{\text{sym}}^{\text{FCI}} = & \bar{\psi}_a \psi_b \bar{\psi}_c \psi_d [C'_1(SS + PP) \otimes (t_0 t_0 + tt) \otimes \delta\delta + C'_2(VV + AA) \otimes (t_0 t_0 + tt) \otimes \delta\delta \\
& + C'_3(VV + AA) \otimes t_0 t_0 \otimes \delta\delta + C'_4(VV - AA) \otimes t_0 t_0 \otimes \delta\delta]_{ab,cd}.
\end{aligned} \tag{4}$$

Then, after Fierz transformation, coupling constants C'_i can be related to C_i . We can also rewrite the Lagrangian (2a) into a Fierz-invariant form as follows:

$$\begin{aligned}
\mathcal{L}_{\text{sym}}^{\mathcal{F}} = & \frac{1}{2} [\mathcal{L}_{\text{sym}}^{\text{FCI}} + \mathcal{F}_{\bar{q}q}(\mathcal{L}_{\text{sym}}^{\text{FCI}})], \\
= & \frac{1}{2} \bar{\psi}_a \psi_b \bar{\psi}_c \psi_d [G_1(SS + PP) \otimes (t_0 t_0 + tt) \otimes \delta\delta + G_2(VV + AA) \otimes (t_0 t_0 + tt) \otimes \delta\delta \\
& + G_3(VV + AA) \otimes t_0 t_0 \otimes \delta\delta + G_4(VV - AA) \otimes t_0 t_0 \otimes \delta\delta]_{ab,cd} \\
& + \frac{N_c^2}{N_c^2 - 1} \bar{\psi}_a \psi_b \bar{\psi}_c \psi_d \left[\left(-\frac{1}{2N_c} G_1 - \frac{1}{N_f} G_4 \right) (SS + PP) \otimes (t_0 t_0 + tt) \otimes \lambda\lambda \right. \\
& + \left(-\frac{1}{2N_c} G_2 + \frac{1}{2N_f} G_3 \right) (VV + AA) \otimes (t_0 t_0 + tt) \otimes \lambda\lambda \\
& + \left(\frac{N_f}{2} G_2 - \frac{1}{2N_c} G_3 \right) (VV + AA) \otimes t_0 t_0 \otimes \lambda\lambda \\
& \left. + \left(-\frac{N_f}{4} G_1 - \frac{1}{2N_c} G_4 \right) (VV - AA) \otimes t_0 t_0 \otimes \lambda\lambda \right]_{ab,cd},
\end{aligned} \tag{5}$$

where $\mathcal{F}_{\bar{q}q}$ denotes the quark–antiquark channel Fierz transformation and the $G_1 \sim G_4$ relate to $C'_1 \sim C'_4$ through

$$\begin{aligned}
G_1 = C'_1 - \frac{2}{N_f N_c} C'_4, \quad G_2 = C'_2 + \frac{1}{N_f N_c} C'_3, \\
G_3 = C'_3 + \frac{N_f}{N_c} C'_2, \quad G_4 = C'_4 - \frac{N_f}{2N_c} C'_1.
\end{aligned} \tag{6}$$

Similarly, we can obtain the relations between G_i and C_i . In this sense, the eight parameters in (2a) are reduced to four.

Note that any two NJL-type four-fermion interactions that are both chiral and $U(1)_A$ invariant should be physically equivalent if they predict the same values of $G_1 \sim G_4$ in the Fierz-invariant form. Then, combining the determinant interaction, the general NJL model only has five independent couplings $G_1 \sim G_4$ and G . For example, the original NJL

model [47] can be obtained by setting $C'_1 = G/2$ and $C'_2 = C'_3 = C'_4 = 0$ in the Lagrangian (4). In the OGE model, which is widely used in the study of CSC [13,33,60,61],

$$\mathcal{L}_{\text{int}} = -g(\bar{\psi}\gamma^\mu\lambda_a\psi)^2, \quad (7)$$

we have $G_1 = -2G_2 = (N_c^2 - 1)g/N_c^2$ and $G_3 = G_4 = G = 0$.

In order to study the color superconducting phase structure, we should consider the diquark interactions which can be derived via the Fierz transformation by the exchange of ψ_b and ψ_c or ψ_a and ψ_d based on the quark–antiquark channel interactions given in Eq. (4). Explicitly, the Lagrangian takes the form

$$\begin{aligned} \mathcal{F}_{qq}[\mathcal{L}_{\text{sym}}^{\text{FCI}}] = & \bar{\psi}_a\bar{\psi}_b\psi_c\psi_d\{H_1[(i\gamma_5 C)(i\gamma_5 C) + (C)(C)] \otimes t_A t_A \otimes \lambda_A \lambda_A \\ & + H_2[(i\gamma_5 C)(i\gamma_5 C) + (C)(C)] \otimes t_S t_S \otimes \lambda_S \lambda_S \\ & + H_3[(\gamma^\mu\gamma_5 C)(C\gamma_\mu\gamma_5) \otimes t_A t_A + (\gamma^\mu C)(C\gamma_\mu) \otimes t_S t_S] \otimes \lambda_A \lambda_A \\ & + H_4[(\gamma^\mu\gamma_5 C)(C\gamma_\mu\gamma_5) \otimes t_S t_S + (\gamma^\mu C)(C\gamma_\mu) \otimes t_A t_A] \otimes \lambda_S \lambda_S\}_{ab,cd}, \end{aligned} \quad (8)$$

where $\mathcal{F}_{qq}$ denotes the diquark channel Fierz transformation, the indices S and A represent the symmetric and antisymmetric parts of corresponding group generators, and $C = i\gamma^2\gamma^0$. The couplings H_i relate to G_i through

$$\begin{aligned} H_1 &= \frac{N_c}{1 - N_c} \left[G_2 - \frac{1}{N_f} G_3 \right], \\ H_2 &= \frac{N_c}{1 + N_c} \left[G_2 + \frac{1}{N_f} G_3 \right], \\ H_3 &= \frac{N_c}{1 - N_c} \left[\frac{1}{4} G_1 + \frac{1}{2N_f} G_4 \right], \\ H_4 &= \frac{N_c}{1 + N_c} \left[\frac{1}{4} G_1 - \frac{1}{2N_f} G_4 \right]. \end{aligned} \quad (9)$$

Note that $\mathcal{L}_{\text{sym}}^{\text{FCI}}$, $\mathcal{F}_{\bar{q}q}[\mathcal{L}_{\text{sym}}^{\text{FCI}}]$, and $\mathcal{F}_{qq}[\mathcal{L}_{\text{sym}}^{\text{FCI}}]$ are, in fact, equivalent. We only extract their distinct interaction channels through the Fierz transformation.

Although parameters $G_1 \sim G_4$ can be partly determined by the masses and decay constants of mesons in vacuum [62], in this work, in order to investigate the influence of different channels on the 2SC phase structure, we vary their values except the coupling $G_s^{(0)}$ of the scalar interaction $(\bar{\psi}\psi)^2$ which is fixed by the chiral symmetry breaking and the pion properties in vacuum.

Due to the Fierz transformation, the Hartree-Fock and BCS mean field approximation (MFA) of model $\mathcal{L}_{\text{sym}} + \mathcal{L}_{\text{det}}$ can be written as

$$\begin{aligned} \langle \mathcal{L}_{\text{sym}} + \mathcal{L}_{\text{det}} \rangle_{\text{HFB}} = & \langle \mathcal{L}_{\text{sym}} + \mathcal{L}_{\text{det}} \rangle_{\text{H}} + \langle \mathcal{F}_{\bar{q}q}[\mathcal{L}_{\text{sym}} + \mathcal{L}_{\text{det}}] \rangle_{\text{H}} \\ & + \langle \mathcal{F}_{qq}[\mathcal{L}_{\text{sym}} + \mathcal{L}_{\text{det}}] \rangle_{\text{B}}, \end{aligned} \quad (10)$$

where the subindices HFB, H, and B represent the Hartree-Fock and BCS MFAs, Hartree MFA, and BCS MFA, respectively.

Although the derivation preceding Eq. (10) is valid in both two- and three-flavor cases, our subsequent analysis is carried out entirely within the two-flavor framework. This choice is justified because, in the chemical potential range relevant for the 2SC phase ($M_u \lesssim \mu \lesssim M_s \approx 550$ MeV), the s -quark density is nearly zero and its self-energy σ_s varies only weakly, so it can be treated as a background. Under this approximation, the three-flavor determinant interaction with coupling constant K reduces effectively to the two-flavor form with an effective coupling $G = -K\sigma_s$ of determinant interaction, which ensures that the essential features of the 2SC phase can be consistently studied within the two-flavor framework.

In the 2SC phase, the $SU(3)_c$ gauge symmetry is spontaneously broken down to $SU(2)_c$. Focusing on the homogeneous and electrically neutral 2SC phase, the introduction of an electric chemical potential explicitly breaks the isospin symmetry between u and d quarks. Consequently, the residual unbroken symmetry takes the form [64]

$$SO(3) \times SU(2)_c \times SU(2)_A \times U_{\tau_3}(1), \quad (11)$$

where $U_{\tau_3}(1)$ is generated by the third Pauli matrix τ_3 of $SU(2)_V$, and $SO(3)$ is the rotation group.

Although the NJL model includes many interaction channels, after MFA, only few of them contribute due to the symmetry consideration. In the charge-neutral 2SC phase, the following channels survive:

$$\begin{aligned}
\mathcal{L}_{\bar{q}q} = & G_s^{(0)}(\bar{\psi}\psi)^2 + G_s^{(8)}(\bar{\psi}\lambda_a\psi)^2 + G_{sIv}^{(0)}(\bar{\psi}\tau\psi)^2 + G_{sIv}^{(8)}(\bar{\psi}\tau\lambda_a\psi)^2 + G_v^{(0)}(\bar{\psi}\gamma^0\psi)^2 \\
& + G_v^{(8)}(\bar{\psi}\gamma^0\lambda_a\psi)^2 + G_{vIv}^{(0)}(\bar{\psi}\gamma^0\tau_3\psi)^2 + G_{vIv}^{(8)}(\bar{\psi}\gamma^0\tau_3\lambda_a\psi)^2, \\
\mathcal{L}_{qq} = & H(\bar{\psi}i\gamma_5\tau_2\lambda_2\psi^C)(\bar{\psi}^Ci\gamma_5\tau_2\lambda_2\psi) + H_0(\bar{\psi}\gamma^0\gamma_5\tau_2\lambda_2\psi^C)(\bar{\psi}^C\gamma^0\gamma_5\tau_2\lambda_2\psi),
\end{aligned} \tag{12}$$

where the upper indices (0) and (8) denote the color singlet and octet, respectively, and the lower indices s, sIv, v , and vIv represent the Lorentz-scalar, Lorentz-scalar isovector, Lorentz-vector, and Lorentz-vector isovector. The charge-conjugated spinors are defined as $\psi^C = C\bar{\psi}^T$ and $\bar{\psi}^C = \psi^TC$. Here, we did not consider the color-sextet diquark channels since they are in the repulsive channels and the condensates are small [13,32]. The condensation of the temporal component of the vector diquark channel, $\langle\bar{\psi}^C\gamma^0\gamma_5\tau_2\lambda_2\psi\rangle$, breaks chiral

symmetry $SU(2)_A$. This condensate is expected to vanish once chiral symmetry is fully restored. Nevertheless, in the mixed-phase region—where chiral symmetry is only partially restored while the 2SC gap remains finite [65–67]—it can still acquire a nonvanishing value. We neglect the axial-vector counterpart, $\langle\bar{\psi}^Ci\gamma^0\tau_1\lambda_2\psi\rangle$, because it can be shown to vanish identically. In addition, in 2SC, we preserve the residual color symmetry between red and green quarks. The couplings in (12) relate to G and G_i through

$$\begin{aligned}
G_s^{(0)} &= G_1 + \frac{1}{2}G + \frac{1}{4N_c}G, \\
G_s^{(8)} &= \frac{N_c^2}{N_c^2 - 1} \left(-\frac{1}{2N_c}G_1 - \frac{1}{N_f}G_4 \right) + \frac{G}{8}, \\
G_{sIv}^{(0)} &= G_1 - \frac{1}{2}G - \frac{1}{4N_c}G, \\
G_{sIv}^{(8)} &= \frac{N_c^2}{N_c^2 - 1} \left(-\frac{1}{2N_c}G_1 - \frac{1}{N_f}G_4 \right) - \frac{G}{8}, \\
G_v^{(0)} &= G_2 + G_3 + G_4, \\
G_v^{(8)} &= \frac{N_c^2}{N_c^2 - 1} \left(-\frac{1}{2N_c}G_2 + \frac{1}{2N_f}G_3 + \frac{N_f}{2}G_2 - \frac{1}{2N_c}G_3 - \frac{N_f}{4}G_1 - \frac{1}{2N_c}G_4 \right), \\
G_{vIv}^{(0)} &= G_2, \\
G_{vIv}^{(8)} &= \frac{N_c^2}{N_c^2 - 1} \left(-\frac{1}{2N_c}G_2 + \frac{1}{2N_f}G_3 \right), \\
H &= H_1 + \frac{G}{4}, \\
H_0 &= H_3.
\end{aligned} \tag{13}$$

III. CONTRIBUTION FROM DIFFERENT CHANNEL

In this section, we investigate the effect of individual interactions in Eq. (12) on the color superconducting phase.

A. Thermodynamic potential

From Eq. (10), we obtain the thermodynamic potential

$$\Omega = \Omega_M + U + \Omega_e, \tag{14}$$

where Ω_M , U , and Ω_e are the contributions from the quark dynamical, quark condensate, and electron parts, respectively. They are expressed as

$$\begin{aligned}
\Omega_M &= - \sum_{i=1}^4 \int_0^\Lambda \frac{p^2 dp}{\pi^2} [\epsilon_r^i + 2T \ln(1 + e^{-\epsilon_r^i/T})] \\
&\quad - \sum_{f=u,d} \int_0^\Lambda \frac{p^2 dp}{\pi^2} E_{fb} + T \ln[1 + e^{-(E_{fb} + \hat{\mu}_{fb})/T}] + T \ln[1 + e^{-(E_{fb} - \hat{\mu}_{fb})/T}], \\
U &= G_s^{(0)} \phi^2 + G_s^{(8)} \phi_8^2 + G_{slv}^{(0)} \phi_I^2 + G_{slv}^{(8)} \phi_{I8}^2 + G_v^{(0)} n^2 + G_v^{(8)} n_8^2 + G_{vlv}^{(0)} n_I^2 + G_{vlv}^{(8)} n_{I8}^2 + H|\delta|^2 + H_0|\delta_0|^2, \\
\Omega_e &= - \int \frac{p^2 dp}{\pi^2} [T \ln(1 + e^{-(p+\mu_Q)/T}) + T \ln(1 + e^{-(p-\mu_Q)/T})].
\end{aligned} \tag{15}$$

Here, $E_{fc} = \sqrt{p^2 + M_{fc}^2}$ for $f = \{u, d\}$, $c = \{r, g, b\}$. $\phi = \langle \bar{\psi}\psi \rangle$, $\phi_8 = \langle \bar{\psi}\lambda_8\psi \rangle$, $\phi_I = \langle \bar{\psi}\tau_3\psi \rangle$, and $\phi_{I8} = \langle \bar{\psi}\tau_3\lambda_8\psi \rangle$ represent the Lorentz-scalar density in color-singlet, color-octet, isovector, and isovector-color-octet channels, respectively. $n = \langle \bar{\psi}\gamma^0\psi \rangle$, $n_8 = \langle \bar{\psi}\gamma^0\lambda_8\psi \rangle$, $n_I = \langle \bar{\psi}\gamma^0\tau_3\psi \rangle$, and $n_{I8} = \langle \bar{\psi}\gamma^0\tau_3\lambda_8\psi \rangle$ denote quark number density in color-singlet, color-octet, isovector, and isovector-color-octet channels, respectively. $\delta = \langle \bar{\psi}^C\gamma_5\tau_2\lambda_2\psi \rangle$ is diquark condensate Lorentz-scalar channel and $\delta_0 = \langle \bar{\psi}^C\gamma^0\gamma_5\tau_2\lambda_2\psi \rangle$ stands for the diquark condensate in Lorentz-vector channel. $\hat{\mu}_{fc}$ is the effective chemical potential for f -flavor c -color quark and $\mu_e = -\mu_Q$ is the chemical potential of electrons. The spectra ϵ_r^i in red and green subspace is the eigenvalue of the characteristic equation

$$\begin{aligned}
& [(-\lambda + \delta\mu)^2 - p^2 - \bar{M}_r^2 - \bar{\mu}^2 - \Delta^2 - \Delta_0^2 + (\delta M_r)^2]^2 \\
& - 4[(-\delta M_r(-\lambda + \delta\bar{\mu}) + \bar{M}_r\bar{\mu} - \Delta\Delta_0)^2 \\
& + p^2(\Delta_0^2 + \bar{\mu}^2 - \delta M_r^2)] = 0,
\end{aligned} \tag{16}$$

where $\Delta = -2H\delta$, $\Delta_0 = 2H_0\delta_0$ and

$$\begin{aligned}
\bar{\mu} &= \frac{\hat{\mu}_{ur} + \hat{\mu}_{dr}}{2}, & \delta\mu &= \frac{\hat{\mu}_{ur} - \hat{\mu}_{dr}}{2}, \\
\bar{M}_r &= \frac{M_{ur} + M_{dr}}{2}, & \delta M_r &= \frac{M_{ur} - M_{dr}}{2}.
\end{aligned} \tag{17}$$

Due to the degeneracy between the red and green quarks, we have $M_{fg} = M_{fr}$ and $\hat{\mu}_{fg} = \hat{\mu}_{fr}$ in the thermodynamic potential (15) and in the characteristic equation (16). Thus, we obtain the following relations:

$$\begin{aligned}
\frac{\delta\Omega_M}{\delta M_{fr}} &= \phi_{fr} + \phi_{fg} = 2\phi_{fr}, \\
-\frac{\delta\Omega_M}{\delta \hat{\mu}_{fr}} &= n_{fr} + n_{fg} = 2n_{fr},
\end{aligned} \tag{18}$$

with $f = u, d$ quarks.

In Refs. [36,37], the criterion for a g2SC phase is $\Delta \leq |\delta\mu|$. But this criterion is not applicable here when

we consider the contributions of δM_r and Δ_0 . At present, the criterion for the g2SC phase is that one of the spectra $\epsilon_r^i(p)$ —eigenvalues of the characteristic equation (16)—becomes zero at some momentum p . This implies that Eq. (16) has a real number solution about momentum p when we take $\lambda = 0$. Equivalently,

$$\begin{aligned}
& \Delta_0^2(\delta\mu^2 - \bar{M}_r^2) + (\bar{M}_r\delta M_r - \bar{\mu}\delta\mu)^2 + 2\Delta\Delta_0(\delta M_r\delta\mu - \bar{M}_r\bar{\mu}) \\
& + \Delta^2(\delta M_r^2 - \bar{\mu}^2) \geq 0.
\end{aligned} \tag{19}$$

If both δM_r and Δ_0 vanish, the above inequality reduces to $\Delta \leq |\delta\mu|$.

When we take HFB MFA, the constituent quark mass M_{fc} , effective chemical potential $\hat{\mu}_{fc}$, and 2SC gaps Δ , Δ_0 are related to condensates and densities through

$$\begin{aligned}
M_{fc} &= m - 2G_s^{(0)}\phi - 2G_s^{(8)}(\lambda_8)_{cc}\phi_8 - 2G_{slv}^{(0)}(\tau_3)_{ff}\phi_I \\
&\quad - 2G_{slv}^{(8)}(\tau_3)_{ff}(\lambda_8)_{cc}\phi_{I8}, \\
\hat{\mu}_{fc} &= \mu + 2G_v^{(0)}n + Q_f\mu_Q + (\lambda_8)_{cc}(\mu_8 + 2G_v^{(8)}n_8) \\
&\quad + 2G_{vlv}^{(0)}(\tau_3)_{ff}n_I + 2G_{vlv}^{(8)}(\tau_3)_{ff}(\lambda_8)_{cc}n_{I8},
\end{aligned} \tag{20}$$

with $Q_u = 2/3$, $Q_d = -1/3$ and $\lambda_8 = \frac{1}{\sqrt{3}}\text{diag}(1, 1, -2)$. The condensates and densities in the above equations can be determined from the extreme of the thermodynamic potential

$$\begin{aligned}
\frac{\delta\Omega}{\delta\phi} &= \frac{\delta\Omega}{\delta\phi_8} = \frac{\delta\Omega}{\delta\phi_I} = \frac{\delta\Omega}{\delta\phi_{I8}} = \frac{\delta\Omega}{\delta n} \\
&= \frac{\delta\Omega}{\delta n_8} = \frac{\delta\Omega}{\delta n_I} = \frac{\delta\Omega}{\delta n_{I8}} = \frac{\delta\Omega}{\delta\delta} = \frac{\delta\Omega}{\delta\delta_0} = 0.
\end{aligned} \tag{21}$$

Because of the charge and color neutrality, we have

$$\begin{aligned}
n_Q &= -\frac{\delta\Omega}{\delta\mu_Q} = 0, \\
n_8 &= -\frac{\delta\Omega}{\delta\mu_8} = 0.
\end{aligned} \tag{22}$$

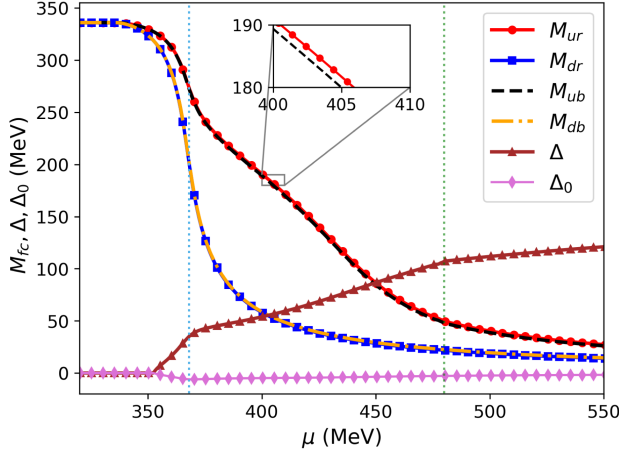


FIG. 1. The chemical potential dependence of the constituent quark mass M_{fc} and 2SC gap Δ, Δ_0 . The M_{db} (orange dash-dotted line) and M_{dr} (blue solid-square line) coincide. The vertical blue-dotted line at $\mu \approx 370$ MeV and green-dotted line at $\mu \approx 480$ MeV denote, respectively, the locations of the chiral phase transition and the g2SC to 2SC phase transition.

These neutrality conditions, which have not been seriously considered before, yield relations among the chemical potentials $\{\mu, \mu_Q \mu_8\}$, such that there is only one independent chemical potential μ .

B. Gap equation

To self-consistently analyze the contribution from a specific channel, we solve the gap equations (20)–(22) in the OGE model (7), which has only independent coupling g . The other couplings relate to g through Eq. (13). The model parameters are cutoff $\Lambda = 631$ MeV in Eq. (15), current quark mass $m = 5.5$ MeV in Eq. (20), and $G_s^{(0)} = 2.188/\Lambda^2$ [68], which were obtained by fitting $\langle \bar{u}u \rangle^{1/3} = -246$ MeV, $f_\pi = 93$ MeV, and $m_\pi = 138$ MeV. The coupling g is chosen such that $G_s^{(0)} = 2.188/\Lambda^2$.

The constituent mass of quarks and 2SC gap are shown in Fig. 1. From this figure, we find that the constituent mass of quarks is nearly independent of color. Explicitly, the mass difference between the red and blue components of up quarks is less than 5 MeV, while that of down quarks is less than 1 MeV. These mass differences are related to the condensates ϕ_8 and ϕ_{I8} through $M_{ur} - M_{ub} \sim \phi_8 + \phi_{I8}$ and $M_{dr} - M_{db} \sim \phi_8 - \phi_{I8}$. Then, the smallness of mass difference induced by color can be understood from Fig. 2, which shows that both condensates ϕ_8 and ϕ_{I8} are very small. Compared to ϕ_8 and ϕ_{I8} , the condensates ϕ_I and ϕ are big. Thus, we can ignore contributions from the channels $(\bar{\psi}\lambda_a\psi)^2$ and $(\bar{\psi}\tau\lambda_a\psi)^2$. The contribution of the $(\bar{\psi}\tau\psi)^2$ channel, ϕ_I^2 , has a maximum magnitude at about $\mu = 390$ MeV, and then decreases with chemical potential; therefore, this channel may have some effects on the 2SC phase transition at relatively low chemical

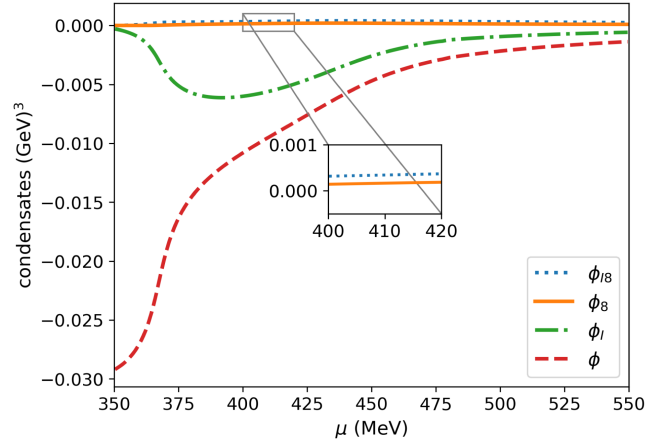


FIG. 2. The chemical potential dependence of different channel condensates.

potential (or density), but is expected to vanish as the chemical potential increases. The mass splitting between up and down quarks induced by the ϕ_I channel can reach up to 140 MeV, as can be seen from Fig. 1. In addition, this figure shows that, in the 2SC phase, the 2SC gap parameters Δ are much larger than Δ_0 , which agrees with the discussion in Ref. [60].

It is shown in Fig. 1 that, as the chemical potential increases, this model undergoes two transitions. The first is the chiral transition occurring at $\mu \approx 370$ MeV, which is estimated at the maximum value of the slope of effective mass M_{fc} —the susceptibilities [69], or equivalently, the slope of ϕ . In Ref. [65], this transition is first or second order because the chiral condensate is either discontinuous or nonsmooth. In our case, however, this transition behaves more like a crossover, as all condensates and densities remain smooth. This difference is due to the vector interaction channel included here, which can make the chiral transition smoother. The second is the transition from the g2SC phase to the 2SC phase, which occurs at $\mu \approx 480$ MeV according to the criterion (19). This result is different from that of Ref. [70], which stated that there is a phase transition from 2SC to normal quark phase (NQ). Although the ratio $H/G_s^{(0)} = 3/4$ used in Ref. [70] is equal to the OGE model considered here, the contribution of channel $(\bar{\psi}\gamma^0\tau\psi)^2$ was not considered. As we will discuss later, this channel will make the 2SC phase more stable than the NQ phase.

The color and charge neutrality conditions are

$$\begin{aligned} \frac{2}{3}n_u - \frac{1}{3}n_d &= n_e, \\ n_r &= n_g = n_b. \end{aligned} \quad (23)$$

Using the identities

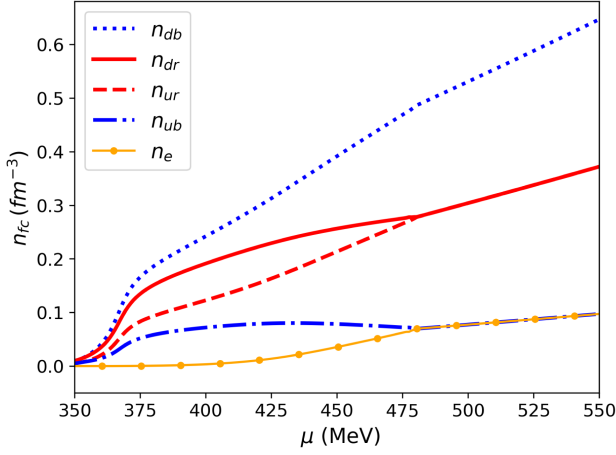


FIG. 3. The chemical potential dependence of the quark and electron densities.

$$\begin{aligned}
 n_r &= n_{ur} + n_{dr}, \\
 n_b &= n_{ub} + n_{db}, \\
 n_d &= n_{dr} + n_{dg} + n_{db}, \\
 n_u &= n_{ur} + n_{ug} + n_{ub}, \\
 n_{ur} &= n_{ug}, \quad n_{dr} = n_{dg},
 \end{aligned} \tag{24}$$

we find that the neutrality conditions are equivalent to

$$\begin{aligned}
 n_{dr} + n_{ur} &= n_{ub} + n_{db}, \\
 n_{dr} - n_{ur} &= n_{ub} - n_e.
 \end{aligned} \tag{25}$$

With respect to the constraints of color and charge neutrality, the densities of quarks with different flavors and colors and the density of electrons are displayed in Fig. 3. One can see that, in the gapless phase, the difference between n_{ur} and n_{dr} first increases and then decreases with chemical potential, until they are equal where the phase changes from g2SC to gapped 2SC. In the 2SC phase, the mismatch between the fermion surfaces of the u -red (u -green) quark and d -green (d -red) quark vanishes, and $n_{ug} = n_{ur} = n_{dg} = n_{dr}$. Thus, $n_{ub} = n_e$.

We plot in Fig. 4 the particle density fractions of quarks and electrons such that the charge neutrality is more clearly illustrated. According to Eq. (25), the charge neutrality requires that

$$n_{dr} - n_{ub} = n_{db} - n_{ur} = n_{ur} - n_e, \tag{26}$$

which is illustrated in Fig. 4 as the gaps between corresponding lines. The interesting thing is that $n_{dr} - n_{ub}$ is roughly proportional to the quark number density n in both the g2SC and 2SC phases. The reason for this observation will be discussed later.

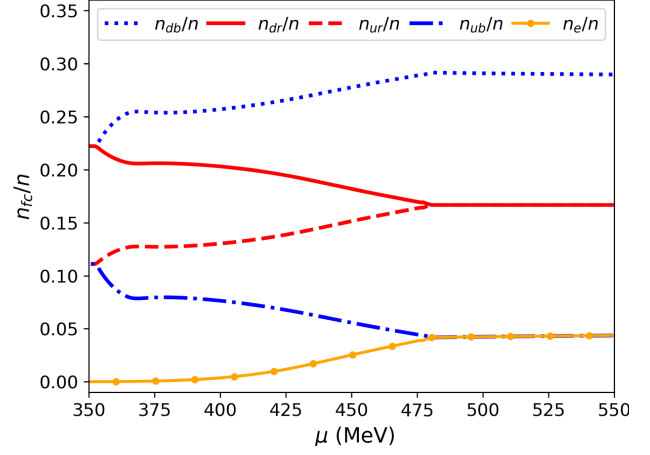


FIG. 4. The particle density fractions of quarks and electrons.

In the 2SC phase, $n_{ub} = n_e$, and therefore we have $\hat{\mu}_{ub} \approx \mu_Q = -\mu_Q$ in the chiral limit. Additionally, according to the chemical potential relations (20), we have

$$\hat{\mu}_{db} \approx 2\mu_e - 4 \left(G_{vIv}^{(0)} + \frac{4}{3} G_{vIv}^{(8)} \right) n_I. \tag{27}$$

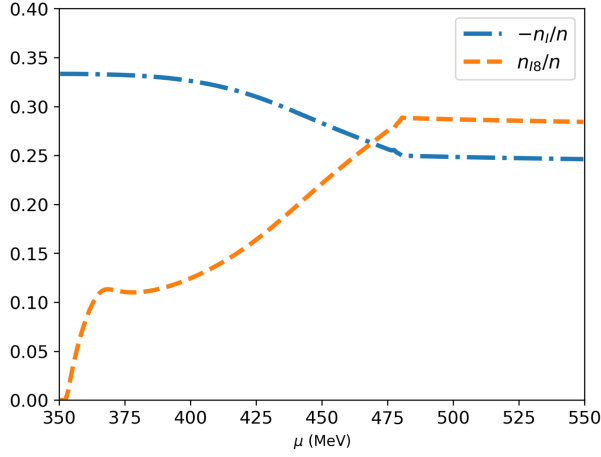
Because of

$$n = 2(n_{ur} + n_{dr}) + n_{ub} + n_{db} = 3(n_{ub} + n_{db}), \tag{28}$$

we obtain the fraction of electrons at zero temperature as

$$\begin{aligned}
 \frac{n_e}{n} &= \frac{1}{3 \left[2 - 4 \left(G_{vIv}^{(0)} + \frac{4}{3} G_{vIv}^{(8)} \right) \frac{n_I}{\mu_e} \right]^3 + 3} \\
 &\approx \frac{1}{27} + \frac{16}{81} \left[G_{vIv}^{(0)} + \frac{4}{3} G_{vIv}^{(8)} \right] \frac{n_I}{\mu_e} \\
 &\approx \frac{1}{27} - \frac{16}{81} \times \frac{7}{3\pi^2} \left[G_{vIv}^{(0)} + \frac{4}{3} G_{vIv}^{(8)} \right] \mu_e^2 \\
 &\approx \frac{1}{27} - \frac{112}{243\pi^2} \times \left(\frac{3}{5} \right)^2 \left[G_{vIv}^{(0)} + \frac{4}{3} G_{vIv}^{(8)} \right] \mu_e^2.
 \end{aligned} \tag{29}$$

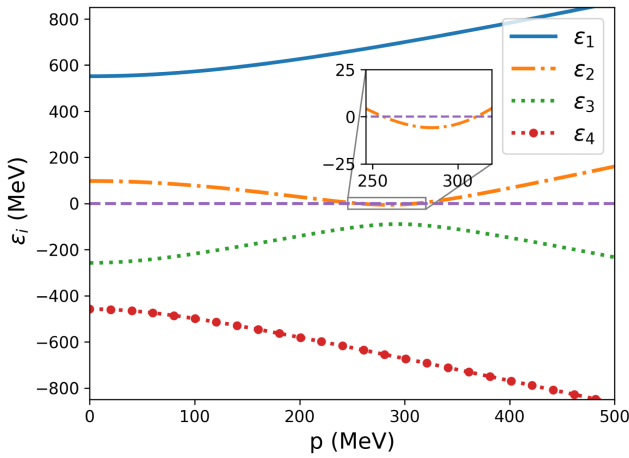
If we ignore the second term since the coupling constants $G \sim 1/\Lambda^2$ imply that the second term is much smaller than the first one, we have $n_e/n \approx 1/27$. The fraction of $n_{db}/n \approx 8/27$ is directly obtained from the neutrality constraints (25). In the 2SC phase, we can obtain the approximate ratios of quarks and electrons as $n_{db}:n_{ur}:n_{dr}:n_{ub}:n_e \approx 8:4.5:4.5:1:1$. In the NQ phase, we can also obtain the fractions $n_{dc}/n \approx 2/9$ and $n_{uc}/n \approx 1/9$ for $c = \{r, g, b\}$. The electrons in the NQ phase are negligible, $n_e/n \approx 0$. In the g2SC phase, the particle fractions lie between those of NQ phase and 2SC phase, namely

FIG. 5. The contributions of n_I and n_{I8} channels.

$$\begin{aligned}
 0 &\lesssim \frac{n_e}{n} \lesssim \frac{1}{27}, \\
 \frac{1}{27} &\lesssim \frac{n_{ub}}{n} \lesssim \frac{1}{9}, \\
 \frac{1}{9} &\lesssim \frac{n_{ur}}{n} \lesssim \frac{1}{6}, \\
 \frac{1}{6} &\lesssim \frac{n_{dr}}{n} \lesssim \frac{2}{9}, \\
 \frac{2}{9} &\lesssim \frac{n_{db}}{n} \lesssim \frac{8}{27}.
 \end{aligned} \quad (30)$$

From these relations, we can obtain that $0.11 \lesssim (n_{dr} - n_{ub})/n \lesssim 0.13$, which hardly changes with the chemical potential. In this model, including the second term of Eq. (29) results in a slight increase in the electron fraction from $1/27 \approx 0.037$ to 0.045 when $\mu = 500$ MeV.

The contributions of n_I and n_{I8} channels are displayed in Fig. 5. This figure shows that, in the g2SC phase, n_I is larger than n_{I8} and thus the effect of n_{I8} is insignificant,

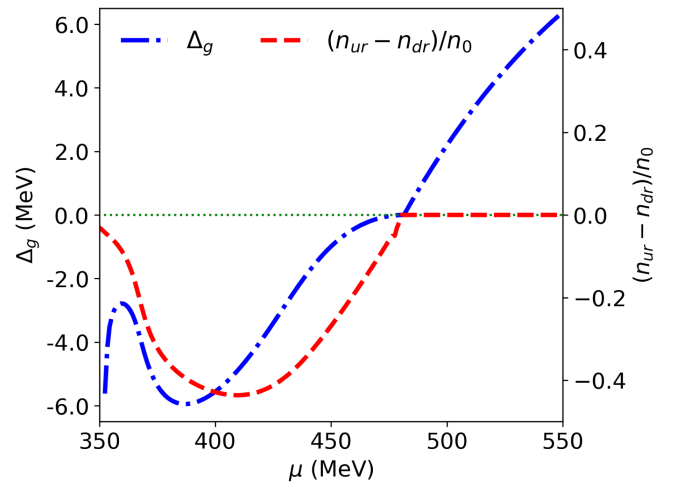
FIG. 6. Dispersion relations $\epsilon_r^i(p)$ at $\mu = 380$ MeV.

while in the 2SC phase, n_{I8} is proportional to n_I as $n_{I8} = -2n_I/\sqrt{3}$, and both effects cannot be neglected. Furthermore, in the gap equations (20), the contributions of n_I and n_{I8} are always added together, so the effects of n_{I8} can be mimicked by n_I totally. Considering the above reasons, in the following discussion, we will not address the impact of n_{I8} individually but focus solely on the effect of n_I .

Figure 6 displays the spectra $\epsilon_r^i(p)$ of the gap equations at $\mu = 380$ MeV. In this diagram, the ϵ_r^2 and ϵ_r^3 are particle modes, and ϵ_r^1 and ϵ_r^4 are antiparticle modes. If we ignore Δ_0 and δM_r , they are expressed as

$$\begin{aligned}
 \epsilon_r^1(p) &= \sqrt{(E_r(p) + \mu)^2 + \Delta^2} - |\delta\mu|, \\
 \epsilon_r^2(p) &= \sqrt{(E_r(p) - \mu)^2 + \Delta^2} - |\delta\mu|, \\
 \epsilon_r^3(p) &= -\sqrt{(E_r(p) - \mu)^2 + \Delta^2} - |\delta\mu|, \\
 \epsilon_r^4(p) &= -\sqrt{(E_r(p) + \mu)^2 + \Delta^2} - |\delta\mu|.
 \end{aligned} \quad (31)$$

Therefore, if there is no mismatch between Fermi surfaces of u -red (u -green) and d -green (d -red), i.e., $\delta\mu = 0$, the minimum energy required to break up a Cooper pair and excite a quasiparticle is Δ (see the ϵ_r^2 in above equations). However, the mismatch of the Fermi surfaces hinders the formation of Cooper pairs and makes it easier to excite a quasiparticle. When $|\delta\mu| \geq \Delta$, i.e., ϵ_r^i passes through the zero value such as the ϵ_r^2 in Fig. 6 and there is no gap to excite the quasiparticle, this is the so-called gapless phase. We denote the minimal of the ϵ_r^2 in Fig. 6 as Δ_g and draw its value as a function of μ in Fig. 7. From this picture we can see that, in the g2SC phase, the density difference between

FIG. 7. Chemical potential dependence of gap Δ_g of exciting a quasiparticle and the density difference between the u red and d red quarks. $n_0 \simeq 0.16 \text{ fm}^{-3}$ is the nuclear saturation density.

u red and d red quarks and the gap Δ_g show the same trend as μ changes, but former is slightly lagged. In the 2SC phase, the pairing between u -red (u -green) and d -green (d -red) quarks partially smooths out the mismatch of their Fermi surfaces. This “smearing” effect tends to equalize the Fermi surfaces of u -red and d -green quarks, making their particle numbers nearly identical. In the g2SC phase, however, the Fermi-surface mismatch is larger and cannot be completely compensated by the pairing, so a particle-number difference appears. If we neglect the contributions from δM and Δ_0 , Eq. (31) shows that $\Delta_g = \Delta - |\delta\mu|$. Here, Δ_g can be regarded as the net Fermi-surface mismatch between u -red and d -green quarks after being partially compensated by the pairing. Therefore, the particle-number difference between u -red and d -green quarks can be estimated as

$$n_{ur} - n_{dg} = n_{ur} - n_{dr} \approx \frac{2\bar{\mu}^2}{\pi^2} \Delta_g \theta(-\Delta_g), \quad (32)$$

where $\theta(x)$ is the Heaviside step function.

IV. INFLUENCE OF DIFFERENT CHANNELS ON 2SC PHASE STRUCTURE

Based on the aforementioned analysis, it becomes evident that, besides the diquark channel Δ and vector channel n , the channels ϕ_I and n_I may have considerable influence on the 2SC phase diagram. In order to see their distinct impacts on the phase diagram, we systematically modulate the coupling constants associated with these channels. We vary the ratios of the coupling constants of these channels to that of the scalar channel, and separately discuss the effects of these channels while neglecting the contributions from other channels. In this sense, the results in this section are beyond the OGE model.

A. Phase diagram of 2SC

The phase diagram with different values of $H/G_s^{(0)}$ is shown in Fig. 8. Here, we only take the diquark channel into consideration. In this figure, we can see that the g2SC phase lies in the range of $0.67 \lesssim H/G_s^{(0)} \lesssim 0.81$ at low chemical potential $\mu \lesssim 400$ MeV, consistent with Ref. [36], which was obtained in the chiral limit and with the approximations $\Delta \ll \bar{\mu} \ll \Lambda$. The higher values of $H/G_s^{(0)}$ correspond to the gapped 2SC phase, while the lower values of $H/G_s^{(0)}$ correspond to normal quark phase. This observation indicates that the current quark mass does not affect the lower boundary of the g2SC phase so much. However, the upper bound of the g2SC phase differs somewhat from that of Ref. [36]. The reason for this discrepancy is that the approximation $\Delta \ll \bar{\mu} \ll \Lambda$ used in Ref. [36] is no longer valid when μ approaches the cutoff Λ .

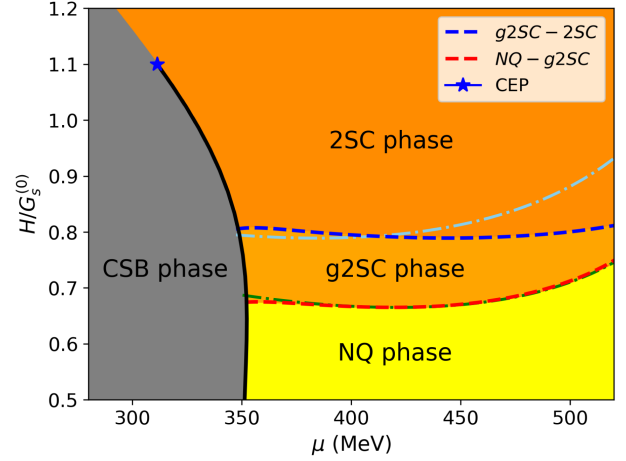


FIG. 8. The phase diagram in the $\mu - H/G_s^{(0)}$ plane. The solid line denotes the first-order chiral phase transition, and the blue and red dashed lines represent the two boundaries of the g2SC phase. For comparison, the boundaries of g2SC phase obtained in Ref. [36] are plotted as dash-dotted lines.

Moreover, Fig. 8 also shows that the increasing value of $H/G_s^{(0)}$ decreases the chemical potential of chiral phase transition. In order to understand the reason for this effect, we analyze the impact of the diquark channel with fixed chemical potential in Fig. 9. This figure illustrates that in the g2SC phase, the diquark gap parameter Δ slightly increases the constituent quark mass, while in the 2SC phase, it decreases the constituent quark mass. For this reason, in 2SC phase, the increasing of Δ will cause the phase transition to occur earlier. Especially when $H/G_s^{(0)}$ is larger than 1.1, the chiral phase transition becomes a crossover instead of first order. This result is quite different from Ref. [65], where the charge and color neutrality constraints were not taken into account. As shown in Fig. 10, the chiral phase transition is of first order when $H/G_s^{(0)} = 1.0$. As

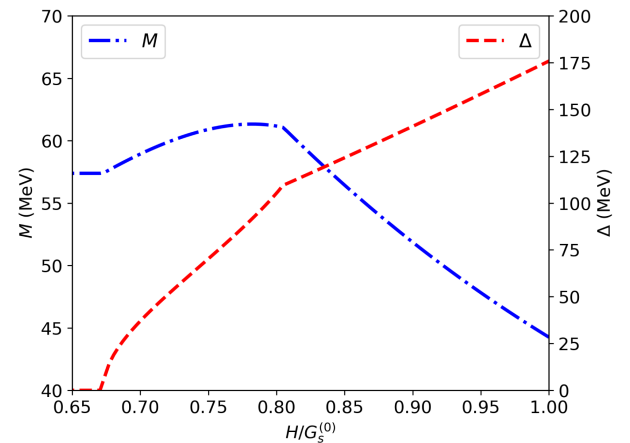


FIG. 9. $H/G_s^{(0)}$ effect on the constituent quark mass M and diquark gap parameter Δ at $\mu = 370$ MeV.

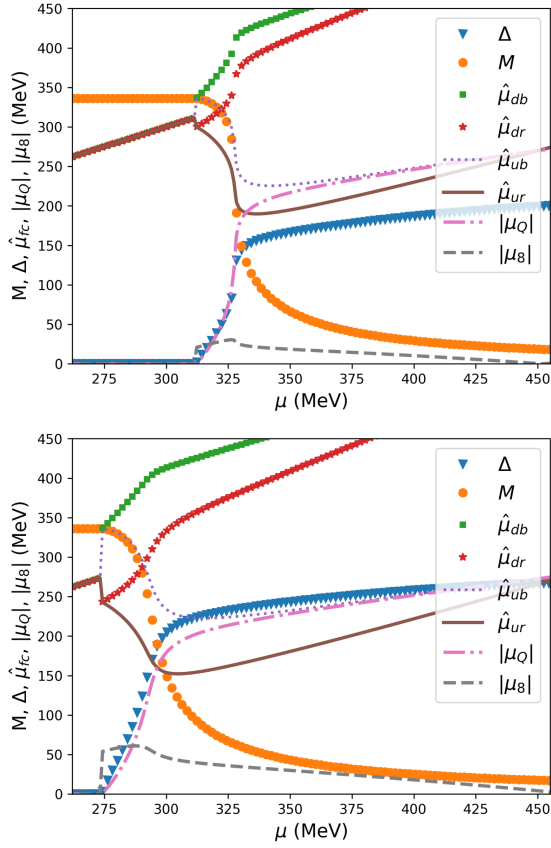


FIG. 10. The chemical potential dependence of the constituent quark mass M , 2SC gap Δ , effective chemical potentials $\hat{\mu}_{fc}$, $|\mu_Q|$, and $|\mu_8|$, with different values of $H/G_s^{(0)}$. The upper panel corresponds to $H/G_s^{(0)} = 1.0$, and the lower panel to $H/G_s^{(0)} = 1.2$.

$H/G_s^{(0)}$ increases, the transition becomes a crossover, rather than a second-order phase transition suggested in Ref. [65]. In our case—for example, at $H/G_s^{(0)} = 1.2$ —all quantities appear to vary smoothly with the chemical potential in the vicinity of the chiral phase transition, indicating that the transition is a crossover. The CEP is located at $\mu = 312$ MeV, $H/G_s^{(0)} = 1.1$, and the corresponding $\Delta = 138$ MeV.

B. Effects of other channels on 2SC phase structure

In order to show the influences of the different channels, specifically the ϕ_I , n , and n_I channels, on the phase diagram, we separately solve the gap equations with some typical values of couplings.

In Fig. 11, we analyze the effect of the ϕ_I channel with $G_{slv}^{(0)}/G_s^{(0)} = 1$. We find that the ϕ_I channel decreases the chemical potential of chiral phase transition slightly. In addition, it expands the gapless phase range from $0.67 - 0.8$ to $0.67 - 0.85$ in the chemical potential region shortly after the chiral phase transition. The ϕ_I channel mainly affects

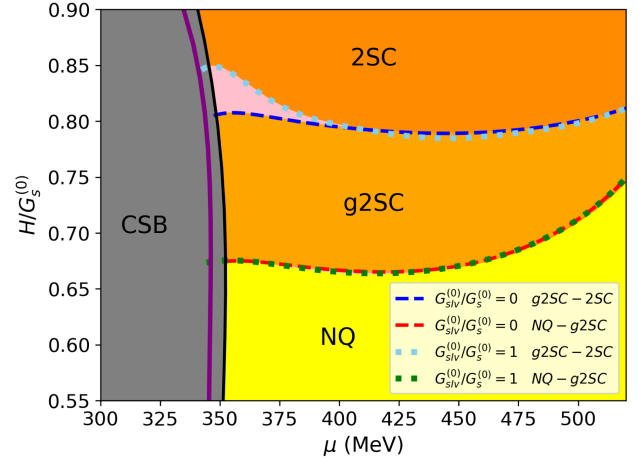


FIG. 11. Phase diagram in the $\mu - H/G_s^{(0)}$ plane. The black and purple solid lines correspond to the first-order chiral phase transitions without and with considering the ϕ_I channel interactions, respectively. The dashed and dotted lines represent the phase boundaries of the g2SC phase, with and without the ϕ_I channel contributions, respectively.

the mass difference between up and down quarks. This mass difference contributes a value of $M_r \delta M_r / E_r(p)$ to $|\delta\mu|$ in dispersion relation $\epsilon_r^2(p)$ in Eq. (31), transforming the original gapped 2SC into the g2SC region. Meanwhile, the ϕ_I channel hardly changes the lower boundary of the g2SC phase and the upper boundary at high chemical potential.

The effect of the n channel on the 2SC phase is shown in Fig. 12 with $G_v^{(0)}/G_s^{(0)} = -0.3$. We choose a negative $G_v^{(0)}$ because, after bosonizing the NJL Lagrangian, $-1/G_v^{(0)}$ is proportional to the squared mass of the vector meson times

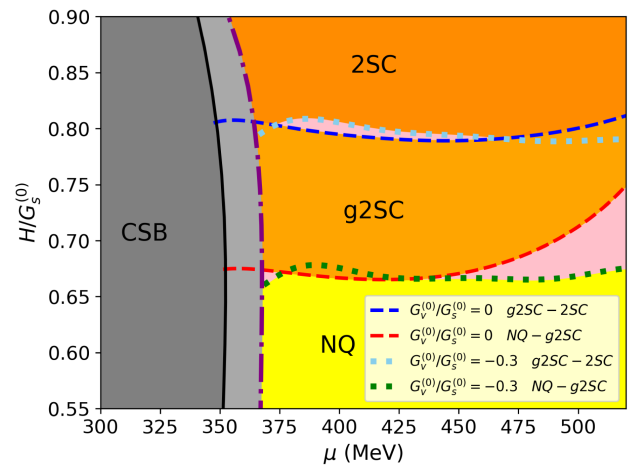


FIG. 12. Effects of n channel on the phase boundaries of the g2SC phase. The purple dash-dotted line denotes the crossover chiral phase transition. The dotted and dashed lines indicate the g2SC phase boundaries with and without the n channel interactions, respectively.

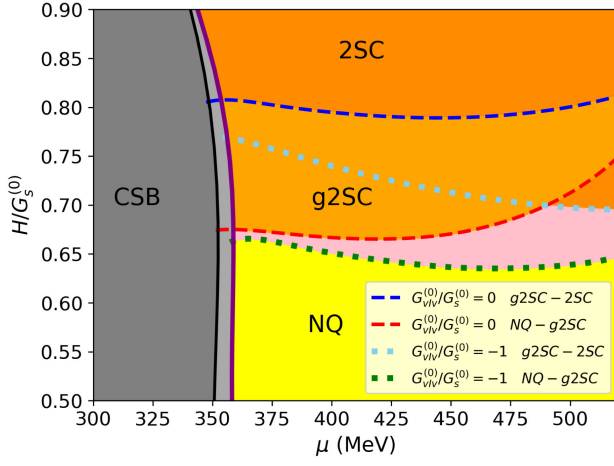


FIG. 13. Comparison of the g2SC phase boundaries with different n_I channel coupling strengths. The dashed and dotted lines correspond to the g2SC phase boundaries without and with the inclusion of the n_I channel contributions, respectively.

a positive constant [71]. The main feature of the n channel is to shift the entire phase diagram along the high chemical potential direction. At the same time, the type of chiral phase transition changes from a first-order phase transition to a crossover. From the gap equation (20) we can see that, due to the negative coupling $G_v^{(0)}$, the density n decreases the effective chemical potential, which is equivalent to an increase of the chemical potential. Consequently, this channel shifts the entire diagram to the right. As mentioned in Ref. [70], when the chemical potential nears the cutoff Λ , the diquark condensate starts to be affected by the momentum cutoff, declining to zero, and eventually triggers a transition to the NQ phase. The n channel's contribution removes this limitation, stabilizing the 2SC phase at high chemical potentials.

Finally, we show the effects of the n_I channel in Fig. 13 by using $G_{vIv}^{(0)}/G_s^{(0)} = -1$. This figure shows that the n_I channel reduces the upper and lower boundaries of the g2SC phase, with the effect more significant as the chemical potential increases. The gap equation (20) implies that the mismatch $\delta\mu$ of the chemical potential between the u -red and d -green quarks is proportional to $G_{vIv}^{(0)}n_I$. Because we take $G_{vIv}^{(0)} < 0$, n_I decreases the value of $|\delta\mu|$, then decreases the g2SC upper boundary. As we mentioned above, there is a phase transition from g2SC to 2SC instead of to NQ phase at $H/G_s^{(0)} \sim 0.75$. The g2SC to NQ phase transition disappears, replaced by a g2SC to 2SC phase transition.

V. SUMMARY AND DISCUSSION

In this paper, we systematically investigated the effects of various interaction channels within the NJL model on the

2SC phase at zero temperature by seriously considering the constraints from color neutrality and charge neutrality. We started from the general NJL model and derived the relations among the couplings using Fierz transformations. All these couplings are controlled by only five independent couplings.

To figure out the dominant channels, we adopted the traditional OGE model. The results showed that in addition to the diquark and vector channels widely discussed in the literature, due to the inclusion of both charge/color neutrality constraints and self-consistent interaction contributions, scalar-isovector, vector-isovector, and vector-isovector-color-octet channels are also found to be important, while other channels can be neglected. Furthermore, our analysis showed that the color and charge neutrality conditions give rise to the emergence of the g2SC phase between the NQ and 2SC phases, while the contribution from the ϕ_I channel induces the phase transition from g2SC to 2SC. The particle fractions of quarks with different colors and flavors were estimated as $n_{db}:n_{ur}:n_{dr}:n_e \approx 8:4.5:4.5:1:1$ in the 2SC phase. Moreover, the ϕ_{slv} channel results in a mass splitting of up to 140 MeV between the u and d quarks. Additionally, the contribution from the n channel turns the chiral phase transition into a crossover, rather than a first-order, transition.

Next, we discussed the impacts of some channels on the phase structure beyond the OGE approximation. We found that a gapless phase lies in the range of $0.67 \lesssim H/G_s^{(0)} \lesssim 0.81$, which agrees with the results of Ref. [36]. Moreover, the increasing value of $H/G_s^{(0)}$ decreases the chemical potential of chiral phase transition. This result is caused by the competition between the diquark condensate and chiral condensate in the 2SC phase [61,65]. In addition, we found the CEP on the $\mu - H/G_s^{(0)}$ plane located at $\mu = 312$ MeV, $H/G_s^{(0)} = 1.1$ with $\Delta = 138$ MeV. The chiral transition is crossover instead of first order when $H/G_s^{(0)} > 1.1$. For the influence of other channels on the 2SC phase diagram, we found that the scalar-isovector channel ϕ_I expands the gapless phase from $0.67 \lesssim H/G_s^{(0)} \lesssim 0.8$ to $0.67 \lesssim H/G_s^{(0)} \lesssim 0.85$ in the chemical potential region shortly after the chiral phase transition. The vector channel shifts the entire phase diagram along the larger μ direction. The isovector channel reduces the $H/G_s^{(0)}$ values of both the upper and lower boundaries of the g2SC phase.

The above results indicate that when studying color-superconducting phases and their applications in neutron star physics, these interaction channels should also be considered.

We note, however, that the critical density of the chiral phase transition illustrated in Fig. 1 in our calculation is about $1.1\rho_0$, only slightly above nuclear saturation density.

In this low-density region, the NJL model is not reliable due to the absence of confinement and explicit baryonic degrees of freedom, and the transition shown in Fig. 1 should not be interpreted as quantitatively reliable at such low densities. Our results should thus be understood as illustrating the qualitative interplay between chiral and diquark condensates within the quark matter framework, rather than as quantitative predictions near nuclear saturation density.

In future work, we plan to incorporate more realistic nuclear matter equations of state in order to make contact with neutron star physics, and to explore in detail the possible window for the existence of the g2SC phase in neutron star cores.

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DATA AVAILABILITY

All data generated or analyzed during this study are included in this published article.

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