

Coupling light waves to gravitational waves

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We demonstrate analytically that gravitational waves, upon interacting with copropagating electromagnetic radiation in a plasma, induce distinctive sidebands on the modulated light, thereby providing a detectable signature of their presence. Employing a fully covariant coupled-wave framework, we envision gravitational waves as phase-insensitive luminal moving gratings and derive explicit phase-matching conditions that articulate such an interaction while conserving both energy and momentum. Beyond preserving the directional signature of gravitational waves, the coupling mechanism imposes no coherence requirements on the photon-by-graviton scattering, hence enabling possibilities for exploiting cosmic microwave background radiation. Although detection at low frequencies is constrained by the requirement of long interaction lengths, advances in laser technology are poised to enable high-frequency gravitational wave detection, potentially unveiling insights into the primordial spacetime ripples that have been traversing the cosmos since the inflationary epoch.

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I. INTRODUCTION

As early as 1893, Heaviside proposed an analogy between electromagnetic waves (EMWs) and gravitational waves (GWs) [1], a concept later formalized through Einstein's general theory of relativity [2]. While pioneering efforts in the 1960s laid the groundwork for GW detection [3], only recently have advanced interferometric techniques achieved the direct detection of GWs, primarily at low frequencies (10–1000 Hz) [4]. The detection of high-frequency GWs in the MHz–GHz range, however, remains an uncharted experimental frontier, with the potential to unveil early-Universe phenomena such as cosmic strings [5]. While resonant microwave cavities [6] and axion haloscope detectors [7] have shown promise, experimental confirmation remains a formidable challenge.

The interaction between EMWs and GWs has captivated physicists since the prediction of the Gertsenshtein effect, whereby an EMW traversing a strong transverse magnetic field generates a GW of the same frequency [8], provided both waves are coherent [9]. Despite its theoretical appeal, this effect is limited by weak coupling, which has hindered observational prospects. The inverse Gertsenshtein effect—whereby spacetime distortions within a static magnetic

field generate EMWs—further highlights the complexity of such EMW-GW interactions [10].

Akin to the paradigm of “synthetic traveling-wave modulation” in metaoptics [11] and the notion of a “luminal mirror” [12], we propose in this communication to conceptualize GWs as “luminal moving gratings”—see Fig. 1. Employing a

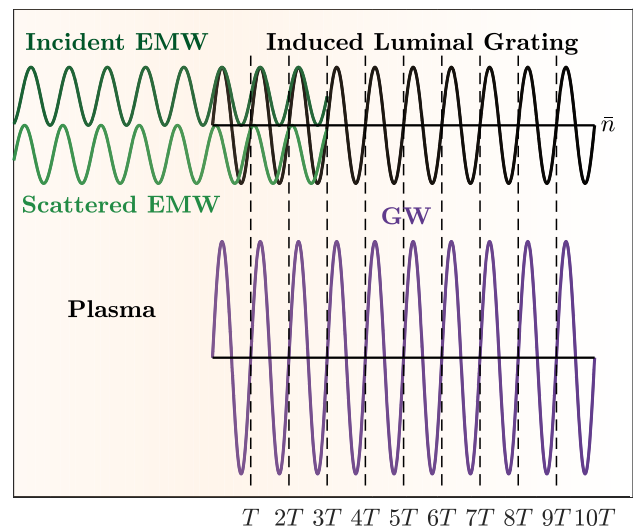


FIG. 1. Luminal grating induced by a gravitational wave. A plane GW, propagating at the speed of light, induces a periodic (period: T) spatiotemporal modulation in the refractive index (mean value: $\bar{n}$), forming a dynamic scaffold that facilitates forward scattering of EMWs in a plasma. The emerging subluminal EMWs encode the grating's characteristics, thus enabling indirect GW detection, while preserving its directional signature without necessitating coherence.

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fully covariant coupled-wave approach, we establish a rigorous framework for EMW-GW interactions, wherein GWs are modeled as archetypal spacetime perturbations propagating at the speed of light. These dynamic disturbances induce effective refractive index modulations, which interact with EMWs in a plasma under specific phase-matching conditions that conserve both energy and momentum. In this setting, GWs copropagating with EMWs in a plasma generate characteristic sidebands in the spectrum, offering a distinctive signature of their presence. Our scheme aims to extend the detectable GW frequency spectrum beyond the limitations of current interferometric techniques and provides a pathway toward the long-sought directional detection of high-frequency GWs, while relaxing the requirement for coherence between the interacting waves.

The present article is organized as follows: in Sec. II we outline the mathematical framework for the propagation of EMWs in curved spacetime, specifically under the influence of a plane GW. Building on this foundation, Sec. III develops a four-dimensional coupled-wave theory to describe the modulation of EMWs by GWs, in particular the generation of sidebands in the electromagnetic spectrum. Using the analytic expressions derived for the interaction lengths required to obtain detectable scattered signals, Sec. IV examines the feasibility of coupling coherent and incoherent EMW sources to GWs. Finally, Sec. V summarizes the results and discusses the detection challenges posed by the intrinsically weak coupling.

II. ELECTROMAGNETIC FIELDS IN CURVED SPACETIME

In the presence of a weak gravitational field, the spacetime metric can be treated as a small perturbation of the flat Minkowski metric $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$. Thus in nearly Lorentzian coordinates we have $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $h_{\mu\nu}$ is the GW perturbation, with $|h_{\mu\nu}|$ and all its derivatives $\ll 1$ [13]. For a plane GW propagating along the z -axis, the metric perturbation assumes the form $h^{\mu\nu} = (1/2)\mathcal{H}^{\mu\nu}e^{iK_\rho x^\rho} + \text{c.c.}$, where $\mathcal{H}^{\mu\nu}$ is a Hermitian polarization tensor, $K_\mu = \Omega(-1, 0, 0, \pm 1)$ is the 4-wavevector of the GW, and complex conjugation ensures real solutions; Ω is the GW frequency and the “ $\pm$ ” sign indicates the direction of phase propagation.

In the absence of sources the electromagnetic field in curved spacetime obeys a wave equation for the 4-potential A^μ , namely [16]

$$-A^{\alpha;\mu}_{;\mu} + A^\mu_{;\mu}{}^{;\alpha} + R^\alpha_\mu A^\mu = 0, \quad (1)$$

where R^α_μ is the Ricci tensor; in the linearized regime $R_{\mu\nu} \approx (h_{\nu}{}^\alpha{}_{;\mu} + h_{\mu}{}^\alpha{}_{;\nu} - h_{\mu\nu}{}^{;\alpha} - h_{\mu\nu}{}_{;\alpha})/2$. Under the Lorenz gauge, $A^\mu_{;\mu} = 0$, Eq. (1) simplifies to $A^{\alpha;\mu}_{;\mu} = R^\alpha_\mu A^\mu$. As it turns out, in transverse-traceless (TT) gauge the Ricci tensor vanishes. Indeed, in the TT gauge $h = \eta^{\alpha\beta} h_{\alpha\beta} = h^\alpha_\alpha = 0$

whereby substituting the GW ansatz into the Ricci tensor yields $2R_{\mu\nu} = -K_\mu K_\nu h^\alpha_\alpha - K_\nu K_\alpha h_\mu^\alpha + K_\alpha K^\alpha h_{\mu\nu} - h^\alpha_\alpha{}_{;\mu\nu}$. The first two terms vanish on account of transversality, the third term vanishes since K_α is null, and the fourth term vanishes due to tracelessness. The wave equation thus reduces to $A^{\alpha;\mu}_{;\mu} = 0$ [17]. Subsequently the covariant derivatives can be expanded as $(A^{\alpha;\mu} + \eta^{\mu\lambda}\Gamma^\alpha_{\nu\lambda}A^\nu)_{;\mu} = 0$, with $\Gamma^\alpha_{\nu\lambda}$ being the Christoffel symbols; the Einstein summation convention is herein implicit. Expanding the latter leads to

$$A^{\alpha;\mu}_{;\mu} + \eta^{\mu\lambda}\Gamma^\alpha_{\nu\lambda,\mu}A^\nu + \Gamma^\alpha_{\nu\lambda}A^{\nu,\lambda} + (A^{\beta;\mu} + \eta^{\mu\lambda}\Gamma^\beta_{\nu\lambda}A^\nu)\Gamma^\alpha_{\beta\mu} = 0.$$

As detailed in Sec. V of Supplemental Material [13], the Christoffel symbols in the linearized regime retain only $\mathcal{O}(h)$ terms, thereby leading to

$$A^{\alpha;\mu}_{;\mu} + \eta^{\mu\lambda}\Gamma^\alpha_{\nu\lambda,\mu}A^\nu + 2\Gamma^\alpha_{\nu\lambda}A^{\nu,\lambda} = 0. \quad (2)$$

III. FOUR-DIMENSIONAL COUPLED-WAVE THEORY

Considering first-order diffracted sidebands, the total EMW potential is formulated as a superposition of an incident $\mathcal{A}_{(0)}$ and two scattered, $\mathcal{A}_{(-1)}$ and $\mathcal{A}_{(1)}$, wave potentials: $A^\alpha = \sum_j \mathcal{A}_{(j)}^\alpha e^{ik_{(j)\mu}x^\mu}$. For a plane EMW propagating in a plasma along the z -axis, $k_{(j)\mu} = \omega_{(j)}(-1, 0, 0, \pm n)$ is the 4-wavevector, with $\omega_{(j)}$ being both the temporal and the spatial frequency in units where the phase velocity of light in vacuum is unity; $j = \{-1, 0, 1\}$ labels the incident, $j = 0$, and scattered, $j = \pm 1$, EMWs. In a plasma, dispersion modifies the refractive index, which in the high-frequency limit may be approximated as $n(\omega) = 1 - \delta n$, where $0 < \delta n \ll 1$. This slight departure from lightlike propagation nullifies the vacuum null condition: $k_{(j)}^\mu k_{(j)\mu} = \omega_{(j)}^2(n^2 - 1) \approx -2\delta n\omega_{(j)}^2$. In typical interstellar environments, $\delta n \sim 10^{-8}$ at radio frequencies [18]—a small yet consequential deviation permitting interactions with GWs otherwise forbidden in the luminal regime [19,20].

Based on the EMW ansatz, we calculate

$$A^{\nu,\lambda} = \sum_j \left(\mathcal{A}_{(j)}^{\nu,\lambda} + ik_{(j)}^\lambda \mathcal{A}_{(j)}^\nu \right) e^{ik_{(j)\mu}x^\mu}, \quad (3a)$$

$$A^{\alpha;\mu}_{;\mu} \approx \sum_j \left(2ik_{(j)}^\mu \mathcal{A}_{(j),\mu}^\alpha - k_{(j)}^\mu k_{(j)\mu} \mathcal{A}_{(j)}^\alpha \right) e^{ik_{(j)\nu}x^\nu} \quad (3b)$$

and note that the simplification of Eq. (3b) stems from the slowly varying envelope approximation [21]. Whence, substituting the 4-potential ansatz together with Eqs. (3a)

and (3b) into Eq. (2), we obtain

$$\begin{aligned} & \sum_j \left[2ik_{(j)}^\mu \mathcal{A}_{(j),\mu}^\alpha + i \frac{\eta^{\alpha\beta}}{2} \left(\Delta\Gamma_{\beta\nu\lambda} e^{iK_\rho x^\rho} - \Delta\Gamma_{\beta\nu\lambda}^* e^{-iK_\rho x^\rho} \right) \mathcal{A}_{(j)}^{\nu\lambda} \right] e^{ik_{(j)\gamma} x^\gamma} \\ &= \sum_j \left[\frac{\eta^{\alpha\beta}}{2} \left(\Delta\Gamma_{\beta\nu\lambda} e^{iK_\rho x^\rho} - \Delta\Gamma_{\beta\nu\lambda}^* e^{-iK_\rho x^\rho} \right) k_{(j)}^\lambda - \frac{\eta^{\mu\lambda} \eta^{\alpha\beta}}{4} \left(\Delta\Gamma_{\beta\nu\lambda\mu} e^{iK_\rho x^\rho} + \Delta\Gamma_{\beta\nu\lambda\mu}^* e^{-iK_\rho x^\rho} \right) \right] \mathcal{A}_{(j)}^{\nu\lambda} e^{ik_{(j)\mu} x^\mu}, \end{aligned} \quad (4)$$

where $\Delta\Gamma_{\beta\nu\lambda} = \mathcal{H}_{\beta\lambda} K_\nu + \mathcal{H}_{\nu\beta} K_\lambda - \mathcal{H}_{\nu\lambda} K_\beta$ and $\Delta\Gamma_{\beta\nu\lambda\mu} = \mathcal{H}_{\nu\lambda} K_\beta K_\mu - \mathcal{H}_{\beta\lambda} K_\nu K_\mu - \mathcal{H}_{\nu\beta} K_\lambda K_\mu$.

Scrutinizing Eq. (4) one can forthwith identify that synchronous sidebands must have 4-wavevectors given by

$$k_{(1)\mu} = k_{(0)\mu} + K_\mu \quad \text{and} \quad k_{(-1)\mu} = k_{(0)\mu} - K_\mu, \quad (5)$$

hence reaffirming the simulations of Ref. [22] and consistent with 4-momentum conservation required for single-graviton absorption in the dilute regime of Ref. [24]. The phase-matching conditions of Eq. (5) encapsulate the requirement that for this subluminal [25] coupling process both energy ($\mu = 0$) and momentum ($\mu = 3$) must be simultaneously conserved, dictating moreover that for weak δn all EMWs and GWs must be copropagating. It is this aspect of the coupling that conveys directional information about the GWs, with the angular spectrum of the generated sidebands being, in practice, dictated by the bandwidths of both the GW and the primary EMW.

Manifestly, the relationships in Eq. (5) represent an intermediate regime between a static Bragg grating, which conserves energy but not momentum [26], and a temporal Bragg grating, which conserves momentum but not energy [27]. This intermediary scenario pertains to traveling-wave modulation [11,28], with Eq. (5) aligning with the frame-specific conservation laws derived in Ref. [29]. A key distinction from the preceding cases lies in the absence of a resonant (or indeed synchronization) phenomenon *per se*, evincing that the “scattering” effect is *not* contingent upon any specific resonant frequency.

Given the heretofore prevailing technological limitations for GW detection, we assume $|K_\mu| \ll |k_{(j)\mu}|$, indicating that light couples with a GW of considerably lower frequency. Under this assumption, $\omega_{(0)} \approx \omega_{(1)} \approx \omega_{(-1)} \equiv \omega$, substantiating that the outgoing wave phases copropagate with respect to the incident wave phase. Nevertheless, GWs are either predicted or observed across a wide spectral range. Therefore, by relaxing this presupposition and setting, say, $\Omega = 2\omega_{(0)}$, Eq. (5) entails that the spatial and temporal directions of the “(−1)” generated sideband are reversed from $k_{(-1)\mu} = \omega_{(0)}(-1, 0, 0, +n)$ to $k_{(-1)\mu} = \omega_{(0)}(1, 0, 0, -1 - \delta n)$. Since EMWs are described by the real part of phasors, the “(−1)” generated sideband will remain copropagating with respect to the incident wave, underpinning yet another fundamental distinction

from conventional Bragg scattering: for most natural environments, where δn is weak, no backscattering as such is supported.

For copropagating waves in a plasma, backscattering of a frequency downshifted “(−1)” EMW requires $\delta n > 1 - \Omega/\omega_{(0)}$, provided that $\omega_{(0)} > \Omega$. Conversely, for counterpropagating waves in a plasma, backscattering of a frequency upshifted “(+1)” EMW requires the same inequality, which in this instance holds for arbitrary values of $\omega_{(0)}$ and Ω . One might hastily conclude that setting $\delta n = 0$ always permits backscattering of counterpropagating EMWs and GWs; however, this is incorrect, as in the luminal regime the k_0 and k_3 components of the 4-wavevector must be equal in magnitude.

Applying the phase-matching conditions from Eq. (5) we can model the interaction between the impinging EMW and the forward-scattered EMWs in a plasma, mediated by the GW, via a system of coupled-wave equations. Collecting all terms that match $k_{(0)}$ we obtain

$$\begin{aligned} & k_{(0)}^\mu \mathcal{A}_{(0),\mu}^\alpha + C_{\nu}^{\alpha\mu} \mathcal{A}_{(-1),\mu}^\nu - (C_{\nu}^{\alpha\mu})^* \mathcal{A}_{(1),\mu}^\nu \\ &= P_{\nu}^{\alpha} \mathcal{A}_{(-1)}^\nu + Q_{\nu}^{\alpha} \mathcal{A}_{(1)}^\nu. \end{aligned} \quad (6a)$$

Subsequently the terms that match $k_{(1)}$ are written as

$$k_{(1)}^\mu \mathcal{A}_{(1),\mu}^\alpha + C_{\nu}^{\alpha\mu} \mathcal{A}_{(0),\mu}^\nu = M_{\nu}^{\alpha} \mathcal{A}_{(0)}^\nu + iN_{\nu}^{\alpha} \mathcal{A}_{(0)}^\nu, \quad (6b)$$

while those that match $k_{(-1)}$ as

$$\begin{aligned} & k_{(-1)}^\mu \mathcal{A}_{(-1),\mu}^\alpha - (C_{\nu}^{\alpha\mu})^* \mathcal{A}_{(0),\mu}^\nu \\ &= (M_{\nu}^{\alpha})^* \mathcal{A}_{(0)}^\nu + i(N_{\nu}^{\alpha})^* \mathcal{A}_{(0)}^\nu. \end{aligned} \quad (6c)$$

The introduced tensors are defined as follows: $C_{\nu}^{\alpha\mu} = (\eta^{\alpha\beta}/4) \Delta\Gamma_{\beta\nu\lambda} \eta^{\lambda\mu}$, $M_{\nu}^{\alpha} = -i(\eta^{\alpha\beta}/4) k_{(0)}^\lambda \Delta\Gamma_{\beta\nu\lambda}$, $N_{\nu}^{\alpha} = (\eta^{\mu\lambda} \eta^{\alpha\beta}/8) \Delta\Gamma_{\beta\nu\lambda\mu}$, $Q_{\nu}^{\alpha} = (i/4) [\eta^{\alpha\beta} k_{(1)}^\lambda \Delta\Gamma_{\beta\nu\lambda}^* + (\eta^{\mu\lambda} \eta^{\alpha\beta}/2) \Delta\Gamma_{\beta\nu\lambda\mu}^*]$, and $P_{\nu}^{\alpha} = -(i/4) [\eta^{\alpha\beta} k_{(-1)}^\lambda \Delta\Gamma_{\beta\nu\lambda} - (\eta^{\mu\lambda} \eta^{\alpha\beta}/2) \Delta\Gamma_{\beta\nu\lambda\mu}]$.

Considering transverse waves traversing a plasma along the z -axis we may define $\mathbf{A} = [\mathcal{A}_{(-1)}^x \mathcal{A}_{(-1)}^y \mathcal{A}_{(0)}^x \mathcal{A}_{(0)}^y \mathcal{A}_{(1)}^x \mathcal{A}_{(1)}^y]^\top$, where $^\top$ denotes transpose, and calculate the nonzero independent components of the aforementioned tensors [30]. Thence, by setting $\alpha \equiv x$ and $\alpha \equiv y$ we can

concisely cast Eqs. (6a)–(6c) in a matrix notation, viz.,

$$\mathbf{L} \cdot d_u \mathbf{A} = \mathbf{S} \cdot \mathbf{A}, \quad (7)$$

where the operator $d_u = \partial/\partial t + n\partial/\partial z$ differentiates along the timelike curve tracking the EMWs; u is an affine parameter. Additionally we have

$$\mathbf{L} = \begin{bmatrix} \omega_{(-1)}\mathbb{I} & \frac{\Omega}{4}\mathbf{S}_{+, \times}^* & \mathbf{0} \\ -\frac{\Omega}{4}\mathbf{S}_{+, \times} & \omega_{(0)}\mathbb{I} & \frac{\Omega}{4}\mathbf{S}_{+, \times}^* \\ \mathbf{0} & \frac{\Omega}{4}\mathbf{S}_{+, \times} & \omega_{(1)}\mathbb{I} \end{bmatrix}$$

and

$$\mathbf{S} = i\delta n \frac{\Omega}{4} \begin{bmatrix} \mathbf{0} & -\omega_{(0)}\mathbf{S}_{+, \times}^* & \mathbf{0} \\ \omega_{(-1)}\mathbf{S}_{+, \times} & \mathbf{0} & -\omega_{(1)}\mathbf{S}_{+, \times}^* \\ \mathbf{0} & \omega_{(0)}\mathbf{S}_{+, \times} & \mathbf{0} \end{bmatrix};$$

$\mathbb{I}$ is the 2×2 identity, $\mathbf{0}$ is the 2×2 null matrix, and

$$\mathbf{S}_{+, \times} = \begin{bmatrix} \mathcal{H}_+ & \mathcal{H}_\times \\ \mathcal{H}_\times & -\mathcal{H}_+ \end{bmatrix},$$

with $\mathcal{H}_+$ and $\mathcal{H}_\times$ being the amplitudes of the “plus” and “cross” polarization states of the GW, respectively [13].

Insofar as $\mathbf{L}$ is invertible, we may set $\mathbf{W} = \mathbf{L}^{-1} \cdot \mathbf{S}$ so that the coupled-wave system in Eq. (7) becomes

$$d_u \mathbf{A} = \mathbf{W}(\delta n, \omega_{(0)}, \Omega, \mathcal{H}^{\mu\nu}) \cdot \mathbf{A}, \quad (8)$$

wherein the coupling matrix is

$$\mathbf{W} = \begin{bmatrix} 0 & 0 & i\kappa_{-1,0}^{x,x} & i\kappa_{-1,0}^{x,y} & 0 & 0 \\ 0 & 0 & i\kappa_{-1,0}^{x,y} & -i\kappa_{-1,0}^{x,x} & 0 & 0 \\ i\kappa_{0,-1}^{x,x} & i\kappa_{0,-1}^{x,y} & 0 & 0 & i\kappa_{0,1}^{x,x} & i\kappa_{0,1}^{x,y} \\ i\kappa_{0,-1}^{x,y} & -i\kappa_{0,-1}^{x,x} & 0 & 0 & i\kappa_{0,1}^{x,y} & -i\kappa_{0,1}^{x,x} \\ 0 & 0 & i\kappa_{1,0}^{x,x} & i\kappa_{1,0}^{x,y} & 0 & 0 \\ 0 & 0 & i\kappa_{1,0}^{x,y} & -i\kappa_{1,0}^{x,x} & 0 & 0 \end{bmatrix},$$

and the identified coupling coefficients read

$$\begin{aligned} \kappa_{-1,0}^{x,x} &= \delta n \frac{\omega_{(0)}\Omega}{4\omega_{(0)} - 4\Omega} \mathcal{H}_+^*, & \kappa_{-1,0}^{x,y} &= \delta n \frac{\omega_{(0)}\Omega}{4\omega_{(0)} - 4\Omega} \mathcal{H}_\times^*, \\ \kappa_{0,-1}^{x,x} &= \delta n \frac{\Omega^2 - \omega_{(0)}\Omega}{4\omega_{(0)}} \mathcal{H}_+, & \kappa_{0,-1}^{x,y} &= \delta n \frac{\Omega^2 - \omega_{(0)}\Omega}{4\omega_{(0)}} \mathcal{H}_\times, \\ \kappa_{0,1}^{x,x} &= \delta n \frac{\omega_{(0)}\Omega + \Omega^2}{4\omega_{(0)}} \mathcal{H}_+^*, & \kappa_{0,1}^{x,y} &= \delta n \frac{\omega_{(0)}\Omega + \Omega^2}{4\omega_{(0)}} \mathcal{H}_\times^*, \\ \kappa_{1,0}^{x,x} &= -\delta n \frac{\omega_{(0)}\Omega}{4\omega_{(0)} + 4\Omega} \mathcal{H}_+, & \kappa_{1,0}^{x,y} &= -\delta n \frac{\omega_{(0)}\Omega}{4\omega_{(0)} + 4\Omega} \mathcal{H}_\times. \end{aligned}$$

Although the linear system in Eq. (8) is autonomous and thus admits analytic closed-form solutions, its foremost merit resides in yielding straightforward insights. Indeed, for the specified polarization, we could approximate the coupling strengths between the impinging and the forward-scattered EMWs as, say, $\kappa_{(\pm 1)} \approx \delta n [\omega_{(0)}\Omega / (4\omega_{(\pm 1)})] h_s$; $h_s = |\mathcal{H}_+|$ or $|\mathcal{H}_\times|$. Whence the relative sideband amplitudes are $|\mathcal{A}_{(\pm 1)} / \mathcal{A}_{(0)}| \approx (4/\pi) \kappa_{(\pm 1)} L_{c,(\pm 1)}$, with $L_{c,(\pm 1)}$ being the characteristic interaction length of each diffraction order [31]. Translating this into an observable intensity ratio, $\mathcal{I}_{(j)} \approx |\mathcal{A}_{(j)}|^2$, just above the noise threshold relative to the primary laser intensity, we estimate the interaction lengths as

$$L_{c,(\pm 1)} \approx \frac{1}{2\delta n} \frac{\lambda_{(0)}}{\lambda_{(\pm 1)}} \frac{\Lambda}{h_s} \sqrt{\frac{\mathcal{I}_{(\pm 1)}}{\mathcal{I}_{(0)}}}. \quad (9)$$

Here $\Lambda = 2\pi/\Omega$ and $\lambda_{\{(0),(\pm 1)\}} = 2\pi/\omega_{\{(0),(\pm 1)\}}$ are the wavelengths of the GW and the EMWs, respectively.

IV. GRAVITATIONAL WAVE COUPLING TO COHERENT AND INCOHERENT ELECTROMAGNETIC SOURCES

Since the identified mechanism does not rely on phase coherence, incoherent sources, such as the cosmic microwave background (30–600 GHz), become viable probes of high-frequency GWs. At the indicative wavelength of $\lambda_{(0)} = 2 \times 10^{-3}$ m propagation requires $\omega_{(0)} > \omega_p$, where ω_p is the plasma frequency—a condition satisfied in Galactic disk regions with electron densities $n_e \sim 10^{-2} \text{ cm}^{-3}$ yielding a dispersion parameter $\delta n \sim 10^{-17}$ [18]. For currently detectable GWs we adopt $\Lambda = 10^3$ m and $h_s = 10^{-21}$; then, assuming $[\mathcal{I}_{(\pm 1)} / \mathcal{I}_{(0)}]_{\text{thresh}} = 10^{-10}$, Eq. (9) yields $L_{c,(\pm 1)} \sim 10^{35}$ m for both sidebands. This distance, though finite, is alas too large, even by cosmological standards (the diameter of the observable Universe is $D_U \approx 8.8 \times 10^{26}$ m). A coupled-wave theory estimate of the interaction lengths required for the cosmic microwave background radiation to generate sidebands that could, in principle, be detected with current technology is displayed in Table I.

TABLE I. Order-of-magnitude estimates of the interaction lengths required in various astrophysical environments to produce detectable sideband levels. A representative wavelength of $\lambda_{(0)} = 2 \times 10^{-3}$ m is assumed for cosmic microwave background radiation. Evidently, nearly all interaction lengths far exceed the diameter of the observable Universe.

Environment	$n_e (\text{cm}^{-3})$	$\sim \delta n$	$\sim L_c (\text{m})$
Intergalactic space	$< 10^{-4}$	$< 10^{-19}$	$> 10^{37}$
Galactic disk	10^{-2}	10^{-17}	10^{35}
H II region	10^4	10^{-11}	10^{29}
Stellar wind	10^8	10^{-7}	10^{25}

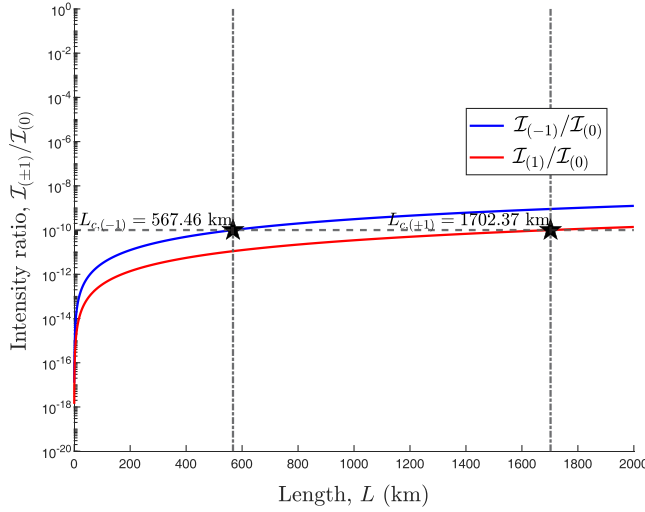


FIG. 2. Intensity ratio vs propagation length. The intensity ratios, $\mathcal{I}_{(\pm 1)}/\mathcal{I}_{(0)}$, are plotted as functions of the corresponding propagation length L on a semilogarithmic scale. The solid lines represent the calculated intensity ratios based on numerical integration of Eq. (8), while the dashed line shows the noise threshold, $[\mathcal{I}_{(\pm 1)}/\mathcal{I}_{(0)}]_{\text{thresh}} = 10^{-10}$, taken from the intensity noise spectrum of the ultralow-noise solid-state laser proposed in Ref. [33]. Within dense layers of the stellar atmosphere we take $\delta n = 0.14$, and the parameters achievable with the next-generation National Ignition Facility are $\lambda_{(0)} = 352.94$ nm, $\Lambda = 176.47$ nm, and $h_s = 10^{-17}$ (cf. Fig. 2 in Ref. [32], wherein resonant enhancement enables sensitivity to such strains without violating energy conservation). Modulo strain-enhancement considerations, the condition $\Omega = 2\omega_{(0)}$ is merely illustrative and not required, unlike in cases of parametrically amplified GWs [19]. N.B.—In this nearly luminal scenario we approximate the affine parameter u in the quasistatic limit as $u \equiv nz \equiv t$.

At optical frequencies, $\omega_{(0)} = 17\pi \times 10^{14}$ Hz, the same mechanism—now acting in extremely dense stellar atmospheres with $n_e \sim 10^{21} \text{ cm}^{-3}$ —yields an augmented dispersion parameter $\delta n = 0.14$. Assuming the presence of a high-frequency GW with $\Omega = 2\omega_{(0)}$ and $h_s = 10^{-17}$ (cf. Fig. 2 in Ref. [32], where strain sensitivity benefits from a resonant enhancement factor $\omega_{(0)}^2 \tau^2$, with τ being the interaction time), we obtain $\lambda_{(0)} = 352.94$ nm and $\Lambda = 176.47$ nm. These yield interaction lengths $L_{c,(-1)} \approx 1.89 \times 10^6$ m and $L_{c,(+1)} \approx 6.29 \times 10^5$ m for the “(−)” and “(+)” sidebands, respectively. Although considerable, these lengths could be reduced through resonant enhancement, the use of optical cavities (with a plasma generating a strong δn), or multipass interferometric techniques, thus enabling terrestrial detection (for comparison, Earth’s diameter is $D_\oplus \approx 1.27 \times 10^7$ m).

The estimated interaction lengths closely align with the computed values determined via brute-force numerical integration of Eq. (8). In fact, the exact values are indicated by the starred points in Fig. 2, demonstrating their

consistency with current interferometric capabilities and corroborating our estimate. The exponential amplification-like behavior exhibited by the curves in Fig. 2 accords with the “compression of lines of force” gain mechanism of synthetic luminal gratings [11].

V. DISCUSSION

In this article, we extend coupled-wave theory into the temporal domain to develop a fully covariant framework for modeling the interaction of copropagating subluminal EMWs in a plasma with luminal GWs. As expected, the coupling between these two fields is extremely weak; nevertheless, it is not negligible. Our formulation shows that the interaction generates sidebands in the electromagnetic spectrum, governed by scaling relations of the form $|\mathcal{A}_{(\pm 1)}/\mathcal{A}_{(0)}|^2 \approx (\kappa_{(\pm 1)} L_{c,(\pm 1)})^2$, where the coupling coefficient is directly proportional to δn , which quantifies the deviation of the EMWs from the luminal regime. This establishes clear physical conditions under which synchronism can be tuned and sideband generation amplified. In plasmas, for instance, where δn can be appreciable, dispersion can be exploited to reduce the required interaction length. Additional enhancement can be achieved using multipath high-finesse optical cavities, where repeated circulation of light effectively multiplies the interaction length by the finesse.

Of paramount importance is the connection between our covariant framework and measurable quantities. Indeed, while the TT gauge provides calculational clarity, physical observables are defined in the detector’s local Lorentz frame, characterized by its 4-velocity u^μ . For instance, the electric field component E^j , $j = \{1, 2, 3\}$, measured in the laboratory is obtained via the projection $E^j = (A^{\beta,j} - A^{j,\beta})u_\beta$. The observed sideband frequencies then follow directly from 4-vector synchronism, $\omega_{(\pm 1)}^{(\text{obs})} = -k_\mu^{(\pm 1)} u^\mu = \omega_{(0)}^{(\text{obs})} \pm \Omega^{(\text{obs})}$, with $\Omega^{(\text{obs})} = -K_\mu u^\mu$. Hence detunings and power ratios are expressed entirely in terms of invariants formed from A^μ , k_μ , K_μ , and u_μ and are therefore independent of gauge choice. These considerations enable our formalism to be directly linked to standard detector-response analyses [32], and they underscore the crucial next step of performing a realistic analysis of GW sources reaching Earth, which will be indispensable for proposing a practical detection scheme.

Realizing that astrophysical detection is severely hindered by the extremely weak coupling between EMWs and GWs, we argue that a strong proof of concept is essential before efforts can focus on designing dedicated detection equipment. A promising route to simulating such a mechanism is offered by quasistatic traveling-wave modulations implemented through synthetic spatiotemporal gratings [11,28,34]. These systems could enable all-optical analog experiments, as proposed in Ref. [35], providing a

controlled setting in which GW propagation is emulated on the optical bench via a traveling grating—namely, a spatiotemporal modulation of the permittivity and permeability of an optical medium. Furthermore, because the interaction is intrinsically phase-insensitive, incoherent electromagnetic sources may be employed, including the cosmic microwave background, where B-mode patterns are hypothesized to encode imprints of primordial GWs [36]. In such contexts, stochastic GW backgrounds become especially relevant, making it crucial to address the intrinsic variance of uniform fields [37]. Together, these avenues underscore how the present framework can guide both experimental analogues and cosmological applications.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

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