

dS/CFT correspondence from a defect operator

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We perform a Wick rotation and analytic continuation from global AdS_{d+1} to static dS_{d+1} , yielding CFT_d generators with a nonstandard adjoint action tied to dS bulk coordinates. To reproduce the real-scalar two-point function, we introduce a global defect operator that twists the inner product. We further show that PT symmetry is spontaneously broken in CFT_2 vacua with a central charge having an imaginary part. Finally, we derive integral identities for bulk and defect correlators, providing a unified framework for computing CFT_d observables in the presence of global and local defects.

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I. INTRODUCTION

The anti-de Sitter/conformal field theory (AdS/CFT) correspondence provides a nonperturbative definition of quantum gravity in asymptotically AdS spacetimes [1–3]. A central question is whether a similar framework can be formulated for dS space. One approach is analytic continuation from AdS to dS [4,5]; however, such continuation alone does not fully determine the quantum structure of the dual theory.

Because a conformal field theory possesses scale invariance, it is often better defined than the corresponding bulk gravitational theory. This motivates an alternative perspective in which gravity is emergent rather than fundamental. In this approach, a gravitational description arises only in the large-central-charge limit of the CFT. The proper quantum gravity regime should be explored—or even defined—entirely within the CFT framework [6,7]. Within this program, an important objective is to understand the mechanism by which a bulk field emerges and to determine the specific conditions a CFT must satisfy to exhibit such emergent gravitational behavior. A central tool in this investigation is the Hamilton-Kabat-Lifschytz-Lowe (HKLL) bulk reconstruction procedure [8–11]. This method reconstructs bulk AdS fields from nonlocal combinations of

boundary CFT operators, thereby providing a concrete map between bulk dynamics and boundary observables and enabling the study of gravitational phenomena directly from the CFT perspective.

To understand our Universe from the perspective of quantum gravity, it is essential to study CFT associated with a de Sitter (dS) background. One approach is to *analytically continue* the AdS curvature radius [4],

$$L \rightarrow iL, \quad (1)$$

and correspondingly continue the positive central charge of CFT_d [12–15],

$$c \propto \frac{L^{d-1}}{G_N} \longrightarrow i^{d-1} \frac{L^{d-1}}{G_N}, \quad (2)$$

where G_N is the physical gravitation constant [4]. This continuation implies, in particular, that the central charge of CFT_2 has an imaginary part [4]. Indeed, the *adjoint* operation of the CFT_2 generators, when represented in dS_3 bulk coordinates, exhibits *exotic* properties [16].

To reproduce the correct real scalar two-point function [17–20], it becomes necessary to *modify* the inner-product structure of the bulk local state space [18]. This requires introducing a biorthogonal basis of eigenstates [16]. The need for such a modification already follows from the dS bulk isometry group, which signals both the alteration of the inner product and the *loss* of conventional hermiticity. By incorporating parity (P) and time-reversal (T) operations as part of the adjoint structure for bulk local states, one is naturally led to define a *global* defect operator G_D acting on the vector space $\mathcal{H}$, characterized by the involutive property

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$$G_D^2 = 1. \quad (3)$$

Consequently, the dS/CFT correspondence can be formulated in the language of *non-Hermitian* quantum theory [21–26] and in terms of *defect* operators.

We have another kind of defect operator: the *local* defect operator, which resides on a p -dimensional submanifold, exhibits a remarkably universal structure governing how broken internal symmetries imprint themselves on local defect observables. Ward-identity analyses show that CFT correlators obey integral relations [27] connecting *higher*-point correlators to *lower*-point data, enabling powerful nonperturbative constraints across diverse models [28] and informing perturbative studies of $O(N)$ line defects [28,29]. Complementary geometric paradigms elucidate these relationships as reflections of a defect conformal manifold, within which the Zamolodchikov metric and curvature are intricately encoded in the two- and four-point functions of precisely marginal defect operators [27,30,31]. Analytic bootstrap techniques further demonstrate that such integral identities sharpen operator unmixing and constrain anomalous dimensions in local defect spectra [32].

The presence of a defect explicitly breaks a global symmetry to a subgroup, resulting in a modified Ward identity. The integral identity provides a general and systematic framework for classifying these modifications [28]. We use the term *anomaly* in the standard sense: a symmetry of the classical theory that is not preserved at the quantum level due to regularization effects. In defect CFTs, the nonconservation of the bulk current is localized on the defect and governed by a specific operator whose transformation properties uniquely determine the defect contribution to the Ward identity. This defect-localized term captures a universal obstruction to extending the bulk CFT symmetry across the defect. Crucially, the anomaly terms, such as the Weyl anomaly, can be addressed generically only through the integral identity, which provides a computationally advantageous tool unmatched by alternative methods [28]. Consequently, the integral identity serves as a powerful method for computing higher-point connected correlators without loss of generality [28]. This analysis unifies geometric and algebraic perspectives on defect conformal manifolds, yielding explicit expressions for anomaly coefficients and new constraints on defect CFT data.

Most previous $\text{AdS} \rightarrow \text{dS}$ continuation focus on the bulk partition functions [4], wave function coefficients [33], cosmological correlators [34–36], and the higher-spin dualities [5]. The present work instead focuses on Hilbert-space structure and the adjoint operation. We show that the analytical continuation changes the adjoint structure of conformal generators. Because the ordinary Hermitian conjugation fails to reproduce the dS scalar Green’s function, a modified inner product is required. A global defect operator can represent this modification. The

resulting theory naturally fits into a non-Hermitian/ PT -symmetric framework. Unlike previous continuation programs, the essential new ingredient is not merely the continuation of correlators but the continuation of the Hilbert-space metric itself. In higher-spin dS/CFT, the dual CFT is nonunitary and possesses unusual Hilbert-space properties. Our analysis suggests that such nonunitarity possibly be understood more generally as arising from a modified inner product encoded by a global defect operator. Although we do not construct the higher-spin dual explicitly, the PT -defect framework developed here may provide a concrete realization of the Hilbert-space structure that has remained somewhat mysterious in higher-spin dS/CFT. In the cosmological holography program, analytic continuation relates AdS correlation functions to inflationary observables and late-time wave functions [34,37,38]. Our goal differs conceptually. In contrast, we study the realization of the conformal algebra itself and investigate the resulting adjoint operation. The necessity of introducing a PT defect therefore appears at the level of Hilbert-space structure rather than at the level of correlation functions alone.

A. Summary of results

In this paper, we investigate the symmetry perspective of the dS/CFT correspondence obtained through Wick rotation and analytic continuation. Although considerations of symmetry alone are generally inadequate to ascertain a comprehensive quantum theory—owing to quantum corrections or anomalies—we assert that Wick rotation and analytic continuation can nevertheless be meaningfully employed within a circumscribed set of structures. Our analysis focuses specifically on the metric, the CFT generators associated with the bulk isometry group, and physical observables defined through expectation values. The action, by contrast, is subtle under such continuations because quantum effects can modify it in nontrivial ways. Nevertheless, the restricted continuation we adopt suffices to extract structural information about de Sitter physics directly from the CFT side, and our derivations remain valid for CFTs in arbitrary dimensions. While analytic continuation from AdS to dS has been extensively studied, our work identifies an additional necessary structure: the modification of the inner product in the dual CFT. We show that this modification can be naturally formulated as a global defect operator that twists the Hilbert space structure. This leads to a non-Hermitian realization of the conformal algebra and provides a consistent framework for reproducing de Sitter bulk correlators. Our main results are summarized as follows (as in Fig. 1):

- (i) We first derive the static patch of dS_{d+1} from Lorentzian global AdS_{d+1} via Wick rotation and analytic continuation, and then identify the corresponding CFT_d generators. We show that the adjoint operation in this setting differs from the familiar

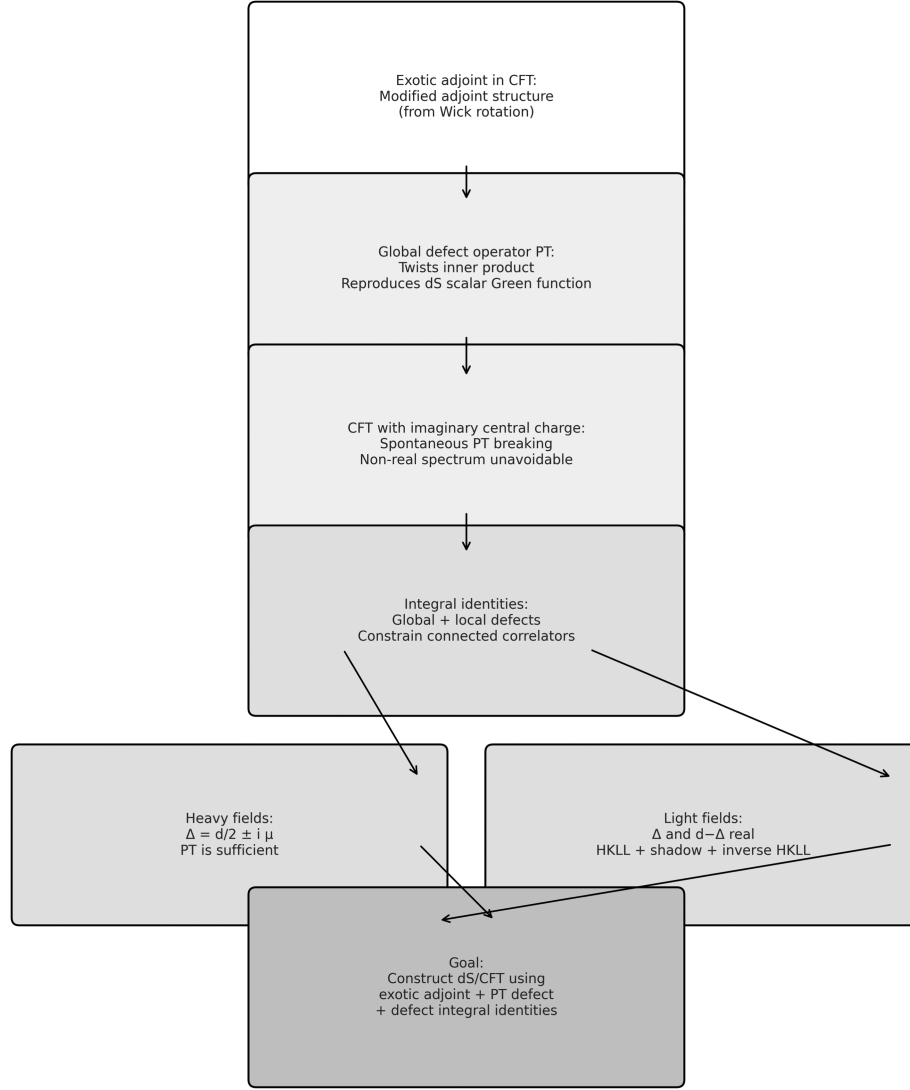


FIG. 1. Schematic summary of the dS/CFT construction in this work: from Wick-rotated AdS to static dS, exotic adjoint, and global PT defect, spontaneous PT symmetry-breaking, and integral identities with global and local defects constraining CFT bulk and defect correlators.

structure appearing in the AdS/CFT correspondence.

- (ii) We study bulk local states in the static patch of dS_{d+1} , noting that the same construction applies to the global patch as well. By computing the two-point Green's function of a real scalar field, we demonstrate that reproducing this result from the CFT_d side requires introducing a global defect—or equivalently, a PT operator—to twist the CFT inner product. The introduction of a global defect is not optional but required: without modifying the inner product, the CFT cannot reproduce the correct de Sitter Green's functions.
- (iii) We show that PT symmetry must be spontaneously broken for any vacuum state in a CFT_2 with a central charge having an imaginary part. This implies the

unavoidable presence of nonreal operator dimensions in the dS/CFT correspondence.

- (iv) We demonstrate how such a CFT remains computable under integral identities in the presence of both global and local defects. This clarifies how global defects are incorporated beyond two-point functions and how conformal symmetry continues to impose constraints on the defect conformal group.

The structure of the paper is as follows. In Sec. II, we derive the static dS_{d+1} geometry via Wick rotation and describe the associated CFT_d generators and adjoint operation. In Sec. III, we construct bulk local states for both the static and global patches of dS. In Sec. IV, we introduce the global defect operator PT and show how it twists the CFT_d inner product to reproduce the dS_{d+1} scalar two-point function. In Sec. V, we prove that spontaneous PT

symmetry breaking is inevitable in any CFT_2 with a central charge having an imaginary part. In Sec. VI, we present the integral identity and demonstrate its application in the presence of global and local defects. We conclude in Sec. VII with a summary and discussion of our results.

II. STATIC DS FROM GLOBAL ADS

In this section, we first review some basics like the static dS metric for later convenience. The main result is to show that the conformal generators for dS/CFT in general dimensions obey a nonstandard adjoint operation. Let us begin introducing the embedding coordinates X^A ,

$$\eta^{AB} X^A X^B = -X_{-1}^2 - X_0^2 + \sum_{j=1}^d X_j^2 = -L^2, \quad (4)$$

where the metric is

$$\eta^{AB} = \text{diag}(-1, -1, 1, 1, \dots, 1), \quad (5)$$

and the L is the radius of curvature. We label the embedding coordinates' indices as $A = -1, 0, 1, \dots, d$. The global coordinates $(\tau, \rho, \Omega_{d-1})$ are

$$\begin{aligned} X_{-1} &= L \cosh(\rho) \cos(\tau); & X_0 &= L \cosh(\rho) \sin(\tau); \dots; \\ X_j &= L \sinh(\rho) \Omega_j, \end{aligned} \quad (6)$$

where

$$\begin{aligned} L_0 &= \frac{i}{2}(\partial_\tau + \partial_\phi), & L_{\pm 1} &= \frac{i}{2}e^{\pm i(\tau+\phi)} \left(\frac{\sinh \rho}{\cosh \rho} \partial_\tau + \frac{\cosh \rho}{\sinh \rho} \partial_\phi \mp i \partial_\rho \right); \\ \tilde{L}_0 &= \frac{i}{2}(\partial_\tau - \partial_\phi), & \tilde{L}_{\pm 1} &= \frac{i}{2}e^{\pm i(\tau-\phi)} \left(\frac{\sinh \rho}{\cosh \rho} \partial_\tau - \frac{\cosh \rho}{\sinh \rho} \partial_\phi \mp i \partial_\rho \right). \end{aligned} \quad (11)$$

The algebra satisfies

$$[L_m, L_n] = (m-n)L_{m+n}; \quad [\tilde{L}_m, \tilde{L}_n] = (m-n)\tilde{L}_{m+n}. \quad (12)$$

The Hermitian conjugate in the algebra shows

$$(L_n)^\dagger = L_{-n}; \quad (\tilde{L}_n)^\dagger = \tilde{L}_{-n}. \quad (13)$$

We can perform the Wick rotation and analytical continuation from the global AdS_{d+1} to the static dS_{d+1} ,

$$\tau \rightarrow it; \quad \rho \rightarrow i\theta; \quad L^2 \rightarrow -L^2, \quad (14)$$

to obtain the static dS_{d+1} metric

$$\sum_{j=1}^d \Omega_j^2 = 1. \quad (7)$$

The range of τ is from $-\infty$ to ∞ , the range of ρ is $[0, \infty)$, and $\Omega_j \in S^{d-1}$. The AdS_{d+1} metric in the global coordinates is

$$ds^2 = L^2(-\cosh^2(\rho)d\tau^2 + d\rho^2 + \sinh^2(\rho)d\Omega_{d-1}^2). \quad (8)$$

We can consider $\rho \rightarrow \infty$ to approach the AdS_{d+1} conformal boundary $\mathbb{R} \times S^{d-1}$ (a cylinder). The isometry group is $\text{SO}(d, 2)$, and the boundary conformal group is also $\text{SO}(d, 2)$. For $d = 2$, the AdS_3 has the isometry group $\text{SO}(2, 2) \cong \text{SL}(2, \mathbb{R})_L \otimes \text{SL}(2, \mathbb{R})_R$. This represents the overarching symmetry of AdS_3 , about the global conformal group of CFT_2 . Three elements can generate each $\text{SL}(2, \mathbb{R})$ factor:

$$(L_0, L_1, L_{-1}); \quad (\tilde{L}_0, \tilde{L}_1, \tilde{L}_{-1}). \quad (9)$$

The conformal algebra $\text{so}(2, 2)$ can also be written in terms of the $\text{so}(2, 1) \oplus \text{so}(2, 1)$ generators, each satisfying

$$[L_0, L_{\pm 1}] = \mp L_{\pm 1}; \quad [L_1, L_{-1}] = 2L_0. \quad (10)$$

We write the CFT_2 generators when using the AdS bulk coordinates representation:

$$ds^2 = L^2(-\cos^2(\theta)dt^2 + d\theta^2 + \sin^2(\theta)d\Omega_{d-1}^2), \quad (15)$$

where

$$0 < \theta < \pi; \quad -\infty < t < \infty. \quad (16)$$

In the dS/CFT correspondence, the dual conformal field theory is posited on the sphere at the asymptotic future and past infinities, represented as $t = \pm\infty$. We also apply the Wick rotation and analytical continuation to the CFT_2 generators

$$\begin{aligned} L_0 &= \frac{1}{2}(\partial_t + i\partial_\phi), & L_{\pm 1} &= \frac{i}{2}e^{\pm i(-t+i\phi)}\left(\frac{\sin\theta}{\cos\theta}\partial_t - i\frac{\cos\theta}{\sin\theta}\partial_\phi \mp i\partial_\rho\right); \\ \tilde{L}_0 &= \frac{1}{2}(\partial_t - i\partial_\phi), & \tilde{L}_{\pm 1} &= \frac{i}{2}e^{\pm i(-t-i\phi)}\left(\frac{\sin\theta}{\cos\theta}\partial_t + i\frac{\cos\theta}{\sin\theta}\partial_\phi \mp i\partial_\rho\right). \end{aligned} \quad (17)$$

The algebra satisfies

$$[L_m, L_n] = (m-n)L_{m+n}; \quad [\tilde{L}_m, \tilde{L}_n] = (m-n)\tilde{L}_{m+n}. \quad (18)$$

The Hermitian conjugate in the algebra shows the non-conventional property [16]:

$$\begin{aligned} (L_0)^\dagger &= -\tilde{L}_0, & (L_{\pm 1})^\dagger &= \tilde{L}_{\pm 1}; \\ (\tilde{L}_0)^\dagger &= -L_0; & (\tilde{L}_{\pm 1})^\dagger &= L_{\pm 1}. \end{aligned} \quad (19)$$

To consider a general dimension, we have the CFT_d generators: P_μ are the translation generators; $M_{\mu\nu}$ are the Lorentz generators; D generates the scaling transformations (dilations); K_μ generate the special conformal transformations. In general dimensions, a real scalar field with a given mass m , which satisfies

$$m^2 L^2 > \frac{d^2}{4}, \quad (20)$$

we have two conformal dimensions for the dS_{d+1} case

$$\Delta_\pm = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} - m^2 L^2} \equiv \frac{d}{2} \pm i\mu, \quad (21)$$

which can be obtained via Wick rotation and analytical continuation from the AdS_{d+1} case. The conformal dimensions are the eigenvalues of the D operator. The adjoint operation acting on these generators for the dS_{d+1} bulk coordinate representation is

$$D^\dagger = -D; \quad K_\mu^\dagger = K_\mu; \quad P_\mu^\dagger = P_\mu. \quad (22)$$

When the conformal generators act on the primary states $|\Delta_\pm\rangle$, which is the lowest-weight state, the adjoint operation does not have the conventional form:

$$\begin{aligned} \langle \hat{\Delta}_\pm | D^\dagger &= \langle \Delta_\pm | \hat{\Delta}_\mp; D^\dagger | \hat{\Delta}_\pm \rangle = \Delta_\mp | \hat{\Delta}_\pm \rangle; & 0 &= \langle \hat{\Delta}_\pm | K_\mu^\dagger = P_\mu^\dagger | \hat{\Delta}_\pm \rangle; \\ \langle \hat{\Delta}_\pm | D &= -\langle \hat{\Delta}_\pm | \Delta_\mp; D | \hat{\Delta}_\pm \rangle = -\Delta_\mp | \hat{\Delta}_\pm \rangle; & 0 &= \langle \hat{\Delta}_\pm | K_\mu = P_\mu | \hat{\Delta}_\pm \rangle, \end{aligned} \quad (23)$$

where the normalization is

$$\langle \Delta_j | \Delta_k \rangle = \langle \hat{\Delta}_j | \hat{\Delta}_k \rangle = \delta_{jk}; \quad \langle \Delta_j | \hat{\Delta}_k \rangle = \langle \hat{\Delta}_j | \Delta_k \rangle = 0. \quad (24)$$

We define the adjoint bra state as

$$(|\Delta_\pm\rangle)^\dagger \equiv \langle \hat{\Delta}_\pm|; \quad (\langle \Delta_\pm|)^\dagger = |\hat{\Delta}_\pm\rangle. \quad (25)$$

III. BULK LOCAL STATE

In this section, we construct bulk local states in dS_{d+1}, which are CFT states corresponding to local scalar field operators in the bulk. We also demonstrate that antipodal symmetry exchanges the two conformal dimensions $\Delta_\pm$ related to the same mass. The bulk local state for any dS_{d+1} is [19]

$$|\Psi_\Delta\rangle = \sum_{n=0}^{\infty} (-1)^n C_n (P^2)^n |\Delta\rangle, \quad (26)$$

where $|\Delta\rangle$ is a conformal primary state, and

$$P^2 = \sum_{a=0}^{d-1} P_a P_a; \quad C_n = \prod_{k=1}^n \frac{1}{4k\Delta + 4k^2 - 2kd}. \quad (27)$$

It satisfies [19]

$$M_{\mu\nu} |\Psi_\Delta\rangle = 0; \quad (P_\mu + K_\mu) |\Psi_\Delta\rangle = 0. \quad (28)$$

The bulk local state is equivalent to using an AdS_{d+1} bulk scalar field acting on the vacuum state [19]. We can use the Wick rotation and the analytical continuation to transition to the static dS_{d+1}, and find that the bulk local state satisfies the same relation (28). Indeed, we can perform the coordinate transformation from the static coordinate system to the global coordinate system. Therefore, the bulk local states are the same between the static and global coordinates [16]. Hence, our results using the bulk local state will be the same between the static and global dS_{d+1} [16].

We can apply $\exp(Dt)\mathcal{R}\exp(i\theta J)$, where

$$J \equiv \frac{1}{2}(K_0 - P_0), \quad (29)$$

to a bulk local state to take any point to anywhere on a sphere, S^{d-1} . Since we have

$$e^{i\pi J} D e^{-i\pi J} = -D, \quad (30)$$

we obtain

$$\langle \hat{\Delta}_\pm | e^{i\pi J} D = \Delta_\mp \langle \hat{\Delta}_\pm | e^{i\pi J}, \quad (31)$$

which shows that $\langle \hat{\Delta}_\pm | e^{i\pi J}$ is proportional to a conjugate transpose of the primary state $\langle \Delta_\mp |$,

$$\langle \hat{\Delta}_\pm | e^{i\pi J} = \nu_\mp \langle \Delta_\mp |. \quad (32)$$

Taking the conjugate transpose, we obtain

$$e^{-i\pi J} |\Delta_\pm\rangle = \nu_\mp^* |\hat{\Delta}_\mp\rangle. \quad (33)$$

Hence, we obtain that

$$\nu_\pm \langle \Delta_\pm | e^{-2\pi J} |\Delta_\pm\rangle = \nu_\mp^*. \quad (34)$$

When we set all spacetime variables to zero, except for θ , we can find that $\exp(i\theta J)$ acting on the bulk local state gives the shift $\theta \rightarrow \theta + \pi$, which implies that the operation maps a dS bulk point to the antipodal point. The antipodal and dS bulk points in a sphere are the endpoints of a diameter that a straight line segment connects them through the sphere's center. Because $\exp(i\pi J)|\Psi_\Delta\rangle$ remains a bulk local state, the state is the eigenstate of the operator, and we then use $\exp(Dt)\mathcal{R}$ to take a dS bulk point to anywhere, these operations realize the following equality:

$$|\Psi_{\Delta_\pm}(t, \theta + \pi, \Omega)\rangle = \lambda_\pm |\Psi_{\Delta_\pm}(t, \theta, \Omega)\rangle. \quad (35)$$

The sum in the definition of the bulk local state (26) can be carried out and gives [19]

$$|\Psi_\Delta\rangle = \Gamma\left(\Delta - \frac{d}{2} + 1\right) \left(\frac{\sqrt{P^2}}{2}\right)^{\frac{d}{2}-\Delta} J_{\Delta-\frac{d}{2}}(\sqrt{P^2})|\Delta\rangle, \quad (36)$$

which agrees with the known HKLL form [8–11] (in momentum space representation) of a bulk scalar field acting on a vacuum state. It can be further shown from the dictionary of the AdS/CFT correspondence [3] that this particular form is enough to get the bulk-to-boundary propagator, and hence the two-point function of a real scalar

$$G(x, y) = \frac{\Gamma(\Delta)}{2\pi^{\frac{d}{2}}\Gamma(\Delta - \frac{d}{2} + 1)} \times e^{-\Delta D_{\text{AdS}}(x, y)} {}_2F_1\left(\Delta, \frac{d}{2}, \Delta + 1 - \frac{d}{2}; e^{-2D_{\text{AdS}}(x, y)}\right), \quad (37)$$

where the Gamma function is

$$\Gamma(z) \equiv \int_0^\infty dt t^{z-1} e^{-t}, \text{Re}(z) > 0, \quad (38)$$

and the hypergeometric function is

$${}_2F_1(a, b, c; z) \equiv \sum_{n=0}^\infty \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!}, |z| < 1. \quad (39)$$

The Pochhammer symbol is defined as

$$(a)_n \equiv \begin{cases} 1, & n = 0. \\ a(a+1) \cdots (a+n-1), & n > 0. \end{cases} \quad (40)$$

The $D_{\text{AdS}}(x, y)$ is the geodesic line for the AdS_{d+1} background between two points, x and y . When $|z| \geq 1$, we define the hypergeometric function by the analytical continuation directly. We can use the Wick rotation and the analytical continuation to obtain the following Green's function:

$$G_{\Delta_\pm}(x, y) = \frac{\Gamma(\Delta_\pm)}{2\pi^{\frac{d}{2}}\Gamma(\Delta_\pm - \frac{d}{2} + 1)} \times e^{-i\Delta_\pm D_{\text{dS}}(x, y)} {}_2F_1\left(\Delta_\pm, \frac{d}{2}, \Delta_\pm + 1 - \frac{d}{2}; e^{-2iD_{\text{dS}}(x, y)}\right), \quad (41)$$

where $D_{\text{dS}}(x, y)$ is the geodesic line for the dS_{d+1} background connecting two points x and y , and it connects to $D_{\text{AdS}}(x, y)$ as

$$D_{\text{AdS}}(x, y) = iD_{\text{dS}}(x, y). \quad (42)$$

According to Eq. (41), we can read

$$\frac{1}{\lambda_\pm} = (-1)^{\frac{d}{2}} e^{\pm\pi\mu} \quad (43)$$

from:

$$\begin{aligned} & \langle \Psi_{\Delta_\pm}(0, \theta + \pi, 0) | \Psi_{\Delta_\pm}(0, 0, 0) \rangle \\ &= \langle \Psi_{\Delta_\pm}(0, \theta, 0) | e^{-i\pi J} | \Psi_{\Delta_\pm}(0, 0, 0) \rangle \\ &= (-1)^{\frac{d}{2}} e^{\pm\pi\mu} \langle \Psi_{\Delta_\pm}(0, \theta, 0) | \Psi_{\Delta_\pm}(0, 0, 0) \rangle. \end{aligned} \quad (44)$$

Hence, we can obtain

$$\langle \Delta_\pm | e^{-2\pi J} | \Delta_\pm \rangle = (-1)^d e^{\pm 2\pi\mu}, \quad (45)$$

and obtain

$$\nu_{\pm} = (-1)^{\frac{d}{2}+1} e^{\mp \pi \mu}. \quad (46)$$

We can take the conjugate transpose of the bulk local state to get

$$\begin{aligned} \langle \hat{\Psi}_{\Delta_{\pm}} | &= \left\langle \hat{\Delta}_{\pm} \left| \sum_{n=0}^{\infty} (-1)^n C_n(\Delta_{\mp}) (P^2)^n e^{-i\theta J} \mathcal{R}^{-1} e^{-Dt} \right. \right. \\ &= \left\langle \hat{\Delta}_{\pm} \left| \sum_{n=0}^{\infty} (-1)^n C_n(\Delta_{\mp}) (P^2)^n e^{i\pi J} e^{-i\pi J} e^{-i\theta J} \mathcal{R}^{-1} e^{-Dt} \right. \right. \\ &= \left\langle \hat{\Delta}_{\pm} \left| e^{i\pi J} \sum_{n=0}^{\infty} (-1)^n C_n(\Delta_{\mp}) (K^2)^n e^{-i(\theta+\pi)J} \mathcal{R}^{-1} e^{-Dt} \right. \right. \\ &= \nu_{\mp} \left\langle \Delta_{\pm} \left| \sum_{n=0}^{\infty} (-1)^n C_n(\Delta_{\mp}) (K^2)^n e^{-i(\theta+\pi)J} \mathcal{R}^{-1} e^{-Dt} \right. \right. \\ &= \nu_{\mp} \langle \Psi_{\Delta_{\mp}}(t, \theta + \pi, \Omega) |. \end{aligned} \quad (47)$$

Hence, we show that the inner product of the nonvanishing bulk local states is

$$\langle \hat{\Psi}_{\Delta_{\mp}}(x) | \Psi_{\Delta_{\pm}}(y) \rangle = \frac{\nu_{\pm} \Gamma(\Delta_{\pm})}{2\pi^{\frac{d}{2}} \Gamma(\Delta_{\pm} - \frac{d}{2} + 1)} e^{-i\Delta_{\pm} D_{\text{ds}}(x_A, y)} {}_2F_1\left(\Delta_{\pm}, \frac{d}{2}, \Delta_{\pm} + 1 - \frac{d}{2}; e^{-2iD_{\text{ds}}(x_A, y)}\right), \quad (48)$$

where x_A is the antipodal point, with respect to the dS bulk point x , and the other inner product of the bulk local states vanishes,

$$\langle \hat{\Psi}_{\Delta_{\pm}}(x) | \Psi_{\Delta_{\pm}}(y) \rangle = 0. \quad (49)$$

The bulk local states $|\Psi_{\Delta_{\pm}}\rangle$ both correspond to the same mass, and we want to make a linear combination to obtain another Green's function with the dual to the dS_{d+1} free scalar field that we explain in the next section.

IV. GREEN'S FUNCTION

The principal result of this section is to show that the correct Wightman function can be reproduced from the inner product between bulk local states only after introducing a PT twist. The Green's function (Wightman function) for a real scalar field in a Bunch-Davies vacuum state takes the form [17]

$$G_E(x, y) = \frac{\Gamma(\Delta_+) \Gamma(\Delta_-)}{(4\pi)^{\frac{d+1}{2}} \Gamma(\frac{d+1}{2})} {}_2F_1\left[\Delta_+, \Delta_-, \frac{d+1}{2}; \cos^2\left(\frac{D_{\text{ds}}(x, y)}{2}\right)\right]. \quad (50)$$

We can use the following identities:

$$\cos^2\left(\frac{D_{\text{ds}}(x_A, y)}{2}\right) = \sin^2\left(\frac{D_{\text{ds}}(x, y)}{2}\right) = 1 - \cos^2\left(\frac{D_{\text{ds}}(x, y)}{2}\right), \quad (51)$$

where

$$D_{\text{ds}}(x_A, y) = \pi - D_{\text{ds}}(x, y), \quad (52)$$

$${}_2F_1\left(a, b, \frac{a+b+1}{2}; 1-z\right) = {}_2F_1\left(\frac{a}{2}, \frac{b}{2}, \frac{a+b+1}{2}; 4z(1-z)\right) \quad (53)$$

with

$$z = \cos^2\left(\frac{D_{\text{ds}}(x, y)}{2}\right) \quad (54)$$

to obtain

$$G_E(x_A, y) = \frac{\Gamma(\Delta_+)\Gamma(\Delta_-)}{(4\pi)^{\frac{d+1}{2}}\Gamma(\frac{d+1}{2})} {}_2F_1\left(\frac{\Delta_+}{2}, \frac{\Delta_-}{2}, \frac{d+1}{2}; \sin^2(D_{\text{ds}}(x, y))\right). \quad (55)$$

We then apply the identity

$${}_2F_1(a, b, 2b; z) = (1-z)^{-\frac{a}{2}} {}_2F_1\left(\frac{a}{2}, b - \frac{a}{2}, b + \frac{1}{2}; \frac{z^2}{4(z-1)}\right) \quad (56)$$

to rewrite $G_E(x_A, y)$ as

$$G_E(x_A, y) = \frac{\Gamma(\Delta_+)\Gamma(\Delta_-)}{(4\pi)^{\frac{d+1}{2}}\Gamma(\frac{d+1}{2})} e^{i\Delta_+ D_{\text{ds}}} {}_2F_1\left(\Delta_+, \frac{d}{2}, d; 1 - e^{2iD_{\text{ds}}}\right). \quad (57)$$

The $G_E(x_A, y)$ can also be rewritten as the symmetric form by exchanging Δ_+ and Δ_- ,

$$\begin{aligned} G_E(x_A, y) &= \frac{\Gamma(\Delta_-)\Gamma(\frac{d}{2} - \Delta_-)}{4(\pi)^{\frac{d}{2}+1}} e^{i\Delta_- D_{\text{ds}}(x, y)} {}_2F_1\left(\Delta_-, \frac{d}{2}, -\frac{d}{2} + \Delta_- + 1; e^{2iD_{\text{ds}}(x, y)}\right) \\ &+ \frac{\Gamma(\Delta_+)\Gamma(\frac{d}{2} - \Delta_+)}{4(\pi)^{\frac{d}{2}+1}} e^{i\Delta_+ D_{\text{ds}}(x, y)} {}_2F_1\left(\Delta_+, \frac{d}{2}, -\frac{d}{2} + \Delta_+ + 1; e^{2iD_{\text{ds}}(x, y)}\right) \end{aligned} \quad (58)$$

by using the following formula:

$$\begin{aligned} {}_2F_1(a, b, c; z) &= \frac{(1-z)^{-a-b+c}\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} {}_2F_1(c-a, c-b; -a-b+c+1; 1-z) \\ &+ \frac{\Gamma(c)\Gamma(-a-b+c)}{\Gamma(c-a)\Gamma(c-b)} {}_2F_1(a, b, a+b-c+1; 1-z). \end{aligned} \quad (59)$$

From this symmetric form, we can use the linear combination of the primaries $\Delta_{\pm}$ to write the dual wave function

$$|\Psi_E(x)\rangle = \frac{1}{\sqrt{2\sinh(\pi\mu)}} \left(\frac{1}{\sqrt{i}} |\Psi_{\Delta_+}(x)\rangle + \sqrt{i} |\Psi_{\Delta_-}(x_A)\rangle \right), \quad (60)$$

and then $G_E(x, y)$ can be obtained by replacing x with x_A in the $D_{\text{ds}}(x, y)$ from $G_E(x_A, y)$ and can be written as an inner product:

$$G_E(x, y) = \langle \Psi_E(x) | \Psi_E(y) \rangle = \frac{i}{2\sinh(\pi\mu)} (\langle \hat{\Psi}_{\Delta_+}(x) | \Psi_{\Delta_-}(y_A) \rangle - \langle \hat{\Psi}_{\Delta_-}(x_A) | \Psi_{\Delta_+}(y) \rangle), \quad (61)$$

where

$$\langle \Psi_E(x) | = \frac{1}{\sqrt{2\sinh(\pi\mu)}} \left(\sqrt{i} \langle \hat{\Psi}_{\Delta_+}(x) | + \frac{1}{\sqrt{i}} \langle \hat{\Psi}_{\Delta_-}(x_A) | \right). \quad (62)$$

Finally, let us discuss the implications of the result from the perspective of the inner product. We observe that the bra state can be obtained from the PT transformation,

$$\nu_- \langle 0 | PT (PT)^{-1} \Phi_{\Delta_+}(t, \theta, \Omega) PT = \nu_- \langle 0 |_{PT} \Phi_{\Delta_-}(t, \theta + \pi, \Omega) = \langle \hat{\Psi}_{\Delta_+}(x) |, \quad (63)$$

where

$$\langle 0|_{PT} \equiv \langle 0|_{PT}. \quad (64)$$

The $\nu_{\mp}$ can be thought of as the linear combination of the coefficients for the states

$$\langle \Psi_E(x) | = \frac{1}{\sqrt{2 \sinh(\pi \mu)}} \left(\sqrt{i} \nu_- \langle \hat{\Psi}_{\Delta_+}(x) | + \frac{\nu_+}{\sqrt{i}} \langle \hat{\Psi}_{\Delta_-}(x_A) | \right), \quad (65)$$

where

$$\nu_{\mp} \langle \hat{\Psi}_{\Delta_{\pm}} | = \langle \hat{\Psi}_{\Delta_{\pm}} |. \quad (66)$$

We can also use the conventional PT inner product to define $\langle \hat{\Psi}_{\Delta_{\pm}} |$,

$$\langle \hat{\Psi}_{\Delta_{\pm}}(x_A) | = \langle \Psi_{\Delta_{\pm}}(x) |_{PT}. \quad (67)$$

Hence, it is equivalent to twisting the inner product space by the PT transformation. The PT transformation maps a dS bulk point to an antipodal point and also exchanges the conformal dimensions, $\Delta_+ \leftrightarrow \Delta_-$. We then observe that the PT operation acting on the $|\Psi_E\rangle$ is invariant due to the fact that

$$\begin{aligned} PT \left(\frac{1}{\sqrt{i}} |\Psi_{\Delta_+}(x)\rangle \right) &\leftrightarrow \sqrt{i} |\Psi_{\Delta_-}(x_A)\rangle; \\ PT(\sqrt{i} |\Psi_{\Delta_-}(x_A)\rangle) &\leftrightarrow \frac{1}{\sqrt{i}} |\Psi_{\Delta_+}(x)\rangle. \end{aligned} \quad (68)$$

Hence, the linear combination makes the PT invariant state, so that we can make the conjugation transpose for obtaining the bra state, but the complex conjugate of $|\Psi_{\Delta_{\pm}}\rangle$ corresponds to the different conformal dimensions, so that we need to twist the inner product by PT operator, which is one class of the global defect for spinless particles

$$(PT)^2 = 1. \quad (69)$$

Because the defect operator corresponds to a symmetry of a theory, the global defect acting on a vacuum state is proportional to the vacuum state if we do not have spontaneous symmetry breaking. However, the sphere has antipodal symmetry, so we expect the vacuum state to have degeneracies, and we should observe that each vacuum state's conjugate spectrum corresponds to another vacuum state's spectrum. In other words, we expect that spontaneous symmetry breaking is generic for the CFT's vacuum states. We demonstrate the proof for the vacuum states in any CFT_2 .

V. SPONTANEOUS PT SYMMETRY BREAKING

The main theorem of this section is that a CFT_2 with complex central charge cannot possess a PT -invariant vacuum. Since the Virasoro generators are

$$L_n = \frac{1}{2\pi i} \oint_{|z|=1} dz z^{n+1} T(z), \quad (70)$$

the parity transformation reverses the contour, and the time reversal transformation conjugates the coefficient:

$$\begin{aligned} (PT)L_n(PT)^{-1} &= \frac{1}{2\pi i} \oint_{|\bar{z}|=1} d\bar{z} \bar{z}^{n+1} (PT)T(z)(PT)^{-1} \\ &= \frac{1}{2\pi i} \oint_{|\bar{z}|=1} d\bar{z} \bar{z}^{n+1} T^{\dagger}(\bar{z}) \\ &= \frac{1}{2\pi i} \oint_{|z|=1} dz z^{n+1} T^{\dagger}(z) = L_n^{\dagger}. \end{aligned} \quad (71)$$

The Virasoro algebra is

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0}, \quad (72)$$

where c is a central charge with an imaginary part. After we apply the PT transformation on the left and right sides, the Virasoro algebra becomes

$$[L_m^{\dagger}, L_n^{\dagger}] = (m-n)L_{m+n}^{\dagger} + \frac{c^*}{12}(m^3 - m)\delta_{m+n,0}. \quad (73)$$

The physical vacuum must satisfy

$$L_n|0\rangle = 0, \quad n \geq 0. \quad (74)$$

If the vacuum state is also a PT eigenstate

$$PT|0\rangle = \lambda|0\rangle, \quad (75)$$

we apply the PT operator to obtain

$$0 = PTL_{n \geq 0}|0\rangle = (PT)L_{n \geq 0}(PT)^{-1}(PT)|0\rangle = L_{n \geq 0}^{\dagger}\lambda|0\rangle. \quad (76)$$

Hence, we get the condition

$$L_{n \geq 0}^{\dagger}|0\rangle = 0. \quad (77)$$

Because the algebra of L_n is the Virasoro algebra with a central charge c , and the algebra of $L_n^{\dagger}$ is the Virasoro algebra with central charge c^* , and the vacuum cannot simultaneously satisfy the highest weight conditions for c and c^* , the vacuum state cannot be the PT invariant or in the PT symmetric phase for the central charge with an imaginary part. The precise mathematical statement is that

we use the relation

$$\langle 0 | [L_{-m} - L_{-m}^\dagger, L_{m \geq 0} - L_{m \geq 0}^\dagger] | 0 \rangle = \frac{c - c^*}{12} m(m^2 - 1) = 0 \quad (78)$$

to prove that the PT invariant vacuum state only exists when the central charge is a real number ($c = c^*$). Hence, the vacuum state must transform into a different state under the PT transformation, implying that the vacuum state is not an eigenstate of the PT operator. This proof implies that the nonreal spectrum is unavoidable in the dS/CFT correspondence.

VI. INTEGRAL IDENTITY OF CONNECTED CORRELATORS

In this section, we demonstrate that the global defect operator can be incorporated into the integral-identity formalism of defect CFTs. We introduce the CFT bulk operators and global and local defect operators simultaneously for the most general situations. Since the introduction of a global defect operator in CFT_d should depend on what bulk theory we consider, we generically use G_D as the global defect for twisting an inner product. For the free dS bulk scalar case, the theory respects the PT symmetry, and we introduce the global defect operator, PT . However, even for the free theory with charges, we expect the inner product to change upon introducing the charge-conjugation operator. Hence, we consider the most generic approach here. We introduce the flat defect, the generating function, and the integral identity in the presence of the local defect [28]. We then discuss how to introduce global defect operators to have the dS_{d+1} dual from the CFT_d and demonstrate how the conformal symmetry is still applicable to determine the connected correlators for the conformal defect group case.

A. Flat defect

We consider a d -dimensional Euclidean CFT with a global internal symmetry group G and insert a flat p -dimensional conformal defect W (bare defect operator) along a flat submanifold

$$\mathbb{R}^p \subset \mathbb{R}^d \quad (79)$$

and coordinates

$$x = (\tau^a, x_\perp^j), \quad (80)$$

where the full symmetry index (CFT bulk index) is $a = 1, 2, \dots, p$, and the index for the broken directions (associated to the broken generators $h_\perp$) is $j = 1, 2, \dots, d - p$. The $\tau \in \mathbb{R}^p$ are the coordinates along the defect, and $x_\perp \in \mathbb{R}^{d-p}$ are the coordinates transverse to the defect. The

defect operators are at $x_\perp = 0$. The bare defect operator W is simply the operator that implements the presence of the defect in the path integral before any defect operators are inserted, which is analogous to inserting

$$W(1) = \exp(-S_{\text{defect}}(\phi)) \quad (81)$$

on $\mathbb{R}^p$.

Starting from the bare defect insertion $W(1)$, one can deform it by turning on sources of the operator $\hat{i}(\tau)$, $w^j(\tau)$,

$$W(e^{\int d^p \tau w_j(\tau) \hat{i}_j(\tau)}), \quad (82)$$

which implies the compact notation

$$W(\cdots) = \sum_{n=0}^{\infty} \frac{1}{n!} \int d\tau_1 d\tau_2 \cdots d\tau_n w_{i_1}(\tau_1) w_{i_2}(\tau_2) \cdots \times w_{i_n}(\tau_n) W(1) \hat{i}_{i_1} \hat{i}_{i_2} \cdots \hat{i}_{i_n}. \quad (83)$$

The presence of the local defect breaks the symmetry group from G to H , which means that only a subgroup H of the CFT bulk symmetry survives. The global symmetry is explicitly broken, and locality is violated along the defect directions. The current conservation equation must be changed only at $x_\perp = 0$, which forces a delta function structure

$$\partial_\mu J_a^\mu(x) = \delta^{(d-p)}(x_\perp) P_a^j \hat{i}_j(\tau), \quad (84)$$

where P_a^j is a projector extracting the part of the generator a lying in the broken direction j . This is a universal identity for internal-symmetry breaking by a local defect. The local defect has degrees of freedom that encode its response to the broken symmetry. The $\hat{i}_j(\tau)$ is the response that represents the infinitesimal action of the broken internal symmetry from the local defect operators. Hence, the name tilt operator.

B. Generating function

The generating functional is [28]

$$Z[r, w] = \int \mathcal{D}\phi e^{-S} \exp\left(\int d^d x r_a(x) \mathcal{O}_a(x)\right) \times W(e^{\int d^p \tau w_j(\tau) \hat{i}_j(\tau)}), \quad (85)$$

where S is the CFT bulk action, and $\mathcal{O}_a$ are the CFT bulk correlators. We can use functional derivatives to consider the most general correlators that do not involve global defect operators.

A group element $g \in G$ acts on the sources by a map

$$(r, w) \mapsto (L_g r, L_g w). \quad (86)$$

For an infinitesimal transformation $g = \exp(\lambda)$ with λ in the Lie algebra, we define

$$L_{e^\lambda} r = r + \rho(\lambda, r) + \mathcal{O}(\lambda^2); \quad L_{e^\lambda} w = w + l(\lambda, w) + \mathcal{O}(\lambda^2), \quad (87)$$

where $\rho(\lambda, r)$ and $l(\lambda, w)$ are the infinitesimal variations of the CFT bulk and local defect couplings. The local defect variation $l(\lambda, w)$ is expanded as a formal power series in w ,

$$l(\lambda, w) = \sum_{k=0}^{\infty} \frac{1}{k!} l_k(\lambda; w, w, \dots, w), \quad (88)$$

where $l_k(\lambda; w, w, \dots, w)$ is a multilinear functional of k copies of w . The $l_0(\lambda)$ is independent of w , which is just a vector in the broken directions, and the $l_1(\lambda; w)$ is linear in w , etc.

We first introduce the shorthand notation:

$$\mathcal{O}(r) \equiv \int d^d x r_a(x) \mathcal{O}_a(x); \quad \hat{i}(w) \equiv \int d^p \tau w_j(\tau) \hat{i}_j(\tau). \quad (89)$$

The logarithm of the generating functional about the connected correlators is

$$\ln Z[r, w] = \sum_{m, n \geq 0} \frac{1}{m!n!} \langle (\mathcal{O}(r))^m (\hat{i}(w))^n \rangle_c, \quad (90)$$

where the coefficient $1/(m!n!)$ in this expansion is precisely the connected correlator with m insertions of $\mathcal{O}(r)$ and n insertions of $\hat{i}(w)$. Under the infinitesimal transformation, the logarithm of the partition function becomes [28]

$$\ln Z[L_g r, L_g w] = \ln Z[r, w] + A[\lambda, w], \quad (91)$$

where $A[\lambda, w]$ is a defect-local anomaly functional, which can be expanded as

$$A[\lambda, w] = \sum_{n=1}^{\infty} \frac{1}{n!} A_n[\lambda; w, w, \dots, w], \quad (92)$$

where each $A_n[\lambda; w_1, w_2, \dots, w_n]$ is multilinear and symmetric in the w_j 's.

If we now set $w = 0$, it corresponds to turning off all explicit symmetry-breaking local defect couplings

$$\ln Z[L_g r, 0] = \ln Z[r, 0] + A_0[\lambda], \quad (93)$$

where

$$A_0[\lambda] = A[\lambda, w = 0]. \quad (94)$$

In this situation, by assumption, the theory (bulk + local defect with $w = 0$) has an exact global symmetry G . This means that the generating functional is invariant, which implies [28]

$$A_0[\lambda, 0] = 0. \quad (95)$$

Hence, the A_0 is simply the anomaly evaluated at zero defect source, and since the theory with $w = 0$ is genuinely symmetric (no defect couplings, no anomaly), that anomaly must vanish. The A_1 is linear in w , which can be eliminated by a local redefinition of the local defect couplings (a local counterterm), so only $A_{n \geq 2}$ contains nontrivial physical anomaly data [28].

C. Integral identity

We expand the logarithm of the partition function up to the first order in λ ,

$$\delta_\lambda \ln Z = A[\lambda, w], \quad (96)$$

where

$$\delta_\lambda \ln Z = \ln(Z[r + \rho(\lambda, r), w + l(\lambda, w)]) - \ln(Z[r, w]), \quad (97)$$

and it is linear in λ . We can now plug in the expansion

$$\ln Z[r, w] = \sum_{m, n \geq 0} \frac{1}{m!n!} \langle (\mathcal{O}(r))^m (\hat{i}(w))^n \rangle_c, \quad (98)$$

for $\ln Z$ and keep only the linear terms in λ . To consider the variation of the bulk resource [28]

$$r \rightarrow r + \rho(\lambda, r) \quad (99)$$

and up to the first order in λ , we obtain [28]

$$\ln Z[r + \rho(\lambda, r), w] = \sum_{m, n \geq 0} \langle (\mathcal{O}(r) + \mathcal{O}(\rho(\lambda, r)))^m (\hat{i}(w))^n \rangle_c. \quad (100)$$

Hence, the bulk variation of the logarithm of the partition function is [28]

$$\delta_\lambda^{\text{bulk}} \ln Z = \sum_{m, n \geq 0} \frac{1}{m!n!} \langle \mathcal{O}(\rho(\lambda, r)) (\mathcal{O}(r))^m (\hat{i}(w))^n \rangle_c, \quad (101)$$

where $\langle \dots \rangle_c$ is the expectation value of the connected correlators. We can extract the term [28]

$$\frac{1}{n!} \sum_{j=1}^m \langle \mathcal{O}(r_1) \mathcal{O}(r_2) \cdots \mathcal{O}(\rho(\lambda, r_j)) \cdots \mathcal{O}(r_m) (\hat{t}(w))^n \rangle_c. \quad (102)$$

We now vary the local defect source [28]

$$w \rightarrow w + l(\lambda, w) \quad (103)$$

up to the first order in λ to show [28]

$$\ln Z[r, w + l(\lambda, w)] = \sum_{m,n \geq 0} \frac{1}{m!n!} \langle (\mathcal{O}(r))^m (\hat{t}(w) + \hat{t}(l(\lambda, w)))^n \rangle_c. \quad (104)$$

The local defect variation of the logarithm of the partition function is [28]

$$\delta_\lambda^{\text{defect}} \ln Z = \sum_{m,n \geq 0} \frac{1}{m!n!} \sum_{k \geq 0} \frac{1}{k!} \langle (\mathcal{O}(r))^m \hat{t}(l_k(\lambda; w, w, \dots, w)) (\hat{t}(w))^n \rangle_c. \quad (105)$$

We extract the terms with a fixed order in n for w [28],

$$\frac{1}{n!} \sum_{k=0}^n \frac{n!}{k!(n-k)!} \langle \mathcal{O}(r_1) \mathcal{O}(r_2) \cdots \mathcal{O}(r_m) \hat{t}(l_k(\lambda; w, w, \dots, w)) (\hat{t}(w))^{n-k} \rangle_c. \quad (106)$$

The coefficient is given by the k choice in the $l_k(\lambda; w, w, \dots, w)$ and also the remaining $n - k$ choices in $\hat{t}(w)$ [28]. We combine the CFT bulk and local defect variation of the sources and the defect anomaly, and we then obtain the integral identities for the connected correlators [28]

$$\begin{aligned} & \sum_{j=1}^m \langle \mathcal{O}(r_1) \mathcal{O}(r_2) \cdots \mathcal{O}(\rho(\lambda, r_j)) \cdots \mathcal{O}(r_m) (\hat{t}(w))^n \rangle_c \\ & + \sum_{k=0}^n \frac{n!}{k!(n-k)!} \langle \mathcal{O}(r_1) \mathcal{O}(r_2) \cdots \mathcal{O}(r_m) \hat{t}(l_k(\lambda; w, w, \dots, w)) (\hat{t}(w))^{n-k} \rangle_c \\ & = A_n[\lambda; w, w, \dots, w]. \end{aligned} \quad (107)$$

Let us illustrate some examples. For $n = 0$, the sum over k has only one term, $k = 0$ [28],

$$\langle \hat{t}(l_0(\lambda)) \rangle = A_0[\lambda]. \quad (108)$$

Because $l_0(\lambda)$ is the w -independent part of the defect source variation, we get [28]

$$\hat{t}(l_0(\lambda)) = \hat{t}(\lambda). \quad (109)$$

Hence, we get [28]

$$\langle \hat{t}(\lambda) \rangle_c = 0. \quad (110)$$

We now do the variation for λ_j and then set $\lambda = 0$ to show [28]

$$\left. \frac{\delta}{\delta \lambda_j} \langle \hat{t}(\lambda) \rangle_c \right|_{\lambda=0} = \int d^p \tau \langle \hat{t}_j(\tau) \rangle_c = 0. \quad (111)$$

When considering the conformal defect group, the one-point tilt function vanishes

$$\langle t_j(\tau) \rangle = 0, \quad (112)$$

which automatically satisfies the integral identity [28]. Now, we go to $n = 1$ for the defect-only identity [28],

$$\langle \hat{t}(\lambda) \hat{t}(w) \rangle_c + \langle \hat{t}(l_1(\lambda; w)) \rangle_c = 0. \quad (113)$$

We continue to consider the conformal defect group case, and the defect one-point function vanishes, and then it shows [28]

$$\langle \hat{t}(\lambda) \hat{t}(w) \rangle_c = 0. \quad (114)$$

We then take functional derivatives for the sources and then set them to zero [28],

$$\frac{\delta}{\delta \omega_k(\tau_2)} \frac{\delta}{\delta \lambda_j} \langle \hat{t}(\lambda) \hat{t}(w) \rangle_c \Big|_{\lambda=w=0} = \int d^p \tau_1 \langle \hat{t}_j(\tau_1) \hat{t}_k(\tau_2) \rangle_c = 0. \quad (115)$$

Now, we get an integral constraint to the two-point tilt function [28]

$$\int d^p \tau_1 \langle \hat{t}_j(\tau_1) \hat{t}_k(\tau_2) \rangle_c = 0. \quad (116)$$

Let us now consider the integral identity for $(m, n) = (1, 0)$ [28],

$$\langle \mathcal{O}(\rho(\lambda, r)) \rangle_c + \langle \mathcal{O}(r) \hat{t}(\lambda) \rangle_c = 0. \quad (117)$$

The infinitesimal action on the bulk source is [28]

$$\rho(\lambda, r)_\beta(x) = \lambda_j r_\alpha(x) (T_j)_{\beta\alpha}, \quad (118)$$

where $(T_j)_{\beta\alpha}$ are the representation matrices of the broken generators. We take the functional derivatives [28]:

$$\begin{aligned} \frac{\delta}{\delta \lambda_j} \frac{\delta}{\delta r_\alpha(x_0)} \langle \mathcal{O}(\rho(\lambda, r)) \rangle_c \Big|_{\lambda=r=0} &= (T_j)_{\beta\alpha} \langle \mathcal{O}_\beta(x_0) \rangle_c; \\ \frac{\delta}{\delta \lambda_j} \frac{\delta}{\delta r_\alpha(x_0)} \langle \mathcal{O}(r) \hat{t}(\lambda) \rangle_c \Big|_{\lambda=r=0} &= \int d^p \tau_1 \langle \mathcal{O}_\alpha(x_0) \hat{t}_j(\tau_1) \rangle_c. \end{aligned} \quad (119)$$

Combining the results that we get shows that [28]

$$(T_j)_{\beta\alpha} \langle \mathcal{O}_\beta(x_0) \rangle_c + \int d^p \tau_1 \langle \mathcal{O}_\alpha(x_0) \hat{t}_j(\tau_1) \rangle_c = 0. \quad (120)$$

The result implies that the generator acting on the one-point function plus integrated bulk-tilt two-point function is zero. Hence, our examples demonstrate the constraints on the connected correlators and how the connected correlator can be determined from the lower-point connected correlator [28].

D. Conformal defect group

To apply the conformal symmetry to determine the low-point correlators, we consider the infinite planar defect. The full conformal group in d dimensions is $\text{SO}(d+1, 1)$. A flat p -dimensional local defect preserves a conformal group along the defect, $\text{SO}(p+1, 1)$, which includes

(i) translations along the defect

$$\tau^a \rightarrow \tau^a + c^a; \quad (121)$$

(ii) rotations within the defect $\text{SO}(p)$ and rotations in directions normal to the defect, $\text{SO}(d-p)$;

(iii) dilatations: $(\tau, x_\perp) \rightarrow (\lambda\tau, \lambda x_\perp)$; and

(iv) special conformal transformations tangent to the defect.

Hence, the preserved symmetry group is $\text{SO}(p+1, 1) \times \text{SO}(d-p)$, which is the standard “defect conformal group.” The title operator $\hat{t}_j$ arises because the defect breaks an internal symmetry $G \rightarrow H$. For the broken generators $j \in h_\perp$, the current conservation equation becomes the defect-localized Ward identity

$$\partial_\mu J_j^\mu(x) = \delta^{(d-p)}(x_\perp) \hat{t}_j(\tau). \quad (122)$$

Hence, the tilt operator is a local operator with scaling dimension $\Delta_{\hat{t}} = p > 0$ that lives on the defect. It transforms as a vector in the broken symmetry space $h_\perp$.

Since we have the conformal group for each CFT bulk and defect space, respectively, we can introduce the same adjoint relations to the generators for the CFT bulk and defect direction indices as our previous analysis (whole conformal group in CFT_d), respectively. To compute higher-point correlators with a proper inner product, we can introduce the global defect only via an invariant state under a global defect operator. Suppose a state is invariant under a global defect operator. In that case, we can take its complex conjugate to see how the global defect must be introduced. Because the vacuum state is not an invariant state under a global defect operator, the CFT bulk or defect operators need to act on the vacuum state via a linear combination to form an invariant state under a global defect operator. Hence, only the operator acting on the bra state is affected by a global defect operator. Another reason for twisting the inner product in CFT is that the dS bulk operator can be translated to the CFT operators through the HKLL bulk reconstruction procedure [12–15, 20]. It should imply that the computation of CFT correlators requires the introduction of the same global defect to twist the inner product. Because we twist the inner product from a global defect, the symmetry needs to be enlarged. If we consider the global defect operator as PT , we can enlarge the defect conformal group to $\text{O}(p+1, 1) \times \text{O}(d-p)$.

When considering the defect conformal group, we have the maximum possible conformal symmetry. We can use translational symmetry to constrain the expectation value of the one-point tilt operator to be a constant. We then use the scaling symmetry to conclude that the constant must be zero, thereby automatically satisfying the integral constraint (111). Hence, the defect still explicitly breaks the symmetry, rather than breaking it spontaneously.

For the two-point tilt operators, we can first consider translational and rotational symmetries along the defect directions to show that the two-point Green's function of the tile operators depends only on the distance in defect coordinates. We then apply the scaling to the two-point tilt operators, which implies

$$\langle \hat{t}_j(\tau_1) \hat{t}_k(\tau_2) \rangle_c = \frac{C_{jk}}{|\tau_{12}|^{2p}}, \quad (123)$$

where

$$\tau_{12} \equiv \tau_1 - \tau_2. \quad (124)$$

Because the tile operators transform as a vector under the internal representation, and the vacuum state is invariant under the internal symmetry, the only invariant tensor is proportional to the identity matrix

$$C_{jk} = C_t \delta_{jk}. \quad (125)$$

Hence, the two-point tilt function also satisfies the integral constraint (116). Let us specifically consider the global defect operator PT . Only the parity operator acts on the defect directions. The expectation value of the two-point tilt operators does not vanish only when two operators have the same parity. The same analysis applies to other one- and two-point correlators, and the integral constraints help connect higher-point connected correlators, such as Eq. (120). Hence, the introduction of global defect operators enlarges the symmetry group and imposes additional constraints on the CFT correlators. The introduction of the global defect operator, which twists the inner product, simplifies the computation of correlators.

VII. DISCUSSION AND CONCLUSION

In this work, we have investigated the symmetry-based perspective of the dS/CFT correspondence using Wick rotation and analytic continuation. Beginning with global AdS_{d+1} in Lorentzian signature, we demonstrated how these operations yield the static patch of dS_{d+1} together with the appropriate bulk dS_{d+1} isometry group, from which one can read off the corresponding CFT_d generators. The resulting adjoint operation for these generators in the dS bulk coordinate representation differs from the familiar AdS/CFT case, leading to an exotic adjoint structure that naturally introduces a global defect operator PT . This operator is essential for reproducing the correct Green's function of a real scalar field in dS_{d+1} . Motivated by the antipodal symmetry inherent to the sphere, we posited that vacuum degeneracy should be anticipated within the dS/CFT framework. Furthermore, we explicitly demonstrated that any CFT_2 with a central charge possessing an imaginary component must necessarily exhibit spontaneous PT -symmetry breaking of its vacuum state.

Finally, we scrutinized connected correlators in the presence of both global and local defects using integral identities, elucidating how global defect operators can be seamlessly incorporated into these identities and how the defect conformal group must be appropriately expanded in the dS/CFT correspondence.

We applied Wick rotation and analytic continuation only to the metric, CFT generators, and Green's functions, and we found that this restricted procedure works consistently in arbitrary dimensions. However, these operations remain subtle in general quantum field theories, particularly in curved spacetime. When applied directly to the Lagrangian, difficulties arise in the path integral and regularization. Our examination reveals that Wick rotation and analytic continuation can be executed reliably on numerical quantities—such as Green's functions post-evaluation—yet cannot be applied directly to bra states. While the continuation generates expressions akin to Eq. (41), such expressions ought to correspond to expectation values of vacuum states whose conjugate spectra are distinct, thereby indicating that the continuation cannot be uniformly applied to all states. Instead, the exotic adjoint structure we introduced provides a consistent realization of observables through the Wick rotation and analytical continuation. This observation is promising for the study of dS_{d+1} quantum gravity from CFT_d data. Once expectation values of all connected correlators are known, the dS bulk theory is fully determined, even without explicit knowledge of its Lagrangian. The central lesson of this work is that analytic continuation from AdS to dS modifies not only correlation functions but also the notion of adjointness and the Hilbert-space metric. The resulting PT -defect structure provides a possible bridge between analytic continuation approaches, nonunitary dS dualities [4], and higher-spin realizations of dS/CFT [5].

Our analysis revealed the necessity of introducing a PT operator from the two-point function of a real scalar field in a de Sitter background. Since charge conjugation is trivial for real scalars, a more general global defect structure likely requires considering fields with nontrivial spin or internal charges. The PT operator is consequently anticipated to be exclusive to real scalar fields, and supplementary global defects will necessitate corresponding augmentations of the conformal symmetry group. Furthermore, it is imperative to examine higher-point connected correlators—such as conformal blocks—beyond the realm of the two-point de Sitter bulk Green's function. While the two-point function can be portrayed as the inner product of two bulk local states, this assertion does not hold for higher-point functions; thus, comprehending their structure is vital for ascertaining how the inner product ought to be twisted in the de Sitter/conformal field theory correspondence. Such higher-point data also encode information about interactions in the dS bulk theory. For a well-defined theory, the global defect should act as a metric operator [23]. However, we focused on PT symmetry because the free theory respects it;

interacting theories may require different choices. Exploring these possibilities provides a compelling direction for formulating a consistent, physically meaningful dS quantum gravity theory from CFT principles.

In the PT -broken phase of our non-Hermitian defect CFT, the one-point function of the tilt operator acquires a nonzero value, signaling that the vacuum is no longer invariant under the combined defect conformal symmetry and PT symmetry. This nonvanishing one-point tilt function acts as an order parameter that selects a specific PT -broken vacuum from a degenerate manifold, encoding how the defect sources bulk fields and breaks scale invariance along the defect worldvolume. Because the AdS and dS situations differ substantially in both symmetry and analytic structure, this disorder parameter provides a natural diagnostic for distinguishing the two phases. Notably, although the background expectation value reflects spontaneous PT -symmetry breaking, the fluctuating part of the defect operator continues to transform covariantly under the unbroken subgroup of the defect conformal group. Consequently, higher-point connected correlators—such as two-point functions of defect fluctuations—remain constrained by standard defect conformal symmetry, even when the whole theory resides in a symmetry-broken phase. Thus, while the nonzero one-point function enriches defect physics by introducing a symmetry-breaking background, the conformally invariant structure of fluctuations persists. The PT -breaking studied here represents the

simplest example of a nonvanishing one-point tilt function beyond the conventional defect conformal group. It would be compelling to apply the integral identity of Ref. [28] without assuming the vanishing one-point tilt function to analyze higher-point functions in this expanded framework.

In this work, we have realized the dS/CFT correspondence for heavy bulk particles whose conformal dimensions possess imaginary parts. In contrast, light particles have real-valued conformal dimensions Δ and $d - \Delta$. This structure is precisely what appears in the study of light fields in cosmology, where such mixed-dimension behavior plays a central role in inflationary correlators. It would therefore be exciting to explore the interplay between bulk reconstruction and non-Hermitian defect structures in the light-particle regime of the dS/CFT correspondence.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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