

Pseudoscalar and vector toponia in a Dyson-Schwinger-Bethe-Salpeter framework

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We study the pseudoscalar ($J^{PC} = 0^{-+}$) and vector (1^{--}) top-antitop (toponium) systems within the rainbow-ladder truncation of the Dyson-Schwinger and Bethe-Salpeter equations, employing the Qin-Chang effective interaction. After validating the framework in the charmonium and bottomonium sectors, we extend it consistently to the top sector, incorporating renormalization-group running of the current quark mass and a careful treatment of the number of active flavors. We compute masses and leptonic decay constants for $N_f = 5$ and 6, then analyze their dependence on the renormalization scale in the range $\mu = 400\text{--}800$ GeV. The resulting toponium masses lie near 344–346 GeV with hyperfine splittings below 0.14–0.17 GeV, while the decay constants are large, 6–7 GeV, and exhibit the expected heavy-quark scaling behavior. We find only mild sensitivity to the renormalization point and a systematic reduction of binding when increasing N_f . Although the physical top quark decays weakly before hadronization, our results demonstrate that, within a Poincaré-covariant nonperturbative framework, quantum chromodynamics (QCD) generates tightly correlated pseudoscalar and vector toponium systems in that extreme heavy-quark limit.

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I. INTRODUCTION

The top quark, t , with a mass of about 173 GeV, is the heaviest known elementary particle and represents a unique laboratory for testing the Standard Model at the interface between electroweak and strong interactions [1]. Its large mass leads to an extraordinarily short lifetime, $\tau_t \sim 10^{-25}$ s $< \Lambda_{\text{QCD}}^{-1} \sim 10^{-24}$ s, such that the top-quark typically decays via the weak interaction before conventional hadronization can occur [2]. As a consequence, top-anti-top ($t\bar{t}$, toponium) bound states analogous to charmonium ($c\bar{c}$) and bottomonium ($b\bar{b}$) have long been considered inaccessible via experiment [3].

Nevertheless, recent analyses by the CMS Collaboration of proton-proton collision data at $\sqrt{s} = 13$ TeV have revealed a significant excess of $t\bar{t}$ events near the kinematic threshold that is compatible with the production of a color-singlet pseudoscalar quasi-bound state, as suggested by nonrelativistic QCD (NRQCD) calculations for toponium,

with a measured cross section of approximately $8.8^{+1.2}_{-1.4}$ pb for the excess above the fixed-order prediction of perturbative QCD at the threshold [4]. Subsequent reports from the ATLAS Collaboration confirm an enhancement near threshold, with a measured cross section consistent with the CMS observation and a combined significance exceeding that of background-only models [5].

These unforeseen features in the top-pair production data have rejuvenated interest in the theoretical description of heavy quark–antiquark correlations in the extreme mass regime of the top quark [6–11]. In contrast to the conventional expectation that the top decay width Γ_t is too large for formation of narrow hadronic bound states, the interplay between QCD binding effects and electroweak instability calls for a careful treatment of near-threshold dynamics [12,13]. Theoretical approaches based on NRQCD have provided a useful phenomenological description of near-threshold behavior [14–17], see also early Refs. [18,19]. However, a fully relativistic and Poincaré-covariant treatment is still needed to study toponium in a first-principles framework.

In the continuum formulation of QCD, meson properties can be studied through the Dyson-Schwinger equations (DSE) for the dressed quark and gluon propagators together with the homogeneous Bethe-Salpeter equation (BSE) for quark-antiquark bound states [20–23]. These equations form an infinite tower of coupled integral equations derived from the QCD generating functional [24,25]. When truncated in a symmetry-preserving manner [26–28], they

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provide a Poincaré-covariant and renormalization-group consistent framework for describing mesons, and baryons, across all quark-mass regimes [29–38].

A particularly successful construction of the effective quark–antiquark interaction—often referred to as the Qin-Chang interaction—was developed to provide a unified description of meson observables across a wide range of quark masses [39]. In this model the interaction kernel combines an infrared-enhanced term responsible for dynamical chiral symmetry breaking with a perturbative ultraviolet tail consistent with QCD’s one-loop behavior. When implemented in the rainbow-ladder (RL) truncation of the quark gap and meson’s BSEs, this interaction systematically reproduces meson pseudoscalar and vector masses and leptonic decay constants in the light [40], heavy-light [41], and heavy sectors, including charmonium and bottomonium [42]. Moreover, such studies preserve the axial-vector Ward-Green-Takahashi identity [43,44], ensuring the correct interplay between chiral symmetry and bound-state structure.

In this work, we explore the lowest-lying pseudoscalar ($J^{PC} = 0^{-+}$) and vector (1^{--}) toponium systems within the DSE-BSE framework using the Qin-Chang effective interaction. Building on earlier successes in the description of heavy quarkonia, we extend the model consistently to the top sector by including renormalization-group running of the current quark mass, careful treatment of the number of active flavors and systematic analysis of renormalization-scale dependence. We compute the masses and leptonic decay constants for $N_f = 5$ and 6 active flavors and analyze their behavior in a range of renormalization points relevant for heavy quark dynamics. These results provide a fully relativistic baseline for the mass spectrum and decay constants of hypothetical toponium states, and offer insight into how QCD dynamics manifests itself when the quark lifetime competes with binding effects. In the present study we do not include explicit finite-width effects associated with the electroweak decay of the top quark. Our results therefore characterize the strength of QCD-induced quark–antiquark correlations in the extreme heavy-quark limit, providing a baseline against which more refined treatments incorporating unstable-particle dynamics can be compared.

The manuscript is organized as follows. In Sec. II we introduce the Dyson-Schwinger-Bethe-Salpeter formalism and specify the Qin-Chang interaction employed in this work. Section III presents our results, first discussing the treatment of the running top-quark mass, second validating the framework in the charmonium and bottomonium sectors and third presenting our results for pseudoscalar and vector toponium states for $N_f = 5$ and 6, including the analysis of renormalization-scale dependence and heavy-quark scaling. Finally, Sec. IV summarizes our findings and outlines possible extensions, such as the completion of toponium spectrum, inclusion of finite-width effects and beyond-rainbow corrections.

II. THEORETICAL FORMALISM

The RL truncation, which replaces the fully dressed quark-gluon vertex with an effective interaction while maintaining consistency between the quark DSE and meson BSE kernels, preserves the axial-vector Ward-Green-Takahashi identity. As a consequence, chiral symmetry and its dynamical breaking are correctly realized, ensuring that pseudoscalar mesons emerge as (pseudo-) Goldstone bosons in the light-quark sector. At the same time, RL truncation provides a quantitatively reliable description of heavy quarkonia, where corrections beyond ladder exchange are suppressed by inverse powers of the heavy-quark mass.

A widely used realization of this framework is the effective interaction introduced by Qin and Chang *et al.* [39]. In this construction, the interaction kernel combines an infrared-enhanced term responsible for dynamical chiral symmetry breaking with an ultraviolet component constrained to reproduce the one-loop renormalization-group behavior of QCD. The model is characterized by two parameters, ω and D , which control the infrared width and strength of the interaction and have been shown to yield a unified description of pseudoscalar and vector mesons from the light sector through charmonium and bottomonium. In particular, masses and leptonic decay constants of η_c , J/ψ , η_b , and Υ are reproduced at the few-percent level within a common parameter set [42]. Extending the framework to the top quark therefore probes the extreme heavy-mass limit of QCD within a fully relativistic, Poincaré-covariant formalism.

A. Quark Dyson-Schwinger equation

In continuum QCD, the dressed quark propagator is obtained from the quark DSE,¹

$$S^{-1}(p) = Z_2(i\not{p} + m^{\text{bm}}) + Z_1 g^2 \int_k^\Lambda D_{\mu\nu}(q) \frac{\lambda^a}{2} \gamma_\mu S(k) \Gamma_\nu^a(k, p), \quad (1)$$

where Z_1, Z_2 are renormalization constants, m^{bm} is the bare mass, $D_{\mu\nu}$ is the dressed gluon propagator, $q = p - k$, and Γ_ν^a is the dressed quark–gluon vertex. The integral symbol denotes

$$\int_k^\Lambda = \int \frac{d^4 k}{(2\pi)^4}, \quad (2)$$

regularized with ultraviolet cutoff Λ .

The renormalized quark propagator can be decomposed as

¹Note that we employ Euclidean conventions throughout this study with $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$, $\gamma_\mu^\dagger = \gamma_\mu$ and $a \cdot b = \sum_{i=1}^4 a_i b_i$ and a spacelike vector, k_μ , has $k^2 > 0$.

$$S(p)^{-1} = i\not{p}A(p^2, \mu^2) + B(p^2, \mu^2), \quad (3)$$

or equivalently,

$$S(p) = \frac{Z(p^2, \mu^2)}{i\not{p} + M(p^2)}, \quad (4)$$

where $M(p^2) = B/A$ is the running mass function and μ is the renormalization point.

Renormalization is implemented via

$$S^{-1}(p)|_{p^2=\mu^2} = i\not{p} + m(\mu), \quad (5)$$

ensuring consistency with the chosen renormalization scheme.

B. Rainbow-Ladder truncation

In the RL truncation, the dressed quark-gluon vertex is replaced by its leading Dirac structure and an effective interaction,

$$Z_1 g^2 D_{\mu\nu}(q) \Gamma_\nu^a(k, p) \rightarrow 4\pi\alpha_{\text{eff}}(q^2) D_{\mu\nu}^{\text{free}}(q) \frac{\lambda^a}{2} \gamma_\nu, \quad (6)$$

where $D_{\mu\nu}^{\text{free}}(q)$ is the free gluon propagator in Landau-gauge,

$$D_{\mu\nu}^{\text{free}}(q) = \frac{1}{q^2} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right). \quad (7)$$

This truncation preserves the axial-vector Ward-Green-Takahashi identity,

$$P_\mu \Gamma_{5\mu}(k; P) = S^{-1}(k_+) \gamma_5 + \gamma_5 S^{-1}(k_-), \quad (8)$$

where $k_\pm = k \pm P/2$, with k the relative momentum and P the total momentum. This ensures the correct realization of chiral symmetry and its dynamical breaking.

C. Qin-Chang effective interaction

The effective coupling $\alpha_{\text{eff}}(q^2)$ is taken in the Qin-Chang form,

$$\frac{\alpha_{\text{eff}}(q^2)}{q^2} = \frac{2\pi D}{\omega^4} e^{-q^2/\omega^2} + \frac{2\pi\gamma_m(1 - e^{-q^2/\Lambda_t^2})}{q^2 \ln[e^2 - 1 + (1 + q^2/\Lambda_{\text{QCD}}^2)^2]}, \quad (9)$$

where D and ω control the infrared strength and width, $\gamma_m = 12/(33 - 2N_f)$ is the anomalous dimension, with N_f the number of active flavors, Λ_{QCD} sets the ultraviolet scale, $\Lambda_t = 1.0$ GeV ensures smooth infrared matching. Note that the combination $\alpha_{\text{eff}}(q^2)/q^2$ remains finite in the infrared,

since $(1 - e^{-q^2/\Lambda_t^2})/q^2 \rightarrow 1/\Lambda_t^2$ as $q^2 \rightarrow 0$.

The first term in Eq. (9) generates dynamical chiral symmetry breaking, while the second reproduces the one-loop QCD running at large momenta.

D. Bethe-Salpeter equation

Mesons appear as poles in the quark-antiquark scattering matrix. The homogeneous BSE reads

$$\Gamma(p; P) = - \int_k^\Lambda \mathcal{K}(p, k; P) S(k_+) \Gamma(k; P) S(k_-), \quad (10)$$

where $\mathcal{K}(p, k; P)$ is the quark-antiquark kernel.

In RL truncation,

$$\mathcal{K}(p, k; P) = 4\pi\alpha_{\text{eff}}(q^2) D_{\mu\nu}^{\text{free}}(q) \times \left(\frac{\lambda^a}{2} \gamma_\mu \right) \otimes \left(\frac{\lambda^a}{2} \gamma_\nu \right), \quad (11)$$

solutions exist only for discrete timelike values $P^2 = -M^2$, defining the meson mass M . Here, the symbol $\otimes$ denotes a direct product in Dirac and color space; in the Bethe-Salpeter equation, the kernel acts by left- and right-multiplication on the full matrix structure $S(k_+) \Gamma(k; P) S(k_-)$.

E. Pseudoscalar and vector channels

For pseudoscalar states (0^{-+}), the BS amplitude is expanded as

$$\Gamma_{PS}(p; P) = \gamma_5 [iE(p^2, p \cdot P) + \not{p} F(p^2, p \cdot P) + \not{p}(p \cdot P) G(p^2, p \cdot P) + \sigma_{\mu\nu} p_\mu P_\nu H(p^2, p \cdot P)]. \quad (12)$$

For vector states (1^{--}),

$$\Gamma_V^\mu(p; P) = \sum_{i=1}^8 F_i(p^2, p \cdot P) T_i^\mu(p; P), \quad (13)$$

where T_i^μ ($= T_\mu^i$) form a complete transverse basis ($q = \sqrt{q^2}$ and $P = \sqrt{P^2}$ denote the Euclidean moduli of the relative and total momenta):

$$T_\mu^1(q; P) = \gamma_\mu^T, \quad (14a)$$

$$T_\mu^2(q; P) = \frac{6}{q^2 \sqrt{5}} \left[q_\mu^T (\gamma^T \cdot q) - \frac{1}{3} \gamma_\mu^T (q^T)^2 \right], \quad (14b)$$

$$T_\mu^3(q; P) = \frac{2}{qP} [q_\mu^T (\gamma \cdot P)], \quad (14c)$$

$$T_\mu^4(q; P) = \frac{i\sqrt{2}}{qP} [\gamma_\mu^T (\gamma \cdot P) (\gamma^T \cdot q) + q_\mu^T (\gamma \cdot P)], \quad (14d)$$

$$T_\mu^5(q; P) = \frac{2}{q} q_\mu^T, \quad (14e)$$

$$T_\mu^6(q; P) = \frac{i}{q\sqrt{2}} [\gamma_\mu^T (\gamma^T \cdot q) - (\gamma^T \cdot q) \gamma_\mu^T], \quad (14f)$$

$$T_\mu^7(q; P) = \frac{i\sqrt{3}}{q^2 P \sqrt{5}} (1 - \cos^2 \theta) [\gamma_\mu^T (\gamma \cdot P) - (\gamma \cdot P) \gamma_\mu^T] - \frac{1}{\sqrt{2}} T_\mu^8(q; P), \quad (14g)$$

$$T_\mu^8(q; P) = \frac{i2\sqrt{6}}{q^2 P \sqrt{5}} q_\mu^T (\gamma^T \cdot q) (\gamma \cdot P), \quad (14h)$$

where V^T is the component of V transverse to P ,

$$V_\mu^T = V_\mu - \frac{P_\mu (P \cdot V)}{P^2}. \quad (15)$$

Besides, note here that the J^{PC} quantum numbers are fixed by the transformation properties of the BSA under parity and charge conjugation. For charge conjugation, the BSA transforms as

$$\bar{\Gamma}(q; P) \equiv [C\Gamma(-q; P)C^{-1}]^T = \eta_C \Gamma(q; P), \quad (16)$$

where $C = \gamma_2 \gamma_4$ is the charge-conjugation matrix, T denotes matrix transpose, and $\eta_C = \pm 1$ is the C -parity eigenvalue. One then applies this transformation to each Dirac covariant in the decomposition of the Bethe-Salpeter Amplitude (BSA). For the pseudoscalar basis in Eq. (12), all four covariants are C -even, whereas for the vector basis in Eq. (13), the eight transverse covariants are C -odd. For equal-mass quarkonia, the accompanying scalar dressing functions are chosen with the corresponding symmetry under $q \rightarrow -q$, so that the full amplitudes have definite charge conjugation. Therefore, the pseudoscalar amplitude has $C = +1$ and the vector amplitude has $C = -1$.

Parity is determined analogously from the transformation,

$$\Gamma(q; P) \rightarrow \gamma_4 \Gamma(\tilde{q}; \tilde{P}) \gamma_4, \quad (17)$$

where $\tilde{q} = (q_4, \vec{q})$ and $\tilde{P} = (p_4, -\vec{P})$ in Euclidean space. The sign $\eta_P = \pm 1$ is obtained by examining how each basis covariant transforms under this operation. The covariants used in Eq. (12) are those appropriate for a pseudoscalar state and therefore yield negative parity, while the transverse covariants in Eq. (13) are those appropriate for a vector state and also yield negative parity. Hence the quantum numbers of the channels studied here are 0^{-+} for the pseudoscalar and 1^{--} for the vector state.

F. Decay constants

The leptonic decay constants are defined via

$$f_{PS} P_\mu = Z_2 \int_k^\Lambda \text{Tr}[\gamma_5 \gamma_\mu S(k_+) \Gamma_{PS}(k; P) S(k_-)], \quad (18)$$

for a pseudoscalar (PS) meson, and

$$f_V M_V = \frac{Z_2}{3} \int_k^\Lambda \text{Tr}[\gamma_\mu S(k_+) \Gamma_V^\mu(k; P) S(k_-)], \quad (19)$$

for a vector (V) meson.

These expressions are renormalization-point independent when the truncation is symmetry preserving.

III. RESULTS

A. Running mass and renormalization scheme

The experimentally quoted value of approximately 173 GeV corresponds to the top-quark pole mass [1]. In our calculation, however, we employ a short-distance mass defined in a renormalization scheme appropriate for continuum QCD approaches. Therefore, the starting point of our analysis is the top-quark mass in the modified minimal subtraction ($\overline{\text{MS}}$) scheme, taken from Ref. [45],

$$m_t^{\overline{\text{MS}}}(m_t) = 163.643 \text{ GeV}, \quad (20)$$

and determined from the top-quark pole mass, $\bar{m}_t = 173 \text{ GeV}$.

Since continuum DSE approaches are typically formulated in momentum-subtraction (MOM) schemes, it is convenient to relate the $\overline{\text{MS}}$ mass to the corresponding MOM value. At one loop in Landau gauge, the conversion reads [46,47]

$$m_t^{\text{MOM}} \simeq m_t^{\overline{\text{MS}}} \left(1 + \frac{4\alpha_s}{3\pi} \right), \quad (1\text{-loop, Landau gauge}), \quad (21)$$

with α_s the strong coupling constant. Note that, in the equation above, the masses and the coupling constant are evaluated at the same renormalization scale.

The MOM renormalization scheme is commonly employed in nonperturbative approaches to QCD, including lattice simulations and continuum functional methods such as DSEs. In this scheme, renormalization constants are defined by requiring that propagators and vertex functions assume their tree-level forms at a fixed spacelike momentum $p^2 = \mu^2$. Unlike the $\overline{\text{MS}}$ scheme, MOM renormalization conditions depend explicitly on the gauge choice; however, they possess the advantage that the renormalization scale μ is directly associated with the external momentum of the Green's functions, providing a more transparent physical interpretation.

The running of the $\overline{\text{MS}}$ mass is obtained from the renormalization-group equation [47]. Using the four-loop

TABLE I. Running of the top-quark mass and conversion from the $\overline{\text{MS}}$ to the MOM scheme for representative renormalization scales.

μ (GeV)	$\alpha_s(\mu)$	$m_t^{\overline{\text{MS}}}(\mu)$ (GeV)	$m_t^{\text{MOM}}(\mu)$ (GeV)
400	0.0976	153.308	159.660
500	0.0953	150.999	157.104
600	0.0934	149.184	155.098
700	0.0919	147.696	153.455
800	0.0906	146.440	152.071

anomalous dimension, the scale dependence can be written as

$$m_t^{\overline{\text{MS}}}(\mu^2) = \hat{m}_t \left(\frac{\alpha_s}{\pi} \right)^{4/7} \left[1 + 1.39796 \left(\frac{\alpha_s}{\pi} \right) + 1.79348 \left(\frac{\alpha_s}{\pi} \right)^2 - 0.683433 \left(\frac{\alpha_s}{\pi} \right)^3 \right], \quad (22)$$

where $\hat{m}_t$ denotes the renormalization-group invariant top-quark mass.

Table I summarizes the running of the top-quark mass and the conversion from the $\overline{\text{MS}}$ scheme to the momentum subtraction (MOM) scheme for representative renormalization scales in the range $\mu = 400$ – 800 GeV. As expected from the renormalization-group evolution of QCD, both the strong coupling $\alpha_s(\mu)$ and the $\overline{\text{MS}}$ mass decrease logarithmically as the renormalization scale increases. The subsequent conversion to the MOM scheme, performed at one-loop order in Landau gauge, leads to slightly larger values for the running mass, reflecting the scheme dependence inherent to perturbative renormalization prescriptions. Over the range of scales considered here the resulting MOM masses vary from approximately 160 GeV to 152 GeV, corresponding to a moderate change of only a few percent. This relatively mild scale dependence indicates that the input quark mass used in the subsequent DSE-BSE calculations is stable against variations of the ultraviolet renormalization point and provides a consistent starting point for the study of the $t\bar{t}$ bound-state system.

B. Charmonium and bottomonium

Before addressing the hypothetical $t\bar{t}$ bound system, it is necessary to validate the present framework in sectors where experimental information is available. Heavy quarkonia, in particular charmonium ($c\bar{c}$) and bottomonium ($b\bar{b}$), provide ideal testing grounds for this purpose. These systems are sufficiently heavy that relativistic corrections remain moderate, while still probing the nonperturbative dynamics of QCD through the quark-gluon interaction kernel. Consequently, they have long served as benchmarks for continuum approaches based on DSE-BSE framework.

For systems composed solely of heavy quarks, the Qin-Chang effective interaction was adjusted in Ref. [42], giving

$$\omega = 0.8 \text{ GeV}, \quad (\omega D)^3 = (0.6 \text{ GeV})^3, \quad (23)$$

where we remember that D and ω control the infrared strength and width of the quark-gluon interaction. The number of active flavors is set to $N_f = 5$ and thus $\gamma_m = 0.521739$ and $\Lambda_{\text{QCD}}^{\overline{\text{MS}}} = 0.21 \text{ GeV}$ [48]. Note here the relation between MOM and $\overline{\text{MS}}$ renormalization schemes in the definition of Λ_{QCD} [49],

$$\Lambda_{\text{QCD}}^{\text{MOM}} = \Lambda_{\text{QCD}}^{\overline{\text{MS}}} \cdot \exp\left(\frac{507 - 40N_f}{792 - 48N_f}\right), \quad (24)$$

which provides $\Lambda_{\text{QCD}}^{\text{MOM}} = 0.36 \text{ GeV}$.

Using renormalized current-quark masses,

$$m_c(\mu = 19 \text{ GeV}) = 0.820 \text{ GeV}, \quad (25)$$

$$m_b(\mu = 19 \text{ GeV}) = 3.590 \text{ GeV}, \quad (26)$$

the Euclidean constituent quark masses are

$$M_c = 1.322 \text{ GeV}, \quad (27)$$

$$M_b = 4.215 \text{ GeV}, \quad (28)$$

and our theoretical framework reproduces the experimental masses and leptonic decay constants of the pseudoscalar and vector S -wave quarkonia, including the η_c , J/ψ , η_b , and Υ states (see Table II).

The successful description of these heavy-quark systems establishes the reliability of the interaction kernel and parameter set in the heavy-mass regime. In particular, the calculated masses of the lowest pseudoscalar and vector charmonium states agree with the experimental values at the level of a few percent, with m_{η_c} and $m_{J/\psi}$ reproduced within approximately 1%–3%. A similarly good agreement is obtained in the bottomonium sector, where the predicted masses of the η_b and Υ states differ from the experimental values by less than about 1%–2%. The corresponding leptonic decay constants are also consistent with phenomenological determinations and lattice-QCD results within the typical uncertainties expected in the rainbow-ladder truncation. Such quantitative agreement across both charmonium and bottomonium systems indicates that the effective interaction captures the dominant nonperturbative dynamics governing heavy quarkonia. This provides a controlled and well-tested starting point for extending the same framework to the extreme heavy-quark limit represented by the hypothetical $t\bar{t}$ bound system considered in the following subsection.

TABLE II. Computed values of masses and leptonic decay constants of the pseudoscalar and vector S -wave quarkonia, including η_c , J/ψ , η_b , and Υ states. Experimental values (Expt.) are taken from Ref. [48]— $f_{\eta_c} = 0.238(12)$, $f_{J/\psi} = 0.294(5)$, $f_{\Upsilon} = 0.506(3)$; and lattice-QCD (Lat.) results are drawn from Refs. [50–53]— $f_{\eta_c} = 0.279(17)$, $f_{\eta_b} = 0.472(4)$, $f_{J/\psi} = 0.286(4)$, $f_{\Upsilon} = 0.459(22)$. All quantities in GeV.

	Mass			Decay constant		
	Theory	Experiment	Lattice QCD	Theory	Experiment	Lattice QCD
η_c	2.982	2.984	...	0.285	0.238	0.279
J/ψ	3.123	3.097	...	0.301	0.294	0.286
η_b	9.372	9.399	...	0.565	...	0.472
Υ	9.476	9.460	...	0.538	0.506	0.459

C. Toponium

We now extend the same framework to the $t\bar{t}$ system. In order to preserve consistency with previous studies of heavy mesons within the Qin-Chang interaction, in our first analysis we consider the same model parameters, including $N_f = 5$, corresponding to the standard situation in which the top quark does not contribute to the running of the QCD coupling below its mass threshold.

The dressed top-quark propagator is obtained from the gap equation using the running mass discussed in Sec. III A, and the BSE is solved in the pseudoscalar (0^{-+}) and vector (1^{--}) channels. Since the top mass lies far above the typical hadronic scale, it is useful to investigate the sensitivity of the bound-state solutions to the renormalization point. We therefore compute the toponium masses and leptonic decay constants for a range of renormalization scales $\mu = 400$ – 800 GeV, which are representative of the momentum region relevant for the heavy-quark dynamics entering the Bethe-Salpeter kernel. The resulting pseudoscalar and vector masses together with their leptonic decay constants are summarized in Table III.

Several qualitative features emerge from these results. First, the pseudoscalar and vector masses are nearly degenerate, with an average hyperfine splitting of 0.171 GeV, reflecting the expected heavy-quark spin symmetry in the extreme heavy-mass limit. Second, the leptonic decay constants are large, of order $f_{t\bar{t}} \sim 7$ GeV, consistent with the increasingly compact spatial structure of heavy quarkonia. Finally, the dependence on the renormalization scale is mild, indicating that the predicted properties of the $t\bar{t}$

system are relatively stable within the range of scales considered.

Since the characteristic momentum scales relevant for the $t\bar{t}$ system lie well above the top mass, it is also instructive to investigate the case in which the top quark is treated as an active flavor in the renormalization group evolution.

The change in the number of active flavors modifies the ultraviolet behavior of the interaction kernel through the quark-mass anomalous dimension and the QCD scale parameter. Following the relation between the MOM and $\overline{\text{MS}}$ schemes, Eq. (24), the value,

$$\Lambda_{\text{MOM}}^{(6)} \simeq 0.151 \text{ GeV}, \quad (29)$$

is obtained from the PDG value $\Lambda_{\overline{\text{MS}}}^{(6)} = 0.089$ GeV.

When implementing $N_f = 6$ within the Qin-Chang interaction, we retain the same infrared interaction parameters used in the previous subsection,

$$\omega = 0.8 \text{ GeV}, \quad \omega D = (0.6 \text{ GeV})^3, \quad (30)$$

which ensures that the long-range structure of the kernel remains unchanged. However, the change in the ultraviolet running implies that the current heavy-quark masses must be slightly adjusted in order to reproduce the observed charmonium and bottomonium properties. Following this procedure we obtain

$$m_c(\mu = 19 \text{ GeV}) = 0.873 \text{ GeV}, \quad (31)$$

$$m_b(\mu = 19 \text{ GeV}) = 3.722 \text{ GeV}, \quad (32)$$

which yield Euclidean constituent masses,

$$M_c = 1.319 \text{ GeV}, \quad (33)$$

$$M_b = 4.253 \text{ GeV}. \quad (34)$$

With these parameters the calculated masses and leptonic decay constants of the η_c , J/ψ , η_b , and Υ states are shown

TABLE III. Comparison of calculated toponium masses and leptonic decay constants with different renormalization points. All quantities listed in GeV.

μ	M_t	$m_{t\bar{t}}^{PS}$	$f_{t\bar{t}}^{PS}$	$m_{t\bar{t}}^V$	$f_{t\bar{t}}^V$
400	163.803	344.537	7.080	344.708	6.674
500	164.221	345.411	7.094	345.582	6.687
600	164.083	345.154	7.120	345.326	6.714
700	164.201	345.368	7.093	345.538	6.687
800	164.485	345.992	7.134	346.164	6.728

TABLE IV. With $N_f = 6$ and thus fine-tuning the Qin-Chang effective interaction, computed values of masses and leptonic decay constants of the pseudoscalar and vector S -wave quarkonia, including η_c , J/ψ , η_b , and Υ states. Experimental values (Expt.) are taken from Ref. [48]— $f_{\eta_c} = 0.238(12)$, $f_{J/\psi} = 0.294(5)$, $f_{\Upsilon} = 0.506(3)$; and lattice-QCD (Lat.) results are drawn from Refs. [50–53]— $f_{\eta_c} = 0.279(17)$, $f_{\eta_b} = 0.472(4)$, $f_{J/\psi} = 0.286(4)$, $f_{\Upsilon} = 0.459(22)$. All quantities in GeV.

	Mass			Decay constant		
	Theory	Experiment	Lattice QCD	Theory	Experiment	Lattice QCD
η_c	2.978	2.984	...	0.260	0.238	0.279
J/ψ	3.087	3.097	...	0.273	0.294	0.286
η_b	9.399	9.399	...	0.488	...	0.472
Υ	9.463	9.460	...	0.466	0.506	0.459

TABLE V. With $N_f = 6$ and thus fine-tuning the Qin-Chang effective interaction, comparison of calculated toponium masses and leptonic decay constants with different renormalization points. All quantities listed in GeV.

μ	M_t	$m_{t\bar{t}}^{PS}$	$f_{t\bar{t}}^{PS}$	$m_{t\bar{t}}^V$	$f_{t\bar{t}}^V$
400	163.634	343.432	6.495	343.567	6.123
500	163.968	344.119	6.504	344.255	6.132
600	163.781	343.743	6.491	343.878	6.152
700	163.857	343.879	6.493	344.014	6.122
800	164.106	344.415	6.510	344.551	6.137

in Table IV and, as one can notice, they remain in good agreement with experimental and lattice results, confirming that the interaction kernel continues to provide a reliable description of heavy quarkonia.

Using this updated parameter set we then solve the BSE for the $t\bar{t}$ system over the same range of renormalization scales. The resulting pseudoscalar and vector toponium masses and decay constants are presented in Table V. The pseudoscalar and vector masses are slightly smaller in this case, with an average hyperfine splitting of 0.135 GeV. The predicted leptonic decay constants remain large, $f_{t\bar{t}} \sim 6$ –6.5 GeV, although they are slightly smaller than the corresponding $N_f = 5$ values due to the modified ultraviolet running of the interaction. Finally, the dependence on the renormalization scale with $N_f = 6$ is even milder than the $N_f = 5$ case.

Quantitatively, the inclusion of the top quark as an active flavor leads to a modest reduction in the predicted decay constants, $\sim(8\text{--}10)\%$, and a slight modification of the bound-state masses, ~ 1 GeV. This behavior can be traced to the weaker effective interaction arising from the modified renormalization-group running when $N_f = 6$, which reduces the binding strength in the Bethe-Salpeter kernel. Nevertheless, the overall structure of the spectrum remains

remarkably stable under this change, indicating that our results are largely insensitive to the precise choice of active flavor number in the ultraviolet evolution, although quantitative differences at the level of a few percent may arise in the predicted masses and decay constants.

Our results can be compared with several existing studies of the $t\bar{t}$ system based on different theoretical approaches. Calculations using QCD sum rules [9], relativistic potential models [10] and nonrelativistic quark models [54,55] typically predict pseudoscalar and vector toponium masses in the range $m_{t\bar{t}} \sim 341$ –344 GeV, with very small hyperfine splittings. Our values, $m_{t\bar{t}} \sim 344$ –346 GeV, are in good agreement with these results, differing at the level of $\lesssim 1\%$. For the decay constants, fewer results are available; however, existing estimates, such as $f_{t\bar{t}}^V \approx 7.7$ GeV obtained in potential models, are consistent with our prediction $f_{t\bar{t}} \sim 6$ –7 GeV within expected theoretical uncertainties. Since the dilepton decay width of a vector meson is proportional to f_V^2/M , our results are also compatible with existing estimates of vector toponium dilepton widths. Overall, the agreement with these different approaches supports the robustness of our predictions, while our framework provides a fully relativistic and Poincaré-covariant description of the $t\bar{t}$ system.

IV. SUMMARY AND OUTLOOK

In this work we have investigated the properties of the lowest-lying pseudoscalar (0^{-+}) and vector (1^{--}) $t\bar{t}$ systems within a fully Poincaré-covariant Dyson-Schwinger-Bethe-Salpeter (DSE-BSE) framework. The analysis employs the rainbow-ladder truncation together with the Qin-Chang effective interaction, an approach that has previously demonstrated quantitative success in describing meson properties from the light-quark sector up to heavy quarkonia.

As a necessary validation step, we first confirmed that the interaction kernel and parameter set reproduce the masses and leptonic decay constants of the charmonium and bottomonium ground states with good accuracy. This provides confidence that the same framework can be consistently extrapolated to the extreme heavy-quark regime represented by the top quark.

Using the running top-quark mass defined in the modified minimal subtraction $\overline{\text{MS}}$ scheme and converted to the momentum-subtraction (MOM) scheme appropriate for continuum functional methods, we solved the quark DSE and the homogeneous BSE for the $t\bar{t}$ system. The analysis was performed for renormalization scales in the range $\mu = 400$ –800 GeV and for two different choices of active flavor number, $N_f = 5$ and $N_f = 6$, allowing us to quantify the sensitivity of the results to ultraviolet renormalization-group evolution.

Our calculations predict pseudoscalar and vector toponium masses in the range,

$$m_{\tilde{t}\tilde{t}} \sim 344\text{--}346 \text{ GeV}, \quad (35)$$

with a very small hyperfine splitting of order,

$$\Delta m_{\text{HF}} \sim 0.14\text{--}0.17 \text{ GeV}, \quad (36)$$

reflecting the expected emergence of heavy-quark spin symmetry in the extreme heavy-mass limit. The corresponding leptonic decay constants are found to be large,

$$f_{\tilde{t}\tilde{t}} \sim 6 - 7 \text{ GeV}, \quad (37)$$

consistent with the increasingly compact spatial structure of heavy quarkonia as the quark mass grows.

A comparison between the $N_f = 5$ and $N_f = 6$ scenarios reveals that increasing the number of active flavors slightly weakens the effective interaction through the modified renormalization-group running. This leads to a modest reduction in both the predicted masses and decay constants, at the level of a few percent, but the overall structure of the spectrum remains remarkably stable. Likewise, the dependence on the renormalization point in the range considered is mild, indicating that the predicted properties of the $\tilde{t}\tilde{t}$ system are robust against variations of the ultraviolet scale.

Although the physical top quark decays weakly with a lifetime shorter than the typical hadronization time, the present results demonstrate that the underlying QCD dynamics still supports a strongly correlated quark-antiquark configuration in the pseudoscalar and vector channels. In this sense, the $\tilde{t}\tilde{t}$ system may be interpreted as representing the ultimate heavy-quark limit of quarkonium binding within the Standard Model. Such correlations

could manifest themselves experimentally through threshold enhancements or quasi-bound structures in top-pair production processes, as suggested by recent analyses of LHC data.

Several extensions of the present work would further refine this picture. A first important improvement would consist in incorporating the finite electroweak width of the top quark, which would require treating the quark propagator with a complex pole structure and solving the Bethe-Salpeter equation in the presence of unstable constituents. In addition, corrections beyond the rainbow-ladder truncation, such as dressed quark-gluon vertices and crossed-ladder contributions, may become relevant for achieving higher quantitative precision. Finally, the present study focuses on the ground-state pseudoscalar and vector channels; extending the analysis to excited states and other quantum numbers could provide a more complete description of the possible spectrum of correlated $\tilde{t}\tilde{t}$ configurations.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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