

Convergence in charmonium structure: Light-front wave functions from basis light-front quantization and Dyson-Schwinger equations

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We present a systematic comparison of charmonium light-front wave functions obtained through two complementary nonperturbative approaches: basis light-front quantization (BLFQ) and Dyson-Schwinger equations (DSEs). Key observables include the charge form factor, gravitational form factors, light-cone distribution amplitudes, decay constants, and two-photon transition form factors. Despite their distinct theoretical foundations and model parameters, the predictions from BLFQ and DSEs exhibit remarkable agreement across all observables. This convergence validates both frameworks for studying charmonium structure and highlights the complementary strengths of Hamiltonian-based (BLFQ) and Lagrangian-based (DSEs) methods in addressing nonperturbative QCD.

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The charmonium system, bound states of charm and anticharm quarks, serves as a critical laboratory for exploring nonperturbative QCD [1]. It is relatively simple compared to light hadrons—owing to the heavy quark mass, making it an ideal test bed for theoretical frameworks that bridge perturbative and nonperturbative physics.

In experimental measurements, charmonia are precise probes for extracting information from the system due to their distinct signals [2]. For instance, the vector charmonium J/ψ can be used to extract the generalized parton distribution of the nucleon in deeply virtual meson production and the gluon distribution in diffractive vector meson production [3,4]. The P-wave charmonia χ_{c0} and χ_{c2} are sensitive to pomeron and odderon exchanges in ep collisions and in ultraperipheral heavy-ion collisions [5]. Additionally, the evolution of charmonia in the quark-gluon plasma serves as a thermometer for hot quark matter [6]. All of these applications hinge on a comprehensive understanding of charmonium structure, particularly in the high-energy limit,

i.e., on the light front ($ct + z = 0$). Such information is fully encoded within the light-front wave functions (LFWFs), which describe hadron structures in terms of partonic degrees of freedom, enabling direct computation of observables such as form factors, distribution amplitudes, and parton distribution functions [7].

In recent years, there has been intense interest to access the LFWFs directly from QCD [8]. Two nonperturbative approaches—basis light-front quantization (BLFQ) and Dyson-Schwinger equations (DSEs)—have emerged as powerful tools for investigating charmonium [8]. BLFQ diagonalizes the light-front QCD Hamiltonian in a symmetry-preserving basis, providing direct access to hadron spectra and wave functions [9,10]. BLFQ with effective interactions inspired by holographic QCD has been used to investigate a variety of systems, such as heavy quarkonia [11], heavy-light mesons [12,13], light mesons [14–16], nucleons [17–19], and tetraquarks [20], and to investigate a number of observables including mass spectra [21], form factors [17,22–24], radiative transitions [25–28], parton distributions [18,29–38], spin structures [39,40], and transverse momentum distributions [41–43].

In contrast, DSEs solve QCD's Green's functions in the continuum, emphasizing symmetry and Lorentz covariance [44–46]. The Maris-Tandy (MT) model combined with rainbow-ladder (RL) truncation exposes the key role of the

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dynamical chiral symmetry breaking (D χ SB) in describing the structure of the pion as a Goldstone boson [47]. This approach, including the subsequent improvements of the MT model [48], has been successfully applied to investigate the meson spectrum, baryon spectrum, decay constants, electromagnetic form factors, axial form factors, gravitational form factors, and radiative widths [47,49–51]. See Ref. [51] for a recent review.

Recently, it was shown that $k_\perp$ -dependent moments can be used to project the covariant Bethe-Salpeter amplitudes (BSA) to the light cone [52,53]. LFWFs extracted from this method have been used to investigate a number of partonic observables, including parton distribution amplitudes and one-dimensional and three-dimensional parton distribution functions [54–59].

Since both methods have independently reproduced key charmonium properties, despite their conceptual differences, there is a strong motivation to scrutinize the underlying LFWFs—the fundamental entities encoding hadron structure—obtained from these two approaches through common observables. Similar comparisons were performed in the literature between various approaches and for various systems, e.g., Refs. [38,60]. In this work, we address this question by comparing BLFQ and DSE predictions across five sets of structural probes: (i) charge form factor, (ii) gravitational form factors, (iii) light-cone distribution amplitudes, (iv) decay constants, and (v) two-photon transition form factors. The same LFWFs—obtained from BLFQ and DSEs respectively—are used for each set of observables, to avoid additional phenomenological modifications. The observed agreement, despite differing model parameters (e.g., regulator scales, interaction kernels), highlights the universality of QCD-driven charmonium properties and strengthens confidence in both approaches for future studies of more challenging systems and more complicated observables.

The remainder of this article is structured as follows: Sec. I briefly reviews the theoretical framework and properties of the light-front wave functions. Section II presents the hadronic form factors. Section III compares the light-cone distribution amplitudes, and the two-photon transition form factors closely related to the light-cone distribution amplitude (LCDA). Finally, we summarize in Sec. IV.

I. LIGHT-FRONT WAVE FUNCTIONS

The LFWFs are the amplitudes of the hadronic state vector in Fock space at fixed light-front time x^+ [61]. Specifically, the charmonium hadronic state vector on the light front can be written as

$$|\psi_h(p, j, \sigma)\rangle = \sum_{s, \bar{s}} \int_0^1 \frac{dx}{2x(1-x)} \int \frac{d^2 k_\perp}{(2\pi)^3} \psi_{s\bar{s}/h}^\sigma(x, \vec{k}_\perp) \times \frac{1}{\sqrt{N_c}} \sum_i b_{si}^\dagger(p_1) d_{\bar{s}i}^\dagger(p_2) |0\rangle + \dots \quad (1)$$

where p is the 4-momentum of the particle, and j and σ are the particle's total angular momentum and magnetic projection, respectively. The coefficients $\psi_{s\bar{s}/h}^\sigma(x, \vec{k}_\perp)$ are the valence sector LFWFs, with $s(\bar{s})$ representing the spin of the quark (antiquark). $x = p_1^+/p^+$ is the longitudinal momentum fraction, $\vec{k}_\perp = \vec{p}_{1\perp} - x\vec{p}_\perp$ is the relative transverse momentum and $p_1^2 = p_2^2 = m_q^2$. The ellipsis represents the nonvalence Fock contributions. The quark and antiquark creation operators satisfy the anticommutation relations, as follows:

$$\{b_{si}(p), b_{s'i'}^\dagger(p')\} = \{d_{si}(p), d_{s'i'}^\dagger(p')\} = 2p^+(2\pi)^3 \delta^{(3)}(p - p') \delta_{ss'} \delta_{ii'}. \quad (2)$$

Here, i denotes the color charge of quark and antiquark, and the total color number is $N_c = 3$.

In principle, the hadronic state vector, hence the LFWFs, can be directly obtained from diagonalizing the light-cone Hamiltonian operator $H_{LC} = P^\mu P_\mu \equiv P^+ P^- - \vec{P}_\perp^2$ [61], as follows:

$$H_{LC} |\psi_h(p)\rangle = M_h^2 |\psi_h(p)\rangle. \quad (3)$$

This is the starting point of BLFQ. On the other hand, from Eq. (1) the LFWFs can also be written as the hadronic amplitude of the appropriate bilocal operators [62]. For example, the spin-flip LFWF of a pseudoscalar meson P can be expressed as

$$\psi_{\uparrow\downarrow-\downarrow\uparrow/P}(x, \vec{k}_\perp) = \frac{1}{2p^+} \int d^3 z e^{\frac{i}{2} x p \cdot z - i \vec{k}_\perp \cdot \vec{z}_\perp} \times \langle 0 | \bar{\psi}(0) \gamma^+ \gamma_5 \psi(z) | P(p) \rangle_{z^+=0}, \quad (4)$$

where z is the four-vector in Minkowski space. In this way, the LFWFs can also be obtained from the covariant Bethe-Salpeter amplitudes.

In both the BLFQ and the DSE approaches, the symmetries of individual LFWFs are fully retained. For example, the pseudoscalar LFWF consists of two independent structures [63],

$$\psi_{s\bar{s}/P}(x, \vec{k}_\perp) = \bar{v}_{\bar{s}}(p_2) \left[\gamma^5 \phi_1(x, \vec{k}_\perp^2) + \frac{\gamma^+ \gamma^5}{p^+} \phi_2(x, \vec{k}_\perp^2) \right] u_s(p_1). \quad (5)$$

The second term appears because the LFWFs in general depend on the orientation of light front ω , which is a null vector ($\omega^2 = 0$, and $\omega \cdot \gamma = \gamma^+$). In some phenomenological applications, the second term is dropped [64–70]. However, we have shown that this term plays an important role in chiral symmetry breaking [71]. In both BLFQ and DSEs, the maximal kinematical symmetries of the LFWFs are retained and both approaches produce two independent

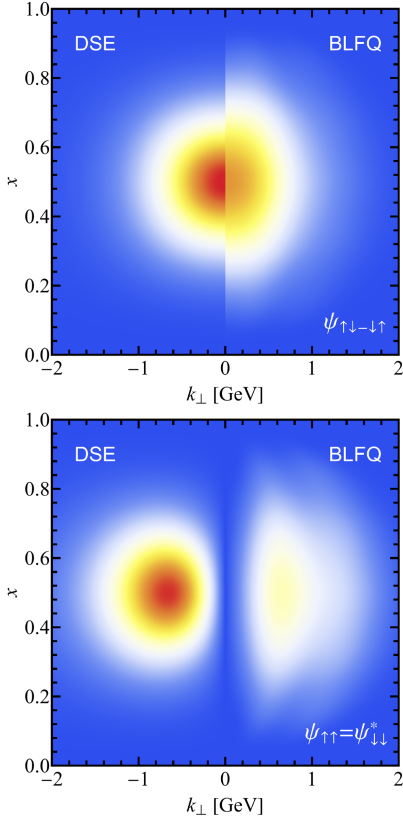


FIG. 1. Comparison of the light-front wave functions for the ground-state charmonium η_c computed from BLFQ and from DSEs. The negative $k_\perp$ region shows the DSE results. The positive $k_\perp$ region shows the BLFQ results with $N_{\max} = 8$. The spin singlet component of the LFWF is defined as $\psi_{\uparrow\downarrow-\downarrow\uparrow} = (\psi_{\uparrow\downarrow} - \psi_{\downarrow\uparrow})/\sqrt{2}$.

structures. Figure 1 compares the valence LFWFs of η_c as obtained from BLFQ and from DSEs. The LFWFs of BLFQ and DSEs are accessible from Refs. [72] and [73], respectively. For both spin-singlet $\psi_{\uparrow\downarrow-\downarrow\uparrow}$ and spin-triplet $\psi_{\uparrow\uparrow}/\psi_{\downarrow\downarrow}$ components, the LFWFs from BLFQ are broader than DSEs in the transverse momentum ($k_\perp$) direction and longitudinal (x) direction. However, the normalizations of each of the spin configurations are qualitatively similar in these two frameworks. The normalization for each spin configuration is defined as

$$I_{s\bar{s}} = \int_0^1 \frac{dx}{2x(1-x)} \int \frac{d^2k_\perp}{(2\pi)^3} |\psi_{s\bar{s}/h}^\sigma(x, \vec{k}_\perp)|^2. \quad (6)$$

For BLFQ, we find $I_{\uparrow\uparrow} + I_{\downarrow\downarrow} = 0.074$ and $I_{\uparrow\downarrow-\downarrow\uparrow} = 0.926$, whereas the DSE LFWFs give $I_{\uparrow\uparrow} + I_{\downarrow\downarrow} = 0.140$ and $I_{\uparrow\downarrow-\downarrow\uparrow} = 0.860$. In both frameworks, the spin-singlet component is dominant.

A comment is in order here. The valence Fock sector LFWFs projected from the DSEs are not automatically normalized to 1 due to contributions from high Fock states in the BSA. However, for heavy quarkonia studied in this

paper, the valence Fock sector contributes more than 90% of the total normalization [55]. To be consistent with the BLFQ approach, we have normalized the DSE LFWFs to 1. All subsequent calculations will use these normalized LFWFs.

In practical calculations, the LFWFs within the BLFQ are expanded in a basis of two-dimensional harmonic oscillator functions in the transverse directions and Jacobi functions in the longitudinal momentum fraction x [21]. To render the basis finite for numerical implementation, the quantum numbers are truncated according to $2n + |m| + 1 \leq N_{\max}$ and $0 \leq l \leq L_{\max}$, where n, m denote the radial and angular quantum numbers of the two-dimensional harmonic oscillator modes and l labels the order of the Jacobi functions. This $N_{\max}$ -truncation naturally introduces both UV and infrared IR cutoffs: $\Lambda_{\text{UV}} = b\sqrt{N_{\max}}$, $\lambda_{\text{IR}} = b/\sqrt{N_{\max}}$, where b is the oscillator basis scale. Meanwhile, $L_{\max}$ controls the resolution in the longitudinal momentum fraction $\Delta x \approx L_{\max}^{-1}$. In contrast, the DSE approach adopts an explicit UV cutoff Λ on the Euclidean momentum k_E of the Green's functions. In practice, Λ is chosen sufficiently large to ensure convergence while minimizing regulator artifacts [55].

II. FORM FACTORS AND HADRONIC DENSITIES

Hadronic form factors are defined from the hadronic matrix elements of local current operators. In LFWF representation, they are related to the Fourier transform of the one-body hadronic densities. In this section, we focus on the charge form factor and the gravitational form factor.

The charge form factor $F(Q^2)$ is defined from the hadronic matrix elements of the electromagnetic vector current $J^\mu(x)$, as follows:

$$\langle \psi(p') | J^\mu(0) | \psi(p) \rangle = 2P^\mu F(Q^2), \quad (7)$$

where, $P = (p' + p)/2$, $q = p' - p$ and $Q^2 = -q^2$. In light-front dynamics, $F(Q^2)$ can be interpreted as the Fourier transform of the transverse charge density [74]:

$$\rho(r_\perp) = \int \frac{d^2q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{r}_\perp} F(q_\perp^2). \quad (8)$$

Unlike the Breit frame density, the above charge distribution is frame independent and is a genuine distribution [75,76]. The Drell-Yan-West formula expresses the transverse charge density as a one-body density (OBD) using LFWFs [77–79], as follows:

$$\rho(r_\perp) = \left\langle \sum_i e_i \delta^2(r_\perp - r_{i\perp}) \right\rangle, \quad (9)$$

where, the $\langle \cdots \rangle$ represents the quantum average:

$$\begin{aligned} \langle O \rangle &= \sum_n \frac{1}{S_n} \sum_{s_i} \prod_i^n \int \frac{dx_i}{4\pi x_i} \int d^2 r_{i\perp} 4\pi \delta\left(\sum_j x_j - 1\right) \\ &\times \delta^2\left(\sum_j x_j \vec{r}_{j\perp}\right) |\tilde{\psi}_n(\{x_i, \vec{r}_{i\perp}, s_i\})|^2 O(\{x_i, \vec{r}_{i\perp}, s_i\}). \end{aligned} \quad (10)$$

Here, S_n is the symmetry factor. $\psi_n(\{x_i, \vec{r}_{i\perp}, s_i\})$ is the transverse coordinate space wave function, defined as

$$\begin{aligned} \tilde{\psi}_n(\{x_i, \vec{r}_{i\perp}, s_i\}) &= \prod_{i=1}^n \int \frac{d^2 k_{i\perp}}{(2\pi)^2} (2\pi)^2 \delta^2\left(\sum_i k_{i\perp}\right) \\ &\times e^{-i\sum_i \vec{k}_{i\perp} \cdot \vec{r}_{i\perp}} \psi_n(\{x_i, \vec{k}_{i\perp}, s_i\}). \end{aligned} \quad (11)$$

For charmonium, the physical charge form factor vanishes due to the charge conjugation symmetry. Nevertheless, it is custom to define a fictitious charge form factor, where the photon couples to the quark and antiquark differently, similar to the charged pion $\pi^\pm$ and B_c , while the quark masses remain unchanged $m_q = m_{\bar{q}} = m_c$ [80]. Figure 2 compares the charge form factor computed using LFWFs from BLFQ and from DSEs. Following our previous analyses for dilepton, diphoton, and radiative transitions, hereafter we adopt $N_{\max} = 8$ LFWFs for central values of BLFQ results, which corresponds to a UV scale $\Lambda_{UV} = 2.8 \text{ GeV} \approx M_{c\bar{c}}$, and quote the difference between $N_{\max} = 8$ and $N_{\max} = 16$ as the basis sensitivity [21,27,28], where $N_{\max}$ denotes the basis truncation parameter. This choice is motivated by the competition between the needs for both a better basis resolution and a lower UV scale to suppress the radiative corrections. The primary source of uncertainty in the DSEs results stems from the RL approximation, which is difficult

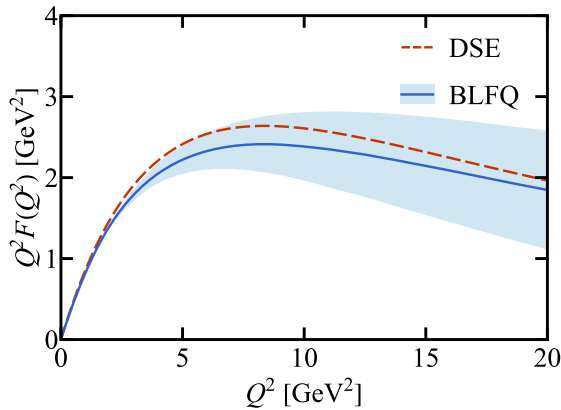


FIG. 2. Comparison of the (fictitious) charge form factor $F(Q^2)$ for the ground-state charmonium η_c computed using LFWFs from BLFQ and from DSEs. The dashed line is the DSEs result. The solid line is the BLFQ result with $N_{\max} = 8$. And the band represents the uncertainty associated with the basis sensitivity in BLFQ as computed from the difference between $N_{\max} = 8$ and $N_{\max} = 16$ results. See the text for more details.

to estimate. Consequently, only the central values are presented here and in the subsequent calculations. As shown in Fig. 2, the BLFQ and DSEs results are in good agreement with each other including at high Q^2 ($Q^2 \gtrsim 10 \text{ GeV}^2$).

The gravitational form factors are defined via the hadronic matrix elements of the energy-momentum tensor $T^{\mu\nu}(x)$, as follows:

$$\begin{aligned} \langle \psi(p') | T^{\mu\nu}(0) | \psi(p) \rangle &= 2P^\mu P^\nu A(Q^2) \\ &+ \frac{1}{2} (q^\mu q^\nu - g^{\mu\nu} q^2) D(Q^2). \end{aligned} \quad (12)$$

In light-front dynamics, A and D are related to the Fourier transform of the transverse flow density $\mathcal{A}(r_\perp)$, and transverse shear density $\mathcal{T}^{12}(r_\perp) = \mathcal{S}(r_\perp)/(2P^+)$, as follows:

$$\mathcal{A}(r_\perp) = \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{r}_\perp} A(q_\perp^2), \quad (13)$$

$$\mathcal{S}(r_\perp) = \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{r}_\perp} \frac{1}{2} q_\perp^1 q_\perp^2 D(q_\perp^2). \quad (14)$$

The Brodsky-Hwang-Ma-Schmidt formula gives the transverse flow density as an OBD using LFWFs [81], as follows:

$$\mathcal{A}(r_\perp) = \left\langle \sum_i x_i \delta^2(r_\perp - r_{i\perp}) \right\rangle. \quad (15)$$

Recently, some of us provided the LFWF representation of the transverse shear density as an OBD [82–84], as follows:

$$\mathcal{S}(r_\perp) = \left\langle \sum_i \frac{i\vec{\nabla}_{i\perp}^1 i\vec{\nabla}_{i\perp}^2 - i\nabla_{i\perp}^1 i\nabla_{i\perp}^2}{2x_i} \delta^2(r_\perp - r_{i\perp}) \right\rangle, \quad (16)$$

where $\vec{\nabla}_\perp$ only acts on the suppressed wave functions and $f\vec{\nabla}g = f\nabla g - (\nabla f)g$.

Figure 3 compares the gravitational form factors $A(Q^2)$ and $D(Q^2)$ computed using LFWFs from BLFQ and from DSEs. As one can see, the BLFQ and DSEs results are in good agreement. The D -term, defined as $D \equiv D(0)$, is known to be sensitive to the interaction. In Ref. [85], the authors argue that the D -term of the pseudoscalar meson should lie within $(-1, -1/3)$, which are the values in the chiral limit and the infinite mass limit respectively. Using DSEs with a contact interaction, they obtain $D = -0.58$ for η_c . In contrast, the values extracted from BLFQ and from DSE LFWFs in this work are $D_{\text{BLFQ}} = -4.6$ and $D_{\text{DSE}} = -4.1$.

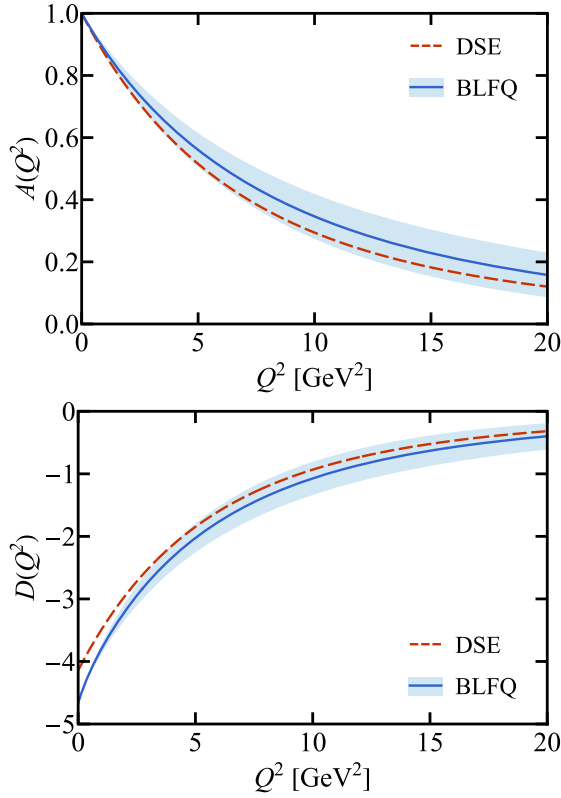


FIG. 3. Comparison of the gravitational form factors $A(Q^2)$ and $D(Q^2)$ for the ground-state charmonium η_c computed using LFWFs from BLFQ ($N_{\max} = 8$, solid lines) and from DSEs (dashed lines). The uncertainty bands are computed from the difference between $N_{\max} = 8$ and $N_{\max} = 16$ results in BLFQ. See the text for more details.

III. LIGHT-CONE DISTRIBUTION AMPLITUDES AND RADIATIVE TRANSITIONS

While the form factors probe the transverse structures of the system, the LCDAs describe its longitudinal structure. LCDAs are defined as the bilocal lightlike vacuum-to-hadron transition amplitudes. For pseudoscalar P , the leading-twist LCDA $\phi_P(x, \mu)$ is

$$\begin{aligned} & \langle 0 | \bar{\psi} \left(-\frac{1}{2} z \right) \gamma^+ \gamma_5 \psi \left(+\frac{1}{2} z \right) | P(p) \rangle_\mu \\ &= i f_P p^+ \int_0^1 dx e^{\frac{i}{2} x p^+ z^-} \phi_P(x, \mu), \end{aligned} \quad (17)$$

where f_P is the decay constant, and z is the same as in Eq. (4). Using LFWFs, the above LCDA can be expressed as

$$\begin{aligned} & \frac{f_P}{2\sqrt{2N_c}} \phi_P(x, \mu) \\ &= \frac{1}{2\sqrt{x(1-x)}} \int \frac{d^2 k_\perp}{(2\pi)^3} \psi_{\uparrow\downarrow-\downarrow\uparrow/P}(x, \vec{k}_\perp). \end{aligned} \quad (18)$$

Figure 4 compares the leading-twist LCDA of η_c as obtained from BLFQ and from DSEs and evolved to $\mu = 2.8$ GeV. The DSE result appears narrower than the BLFQ result. However, both are broad enough to distinguish from the nonrelativistic limit. The decay constant can be derived after integrating over x in Eq. (18), as follows:

$$\frac{f_P}{2\sqrt{2N_c}} = \int_0^1 \frac{dx}{2\sqrt{x(1-x)}} \int \frac{d^2 k_\perp}{(2\pi)^3} \psi_{\uparrow\downarrow-\downarrow\uparrow}(x, \vec{k}_\perp). \quad (19)$$

The BLFQ LFWFs of η_c yield a decay constant $f_P = 0.414(78)$ GeV while the DSE LFWFs give $f_P = 0.396$ GeV. Both results are close to the lattice QCD result $f_P = 0.395(2)$ GeV [86].

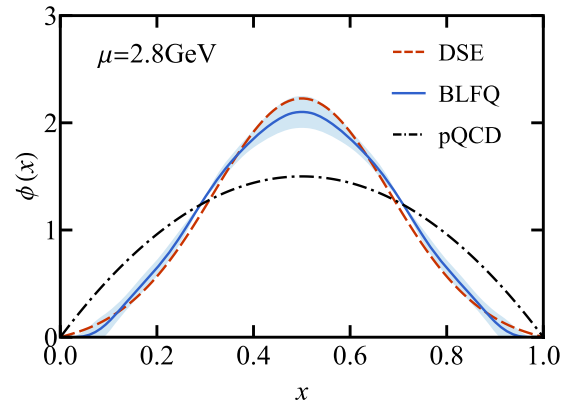


FIG. 4. Comparison of the leading-twist LCDA of η_c as obtained from BLFQ ($N_{\max} = 8$, solid line) and from DSEs (dashed line). The uncertainty bands are computed from the difference between $N_{\max} = 8$ and $N_{\max} = 16$ results in BLFQ. The dot-dashed line corresponds to the perturbative QCD (pQCD) asymptotic form $6x(1-x)$ [7]. The DSE results are evolved from $\mu_0 = 2.6$ GeV to $\mu = 2.8$ GeV to match the BLFQ energy scale.

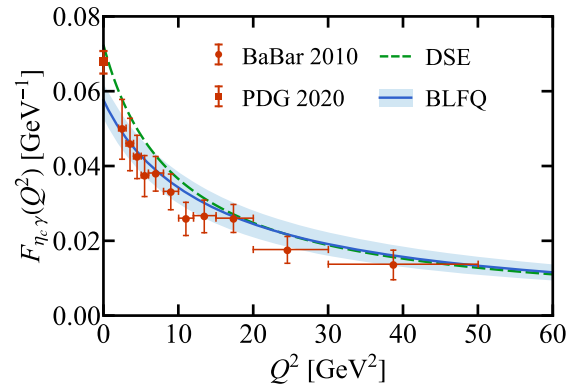


FIG. 5. Comparison of the transition form factor of η_c as obtained from BLFQ ($N_{\max} = 8$, solid line) and from DSEs (dashed line). The uncertainty bands are computed from the difference between $N_{\max} = 8$ and $N_{\max} = 16$ results in BLFQ.

The LCDA is an essential tool for describing exclusive processes in high-energy scattering $Q^2 \gg \Lambda_{\text{QCD}}^2$. One of the observables that can be described by LCDA is the two-photon transition form factor (TFF), as shown by Lepage and Brodsky [7]. On the other hand, the evaluation of the TFF can be extended to low Q^2 via LFWFs [7,27,87]. Figure 5 compares the evaluation of the TFF $F_{\eta_c\gamma}(Q^2)$ for the process $\gamma^*\gamma \rightarrow \eta_c$ using LFWFs obtained from BLFQ and from DSEs. The theoretical results, without tuning of parameters, are in good agreement with *BABAR* measurements.

IV. SUMMARY AND CONCLUSIONS

In this work, we have presented a comprehensive comparison of charmonium LFWFs computed using BLFQ and DSEs approaches, evaluating five key observables that probe different aspects of hadron structure. The remarkable agreement between the two approaches, despite their independent methodologies and model parameters, underscores the robustness of their respective nonperturbative QCD predictions for heavy quarkonia. This work not only validates the BLFQ and DSEs frameworks but also provides a foundation for future studies of more complex systems and processes in hadronic physics.

This work also validates the reliability of the BSA projection method in heavy quarkonia. The observed agreement demonstrates that the leading Fock sector LFWFs projected from BSA are sufficiently accurate to

describe the hadron structure. Actually, direct calculation of the normalization indicates that the valence Fock sector is dominant for heavy quarkonia. However, for light mesons like the pion, higher Fock states are more important.

This study demonstrates shared features and predictive power within Hamiltonian-based and Lagrangian-based approaches toward LFWFs, providing a unified perspective on charmonium structure. The agreement also corroborates the reliability of these frameworks for future investigations of more complex systems, such as exotic hadrons, heavy-light mesons, and even nuclei. Furthermore, the consistency between BLFQ and DSEs predictions strengthens the reliability of both nonperturbative QCD methods for interpreting experimental data from facilities like Jefferson Lab, the LHC, and the future electron-ion colliders.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [72,73].

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