

Addendum to “Resolved $1/m_b$ contributions to $\bar{B} \rightarrow X_{s,d} \ell^+ \ell^-$ and $\bar{B} \rightarrow X_s \gamma$ ”

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In all previous calculations of the nonlocal subleading contribution to the inclusive penguin decay $\bar{B} \rightarrow X_s \gamma$ due to the interference of the electroweak operators $\mathcal{O}_1^c - \mathcal{O}_{7\gamma}$ the local Voloshin term was subtracted. In view of the ongoing analysis at order α_s , we present a calculation of the complete nonlocal contribution which takes into account the high correlation between the uncertainties of the local Voloshin and the nonlocal term of the previous analyses. The new calculation has a high impact on the range of the nonlocal contribution.

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I. STRATEGY OF THE CALCULATION

In Ref. [1] we presented a systematic analysis of the dominant resolved contribution due to the interference $\mathcal{O}_1^c - \mathcal{O}_{7\gamma}$ to the inclusive penguin decays $\bar{B} \rightarrow X_s \gamma$ following the method proposed in Ref. [2], based on previous analyses of these contributions in both inclusive penguin modes, $\bar{B} \rightarrow X_s \gamma$ and $\bar{B} \rightarrow X_s \ell \ell$, to leading order in the strong coupling constant α_s [3–7].

The calculation is done for large photon energies near the kinematical end point, $m_b - 2E_\gamma = \mathcal{O}(\Lambda_{\text{QCD}}) \ll m_b$. In this region, the hadronic state X_s has a large energy of the order $\mathcal{O}(m_b)$ but small invariant mass of the order $\mathcal{O}(m_b \Lambda_{\text{QCD}}) \ll m_b^2$, which is still a perturbative scale. A local operator product expansion (OPE) fails to describe this jetlike configuration, but the latter can be analyzed with factorization methods from soft-collinear effective theory (SCET). Moreover, the effects of the resolved contributions still have to be described in terms of nonlocal operator matrix elements when the photon cut is moved outside the end-point region [3–7]. In addition, the support properties of the soft functions imply that the resolved contributions, except the $\mathcal{O}_8 - \mathcal{O}_8$ one, are almost cut independent. Therefore all the resolved contributions represent an

irreducible uncertainty. In Ref. [8] a previous failure of factorization in a specific resolved contribution was healed by using new refactorization techniques [9–11].

Using the original notation of Ref. [4] one can write the resolved contribution due to the interference $\mathcal{O}_1^c - \mathcal{O}_{7\gamma}$ normalized to the perturbative leading-order (LO) result as

$$\mathcal{F}_{b \rightarrow s\gamma}^{17} = \frac{C_1(\mu) C_{7\gamma}(\mu)}{C_{7\gamma}(\mu_{\text{OPE}})^2} \frac{\Lambda_{17}(m_c^2/m_b, \mu)}{m_b}, \quad (1)$$

where μ_{OPE} denotes the hard scale within the local OPE and μ the scale within the resolved contribution. As in the original analysis in Ref. [4], the perturbative decay rate at the hard scale at LO accuracy is used as normalization, meaning that also the Wilson coefficient is taken at the hard scale μ_{OPE} . Since no α_s corrections or any RG improvements are considered yet in the calculation of the resolved power corrections, the scale choice is ambiguous. One first fixes the Wilson coefficients in the resolved contribution at the hard scale. Then one varies the scale of the Wilson coefficients in the resolved contributions between the hard and the hard-collinear scale to make the scale ambiguity of the resolved contributions manifest.

At subleading power one finds [4]

$$\begin{aligned} \Lambda_{17}\left(\frac{m_c^2}{m_b}, \mu\right) &= e_c \text{Re} \int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1} \\ &\times \left[1 - F\left(\frac{m_c^2 - i\epsilon}{m_b \omega_1}\right) + \frac{m_b \omega_1}{12 m_c^2} \right] h_{17}(\omega_1, \mu), \end{aligned} \quad (2)$$

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with the penguin function $F(x) = 4x \arctan^2(1/\sqrt{4x-1})$. The shape function h_{17} is connected to the original shape function g_{17} (see Ref. [4] for the precise definition) by

$$h_{17}(\omega_1, \mu) = \int_{-\infty}^{\bar{\Lambda}} d\omega g_{17}(\omega, \omega_1, \mu), \quad (3)$$

where ω_1 corresponds to the soft gluon momentum and $\bar{\Lambda} = M_B - m_b$.

In Eq. (2) the third term in the brackets corresponds to the local Voloshin term [12] (multiplied by -1), which is traditionally subtracted from the $\mathcal{O}_1^c - \mathcal{O}_{7\gamma}$ resolved contribution. This term is proportional to the zeroth moment of the shape function g_{17} which can be expressed by the well-known heavy-quark effective theory parameter λ_2 :¹

$$\begin{aligned} \int_{-\infty}^{\bar{\Lambda}} d\omega \int_{-\infty}^{\infty} d\omega_1 g_{17}(\omega, \omega_1, \mu) &= \frac{\langle \bar{B} | \bar{h} \tilde{n} i \gamma_\alpha^\perp \bar{n}_\beta g G_s^{\alpha\beta} h | \bar{B} \rangle}{2M_B} \\ &= 2\lambda_2. \end{aligned} \quad (4)$$

Integrating the third term and taking into account the minus sign we get

$$\Lambda_{17}^{\text{Voloshin}} = (-1) \frac{m_b \lambda_2}{9m_c^2}. \quad (5)$$

This formula translates via Eq. (1) into

$$\mathcal{F}_{17}^{\text{Voloshin}} = -\frac{C_1(\mu) C_{7\gamma}(\mu) \lambda_2}{C_{7\gamma}(\mu_{\text{OPE}})^2 9m_c^2} = +3.3\%. \quad (6)$$

This normalized central value of the Voloshin term was evaluated using the LO Wilson coefficients and the hard OPE scale in the resolved part.

From now on we denote the quantity in Eq. (2) as shape function term $\Lambda_{17}^{\text{Shape}}$. We can then write the complete resolved contribution as

$$\begin{aligned} \Lambda_{17}^{\text{Complete}} &= \Lambda_{17}^{\text{Shape}} + \Lambda_{17}^{\text{Voloshin}} \\ &= e_c \text{Re} \int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1} \left[1 - F\left(\frac{m_c^2 - i\epsilon}{m_b \omega_1}\right) \right] h_{17}(\omega_1, \mu). \end{aligned} \quad (7)$$

The separate evaluation of the local Voloshin term $\Lambda_{17}^{\text{Voloshin}}$ within the $\mathcal{O}_1 - \mathcal{O}_7$ resolved contribution was presented a long time ago [12–15]. The latest calculation of the shape

function term $\Lambda_{17}^{\text{Shape}}$ was done in Ref. [1] and, including uncertainties due to all parameters², leads to

$$-61 \text{ MeV} \leq \Lambda_{17}^{\text{Shape}} \leq +5 \text{ MeV}, \quad (8)$$

which translates via Eq. (1) into

$$\mathcal{F}_{17}^{\text{Shape}} \in [-0.4\%, 4.7\%], \quad (9)$$

where we used again the LO Wilson coefficients and $m_b = 4.58 \text{ GeV}$.

Adding the central value of the Voloshin term without uncertainties [see Eq. (6)] we get

$$\mathcal{F}_{17}^{\text{Shape}} + \mathcal{F}_{17}^{\text{Voloshin}} \in [2.9\%, 8\%]. \quad (10)$$

In addition, a substantial scale ambiguity must be considered. At LO, the scale of the hard functions, i.e. of the Wilson coefficients in the resolved contribution, is not fixed. We make this scale ambiguity manifest by shifting the scale of the Wilson coefficients from the hard scale to the hard-collinear scale. This results in an increase of the range by more than 40% to $[4.2\%, 11.7\%]$. The overall range including the scale ambiguity is then

$$\begin{aligned} \mathcal{F}_{17}^{\text{Shape}} + \mathcal{F}_{17}^{\text{Voloshin}} &\in [2.9\%, 11.7\%] \\ &(\text{scale ambiguity included}). \end{aligned} \quad (11)$$

We emphasize that the uncertainties of the Voloshin term due to parameter variation are not taken into account yet. Furthermore, Eq. (7) shows that the separation of these two terms is unnatural from the SCET point of view and it is clear that an error analysis of the complete $\mathcal{O}_1^c - \mathcal{O}_{7\gamma}$ resolved contribution has to be redone because the uncertainties of the two terms are highly correlated.

II. NUMERICAL ANALYSIS

We calculate now the complete resolved $\mathcal{O}_1^c - \mathcal{O}_{7\gamma}$ contribution without separating this contribution into two pieces. As in our previous analysis in Ref. [1], we follow the systematic method proposed in Ref. [2] and use a complete set of basis functions, namely the Hermite polynomials multiplied by a Gaussian:

$$h_{17}(\omega_1) = \sum_n a_{2n} H_{2n}\left(\frac{\omega_1}{\sqrt{2}\sigma}\right) e^{-\frac{\omega_1^2}{2\sigma^2}}. \quad (12)$$

Here only even polynomials have to be chosen due to the fact that our shape function h_{17} is even. We also allow for

¹This local Voloshin term can be derived from the complete resolved contribution $\mathcal{O}_1^c - \mathcal{O}_{7\gamma}$ by neglecting the shape function effects and under the assumption that the charm quark mass is treated as heavy. It does not account for the complete resolved contribution.

²The authors of Ref. [2] found much smaller values. As discussed in detail in Ref. [1], the main reasons for this difference is the much smaller variation of the charm mass and the noninclusion of an uncertainty due to $1/m_b^2$ corrections in Ref. [2].

higher suppression in the Gaussian because such functions cannot be expressed by the basis of functions given in Eq. (12) and, thus, have to be included within a systematic analysis. This approach avoids any prejudices regarding the unknown functional form of the shape function, but only uses well-established properties like the zeroth and the second moment which were derived in Refs. [2,16], and also the fourth and the sixth moment which are based on dimensional arguments.

All input parameters are varied on our grid within their uncertainties, as done in the previous analyses [1,2], since we cannot assume that the uncertainties are in general Gaussian in the considered ranges and therefore we do not add them in quadrature. The charm quark mass is varied within the range $1.14 \text{ GeV} \leq m_c \leq 1.23 \text{ GeV}$ in ten steps, the bottom quark mass within the range $4.55 \text{ GeV} \leq m_b \leq 4.61 \text{ GeV}$ in three steps. The zeroth moment $m_0 = 2\lambda_2$ is varied from 0.197 GeV^2 to 0.277 GeV^2 in eight steps and the second moment m_2 from 0.03 GeV^4 to 0.27 GeV^4 in 12 steps; the fourth and the sixth moment within $-0.3 \text{ GeV}^6 \leq m_4 \leq 0.3 \text{ GeV}^6$ and within $-0.3 \text{ GeV}^8 \leq m_6 \leq 0.3 \text{ GeV}^8$, respectively, in 30 steps. The parameter σ of the model functions in Eq. (12) is varied from 0 to 1 GeV via 20 steps. For details on the schemes adopted for m_c and m_b we refer the reader to Ref. [1].

For a polynomial of degree 6 with an $\exp(-x^4)$ suppression, like in the calculation of the shape function term in Ref. [1], we find the following range for the complete resolved contribution:

$$-115 \text{ MeV} \leq \Lambda_{17}^{\text{Complete}} \leq -34 \text{ MeV}, \quad (13)$$

which translates via Eq. (1) into

$$\mathcal{F}_{17}^{\text{Complete}} \in [2.6\%, 8.9\%], \quad (14)$$

which can be compared to Eq. (10) where the uncertainty of the Voloshin term was not included. The range increases by 22.5%.

Polynomials of degree 8 or higher suppression factors like $\exp(-x^6)$ do not lead to more extreme values.

It turns out that the grid parameters which lead to the two extreme values for the complete resolved contribution are the same as in the case of the extreme values for the shape function term only, found in our previous analysis in Ref. [1]. The latter result, quoted in Eq. (8), has been also confirmed by the new calculation.

The grid parameters for the largest value are $m_c = 1.14 \text{ GeV}$, $m_b = 4.61 \text{ GeV}$, $\sigma = 0.750 \text{ MeV}$, $m_0 = 0.197 \text{ GeV}^2$, $m_2 = 0.030 \text{ GeV}^4$, $m_4 = -0.100 \text{ GeV}^6$, and $m_6 = -0.180 \text{ GeV}^8$ and for the smallest the parameters are $m_c = 1.14 \text{ GeV}$, $m_b = 4.61 \text{ GeV}$, $\sigma = 0.700 \text{ MeV}$, $m_0 = 0.277 \text{ GeV}^2$, $m_2 = 0.270 \text{ GeV}^4$, $m_4 = 0.280 \text{ GeV}^6$, and $m_6 = 0.300 \text{ GeV}^8$.

A specific pattern has also been seen in our previous analysis of just the shape function term. Both extreme values are found for the smallest m_c value and for the largest m_b value. This can be understood by the fact that these values allow for a maximum overlap of the shape function with the jet function. The fact that the grid parameters corresponding to the extreme values of the complete resolved term and of just the shape function term are the same implies that the increase of the range can be solely traced back to the Voloshin term including its uncertainties.

The large scale-ambiguity of the resolved contribution at LO can be made manifest again by shifting the scale of the Wilson coefficients from the hard to the hard-collinear scale. This leads to the range [3.8%, 13%]. Thus, the overall range including the scale ambiguity is then

$$\mathcal{F}_{17}^{\text{Complete}} \in [2.6\%, 13\%] \text{ (scale ambiguity included)}. \quad (15)$$

This represents our final LO result for the complete resolved $\mathcal{O}_1^c - \mathcal{O}_{7\gamma}$ contribution. We note that the range does not correspond to a Gaussian uncertainty. The estimate should be considered as the range within which we expect the actual values of the resolved contribution to lie, without making a statement about the most likely values within this range, as noted already in Ref. [4].

III. CONCLUSIONS

We presented a calculation of the complete nonlocal resolved contribution due to the interference of the electroweak operators $\mathcal{O}_1^c - \mathcal{O}_{7\gamma}$ at LO. This joint calculation of the shape function term and the local Voloshin term is mandatory due to the fact that the uncertainties are highly correlated. The new calculation leads to a significant increase of the range of the resolved contribution which does not correspond to a Gaussian error. We find $\mathcal{F}_{17}^{\text{Complete}} \in [2.6\%, 8.9\%]$. This represents one of the largest uncertainties of the inclusive radiative penguin decay $\bar{B} \rightarrow X_s \gamma$. The large uncertainty can be traced back to a large charm mass uncertainty in the LO result. If we make the scale ambiguity manifest, we get an even larger range, namely $\mathcal{F}_{17}^{\text{Complete}} \in [2.6\%, 13\%]$. The ongoing α_s calculation [17,18] will establish a well-defined scale dependence diminishing the large scale-ambiguity and may reduce the large charm mass dependence.

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DATA AVAILABILITY

The data are available from the authors upon reasonable request.

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