

CP violation in the HZZ vertex and left-right asymmetries

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We investigate new contributions to the HZZ vertex from the flavor changing neutral current (FCNC) involving the Higgs and Z bosons. Our calculations reveal that the form factors h_2^V and h_3^V ($V = H, Z$) can be induced through these couplings, and we present our results in terms of the Passarino-Veltman scalar functions. Using the current limits on $H\bar{t}c$ and $Z\bar{t}c$ couplings, we determine that the new contributions to the CP -conserving form factor h_2^V are small compared to the Standard Model (SM) predictions. However, for the CP -violating form factor h_3^V , the contributions can reach values as high as 10^{-6} , 5 orders of magnitude larger than in the SM. Furthermore, we examine how these results influence the left-right asymmetries in the processes $H^* \rightarrow ZZ$ and $Z^* \rightarrow ZH$. Our findings suggest that significant deviations from SM predictions may occur when considering FCNC contributions.

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I. INTRODUCTION

In the Standard Model (SM) of particle physics, the Higgs boson, discovered in 2012 at the Large Hadron Collider (LHC) [1,2], is the remnant of the Brout-Englert-Higgs mechanism that gives mass to the gauge bosons and fermions [3–5]. Since the Higgs boson discovery, notable progress has been made in measuring its properties [6,7]. Recently, the couplings of the Higgs boson with weak bosons have attracted considerable attention, particularly after the LHC reported that the signal strength of the $H \rightarrow Z\gamma$ decay is twice the predicted value by the SM [8,9]. Furthermore, for the first time, the evidence of a pair of Z bosons produced via an off-shell Higgs boson was announced by the ATLAS and CMS collaborations [10,11]. This finding also facilitated the determination of Γ_H by analyzing the ratio between the on-shell and off-shell Z pair production rates [12,13]. Moreover, the HZZ vertex has been proposed to be sensitive to quantum entanglement effects at the LHC [14–18].

The HZZ vertex with anomalous couplings can be induced through the following effective Lagrangian:

$$\mathcal{L} = \frac{g}{c_W} m_Z \left[\frac{(1-a_Z)}{2} H Z_\mu Z^\mu + \frac{1}{2m_Z^2} \left\{ \hat{b}_Z H Z_{\mu\nu} Z^{\mu\nu} + \hat{c}_Z H Z_\mu \partial_\nu Z^{\mu\nu} + \tilde{b}_Z H Z_{\mu\nu} \tilde{Z}^{\mu\nu} \right\} \right], \quad (1)$$

where a_Z corresponds to the tree-level correction, while $\hat{b}_Z$ arises at the one-loop level within the SM and is of order 10^{-2} [19–21]. The anomalous coupling $\hat{c}_Z$ is also expected to emerge at the one-loop level; however, it has not been identified yet in SM calculations. The a_Z , $\hat{b}_Z$, and $\hat{c}_Z$ couplings are CP conserving [19], whereas $\tilde{b}_Z$ is CP violating and could be generated at the three-loop level in the SM, with an approximate magnitude of 10^{-11} [22]. At the LHC, bounds on the effective fractional cross sections f_{a_i} have been derived from the decays $H \rightarrow 4\ell$ [10,23] and $H \rightarrow \tau\tau$ [24]. The f_{a_i} approach minimizes uncertainties and is independent of the coupling parametrization [25,26]; however, this methodology restricts the determination of limits to the ratios $\hat{c}_Z/\hat{b}_Z$ and $\tilde{b}_Z/\hat{b}_Z$. In contrast, by integrating the LHC results in Ref. [10] with the theoretical calculation of $\hat{b}_Z$, individual constraints on the $\hat{c}_Z$ and $\tilde{b}_Z$ anomalous couplings of order 10^{-2} – 10^{-4} were established in Ref. [21]. Tighter bounds can be achieved using the stringent constraints on the effective ratios f_{a_i} presented in Ref. [24], which reports an improvement of approximately 20%–50% in these limits through the combination of results from the decays $H \rightarrow 4\ell$ and $H \rightarrow \tau\tau$. Nevertheless, these bounds are of comparable

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order of magnitude to those used in Ref. [21]. Therefore, significant changes for the constraints on $\hat{c}_Z$ and $\tilde{b}_Z$ outlined above are not anticipated. The phenomenology of the H^*ZZ coupling at the LHC and future colliders has been extensively studied by numerous authors [27–33]. Additionally, the cases HZZ^* and HZ^*Z^* have relevant implications at colliders [34–58]. The $H^* \rightarrow ZZ$ decay is included in publicly available codes such as HDECAY [59] and PROPHECY4F [60].

From Lagrangian (1) and using the nomenclature in Fig. 1, the vertex function can be written as follows:

$$\Gamma_{\mu\nu}^{ZZH} = h_1^V(q^2, p_1^2, p_2^2)g_{\mu\nu} + \frac{h_2^V(q^2, p_1^2, p_2^2)}{m_Z^2} p_{1\nu} p_{2\mu} + \frac{h_3^V(q^2, p_1^2, p_2^2)}{m_Z^2} \epsilon_{\mu\nu\alpha\beta} p_1^\alpha p_2^\beta, \quad (2)$$

where V denotes the off-shell boson. Following the kinematics $H^* \rightarrow ZZ$ ($Z^* \rightarrow ZH$), the form factors h_i^V can be expressed as

$$h_1^V(q^2, p_1^2, p_2^2) = 1 + a_Z - \hat{b}_Z \frac{q^2 - p_1^2 - p_2^2}{m_Z^2} + \frac{\hat{c}_Z}{2m_Z^2} p_1^2 + p_2^2, \quad (3)$$

$$h_2^V(q^2, p_1^2, p_2^2) = \pm 2\hat{b}_Z, \quad (4)$$

$$h_3^V(q^2, p_1^2, p_2^2) = \pm 2\tilde{b}_Z. \quad (5)$$

For simplicity, we set $a_Z = 0$ in this work. Since $\hat{b}_Z$ has an imaginary part in the SM [21], we expect that $\hat{c}_Z$, $\hat{b}_Z$, and $\tilde{b}_Z$ are complex quantities. The Lagrangian in Eq. (1) necessitates real anomalous couplings to be Hermitian. However, it uses an effective approach to describe the HZZ interaction, which is valid only at the Born level; thus, the operators that induce the anomalous couplings at higher orders are not included in the HZZ Lagrangian [61]. These operators do not strictly require real anomalous couplings to maintain a Hermitian Lagrangian.

The imaginary part, together with the CP -violating form factor, may give rise to intriguing new physics effects that could be observed through polarized observables at the LHC [21,40,62–66]. The phenomenology involving a pair of polarized Z bosons in the process $pp \rightarrow H^* \rightarrow ZZ$ at the LHC has been addressed in Refs. [67–71]. The polarizations of gauge bosons are particularly noteworthy at the LHC and are currently under investigation across various processes. For instance, the ATLAS collaboration has reported evidence of longitudinally polarized $W^\pm Z$ and ZZ boson pairs [72,73]. Additionally, polarization fractions of the Z bosons have been analyzed by the LHCb, ATLAS, and CMS collaborations [74–77]. Studies involving gauge boson polarizations at the LHC include $W^\pm W^\pm$ production [78], W + Jets events [79,80], $W^\pm Z$ production [81], and the generation of

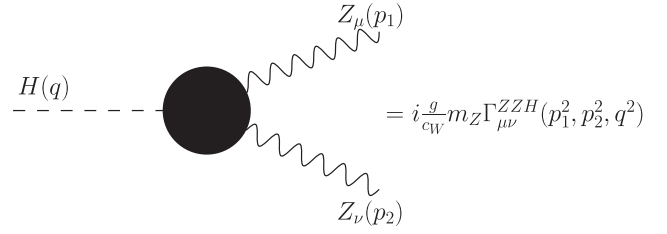


FIG. 1. Nomenclature for the HZZ coupling and the $\Gamma_{\mu\nu}^{ZZH}$ vertex function.

W bosons in $t\bar{t}$ events and top decays [82–85]. The potential for producing polarized gauge bosons has been incorporated into event generators like MadGraph5_aMC@NLO2.6.2 [86] and SHERPA [87].

In this study, we examine the flavor changing neutral current (FCNC) contributions of the Higgs and Z bosons to the HZZ vertex, with a particular focus on the CP -violating form factor h_3^V ($V = H, Z$). Furthermore, we investigate the potential for new left-right asymmetries arising from the polarizations of the Z bosons. The structure of this work is organized as follows: in Sec. II, we calculate the new contributions to the HZZ vertex resulting from FCNC couplings mediated by the H and Z bosons. Next, in Sec. III, we analyze the left-right asymmetries that can be induced by the CP -violating form factor h_3^V . Finally, in Sec. IV, we present a numerical analysis of our results, with our conclusions summarized in Sec. V.

II. FCNC CONTRIBUTIONS TO THE HZZ COUPLING

To generate CP -violating contributions to the HZZ vertex, we consider the following effective Lagrangian, which induces FCNC couplings mediated by the Z and H bosons:

$$\mathcal{L} = \frac{g}{c_W} \bar{f}_j \gamma_\mu (g_V^{ij} - g_A^{ij} \gamma^5) f_i Z^\mu - \frac{g}{2m_W} H \bar{f}_j (g_S^{ij} + g_P^{ij} \gamma^5) f_i + \mathcal{H.C.}, \quad (6)$$

where $f_{i,j}$ correspond to the SM fermions, while g_r^{ij} ($r = V, A, S, P$) are complex couplings. For FCNC couplings involving light quarks mediated by the Higgs boson, the strongest constraints are derived from $B - \bar{B}$, $K^0 - \bar{K}^0$, and $D^0 - \bar{D}^0$ oscillations [88]. The corresponding limits on the $g_{V,A}^{ij}$ couplings are obtained from decays of K^0 and B mesons [89–93]. Bounds on lepton FCNC mediated by the Higgs and Z bosons have been established through the analysis of dipole moments, meson oscillation, and three-body lepton flavor violating decays [88,94]. Moreover, these interactions have been analyzed at the LHC [95–99]. FCNC interactions involving the top quark have been investigated at the LHC through the decays $t \rightarrow Zq$ and $t \rightarrow Hq$ [100,101]. In the SM, the corresponding branching

ratios are highly suppressed, with $\mathcal{B}(t \rightarrow Zq)$ being of order 10^{-14} , and $\mathcal{B}(t \rightarrow Hq)$ 1 order of magnitude smaller [102]. However, these results could be significantly enhanced by new physics contributions [103–115]. Given the large mass of the top quark, we expect the most significant contributions from FCNC interactions involving this particle. Constraints on the couplings $Z\bar{t}q$ and $H\bar{t}q$ ($q = c, u$) have been obtained from LHC data [116,117], which can be summarized as follows:

$$|g_V^{tc}|, \quad |g_A^{tc}| \leq 0.0095, \quad (7)$$

$$|g_S^{tc}|, \quad |g_P^{tc}| \leq 0.25 \text{ GeV}, \quad (8)$$

while the corresponding limits for the up quark are smaller than those presented above, but of the same order of magnitude.

In the rest of this section, we will calculate the one-loop FCNC contributions to the h_2^V and h_3^V from factors arising from Lagrangian (6). To our knowledge, these contributions have not been previously reported. We will consider scenarios that involve an off-shell Higgs boson or an off-shell Z boson, with the following kinematics $H^* \rightarrow ZZ$ and $Z^* \rightarrow ZH$.

A. Analytical results

Two distinct contributions to the HZZ vertex can arise from the FCNC couplings in Lagrangian (6). The first category of diagrams (type I), as illustrated in Fig. 2, involves only flavor-violating couplings mediated by the Z boson. The second category (type II) also includes FCNC couplings of the Higgs boson and is depicted in Fig. 3. For our calculations, we consider additional diagrams arising from both $p_1^\mu \leftrightarrow p_2^\mu$ and $m_i \leftrightarrow m_j$ exchanges. Our results were obtained using the FeynCalc package [118–121] and are expressed in terms of the Passarino-Veltman scalar functions.

1. Diagrams type I

By considering all the contributing Feynman diagrams from Fig. 2, we obtained the FCNC contribution to the CP -conserving form factor h_2^V , which can be expressed as

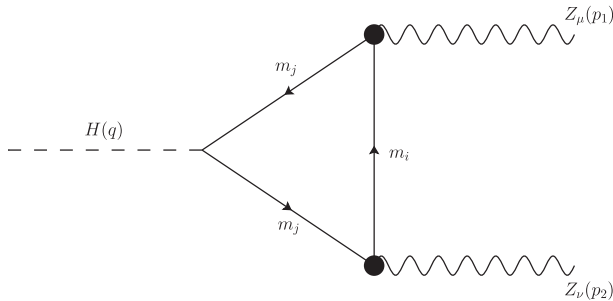


FIG. 2. One-loop contributions of type I, where only FCNC couplings of the Z boson are involved.

$$h_2^V(I) = -\frac{g^2 m_Z N_f}{4\pi^2 c_W m_W} \left\{ |g_V^{ij}|^2 A_V^V(K^2, m_i^2, m_j^2) + |g_A^{ij}|^2 A_A^V(K^2, m_i^2, m_j^2) \right\},$$

$$V = H, Z, \quad K = Q, P, \quad (9)$$

where N_f corresponds to the number of colors. The functions $A_{V,A}^V$ ($V = H, Z$) can be found in Appendix A 1. For the off-shell H case, $A_{V,A}^H$ depend on Q , which is defined as $Q \equiv \|q\|$. In the case of the off-shell Z boson, the functions $A_{V,A}^Z$ depend on P , with $P = \|p_1\|$. When $m_j = m_i$, the form factor h_2^V ($V = H, Z$) reduces to twice the SM results [21], due to the inclusion of double diagrams corresponding to the $m_i \leftrightarrow m_j$ exchange.

For the CP -violating form factor h_3^V , the FCNC contribution of type I is given as follows:

$$h_3^V(I) = -\frac{g^2 m_i m_j m_Z N_f}{2\pi^2 c_W m_W} \text{Im} \left[g_V^{ij} g_A^{ij*} \right] \tilde{\mathcal{F}}^V(K^2, m_i^2, m_j^2),$$

$$V = H, Z, \quad K = Q, P, \quad (10)$$

where the functions $\tilde{\mathcal{F}}^V$ ($V = H, Z$) can also be found in Appendix A 1. Notably, at least one complex coupling in the $Z\bar{f}_j f_i$ vertex is required to induce CP violation in the HZZ vertex. For the flavor-conserving scenario $m_j = m_i$, the form factor h_3^V vanishes. Additionally, we anticipate negligible contributions for FCNC involving only light fermions, as h_3^V is proportional to the product $m_i m_j$.

The $A_{V,A}^V$ and $\tilde{\mathcal{F}}^V$ ($V = H, Z$) functions presented in Eqs. (9) and (10) are free of divergences.

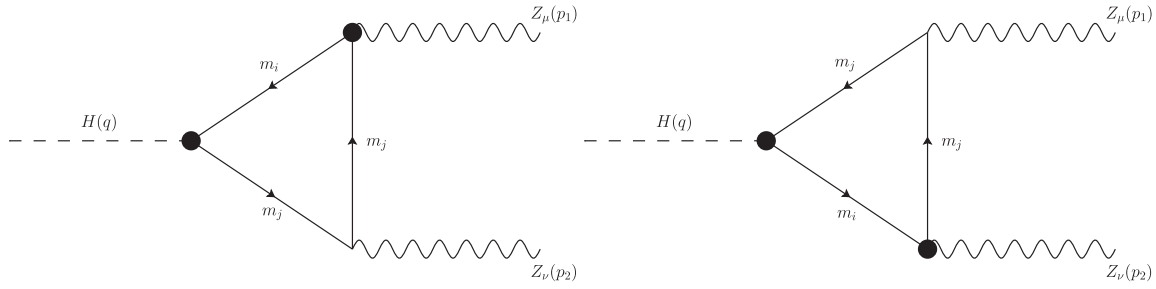
2. Diagrams type II

From diagrams of type II in Fig. 3, the resulting contribution to the CP -conserving form factor can be expressed as

$$h_2^V(II) = -\frac{g^2 m_Z N_f}{2\pi^2 c_W m_W} \left\{ g_V \text{Re} \left[g_V^{ij} g_S^* \right] R_1^V(K^2, m_i, m_j) + g_A \text{Re} \left[g_V^{ij} g_P^* \right] R_2^V(K^2, m_i, m_j) + g_A \text{Re} \left[g_A^{ij} g_S^* \right] R_3^V(K^2, m_i, m_j) + g_V \text{Re} \left[g_A^{ij} g_P^* \right] R_4^V(K^2, m_i, m_j) \right\},$$

$$V = H, Z, \quad K = Q, P, \quad (11)$$

where g_V and g_A correspond to the SM vector and axial couplings of the Z boson with fermions. The functions R_i^V ($i = 1, 2, 3, 4$) are presented in Appendix A 2. For the case where $m_j = m_i$, $g_S = m_i$, and $g_P = 0$, our expression

FIG. 3. One-loop contributions of type II, where FCNC couplings of the Z and H bosons are involved.

simplifies to 4 times the SM contribution due to the additional Feynman diagrams from the $m_i \leftrightarrow m_j$ and $Z\bar{f}_j f_i$ exchanges.

The contribution to the form factor associated with CP violation can be expressed as follows:

$$\begin{aligned}
 h_3^V(II) = & \frac{g^2 m_Z N_f}{2\pi^2 c_W m_W} \left\{ g_A \text{Im} \left[g_V^{ij} g_S^* \right] T_1^V(K^2, m_i, m_j) \right. \\
 & + g_V \text{Im} \left[g_V^{ij} g_P^* \right] T_2^V(K^2, m_i, m_j) \\
 & + g_V \text{Im} \left[g_A^{ij} g_S^* \right] T_3^V(K^2, m_i, m_j) \\
 & \left. + g_A \text{Im} \left[g_A^{ij} g_P^* \right] T_4^V(K^2, m_i, m_j) \right\}, \\
 V = H, Z, \quad K = Q, P.
 \end{aligned} \tag{12}$$

The functions T_i^V ($i = 1, 2, 3, 4$) in Eq. (12) are also shown in Appendix A 2. Notably, the pseudoscalar coupling is not necessary to induce CP violation. For the flavor-conserving scenario ($m_j = m_i$), and considering $g_P = 0$, the form factor h_3^V vanishes. However, the functions $T_{2,4}^V$, which are proportional to terms involving the pseudoscalar coupling g_P , are not zero when $m_j = m_i$. Therefore,

generating CP violation without FCNC contributions remains feasible if we include a pseudoscalar coupling. The R_i^V and T_i^V ($i = 1, 2, 3, 4$) functions in Eqs. (11) and (12) are free of divergences.

The new contributions to the form factor h_3^H can lead to left-right asymmetries in the $H^* \rightarrow ZZ$ process [21,66]. Furthermore, the FCNC contributions to h_2^V and h_3^V ($V = H, Z$) of types I and II have not been reported previously.

III. LEFT-RIGHT ASYMMETRIES OF THE HZZ VERTEX

The polarized observables of the HZZ vertex have been explored in multiple contexts [21,40,65,66], where the left-right asymmetry $\mathcal{A}_{LR}^H$ for the $H^* \rightarrow ZZ$ process is defined as follows:

$$\mathcal{A}_{LR}^H = \frac{\Gamma_{H^* \rightarrow Z_L Z_L}^L - \Gamma_{H^* \rightarrow Z_R Z_R}^R}{\Gamma_{H^* \rightarrow Z_L Z_L}^L + \Gamma_{H^* \rightarrow Z_R Z_R}^R}. \tag{13}$$

By considering the form factors h_i^H as complex, the $\mathcal{A}_{LR}^H$ asymmetry has been computed in Ref. [21] in terms of the real and imaginary parts of h_1^H and h_3^H as follows:

$$\mathcal{A}_{LR}^H = 4m_Z^2 \mathcal{K}(Q) \frac{\text{Re}[h_1^H] \text{Im}[h_3^H] - \text{Re}[h_3^H] \text{Im}[h_1^H]}{\mathcal{K}^2(Q) \left\{ \text{Re}[h_3^H]^2 + \text{Im}[h_3^H]^2 \right\} + 4m_Z^4 \left\{ \text{Re}[h_1^H]^2 + \text{Im}[h_1^H]^2 \right\}}, \tag{14}$$

with

$$\mathcal{K}(Q) = \sqrt{Q^2(Q^2 - 4m_Z^2)}. \tag{15}$$

Motivated by this result, we expect that a nonzero left-right asymmetry can also arise in the process $Z^* \rightarrow ZH$. With this aim, we define $\mathcal{A}_{LR}^Z$ as

$$\mathcal{A}_{LR}^Z = \frac{\Gamma_{Z^* \rightarrow Z_L H}^L - \Gamma_{Z^* \rightarrow Z_R H}^R}{\Gamma_{Z^* \rightarrow Z_L H}^L + \Gamma_{Z^* \rightarrow Z_R H}^R}. \tag{16}$$

To obtain the analytic expression for the $\mathcal{A}_{LR}^Z$ asymmetry, we begin by calculating the polarized width decays $\Gamma_{Z^* \rightarrow Z_L H}^L$ under the following kinematic conditions: the process $Z^*(p_1) \rightarrow Z(p_2)H(q)$ occurs in the rest frame of the $Z^*(p_1)$ boson, with the $Z(p_2)$ moving along the x axis. In this context, the polarization vectors of the $Z(p_2)$

boson are

$$\epsilon(0) = \frac{1}{2m_Z P} \left(\sqrt{(P^2 + m_Z^2 - m_H^2)^2 - 4m_Z^2 P^2}, P^2 + m_Z^2 - m_H^2, 0, 0 \right), \quad (17)$$

$$\epsilon(R/L) = \frac{1}{\sqrt{2}} \left(0, 0, -i, \pm 1 \right). \quad (18)$$

Then, the polarized width decays can be expressed as

$$\Gamma_{Z^* \rightarrow Z_i H}^\lambda = \frac{\sqrt{(P^2 + m_Z^2 - m_H^2)^2 - 4m_Z^2 P^2}}{16\pi P^3} \mathcal{M}^2(\lambda), \quad \lambda = L, R, 0, \quad (19)$$

where λ denotes the polarization of the on-shell Z boson and $\mathcal{M}^2(\lambda)$ is the squared polarized amplitude. In Eq (19), we do not average over the initial polarizations because the off-shell $Z(p_1)$ boson corresponds to a propagator in a collider process.

The right and left polarized amplitudes are given as

$$\mathcal{M}^2(R/L) = -\frac{g^2}{4c_W^2 m_Z^4} \left\{ m_Z^2 \left[-4m_Z^4 (\text{Re}[h_1^Z]^2 + \text{Im}[h_1^Z]^2) + (\text{Re}[h_3^Z]^2 + \text{Im}[h_3^Z]^2) (2m_H^2(m_Z^2 + P^2) - m_H^4 - (m_Z^2 - P^2)^2) \right] \right. \\ \left. \pm 4m_Z^4 (\text{Re}[h_1^Z] \text{Im}[h_3^Z] - \text{Im}[h_1^Z] \text{Re}[h_3^Z]) \sqrt{(P^2 + m_Z^2 - m_H^2)^2 - 4m_Z^2 P^2} \right\}. \quad (20)$$

We observe that analogous to the $H^* \rightarrow ZZ$ process, the amplitudes for transversely polarized states only exhibit a dependence on the h_1^Z and h_3^Z form factors [21].

To ensure a comprehensive analysis, we have computed the amplitude corresponding to longitudinal polarization:

$$\mathcal{M}^2(0) = \frac{g^2}{16c_W^2 m_Z^6} \left\{ 4m_Z^4 \left(\text{Re}[h_1^Z]^2 + \text{Im}[h_1^Z]^2 \right) (-2m_H^2(m_Z^2 + P^2) + m_H^4 - 2P^2 m_Z^2 + 5m_Z^4 + P^4) \right. \\ + \left[\text{Re}[h_2^Z]^2 + \text{Im}[h_2^Z]^2 \right] [(m_H - m_Z)^2 - P^2][(m_H + m_Z)^2 - P^2][m_H^2 - 3m_Z^2 - P^2][m_H^2 + m_Z^2 - P^2] \\ \left. + 4m_Z^2 \left(\text{Re}[h_1^Z] \text{Re}[h_2^Z] + \text{Im}[h_1^Z] \text{Im}[h_2^Z] \right) (-m_H^2 + m_Z^2 + P^2)(-2m_H^2(m_Z^2 + P^2) + m_H^4 + (m_Z^2 - P^2)^2) \right\}. \quad (21)$$

The absence of the CP -violating form factor h_3^Z in Eq. (21) suggests that the longitudinal polarization of the Z boson does not offer an opportunity for detecting CP -violating effects. On the other hand, we note a difference in sign between the left- and right-polarized amplitudes in the last term of Eq. (20). This finding leads to a nonzero $\mathcal{A}_{LR}^Z$ asymmetry, which is sensitive to CP violation and can be expressed as follows:

$$\mathcal{A}_{LR}^Z = 4m_Z^2 \mathcal{K}(P) \frac{\text{Im}[h_3^Z] \text{Re}[h_1^Z] - \text{Im}[h_1^Z] \text{Re}[h_3^Z]}{\mathcal{K}^2(P) \left\{ \text{Re}[h_3^Z]^2 + \text{Im}[h_3^Z]^2 \right\} + 4m_Z^4 \left\{ \text{Im}[h_1^Z]^2 + \text{Re}[h_1^Z]^2 \right\}}, \quad (22)$$

where the $\mathcal{K}(P)$ function is given by

$$\mathcal{K}(P) = \sqrt{(P^2 + m_Z^2 - m_H^2)^2 - 4m_Z^2 P^2}. \quad (23)$$

The $\mathcal{A}_{LR}^Z$ asymmetry exhibits a structure similar to that reported in the $H^* \rightarrow ZZ$ case. Additionally, we observe that to achieve nonvanishing $\mathcal{A}_{LR}^V$ ($V = H, Z$) asymmetries, CP -violating and complex anomalous couplings are necessary. In the SM, the h_1^V ($V = H, Z$) form factor is known to be complex [21]. Therefore, a nonzero h_3^V ($V = H, Z$) would give rise to the left-right asymmetries discussed in

this section. This work presents the $\mathcal{A}_{LR}^Z$ asymmetry for the first time.

IV. NUMERICAL ANALYSIS

We will now assess the FCNC contributions to the h_2^V and h_3^V ($V = H, Z$) form factors, as well as the $\mathcal{A}_{LR}^V$ asymmetries. Given that we anticipate significant results from FCNC couplings involving the top quark, we will consider the bounds outlined in Eqs. (7) and (8). Furthermore, our analysis will focus exclusively on energy regions where the ZZ and HZ pairs can be produced on shell. To numerically evaluate the Passarino-Veltman scalar functions, we utilized the LoopTools package [122].

A. Contributions of type I

We begin by analyzing the contributions of type I for both scenarios: the H^*ZZ and Z^*ZH vertex. To maximize the contributions to the CP -conserving form factor, we will use the upper bounds on the norms of the g_V^{tc} and g_A^{tc} couplings in Eq. (7).

For the CP -violating form factor, it is useful to define

$$\hat{h}_3^V(I) = \frac{h_3^V(I)}{\text{Im}[g_V^{ij}g_A^{ij*}]}, \quad V = H, Z, \quad (24)$$

which allows us to analyze its contributions in a model-independent manner.

1. H^*ZZ vertex

In Fig. 4, we present the behavior of the real and imaginary parts of $h_2^H(I)$ (left plot) and $\hat{h}_3^H(I)$ (right plot) as a function of Q . Our analysis focuses solely on the $Z\bar{t}c$ contributions, while the contributions from lighter fermions will be addressed later. The absorptive parts emerge when the particles in the loop that couple to the V^* boson can be on shell [123,124]. For the CP -conserving form factor h_2^H , we find that its contributions can reach values up to order 10^{-6} , which is 4 and 3 orders of magnitude smaller than those predicted in the SM and the current limits [21], respectively. The real and imaginary parts exhibit a decreasing trend at high energies. Notably, we observe that below the threshold energy of $Q = 2m_t$, the absorptive part of h_2^H is approximately of order 10^{-10} . The contributions to the imaginary part at these lower energy levels are exclusively attributed to the diagrams depicted in Fig. 2, wherein the Higgs boson interacts with a $\bar{c}c$ pair. Beyond this threshold energy, additional diagrams involving the $H\bar{t}t$ coupling contribute significantly. As a result, the magnitude of $\text{Im}[h_2^H]$ is enhanced and becomes comparable

to that of the real part. Remarkably, around $Q \approx 500$ GeV, the absorptive part exceeds the magnitude of $\text{Re}[h_2^H]$.

For the CP -violating form factor, we find that the contributions to its real and absorptive parts are of order 10^{-4} . In contrast to the form factor h_2^H , the imaginary part of the CP -violating form factor exhibits significantly larger contributions for energies $Q \leq 2m_t$. Furthermore, its magnitude dominates at both low and high energy levels. However, at energy levels around 400 GeV, the real part is the main contribution. From the limits in Eq. (7), we estimate that $\text{Im}[g_Vg_A^*]$ can reach values of order 10^{-5} . In this context, the real and imaginary parts of h_3^H could achieve magnitudes between 10^{-8} and 10^{-9} . While this result is quite small compared to the current constraints on h_3^H [21], it is 3 orders of magnitude larger than the SM prediction [22].

For the light fermion FCNC contributions, we examine an optimistic scenario where their couplings are comparable to the values used for the $Z\bar{t}c$ interaction, i.e., they are of order 10^{-3} . By neglecting the mass of the light quarks to avoid nonperturbative effects, we find that the contributions from the $Z\bar{t}u$ and $Z\bar{c}u$ couplings to the form factor $h_2^H(I)$ are 2 and 6 orders of magnitude smaller than in Fig. 4, respectively. For the contributions from FCNC involving down-type quarks and leptons, we estimate values of orders 10^{-10} – 10^{-12} . However, the constraints on these FCNC couplings are tighter than the values considered within this analysis [88,93,94,115]. As a result, their contributions to $h_2^H(I)$ will be significantly smaller than those outlined above.

Regarding the CP -violating form factor, we find that FCNC contributions from light fermions are negligible compared with those obtained in Fig. 4, since $h_3^V \sim m_i m_j$ in Eq. (10). We observe a similar pattern for the contributions of light fermions to the Z^*ZH vertex and type II diagrams. Therefore, these contributions will not be examined further in this work.

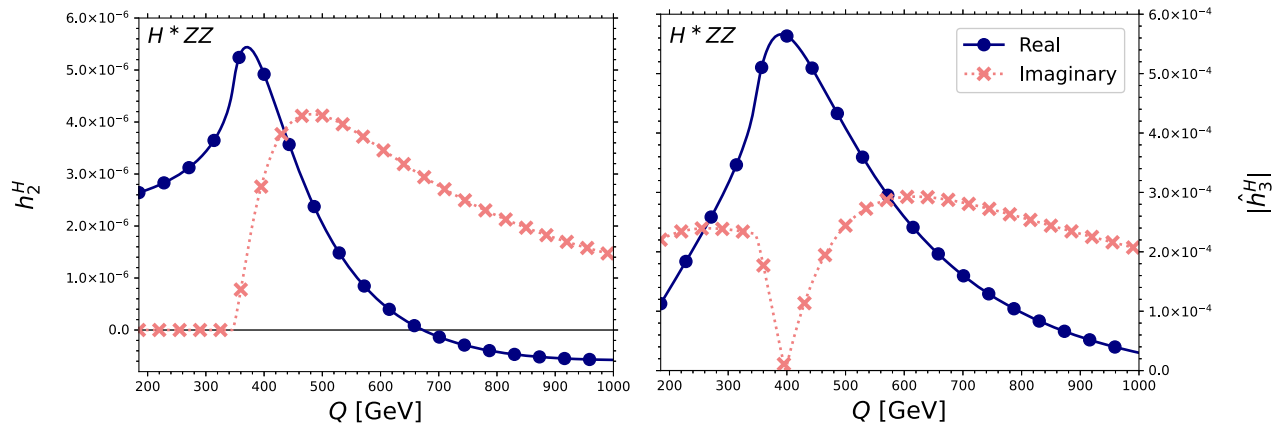


FIG. 4. The FCNC contributions of the type I to the form factors h_2^H (left plot) and $\hat{h}_3^H$ (right plot) as a function of Q . We only consider the $Z\bar{t}c$ coupling.

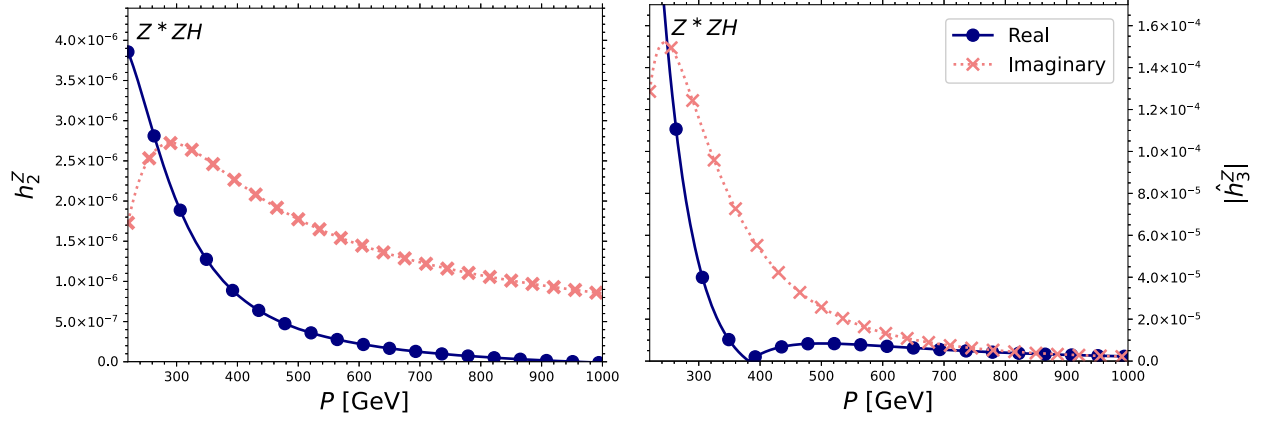


FIG. 5. The FCNC contributions of the type I to the form factors h_2^Z (left plot) and $\hat{h}_3^Z$ (right plot) as a function of P . We only consider the $Z\bar{t}c$ coupling.

2. Z^*ZH vertex

We now show the behavior of $h_2^Z(I)$ (left plot) and $\hat{h}_3^Z(I)$ (right plot) as a function of P , in Fig. 5. For h_2^Z , we observe a pattern similar to the previous case. Both the real and imaginary parts reach magnitudes of order 10^{-6} , with the absorptive part dominating at high energies. However, for values of P less than $2m_t$, the magnitudes of both the imaginary and real parts are comparable, contrasting with the behavior observed for the scenario involving an off-shell Higgs boson. In the case of the Z^*ZH vertex, the amplitudes of the contributing diagrams behave differently, as all exhibit an imaginary component. This phenomenon arises because the $\bar{t}c$ and $\bar{c}t$ pairs, which couple to the Z^* gauge boson in Fig 2, can be on shell for energies $P < 2m_t$. Therefore, in the Z^*ZH vertex, the absorptive part is not negligible at low values of P .

For the CP -violating form factor $\hat{h}_3^Z(I)$, both the real and imaginary parts can be as large as those found in the H^*ZZ scenario, although they are significantly smaller as P increases. The imaginary part becomes the dominant contribution in the energy range $300 \text{ GeV} \lesssim P \lesssim 700 \text{ GeV}$. At high values of P , the magnitudes of the real and absorptive parts become similar but of order 10^6 . For $\text{Im}[g_V g_A^*] \sim 10^{-5}$, the form factor h_3^Z can achieve values of order 10^{-8} , which is 3 orders of magnitude larger than the SM prediction.

B. Contributions of type II

The form factors obtained from diagrams of type II require a different approach, as they can be expressed through four combinations of the g_r ($r = V, A, S, P$) couplings. To assess the FCNC contributions numerically, we will examine four scenarios that align with the bounds outlined in Eqs. (7) and (8).

- (i) *Scenario I.* As the pseudoscalar coupling in Lagrangian (6) is not essential for inducing CP violation, we focus on the case where $g_P = 0$. We

employ the upper limits from Eqs. (7) and (8) to determine the values of the real and imaginary parts of the various combinations involved in the form factors $h_2^V(II)$ and $h_3^V(II)$. The resulting values are

$$\text{Re}[g_r g_S^*] = 0.00237 \text{ GeV}, \quad r = V, A, \quad (25)$$

$$\text{Im}[g_r g_S^*] = 0.00237 \text{ GeV}, \quad r = V, A. \quad (26)$$

- (ii) *Scenario II.* In this scenario, we analyze the impact of negative couplings on the behavior of the form factors. The real and imaginary parts that include the pseudoscalar coupling will be negative. Furthermore, we will consider the magnitudes of the couplings to be half of the values used in the previous case:

$$\text{Re}[g_r g_S^*] = -\text{Re}[g_r g_P^*] = 0.0011 \text{ GeV}, \quad r = V, A, \quad (27)$$

$$\text{Im}[g_r g_S^*] = -\text{Im}[g_r g_P^*] = 0.0011 \text{ GeV}, \quad r = V, A. \quad (28)$$

- (iii) *Scenario III.* Similar to the previous case, but with different negative values of the real and imaginary parts:

$$\begin{aligned} \text{Re}[g_V g_S^*] &= \text{Re}[g_A g_P^*] = -\text{Re}[g_A g_S^*] \\ &= -\text{Re}[g_V g_P^*] = 0.0011 \text{ GeV}, \end{aligned} \quad (29)$$

$$\begin{aligned} \text{Im}[g_V g_S^*] &= \text{Im}[g_A g_P^*] = -\text{Im}[g_A g_S^*] \\ &= -\text{Im}[g_V g_P^*] = 0.0011 \text{ GeV}. \end{aligned} \quad (30)$$

- (iv) *Scenario IV.* In this scenario, we only consider the contributions from the pseudoscalar coupling ($g_S = 0$). Furthermore, as the CP -violating form

factor h_3^V is not vanishing for the flavor-conserving case, we analyze the contributions to h_3^V that involve only top quarks in the loop. For the h_2^V ($V = H, Z$) form factor, these contributions are of order 10^{-20} and can be neglected. We utilize the same values as those considered in scenario *I*:

$$\text{Re}[g_r g_P^*] = 0.00237 \text{ GeV}, \quad r = V, A, \quad (31)$$

$$\text{Im}[g_r g_P^*] = 0.00237 \text{ GeV}, \quad r = V, A. \quad (32)$$

1. H^*ZZ

In Fig. 6, we show the behavior of $h_2^H(II)$ as a function of Q for the scenarios *I–III*. At low values of Q , the dominant contributions correspond to the real part, while the absorptive part is of a similar order of magnitude. This behavior differs from that observed in contributions of type I, where the imaginary part becomes relevant only for energies above the threshold $Q = 2m_t$. In the case of type II diagrams in Fig. 3, the Higgs boson is always coupled to a $\bar{t}c$ pair, which can be on shell behind the threshold energy. Consequently, at low energies, the amplitudes of all of the

diagrams of type II develop an absorptive part, resulting in the real and imaginary parts being comparable in magnitude. As the energy increases, the imaginary part emerges as the dominant contribution. Notably, the most significant results are obtained in scenarios *I* and *III*, of order 10^{-8} . These values are 3 orders of magnitude larger than in the SM [21] and 2 orders of magnitude less than the contributions from type I diagrams. Additionally, we observe distinct patterns in scenarios *II* and *III*, indicating the significant impact of negative couplings on the behavior of $h_2^H(II)$. We do not present scenario IV, which involves only top quarks running in the loop, as h_2^H is of order 10^{-20} .

For the CP -violating form factor, we present in Fig. 7 the absolute value of the real and absorptive part of $h_3^H(II)$ as a function of Q . We observe that both contributions alternate in dominance across different energy regions. Furthermore, they decrease in magnitude and converge to similar values at high energies. In scenario *IV*, the absorptive part is zero for $Q < 2m_t$, as only top quarks are included in the loop. We find the most significant values in scenarios *I* and *IV*, where the real and imaginary parts can reach magnitudes of order 10^{-7} . These values are 4 and 1 orders of magnitude larger than the prediction in the SM [22] and those obtained from

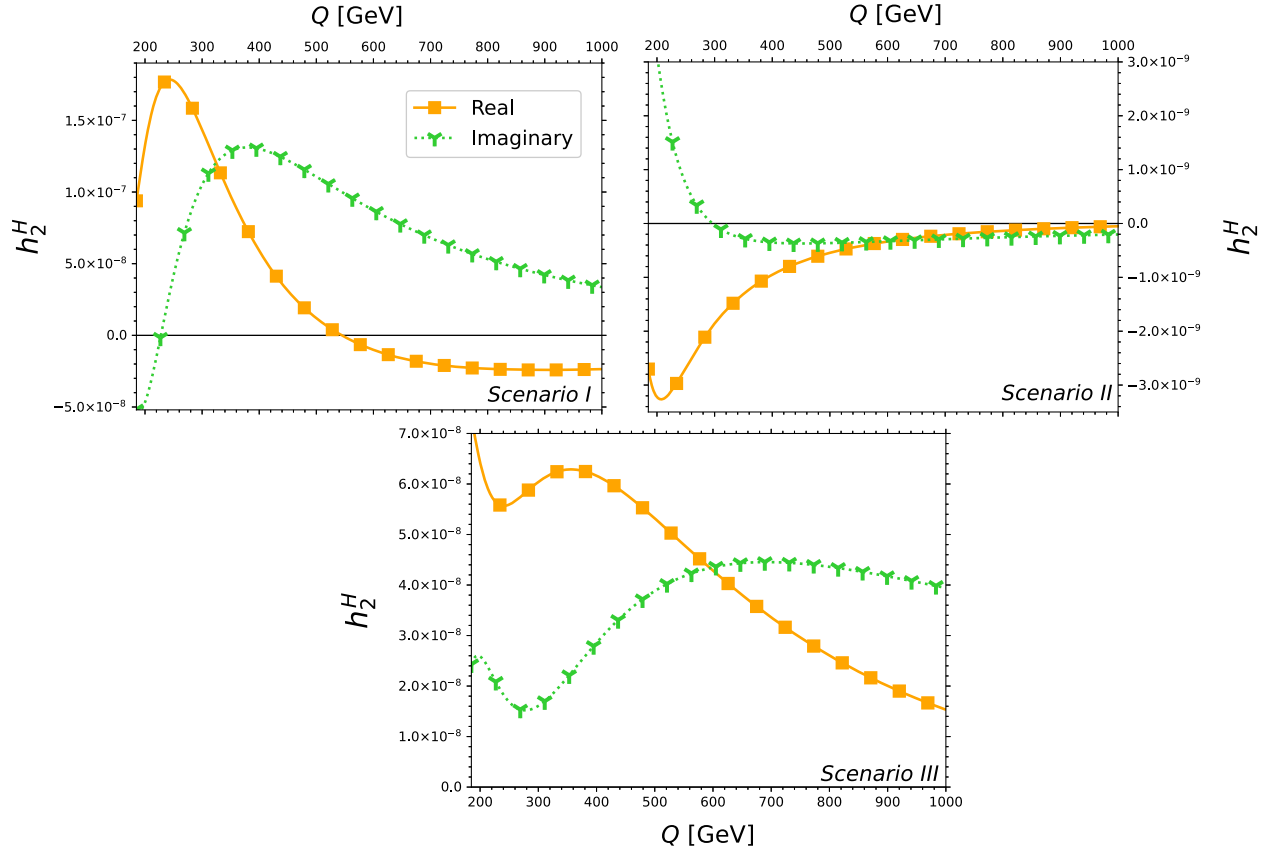


FIG. 6. FCNC contributions of the type II to the h_2^H form factor as a function of Q . For the values of the different couplings, we consider the scenarios *I–III*, whereas the contribution in scenario *IV* is tiny and not shown. We only consider the $Z\bar{t}c$ and $H\bar{t}c$ couplings.

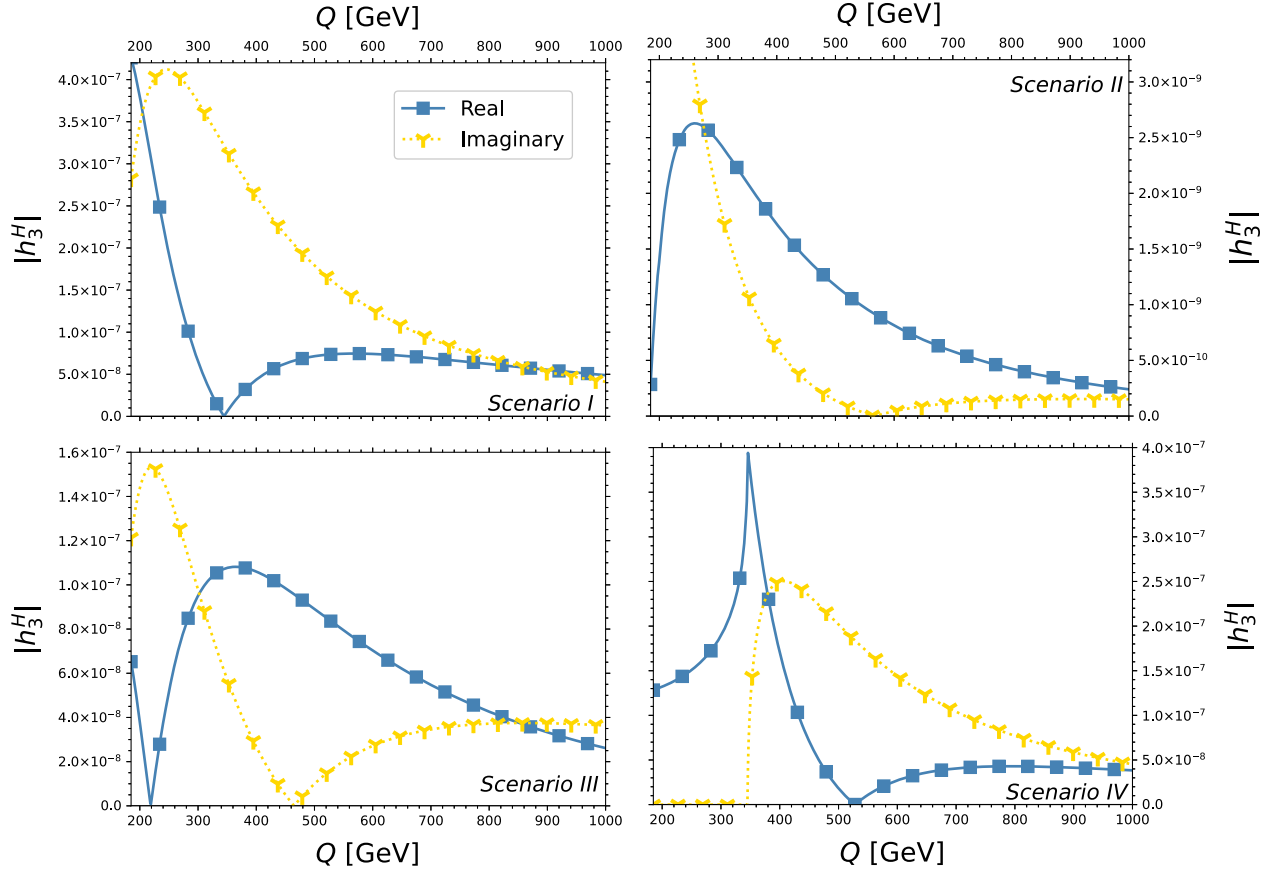


FIG. 7. FCNC contributions of the type II to the h_3^H form factor as a function of Q . For the values of the different couplings, we consider the scenarios I–IV discussed in this section. We only consider the $Z\bar{t}c$ and $H\bar{t}c$ couplings.

type I diagrams, respectively. Hence, relevant contributions to the left-right asymmetries from type II diagrams are feasible. Moreover, distinct patterns are evident in the four scenarios, indicating that negative couplings impact the behavior of $h_3^H(II)$. In contrast with the CP -conserving form factor $h_2^H(II)$, the contributions from the pseudoscalar coupling are significant for flavor-conserving scenarios as they are not negligible in scenario IV.

2. Z^*ZH

For the case of an off-shell Z boson, we present the behavior of $h_2^Z(II)$ as a function of P in Fig. 8. We note that the magnitudes of the real and absorptive parts are comparable, with the imaginary part dominating at high energies. The largest values occur in scenario III, reaching orders of 10^{-7} . In scenario IV, the contributions are of order 10^{-20} and are not shown. Similar to the H^*ZZ case, we find that negative couplings significantly influence the behavior of $h_2^Z(II)$. Our numerical results for $h_2^Z(II)$ are analogous to those with an off-shell Higgs boson. However, they are 1 order of magnitude smaller than those from contributions of type I and are negligible compared to the SM result.

In Fig. 9, we show the absolute value of the real and imaginary parts of the CP -violating form factor $h_3^Z(II)$. In scenarios I–II, the imaginary part dominates, while in scenario III, the real part is the most significant contribution. In scenario IV, the absorptive part is zero for energies below $2m_t$, but it becomes the main contribution at higher energies. The real and absorptive parts of $h_3^Z(II)$ can reach values of order 10^{-7} , 4 and 1 orders of magnitude larger than the SM predictions and those derived from type I diagrams, respectively. As observed in the H^*ZZ vertex, the pseudoscalar coupling plays a relevant role in flavor-violating and flavor-conserving contributions to the CP -violating form factor h_3^V . An abrupt change at the threshold energy of $2m_t$ is noteworthy for both the real and imaginary parts, which differs from the behavior observed in Fig. 7.

In summary, the imaginary and real parts of h_2^V and h_3^V ($V = H, Z$) are comparable for both contributions from diagrams type I and II. This behavior has also been observed in other off-shell couplings, such as trilinear neutral gauge bosons couplings [125,126] and the $g\bar{q}q$ vertex [127,128]. The numerical values obtained for the CP -conserving form factor are negligible compared to those predicted in the SM and the current limits. In contrast,

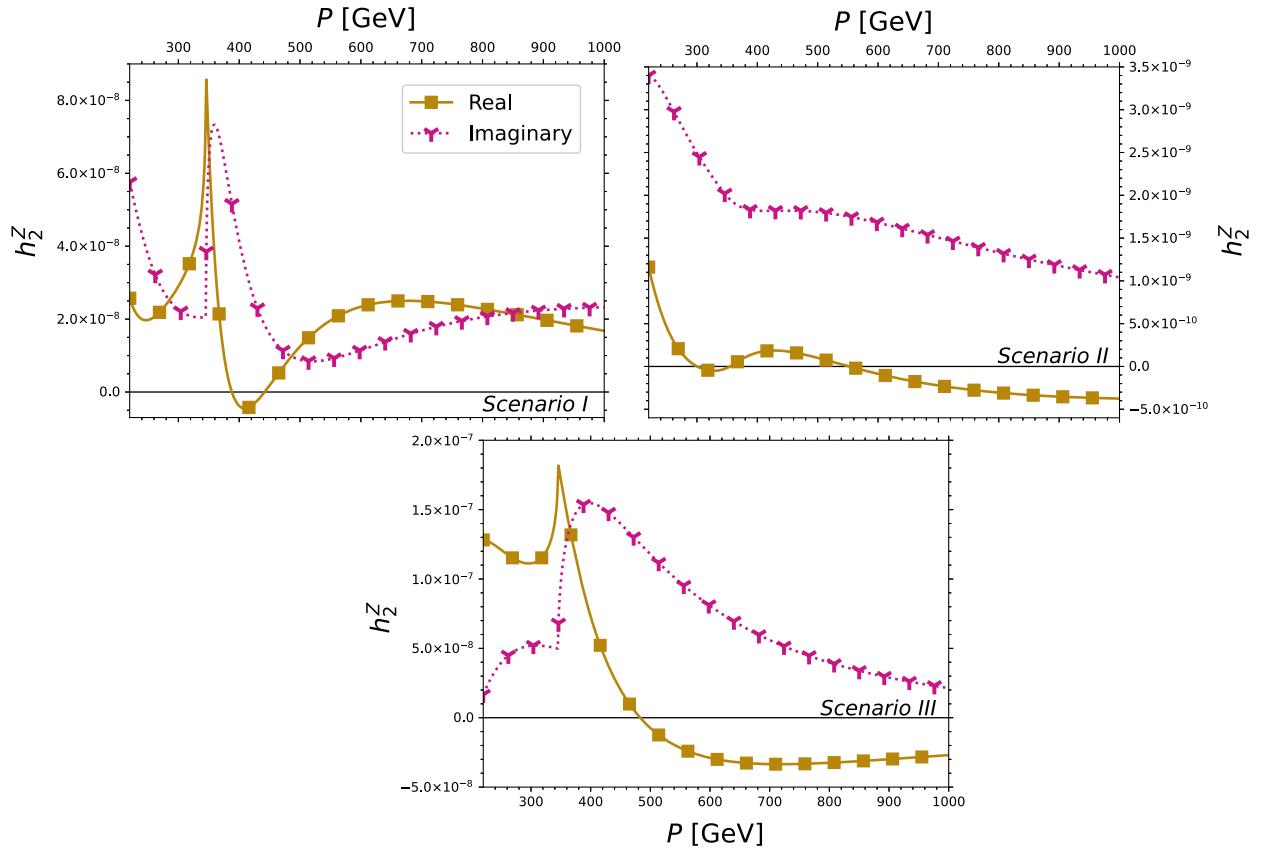


FIG. 8. FCNC contributions of the type II to the h_2^Z form factor as a function of P . For the values of the different couplings, we consider the scenarios I–III, whereas the contribution in scenario IV is tiny and not shown. We only consider the $Z\bar{t}c$ and $H\bar{t}c$ couplings.

the contributions to h_3^V can be up to 5 orders larger than in the SM. This form factor and the SM contributions to h_1^V can lead to nonzero left-right asymmetries, as discussed in Sec. III.

C. Contributions to the $\mathcal{A}_{LR}^V$ ($V=H, Z$) asymmetry

We now perform a numerical evaluation of the left-right asymmetries $\mathcal{A}_{LR}^V$ ($V=H, Z$) discussed in Sec. III.

We note from Eqs. (14) and (22) that only the form factors h_1^V and h_3^V are necessary for analyzing the behavior of the $\mathcal{A}_{LR}^V$ ($V=H, Z$) asymmetries. For the CP -conserving form factor in Eq. (3), we will only consider the tree-level and the SM one-loop contribution to $\hat{b}_Z$, as reported in Ref. [21]. Regarding the CP -violating form factor h_3^V ($V=H, Z$), we will utilize the expressions derived from FCNC contributions of both type I and II, discussed in Sec. II.

1. Type I

In Fig. 10, we present the $\mathcal{A}_{LR}^V$ ($V=H, Z$) asymmetries as a function of Q and P , considering the FCNC contributions of type I. Using the upper bounds stated in Eq. (7), we find that $\text{Im}[g_V^c g_A^c]$ can reach magnitudes of

order 10^{-5} . Accordingly, we utilize values of -8×10^{-5} and 10^{-5} for $\text{Im}[g_V^c g_A^c]$ in Eq. (10). The first value leads to the largest contributions for both asymmetries, of order 10^{-7} for the case involving an off-shell Higgs and 10^{-8} for the off-shell Z boson. In contrast, the latter value of $\text{Im}[g_V^c g_A^c]$ results in asymmetries that are 1 order of magnitude smaller. Additionally, we observe distinct patterns between $\mathcal{A}_{LR}^H$ and $\mathcal{A}_{LR}^Z$. Both asymmetries exhibit an inflection point at $Q=2m_t$, where the one-loop SM contributions develop a notable imaginary part.

The SM prediction for the CP -violating form factor h_3^H is approximately of order 10^{-11} [22]. Based in this estimate, the $\mathcal{A}_{LR}^H$ asymmetry has been calculated to be of order 10^{-8} – 10^{-9} [21]. If we assume that the form factor h_3^Z is similar in magnitude to that of an off-shell H boson, we find that $\mathcal{A}_{LR}^Z$ can fall within the range 10^{-11} – 10^{-12} in the SM. Thus, our results in Fig. 10 are 1 to 3 orders of magnitude larger than those predicted by the SM.

2. Type II

Similar to the previous case, we draw in Fig. 11 the $\mathcal{A}_{LR}^V$ ($V=H, Z$) asymmetries for the contributions of type II. We have examined the four scenarios introduced in

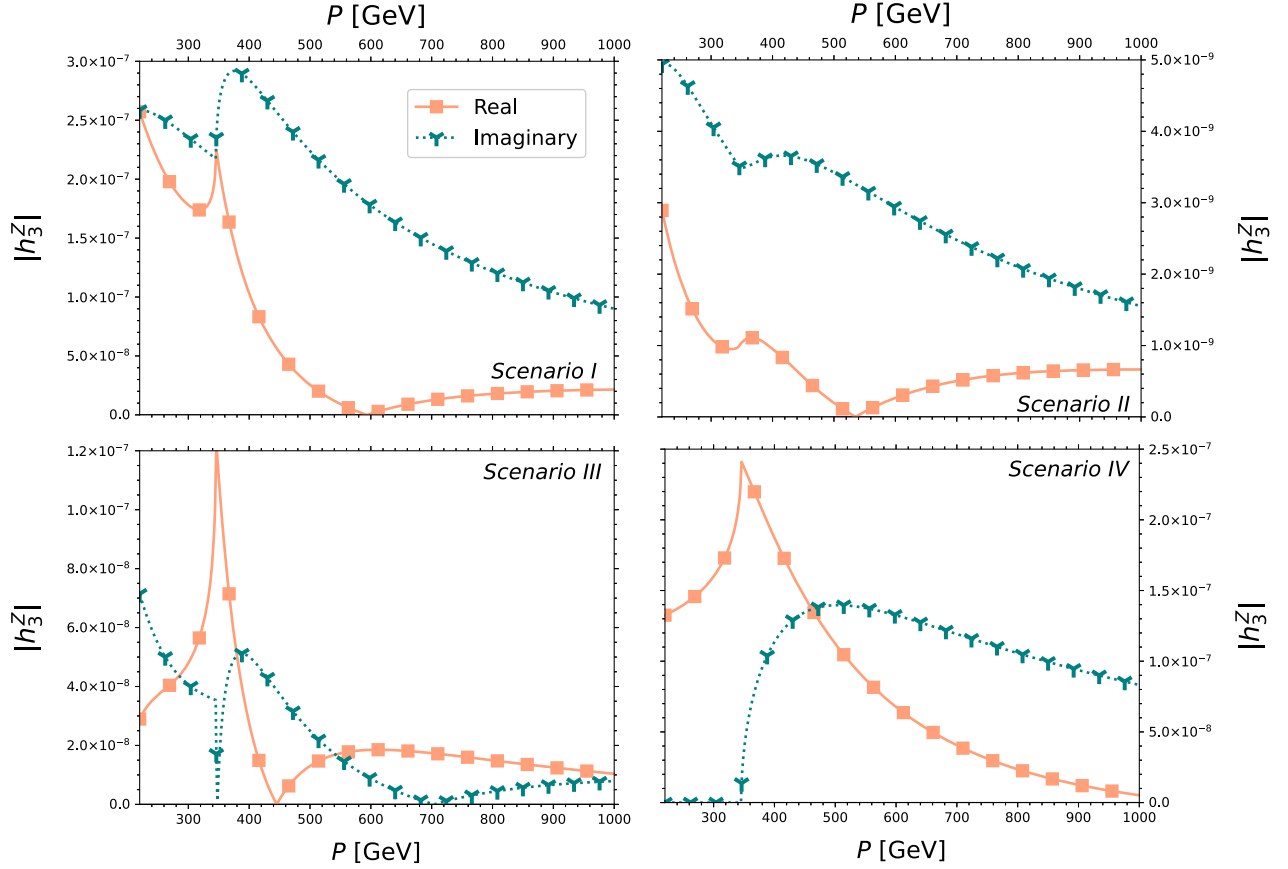


FIG. 9. FCNC contributions of the type II to the h_3^Z form factor as a function of P . For the values of the different couplings, we consider the scenarios *I–IV* discussed in this section. We only consider the $Z\bar{t}c$ and $H\bar{t}c$ couplings.

Sec. IV B. In both asymmetries, scenario *II* is negligible, while the remaining scenarios yield values ranging from 10^{-6} to 10^{-7} . The most significant results are obtained in scenarios *I* and *IV*, where $h_3^V(II)$ also reaches its highest values. Comparing both asymmetries, we observe that in

scenarios *I* and *III*, the $\mathcal{A}_{LR}^H$ asymmetry does not exhibit an inflection point at $Q = 2m_t$. This phenomenon can be explained by the absence of any significant change in the real and imaginary parts of $h_3^H(II)$ at the threshold energy of $2m_t$, as illustrated in Fig. 7.

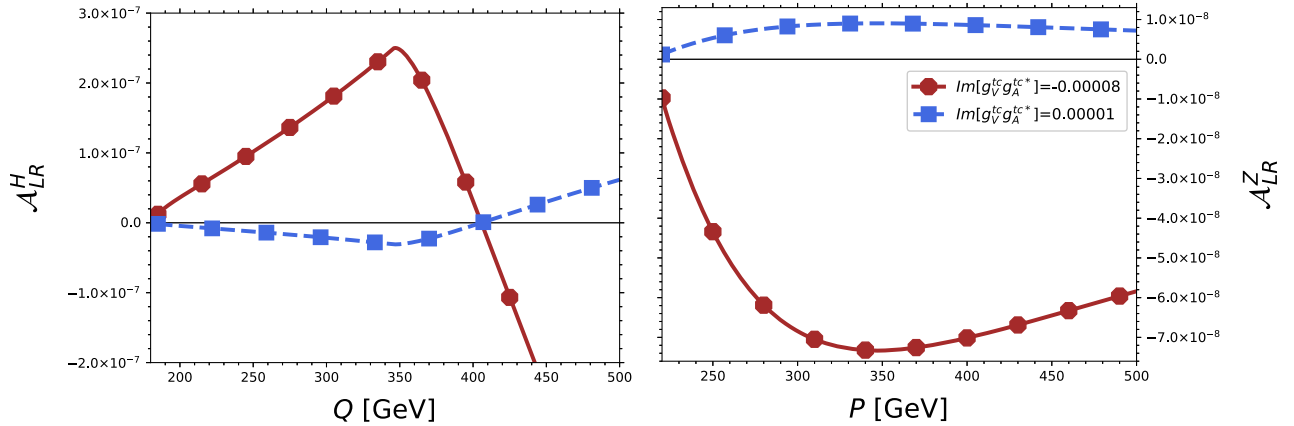


FIG. 10. The asymmetries $\mathcal{A}_{LR}^H$ and $\mathcal{A}_{LR}^Z$ as a function of Q and P , respectively. For the CP -conserving form factor h_2^V ($V = H, Z$), we have considered the one-loop level contributions from the SM, whereas for h_3^V we used the results obtained for FCNC contributions of type I.

In scenario IV, the asymmetries become relevant at $Q = 2m_t$, which aligns with the energy level where the h_1^V and h_3^V ($V = H, Z$) form factors also achieve considerable values. This finding indicates that the absorptive parts of the form factors notably influence the behavior of the asymmetries and should be considered in future analyses. We observe similar patterns between the two asymmetries, contrasting with the results obtained in Fig. 10.

The $\mathcal{A}_{LR}^H$ and $\mathcal{A}_{LR}^Z$ asymmetries in Fig. 11 are 1 and 2 orders of magnitude larger than the corresponding from FCNC contributions of type I, respectively. Compared with the SM predictions, our results are greater by 2 to 5 orders of magnitude.

Finally, the feasibility of measuring the left-right asymmetries discussed in this paper may be questioned. Recently, two methods have been developed to study polarized ZZ final states at the LHC. The first method is the matrix-element reweighting method [69], which utilizes unpolarized events to obtain polarized observables through a fully differential reweighting technique. The second method can be easily integrated into existing public simulation tools and treats the polarizations (λ) of the Z bosons as the propagators of new V_λ bosons [70]. In both methods, the definition of the polarized intermediate states is based on the completeness relationship:

$$g_{\mu\nu} - \frac{q_\mu q_\nu}{M_V^2} = \sum_\lambda \epsilon_\mu(q, \lambda) \epsilon_\nu^*(q, \lambda), \quad \lambda = L, R, 0, \quad (33)$$

which is introduced in the four-lepton decay amplitude by substituting the Z boson propagators. In the case of an off-shell Z boson, this relationship can be extended by incorporating an auxiliary polarization ϵ_A^μ , which becomes zero when the boson is resonant [68]. To derive the polarized observables from the unpolarized amplitude in the $H^* \rightarrow ZZ \rightarrow 4\ell$ process, one must consider only the

amplitudes with on-shell Z . However, this selection breaks gauge invariance, which can be restored by employing the narrow-width or double-pole approximation [87].

The decay channel $H \rightarrow ZZ^* \rightarrow 4\ell$ has been studied at the LHC [26, 129]. The signal region (SR) for this process is defined within the range of $115 \text{ GeV} < m_{4\ell} < 130 \text{ GeV}$, where $m_{4\ell}$ represents the four-lepton invariant mass. For the lepton pairs, the invariant masses must satisfy the conditions: $50 \text{ GeV} < m_{12} < 106 \text{ GeV}$ and $12 \text{ GeV} < m_{34} < 115 \text{ GeV}$. The m_{12} requirement ensures that the lepton pair originates from an on-shell Z boson. Furthermore, the leading three leptons with the highest transverse momentum must have p_T values exceeding 20, 15, and 10 GeV, respectively. The four leptons also require an angular separation of $\Delta R > 0.10$. For the process $H^* \rightarrow ZZ \rightarrow 4\ell$, the SR is defined above the Higgs mass at $180 \text{ GeV} < m_{4\ell}$ [11]. The same transverse momentum criteria of the leading three leptons established in the $H \rightarrow ZZ^*$ case are maintained. However, the four-lepton invariant masses conditions must satisfy $50 \text{ GeV} < m_{12} < 106 \text{ GeV}$ and $50 \text{ GeV} < m_{34} < 115 \text{ GeV}$ for $190 \text{ GeV} < m_{4\ell}$, ensuring that both Z bosons are on shell. These SRs have been used to assess the expected sensitivity to new physics contributions in polarized observables at the LHC [69, 70].

The observation of the left-right asymmetries necessitates precise measurements on polarized amplitudes. For run 3 at the LHC, with an integrated luminosity of $\mathcal{L} = 500 \text{ fb}^{-1}$, the anticipated sensitivity to the ratios ($\mathcal{R}_{L,R}$) between new physics and SM production of transversal polarizations is projected to lie within $0.5 \lesssim \mathcal{R}_{L,R} \lesssim 1.7$ ($0.7 \lesssim \mathcal{R}_{L,R} \lesssim 1.5$) at 95% (68%) CL, taking into account both systematic and statistical uncertainties associated with the $H^* \rightarrow ZZ$ process [70]. In Fig. 12, we show the behavior of the ratios $\mathcal{R}_{L,R}$ as a function of $m_{4\ell}$ for different values of h_3^H within the range 10^{-2} – 10^{-5} . These values align with the current constraints on the HZZ

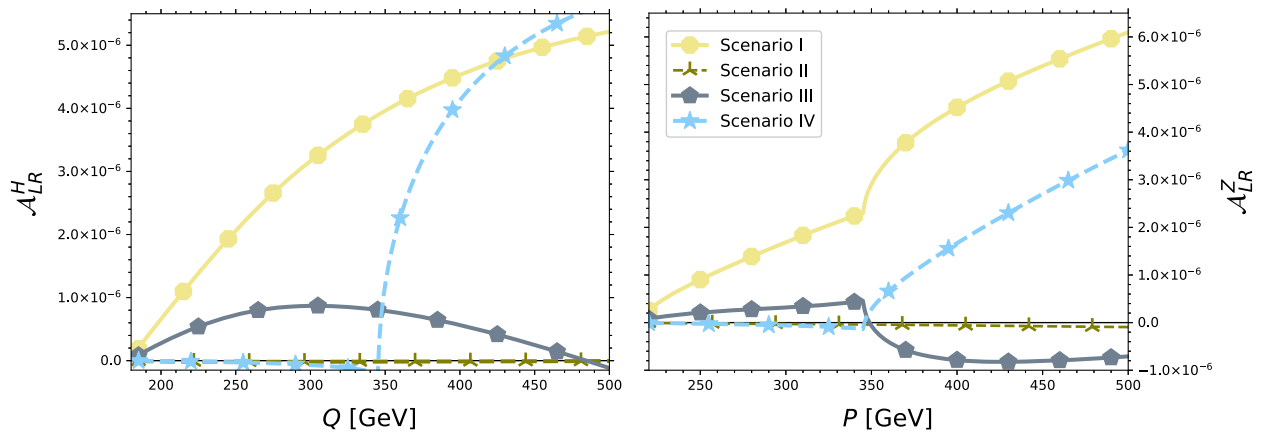


FIG. 11. The asymmetries $\mathcal{A}_{LR}^H$ and $\mathcal{A}_{LR}^Z$ as a function of Q and P , respectively. For the CP -conserving form factor h_2^V ($V = H, Z$), we have considered up the one-loop level contributions from the SM, whereas for h_3^V we used the results obtained for FCNC contributions of type II.

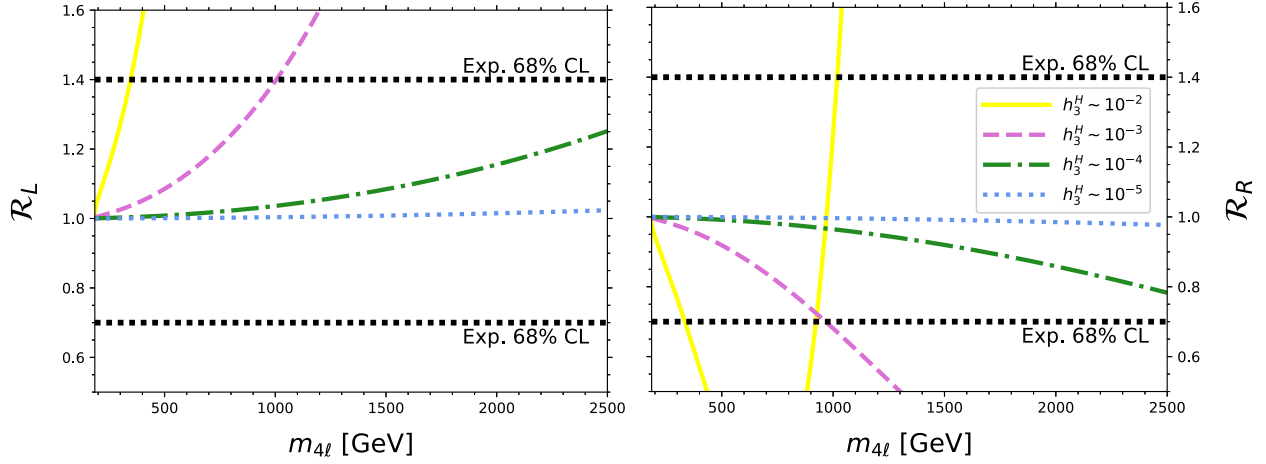


FIG. 12. Ratios between new physics and SM production of transversal polarizations as a function of the four-lepton invariant mass. We also plot the upper and lower expected limits at 68% CL [70].

anomalous couplings [10,24]. Notably, the CP -violating contributions in the left- and right-polarized amplitudes become more significant as the four-lepton invariant mass increases [21]. We find that the effects corresponding to h_3^H values of order 10^{-2} and 10^{-3} can be achieved at the 68% CL for $m_{4\ell}$ less than 500 and 1000 GeV, respectively. For $h_3^H \sim 10^{-4}$, invariant masses above 2500 GeV are required to distinguish the consequences of CP violation. The projected sensitivity could be further improved using deep neural networks, similar to those implemented by the ATLAS or CMS collaborations [70]. Nevertheless, we estimate that, to observe effects of CP -violating contributions of order 10^{-5} for low values of $m_{4\ell}$, the sensitivity on $\mathcal{R}_{L,R}$ would need to be increased by at least 2 orders of magnitude.

As both theoretical and phenomenological methods for studying polarized processes continue to advance [67–71,130–134], we expect a significant enhancement in sensitivity to these observables soon. In this context, the asymmetries $\mathcal{A}_{LR}^V$ ($V = H, Z$) that arises from small CP -violating contributions may become measurable. The observation of a nonzero $\mathcal{A}_{LR}^V$ ($V = H, Z$) asymmetry would indicate the presence of new sources of CP violation.

V. CONCLUSIONS

In this work, we computed new contributions to the H^*ZZ and Z^*ZH vertices that arise from FCNC couplings mediated by the Z and Higgs bosons. We identified two distinct categories of contributing Feynman diagrams. The first contribution, type I, results from considering only the FCNC couplings mediated by the Z boson. The second contribution, type II, emerges when the FCNC couplings of the Higgs bosons are also included. From both scenarios, we found new contributions to the form factors h_2^V and h_3^V ($V = H, Z$), with the results expressed in terms of the

Passarino-Veltman scalar functions. Notably, the CP -violating form factor h_3^V can even be induced without flavor violation in a pseudoscalar coupling scenario. Using bounds on the FCNC couplings of the top quark, we determined that the contributions to h_2^V can be of order 10^{-6} – 10^{-7} , which are significantly small compared with those in the SM. In contrast, the CP -violating form factor h_3^V can reach values of 10^{-8} and 10^{-7} for type I and II contributions, respectively. These estimates are 3 and 4 orders of magnitude larger than the predictions of the SM. Additionally, we observed that the absorptive part is comparable to the real part and can dominate in certain energy regions. The contributions of FCNC mediated by the Z and H bosons to the HZZ vertex have not been reported previously.

Furthermore, we calculated the left-right asymmetry $\mathcal{A}_{LR}^Z$ for the $Z^* \rightarrow ZH$ process for the first time. Our results show that the $\mathcal{A}_{LR}^Z$ shares a similar form to the reported for the $H^* \rightarrow ZZ$ process, as both are in terms of the form factors h_1^V and h_3^V . Using the SM one-loop level expressions for h_1^V along with the contributions from FCNC couplings to h_3^V , we found that $\mathcal{A}_{LR}^H$ and $\mathcal{A}_{LR}^Z$ can reach significantly larger values than those derived from SM contributions alone. The most notable results arise from contributions of type II, where the asymmetries can be up to 5 orders of magnitude larger than in the SM. Additionally, we observe that the imaginary parts of the form factors considerably influence the behavior of the asymmetries. This finding suggests that the absorptive parts should not be ignored when studying physical observables. The detection of a nonzero left-right asymmetry $\mathcal{A}_{LR}^V$ ($V = H, Z$) would indicate the presence of CP violation in the HZZ vertex. However, the realization of such an observation requires enhanced sensitivity to polarized observables. We encourage studies focused on precise measurements of these observables.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX: ANALYTICAL FORMS

This appendix presents the analytical expressions for the functions appearing in the h_2^V and h_3^V ($V = H, Z$) form factors. Our results are in terms of the Passarino-Veltman

scalar functions. We introduce the shorthand notation:

$$B_{ij}(c^2) = B_0(c^2, m_i^2, m_j^2), \quad (A1)$$

$$C_{ijk}(Q^2) = C_0(m_Z^2, m_Z^2, Q^2, m_i^2, m_j^2, m_k^2), \quad (A2)$$

$$C_{ijk}(P^2) = C_0(m_H^2, m_Z^2, P^2, m_i^2, m_j^2, m_k^2), \quad (A3)$$

where B_0 and C_0 are the usual two- and three-point Passarino-Veltman scalar functions. The following symmetry relations will also be useful:

$$B_{ij}(c^2) = B_{ji}(c^2), \quad (A4)$$

$$C_{ijk}(c^2) = C_{kji}(c^2). \quad (A5)$$

1. Diagrams type I

The functions A_V^V and A_A^V ($V = H, Z$) for contributions of type I are given as follows:

$$\begin{aligned} A_V^H(Q^2, m_i^2, m_j^2) = & \frac{1}{Q^2(Q^2 - 4m_Z^2)^2} \left\{ \left[4m_i^2(m_i^2 - m_j^2)(4m_Z^2 - Q^2) \right] B_{ii}(0) + \left[4m_j^2(m_j^2 - m_i^2)(4m_Z^2 - Q^2) \right] B_{jj}(0) \right. \\ & + \left[2Q^4(m_i - m_j)^2 - 4Q^2\{m_Z^2(-4m_i m_j + m_i^2 + m_j^2) + (m_i^2 - m_j^2)^2\} \right. \\ & + 8m_Z^4(m_i^2 + m_j^2) - 8m_Z^2(m_i^2 - m_j^2)^2 \left. \right] B_{ij}(m_Z^2) + 2m_i \left[Q^4(m_j - m_i) \right. \\ & + 2Q^2\{m_Z^2(m_i - 2m_j) + 2m_i(m_i - m_j)(m_i + m_j)\} - 4m_i m_Z^2(m_i^2 - m_j^2 + m_Z^2) \left. \right] B_{ii}(Q^2) \\ & + 2m_j \left[Q^4(m_i - m_j) + 2Q^2\{m_Z^2(m_j - 2m_i) + 2m_j(m_j - m_i)(m_j + m_i)\} \right. \\ & - 4m_j m_Z^2(m_j^2 - m_i^2 + m_Z^2) \left. \right] B_{jj}(Q^2) + m_i \left[2Q^4\{-m_Z^2(m_i + 3m_j) - m_i^2 m_j - 3m_i m_j^2 \right. \\ & + m_i^3 + m_j^3\} + 8Q^2\{m_Z^2(m_i^2 m_j + 2m_i m_j^2 + m_i^3 - m_j^3) - m_i(m_i^2 - m_j^2)^2 + m_j m_Z^4\} \\ & + 8m_i m_Z^2(m_i - m_j - m_Z)(m_i + m_j - m_Z)(m_i - m_j + m_Z)(m_i + m_j + m_Z) + Q^6 m_j \left. \right] C_{iji}(Q^2) \\ & + m_j \left[2Q^4\{-m_Z^2(m_j + 3m_i) - m_j^2 m_i - 3m_j m_i^2 + m_j^3 + m_i^3\} + 8Q^2\{m_Z^2(m_j^2 m_i + 2m_j m_i^2 \right. \\ & + m_j^3 - m_i^3) - m_j(m_j^2 - m_i^2)^2 + m_i m_Z^4\} + 8m_j m_Z^2(m_j - m_i - m_Z)(m_j + m_i - m_Z) \\ & \times (m_j - m_i + m_Z)(m_j + m_i + m_Z) + Q^6 m_i \left. \right] C_{jij}(Q^2) \\ & \left. + 2 \left[4m_Z^2 - Q^2 \right] \left[m_i^2(-4m_j^2 - 2m_Z^2 + Q^2) + 2m_i^4 + m_j^2(2m_j^2 - 2m_Z^2 + Q^2) \right] \right\} \quad (A6) \end{aligned}$$

$$\begin{aligned} A_A^H(Q^2, m_i^2, m_j^2) = & A_V^H(Q^2, m_i^2, m_j^2) - \frac{1}{Q^2(Q^2 - 4m_Z^2)^2} 2Q^2 m_i m_j (Q^2 - 4m_Z^2) \left[-4B_{ij}(m_Z^2) + 2B_{ii}(Q^2) + 2B_{jj}(Q^2) \right. \\ & \left. + \{Q^2 - 2(m_i^2 - m_j^2 + m_Z^2)\} C_{ij}(Q^2) + \{2m_i^2 - 2(m_j^2 + m_Z^2) + Q^2\} C_{ji}(Q^2) \right] \quad (A7) \end{aligned}$$

$$\begin{aligned}
A_V^Z(P^2, m_i^2, m_j^2) = & \frac{1}{((m_H - m_Z)^2 - P^2)((m_H + m_Z)^2 - P^2)} \left\{ -4m_i^2(m_i - m_j)(m_i + m_j)B_{ii}(0) \right. \\
& + m_i \left[\frac{3m_H m_i(m_Z(m_H + m_Z) + m_i^2 - m_j^2)}{m_H + m_Z - P} + \frac{3m_H m_i(m_Z(m_Z - m_H) + m_i^2 - m_j^2)}{m_H - m_Z + P} \right. \\
& - \frac{3m_H m_i(m_Z(m_Z - m_H) + m_i^2 - m_j^2)}{-m_H + m_Z + P} + \left. \frac{3m_H m_i(m_Z(m_H + m_Z) + m_i^2 - m_j^2)}{m_H + m_Z + P} \right. \\
& - 2(m_i(m_H^2 - 2m_j^2 + m_Z^2) - m_H^2 m_j + 2m_i^3) - 2P^2 m_i \left. \right] B_{ii}(m_H^2) \\
& + \left[\frac{3m_Z(-m_H m_Z(m_i^2 + m_j^2) + m_Z^2(m_i^2 + m_j^2) + (m_i^2 - m_j^2)^2)}{-m_H + m_Z - P} \right. \\
& + \frac{3m_Z(m_i^2(m_Z(m_H + m_Z) - 2m_j^2) + m_j^2(m_Z(m_H + m_Z) + m_j^2) + m_i^4)}{m_H + m_Z - P} \\
& + \frac{3m_Z(-m_H m_Z(m_i^2 + m_j^2) + m_Z^2(m_i^2 + m_j^2) + (m_i^2 - m_j^2)^2)}{-m_H + m_Z + P} \\
& + \frac{3m_Z(m_i^2(m_Z(m_H + m_Z) - 2m_j^2) + m_j^2(m_Z(m_H + m_Z) + m_j^2) + m_i^4)}{m_H + m_Z + P} - 2m_H^2 m_i m_j \\
& + m_H^2 m_i^2 + m_H^2 m_j^2 - P^2(m_i - m_j)^2 - 2m_i m_j m_Z^2 + 4m_i^2 m_j^2 - 9m_i^2 m_Z^2 - 2m_i^4 - 9m_j^2 m_Z^2 \\
& - 2m_j^4 \left. \right] \frac{B_{ij}(m_Z^2)}{2} + \frac{1}{((m_H - m_Z)^2 + P^2)((m_H + m_Z)^2 - P^2)} \left[-2m_i m_j((m_H - m_Z)^2 - P^2) \right. \\
& \times ((m_H + m_Z)^2 - P^2)(m_H^2 - m_Z^2 + P^2) + m_i^2(-5P^4(m_H^2 + 4m_j^2 - m_Z^2) \\
& + P^2(16m_j^2(m_H^2 + m_Z^2) + 6m_H^2 m_Z^2 + m_H^4 - 7m_Z^4) + (m_H^2 - m_Z^2)^2(m_H^2 + 4m_j^2 - m_Z^2) + 3P^6) \\
& - 2m_i^4(4P^2(m_H^2 + m_Z^2) + (m_H^2 - m_Z^2)^2 - 5P^4) + m_j^2(5P^4(-m_H^2 + 2m_j^2 + m_Z^2) \\
& + P^2(-8m_j^2(m_H^2 + m_Z^2) + 6m_H^2 m_Z^2 + m_H^4 - 7m_Z^4) + (m_H^2 - m_Z^2)^2(m_H^2 - 2m_j^2 - m_Z^2) \\
& + 3P^6) \left. \right] \frac{B_{ij}(P^2)}{2} + \frac{m_i}{((m_H - m_Z)^2 - P^2)((m_H + m_Z)^2 - P^2)} \left[2m_i^3(m_j^2(-4m_H^2(m_Z^2 + P^2) \right. \\
& + 8m_H^4 - 4(m_Z^2 - P^2)^2) + 2m_H^4(m_Z^2 + P^2) + m_H^2(6P^2 m_Z^2 - 7m_Z^4 - 7P^4) + m_H^6 \\
& + 4(m_Z^2 - P^2)^2(m_Z^2 + P^2)) + m_i(4m_j^4(m_H^2(m_Z^2 + P^2) - 2m_H^4 + (m_Z^2 - P^2)^2) \\
& + 2m_j^2(4m_H^4(m_Z^2 + P^2) + m_H^2(-10P^2 m_Z^2 + m_Z^4 + P^4) - 3m_H^6 - 2(m_Z^2 - P^2)^2(m_Z^2 + P^2)) \\
& + (-m_H^2 + m_Z^2 + P^2)(-P^4(2m_H^2 + m_Z^2) + P^2(8m_H^2 m_Z^2 + m_H^4 - m_Z^4) + (m_Z^3 - m_H^2 m_Z)^2 \\
& + P^6)) - 2m_H^2 m_i^2 m_j((m_H - m_Z)^2 - P^2)((m_H + m_Z)^2 - P^2) + 4m_i^5(m_H^2(m_Z^2 + P^2) \\
& - 2m_H^4 + (m_Z^2 - P^2)^2) + m_H^2 m_j((m_H - m_Z)^2 - P^2)((m_H + m_Z)^2 - P^2)(m_H^2 + 2m_j^2 - m_Z^2 \\
& - P^2) \left. \right] C_{iii}(P^2) + (m_i^2(-m_H^2 + 4m_j^2 + m_Z^2 + P^2) + m_j^2(-m_H^2 - 2m_j^2 + m_Z^2 + P^2) - 2m_i^4) \\
& + (i \leftrightarrow j) \left. \right\}. \tag{A8}
\end{aligned}$$

$$\begin{aligned}
A_A^Z(p^2, m_i^2, m_j^2) = & A_V^Z(P^2, m_i^2, m_j^2) - \frac{2m_i m_j}{((m_H - m_Z)^2 - P^2)((m_H + m_Z)^2 - P^2)} \left[-2m_H^2 \{B_{ij}(P^2) + B_{ij}(m_Z^2)\} \right. \\
& + 2m_H^2 \{B_{ii}(m_H^2) + B_{jj}(m_H^2)\} + 2P^2 \{B_{ij}(m_Z^2) - B_{ij}(P^2)\} + 2m_Z^2 \{B_{ij}(P^2) - B_{ij}(m_Z^2)\} \\
& \left. + m_H^2(m_H^2 - 2m_i^2 + 2m_j^2 - m_Z^2 - P^2)C_{iij}(P^2) + m_H^2(m_H^2 + 2m_i^2 - 2m_j^2 - m_Z^2 - P^2)C_{jji}(P^2) \right]. \quad (A9)
\end{aligned}$$

The functions $\tilde{\mathcal{F}}^V$ ($V = H, Z$) for contributions of type I are expressed as follows:

$$\begin{aligned}
\tilde{\mathcal{F}}^H(Q^2, m_i, m_j) = & \frac{1}{(4m_Z^2 - Q^2)} \left\{ 2 \left[B_{jj}(Q^2) - B_{ii}(Q^2) \right] + \left[2(m_i^2 - m_j^2 + m_Z^2) - Q^2 \right] C_{iji}(Q^2) \right. \\
& \left. + \left[2(m_i^2 - m_j^2 - m_Z^2) + Q^2 \right] C_{jij}(Q^2) \right\}, \quad (A10)
\end{aligned}$$

$$\begin{aligned}
\tilde{\mathcal{F}}^Z(Q^2, m_i, m_j) = & -\frac{m_H^2}{((m_H - m_Z)^2 - P^2)((m_H + m_Z)^2 - P^2)} \left\{ 2 \left[B_{jj}(m_H^2) - B_{ii}(m_H^2) \right] \right. \\
& \left. + (-m_H^2 + 2m_i^2 - 2m_j^2 + m_Z^2 + P^2)C_{iij}(P^2) + (m_H^2 + 2m_i^2 - 2m_j^2 - m_Z^2 - P^2)C_{jji}(P^2) \right\}. \quad (A11)
\end{aligned}$$

2. Diagrams type II

The functions $R_{1,2,3,4}^V$ ($V = H, Z$) for contributions of type II are given as follows:

$$\begin{aligned}
R_1^H(Q^2, m_i, m_j) = & \frac{1}{Q^4(Q^2 - 4m_Z^2)^2} \left\{ m_Z^2 \left[6m_i^2 m_j (Q^2 - 2m_Z^2) - m_i (6m_j^2 (Q^2 - 2m_Z^2) - 10Q^2 m_Z^2 + Q^4) \right. \right. \\
& + 6m_i^3 (Q^2 - 2m_Z^2) + 3m_j (Q^2 - 2m_j^2)(Q^2 - 2m_Z^2) \left. \right] B_{ii}(m_Z^2) \\
& + Q^2 m_Z^2 (m_i + m_j)(2m_Z^2 + Q^2) [B_{ij}(m_Z^2) - 2B_{ij}(Q^2)] \\
& + \left[-6m_i^4 m_j m_Z^2 (Q^2 - 2m_Z^2) - 2m_i^3 (Q^6 - m_Z^2 (6m_j^2 + 5Q^2)(Q^2 - 2m_Z^2)) \right. \\
& - 2m_i^2 m_j (Q^2 - 2m_Z^2)(Q^4 - m_Z^2 (6m_j^2 + Q^2)) + m_i (4m_Z^4 (Q^2 m_j^2 + 3m_j^4 + 2Q^4) \\
& + m_Z^2 (2Q^4 m_j^2 - 6Q^2 m_j^4 - 7Q^6) + 4Q^2 m_Z^6 + Q^8) - 6m_i^5 m_Z^2 (Q^2 - 2m_Z^2) \\
& \left. + m_j m_Z^2 (2m_Z^2 - Q^2)(-6Q^2 m_j^2 + 6m_j^4 + 2Q^2 m_Z^2 + Q^4) \right] C_{iij}(Q^2) \\
& \left. - Q^2 (m_i + m_j)(-6Q^2 m_Z^2 + 8m_Z^4 + Q^4) \right\} + (i \leftrightarrow j), \quad (A12)
\end{aligned}$$

$$\begin{aligned}
R_2^H(Q^2, m_i, m_j) = & \frac{1}{Q^4(Q^2 - 4m_Z^2)^2} \left\{ (Q^2 - 2m_Z^2)(m_Z^2(-m_i(6m_j^2 + 5Q^2) - 6m_i^2 m_j + 6m_i^3 - 3Q^2 m_j + 6m_j^3) \right. \\
& + 2Q^4 m_i) B_{ii}(m_Z^2) + Q^2 (m_i - m_j)(-3Q^2 m_Z^2 + 2m_Z^4 + Q^4) [B_{ij}(m_Z^2) - 2B_{ij}(Q^2)] \\
& - (Q^2 - 2m_Z^2)(2Q^2 m_Z^4 (m_i - m_j) + m_Z^2 (2m_i m_j^2 (Q^2 - 6m_i^2) + 6m_j^3 (2m_i^2 + Q^2) \\
& - m_j (-2Q^2 m_i^2 + 6m_i^4 + Q^4) + 6m_i m_j^4 - 3Q^4 m_i - 10Q^2 m_i^3 + 6m_i^5 - 6m_j^5) \\
& + Q^4 m_i (2(m_i - m_j)(2m_i + m_j) + Q^2)) C_{iij}(Q^2) \\
& \left. - Q^2 (m_i - m_j)(-6Q^2 m_Z^2 + 8m_Z^4 + Q^4) \right\} - (i \leftrightarrow j), \quad (A13)
\end{aligned}$$

$$\begin{aligned}
R_3^H(Q^2, m_i, m_j) = & \frac{1}{Q^4(Q^2 - 4m_Z^2)^2} \left\{ (Q^2 - 2m_Z^2)(m_Z^2(-m_i(6m_j^2 + 5Q^2) + 6m_i^2m_j + 6m_i^3 + 3m_j(Q^2 - 2m_j^2))) \right. \\
& + 2Q^4m_i)B_{ii}(m_Z^2) + Q^2(m_i + m_j)(-3Q^2m_Z^2 + 2m_Z^4 + Q^4) \left[B_{ij}(m_Z^2) - 2B_{ij}(Q^2) \right] \\
& - (Q^2 - 2m_Z^2)(2Q^2m_Z^4(m_i + m_j) + m_Z^2(2m_im_j^2(Q^2 - 6m_i^2) - 6m_j^3(2m_i^2 + Q^2) \\
& + m_j(-2Q^2m_i^2 + 6m_i^4 + Q^4) + 6m_im_j^4 - 3Q^4m_i - 10Q^2m_i^3 + 6m_i^5 + 6m_j^5) \\
& + Q^4m_i(2(2m_i - m_j)(m_i + m_j) + Q^2))C_{ij}(Q^2) \\
& \left. - Q^2g_A(m_i + m_j)(-6Q^2m_Z^2 + 8m_Z^4 + Q^4) \right\} + (i \leftrightarrow j), \tag{A14}
\end{aligned}$$

$$\begin{aligned}
R_4^H(Q^2, m_i, m_j) = & \frac{1}{Q^4(Q^2 - 4m_Z^2)^2} \left\{ m_Z^2(-6m_i^2m_j(Q^2 - 2m_Z^2) - m_i(6m_j^2(Q^2 - 2m_Z^2) - 10Q^2m_Z^2 + Q^4) \right. \\
& + 6m_i^3(Q^2 - 2m_Z^2) - 3m_j(Q^2 - 2m_j^2)(Q^2 - 2m_Z^2))B_{ii}(m_Z^2) \\
& + Q^2g_Vm_Z^2(m_i - m_j)(2m_Z^2 + Q^2) \left[B_{ij}(m_Z^2) - 2B_{ij}(Q^2) \right] \\
& + (6m_i^4m_jm_Z^2(Q^2 - 2m_Z^2) - 2m_i^3(Q^6 - m_Z^2(6m_j^2 + 5Q^2)(Q^2 - 2m_Z^2)) \\
& + 2m_i^2m_j(Q^2 - 2m_Z^2)(Q^4 - m_Z^2(6m_j^2 + Q^2)) + m_i(4m_Z^4(Q^2m_j^2 + 3m_j^4 + 2Q^4) \\
& + m_Z^2(2Q^4m_j^2 - 6Q^2m_j^4 - 7Q^6) + 4Q^2m_Z^6 + Q^8) - 6m_i^5m_Z^2(Q^2 - 2m_Z^2) \\
& + m_jm_Z^2(Q^2 - 2m_Z^2)(-6Q^2m_j^2 + 6m_j^4 + 2Q^2m_Z^2 + Q^4))C_{ij}(Q^2) \\
& \left. - Q^2(m_i - m_j)(-6Q^2m_Z^2 + 8m_Z^4 + Q^4) \right\} - (i \leftrightarrow j), \tag{A15}
\end{aligned}$$

$$\begin{aligned}
R_1^Z(Q^2, m_i, m_j) = & \frac{1}{((m_Z + m_H)^2 - P^2)^2((m_Z - m_H)^2 - P^2)^2} \left\{ \frac{P^2}{2} \left[2m_Z^2(m_i(4(m_H^2 + P^2) + 3m_j^2) \right. \right. \\
& - 3m_i^2m_j - 3m_i^3 + 3m_j^3) - (P^2 - m_H^2)(m_i(-m_H^2 - 6m_j^2 + P^2) - 3m_j(-m_H^2 + 2m_j^2 + P^2) \\
& + 6m_i^2m_j + 6m_i^3) + m_Z^4(-(7m_i + 3m_j)) \left. \right] B_{ii}(P^2) + \frac{m_Z^2}{2} \left[2m_Z^2(-3m_j(m_H^2 + m_i^2) \right. \\
& + m_i(m_H^2 - 3m_i^2 + 4P^2) + 3m_im_j^2 + 3m_j^3) - (P^2 - m_H^2)(m_i(-m_H^2 - 6m_j^2 + 7P^2) \\
& + 3m_j(m_H^2 - 2m_j^2 + P^2) + 6m_i^2m_j + 6m_i^3) + m_Z^4(-(m_i - 3m_j)) \left. \right] B_{ii}(m_Z^2) \\
& + \frac{P^2}{2} (m_i + m_j)(4m_Z^2(m_H^2 + P^2) + (P^2 - m_H^2)^2 - 5m_Z^4)B_{ij}(P^2) \\
& + \frac{m_Z^2}{2} (m_i + m_j)(m_H^2(4P^2 - 2m_Z^2) + m_H^4 + 4P^2m_Z^2 + m_Z^4 - 5P^4)B_{ij}(m_Z^2) \\
& - (m_i + m_j)(m_H^4(m_Z^2 + P^2) - 2m_H^2(-4P^2m_Z^2 + m_Z^4 + P^4) + (P^2 - m_Z^2)^2(m_Z^2 + P^2))B_{ij}(m_H^2) \\
& + \frac{1}{4} \left[-m_H^6(2m_i^2m_j + m_i(4m_Z^2 + 3P^2) + 2m_i^3 + P^2m_j) + m_H^4(m_Z^2(6m_i^2m_j - P^2m_i + 6m_i^3 \right. \\
& - 3P^2m_j) + P^2(m_i + m_j)(-4m_im_j + 4m_i^2 + 6m_j^2 + 3P^2) + 6m_im_j^4) \\
& + m_H^2(m_Z^4(-6m_i^2m_j + 5P^2m_i - 6m_i^3 + 3P^2m_j) + 4P^2m_Z^2(m_i + m_j)(2m_im_j - 3m_i^2 + P^2) \\
& - P^2(2m_i^3(P^2 - 6m_j^2) - 6m_i^2m_j(2m_j^2 + P^2) + m_i(4P^2m_j^2 + 6m_j^4 + P^4) + 6m_i^4m_j + 6m_i^5 \\
& \left. + 3m_j(4P^2m_j^2 + 2m_j^4 + P^4)) - 4m_im_H^6) + m_H^8m_i + m_Z^6(2m_i^2m_j - P^2m_i + 2m_i^3 + P^2m_j) \right.
\end{aligned}$$

$$\begin{aligned}
& -P^2 m_Z^4 (m_i + m_j) (4m_i m_j - 8m_i^2 + 6m_j^2 + P^2) + P^2 m_Z^2 (-2m_i^3 (6m_j^2 + 5P^2) \\
& - 2m_j^2 m_j (6m_j^2 + P^2) + m_i (8P^2 m_j^2 + 6m_j^4 + P^4) + 6m_i^4 m_j + 6m_i^5 - P^4 m_j + 6m_j^5) \\
& + P^4 (2(m_i + m_j) (-2P^2 m_i m_j + 3m_j^2 (P^2 - 2m_i^2) + 3m_i^4 + 3m_j^4) + P^4 m_j) + m_i m_j^8 \Big] C_{iji}(P^2) \\
& + \frac{1}{4} \Big[m_Z^6 (-m_j (3m_H^2 + 4m_i^2 + P^2) + m_i (P^2 - m_H^2) + 2m_i m_j^2 + 6m_j^3) \\
& + m_i (P^2 - m_H^2)^3 (-m_H^2 + 2m_i (m_i + m_j) + P^2) + m_Z^4 (m_i + m_j) (2m_H^2 (4m_i m_j - m_i^2 \\
& - 6m_j^2 + 2P^2) + 3m_H^4 + 8P^2 m_i m_j - 2m_i^2 (6m_j^2 + 5P^2) + 6m_i^4 + 6m_j^4 - P^4) \\
& - m_Z^2 (P^2 - m_H^2) (6m_j^3 (m_H^2 + 2m_i^2 + P^2) + 2m_i m_j^2 (m_H^2 + 6m_i^2 + 5P^2) - m_j (4P^2 m_H^2 \\
& + m_H^4 + 4P^2 m_i^2 + 6m_i^4 + P^4) + m_i (4m_H^2 (m_i^2 - P^2) - 3m_H^4 - 8P^2 m_i^2 - 6m_i^4 + P^4) - 6m_i m_j^4 \\
& - 6m_j^5) + m_j m_Z^8 \Big] C_{jii}(P^2) \\
& + (m_i + m_j) ((m_H - m_Z)^2 - P^2) ((m_H + m_Z)^2 - P^2) (-m_H^2 + m_Z^2 + P^2) \Big\} + (i \leftrightarrow j), \tag{A16}
\end{aligned}$$

$$\begin{aligned}
R_2^Z(Q^2, m_i, m_j) = & \frac{1}{((m_Z + m_H)^2 - P^2)^2 ((m_Z - m_H)^2 - P^2)^2} \Big\{ \frac{1}{2} (-m_H^2 + m_Z^2 + P^2) (m_i (m_H^2 (4m_Z^2 + P^2) \\
& - 2m_H^4 + 6P^2 m_j^2 + P^2 m_Z^2 - 2m_Z^4 + P^4) + 3P^2 m_j (m_H^2 - 2m_j^2 + m_Z^2 - P^2) + 6P^2 m_i^2 m_j \\
& - 6P^2 m_i^3) B_{ii}(P^2) - \frac{1}{2} (-m_H^2 + m_Z^2 + P^2) (-m_Z^2 (m_i (m_H^2 + 6m_j^2 + P^2) + 3m_j (m_H^2 - 2m_j^2 \\
& + P^2) + 6m_i^2 m_j - 6m_i^3) + 2m_i (P^2 - m_H^2)^2 + m_Z^4 (- (m_i - 3m_j))) B_{ii}(m_Z^2) \\
& + \frac{(m_i - m_j)}{2} (-m_H^2 + m_Z^2 + P^2) (-m_H^2 (P^2 - 2m_Z^2) - m_H^4 - m_Z^2 (m_Z^2 + P^2) + 2P^4) B_{ij}(P^2) \\
& - \frac{(m_i - m_j)}{2} (-m_Z^4 (P^2 - 3m_H^2) + 2P^2 m_Z^2 (P^2 - m_H^2) + (P^2 - m_H^2)^3 - 2m_Z^6) B_{ij}(m_Z^2) \\
& - (m_i - m_j) (-3m_H^4 (m_Z^2 + P^2) + 4P^2 m_H^2 m_Z^2 + 2m_H^6 + (P^2 - m_Z^2)^2 (m_Z^2 + P^2)) B_{ij}(m_H^2) \\
& - \frac{1}{4} (-m_H^2 + m_Z^2 + P^2) (m_H^4 ((2m_i + m_j) (2m_i (m_j - m_i) + P^2) + m_i m_Z^2) \\
& + m_H^2 (2m_i^2 m_j (P^2 - 2m_Z^2) + m_i (-2m_Z^2 (2m_j^2 + P^2) + 2P^2 m_j^2 + m_Z^4 - P^4) \\
& + 2m_i^3 (4m_Z^2 + P^2) - 2P^2 m_j (3m_j^2 - 2m_Z^2 + P^2)) + m_H^6 (-m_i) \\
& + m_Z^4 (2m_i + m_j) (2m_i (m_j - m_i) + P^2) + P^2 m_Z^2 (-m_i (P^2 - 2m_j^2) + 2m_i^2 m_j + 2m_i^3 \\
& - 2m_j (3m_j^2 + P^2)) + P^2 (6m_j^3 (P^2 - 2m_i^2) - 4m_i m_j^2 (P^2 - 3m_i^2) + m_j (-4P^2 m_i^2 + 6m_i^4 + P^4) \\
& - 6m_i m_j^4 + 2m_i^3 (P^2 - 3m_i^2) + 6m_j^5) - m_i m_Z^6) C_{iji}(P^2) \\
& + \frac{1}{4} (-m_H^2 + m_Z^2 + P^2) (m_Z^4 (m_i (m_H^2 + 4m_j^2 + P^2) + 2m_j (m_H^2 - 3m_j^2 + P^2) + 4m_i^2 m_j - 2m_i^3) \\
& + m_i (P^2 - m_H^2)^2 (m_H^2 + 2(m_i - m_j) (2m_i + m_j) + P^2) - m_Z^2 (-6m_j^3 (m_H^2 + 2m_i^2 + P^2) \\
& + 2m_i m_j^2 (m_H^2 + 6m_i^2 + P^2) + m_j (2m_i^2 (m_H^2 + P^2) + 4P^2 m_H^2 + m_H^4 + 6m_i^4 + P^4) \\
& + 2m_i (m_i^2 (m_H^2 + P^2) - P^2 m_H^2 + m_H^4 - 3m_i^4 + P^4) - 6m_i m_j^4 + 6m_j^5) - m_j m_Z^6) C_{jii}(P^2) \\
& + (m_i - m_j) ((m_H - m_Z)^2 - P^2) ((m_H + m_Z)^2 - P^2) (-m_H^2 + m_Z^2 + P^2) \Big\} - (i \leftrightarrow j), \tag{A17}
\end{aligned}$$

$$\begin{aligned}
R_3^Z(Q^2, m_i, m_j) = & \frac{1}{((m_Z + m_H)^2 - P^2)^2((m_Z - m_H)^2 - P^2)^2} \left\{ \frac{1}{2}(-m_H^2 + m_Z^2 + P^2)(m_i(m_H^2(4m_Z^2 + P^2) \right. \\
& - 2m_H^4 + 6P^2m_j^2 + P^2m_Z^2 - 2m_H^4 + P^4) + 3P^2m_j(-m_H^2 + 2m_j^2 - m_Z^2 + P^2) - 6P^2m_i^2m_j \\
& - 6P^2m_i^3)B_{ii}(P^2) - \frac{1}{2}(-m_H^2 + m_Z^2 + P^2)(-m_Z^2(-3m_j(m_H^2 + 2m_i^2 + P^2) \\
& + m_i(m_H^2 - 6m_i^2 + P^2) + 6m_im_j^2 + 6m_j^3) + 2m_i(P^2 - m_H^2)^2 + m_Z^4(-(m_i + 3m_j)))B_{ii}(m_Z^2) \\
& + \frac{1}{2}(m_i + m_j)(-m_H^2 + m_Z^2 + P^2)(-m_H^2(P^2 - 2m_Z^2) - m_H^4 - m_Z^2(m_Z^2 + P^2) + 2P^4)B_{ij}(P^2) \\
& + \frac{1}{2}(m_i + m_j)(m_Z^4(P^2 - 3m_H^2) + 2P^2m_Z^2(m_H^2 - P^2) - (P^2 - m_H^2)^3 + 2m_Z^6)B_{ij}(m_Z^2) \\
& - (m_i + m_j)(-3m_H^4(m_Z^2 + P^2) + 4P^2m_H^2m_Z^2 + 2m_H^6 + (P^2 - m_Z^2)^2(m_Z^2 + P^2))B_{ij}(m_H^2) \\
& + \frac{1}{4}(-m_H^2 + m_Z^2 + P^2)(m_H^4((2m_i - m_j)(2m_i(m_i + m_j) - P^2) - m_im_Z^2) \\
& + m_H^2(2m_i^2m_j(P^2 - 2m_Z^2) + m_i(2m_Z^2(2m_j^2 + P^2) - 2P^2m_j^2 - m_H^4 + P^4) - 2m_i^3(4m_Z^2 + P^2) \\
& - 2P^2m_j(3m_j^2 - 2m_Z^2 + P^2)) + m_H^6m_i + m_Z^4(2m_i - m_j)(2m_i(m_i + m_j) - P^2) \\
& + P^2m_Z^2(m_i(P^2 - 2m_j^2) + 2m_i^2m_j - 2m_i^3 - 2m_j(3m_j^2 + P^2)) + P^2(4m_im_j^2(P^2 - 3m_i^2) \\
& + 6m_j^3(P^2 - 2m_i^2) + m_j(-4P^2m_i^2 + 6m_i^4 + P^4) + 6m_im_j^4 - 2P^2m_i^3 + 6m_i^5 + 6m_j^5) \\
& + m_im_Z^6)C_{iji}(P^2) + \frac{1}{4}(-m_H^2 + m_Z^2 + P^2)(m_Z^4(-2m_j(m_H^2 + 2m_i^2 + P^2) \\
& + m_i(m_H^2 - 2m_i^2 + P^2) + 4m_im_j^2 + 6m_j^3) + m_i(P^2 - m_H^2)^2(m_H^2 + 2(2m_i - m_j)(m_i + m_j) \\
& + P^2) + m_Z^2(-2m_i^3(m_H^2 + 6m_j^2 + P^2) + 2m_i^2m_j(m_H^2 - 6m_j^2 + P^2) - 2m_i(m_j^2(m_H^2 + P^2) \\
& - P^2m_H^2 + m_H^4 - 3m_j^4 + P^4) + m_j(-6m_j^2(m_H^2 + P^2) + 4P^2m_H^2 + m_H^4 + 6m_j^4 + P^4) \\
& + 6m_i^4m_j + 6m_i^5) + m_jm_Z^6)C_{jii}(P^2) \\
& \left. + (m_i + m_j)((m_H - m_Z)^2 - P^2)((m_H + m_Z)^2 - P^2)(-m_H^2 + m_Z^2 + P^2) \right\} + (i \leftrightarrow j), \tag{A18}
\end{aligned}$$

$$\begin{aligned}
R_4^Z(Q^2, m_i, m_j) = & \frac{1}{((m_Z + m_H)^2 - P^2)^2((m_Z - m_H)^2 - P^2)^2} \left\{ \frac{1}{2}P^2(2m_Z^2(m_i(4(m_H^2 + P^2) + 3m_j^2) + 3m_i^2m_j \right. \\
& - 3m_i^3 - 3m_j^3) - (P^2 - m_H^2)(m_i(-m_H^2 - 6m_j^2 + P^2) + 3m_j(-m_H^2 + 2m_j^2 + P^2) - 6m_i^2m_j \\
& + 6m_i^3) + m_Z^4(3m_j - 7m_i))B_{ii}(P^2) + \frac{1}{2}m_Z^2 \left[2m_Z^2(3m_j(m_H^2 + m_i^2) + m_i(m_H^2 - 3m_i^2 + 4P^2) \right. \\
& + 3m_im_j^2 - 3m_j^3) - (P^2 - m_H^2)(-3m_j(m_H^2 + 2m_i^2 + P^2) + m_i(-m_H^2 + 6m_i^2 + 7P^2) \\
& - 6m_im_j^2 + 6m_j^3) + m_Z^4(-(m_i + 3m_j)) \left. \right] B_{ii}(m_Z^2) \\
& + \frac{1}{2}P^2(m_i - m_j)(4m_Z^2(m_H^2 + P^2) + (P^2 - m_H^2)^2 - 5m_Z^4)B_{ij}(P^2) \\
& + \frac{1}{2}m_Z^2(m_i - m_j)(m_H^2(4P^2 - 2m_Z^2) + m_H^4 + 4P^2m_Z^2 + m_Z^4 - 5P^4)B_{ij}(m_Z^2) \\
& - (m_i - m_j)(m_H^4(m_Z^2 + P^2) - 2m_H^2(-4P^2m_Z^2 + m_Z^4 + P^4) + (P^2 - m_Z^2)^2(m_Z^2 + P^2))B_{ij}(m_H^2)
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{4} \left[m_H^6(2m_i^2 m_j - m_i(4m_Z^2 + 3P^2) - 2m_i^3 + P^2 m_j) + m_H^4(m_Z^2(-6m_i^2 m_j - P^2 m_i \right. \\
& + 6m_i^3 + 3P^2 m_j) + P^2(m_i - m_j)(4m_i m_j + 4m_i^2 + 6m_j^2 + 3P^2) + 6m_i m_Z^4) \\
& + m_H^2(m_Z^4(6m_i^2 m_j + 5P^2 m_i - 6m_i^3 - 3P^2 m_j) + 4P^2 m_Z^2(m_i - m_j)(-2m_i m_j - 3m_i^2 + P^2) \\
& - P^2(2m_i^3(P^2 - 6m_j^2) + 6m_i^2 m_j(2m_j^2 + P^2) + m_i(4P^2 m_j^2 + 6m_j^4 + P^4) - 6m_i^4 m_j + 6m_i^5 \\
& - 3m_j(4P^2 m_j^2 + 2m_j^4 + P^4)) - 4m_i m_Z^6) + m_H^8 m_i - m_Z^6(m_j(2m_i^2 + P^2) + m_i(P^2 - 2m_i^2)) \\
& - P^2 m_Z^4(m_i - m_j)(-4m_i m_j - 8m_i^2 + 6m_j^2 + P^2) + P^2 m_Z^2(-2m_i^3(6m_j^2 + 5P^2) \\
& + 2m_i^2 m_j(6m_j^2 + P^2) + m_i(8P^2 m_j^2 + 6m_j^4 + P^4) - 6m_i^4 m_j + 6m_i^5 + m_j(P^4 - 6m_j^4)) \\
& \left. + P^4(2(m_i - m_j)(2P^2 m_i m_j + 3m_j^2(P^2 - 2m_i^2) + 3m_i^4 + 3m_j^4) - P^4 m_j) + m_i m_Z^8 \right] C_{iji}(P^2) \\
& + \frac{1}{4} \left[m_Z^6(m_i(-m_H^2 + 2m_j^2 + P^2) + m_j(3m_H^2 - 6m_j^2 + P^2) + 4m_i^2 m_j) \right. \\
& + m_i(P^2 - m_H^2)^3(-m_H^2 + 2m_i(m_i - m_j) + P^2) - m_Z^4(m_i - m_j)(2m_H^2(4m_i m_j + m_i^2 + 6m_j^2 \\
& - 2P^2) - 3m_H^4 + 8P^2 m_i m_j + 2m_i^2(6m_j^2 + 5P^2) - 6m_i^4 - 6m_j^4 + P^4) \\
& - m_Z^2(P^2 - m_H^2)(-6m_j^3(m_H^2 + 2m_i^2 + P^2) + 2m_i m_j^2(m_H^2 + 6m_i^2 + 5P^2) \\
& + m_j(4P^2 m_H^2 + m_H^4 + 4P^2 m_i^2 + 6m_i^4 + P^4) + m_i(4m_H^2(m_i^2 - P^2) - 3m_H^4 - 8P^2 m_i^2 \\
& - 6m_i^4 + P^4) - 6m_i m_j^4 + 6m_j^5) - m_j m_Z^8 \left. \right] C_{jii}(P^2) \\
& (m_i - m_j)((m_H - m_Z)^2 - P^2)((m_H + m_Z)^2 - P^2)(-m_H^2 + m_Z^2 + P^2) \Big\} - (i \leftrightarrow j). \tag{A19}
\end{aligned}$$

The functions $T_{1,2,3,4}^V$ ($V = H, Z$) for contributions of type II are given as follows:

$$\begin{aligned}
T_1^H(Q^2, m_i, m_j) = \frac{1}{Q^2(Q^2 - 4m_Z^2)} \Big\{ & 2(m_Z^2(m_i + m_j) - Q^2 m_i) B_{ii}(m_Z^2) + 2Q^2(m_j - m_i) \left[B_{ij}(m_Z^2) - 2B_{ij}(Q^2) \right], \\
& (2m_i^2 - 2m_j^2 + Q^2)(Q^2 m_i - m_Z^2(m_i + m_j)) C_{iij}(Q^2) \Big\} - (i \leftrightarrow j), \tag{A20}
\end{aligned}$$

$$\begin{aligned}
T_2^H(Q^2, m_i, m_j) = \frac{1}{Q^2(Q^2 - 4m_Z^2)} \Big\{ & 2m_Z^2(m_i - m_j) B_{ii}(m_Z^2) + \left[m_Z^2(m_i(2m_j^2 + 3Q^2) + 2m_i^2 m_j - 2m_i^3 \right. \\
& \left. + m_j(Q^2 - 2m_j^2)) - Q^4 m_i \right] C_{iij}(Q^2) \Big\} + (i \leftrightarrow j), \tag{A21}
\end{aligned}$$

$$\begin{aligned}
T_3^H(Q^2, m_i, m_j) = \frac{1}{Q^2(Q^2 - 4m_Z^2)} \Big\{ & 2m_Z^2(m_i + m_j) B_{ii}(m_Z^2) + \left[m_Z^2(-m_j(2m_i^2 + Q^2) + 2m_i m_j^2 + 3Q^2 m_i \right. \\
& \left. - 2m_i^3 + 2m_j^3) - Q^4 m_i \right] C_{iij}(Q^2) \Big\} - (i \leftrightarrow j), \tag{A22}
\end{aligned}$$

$$\begin{aligned}
T_4^H(Q^2, m_i, m_j) = \frac{1}{Q^2(Q^2 - 4m_Z^2)} \Big\{ & -2(m_Z^2(m_j - m_i) + Q^2 m_i) B_{ii}(m_Z^2) - 2Q^2(m_i + m_j) \left[B_{ij}(m_Z^2) - 2B_{ij}(Q^2) \right], \\
& + (2m_i^2 - 2m_j^2 + Q^2)(m_Z^2(m_j - m_i) + Q^2 m_i) C_{iij}(Q^2) \Big\} + (i \leftrightarrow j), \tag{A23}
\end{aligned}$$

$$\begin{aligned}
T_1^Z(Q^2, m_i, m_j) = & \frac{1}{((m_H - m_Z)^2 - P^2)((m_H + m_Z) - P^2)} \left\{ (m_i(m_Z^2 - m_H^2) + P^2 m_j) B_{ii}(P^2) \right. \\
& + (m_i(P^2 - m_H^2) + m_j m_Z^2) B_{ii}(m_Z^2) - \frac{1}{2} (m_i - m_j)(m_H^2 - m_Z^2 + P^2) B_{ij}(P^2) \\
& + \frac{1}{2} (m_i - m_j)(-m_H^2 - m_Z^2 + P^2) B_{ij}(m_Z^2) + 2m_H^2(m_i - m_j) B_{ij}(m_H^2) \\
& + \frac{1}{4} (-m_H^2 - 2m_i^2 + 2m_j^2 - m_Z^2 + P^2)(m_i(m_Z^2 - m_H^2) + P^2 m_j) C_{iji}(P^2) \\
& \left. - \frac{1}{4} (m_H^2 + 2m_i^2 - 2m_j^2 - m_Z^2 + P^2)(m_i(P^2 - m_H^2) + m_j m_Z^2) C_{jii}(P^2) \right\} - (i \leftrightarrow j), \quad (A24)
\end{aligned}$$

$$\begin{aligned}
T_2^Z(Q^2, m_i, m_j) = & \frac{1}{((m_H - m_Z)^2 - P^2)((m_H + m_Z) - P^2)} \left\{ P^2(m_i - m_j) B_{ii}(P^2) + m_Z^2(m_i - m_j) B_{ii}(m_Z^2) \right. \\
& + \frac{1}{4} \left[m_H^2(P^2(m_i + m_j) + 2m_i m_Z^2) + m_H^4(-m_i) + P^2 m_Z^2(m_i + m_j) \right. \\
& \left. - P^2(2(m_i - m_j)^2(m_i + m_j) + P^2 m_j) - m_i m_Z^4 \right] C_{iji}(P^2) \\
& + \frac{1}{4} \left[m_Z^2(m_i + m_j)(m_H^2 - 2(m_i - m_j)^2 + P^2) - m_i(P^2 - m_H^2)^2 - m_j m_Z^4 \right] C_{jii}(P^2) \left. \right\} \\
& + (i \leftrightarrow j), \quad (A25)
\end{aligned}$$

$$\begin{aligned}
T_3^Z(Q^2, m_i, m_j) = & \frac{1}{((m_H - m_Z)^2 - P^2)((m_H + m_Z) - P^2)} \left\{ P^2(m_i + m_j) B_{ii}(P^2) + m_Z^2(m_i + m_j) B_{ii}(m_Z^2) \right. \\
& + \frac{1}{4} \left[m_H^2(m_i(2m_Z^2 + P^2) - P^2 m_j) + m_H^4(-m_i) + P^2 m_Z^2(m_i - m_j) \right. \\
& \left. + P^2(P^2 m_j - 2(m_i - m_j)(m_i + m_j)^2) - m_i m_Z^4 \right] C_{iji}(P^2) \\
& + \frac{1}{4} \left[m_Z^2(m_i - m_j)(m_H^2 - 2(m_i + m_j)^2 + P^2) - m_i(P^2 - m_H^2)^2 + m_j m_Z^4 \right] C_{jii}(P^2) \left. \right\} \\
& - (i \leftrightarrow j), \quad (A26)
\end{aligned}$$

$$\begin{aligned}
T_4^Z(Q^2, m_i, m_j) = & \frac{1}{((m_H - m_Z)^2 - P^2)((m_H + m_Z) - P^2)} \left\{ -(m_i(m_H^2 - m_Z^2) + P^2 m_j) B_{ii}(P^2) \right. \\
& + (m_i(P^2 - m_H^2) - m_j m_Z^2) B_{ii}(m_Z^2) - \frac{1}{2} (m_i + m_j)(m_H^2 - m_Z^2 + P^2) B_{ij}(P^2) \\
& + \frac{1}{2} (m_i + m_j)(-m_H^2 - m_Z^2 + P^2) B_{ij}(m_Z^2) + 2m_H^2(m_i + m_j) B_{ij}(m_H^2) \\
& - \frac{1}{4} (-m_H^2 - 2m_i^2 + 2m_j^2 - m_Z^2 + P^2)(m_i(m_H^2 - m_Z^2) + P^2 m_j) C_{iji}(P^2) \\
& \left. - \frac{1}{4} \text{ga}(m_H^2 + 2m_i^2 - 2m_j^2 - m_Z^2 + P^2)(m_i(P^2 - m_H^2) - m_j m_Z^2) C_{jii}(P^2) \right\} + (i \leftrightarrow j). \quad (A27)
\end{aligned}$$

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