

Addressing uncertainties of model predictions for extensive air showers initiated by high energy cosmic rays

Sergey Ostapchenko¹, Tanguy Pierog², and Günter Sigl¹

¹*Universität Hamburg, II Institut für Theoretische Physik, 22761 Hamburg, Germany*

²*Institute for Astroparticle Physics, Karlsruher Institut für Technologie, Hermann-von-Helmholtz-Platz 1, 76344 Eggenstein-Leopoldshafen, Germany*



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A new Monte Carlo generator of hadronic collisions is applied for studying model uncertainties regarding calculations of the development of extensive air showers (EAS) initiated by interactions of high energy cosmic rays in the atmosphere. More specifically, we investigate possibilities to modify the model predictions for two EAS characteristics mostly used in experimental studies of cosmic ray composition: air shower maximum depth and the muon number at ground level. For all the considered modifications of the model, we discuss in some detail the underlying physics mechanisms and investigate the impact of the changes, regarding a comparison of the model results to relevant accelerator data.

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I. INTRODUCTION

Experimental studies of ultrahigh energy cosmic rays (UHECRs) are performed by indirect methods: reconstructing properties of primary cosmic ray (CR) particles from measured characteristics of extensive air showers (EAS)—nuclear-electromagnetic cascades initiated by interactions of primary CRs in the atmosphere [1,2]. Therefore, an accurate description of the development of such cascades by the corresponding numerical tools, like the CORSIKA program [3], is of significant importance. In particular, studies of mass composition of UHECRs rely substantially on the modeling of interactions with air nuclei of both primary CRs and of secondary hadrons produced in the subsequent cascades of nuclear interactions in the atmosphere, which involves Monte Carlo (MC) generators of hadronic interactions [4].

Consequently, of significant interest is to quantify uncertainties of predictions of such MC generators, regarding EAS development, which was the subject of numerous studies (e.g., [5–8]). Generally, one may get some feeling about such uncertainties by analyzing the spread of the corresponding results of current CR interaction models. Yet, because of potential deficiencies of some approaches, the actual uncertainties may appear to be smaller (e.g., [9,10]). On the contrary, in case some important physics is

not accounted for, all current models may deviate from the true picture.

In [11,12], we presented a new MC generator of CR interactions, Quark-Gluon Strings (basic) (QGSb), which is based on a relatively simple and transparent theoretical formalism, while being characterized by a sufficient parameter freedom. Thanks to its substantial flexibility, this generator can be used to study uncertainties for EAS predictions, within the limits allowed by relevant accelerator data regarding hadron-proton and hadron-nucleus interactions. In the current work, we exemplify such a study, addressing possibilities to vary the predictions for two EAS characteristics mainly employed in UHECR composition studies: the average maximum depth of air shower development, $X_{\max}$, and the EAS muon number at ground level, N_{μ} . In particular, we identify key model parameters relevant to such studies and discuss in some detail the corresponding interaction mechanisms, addressing also the constraints imposed by relevant accelerator data, as well as the uncertainties of the latter.

II. ALTERNATIVE TUNING OF MODEL PARAMETERS

A. Modifying the predicted EAS maximum depth

Let us first investigate possibilities to modify the calculated longitudinal development of extensive air showers. The knowledge of the corresponding model uncertainties, notably regarding predictions for EAS maximum depth $X_{\max}$, is of considerable importance for analyzing and interpreting experimental data from giant air shower arrays. Moreover, recent analysis by the Pierre Auger Collaboration indicates that substantially larger $X_{\max}$

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values, compared to current model predictions, are required for a self-consistent interpretation of the data, in terms of mass composition of primary cosmic rays [13].

1. Aiming at a larger $X_{\max}$

Obviously, the simplest way to change $X_{\max}$ predictions is to modify the energy dependence of the inelastic proton-air cross section, $\sigma_{p\text{-air}}^{\text{inel}}$, which controls the mean free path of the primary CR proton in the atmosphere (e.g., [5]). However, $\sigma_{p\text{-air}}^{\text{inel}}$ is rather strongly constrained by measurements of proton-proton interaction cross sections at the Large Hadron Collider (LHC) since the quantities are closely related to each other by the very successful Glauber-Gribov formalism [14,15]. While the default settings of the QGSb model, hereafter referred to as tune 1, make it consistent with the results of the TOTEM experiment [16] on the total and inelastic pp cross sections, as one can see in Fig. 1, one can alternatively tune the relevant model parameters, using measurements of the ATLAS experiment [17], shown by open symbols in Fig. 1, which indicated a slightly slower energy rise of $\sigma_{pp}^{\text{tot/inel}}$. To this end, we choose a $\approx 5\%$ lower overcriticality of the “semihard Pomeron,” $\Delta_{\text{sh}} = 0.21$, which governs the energy dependence of the calculated cross sections in the very high energy limit [12]. The corresponding results for the total

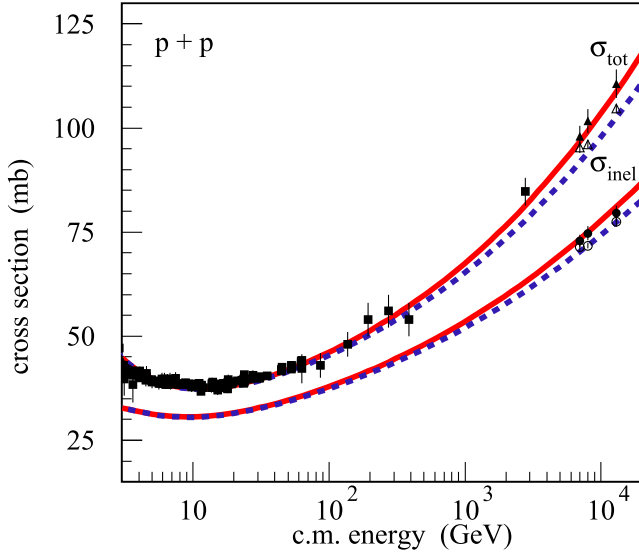


FIG. 1. Center-of-mass (c.m.) energy dependence of the total and inelastic pp cross sections, as indicated in the plot, compared to experimental data [16–18] (points). The calculations performed using the default model (tune 1) or using a smaller value for the overcriticality of the “semihard Pomeron” (tune 2) are shown, respectively, by red solid and blue dashed lines. The results of the TOTEM experiment for σ_{pp}^{tot} and $\sigma_{pp}^{\text{inel}}$ are plotted as filled triangles and filled circles, respectively, while the ones of the ATLAS experiment are shown by the corresponding open symbols.

and inelastic pp cross sections are shown in Fig. 1 as dashed lines. It is noteworthy that such a change of the parameter Δ_{sh} reduces also the predicted multiple scattering rate in proton-proton and proton-nucleus collisions [see Eqs. (3) and (6) in [12]]. Therefore, to stay in agreement with LHC data on charged hadron production in the central rapidity range, we have to make changes in the hadronization procedure of the model by increasing the density of produced hadrons per unit rapidity. This is achieved by choosing a $\approx 30\%$ larger value of the string fragmentation parameter for semihard Pomerons, $\Lambda^{(\text{sh})} = 4.6$ [cf. Eq. (36) and Table 6 in [12]]. The pseudorapidity η distribution of charged hadrons, $dN_{pp}^{\text{ch}}/d\eta$, produced in pp collisions at $\sqrt{s} = 8$ TeV, calculated both with the default model or using the above-discussed modifications of the model parameters (hereafter referred to as tune 2), are plotted in Fig. 2, in comparison to the respective data of the CMS and TOTEM experiments.

Regarding $\sigma_{p\text{-air}}^{\text{inel}}$, it is much more weakly influenced by the considered changes, compared to $\sigma_{pp}^{\text{inel}}$, since proton-nucleus cross sections are largely dominated by the nuclear size, for sufficiently heavy nuclei. This is illustrated in Fig. 3, where we observe up to $\approx 3\%$ difference for the inelastic proton-nitrogen interaction cross section $\sigma_{pN}^{\text{inel}}$ between the tunes 1 and 2 of the model. Such a modification of the proton-air cross section alone would enlarge average $X_{\max}$ by less than 2 g/cm^2 . Yet, as discussed above, using a smaller overcriticality of the semihard Pomeron reduces also the predicted multiple scattering rate in proton-proton and proton-nucleus collisions:

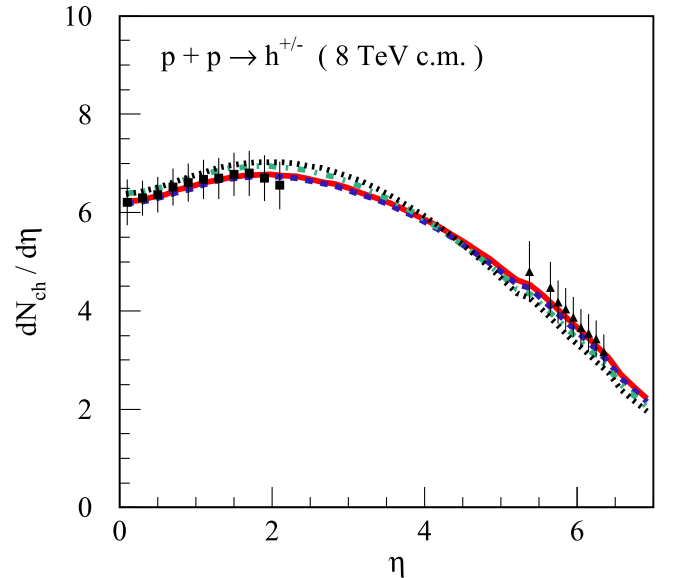


FIG. 2. Pseudorapidity distribution of charged hadrons in c.m. frame, produced in pp collisions at $\sqrt{s} = 8$ TeV, as calculated using tunes 1, 2, 3, and 4 of QGSb—red solid, blue dashed, green dash-dotted, and black dotted lines, respectively, compared to the data of CMS (squares) and TOTEM (triangles) [19].

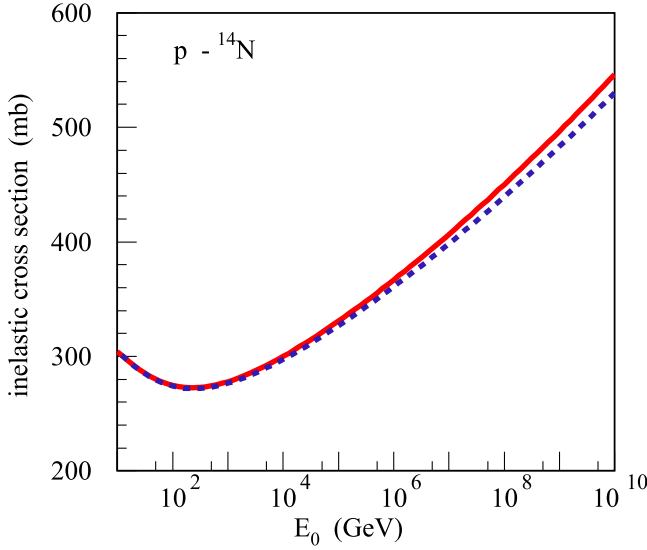


FIG. 3. Laboratory (lab.) energy dependence of the inelastic cross section for proton-nitrogen collisions. The notations for the lines are the same as in Fig. 1.

a smaller number of strings of color field would be attached to the incident proton. Hence, a smaller fraction of the proton energy would be taken by string end partons, thereby reducing the inelasticity K^{inel} (the relative energy loss of “leading,” i.e., most energetic, secondary nucleons) of pp and pA interactions. Indeed, for this model tune, the inelasticity K_{pN}^{inel} of proton-nitrogen collisions is reduced by up to $\approx 2\%$ at the highest energies, compared to the default QGSb (cf. solid and dashed lines in Fig. 4). Both effects combined give rise to a noticeably larger average X_{max} for

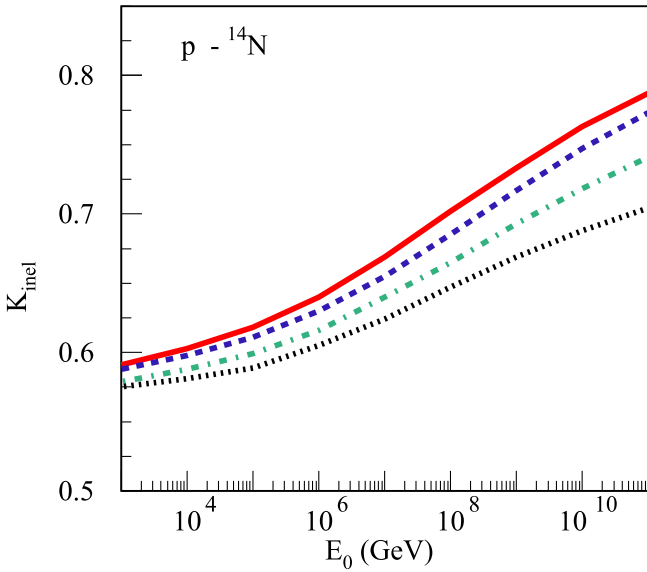


FIG. 4. Lab. energy dependence of the inelasticity K_{pN}^{inel} of proton-nitrogen collisions. The notations for the lines are the same as in Fig. 2.

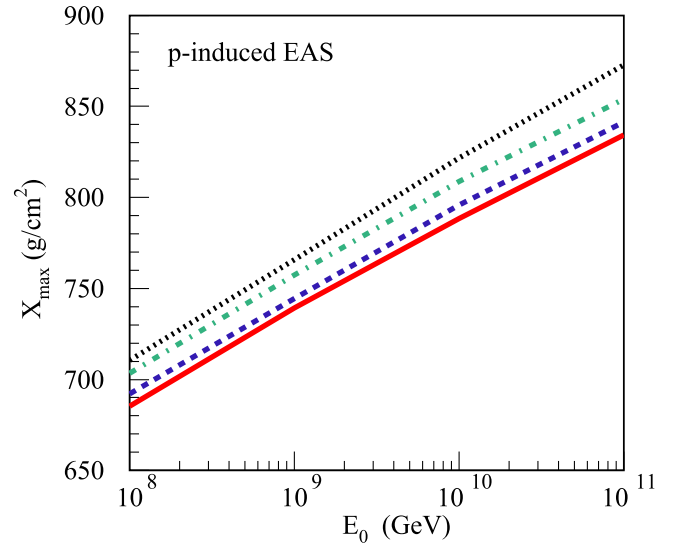


FIG. 5. Dependence of the average shower maximum depth X_{max} , for p -induced vertical EAS, on the primary proton energy E_0 . The notations for the lines are the same as in Fig. 2.

proton-induced EAS by up to $\approx 7 \text{ g/cm}^2$ at $E_0 = 10^{20} \text{ eV}$, as one can see in Fig. 5.

A more radical reduction of the inelasticity of proton-air collisions can be achieved considering softer momentum distributions of constituent partons to which strings of color field are attached [6,7,9,20]. To this end, in addition to using the smaller overcriticality of the semihard Pomeron, $\Delta_{\text{sh}} = 0.21$, we consider larger values of the exponent $\alpha_{\text{sea}}^{(\text{sh})}$ describing the momentum distribution of constituent sea (anti)quarks, $\propto x^{-\alpha_{\text{sea}}^{(\text{sh})}}$ [see Eq. (30) in [12]], to which semihard Pomeron strings are attached, by using $\alpha_{\text{sea}}^{(\text{sh})} = 0.8$ and 0.95 , instead of the default setting $\alpha_{\text{sea}}^{(\text{sh})} = \alpha_{\text{R}} = 0.5$. Here again, to keep the agreement with LHC data on charged hadron production in the central rapidity range, we need to enlarge further the string fragmentation parameter for semihard Pomerons, using, respectively, $\Lambda^{(\text{sh})} = 6.5$ and 9.5 for the two choices of $\alpha_{\text{sea}}^{(\text{sh})}$, which we refer to hereafter as tunes 3 and 4. It is noteworthy that all the considered changes do not affect the model agreement with accelerator data corresponding to hadronic interactions at fixed target energies since those are dominated by soft Pomeron exchanges and by hadronization of soft Pomeron strings. A comparison of the calculated $dN_{pp}^{\text{ch}}/d\eta$ at $\sqrt{s} = 8 \text{ TeV}$ with the data of CMS and TOTEM is shown in Fig. 2, while the obtained energy dependence of the inelasticity of proton-nitrogen collisions is plotted in Fig. 4. As one can see in Fig. 4, K_{pN}^{inel} is reduced by up to 6 and 11%, respectively, for the tunes 3 and 4. In turn, this allowed us to increase X_{max} for proton-induced EAS by up to ≈ 20 and $\approx 40 \text{ g/cm}^2$, for the two cases, correspondingly, as shown in Fig. 5.

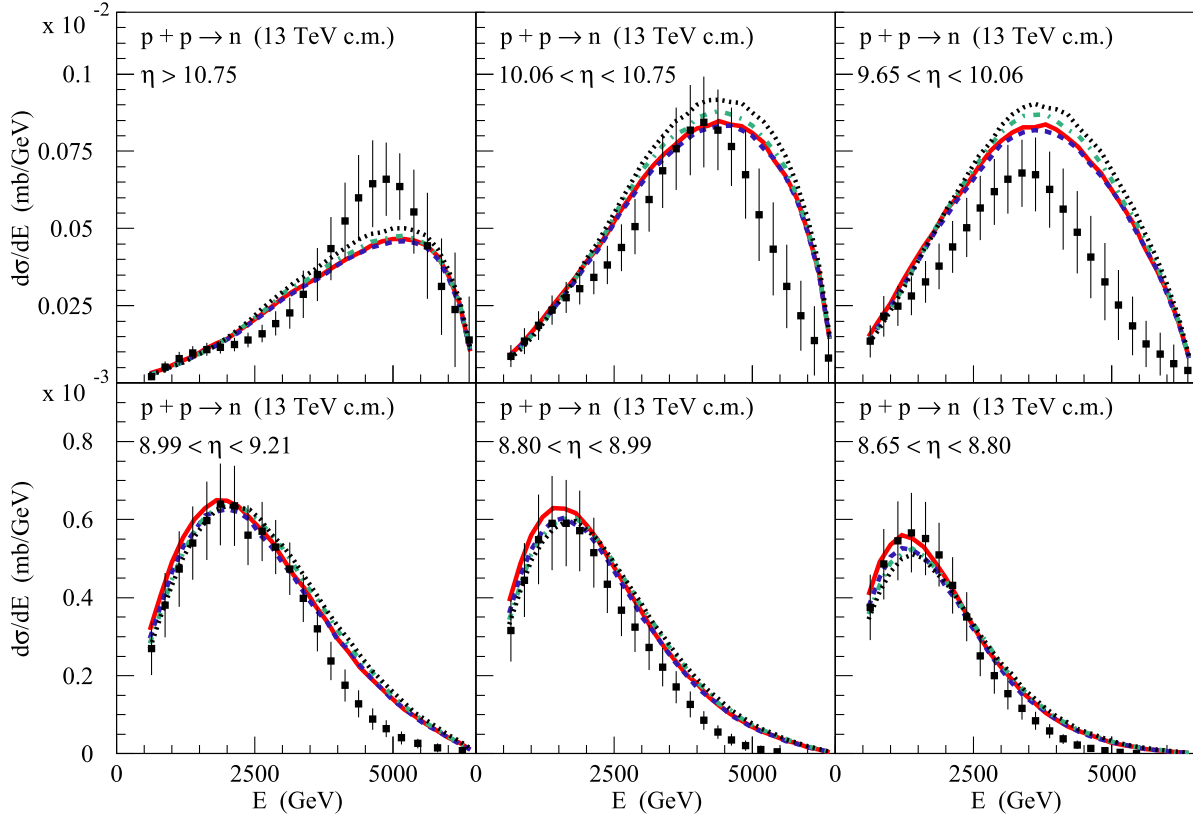


FIG. 6. Neutron energy spectra in c.m. frame, for pp collisions at $\sqrt{s} = 13$ TeV, for different pseudorapidity intervals, as indicated in the plots, in comparison to LHCf data [22] (points). The notations for the lines are the same as in Fig. 2.

Regarding a possibility to discriminate between the considered modifications of the model, promising are measurements of forward neutron spectra by the LHCf experiment [21,22]. As one can see in Fig. 6, the trend toward larger $\alpha_{\text{sea}}^{(\text{sh})}$ is somewhat disfavored by the data: the predicted forward neutron yield exceeds the measured one overall. Yet the corresponding indication is rather weak, and a more definite statement would require both a better tuning of the model to these data and an improvement of the accuracy of the measurement.

A much more decisive discrimination can be made by studying correlations between particle production in the central rapidity range and in the proton fragmentation region since this can allow one to test very basic model assumptions regarding momentum distributions of constituent partons in the proton [9]. Recently, such crucial measurements have been performed by the ALICE experiment at the LHC, for pp collisions at $\sqrt{s} = 13$ TeV: investigating correlations between central pseudorapidity density of charged hadrons, $dN_{\text{ch}}/d\eta|_{\eta=0}$, and a signal strength Z_N in zero degree calorimeters (ZDC), the latter being dominated by the energy deposited in ZDC by secondary neutrons produced with $|\eta| > 8.8$ [23,24]. In Fig. 7, these ALICE results are compared to the predictions of the above-discussed tunes 1, 2, 3, and 4 of QGSb,

regarding the dependence of the relative strength of the ZDC signal, $Z_N/\langle Z_N \rangle_{\text{inel}>0}$, on the relative pseudorapidity density of charged hadrons in the $|\eta| < 0.5$ window, $\frac{dN_{\text{ch}}}{d\eta}|_{|\eta|<0.5} / \langle \frac{dN_{\text{ch}}}{d\eta}|_{|\eta|<0.5} \rangle_{\text{inel}>0}$. Here $\langle Z_N \rangle_{\text{inel}>0}$ and $\langle \frac{dN_{\text{ch}}}{d\eta}|_{|\eta|<0.5} \rangle_{\text{inel}>0}$ are the average values of Z_N and $\frac{dN_{\text{ch}}}{d\eta}|_{|\eta|<0.5}$, respectively, corresponding to the “inel > 0” event selection of the ALICE experiment: at least one charged hadron in the $|\eta| < 1$ window.

The strong anticorrelation between the strength of the ZDC signal and the multiplicity of charged hadrons produced at $|\eta| < 0.5$, observed by ALICE, clearly supports the Regge-inspired choice for the momentum distributions of string end partons, $\alpha_{\text{sea}}^{(\text{sh})} = \alpha_{\text{sea}} = \alpha_{\text{R}} = 0.5$, which is adopted in the tunes 1 and 2, the corresponding results being shown by, respectively, solid and dashed lines in Fig. 7. While the observed anticorrelation is well reproduced by both tunes, a somewhat better overall agreement is provided by the default QGSb (tune 1) characterized by a larger value of the overcriticality of the “semihard Pomeron,” $\Delta_{\text{sh}} = 0.22$. For the tune 2, the smaller $\Delta_{\text{sh}} = 0.21$ gives rise to a somewhat smaller multiple scattering rate at $\sqrt{s} = 13$ TeV, thereby underestimating the energy loss of leading neutrons. On the other hand, for the high multiplicity tail of the distribution of

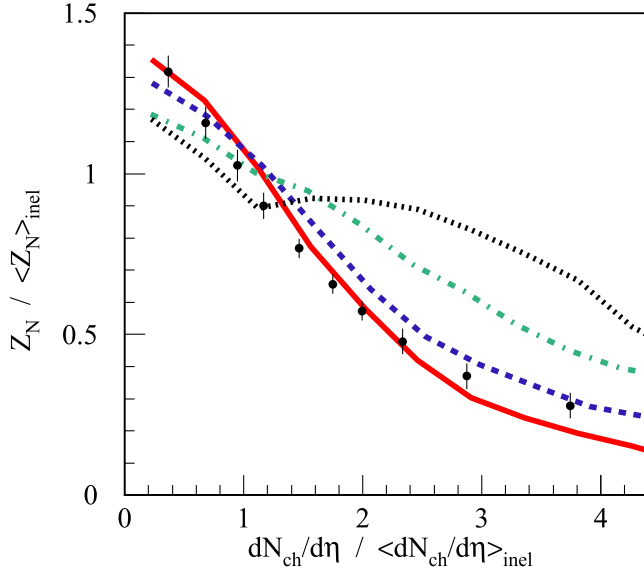


FIG. 7. Dependence of the relative strength of the ZDC signal, $Z_N / \langle Z_N \rangle_{\text{inel} > 0}$, on the relative pseudorapidity density of charged hadrons in the $|\eta| < 0.5$ window, for pp collisions at $\sqrt{s} = 13$ TeV, as calculated using different tunes of the QGSb model, compared to ALICE data [24] (points). The notations for the lines are the same as in Fig. 2.

$dN_{\text{ch}}/d\eta|_{|\eta| < 0.5}$, the strength of the ZDC signal predicted by the default QGSb is slightly smaller than the measured one. Hence the data hint at the energy dependence of multiple scattering rate (and of σ_{pp}^{tot}), which is intermediate between the two tunes.

In contrast, considering softer momentum distributions of string end partons, like in tunes 3 and 4, leads to a much weaker anticorrelation between central hadron production and the forward neutron yield, thereby contradicting the experimental observations (cf. dash-dotted and dotted lines in Fig. 7). Consequently, the extreme modifications of the energy dependence of K_{pN}^{inel} and of X_{max} , obtained using these tunes (dash-dotted and dotted lines in Figs. 4 and 5), are disproved by the above-discussed ALICE measurements. These conclusions can be further cross checked and strengthened by studying correlations between central and forward hadron production in proton-oxygen collisions, using experimental data collected during a recent proton-oxygen run of the Large Hadron Collider.

Additionally, as stressed in [7,25], a discrimination of predictions for the energy dependence of EAS maximum depth can be performed by air shower experiments, based on measurements of the maximal muon production depth X_{max}^{μ} [26]. As one can see in Fig. 8, choosing softer distributions for constituent sea (anti)quarks attached to semihard Pomeron strings, one arrives at substantially larger X_{max}^{μ} values predicted, compared to the default QGSb, by up to $\simeq 40$ and $\simeq 70$ g/cm² for the tunes 3 and 4, respectively. Such a tendency is not supported by the data of the Pierre

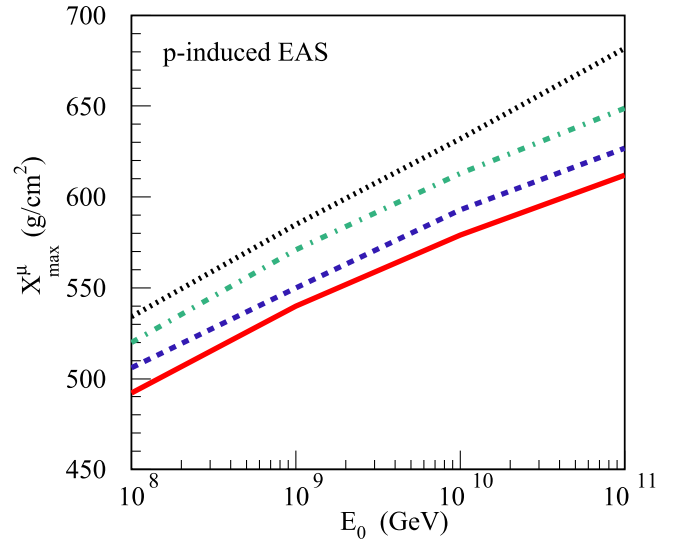


FIG. 8. Dependence of maximal muon production depth X_{max}^{μ} ($E_{\mu} > 1$ GeV), for p -induced vertical EAS, on the proton energy E_0 . The notations for the lines are the same as in Fig. 2.

Auger Observatory [26]: to reach an agreement with the measured maximal muon production depth, one would have to postulate that UHECRs are much heavier than iron.

2. How feasible is a smaller X_{max} ?

One can also study a possibility to shift the predicted shower maximum higher in the atmosphere, i.e., to have a smaller average X_{max} . In fact, there exists a theoretically well motivated mechanism leading to a substantially higher inelasticity of proton-nucleus collisions at high energies, related to the so-called “diquark breaking” in interacting nucleons [27–30]. Up to now, we followed the standard approach of the Quark-Gluon String and Dual Parton models [31,32], assuming that the diquark of an interacting nucleon survives multiple scattering as a coherent entity, i.e., remains in its color antitriplet state. However, as rightfully argued in [27], abundant multiple scattering should destroy the coherence between the two valence quarks forming the diquark, transforming the latter into a color sextet state. As a consequence, the two quarks should hadronize independently, being attached to two different strings of the color field, instead of having a single string coupled to the diquark. The treatment in [27–29] is based on the old idea [33] that the baryon content of a nucleon is not carried by its valence quarks, being rather associated with the so-called string junction. In the above-discussed scenario, the string junction follows a slowed down struck valence quark, with the consequence that a leading nucleon is produced with a substantially smaller fraction of initial light cone momentum ($E \pm p_z$) of the nucleon, compared to the case of diquark hadronization. In [30], a somewhat different approach, though with similar consequences, has been proposed. Namely, one considered a dense gluon

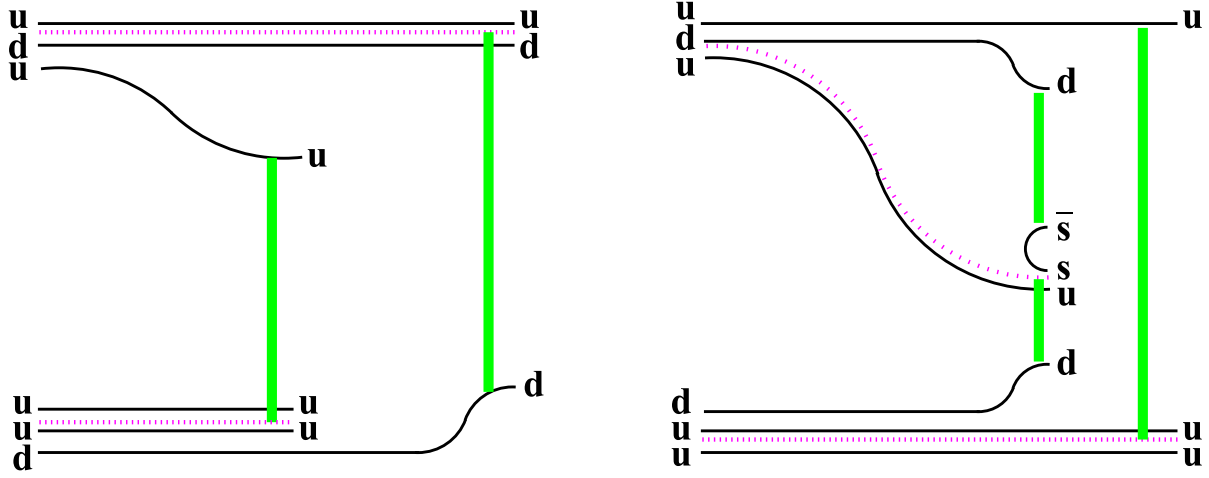


FIG. 9. Left: schematic view of the standard hadronization procedure for a projectile proton. After the proton is split into a slowed down valence quark (here u) and a diquark (here ud), the string junction shown by the (upper) dotted magenta line remains coupled to the diquark and two strings of color field (thick vertical green lines) are stretched between these partons and the corresponding constituents of the target (here uu and d , respectively). Right: a sketch of the diquark breaking mechanism for the projectile proton. The string junction follows the slowed down projectile valence quark (u), while the two remaining valence quarks (d and u) hadronize independently. Upon creation of a quark-antiquark pair (here $\bar{s}s$) from the vacuum, the created quark is coupled to the slowed valence quark and the junction to form a diquark (here us) and three strings of color field are formed: here between d and $\bar{s}$, us and d , u and uu .

cloud incident on a nucleon, in the nucleon rest frame, such that the two valence quarks forming the diquark suffer independent large transverse momentum “kicks” destroying the coherence of the diquark.

Here we choose to follow the approach of [27,28], assuming that the first inelastic rescattering of a nucleon splits it into a valence quark and a diquark, whereas subsequent inelastic scattering processes may, with probability w_{stj} , destroy the coherence of the diquark. Thus, for a nucleon undergoing n inelastic scatterings, i.e., being connected to n cut Pomerons, with the probability $(1 - w_{\text{stj}})^{n-1}$ the diquark survives as a coherent entity containing the string junction, and the standard hadronization procedure exemplified in Fig. 9 (left) takes place. There, two strings of color field are formed between the (here projectile) quark and the diquark [respectively, u and ud in Fig. 9 (left)] and the corresponding constituent partons of the partner (here target) hadron (nucleus) [uu and d in Fig. 9 (left)]. In the other case depicted in Fig. 9 (right), the string junction follows the struck valence quark, while the two remaining valence quarks hadronize independently, being each connected to its own string of color field. We then consider a creation of the quark-antiquark pair from the vacuum [$\bar{s}s$ in the example sketch of Fig. 9 (right)], with the created quark being coupled to the slowed valence quark and the junction to form a diquark. In such a case, we have a formation of three strings: one stretched between one of the remaining valence quarks of the nucleon and the created antiquark [$d - \bar{s}$ in Fig. 9 (right)], one between the created diquark and a quark of the partner hadron [$us - d$ in Fig. 9 (right)], and the last string being formed between the remaining valence quark of the

nucleon and a constituent parton of the partner hadron [$u - uu$ in Fig. 9 (right)].

We choose $w_{\text{stj}} = 0.2$ and, to stay in agreement with measured proton spectra for pp collisions at fixed target energies, use now $\alpha_N(0) = -0.25$ for the intercept of the nucleon Regge trajectory¹ [cf. Eq. (32) and Table 3 in [12]]. In addition, we switch off the special treatment of diquark hadronization in the case of single scattering (see Sec. 4.2 in [12]) by setting the corresponding parameter $\delta = 0$. The results of the so obtained tune 5 of the model, regarding the inelasticity of proton-nitrogen interactions, are plotted in Fig. 10, where we now have a somewhat faster energy rise, compared to tune 1, with up to $\approx 3\%$ larger K_{pN}^{inel} at the highest energies. However, for the calculated average EAS maximum depth X_{max} , we observed no noticeable differences from the default model predictions,² within the statistical uncertainties of the calculation ($\approx 1-2$ g/cm²), in contrast to drastic changes of X_{max} , predicted in [30]. This seemingly paradoxical result can be understood if we realize that the considered mechanism has a sizable impact on the inelasticity of proton-nucleus collisions at not too large impact parameters b only. There, copious multiple scattering already produces a sufficient “stopping” of leading nucleons and the

¹Interestingly, this value of $\alpha_N(0)$ is in agreement with results of standard Regge fits (see, e.g., [34]).

²We verified by an explicit calculation that this remains the case when a higher diquark breaking probability, e.g., $w_{\text{stj}} = 0.5$, is used, provided other model parameters are properly adjusted to keep the agreement with measured proton spectra for pp collisions at fixed target energies.

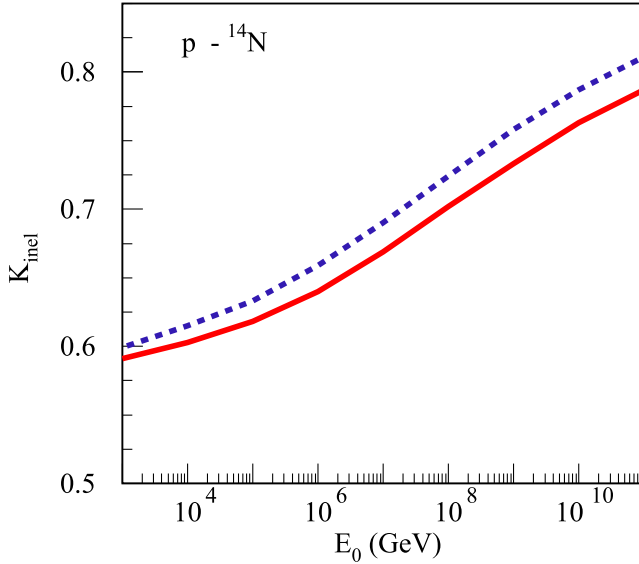


FIG. 10. Lab. energy dependence of the inelasticity K_{pN}^{inel} of proton-nitrogen collisions. The calculations performed using the default model (tune 1) or considering the diquark breaking mechanism (tune 5) are shown, respectively, by red solid and blue dashed lines.

additional “softening” of nucleon spectra is thus of weak importance for the longitudinal EAS development. On the other hand, peripheral proton-air interactions at large b , which contribute to non-Gaussian tails of X_{max} distributions and thereby influence significantly the average X_{max} , remain unaffected. In a sense, the obtained results may be regarded as an indication that the predictions of the default version of QGSb for the average EAS maximum depth are already close to the corresponding lower bound.

B. Aiming at a higher muon content of extensive air showers

Let us now address the muon content of extensive air showers. In particular, we are going to investigate possibilities to enlarge the predicted EAS muon number, N_μ , in view of the reported excess of the number of muons in UHECR-induced air showers, compared to respective predictions of EAS simulations [35,36] (see also the review [37]) regarding the corresponding “muon puzzle”).

As discussed in [8,38,39], model predictions for N_μ are closely correlated with the corresponding results for the moment $\langle x_E^{\alpha_\mu} n_{\text{stable}}^{\pi\text{-air}} \rangle$ of the energy distribution of “stable” secondary hadrons [(anti)nucleons, kaons, and charged pions] in pion-air interactions, defined as

$$\langle x_E^{\alpha_\mu} n_{\text{stable}}^{\pi\text{-air}}(E_0) \rangle = \sum_{h=\text{stable}} \int dx_E x_E^{\alpha_\mu} \frac{dn_{\pi\text{-air}}^h(E_0, x_E)}{dx_E}. \quad (1)$$

The summation in the right-hand side of Eq. (1) is performed over secondary hadrons having significant

chances to interact in the atmosphere, instead of decaying,³ $dn_{\pi\text{-air}}^h/dx_E$ is their distribution, with respect to the energy fraction x_E taken from the parent pion (upon decays of short-lived hadrons), and $\alpha_\mu \simeq 0.9$ is the characteristic exponent for the dependence of N_μ on the primary energy. As is obvious from Eq. (1), $\langle x_E^{\alpha_\mu} n_{\text{stable}}^{\pi\text{-air}} \rangle$ is dominated by forward production of stable hadrons in pion-air collisions, which is also the case for N_μ since more energetic secondaries initiate powerful enough subcascades in the atmosphere and thereby contribute stronger to EAS muon content at ground. On the other hand, N_μ depends rather weakly on the energy dependence of the multiplicity of pion-air interactions, in the very high energy limit, since the energy rise of the multiplicity is driven by hadron production in the central rapidity range, corresponding to small energy fractions x_E , with the contributions of such hadrons to Eq. (1) being damped⁴ by the factor $x_E^{\alpha_\mu}$.

Since α_μ is close to unity, the moment $\langle x_E^{\alpha_\mu} n_{\text{stable}}^{\pi\text{-air}} \rangle$ can be approximated by the average fraction of the parent pion energy taken by all stable secondary hadrons, i.e., by the case $\alpha_\mu = 1$, which is approximately equal to unity minus the energy fraction taken by neutral pions. Therefore, to enhance the predicted EAS muon content, one has to modify the energy balance between stable secondary hadrons and neutral pions, in favor of the former.

As discussed in [40], a particular mechanism which efficiently reduces the energy “leak” into π^0 production is the t -channel pion exchange (see [12] for the details on the implementation of the process in the QGSb model). This is because the process gives rise to forward production of ρ mesons and, upon decays of the latter ($\rho^\pm \rightarrow \pi^\pm \pi^0$, $\rho^0 \rightarrow \pi^+ \pi^-$), shifts the energy balance between secondary charged and neutral pions in favor of the former (see the corresponding discussion in [8]). In QGSb, the rate of virtual pion emission by an incident pion is controlled by the model parameter $g_{\pi/\pi}$, whose magnitude is seriously constrained by measurements of ρ^0 meson production, notably, by the NA61/SHINE experiment [41] (cf. solid lines in Fig. 11). Here we can investigate the effect of a somewhat higher rate for the process, choosing a 50% larger value for this parameter, $g_{\pi/\pi} = 0.225$. As one can see in Fig. 12, such a change (which we refer hereafter as tune 6) improves the agreement with the data of the EHS-NA22 experiment on forward ρ^0 production, while leading to an overestimation of both ρ^+ yield reported by the same experiment and of ρ^0 spectra, compared to other measurements, notably, by NA61/SHINE (cf. Figs. 11–13). Meanwhile, this gives rise to a very small change of the

³Obviously, such a definition of stable hadrons is energy dependent, e.g., $K^\pm$ can be regarded as being stable above ~ 1 TeV only.

⁴This is not the case for π -air interactions at relatively low energies, $E_0 \lesssim 100$ GeV, where the central rapidity range corresponds to relatively large $x_E \gtrsim 0.1$.

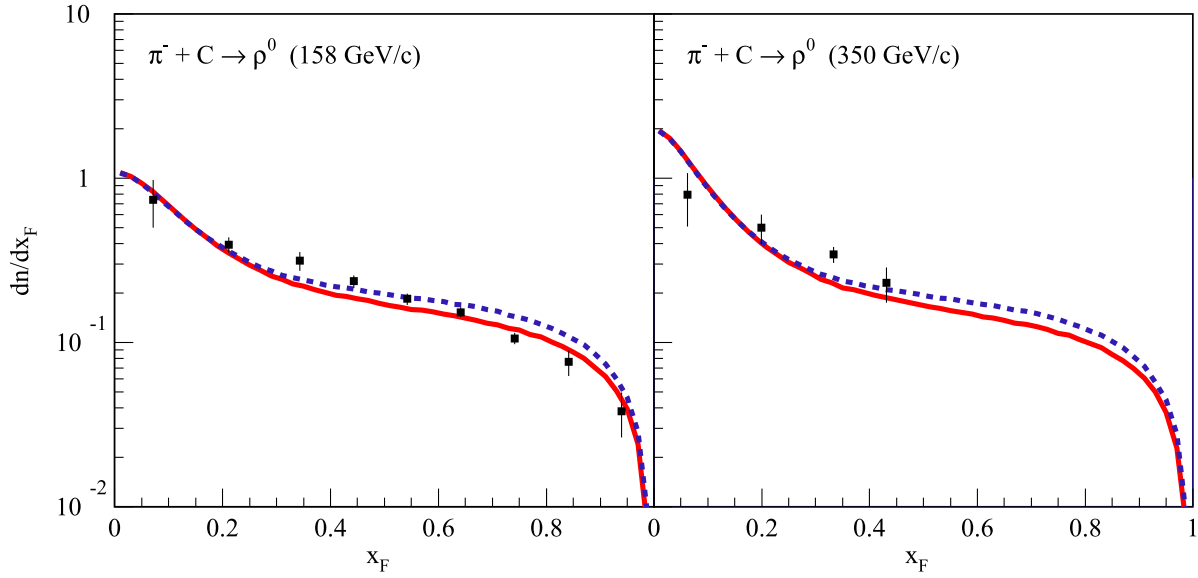


FIG. 11. Feynman x distributions, dn/dx_F , of ρ^0 mesons in c.m. frame, for $\pi^- C$ collisions at 158 GeV/c (left) and 350 GeV/c (right), as calculated using tunes 1 and 6 of QGSb—red solid and blue dashed lines, respectively, compared to NA61/SHINE data [41] (points).

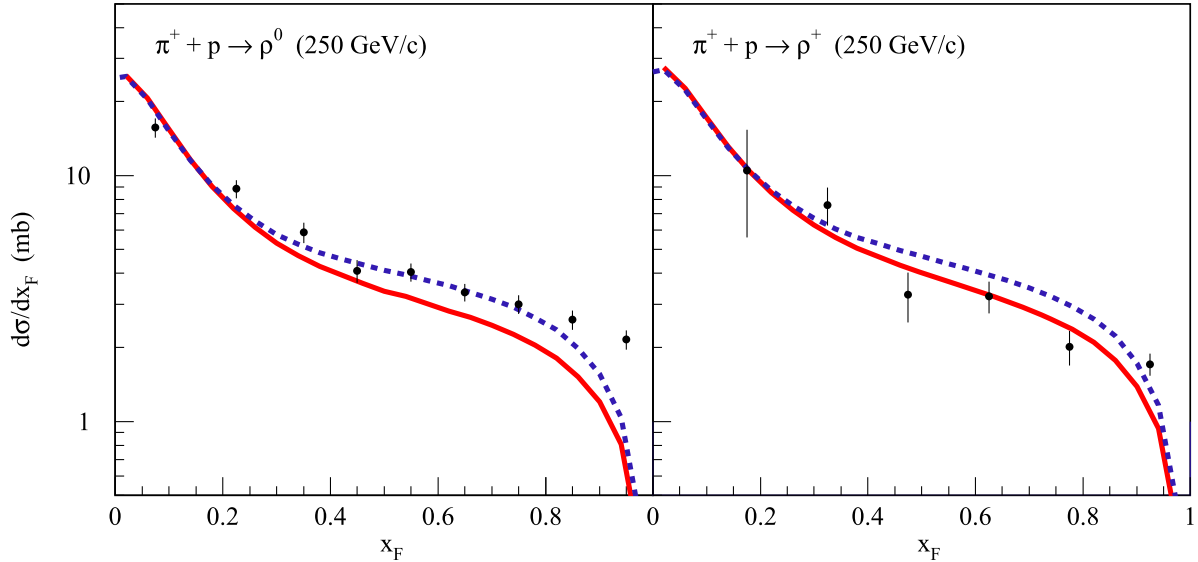


FIG. 12. x_F dependence of production cross sections in c.m. frame for ρ^0 (left) and ρ^+ (right) mesons, for $\pi^+ p$ collisions at 250 GeV/c, compared to EHS-NA22 data [42] (points). The notations for the lines are the same as in Fig. 11.

moment $\langle x_E^{\alpha_\mu} n_{\text{stable}}^{\pi-\text{air}} \rangle$ (less than one per mille) and, correspondingly, to a minor enhancement ($\lesssim 1\%$) of N_μ , compared to the default QGSb results.

Alternatively, the EAS muon content can be enhanced by considering a more abundant production of (anti)baryons and kaons in the projectile fragmentation region in pion-air collisions (e.g., [44,45]), which contributes noticeably to $\langle x_E^{\alpha_\mu} n_{\text{stable}}^{\pi-\text{air}} \rangle$. Those hadrons continue interacting in the atmosphere, thereby keeping the energy more efficiently in the hadronic cascade, compared to pions, since decays of neutral pions in the latter case work as an energy “sink.”

As discussed in [12], forward production of kaons and (anti)nucleons in pion-nucleus interactions, in the QGSb model, was tuned to the corresponding results of the NA61/SHINE experiment [46]. To improve the agreement with the data, we used a special treatment for the fragmentation of strings of color field, attached to “remnant” (anti)quarks of the pion, choosing higher probabilities, compared to the standard string fragmentation procedure, for strange quark-antiquark pairs and diquark-antidiquark pairs creation from the vacuum, characterized by the parameters $a_{s/\pi}$ and $a_{ud/\pi}$, respectively (see Table 4 in [12]). Phenomenologically, this

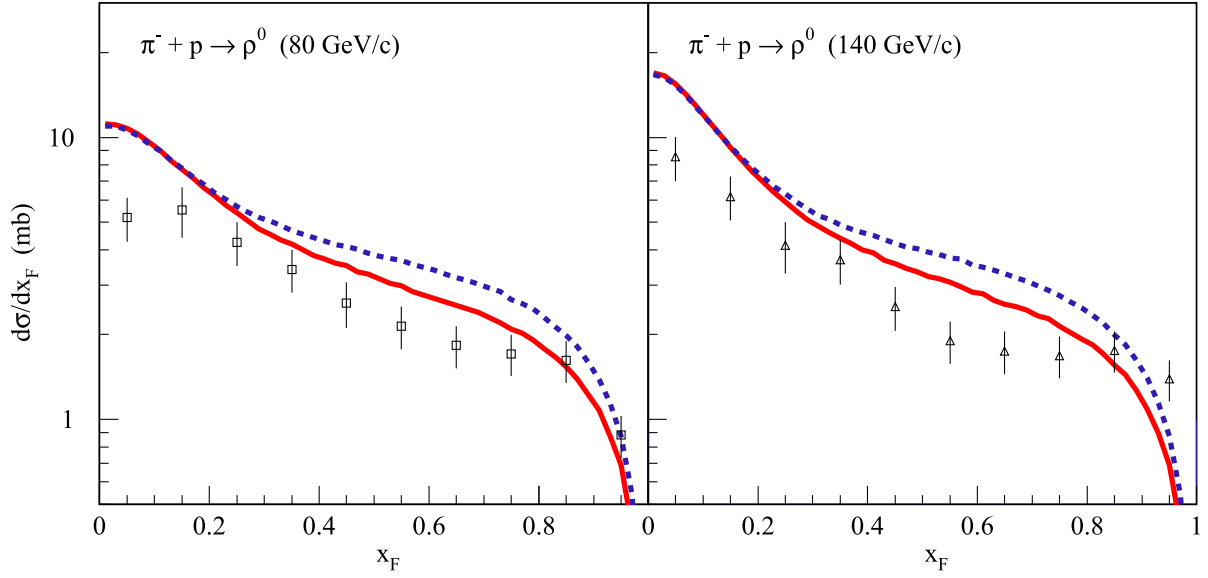


FIG. 13. x_F dependence of production cross sections in c.m. frame for ρ^0 mesons, for $\pi^- p$ collisions at 80 GeV/c (left) and 140 GeV/c (right), compared to the data of the OMEGA Photon Collaboration [43] (points). The notations for the lines are the same as in Fig. 11.

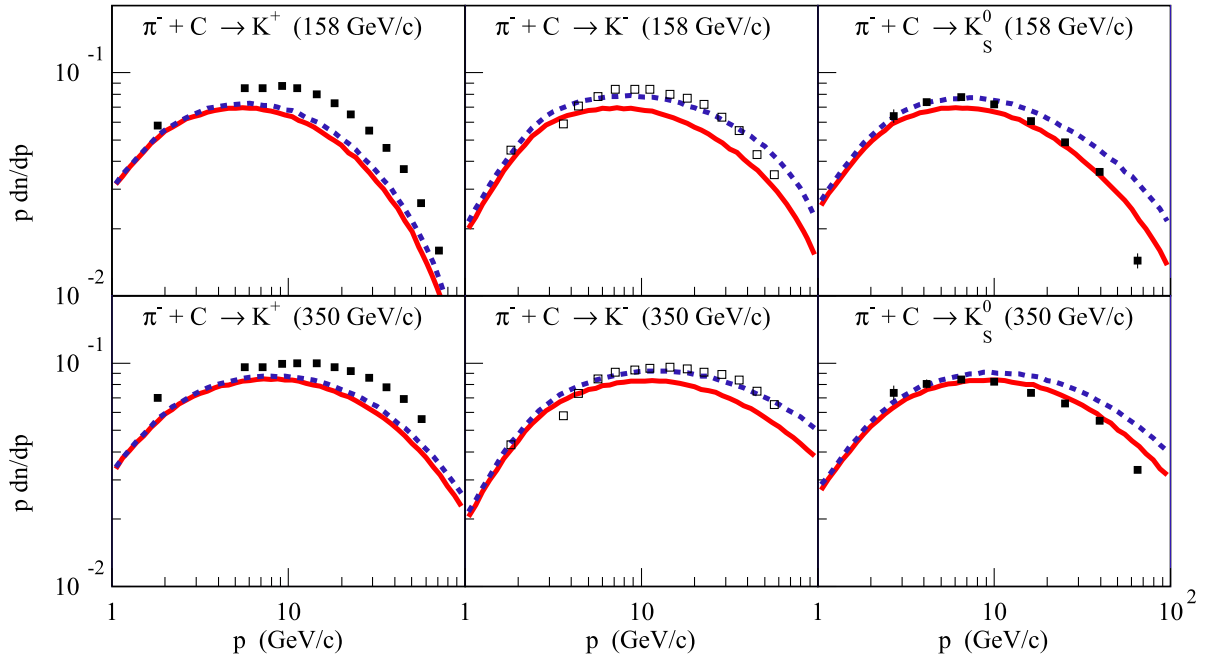


FIG. 14. Lab. momentum distributions of K^+ (left panels), K^- (middle panels), and K_S^0 (right panels) produced in $\pi^- C$ collisions at 158 GeV/c (top row) and 350 GeV/c (bottom row), compared to NA61/SHINE data [46] (points). The calculations performed using tunes 1 and 7 of QGSb are shown, respectively, by red solid and blue dashed lines.

can be interpreted as being related to “intrinsic strangeness” and “intrinsic baryonic content” of pions.⁵

⁵A more theoretically rigorous approach would amount to considering t -channel exchanges of, respectively, K^* and Δ Reggeons, in the Reggeon-Reggeon-Pomeron ($\mathbb{R}\mathbb{R}\mathbb{P}$) configuration ($\mathbb{R} = K^*, \Delta$), similarly to the pion exchange case discussed above.

While this allowed us to reach a reasonable agreement with the data of NA61/SHINE for K_S^0 production in $\pi^- C$ collisions at 158 and 350 GeV/c, the forward yield of charged kaons remained underestimated, compared to the measurements (cf. solid lines in Fig. 14). Leaving aside the question regarding a hypothetical isospin symmetry violation [47], we can study here the impact on N_μ of a more abundant forward kaon production, using a factor 2 larger

value $a_{s/\pi} = 0.4$, while keeping the default values for all other parameters of the model. As one can see in Fig. 14, for the so-obtained tune 7 of the model, we reach a reasonably good agreement with the measured K^- spectra, while still largely underestimating K^+ production and overestimating now the forward yield of K_S^0 . Regarding kaon production in pion-carbon collisions at a smaller incident pion momentum, 60 GeV/c, the agreement with the data is improved both for charged and for neutral kaons, as one can see in Fig. 15. However, for very forward production of K^+ at small polar angles, $\theta < 20$ mrad, the calculated spectra are substantially harder than the measured ones. On the other hand, the obtained model tune largely overestimates forward kaon production, compared to results of other experiments, both for kaon-proton and kaon-nucleus collisions; see Figs. 16 and 17. For the moment $\langle x_E^{\alpha_\mu} n_{\text{stable}}^{\pi-N} \rangle$, $\alpha_\mu = 0.9$, for pion-nitrogen collisions, we obtain with this tune an enhancement, relative to the default QGSb, which ranges between 0.8% at $E_0 = 1$ TeV and 0.4% at the highest energies (cf. solid and dashed lines in Fig. 18). For the muon number N_μ ($E_\mu > 1$ GeV) at sea level, for vertical proton-induced air showers, this leads to an enhancement of up to $\simeq 2\%$, compared to the default QGSb results, as one can see in Fig. 19.

Let us now investigate the impact on EAS muon content of an enhancement of (anti)nucleon production in pion-air interactions. For the default QGSb, we have a reasonable agreement with NA61/SHINE measurements of antiproton spectra in π^-C collisions at 158 and 350 GeV/c, while the calculated forward production of protons is substantially smaller than that observed by the NA61/SHINE experiment (see solid lines in Fig. 20). As already mentioned above, we employed a special treatment for the hadronization of “remnant” (anti)quarks of the pion, using a higher probability for creation of $ud - \bar{u}\bar{d}$ pairs from the vacuum, governed by the parameter $a_{ud/\pi}$, compared to the corresponding parameter a_{ud} of the standard string fragmentation procedure (cf. Table 4 in [12]). Qualitatively, such a prescription can be justified by an analogy to a more rigorous Regge treatment involving an emission of a virtual Δ (anti)baryon by the incident pion, the corresponding contributions being shown schematically in Figs. 21(a) and 21(b), for the particular case of an incident π^- . In Fig. 21(a), upon emission of a Δ^0 baryon, the projectile π^- converts into an antiproton, $\pi^- \xrightarrow{(\Delta^0)} \bar{p}$, while the virtual Δ^0 interacts with the target (not shown in the figure). Likewise, in Fig. 21(b), one has an emission of a virtual $\bar{\Delta}^-$, thereby

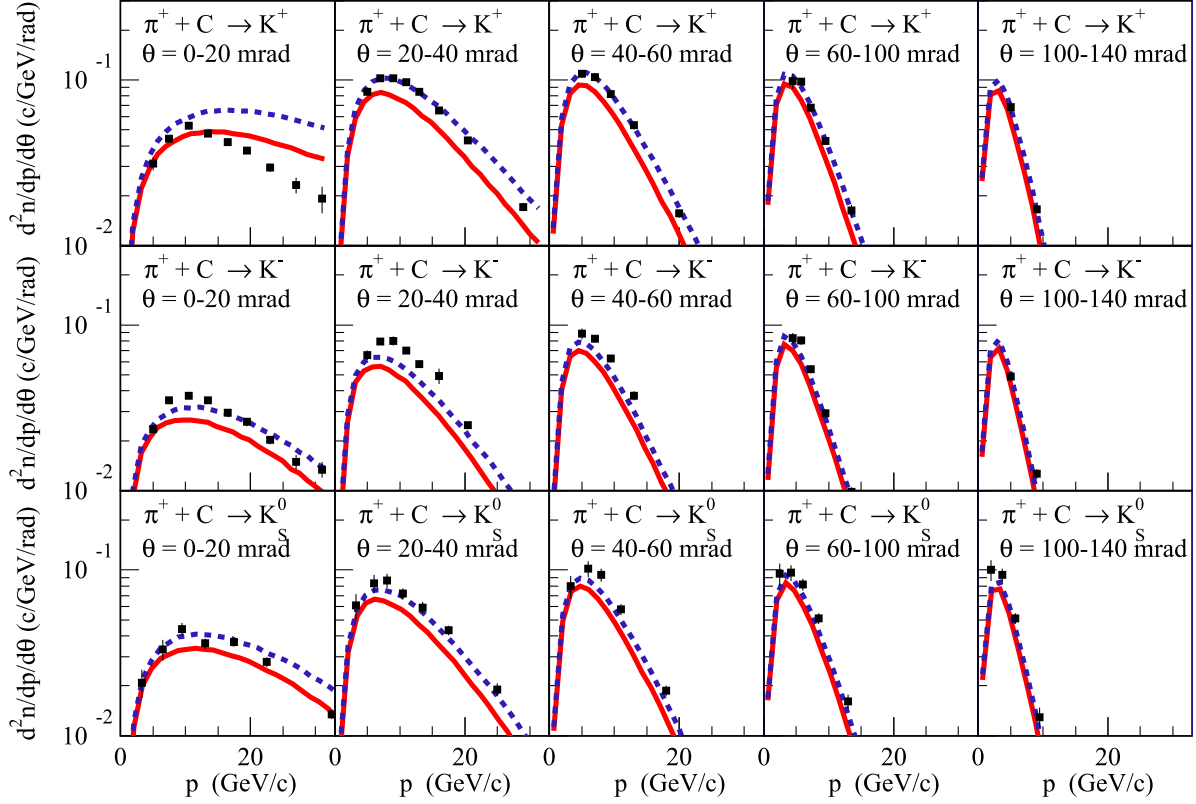


FIG. 15. Lab. momentum distributions of K^+ (top row), K^- (middle row), and K_S^0 (bottom row) produced in π^+C collisions at 60 GeV/c, at different polar angles, as indicated in the plots, compared to NA61/SHINE data [48] (points). The notations for the lines are the same as in Fig. 14.

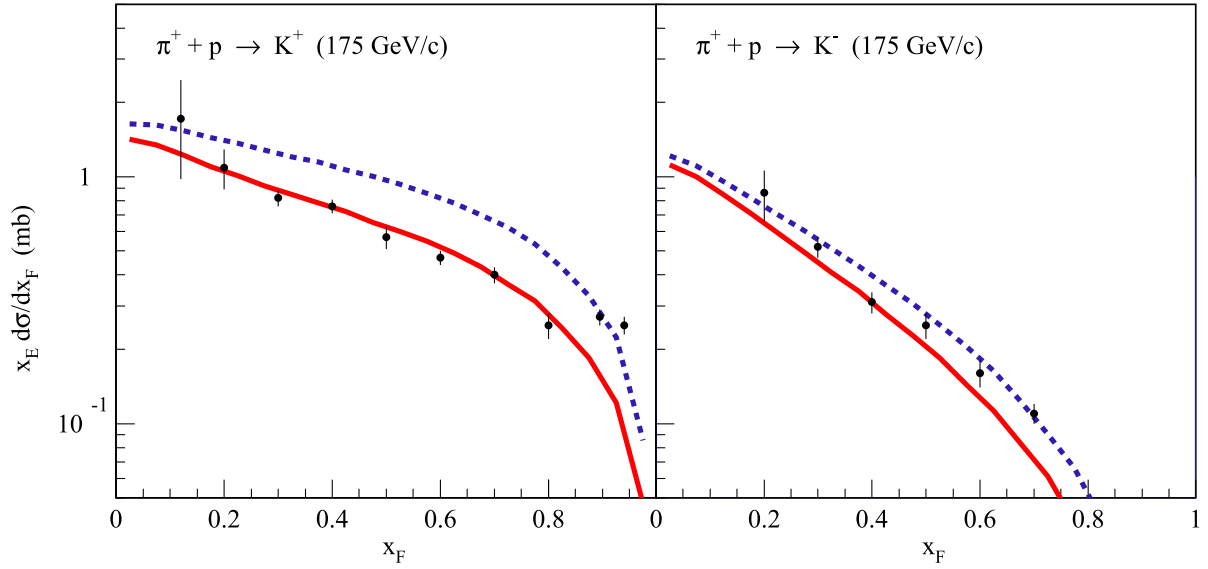


FIG. 16. x_F dependence of transverse momentum p_t integrated invariant cross sections in c.m. frame, $x_E d\sigma/dx_F$ ($x_E = 2E/\sqrt{s}$), for K^+ (left) and K^- (right) production in $\pi^+ p$ collisions at 175 GeV/c, compared to experimental data [49] (points). The notations for the lines are the same as in Fig. 14.

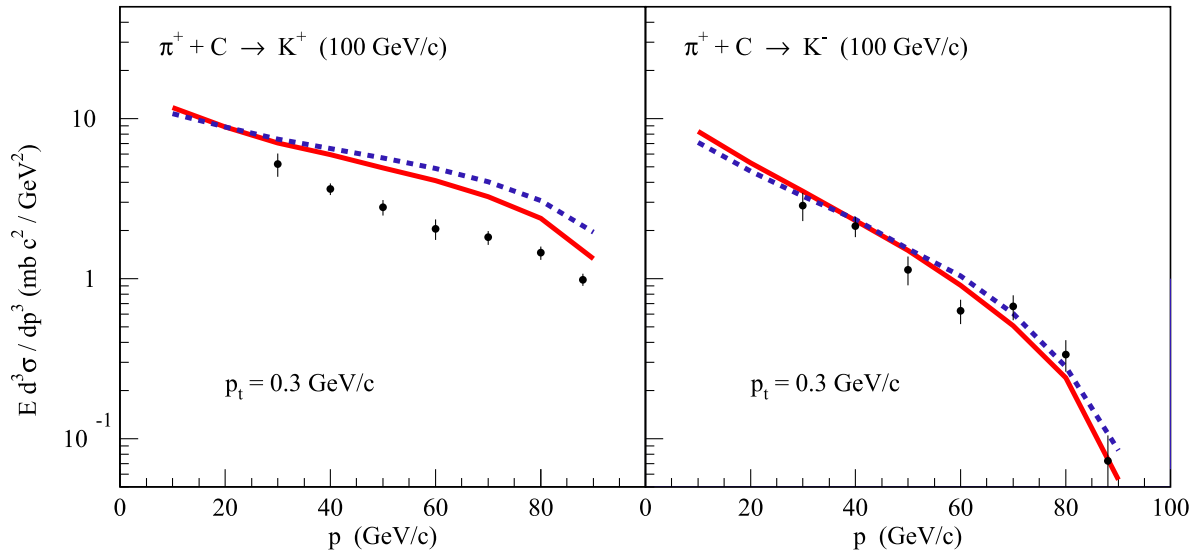


FIG. 17. Lab. momentum dependence of invariant cross sections, $E d^3\sigma/dp^3$, for $p_t = 0.3$ GeV/c, for K^+ (left) and K^- (right) production in $\pi^+ C$ collisions at 100 GeV/c, compared to experimental data [50] (points). The notations for the lines are the same as in Fig. 14.

converting the incident pion into a neutron: $\pi^- \rightarrow n$. Here, pursuing further the analogy to the Regge treatment and invoking the isospin symmetry considerations, one would have to include, with equal probabilities, the processes shown in Figs. 21(c) and 21(d), corresponding to emissions of Δ^- and $\bar{\Delta}^{--}$ by the incident pion. In our phenomenological approach, this would amount to considering a creation of $dd - \bar{d}\bar{d}$ and $uu - \bar{u}\bar{u}$ pairs from the vacuum, with the same probabilities as for $ud - \bar{u}\bar{d}$ pairs, in the hadronization of “remnant” (anti)quarks of the pion.

We study such a modification, using $a_{dd/\pi} = a_{uu/\pi} = a_{ud/\pi} = 0.15$, which we refer to hereafter as tune 8. For the spectra of protons and antiprotons in $\pi^- C$ collisions at 158 and 350 GeV/c, plotted as dashed lines in Fig. 20, we now have a reasonable agreement with NA61/SHINE data. Regarding proton production in pion-carbon collisions at a smaller momentum of the incident pion, 12 GeV/c, the discussed modification improves slightly the agreement with measurements of the HARP experiment [51], as one can see in Fig. 22. However, for p and $\bar{p}$ production in $\pi^- p$

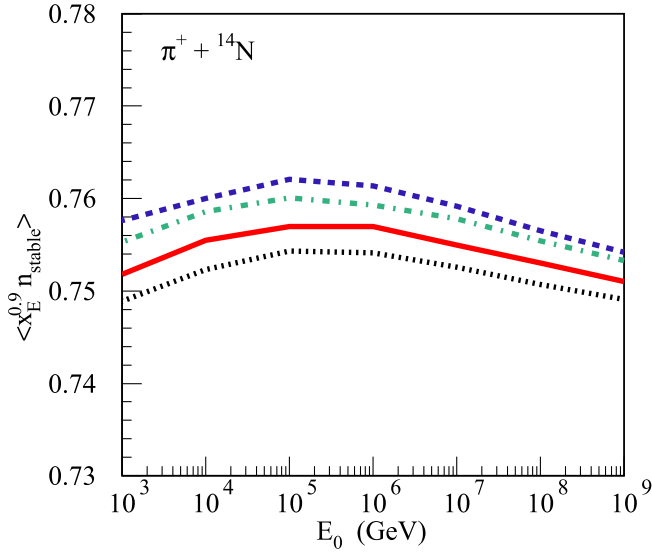


FIG. 18. Lab. energy dependence of the moment $\langle x_E^{0.9} n_{\text{stable}}^{\pi-N} \rangle$, as calculated using the tunes 1, 7, 8, and 9 of the QGSb model—red solid, blue dashed, green dash-dotted, and black dotted lines, respectively.

interactions at 360 GeV/c, we arrive with this tune to a factor 2 excess, compared to the corresponding data of the LEBC-EHS experiment [52]; see Fig. 23. For the moment

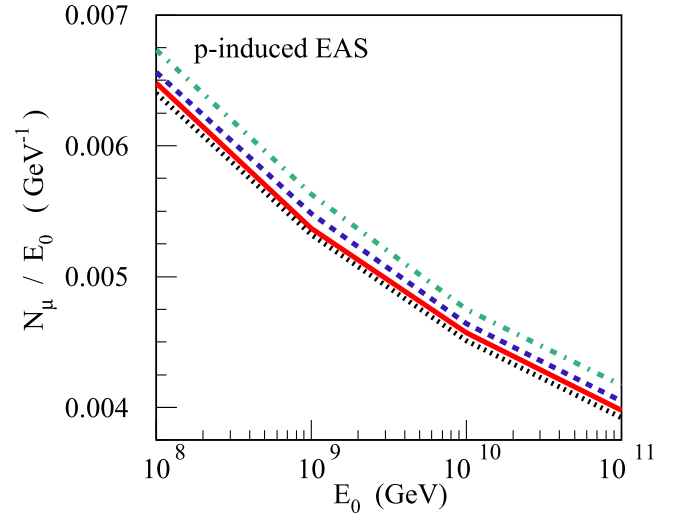


FIG. 19. Dependence on primary energy of muon number N_μ ($E_\mu > 1$ GeV) at sea level, for proton-initiated vertical EAS. The notations for the lines are the same as in Fig. 18.

$\langle x_E^{\alpha_\mu} n_{\text{stable}}^{\pi-N} \rangle$, $\alpha_\mu = 0.9$, the enhancement obtained with this model tune is comparable to the one we had for tune 7 (with enhanced kaon production), ranging between 0.5% and 0.3%, with the corresponding energy dependence being

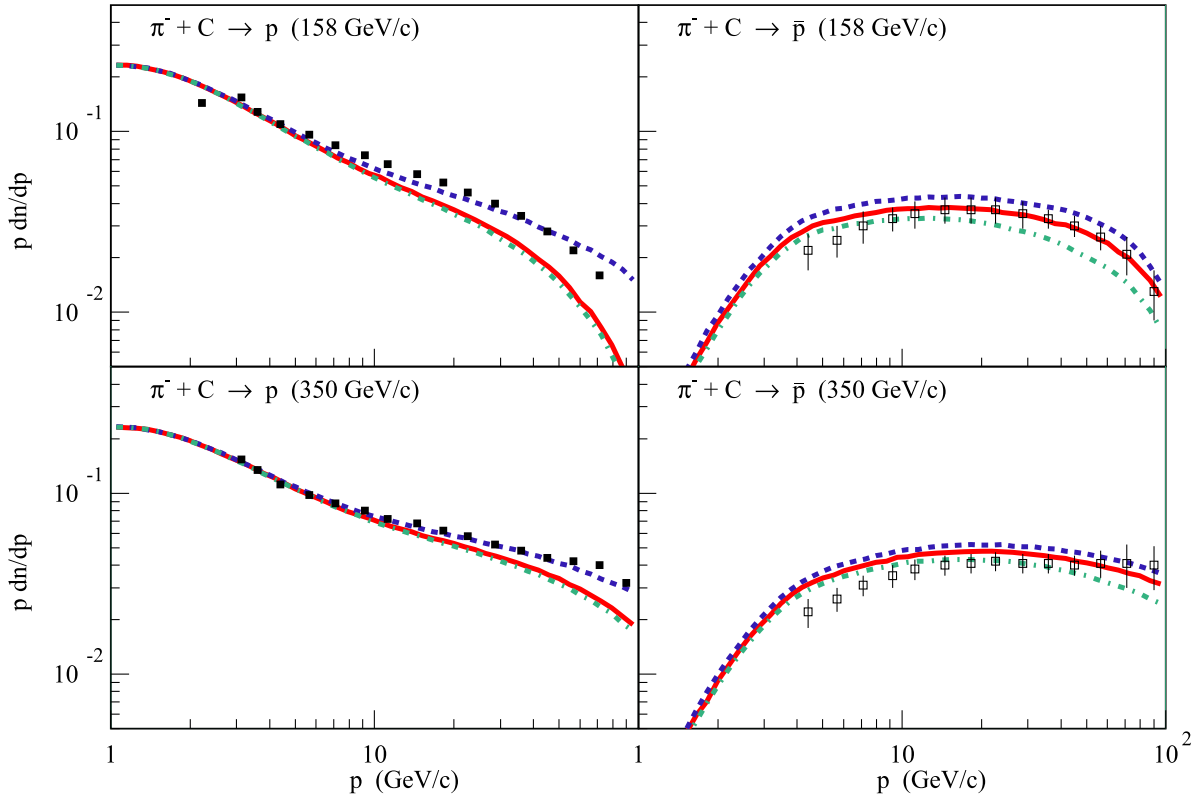


FIG. 20. Lab. momentum distributions of protons (left panels) and antiprotons (right panels), produced in $\pi^- \text{C}$ collisions at 158 GeV/c (top row) and 350 GeV/c (bottom row), compared to NA61/SHINE data [46] (points). The calculations performed using the tunes 1, 8, and 9 of QGSb are shown, respectively, by red solid, blue dashed, and green dash-dotted lines.

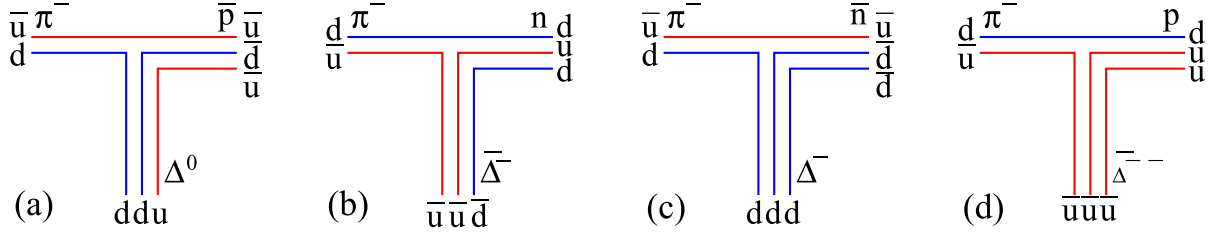


FIG. 21. Schematic view of the contributions of t -channel exchange of Δ Reggeons to forward (anti)nucleon production in pion-proton and pion-nucleus collisions. (a) The ud diquark from a vacuum-created $ud - \bar{u}\bar{d}$ pair couples with the slowed down valence quark d , forming a virtual Δ^0 baryon which subsequently interacts with the target (not shown in the figure). The $\bar{u}\bar{d}$ antiquark is combined with the “remnant” valence antiquark $\bar{u}$ to form a leading antiproton: $\pi^- \xrightarrow{(\Delta^0)} \bar{p}$. (b) Same as in (a) but for an emission of a virtual Δ^- baryon and a formation of a leading neutron: $\pi^- \xrightarrow{(\Delta^-)} n$. The same mechanism proceeds in (c) and (d) via a creation of, respectively, $dd - \bar{d}\bar{d}$ and $uu - \bar{u}\bar{u}$ pairs from the vacuum: $\pi^- \xrightarrow{(\Delta^-)} \bar{n}$ and $\pi^- \xrightarrow{(\Delta^{--})} p$.

plotted in Fig. 18 as dash-dotted line. The calculated EAS muon number N_μ exceeds the corresponding results of the default model by up to 5%, as one can see in Fig. 19.

Finally, we can try the opposite changes: switching down the special treatment of the hadronization of “remnant” (anti)quarks of the pion, i.e., restricting ourselves with the creation of $ud - \bar{u}\bar{d}$ diquark-antidiquark pairs from the vacuum and using one and the same value for the

corresponding model parameters, for fragmentation of all strings (setting $a_{ud/\pi} = a_{ud} = 0.1$). In such a case (referred to as tune 9), we marginally improve the agreement with LEBC-EHS data on p and $\bar{p}$ production (cf. dash-dotted lines in Fig. 23), while further moving away from the corresponding measurements by the NA61/SHINE and HARP experiments, as one can see in Figs. 20 and 22. For this tune, the calculated moment $\langle x_E^{0.9} n_{\text{stable}}^{\pi-N} \rangle$ is smaller

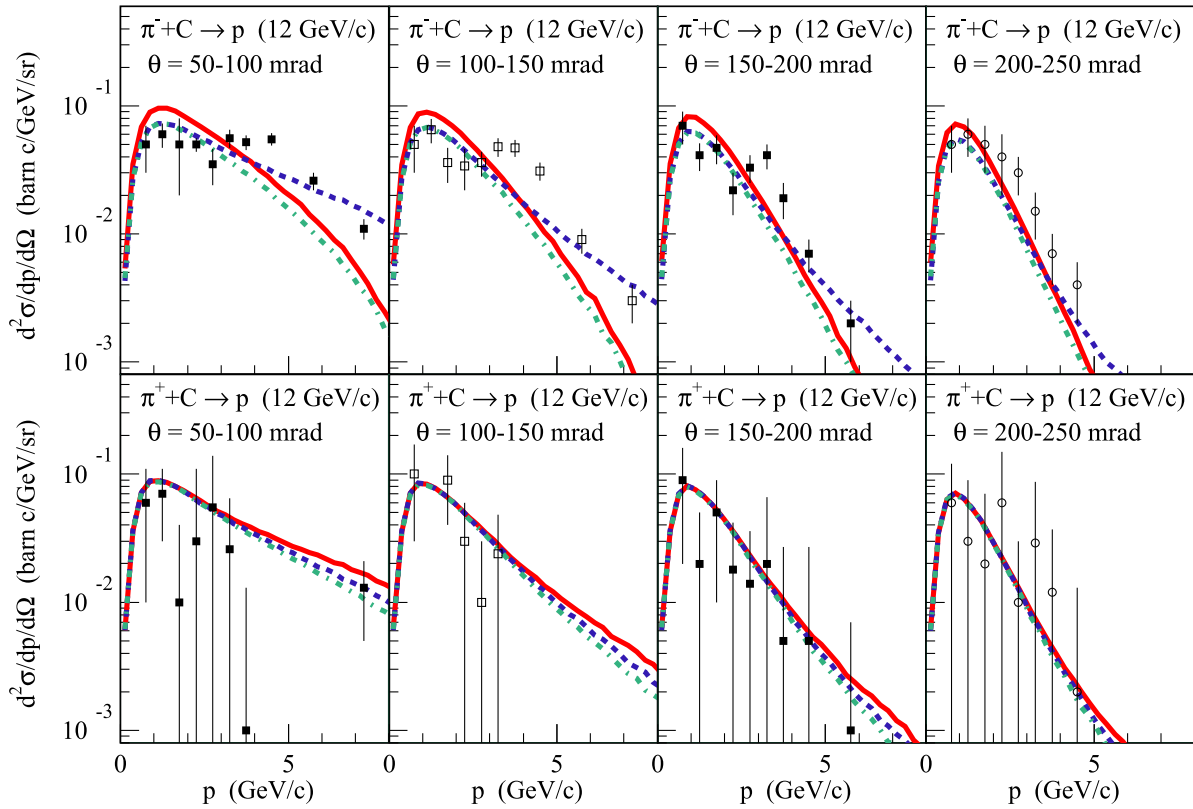


FIG. 22. Lab. momentum dependence of inclusive cross sections for proton production, at different polar angles, as indicated in the plots, in π^-C (top) and π^+C (bottom) collisions at 12 GeV/c, compared to HARP data [51] (points). The notations for the lines are the same as in Fig. 20.

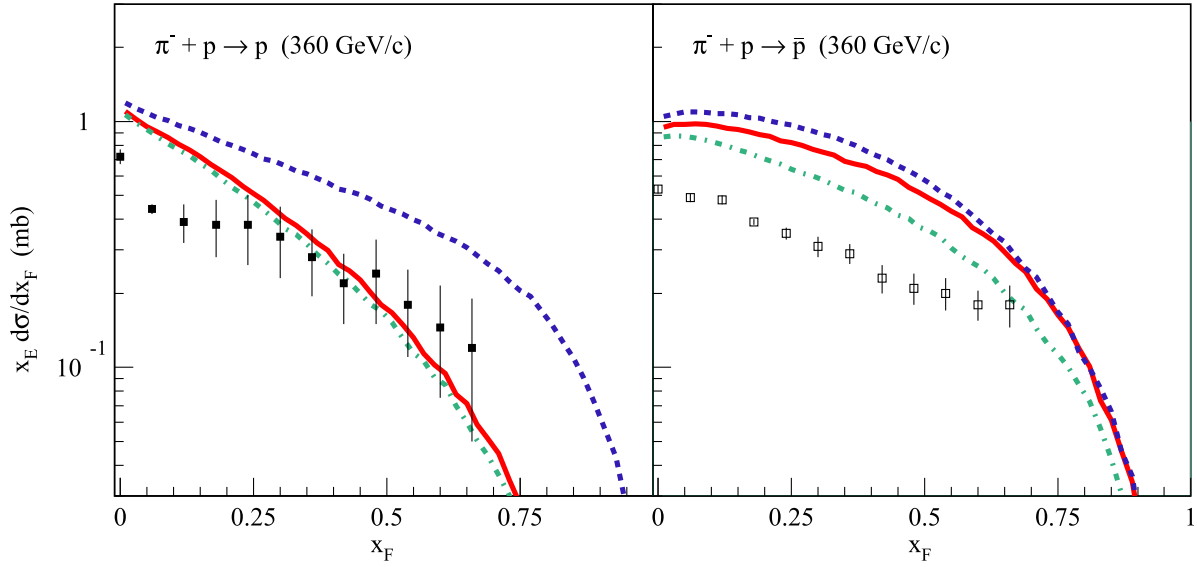


FIG. 23. x_F dependence of p_T -integrated invariant cross sections in c.m. frame, $x_E d\sigma/dx_F$, for proton (left) and antiproton (right) production in $\pi^- p$ collisions at 360 GeV/c, compared to LEBC-EHS data [52] (points). The notations for the lines are the same as in Fig. 20.

than for the default QGSb by $\simeq 0.3\%$ (cf. dotted line in Fig. 18) while the obtained number of muons is decreased by 1%–2%; see Fig. 19.

For all the alternative model tunes studied in this section, the multiplicity of secondary hadrons produced in pion-air collisions remained practically unchanged, the differences to the default model results being at the subpercent level. Hence, the obtained modifications of EAS muon content were caused by changes of the forward spectra of secondary hadrons. Here, the largest impact on N_μ was observed when modifying forward production of (anti)nucleons in pion-air interactions. Partly this is because the corresponding treatment is not sufficiently constrained by relevant accelerator data, due to serious contradictions between different measurements of (anti)proton yields in the fragmentation region of incident pion, in πp and πA collisions.

For convenience, we compile in Table I the parameter changes, for the considered alternative tunes of the model.

III. DISCUSSION AND CONCLUSIONS

We employed the new Monte Carlo generator of hadronic collisions, QGSb, for studying model uncertainties for calculated characteristics of extensive air showers initiated by interactions of high energy cosmic rays in the atmosphere. We specifically concentrated on modifying predictions for two key EAS observables mostly used in experimental studies of nuclear composition of high energy cosmic rays: the shower maximum depth $X_{\max}$ and the muon number at ground level N_μ . In particular, we discussed in some detail the relevant physics mechanisms and the corresponding model parameters, studying also the impact of the considered modifications of the model on the

level of agreement of its predictions with measured characteristics of hadron-proton and hadron-nucleus collisions.

While the studied modifications of the model are not comprehensive, we clearly outlined main directions for an alternative tuning of model parameters and identified relevant accelerator data constraining such changes.

Regarding EAS maximum depth, our main interest was to investigate possibilities to predict a slower air shower development, which could potentially improve the agreement with experimental data of the Pierre Auger Observatory. We demonstrated that the most efficient way for obtaining larger $X_{\max}$ values is to consider softer momentum distributions for hadron constituents to which strings of color field are attached, in agreement with previous studies. However, the most extreme modifications of the kind, studied by us, proved to be incompatible with recent results of the ALICE experiment at LHC, regarding correlations between secondary hadron production at central rapidities and in the proton fragmentation region, in proton-proton collisions. Moreover, those modifications are strongly disfavored by the data of the Pierre Auger Observatory, regarding the maximal muon production depth of UHECR-induced EAS, since a compatibility with the data can only be achieved for primary nuclei heavier than iron. On the other hand, we have not found much freedom for speeding up the hadronic cascade development, i.e., for obtaining sizably smaller $X_{\max}$ values. In particular, contrary to what has been claimed in the literature, implementing the so-called diquark splitting mechanism does not produce any appreciable change of the predicted $X_{\max}$, provided other relevant model parameters are adjusted to keep an agreement with

TABLE I. Summary of the parameter changes, for the alternative tunes of the QGSb model.

	Δ_{sh}	$\alpha_{\text{sea}}^{(\text{sh})}$	$\Lambda^{(\text{sh})}$	$\alpha_N(0)$	δ	$g_{\pi/\pi}$	$a_{s/\pi}$	a_{ud}	$a_{ud/\pi}$	$a_{uu/\pi} = a_{dd/\pi}$	w_{stj}
Default value	0.22	0.5	3.6	-0.05	1	0.15	0.1	0.1	0.19	0	0
Tune 2	0.21		4.6								
Tune 3	0.21	0.8	6.5								
Tune 4	0.21	0.95	9.5								
Tune 5				-0.25	0						0.2
Tune 6						0.225					
Tune 7							0.4				
Tune 8									0.15	0.15	
Tune 9									0.1		

measured proton spectra in pp collisions at fixed target energies.

Overall we end up with $\simeq 10$ g/cm² uncertainty for X_{max} predictions, which is considerably smaller than the spread between the corresponding results of current CR interaction models and, moreover, which excludes the possibility to interpret self-consistently experimental data of the Pierre Auger Observatory. While our analysis has been performed within a particular model framework, our conclusions are generic enough, being grounded by the underlying physics picture. For example, the constraints on X_{max} predictions, imposed by the ALICE measurements of anticorrelations between central and forward hadron production, stem from the same physical mechanism governing such anticorrelations and the energy dependence of the inelasticity of proton-proton collisions. Indeed, while the latter is driven by the energy rise of multiple scattering in pp interactions, the former is governed by variations of multiple scattering rates in such interactions at a given energy, primarily caused by variations of the impact parameter for the collisions. In turn, both the steepness of the energy rise of K_{pp}^{inel} and the magnitude of the above-discussed anticorrelations depend crucially on model assumptions regarding momentum distributions of constituent partons in the proton [9].

However, these conclusions may be compromised by revising the traditional Glauber-Gribov approach regarding the transition from proton-proton to proton-nucleus collisions, aiming at a smaller average number of interacting target nucleons [53]. The corresponding reduction of multiple scattering in proton-air collisions would allow one to reduce the predicted $K_{p\text{-air}}^{\text{inel}}$ and to obtain thereby a larger X_{max} . A measurement of anticorrelations between central and forward hadron production in proton-oxygen collisions may allow one to check whether this is a viable approach.

In relation to the so-called muon puzzle, we investigated possibilities to enlarge the predicted EAS muon content. The latter was shown to depend strongly on the treatment of forward hadron production, notably, on the energy balance between (relatively) stable secondary hadrons and neutral pions. The largest enhancement of N_{μ} , up to $\simeq 5\%$, was obtained considering harder spectra of (anti)nucleons produced in pion-air collisions. A reduction of the

corresponding uncertainty is currently problematic because of serious contradictions between different measurements of (anti)proton production in pion-proton and pion-nucleus interactions.

These results are in line with previous studies [8], showing that the resolution of the muon puzzle is only possible by considering nonstandard approaches to particle production in hadronic collisions. For example, postulating *ad hoc* an enhancement of forward kaon and (anti)nucleon production, with increasing energy, one was able to obtain pretty large values for EAS muon content at ultrahigh energies [54]. A similar, though smaller effect has been observed by considering collective effects in the hadronization procedure of the EPOS-LHC model [55]. A more recent study [56] shows that a significant increase of the muon number can be achieved when using a global approach including all known nuclear effects at all stages in the tuning process of the models. While a discrimination of such approaches is generally problematic, without experimental data on pion-proton and pion-nucleus collisions at very high energies, some constraints do arise from current experimental activities at the LHC. For example, an enhancement of forward kaon production, with increasing energy, has been disfavored by the data of the FASER experiment, based on measurements of muonic and electronic neutrinos at large pseudorapidities [57].

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DATA AVAILABILITY

The code created and used in the reported study will be publicly available within the CONEX extensive air shower simulation program or, alternatively, can be obtained from the authors upon reasonable request.

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