

Unveiling primordial black hole relics through induced gravitational waves

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Black hole relics are of significant interest in cosmology and theoretical physics. In this work, we consider tiny primordial black holes (PBHs) ($M_{\text{PBH}} \lesssim 10^7$ g), which are generated soon after the end of inflation and evaporate and reheat the Universe before big bang nucleosynthesis (BBN), but leave their remnants due to incomplete evaporation. These PBHs remnants may contribute as part or all of the dark matter (DM) today. Assuming that there exist PBH relics, we point out that the number density of PBH today can be directly read from the peak positions of the induced gravitational waves (IGWs) due to the inhomogeneous PBH distribution. If PBH relics are of Planck mass and they form all the DM today, the PBH number density would be of 10^{-25} cm^{-3} with the peak frequency 60 Hz. The peak frequency scales as $f_{\text{relic}}^{1/3}$ where f_{relic} is the fraction of the PBH relics in the total DM density. The peak amplitude carries the information of initial PBH abundance. For monochromatic-mass PBH with the current number density 10^{-41} – 10^{-25} cm^{-3} and initial abundance 10^{-13} – 10^{-7} , the amplitude may be large enough to be detected by planned gravitational wave experiments in the near future.

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I. INTRODUCTION

Black hole relics are hypothetical remnants that may remain after the evaporation of a black hole via Hawking radiation. According to classical black hole thermodynamics and Hawking's seminal work [1–4], black holes are not entirely black but emit radiation due to quantum effects near the event horizon, leading to gradual mass loss and eventual evaporation. However, the final stage of this evaporation process remains uncertain [5–13]. One compelling idea is that instead of evaporating completely, black holes might leave behind stable or metastable remnants known as black hole relics or Planck-mass remnants [14–17].

These relics are of significant interest in theoretical physics because they could provide insight into the unification of quantum mechanics and general relativity, potentially serving

as probes of quantum gravity effects [18–24]. One of the biggest puzzles in theoretical physics is the black hole information paradox: How information about matter that falls into a black hole is preserved or lost after it evaporates via Hawking radiation [25,26]. If black holes evaporate completely, it seems information is lost [27], violating unitarity of quantum theory. Black hole relics provide a way out: If evaporation stops (or slows down) at some stage, the remnant could store the information, preserve unitarity, and resolve the paradox. Theoretical models predicting black hole relics often incorporate modifications to standard physics, such as generalized uncertainty principles or quantum gravity inspired corrections [28]. In these frameworks, black hole relics are thought to form at the end stages of evaporation when quantum gravity effects dominate. Also, it has been recently argued that the black hole evaporation suffers from the memory burden effect, which slows down the evaporation rate significantly [12,13,29–38]. In this case the black hole lifetime can become much larger than the age of the Universe, hence it effectively leaves primordial black hole (PBH) relics that are much heavier than the Planck mass today. Studying relics may offer clues about the nature of quantum gravity and the unification of general relativity with quantum mechanics—a major unsolved problem [12,39].

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Black hole relics are also of great importance in cosmology. PBHs could have formed in the early Universe, by various mechanisms with masses spanning a wide range [40–60]. Hawking radiation predicts that PBHs with initial masses below about $\sim 10^{15}$ g would have evaporated by today [45]. However, if the evaporation stops at a finite mass, these PBHs leave behind black hole relics. The formation and evaporation of PBHs and their relics can influence the early Universe’s thermal history, entropy budget, and matter content. Such relics may contribute to the dark matter (DM) content of the Universe [61–64]. If relics are sufficiently abundant and long-lived, they could account for part or all of DM without conflicting with other cosmological observations. This offers a non-particle-physics, gravity-based candidate for dark matter. Even if they constitute a fraction of DM, depending on their abundance, they could also leave imprints on cosmological observables like the cosmic microwave background, structure formation, and gravitational wave backgrounds [13,34,65–67].

In this work, we consider a scenario where PBHs are formed during radiation dominated era after inflation, with their lifetime much shorter than the time of big bang nucleosynthesis (BBN). We assume the mass spectrum to be monochromatic for simplicity. If PBHs are abundantly produced, they eventually dominate the Universe before evaporation [68–78]. We find that, if PBH remnants exist, one can extract information of the initial PBH abundance and the current number density from the peak of the induced GWs due to the initial inhomogeneous PBH distribution.

The paper is organized as follows. We briefly review the small PBH evaporation and their relics in Sec. II. We present the relation between the initial PBH abundance, the present-day relic number density, and the gravitational wave (GW) spectrum peak induced by PBH density perturbation in Sec. III. We also analyse the capability of detecting these PBH relics in the near future and their influence on people’s normal lives. Section IV is devoted to conclusions.

II. REVIEW OF PRIMORDIAL BLACK HOLE EVAPORATION AND RELICS

PBHs are generated in the radiation dominated era shortly after inflation. The initial mass of the PBHs at formation time is [79]

$$M_{\text{PBH},i} = \gamma \frac{4\pi}{3} (H_f^{-1})^3 3M_{\text{pl}}^2 H_f^2 \approx 1.1 \times 10^{-5} \text{ g} \frac{M_{\text{pl}}}{H_f} \quad (1)$$

where H_f is the Hubble parameter at formation time. These group of PBHs were formed around $t_f \approx 1/(2H_f)$. $M_{\text{pl}} \equiv 1/\sqrt{8\pi G}$ is the reduced Planck mass, which is approximately 2.435×10^{18} GeV, or equivalently 4.341×10^{-6} g [80]. $\gamma \approx 0.2$ is the ratio of mass finally falling into a PBH and horizon mass [47]. For extremely small PBHs

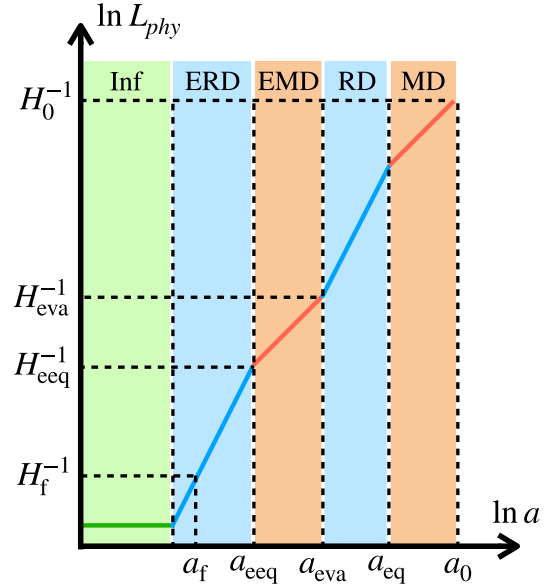


FIG. 1. A schematic diagram of characteristic Hubble scales in the current scenario. The universe undergoes inflation (Inf), followed by an early radiation-dominated era (ERD), and by an early matter (PBH)-dominated era (EMD). The later radiation-dominated (RD) and matter-dominated (MD) eras are those of the standard Big-Bang theory. The recent dark energy dominated era is ignored for simplicity.

that form around the end of inflation, one approximately has $H_f/M_{\text{pl}} \lesssim 10^{-5}$, which implies $M_{\text{PBH},i} \gtrsim 1.1 \text{ g} \sim 2.5 \times 10^5 M_{\text{pl}}$. Define the initial energy density fraction of PBH [81],

$$\beta \equiv \frac{\rho_{\text{PBH},f}}{\rho_{\text{tot},f}} \ll 1. \quad (2)$$

In the conventional scenario, the early Universe is radiation dominated (energy density $\rho_r \propto a^{-4}$, where a is the scale factor) when PBHs are formed. Since PBHs behave like nonrelativistic matter, with energy density scaling as $\rho_{\text{PBH}} \propto a^{-3}$, even if the initial fraction of PBHs in the total energy density is small, their relative contribution grows as the Universe expands. If PBHs are abundant enough, they can easily dominate the Universe and realize a temporary matter-dominated phase during the radiation era. We call it an early matter-dominated era, or the PBH-dominated epoch [69]. The PBH-dominated epoch ends when the energy density of the radiation generated by Hawking radiation exceeds the PBH mass density, and the Universe returns to the radiation-dominated stage.

Requiring that the Hawking evaporation [2] ends before BBN, which occurs at $t_{\text{eva}} \sim 1$ s, the PBH mass should be smaller than $\sim 10^9$ g [43,47]. If there is indeed a PBH-dominated era, we can set a_{eq} as the time of early radiation-PBH equality. The evaporation can happen during the radiation dominated era or the PBH dominated era,

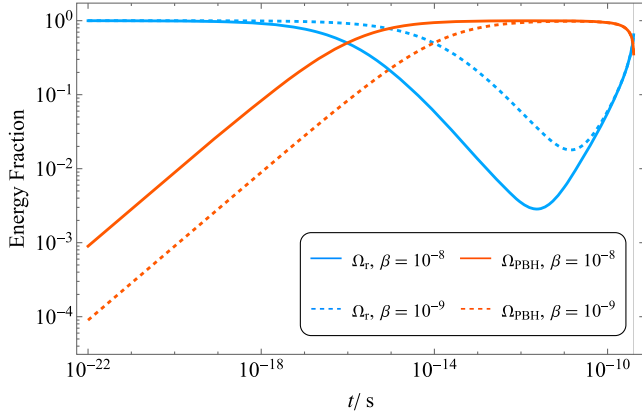


FIG. 2. Energy fractions of PBHs and radiation for two different initial fractions of PBHs. The initial PBH mass is set to $M_{\text{PBH},i} = 10^6$ g. The solid lines are for $\beta = 10^{-8}$, and the dotted lines for $\beta = 10^{-9}$.

depending on the initial energy density fraction of PBH. We focus on the case where PBHs dominate the universe before evaporation. As can be seen from Fig. 1, the condition for the appearance of a PBH dominated era is

$$H_{\text{eva}} \leq H_{\text{eq}}. \quad (3)$$

The minimum PBH initial abundance is therefore

$$\beta_{\min} \approx \left(\frac{H_{\text{eva}}}{\sqrt{2}H_f} \right)^{1/2} \approx 5.3 \times 10^{-13} \left(\frac{M_{\text{PBH},i}}{10^7 \text{ g}} \right)^{-1}. \quad (4)$$

The derivation can be found in Appendix A. The energy density evolution from the PBH formation time to evaporation time for two choices of β is shown in Fig. 2.

We assume that the PBH evaporation leaves behind relics with mass given by

$$m_{\text{relic}} = rM_{\text{pl}}; \quad r \geq 1. \quad (5)$$

Introducing the time when the evaporation stops,

$$M_{\text{PBH}}(t_{\text{end}}) = m_{\text{relic}}, \quad (6)$$

we find the difference between the time t_{eva} and t_{end} is small,

$$\frac{\delta t}{t_{\text{eva}}} = 1 - \frac{t_{\text{end}}}{t_{\text{eva}}} = \frac{m_{\text{relic}}^3}{M_{\text{PBH},i}^3} \ll 1, \quad (7)$$

for $m_{\text{relic}} \ll M_{\text{PBH},i}$.

Now we study the abundance of relics. Let f_{relic} be the fraction of the PBH relics in the total DM density today,

$$f_{\text{relic}} \equiv \frac{\rho_{\text{relic},0}}{\rho_{\text{DM},0}}, \quad (8)$$

where the subscript 0 denotes the present value. The relation between $M_{\text{PBH},i}$ and f_{relic} is [81]

$$\frac{M_{\text{PBH},i}}{10^7 \text{ g}} = 5.3 \times 10^{-2} \left(\frac{r}{f_{\text{relic}}} \right)^{2/5} \times \left(\frac{g_{\star}(T_{\text{eva}})}{106.75} \right)^{3/10} \left(\frac{g_{s\star}(T_{\text{eva}})}{106.75} \right)^{-2/5}. \quad (9)$$

The derivation can be found in Appendix B.

III. RELIC ABUNDANCE HINT FROM IGW

A PBH-dominated era can leave a distinct imprint on the stochastic GW background. The random distribution of PBHs naturally leads to the Poisson power spectrum of the PBH density perturbation, which is isocurvature at the time of PBH formation. It induces GWS at second order in perturbation [69,82–84], which can be potentially detected by future GW observations.

The dimensionless PBH density perturbation spectrum at the formation moment is given by [85]

$$\mathcal{P}_S(k) \equiv \frac{k^3}{2\pi^2} P_S(k) = \frac{k^3}{2\pi^2} \frac{1}{n_{\text{PBH},f} a_f^3} = \frac{2}{3\pi} \left(\frac{k}{k_{\text{uv}}} \right)^3, \quad (10)$$

where $n_{\text{PBH}} = \rho_{\text{PBH}}/M_{\text{PBH}}$ is the PBH number density. The ultraviolet cutoff k_{uv} is given by the mean inter-PBH separation at the formation time,

$$k_{\text{uv}} \equiv a_f \left(\frac{4\pi}{3} n_{\text{PBH},f} \right)^{1/3} = \frac{\beta^{1/3}}{\gamma^{1/3}} a_f H_f. \quad (11)$$

The PBH density perturbations are isocurvature in the beginning, but they evolve to curvature perturbations as PBHs begin to dominate the universe. When the PBH evaporation renders the universe radiation-dominated again, those curvature perturbations start oscillating rapidly and produce GWs. If we assume the complete evaporation, the induced GW spectrum today has a peak at k_{uv} with the peak amplitude [81,85],

$$\Omega_{\text{GW},0}^{\text{peak}} h^2 \approx 2.1 \times 10^{-11} \beta^{16/3} \left(\frac{k_{\text{uv}}}{k_{\text{eva}}} \right)^{17/3} \cdot \left(\frac{g_{\star}(T_{\text{eva}})}{106.75} \right) \left(\frac{g_{s\star}(T_{\text{eva}})}{106.75} \right)^{-4/3}, \quad (12)$$

where k_{eva} is the comoving wave number that crosses the Hubble horizon at the time of evaporation, $k_{\text{eva}} = H(t_{\text{eva}})a(t_{\text{eva}})$.

A. Information contained in the IGW peak

Now we consider the case where the PBH evaporation is incomplete and PBHs leave behind their remnants. The physical frequency today f is given by the comoving wave

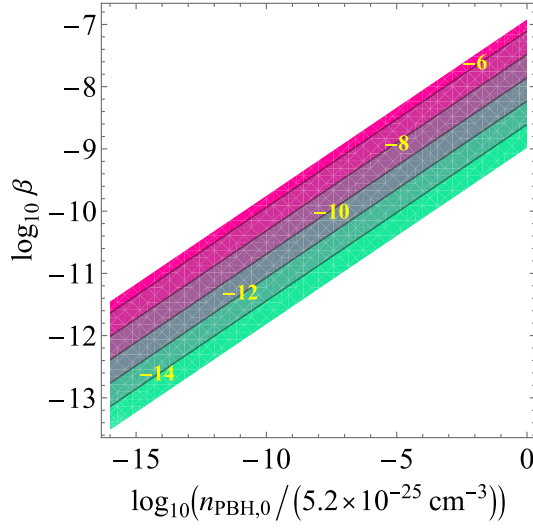


FIG. 3. Contour plot of $\log_{10}(\Omega_{\text{GW},0}^{\text{peak}} h^2)$ in the $(\beta, n_{\text{PBH},0})$ space. We focused on the region $10^{-16} < \Omega_{\text{GW},0}^{\text{peak}} h^2 < 10^{-5}$.

number k as $f = k/(2\pi a_0)$. Taking the ratio of (11) and k_{eva} , we find

$$\frac{k_{\text{uv}}}{k_{\text{eva}}} = \left(\frac{H_f}{\gamma H_{\text{eva}}} \right)^{1/3}. \quad (13)$$

The peak frequency is determined by

$$f_{\text{uv}} \approx 61.9 \text{ Hz} \left(\frac{r}{f_{\text{relic}}} \right)^{-1/3}. \quad (14)$$

The detailed derivation together with a discussion about f_{eva} can be found in Appendix C. This is one of our main findings. The peak frequency of the induced GWs depends only on the relic mass and its current abundance. The case $r = f_{\text{relic}} = 1$ was derived in [81].

Substituting (13) into (12), and using (9), the peak amplitude of the GW spectrum is given as

$$\Omega_{\text{GW},0}^{\text{peak}} h^2 \approx 1.7 \times 10^{-11} \left(\frac{\beta}{10^{-8}} \right)^{16/3} \left(\frac{r}{f_{\text{relic}}} \right)^{68/45} \times \left(\frac{g_{\star}(T_{\text{eva}})}{106.75} \right)^{32/15} \left(\frac{g_{\star}(T_{\text{eva}})}{106.75} \right)^{-128/45}. \quad (15)$$

According to Fig. 5 in [86], $g_{s\star} \approx g_{\star}$ is always a good approximation in the standard model above 1 MeV. For $T > T_{\text{EW}} \sim 200 \text{ GeV}$, $g_{s\star} \approx g_{\star} \approx 106.75$. For $T < 1 \text{ GeV}$, the value of $g_{s\star}$ (or $g_{\star}$) drops down by about one order of magnitude. In this work, in order to avoid unnecessary complications, we focus on $1 \text{ g} \lesssim M_{\text{PBH},i} \lesssim 10^7 \text{ g}$ ($1 \lesssim r \lesssim 10^{12}$), which corresponds to $T_{\text{eva}} \gtrsim 1 \text{ GeV}$. Then we have

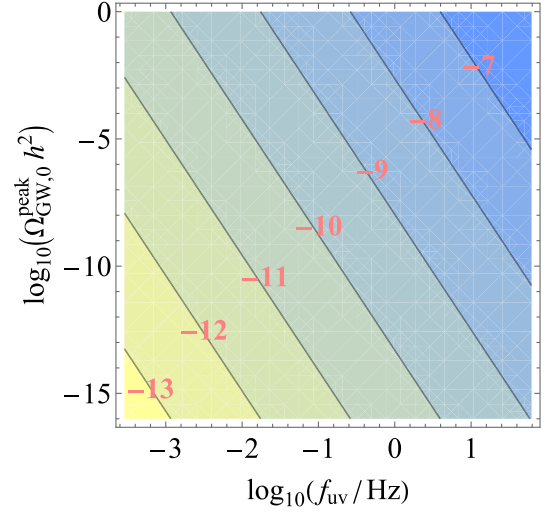


FIG. 4. Contour plot of $\log_{10} \beta$ in the $(f_{\text{uv}}, \Omega_{\text{GW},0}^{\text{peak}} h^2)$ plane.

$$\frac{r}{f_{\text{relic}}} \approx \left(\frac{f_{\text{uv}}}{61.9 \text{ Hz}} \right)^{-3},$$

$$\beta \approx 10^{-8} \left(\frac{\Omega_{\text{GW},0}^{\text{peak}} h^2}{1.7 \times 10^{-11}} \right)^{3/16} \left(\frac{f_{\text{uv}}}{61.9 \text{ Hz}} \right)^{17/20}. \quad (16)$$

The above are the summary of our results. The GW peak frequency determines the ratio of the relic PBH mass and its abundance today, while the amplitude tells us the efficiency of the PBH formation in the early universe.

Moreover, it is useful to mention that there is one-to-one correspondence between r/f_{relic} and the PBH number density today,

$$n_{\text{PBH},0} = \frac{f_{\text{relic}} \rho_{\text{DM},0}}{r M_{\text{pl}}} \approx 5.2 \times 10^{-25} \text{ cm}^{-3} \frac{f_{\text{relic}}}{r}. \quad (17)$$

In terms of the PBH number density, (14) and (15) become

$$f_{\text{uv}} \approx 61.9 \text{ Hz} \left(\frac{n_{\text{PBH},0}}{5.2 \times 10^{-25} \text{ cm}^{-3}} \right)^{1/3},$$

$$\Omega_{\text{GW},0}^{\text{peak}} h^2 \approx 1.7 \times 10^{-11} \left(\frac{\beta}{10^{-8}} \right)^{16/3} \times \left(\frac{n_{\text{PBH},0}}{5.2 \times 10^{-25} \text{ cm}^{-3}} \right)^{-68/45}. \quad (18)$$

We show $\Omega_{\text{GW},0}^{\text{peak}} h^2$ as a function of $(\beta, n_{\text{PBH},0})$ or $(\beta, f_{\text{relic}}/r)$ in Figs. 3 and 4. The relevant frequency band locates in the sensitivity ranges of BBO [87], the Einstein Telescope (ET) [88,89], Cosmic Explorer (CE) [90], DECIGO [91,92], LIGO [93–100], VIRGO [101–106], and KAGRA [107,108].

B. Detectability from GWs surveys

Substituting the sensitivity curves of all kinds of current and future detectors in our main result (16), one obtains the lower bound for the detectability of examining the existence of the PBH dominating era. The gray region in Fig. 5 represents the case when the GW spectrum peak locates within the detection capabilities. Compared with (4), we find that although the allowed parameter space for small PBHs that have dominated the universe is broad, the region that can be tested in the coming decades is limited. Conversely, if no GW spectrum with slope $\propto k^{11/3}$ is detected, Fig. 5 can be treated as the initial abundance constrain for small PBHs. The smallest initial PBH mass we are interested in, $M_{\text{PBH},i} = 5.3 \times 10^5$ g, is obtained from (9) by setting $r = 1$ and $f_{\text{relic}} = 1$. This mass scale is within the sensitivity reach of CE and ET.

One can also consider the induced GW sourced by the primordial (adiabatic) curvature perturbation responsible for the PBH formation, indicated as $\Omega_{\text{adGW},0}h^2$ in Fig. 6. We assume a monochromatic curvature power spectrum when computing the adiabatic induced GWs. For the notational uniformity, below we denote the GW spectrum induced by the isocurvature perturbation by $\Omega_{\text{isGW},0}h^2$. According to (11), the ratio of the two GW spectrum peaks is $(\beta/\gamma)^{-1/3}$. On the small mass side, $\beta \gtrsim 10^{-8}$, the GW spectrum peak corresponding to the PBH formation would be $O(10^{2.7})$ times higher than f_{uv} , hence much higher than 10^2 Hz as can be seen in Fig. 4. Therefore the frequency

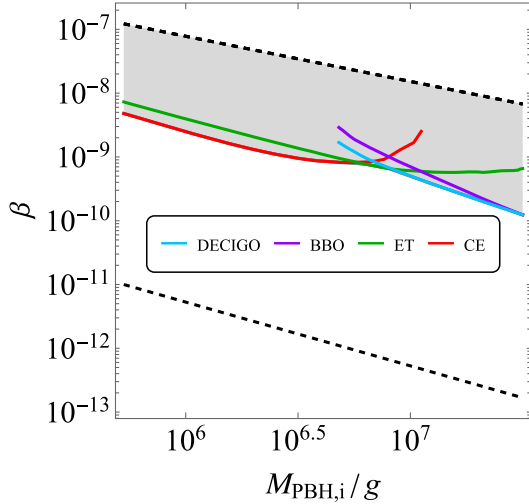


FIG. 5. Testable parameter range of the PBH-dominated scenario using the next generation gravitational wave detections: $5.3 \times 10^5 \text{ g} \lesssim M_{\text{PBH},i} \lesssim 10^{7.5} \text{ g}$. The higher black dashed line is $\Omega_{\text{GW},0}h^2 = 10^{-5}$. The lower black dashed line is the minimum PBH initial abundance required for a PBH-dominated era β_{min} (4). The sensitivity curves are DECIGO [109], BBO [87], ET [88], and CE [90].

would be too high to be detected. On the large mass PBH side, $10^{-10} \lesssim \beta \lesssim 10^{-8}$, the ratio of the peak frequencies could approach $\sim 10^{2.7}$. Nevertheless, since f_{uv} shifts to the frequency lower than 10 Hz, one may expect the peak of $\Omega_{\text{adGW},0}h^2$ to be detected. In Fig. 6, we show the GW spectra for typical examples of large and small mass cases. The primordial curvature perturbation amplitude is determined by the Press-Schechter formalism with the threshold value of 0.306, which is suggested by [53].

In the case that PBH relics constitute all the dark matter today, heavy PBH relics can be captured in the lower-frequency band, see (14). This is also shown in Fig. 7. If PBH relics make up only part of the dark matter, the responsible band goes even lower. The LISA band starts to become relevant if $f_{\text{relic}} \lesssim 10^{-2}$.

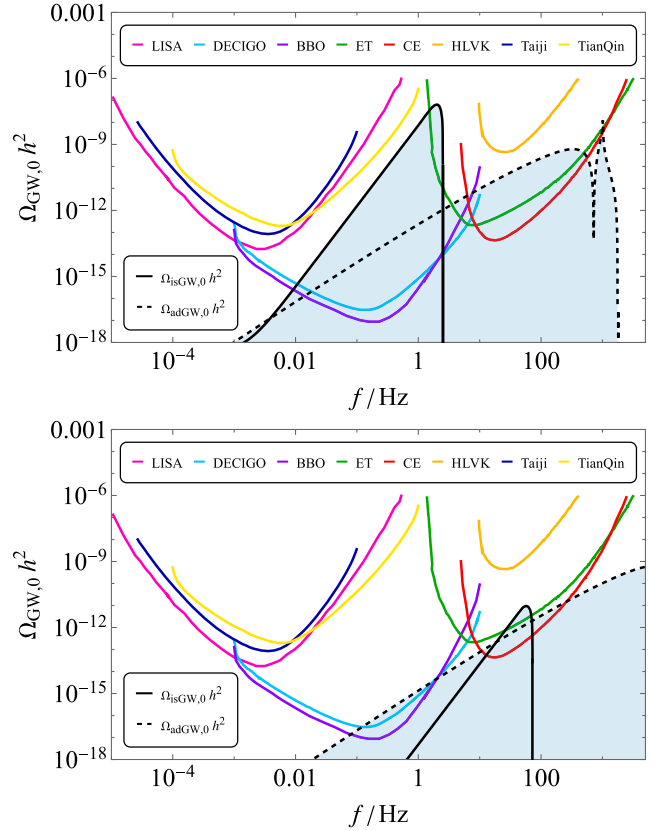


FIG. 6. GW spectra induced by the isocurvature and adiabatic perturbations in the PBH-dominated scenario. The parameters in the top panel are $r/f_{\text{relic}} = 2.4 \times 10^4$, which corresponds to $M_{\text{PBH},i} = 3 \times 10^7 \text{ g}$, and $\beta = 3 \times 10^{-9}$. The parameters in the bottom panel are $r/f_{\text{relic}} = 1$, which corresponds to $M_{\text{PBH},i} = 5.3 \times 10^5 \text{ g}$, and $\beta = 10^{-8}$. The spectrum $\Omega_{\text{isGW},0}h^2$ is the isocurvature-induced GW spectrum due to inhomogeneous PBH distribution, while $\Omega_{\text{adGW},0}h^2$ is the adiabatic-curvature-induced GW spectrum that leads to the PBH formation. The sensitivity curves are LISA [110], HLVK [95,111,112], Taiji [113], TianQin [114], and the same with Fig. 5 for the others.

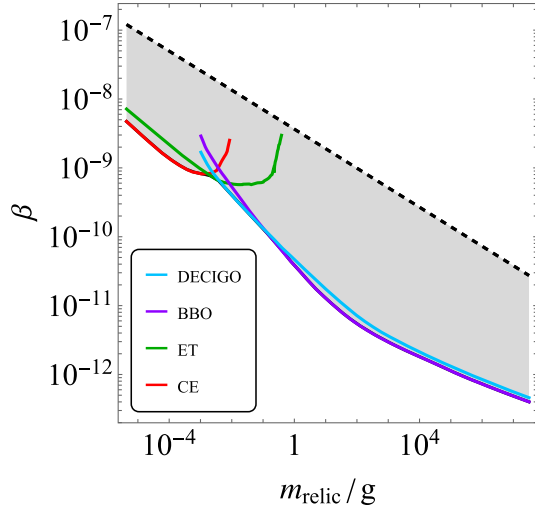


FIG. 7. Detectable PBH relics using the next generation gravitational waves detections when $f_{\text{relic}} = 1$. The black dashed line is $\Omega_{\text{GW},0} h^2 = 10^{-5}$. The sensitivity curves are the same with Fig. 5. The presented relic mass locates at $M_{\text{pl}} \leq m_{\text{relic}} \leq 10^{7.5} \text{ g}$.

C. Laboratory detectors

Now putting aside the fact that the event rate is very low, let us examine if a PBH can be detected when it passes through a detector. We consider the scattering of a PBH relic, treated as the incident particle with mass m_{relic} and velocity v_{DM} , off a target particle in the detector with mass m , initially at rest. We identify the PBH relic velocity with the dark matter velocity dispersion $\sim 200 \text{ km/s}$ [115]. The interaction between the two is governed by the Newtonian gravitational potential $V(r) = -Gm_{\text{relic}}m/r$. This process corresponds to the gravitational analog of Rutherford scattering. In the limit $m_{\text{relic}} \gg m$, the system reduces to the scattering of a light particle in a fixed gravitational potential generated by the heavy PBH relic. The trajectory is hyperbolic, and the scattering angle is given by

$$\theta = 2 \arctan\left(\frac{b_0}{b}\right) \approx 2 \frac{b_0}{b}, \quad (19)$$

where the small-angle approximation has been used. The impact parameter b is the minimum distance between the incoming relic's trajectory and the target detector particle in the absence of interaction. The characteristic gravitational focusing scale is defined as the radius at which the gravitational potential energy becomes comparable to the kinetic energy,

$$b_0 \equiv \frac{Gm_{\text{relic}}}{v_{\text{DM}}^2} \approx \left(\frac{m_{\text{relic}}}{10^7 \text{ g}}\right) 1.7 \times 10^{-15} \text{ cm}. \quad (20)$$

The cross-section for scattering of a black hole relic on any target particle would be $\sigma \propto b_0^2$. For impact parameters much larger than b_0 , the gravitational interaction is weak and the

resulting deflection is small. Assuming elastic scattering, the momentum transfer is given by

$$\Delta p = 2mv_{\text{DM}} \sin\left(\frac{\theta}{2}\right) \approx 2mv_{\text{DM}} \frac{b_0}{b}. \quad (21)$$

Substituting (20), the velocity gained by a particle in the detector is

$$\begin{aligned} \Delta v &\approx 2 \frac{Gm_{\text{relic}}}{b v_{\text{DM}}} \\ &\approx 6.7 \times 10^{-2} \text{ m s}^{-1} \left(\frac{m_{\text{relic}}}{10^7 \text{ g}}\right) \left(\frac{b}{1 \text{ \AA}}\right)^{-1}. \end{aligned} \quad (22)$$

Here we assume a generic solid-state detector, in which the fundamental scattering centers are the atoms in the crystal lattice. We also normalize the impact parameter b to the inter-atomic spacing of the lattice $d_{\text{int-atom}} \sim 1 \text{ \AA} \approx 10^{-8} \text{ cm}$ [116]. The energy exchange into each detector atom is

$$\begin{aligned} \Delta E(b) &\equiv \frac{1}{2} m (\Delta v)^2 \\ &\approx 1.4 \times 10^{-11} \text{ eV} \left(\frac{m}{10^{-24} \text{ g}}\right) \left(\frac{m_{\text{relic}}}{10^7 \text{ g}}\right)^2 \left(\frac{b}{1 \text{ \AA}}\right)^{-2}, \end{aligned} \quad (23)$$

where m is the detector atom mass. Whenever PBH relic travels a small distance dx , it interacts with atoms arranged in concentric shells characterized by different impact parameters b . Energy loss per unit length is

$$\frac{dE}{dx} = n_{\text{atom}} \int db 2\pi b \Delta E(b). \quad (24)$$

$n_{\text{atom}} \sim d_{\text{int-atom}}^{-3} \sim 10^{24} \text{ cm}^{-3}$ is the number density of atoms in detector. Substituting (23), we obtain

$$\frac{dE}{dx} \approx 8.8 \times 10^{-3} \text{ eV cm}^{-1} \left(\frac{m}{10^{-24} \text{ g}}\right) \left(\frac{m_{\text{relic}}}{10^7 \text{ g}}\right)^2 \ln\left(\frac{b_{\text{max}}}{b_{\text{min}}}\right). \quad (25)$$

While (23) describes the energy transfer in a single scattering event, (24) assumes a continuous distribution of scattering centers. This approximation breaks down on scales smaller than the interatomic spacing, and therefore impose a lower cutoff $b_{\text{min}} \sim d_{\text{int-atom}} \sim 1 \text{ \AA}$. The extremely weak gravitational coupling to lattice degrees of freedom leads to a large propagation scale [117], this scale even far exceeds the size of realistic detectors. In practice, the effective upper cutoff is set by the detector size $L \sim 1 \text{ cm}$. Therefore the cumulative energy deposition in the detector is

$$E \approx 1.6 \times 10^{-1} \text{ eV} \left(\frac{m}{10^{-24} \text{ g}} \right) \left(\frac{m_{\text{relic}}}{10^7 \text{ g}} \right)^2 \left(\frac{L}{1 \text{ cm}} \right) \times \left[1 + 0.054 \ln \left(\frac{L}{1 \text{ cm}} \right) \right]. \quad (26)$$

For PBH relics $m_{\text{relic}} \sim 10^7 \text{ g}$ and detector size $L \sim 1 \text{ cm}$, the energy goes above 10^{-1} eV . Once most of the deposited energy is converted into athermal excitations (e.g., phonons or quasiparticles), superconducting detector technologies are expected to achieve such sensitivities [118].

D. Relic-human interactions

Aside from future GW projects and laboratory detectors, one may also discuss the potential effects on daily life. How likely is it that these relics could interact with humans, and under what conditions might such interactions become non-negligible?

The rate of relic-human interactions is given as [119,120]

$$\Gamma = A_{\text{human}} n_{\text{relic}} v_{\text{DM}} \approx 3.3 \times 10^{-21} \xi f_{\text{relic}} \left(\frac{m_{\text{relic}}}{M_{\text{pl}}} \right)^{-1} \text{ yr}^{-1}, \quad (27)$$

where A_{human} is the event cross-section area $\sim 1 \text{ m}^2$, and $\xi \equiv n_{\text{relic}}/n_{\text{PBH},0}$ is the enhancement factor of the local relic number density relative to the cosmic mean (background) number density due to the local dark matter clustering. The local dark matter density in the Milky Way halo (e.g., in the solar neighborhood) is estimated as $\rho_{\text{DM,halo}} \simeq 10^{-1} \text{ GeV/cm}^3$ [121,122], which corresponds to an enhancement factor of $\xi \sim 10^5$, using the mean dark matter density of the Universe today $\rho_{\text{DM},0} \simeq 10^{-6} \text{ GeV/cm}^3$ [80]. Furthermore, taking into account of the total population in the world today, $\sim 8.3 \times 10^9$, the PBH relic-human interaction rate becomes $\sim 10^{-6} \text{ yr}^{-1}$ for the Planck mass size relics. This is very small but not totally negligible. Thus Planck-mass relics may have interacted with humans, while interactions with more massive relics remain more suppressed.

Comparing b_0 with typical intermolecular distance in biological systems $d_{\text{int-mol}} \sim 3 \times 10^{-8} \text{ cm}$ [120], we find that gravitational scattering events are always weak for $m_{\text{relic}} \lesssim 10^7 \text{ g}$. A more detailed calculation following [117,120] gives the damage in a single PBH relic-human interaction event,

$$E \approx \left(\frac{m_{\text{relic}}}{10^7 \text{ g}} \right)^2 5.1 \times 10^{-19} \text{ J}. \quad (28)$$

Such energy may have a slight effect on a human being, considering that an essential chemical bond that composes biological systems typically requires a few eV of energy (on the order of 10^{-19} J) to break [123].

IV. SUMMARY

We have shown that both the initial PBH abundance and the present-day relic number density can be directly inferred from the location and amplitude of the peak of the IGW spectrum, provided that PBHs temporarily dominate the Universe before they evaporate. In this sense, the IGW spectrum acts as a cosmic diagnostic: The spectral peak point encodes the key PBH parameters and the isocurvature fluctuation inherited from their Poisson statistics. Remarkably, we find that the relevant frequency range is broad, making the scenario testable by multiple next-generation gravitational-wave observatories. For the scenario we considered, we conclude that the maximum energy to an individual person would still be roughly below the energy threshold required to damage any biological system. The probability of any scattering effects actually is strongly suppressed. Nevertheless, if a PBH relic passes through a calorimeter, it could be detected, particularly in the case of massive PBH relics.

In addition to gravitational waves, there are many ongoing or future projects to constrain the physics of PBHs; microlensing surveys [124–128], CMB spectral-distortion and anisotropy measurements [129–135], BBN element-abundance constraints [136–141], and high-resolution gamma-ray and x-ray telescopes [142–147]. Together with these, we will have a more solid pathway toward identifying or ruling out the PBH relic scenario with unprecedented precision.

Our present analysis assumed a monochromatic PBH mass spectrum, no clustering, no hierarchical merging, and no accretion-driven mass evolution. These simplifying assumptions made it possible to isolate the cleanest IGW signatures from the PBH isocurvature perturbation. However, this idealized setup also means that any deviation between observations and our predictions encodes valuable information about the true PBH population and evolution. For instance: non-monochromatic mass functions will modify the IGW spectrum [34,148–155]. Clustering may enhance the small-scale density contrast, leading to a larger IGW amplitude [69,156–164]. Mergers and accretion may modify the effective evaporation time, shifting the IGW peak [34,165–173]. We leave all of these for future works.

Therefore, precise measurements of the IGW spectrum—especially its peak position, width, and tail behavior—can serve as a powerful tool for reconstructing PBH mass functions, probing their clustering and merger histories, and testing early-Universe physics such as inflationary models [174–177], phase-transition [178–180], and topological defects [181–188]. In this way, a future detection (or nondetection) provides rich information far beyond the mere presence of PBH relics: it opens a window into the microphysics of the early cosmos.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: DERIVATION OF (4)

The Hawking evaporation rate is proportional to M_{PBH}^{-2} [2],

$$\frac{dM_{\text{PBH}}}{dt} = -\frac{A}{M_{\text{PBH}}^2}, \quad A \approx 2.7M_{\text{pl}}^4. \quad (\text{A1})$$

Hence we have

$$M_{\text{PBH}}(t) = M_{\text{PBH},i} \left(1 - \frac{t}{t_{\text{eva}}}\right)^{1/3}, \quad (\text{A2})$$

where the evaporation time is

$$t_{\text{eva}} = \frac{M_{\text{PBH},i}^3}{3A} \approx 4.1 \times 10^{-7} \text{ s} \left(\frac{M_{\text{PBH},i}}{10^7 \text{ g}}\right)^3. \quad (\text{A3})$$

The ratio of the PBH lifetime to the formation time is

$$\frac{t_{\text{eva}}}{t_{\text{f}}} \approx 3.3 \times 10^{24} \left(\frac{M_{\text{PBH},i}}{10^7 \text{ g}}\right)^2 \gg 1. \quad (\text{A4})$$

If there is indeed a PBH-dominated era, we can set a_{eq} as the time of early radiation-PBH equality. Then at early equality time and PBH formation time, we have

$$3M_{\text{pl}}^2 H_{\text{eq}}^2 = \rho_{\text{PBH,eq}} + \rho_{r,\text{eq}} = 2\rho_{\text{PBH,eq}}, \quad (\text{A5})$$

and

$$\begin{aligned} 3M_{\text{pl}}^2 H_{\text{f}}^2 &= \rho_{\text{PBH,f}} + \rho_{r,\text{f}} \\ &= \rho_{\text{PBH,eq}} \left[\left(\frac{a_{\text{f}}}{a_{\text{eq}}}\right)^{-3} + \left(\frac{a_{\text{f}}}{a_{\text{eq}}}\right)^{-4} \right]. \end{aligned} \quad (\text{A6})$$

Combining them and using the definition (2) of β , we obtain

$$\frac{1}{\beta} = 1 + \left(\frac{a_{\text{f}}}{a_{\text{eq}}}\right)^{-1}. \quad (\text{A7})$$

Considering $\beta \ll 1$, we have

$$\frac{a_{\text{f}}}{a_{\text{eq}}} \approx \beta, \quad (\text{A8})$$

or [81]

$$\frac{H_{\text{eq}}^2}{H_{\text{f}}^2} = \frac{2}{\left(\frac{a_{\text{f}}}{a_{\text{eq}}}\right)^{-3} + \left(\frac{a_{\text{f}}}{a_{\text{eq}}}\right)^{-4}} \approx 2\beta^4. \quad (\text{A9})$$

Noting that $H_{\text{f}}/H_{\text{eva}} = 3t_{\text{eva}}/4t_{\text{f}}$, one obtains (4).

APPENDIX B: DERIVATION OF (9)

Following [81], we derive the relation between $M_{\text{PBH},i}$ and f_{relic} . The key quantity is the fraction of the relic density at the moment of evaporation,

$$\Omega_{\text{relic}}|_{\text{eva}} \equiv \frac{\rho_{\text{relic,eva}}}{\rho_{\text{tot,eva}}}. \quad (\text{B1})$$

The idea is to express this both in terms of $\Omega_{\text{relic},0} = f_{\text{relic}}\Omega_{\text{DM},0}$ and in terms of $\Omega_{\text{PBH,f}} = \beta$, and identify them.

First, we consider the evolution from the time of evaporation to today. Since $\rho_{\text{relic}} \propto a^{-3}$, one has

$$\Omega_{\text{relic}}|_{\text{eva}} = f_{\text{relic}} \frac{\rho_{\text{DM}}(T_0)}{\rho_{\text{tot}}(T_{\text{eva}})} \left(\frac{a_0}{a_{\text{eva}}}\right)^3, \quad (\text{B2})$$

where the scale factor ratio a_0/a_{eva} can be expressed in terms of the temperature ratio as

$$\frac{a_{\text{eva}}}{a_0} = \left(\frac{g_{s\star}(T_0)}{g_{s\star}(T_{\text{eva}})}\right)^{1/3} \frac{T_0}{T_{\text{eva}}}. \quad (\text{B3})$$

Together with

$$\rho_{\text{tot}}(T_{\text{eva}}) = \frac{\pi^2}{30} g_{\star}(T_{\text{eva}}) T_{\text{eva}}^4, \quad (\text{B4})$$

one obtains

$$\begin{aligned} \Omega_{\text{relic}}|_{\text{eva}} &\approx 5.8 \times 10^{-13} f_{\text{relic}} \left(\frac{T_{\text{eva}}}{1 \text{ TeV}}\right)^{-1} \\ &\times \left(\frac{g_{s\star}(T_{\text{eva}})}{106.75}\right) \left(\frac{g_{\star}(T_{\text{eva}})}{106.75}\right)^{-1}, \end{aligned} \quad (\text{B5})$$

where we adopted the values, $\Omega_{\text{DM},0}h^2 = 0.120$ [80], $T_0 \approx 2.35 \times 10^{-13} \text{ GeV}$, and $g_{s\star}(T_0) \approx 3.91$.

The temperature right after the moment of PBH evaporation T_{eva} can be estimated by using

$$3M_{\text{pl}}^2 H_{\text{eva}}^2 = \rho_{\text{tot}}(T_{\text{eva}}) = \frac{\pi^2 g_{\star}(T_{\text{eva}})}{30} T_{\text{eva}}^4. \quad (\text{B6})$$

This gives

$$T_{\text{eva}} \approx 8.9 \times 10^{-1} \text{ GeV} \left(\frac{M_{\text{PBH},i}}{10^7 \text{ g}} \right)^{-3/2} \left(\frac{g_{\star}(T_{\text{eva}})}{106.75} \right)^{-1/4}. \quad (\text{B7})$$

For example, $T_{\text{eva}} \sim 1 \text{ GeV}$ for $M_{\text{PBH},i} \sim 10^7 \text{ g}$. Substituting the above into (B5), one obtains

$$\Omega_{\text{relic}}|_{\text{eva}} \approx 6.6 \times 10^{-10} f_{\text{relic}} \left(\frac{M_{\text{PBH},i}}{10^7 \text{ g}} \right)^{3/2} \times \left(\frac{g_{\star}(T_{\text{eva}})}{106.75} \right)^{-3/4} \left(\frac{g_{s\star}(T_{\text{eva}})}{106.75} \right). \quad (\text{B8})$$

Now we turn to the evolution from the PBH formation to evaporation. The energy density of PBH relics at the evaporation time is

$$\begin{aligned} \rho_{\text{relic,eva}} &= m_{\text{relic}} n_{\text{PBH,eva}} \\ &= m_{\text{relic}} n_{\text{PBH,f}} \left(\frac{a_f}{a_{\text{eva}}} \right)^3, \end{aligned} \quad (\text{B9})$$

where the PBH number density at the formation time is expressed as

$$n_{\text{PBH,f}} \equiv \frac{\rho_{\text{PBH,f}}}{M_{\text{PBH,i}}} = \frac{3\beta}{4\pi\gamma} H_f^3. \quad (\text{B10})$$

From the above, the definition of $\Omega_{\text{relic}}|_{\text{eva}}$ (B1) gives [81]

$$\Omega_{\text{relic}}|_{\text{eva}} = \frac{\beta m_{\text{relic}}}{M_{\text{PBH,i}}} \frac{H_f^2}{H_{\text{eva}}^2} \left(\frac{a_f}{a_{\text{eva}}} \right)^3. \quad (\text{B11})$$

Assuming that the PBH evaporation ends after the universe becomes dominated by PBHs, using (A8) and (A9) one can prove

$$\beta \frac{H_f^2}{H_{\text{eva}}^2} \left(\frac{a_f}{a_{\text{eva}}} \right)^3 = 1. \quad (\text{B12})$$

Thus (B11) gives a very simple formula,

$$\Omega_{\text{relic}}|_{\text{eva}} = \frac{m_{\text{relic}}}{M_{\text{PBH,i}}}. \quad (\text{B13})$$

Combining (B8) and (B13), the initial PBH mass is uniquely determined as a function of r/f_{relic} , one obtains (9).

APPENDIX C: DERIVATION OF (14)

From (B3) and (B6), the frequency f_{eva} today is expressed as

$$f_{\text{eva}} = \frac{H_{\text{eva}}}{2\pi} \left(\frac{g_{s\star}(T_0)}{g_{s\star}(T_{\text{eva}})} \right)^{1/3} \frac{T_0}{T_{\text{eva}}}. \quad (\text{C1})$$

Substitute $T_0 \approx 2.35 \times 10^{-13} \text{ GeV}$, $g_{s\star}(T_0) \approx 3.91$, one finally has

$$\begin{aligned} f_{\text{eva}} &\approx 2.7 \times 10^{-5} \text{ Hz} \left(\frac{T_{\text{eva}}}{1 \text{ TeV}} \right) \\ &\times \left(\frac{g_{\star}(T_{\text{eva}})}{106.75} \right)^{1/2} \left(\frac{g_{s\star}(T_{\text{eva}})}{106.75} \right)^{-1/3}. \end{aligned} \quad (\text{C2})$$

Taking the ratio of (11) and k_{eva} , we find (13). Using (A4), we obtain

$$\frac{k_{\text{uv}}}{k_{\text{eva}}} \approx 2.3 \times 10^8 \left(\frac{M_{\text{PBH,i}}}{10^7 \text{ g}} \right)^{2/3}. \quad (\text{C3})$$

This gives

$$\begin{aligned} f_{\text{uv}} &= f_{\text{eva}} \frac{k_{\text{uv}}}{k_{\text{eva}}} \approx 5.4 \text{ Hz} \left(\frac{M_{\text{PBH,i}}}{10^7 \text{ g}} \right)^{-5/6} \\ &\times \left(\frac{g_{\star}(T_{\text{eva}})}{106.75} \right)^{1/4} \left(\frac{g_{s\star}(T_{\text{eva}})}{106.75} \right)^{-1/3}. \end{aligned} \quad (\text{C4})$$

One obtains (14) by substituting (9) into (C4).

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