

***B* anomalies in a two-Higgs-doublet model with flavor symmetry**Qiaoyi Wen[✉] and Fanrong Xu^{✉*}*Department of Physics, College of Physics and Optoelectronic Engineering, Jinan University, Guangzhou 510632, People's Republic of China*

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The long-standing flavor anomalies in $b \rightarrow s\ell^+\ell^-$ and $b \rightarrow c\ell\nu$ persist. In this work, we explore a specific flavor-gauged two-Higgs-doublet model (FG2HDM) extended by a scalar singlet, with an imposed $U(1)$ flavor symmetry. Compared to the standard model (SM), four additional scalars and one neutral gauge boson Z' are added to the particle spectrum. The special Yukawa coupling matrices, due to the $U(1)$ symmetry, lead to flavor-changing neutral Higgs interactions uniquely occurring in the down-type quark sector, while the new gauge boson Z' induces flavor-changing neutral current interactions in the down-type quark sector as well. The two distinct types of anomalies can be accommodated simultaneously in such a model. Incorporating scalar and vector contributions to $b \rightarrow s\ell^+\ell^-$, $B_s \rightarrow \mu^+\mu^-$, and $B_s^0 - \bar{B}_s^0$ mixing, charged Higgs contributions to $b \rightarrow c\ell\nu$, along with electroweak precision observables and Higgs fermionic decays, we investigate the parameter space in FG2HDM and find substantial room in the solution space. Specifically, in the SM-like model setup, the data suggest that $m_{Z'} < 450$ GeV, $\tan\beta < 28$, and $g'/m_{Z'} > 3.5 \times 10^{-4}$ GeV⁻¹, while there are fewer constraints on heavy scalars, which can be further tested by a combination of other observables.

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Although the anomalies in B meson decays have persisted for over a decade, serving as an indirect way to explore new physics (NP) beyond the standard model (BSM) and attracting significant attention both experimentally and theoretically, the effort to precisely identify these anomalies continues.

The SM-like $R_{K^{(*)}}$ measured by LHCb at the end of 2022 [1], and later observed with large uncertainty by CMS as well [2], indicated that the emergence of NP in terms of a violation of lepton flavor universality (LFU) was not that promising. However, the possibility in binned branching fractions and polarization observables P'_5 still exists. LHCb performed a systematic q^2 unbinned amplitude analysis of $B^0 \rightarrow K^{*0}\mu^+\mu^-$ at the end of 2023 [3,4] and found the deviation of C_9 from the SM to be about 1.9σ and that the nonlocal operator is not sufficient to explain the deviation. In the latest unbinned amplitude analysis of May 2024 [5,6], using the Breit-Wigner improved method [7,8], LHCb obtained C_9 with a 2.1σ

deviation from the SM prediction. The ongoing effort indicates that the NP opportunity in $b \rightarrow s\ell^+\ell^-$ has not yet been concluded [9].

Progress has also been made in the $b \rightarrow c\ell\bar{\nu}$ sector. Recently, LHCb reported that $R_{D^{*+}}^{\text{LHCb}} = 0.249 \pm 0.043(\text{stat}) \pm 0.047(\text{syst})$, and $R_{D^{*+}}^{\text{LHCb}} = 0.402 \pm 0.081(\text{stat}) \pm 0.085(\text{syst})$, with a correlation coefficient $\rho = -0.39$ [10]. The experimental uncertainty of R_{D^*} remains considerable, while the error of R_D decreased, pushing R_D closer to the SM prediction than previously observed [11]. On the other hand, Belle II [12] announced their preliminary results of $R_{D^{(*)}}$ this year as $R_{D^{*+}}^{\text{Belle}} = 0.262^{+0.041}_{-0.039}(\text{stat})^{+0.035}_{-0.032}(\text{syst})$.¹ The preliminary world averages of $R_{D^{(*)}}$ for Moriond 2024, offered by HFLAV [14], with the above Belle II result included [12], are given as

$$R_{D^*}^{\text{avg}} = 0.287 \pm 0.012, \quad R_D^{\text{avg}} = 0.342 \pm 0.026, \quad (1)$$

with the same correlation of -0.39 . Hence, the discrepancy between the SM prediction and the averaged value is at the level of 3.31σ [14]. In addition, the longitudinal polarization fraction of the D^* meson in $B^0 \rightarrow D^{*-}\tau^+\nu_\tau$ provides

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¹The most recent measurements of $R_{D^{(*)}}$ reported by Belle II in 2025, during the review process of this paper, yield a combined result that deviates from the SM prediction by 1.7 standard deviations, taking into account the correlation between the two observables [13].

complementary information to R_{D^*} and has triggered experimental interest. However, the overall q^2 range integrated result reported by LHCb [15], given as $F_L^{D^*} = 0.43 \pm 0.06 \pm 0.03$ by the end of 2023, was found to be SM compatible. Recently, J/ψ has also been employed as a supplement to $D^{(*)}$ mesons in the $b \rightarrow c\ell\nu$ process. A preliminary combined result of $R_{J/\psi}$ announced by CMS collaboration in August 2024 gives $R_{J/\psi}^{\text{CMS}} = 0.49 \pm 0.09(\text{stat}) \pm 0.25(\text{syst})$, with large uncertainty that less information can be extracted at this moment [16,17]. Therefore, R_{D^*} is still the main player of NP in $b \rightarrow c\ell\nu$.

It is commonly known that the two types of anomalies, occurring in $b \rightarrow s\ell^+\ell^-$ and $b \rightarrow c\ell\nu$, are induced by neutral current and charged current interactions, respectively. This indicates that NP is encoded in distinct sectors and can be explored separately. In addition to model-independent global fits in $b \rightarrow s\ell^+\ell^-$ [3–5,18–23] and $b \rightarrow c\ell\nu$ [24–27], various specific NP models have also been discussed, including the Z' models [28–41] and leptoquark models [42–47], to address neutral current anomalies, and charged Higgs [48–54] and others, such as W' [55–58], to address charged current anomalies. Furthermore, efforts to accommodate both types of anomalies with model-independent or model-dependent methods can also be found in references, such as [53,56,59–69].

The two-Higgs-doublet model (2HDM) is a natural extension of the SM and is commonly classified into several types regarding its Yukawa structures. To reduce redundancy in Yukawa coupling matrices, extra symmetry can be introduced. Taking the 2HDM-II as an example, it is imposed with a symmetry for its first scalar doublet and right-handed down quarks ($\Phi_1 \rightarrow -\Phi_1$, $d_R^i \rightarrow d_R^i$), but faces difficulty in explaining the R_{D^*} anomaly [70]. There are still efforts in variant 2HDMs, such as 2HDM-III with Cheng-Sher ansatz [71,72], gauged 2HDM (G2HDM) [73–75], and Branco-Grimus-Lavoura (BGL) 2HDM [76] with special assignment of flavor symmetry, and some early discussion can be found in the references, such as [77]. Here, we will focus on the BGL-type models, with global flavor-symmetry-based Yukawa matrix textures introduced. Although there exists scalar-mediated flavor-changing neutral current (FCNC), it can be suppressed by small off-diagonal entries of the CKM matrix [76]. Recently, the way to localize this BGL flavor symmetry has been proposed [78] and developed [79,80], in which a new gauge boson corresponding to the $U(1)$ gauge group in charge of flavor symmetry and an extra scalar singlet are introduced, in addition to the original two doublets in 2HDM. In this work, we explore how both anomalies can

be accommodated within such a flavor-gauged 2HDM (FG2HDM).

This paper is organized as follows. In Sec. II, we introduce the main structure of FG2HDM. In Sec. III, the model contributions to $b \rightarrow s\ell^+\ell^-$, $b \rightarrow c\ell\nu$, and $B_s^0 - \bar{B}_s^0$ mixing, as well as $Z \rightarrow f\bar{f}$ and $h \rightarrow f\bar{f}$, are calculated. A further numerical analysis and a survey on model parameter space are presented in Sec. IV. We summarize the entire work in Sec. V. Some details are provided in the Appendices, where Appendix A shows exact quantum numbers for a workable FG2HDM, and Appendix B gives details of rotation matrices in the model. In Appendix C, some related phenomenology formulas are exhibited.

II. MODEL SETUP

A. Scalar potential

The scalar potential in the FG2HDM, involving two scalar doublets and one complex singlet, is given by

$$\begin{aligned} V(\Phi_1, \Phi_2, S) &= m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 \\ &\quad + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) \\ &\quad + m_S^2 |S|^2 + \lambda_S |S|^4 + \kappa_1 |\Phi_1|^2 |S|^2 + \kappa_2 |\Phi_2|^2 |S|^2 \\ &\quad + \kappa_3 (\Phi_1^\dagger \Phi_2 S^2 + \Phi_2^\dagger \Phi_1 (S^*)^2), \end{aligned} \quad (2)$$

in which $\Phi_i = (\phi_i^+, (\rho_i + i\eta_i + v_i)/\sqrt{2})^T$ denotes the $SU(2)_L$ scalar doublet, and the singlet is parametrized as $S = (v_S + s_0 + i\chi_0)/\sqrt{2}$. CP -violating phases in the vacuum expectation values (VEV) are neglected. Due to different charge assignments of these doublets under the gauged BGL symmetry, the terms m_{12} and λ_5 vanish. The mass of the physical pseudoscalar arises from the newly introduced singlet after symmetry breaking. By utilizing stationary equations ($\partial_i V_0 \equiv \frac{\partial V_0}{\partial v_i} = 0$, $i = 1, 2, S$) around their vacua,

$$\begin{aligned} (2m_{11}^2 v_1 + \kappa_1 v_1 v_S^2 + \kappa_3 v_2 v_S^2 + \lambda_1 v_1^3 + \lambda_3 v_2^2 v_1 + \lambda_4 v_2^2 v_1) &= 0, \\ (2m_{22}^2 v_2 + \kappa_2 v_2 v_S^2 + \kappa_3 v_1 v_S^2 + \lambda_2 v_2^3 + \lambda_3 v_1^2 v_2 + \lambda_4 v_1^2 v_2) &= 0, \\ (2m_S^2 v_S + 2\lambda_S v_S^3 + \kappa_1 v_1^2 v_S + \kappa_2 v_2^2 v_S + 2\kappa_3 v_1 v_2 v_S) &= 0, \end{aligned} \quad (3)$$

we can derive the mass terms for the scalars,

$$\mathcal{L}_{\phi^\pm} = \frac{1}{2} (v_1 v_2 \lambda_4 + \kappa_3 v_S^2) (\phi_1^-, \phi_2^-) \begin{pmatrix} \frac{v_2}{v_1} & -1 \\ -1 & \frac{v_1}{v_2} \end{pmatrix} \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix},$$

$$\mathcal{L}_\eta = \frac{1}{4} \kappa_3 v_S^2 (\eta_1, \eta_2, \chi_0) \begin{pmatrix} \frac{v_2}{v_1} & -1 & -\frac{2v_2}{v_S} \\ -1 & \frac{v_1}{v_2} & \frac{2v_1}{v_S} \\ -\frac{2v_2}{v_S} & \frac{2v_1}{v_S} & \frac{4v_1 v_2}{v_S^2} \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \\ \chi_0 \end{pmatrix},$$

$$\mathcal{L}_\rho = -\frac{1}{2} (\rho_1, \rho_2, s_0) \begin{pmatrix} \lambda_1 v_1^2 - \frac{1}{2} \kappa_3 \frac{v_2}{v_1} v_S^2 & \lambda_{34} v_1 v_2 + \frac{1}{2} \kappa_3 v_S^2 & v_S (v_1 \kappa_1 + v_2 \kappa_3) \\ \lambda_{34} v_1 v_2 + \frac{1}{2} \kappa_3 v_S^2 & \lambda_2 v_2^2 - \frac{1}{2} \kappa_3 \frac{v_1}{v_2} v_S^2 & v_S (v_2 \kappa_2 + v_1 \kappa_3) \\ v_S (v_1 \kappa_1 + v_2 \kappa_3) & v_S (v_2 \kappa_2 + v_1 \kappa_3) & 2\lambda_S v_S^2 \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \\ s_0 \end{pmatrix}, \quad (4)$$

where $\lambda_{34} = \lambda_3 + \lambda_4$. Assuming CP -even neutral scalar mass terms are block diagonal, i.e., $-\kappa_3 = \kappa_1 v_1/v_2 = \kappa_2 v_2/v_1$, as adopted in [81], we have

$$\mathcal{L}_\rho = -\frac{1}{2} (\rho_1, \rho_2, s_0) \begin{pmatrix} \lambda_1 v_1^2 - \frac{1}{2} \kappa_3 \frac{v_2}{v_1} v_S^2 & \lambda_{34} v_1 v_2 + \frac{1}{2} \kappa_3 v_S^2 & 0 \\ \lambda_{34} v_1 v_2 + \frac{1}{2} \kappa_3 v_S^2 & \lambda_2 v_2^2 - \frac{1}{2} \kappa_3 \frac{v_1}{v_2} v_S^2 & 0 \\ 0 & 0 & 2\lambda_S v_S^2 \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \\ s_0 \end{pmatrix}, \quad (5)$$

giving the CP -even scalars masses

$$m_{h,H}^2 = \frac{1}{2} \left[(m+n) \mp \sqrt{(m-n)^2 + 4l^2} \right], \quad m_{H_s}^2 = 2\lambda_S v_S^2, \quad (6)$$

where m , n , and l are defined as

$$m = \lambda_1 v_1^2 - \frac{1}{2} \kappa_3 \frac{v_2}{v_1} v_S^2, \quad n = \lambda_2 v_2^2 - \frac{1}{2} \kappa_3 \frac{v_1}{v_2} v_S^2, \quad l = \lambda_{34} v_1 v_2 + \frac{1}{2} \kappa_3 v_S^2. \quad (7)$$

Likewise, the mass and gauge eigenstates of other scalar fields can be related via rotation matrices,

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = U_1 \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}, \quad \begin{pmatrix} G^0 \\ H^0 \\ H_A \end{pmatrix} = U_2 \begin{pmatrix} \eta_1 \\ \eta_2 \\ \chi_0 \end{pmatrix}, \quad \begin{pmatrix} H \\ h \\ H_S \end{pmatrix} = U_3 \begin{pmatrix} \rho_1 \\ \rho_2 \\ s_0 \end{pmatrix}, \quad (8)$$

where U_1 , U_2 , and U_3 matrices are detailed in Appendix B. We present the squared masses of the CP -odd mode and charged Higgs explicitly as

$$m_{G^\pm}^2 = m_{G^0}^2 = m_{G'^0}^2 = 0, \quad m_{H_A}^2 = -\frac{1}{2} \kappa_3 \left(4v_1 v_2 + \frac{v^2 v_S^2}{v_1 v_2} \right), \quad m_{H^\pm}^2 = -\frac{1}{2} \left(\lambda_4 v^2 + \frac{\kappa_3 v^2 v_S^2}{v_1 v_2} \right). \quad (9)$$

The Goldstone theorem is preserved, and hence the masses of Goldstone bosons naturally vanish, as shown above.

B. Gauge interactions

The kinetic terms for the scalar fields in the Lagrangian are given by

$$\mathcal{L}_{\text{GS}} = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2) + (D_\mu S)^\dagger (D^\mu S), \quad (10)$$

where the gauge covariant derivatives to the scalar fields are defined as

$$D_\mu S = (\partial_\mu - ig' X_S \hat{Z}'_\mu) S, \\ D_\mu \Phi_j = \left(\partial_\mu - ig_1 Y_j B_\mu - ig' X_{\Phi_j} \hat{Z}'_\mu - ig_2 \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu \right) \Phi_j, \quad (11)$$

with hypercharge under $U(1)_Y$ denoted as $Y_1 = Y_2 = \frac{1}{2}$. The quantum numbers $\mathcal{Q}_j$ of Φ_j , encoded in X_{Φ_j} , under the $U(1)'$ group are detailed in Appendix A.

1. Gauge boson mass

After spontaneous symmetry breaking, the mass terms for the gauge bosons are expressed as

$$\mathcal{L}_m^G = (B \quad W^3 \quad \hat{Z}') \tilde{M} \begin{pmatrix} B \\ W^3 \\ \hat{Z}' \end{pmatrix} = (A \quad Z \quad Z') M_d \begin{pmatrix} A \\ Z \\ Z' \end{pmatrix}, \quad (12)$$

where the $\tilde{M}$ matrix takes the form

$$\tilde{M} = \frac{1}{8}(g_1^2 + g_2^2)v^2 \begin{pmatrix} \sin^2 \theta_W & -\sin \theta_W \cos \theta_W & 2 \sin \theta_W \Delta \\ -\sin \theta_W \cos \theta_W & \cos^2 \theta_W & -2 \cos \theta_W \Delta \\ 2 \sin \theta_W \Delta & -2 \cos \theta_W \Delta & 4\delta \end{pmatrix}. \quad (13)$$

In particular, the notations Δ and δ are defined as

$$\begin{aligned} \Delta &= \sin \xi (Q_1 \cos \beta^2 + Q_2 \sin \beta^2), \\ \delta &= \sin^2 \xi (Q_1^2 \cos^2 \beta + Q_2^2 \sin^2 \beta + \mathcal{R}_v^2 Q_s^2), \end{aligned} \quad (14)$$

with $\mathcal{R}_v^2 = \frac{v_s^2}{v^2}$, $\sin \theta_W = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}$, and $\sin \xi = \frac{g'}{\sqrt{g_1^2 + g_2^2}}$. The determinant of $\tilde{M}$ is zero, implying that one eigenvalue (corresponding to the photon) is zero. The rotation matrix U connecting the mass eigenstates and gauge eigenstates is given by

$$\begin{pmatrix} A \\ Z \\ Z' \end{pmatrix} = U \begin{pmatrix} B \\ W^3 \\ \hat{Z}' \end{pmatrix}, \quad U \equiv U_{\hat{Z}\hat{Z}'}^T U_{A\hat{Z}}^T = \begin{pmatrix} \cos \theta_W & \sin \theta_W & 0 \\ -\sin \theta_W \cos \theta'_2 & \cos \theta_W \cos \theta'_2 & \sin \theta'_2 \\ \sin \theta_W \sin \theta'_2 & -\cos \theta_W \sin \theta'_2 & \cos \theta'_2 \end{pmatrix}, \quad (15)$$

with the matrices $U_{\hat{Z}\hat{Z}'}$ and $U_{A\hat{Z}}$ detailed in Appendix B. The diagonalized mass matrix M_d is then given by

$$M_d = \frac{1}{8}(g_1^2 + g_2^2)v^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & \mu_Z^- & 0 \\ 0 & 0 & \mu_{Z'}^+ \end{pmatrix}, \quad (16)$$

where $\mu_{Z^{(\prime)}}^\mp = \frac{1}{2} [1 + 4\delta \mp \sqrt{(4\delta - 1)^2 + 16\Delta^2}]$. Here, we have assumed $\mathcal{R}_v$ is sufficiently large, making the Z' boson heavier than the SM Z boson.

2. Fermion kinetic terms

The interactions of the new gauge bosons with fermions are obtained from

$$\begin{aligned} \mathcal{L}_{\text{FG}} &= \bar{L}_L^0 i\gamma^\mu D_\mu L_L^0 + \bar{Q}_L^0 i\gamma^\mu D_\mu Q_L^0 + \bar{e}_R^0 i\gamma^\mu (\partial_\mu - ig_1 Y B_\mu - ig' X_{\ell R} \hat{Z}_\mu) e_R^0 \\ &\quad + \bar{u}_R^0 i\gamma^\mu (\partial_\mu - ig_1 Y B_\mu - ig' X_{uR} \hat{Z}_\mu) u_R^0 + \bar{d}_R^0 i\gamma^\mu (\partial_\mu - ig_1 Y B_\mu - ig' X_{dR} \hat{Z}_\mu) d_R^0, \end{aligned} \quad (17)$$

where fermion fields with superscript 0 denote fields in the gauge eigenstate. The neutral current terms in the mass basis are then given by

$$\begin{aligned} \mathcal{L}_{\text{FG}} &= e Q_f A_\mu \bar{f} \gamma^\mu f + \frac{g_2 \cos \theta'_2}{\cos \theta_W} Z_\mu \bar{f} [(I_3^f - Q_f \sin^2 \theta_W) \gamma^\mu P_L - Q_f \sin^2 \theta_W \gamma^\mu P_R] f \\ &\quad - \frac{g_2 \sin \theta'_2}{\cos \theta_W} Z'_\mu \bar{f} [(I_3^f - Q_f \sin^2 \theta_W) \gamma^\mu P_L - Q_f \sin^2 \theta_W \gamma^\mu P_R] f + g' \sin \theta'_2 Z_\mu \bar{f} [Q_{fL} \gamma^\mu P_L + Q_{fR} \gamma^\mu P_R] f \\ &\quad + g' \cos \theta'_2 Z'_\mu \bar{f} [Q_{fL} \gamma^\mu P_L + Q_{fR} \gamma^\mu P_R] f, \end{aligned} \quad (18)$$

where $f = u, d, \nu, \ell$, I_3^f is the weak isospin third component, and Q_f is the electric charge. The couplings to fermions of different chiralities are determined by the $U(1)'$ charges, given generally as

$$\begin{aligned}\mathcal{Q}_{fL} &= U_{fL}^\dagger X_{fL} U_{fL}, \\ \mathcal{Q}_{fR} &= U_{fR}^\dagger X_{fR} U_{fR},\end{aligned}\quad (19)$$

where X_{fR} structures are discussed in Appendix A, and $U_{fL(R)}$ are rotation matrices connecting fermions in the mass eigenstate and weak eigenstate via

$$f_{L(R)}^0 = U_{fL(R)} f_{L(R)}.\quad (20)$$

Hence, the CKM and Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrices are defined as

$$V \equiv V_{\text{CKM}} = U_{uL}^\dagger U_{dL}, \quad U \equiv V_{\text{PMNS}} = U_{\nu L}^\dagger U_{\ell L}, \quad (21)$$

which play crucial roles in gauge interactions and fermion-scalar interactions.

C. Yukawa interaction

The Yukawa interactions in FG2HDM, including both quark and lepton sectors, are generally in the form of

$$\begin{aligned}-\mathcal{L}_Y &= \overline{Q}_L^0 (Y_1^d \Phi_1 + Y_2^d \Phi_2) d_R^0 + \overline{Q}_L^0 (Y_1^u \tilde{\Phi}_1 + Y_2^u \tilde{\Phi}_2) u_R^0 \\ &\quad + \overline{L}_L^0 (Y_1^\ell \Phi_1 + Y_2^\ell \Phi_2) e_R^0 + \overline{L}_L^0 (Y_1^\nu \tilde{\Phi}_1 + Y_2^\nu \tilde{\Phi}_2) \nu_R^0 \\ &\quad + \text{H.c.},\end{aligned}\quad (22)$$

where the superscripts denote the fermion fields in the aforementioned gauge eigenstate. Generically, there exist eight 3×3 Yukawa matrices in total, including the two related to neutrino, which provide Dirac neutrino mass.

1. Fermion mass and Yukawa matrices

After spontaneous symmetry breaking, fermion mass terms take the form

$$-\mathcal{L}_m = \bar{u}_L M_u u_R + \bar{d}_L M_d d_R + \bar{\ell}_L M_\ell \ell_R + \bar{\nu}_L M_\nu \nu_R + \text{H.c.},\quad (23)$$

where M_f ($f = u, d, \ell, \nu$) are diagonal mass matrices, expli-

cally giving $M_u = \text{diag}(m_u, m_c, m_t)$, $M_d = \text{diag}(m_d, m_s, m_b)$, $M_\ell = \text{diag}(m_e, m_\mu, m_\tau)$, and $M_\nu = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$. Note the physical fermion mass is rotated from generally nondiagonal $\tilde{M}_f$ as

$$M_f = U_{fL}^\dagger \tilde{M}_f U_{fR},\quad (24)$$

$$\tilde{M}_f = \frac{1}{\sqrt{2}}(v_1 Y_1^f + v_2 Y_2^f).\quad (25)$$

It is convenient to introduce the auxiliary matrices, defined as

$$N_f = \frac{1}{\sqrt{2}} U_{fL}^\dagger (v_1 Y_2^f - v_2 Y_1^f) U_{fR},\quad (26)$$

for the description of flavor-changing processes involving scalars. Since two types of Yukawa matrices, Y_1 and Y_2 , in FG2HDM are involved, the form of N_f is hence determined by the formats of Y_i^f , which further can be regarded as a consequence of flavor symmetry in this work. Without loss of generality, the N_f matrices can be connected to mass matrices via

$$N_f = -\frac{v_2}{v_1} M_f + \frac{v_2}{\sqrt{2}} \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) U_{fL}^\dagger Y_2^f U_{fR}.\quad (27)$$

Note that the Yukawa coupling matrices have not yet received restrictions so far, and hence keep their most general structure. We will show in the following that the desired FCNC or flavor changing charged current controlled by the CKM matrix can be achieved by special texture of Yukawa coupling matrices, which is a natural consequence of special flavor symmetries.

2. The specific texture for Yukawa coupling matrices

Assigned specific $U(1)'$ quantum numbers detailed in Appendix A, we obtain the following texture for quark Yukawa matrices:

$$Y_1^u = \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad Y_2^u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & * \end{pmatrix}, \quad Y_1^d = \begin{pmatrix} * & * & * \\ * & * & * \\ 0 & 0 & 0 \end{pmatrix}, \quad Y_2^d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ * & * & * \end{pmatrix},\quad (28)$$

where “*” denotes a nonzero arbitrary number. This structure of Yukawa matrices leads to the form of N_f as

$$\begin{aligned}N_u &= -\frac{v_2}{v_1} \text{diag}(m_u, m_c, 0) + \frac{v_1}{v_2} \text{diag}(0, 0, m_t), \\ (N_d)_{ij} &= -\frac{v_2}{v_1} (M_d)_{ij} + \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) (V_{\text{CKM}}^\dagger)_{i3} (V_{\text{CKM}})_{3j} (M_d)_{jj},\end{aligned}\quad (29)$$

where the dummy index is not summed. Clearly, a tree-level FCNC interaction induced by scalars does not occur in the up-type quark sector.

Likewise, by imposing the following lepton Yukawa textures:

$$Y_1^\ell = \begin{pmatrix} 0 & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}, \quad Y_2^\ell = \begin{pmatrix} * & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad Y_1^\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ * & * & * \end{pmatrix}, \quad Y_2^\nu = 0, \quad (30)$$

we find the coupling matrices among leptons and scalars to be diagonal, giving

$$N_\nu = -\frac{v_2}{v_1} M_\nu, \\ N_\ell = -\frac{v_2}{v_1} \text{diag}(0, m_\mu, m_\tau) + \frac{v_1}{v_2} \text{diag}(m_e, 0, 0). \quad (31)$$

Apparently, N_d uniquely introduces tree-level flavor-changing scalar-mediated interactions for down-type quarks among all the interactions between fermions and scalars.

3. The interactions between fermions and scalars

We have learned from Sec. II A that after spontaneous symmetry breaking (SSB), five physical scalar particles (h , H , H_S , H_A , and $H^\pm$) remain in the FG2HDM. In the mass basis, the interaction among fermions and physical scalars can be written as

$$-\mathcal{L} = \frac{\sqrt{2}}{v} H^+ [\bar{u}(V_{\text{CKM}} N_d P_R - N_u^\dagger V_{\text{CKM}} P_L) d + \bar{\nu}(V_{\text{PMNS}} N_\ell P_R - N_\nu^\dagger V_{\text{PMNS}} P_L) \ell] + \text{H.c.} \\ + \frac{1}{v} [\sin(\alpha - \beta) M_f - \cos(\alpha - \beta) N_f] h \bar{f} f - \frac{1}{v} [\cos(\alpha - \beta) M_f + \sin(\alpha - \beta) N_f] H \bar{f} f \\ + \frac{i}{v} [U_\gamma^{21} N_d] H_A \bar{d} \gamma_5 d - \frac{i}{v} [U_\gamma^{21} N_u] H_A \bar{u} \gamma_5 u + \frac{i}{v} [U_\gamma^{21} N_\ell] H_A \bar{\ell} \gamma_5 \ell. \quad (32)$$

The CP -even scalar block mass matrix in Eq. (5) determines the absence of H_S , while H_A enters through rotation. These interactions lay the groundwork for further phenomenological discussions.

III. PHENOMENOLOGY

The semileptonic processes within the B meson system provide crucial windows for investigating NP. The multiple scalars emerged in FG2HDM take part in various interactions at high energy. For instance, the charged Higgs plays a role in $b \rightarrow c \ell \nu$, while neutral scalars may contribute to the $b \rightarrow s \ell \ell$ process. In this section, we will explore in detail the NP contribution to semileptonic B decays, as well as $B_s^0 - \bar{B}_s^0$ mixing.

A. $b \rightarrow c \ell \nu$

By integrating out the heavy scalar, the quark-level decay $b \rightarrow c \bar{\ell} \nu$ can be depicted by the following effective Hamiltonian [27,82]²:

$$\mathcal{H} = 2\sqrt{2} G_F V_{cb} [(1 + C_{V_L}) \mathcal{O}_{V_L} + C_{V_R} \mathcal{O}_{V_R} + C_{S_L} \mathcal{O}_{S_L} + C_{S_R} \mathcal{O}_{S_R} + C_T \mathcal{O}_T] + \text{H.c.}, \quad (33)$$

with four effective operators given by

$$\mathcal{O}_{V_{L,R}} = (\bar{c} \gamma^\mu P_{L,R} b) (\bar{\ell} \gamma_\mu P_L \nu), \quad \mathcal{O}_{S_L} = (\bar{c} P_L b) (\bar{\ell} P_L \nu), \\ \mathcal{O}_{S_R} = (\bar{c} P_R b) (\bar{\ell} P_L \nu), \quad \mathcal{O}_T = (\bar{c} \sigma^{\mu\nu} P_L b) (\bar{\ell} \sigma_{\mu\nu} P_L \nu). \quad (34)$$

Note that all coefficients here represent pure NP contributions since contributions from the standard model (SM) have already been factorized out. In FG2HDM, the contribution from exotic charged Higgs H^+ is naturally generated at tree

²The adopted Hamiltonian form is for the convenience of extensions to incorporate interactions induced by right-handed neutrinos [83,84].

level, while the other contributions from new Z' and neutral H emerge from loop level and have been neglected. Hence, only scalar operators can be generated here, and Wilson coefficients at the electroweak (EW) scale are explicitly given as

$$\begin{aligned} C_{S_L}^{\tau^3} &= -\frac{2}{v^2 m_{H^\pm}^2} (N_u^\dagger V_{\text{CKM}})_{23} (N_\ell)_{\tau^3}^\dagger / (-2\sqrt{2} G_F V_{cb}) = \frac{1}{m_{H^\pm}^2} \left(-\frac{v_2}{v_1} m_c \right) \left(-\frac{v_2}{v_1} m_\tau \right) = \frac{m_c m_\tau}{m_{H^\pm}^2} \tan^2 \beta, \\ C_{S_R}^{\tau^3} &= \frac{2}{v^2 m_{H^\pm}^2} (V_{\text{CKM}} N_d)_{23} (N_\ell)_{\tau^3}^\dagger / (-2\sqrt{2} G_F V_{cb}) = -\frac{m_b m_\tau}{m_{H^\pm}^2} \tan^2 \beta. \end{aligned} \quad (35)$$

By retaining $C_{S_R}^{\tau^3}$, an additional contribution via $C_{S_L}^{\tau^3}$ enters compared to the early estimation of 2HDM-II [85]. For the comparison with the complete 2HDM-II [86] result, a significant difference exists in the relative sign between $C_{S_R}^{\tau^3}$ and $C_{S_L}^{\tau^3}$. This alleviates the tension between the 2HDM (especially 2HDM-II) and the $R_{D^{(*)}}$ measurement [87]. In order to calculate low-energy observables, QCD renormalization group (RG) evolution of $C_{S_{L,R}}$ in the leading approximation is considered. Running from $\Lambda = 100$ GeV to $\mu = m_b$, we have the Wilson coefficients at low energy,

$$C_{S_{L,R}}(\mu) = \underbrace{\left[\frac{\alpha_s(m_t)}{\alpha_s(m_b)} \right]^{-8/(2\beta_0^{(5)})}}_{\lambda_c} C_{S_{L,R}}(\Lambda), \quad (36)$$

with the one-loop β -function $\beta_0^{(n)} = 11 - 2n/3$, $\lambda_c \approx 1.41$, as adopted in [88,89].

Observables related to the $b \rightarrow c \ell \nu$ process, associated with Wilson coefficients evaluated at the $\mu = m_b$, have the following numerical forms [90]:

$$\begin{aligned} R_D &= R_D^{\text{SM}} [1 + 1.07 |C_{S_L}^{\tau^3} + C_{S_R}^{\tau^3}|^2 + 1.528 \Re[C_{S_L}^{\tau^3*} + C_{S_R}^{\tau^3*}]], \\ R_{D^*} &= R_{D^*}^{\text{SM}} [1 + 0.043 |C_{S_L}^{\tau^3} - C_{S_R}^{\tau^3}|^2 - 0.113 \Re[C_{S_L}^{\tau^3*} - C_{S_R}^{\tau^3*}]], \\ F_L^{D^*} &= F_{L,\text{SM}}^{D^*} \left(\frac{R_{D^*}}{R_{D^*}^{\text{SM}}} \right)^{-1} \times (1 + 0.099 |C_{S_L}^{\tau^3} - C_{S_R}^{\tau^3}|^2 - 0.263 \Re[C_{S_L}^{\tau^3*} - C_{S_R}^{\tau^3*}]), \\ P_\tau^{D^*} &= P_{\tau,\text{SM}}^{D^*} \left(\frac{R_{D^*}}{R_{D^*}^{\text{SM}}} \right)^{-1} \times (1 - 0.082 |C_{S_R}^{\tau^3} - C_{S_L}^{\tau^3}|^2 + 0.218 \Re[C_{S_L}^{\tau^3*} - C_{S_R}^{\tau^3*}]), \\ R_{J/\psi} &= R_{J/\psi}^{\text{SM}} \times (1 + 0.043 |C_{S_L}^{\tau^3} - C_{S_R}^{\tau^3}|^2 - 0.113 \Re[C_{S_L}^{\tau^3*} - C_{S_R}^{\tau^3*}]), \\ R_{\Lambda_c} &= R_{\Lambda_c}^{\text{SM}} \times (1 + 0.321 (|C_{S_L}^{\tau^3}|^2 + |C_{S_R}^{\tau^3}|^2) + 0.334 \Re[C_{S_L}^{\tau^3*}] + 0.497 \Re[C_{S_R}^{\tau^3*}] + 0.513 \Re[C_{S_L}^{\tau^3} C_{S_R}^{\tau^3*}]), \\ \mathcal{B}(B_c \rightarrow \tau \bar{\nu}) &= \mathcal{B}(B_c \rightarrow \tau \bar{\nu})_{\text{SM}} \times |1 - 4.35 (C_{S_L}^{\tau^3} - C_{S_R}^{\tau^3})|^2, \\ \mathcal{B}(B_u \rightarrow \tau \bar{\nu}) &= 9.30 |1 + 3.85 * (C_{S_R}^{\tau^3} - C_{S_L}^{\tau^3})_{ub}|^2 \times 10^{-5}, \end{aligned} \quad (37)$$

where $(C_{S_R}^{\tau^3} - C_{S_L}^{\tau^3})_{ub}$ in the last equation indicates that the quark flavor in Eq. (35) should be adjusted accordingly, with u assigned to $C_{S_L}^{\tau^3}$ and b to $C_{S_R}^{\tau^3}$. Although the $b \rightarrow u \ell \nu$ process slightly deviates from the main focus of this subsection, we discuss it later due to its relevance to $H^\pm$.

Recent advances in lattice QCD calculations of the $B \rightarrow D^{(*)}$ form factors [91,92] have led to updated estimates of semileptonic decays of $B \rightarrow D^{(*)}$ [90,93]. These developments refine earlier versions of numerical formulas quoted in [27,48]. In this work, we adopt the results presented in [90] since the full q^2 range, combining the latest calculations from both lattice QCD in the low-recoil region and light-cone sum rules in large hadronic recoil

[94], has been incorporated.³ For the leptonic decay $B_c \rightarrow \tau \bar{\nu}$, we still cite the formula adopted in [27]. The other theoretical predictions and experimental measurements have been summarized and can be found in Sec. IV.

B. $b \rightarrow s \ell^+ \ell^-$

The effective Hamiltonian to describe $b \rightarrow s \ell^+ \ell^-$ is given as

³Some of the relevant numbers in Eq. (37), which are not yet publicly available in the current version of the paper, have been provided through private communications with the authors and will appear in their forthcoming update.

$$\mathcal{H} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \sum_i (C_i \mathcal{O}_i + C'_i \mathcal{O}'_i) + \text{H.c.}, \quad (38)$$

where the effective operators are defined as

$$\begin{aligned} \mathcal{O}_7 &= \frac{m_b}{e} (\bar{s} \sigma_{\mu\nu} P_R b) F^{\mu\nu}, & \mathcal{O}'_7 &= \frac{m_b}{e} (\bar{s} \sigma_{\mu\nu} P_L b) F^{\mu\nu}, \\ \mathcal{O}_8 &= \frac{g_s m_b}{e^2} (\bar{s} \sigma_{\mu\nu} T^a P_R b) G_a^{\mu\nu}, & \mathcal{O}'_8 &= \frac{g_s m_b}{e^2} (\bar{s} \sigma_{\mu\nu} T^a P_L b) G_a^{\mu\nu}, \\ \mathcal{O}_9 &= (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \ell), & \mathcal{O}'_9 &= (\bar{s} \gamma_\mu P_R b) (\bar{\ell} \gamma^\mu \ell), \\ \mathcal{O}_{10} &= (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \gamma_5 \ell), & \mathcal{O}'_{10} &= (\bar{s} \gamma_\mu P_R b) (\bar{\ell} \gamma^\mu \gamma_5 \ell), \\ \mathcal{O}_S &= m_b (\bar{s} P_R b) (\bar{\ell} \ell), & \mathcal{O}'_S &= m_b (\bar{s} P_L b) (\bar{\ell} \ell), \\ \mathcal{O}_P &= m_b (\bar{s} P_R b) (\bar{\ell} \gamma_5 \ell), & \mathcal{O}'_P &= m_b (\bar{s} P_L b) (\bar{\ell} \gamma_5 \ell). \end{aligned} \quad (39)$$

It is known that the decay branching fraction only receives contribution from $\mathcal{O}_{7,8,9,10}$ in the SM, while more operators may contribute in NP models. In FG2HDM, Wilson coefficients of $\mathcal{O}_{9,10}$ receive modifications due to the

FCNC interaction induced by neutral vector bosons, giving

$$\begin{aligned} \Delta C_9 &= -\frac{1}{N} \left[\frac{1}{m_{Z'}^2} \mathcal{A}_L^{bsZ'} \mathcal{B}^{Z'\ell\ell} + \frac{1}{m_Z^2} \mathcal{A}_L^{bsZ} \mathcal{B}^{Z\ell\ell} \right], \\ \Delta C_{10} &= -\frac{1}{N} \left[\frac{1}{m_{Z'}^2} \mathcal{A}_L^{bsZ'} \mathcal{B}_5^{Z'\ell\ell} + \frac{1}{m_Z^2} \mathcal{A}_L^{bsZ} \mathcal{B}_5^{Z\ell\ell} \right], \end{aligned} \quad (40)$$

with $N = 2\sqrt{2}G_F V_{tb} V_{ts}^* e^2 / (16\pi^2)$. The parameters $\mathcal{A}_L^{bsZ^{(\prime)}}$, $\mathcal{B}^{Z^{(\prime)\ell\ell}}$, and $\mathcal{B}_5^{Z^{(\prime)\ell\ell}}$, summarized in Appendix C, represent the corresponding vertices mediated by Z/Z' with P_L on the quark side, and γ^μ and $\gamma^\mu \gamma_5$ on the lepton side, respectively. The absence of $\Delta C'_9$ and $\Delta C'_{10}$ can be interpreted as a consequence of diagonal $\mathcal{Q}_{dR}$.

In addition to a modification of the $\mathcal{O}_{9,10}$ contribution, scalar and pseudoscalar-type operators can be generated at leading order. Due to the interaction in Eq. (32), the flavor-changing neutral Higgs (FCNH) information is encoded in Wilson coefficients of the forms

$$\begin{aligned} \Delta C_S^\ell &= \Delta C_S^{\ell'} \\ &= \frac{1}{N} \frac{2\sqrt{2}G_F V_{tb} V_{ts}^*}{\sin 2\beta} \left\{ \left(\frac{1}{m_H^2} - \frac{1}{m_h^2} \right) [\sin^2(\alpha - \beta) N_\ell + \sin(\alpha - \beta) \cos(\alpha - \beta) M_\ell] + \frac{1}{m_h^2} N_\ell \right\}, \\ \Delta C_P^\ell &= -\Delta C_P^{\ell'} = -\frac{1}{N} \frac{2\sqrt{2}G_F V_{tb} V_{ts}^*}{m_{H_A}^2 \sin 2\beta} [U_\gamma^{21}]^2 N_\ell \approx -\frac{1}{N} \frac{2\sqrt{2}G_F V_{tb} V_{ts}^*}{m_{H_A}^2 \sin 2\beta} N_\ell. \end{aligned} \quad (41)$$

The Wilson coefficients of scalar and pseudoscalar operators should have the same anomalous dimension as quark masses, as stated in [95], indicating that the renormalization group evolution (RGE) equation (36) is also applicable here. In principle, the RGE correction of $C_{9,10}$ running from high-energy to m_b scale should be taken into account. However, such a correction is not sizable from the work in [96], and hence is neglected in the following numerical analysis.

C. $B_s - \bar{B}_s$ mixing

The mixing in the B_s^0 system provides complementary information to semileptonic decay $b \rightarrow s \ell^+ \ell^-$ in view of

pure quark transition $b \rightarrow s$. Constraints on $bs(Z'/H)$ vertices in FG2HDM are aimed to be extracted from experimental data.

Within the state-of-the-art effective Hamiltonian approach, the Hamiltonian in Buras-Misiak-Urbain (BMU) basis [97,98] is given by

$$\mathcal{H}_{\text{eff}}^{B\bar{B}} = N_{B\bar{B}} \sum_i C_i(\mu) \mathcal{O}_i, \quad (42)$$

where the four-quark operators are defined as

$$\begin{aligned} \mathcal{O}_{\text{VLL}} &= (\bar{b} \gamma_\mu P_L q) (\bar{\gamma} \gamma^\mu P_L q), & \mathcal{O}_{\text{VRR}} &= (\bar{b} \gamma_\mu P_R q) (\bar{\gamma} \gamma^\mu P_R q), \\ \mathcal{O}_{\text{LR}}^1 &= (\bar{b} \gamma_\mu P_L q) (\bar{\gamma} \gamma^\mu P_R q), & \mathcal{O}_{\text{LR}}^2 &= (\bar{b} P_L q) (\bar{\gamma} P_R q), \\ \mathcal{O}_{\text{SLL}}^1 &= (\bar{b} P_L q) (\bar{\gamma} P_L q), & \mathcal{O}_{\text{SRR}}^1 &= (\bar{b} P_R q) (\bar{\gamma} P_R q), \\ \mathcal{O}_{\text{SLL}}^2 &= -(\bar{b} \sigma_{\mu\nu} P_L q) (\bar{\gamma} \sigma^{\mu\nu} P_L q), & \mathcal{O}_{\text{SRR}}^2 &= -(\bar{b} \sigma_{\mu\nu} P_R q) (\bar{\gamma} \sigma^{\mu\nu} P_R q), \end{aligned} \quad (43)$$

and the constant is $N_{B\bar{B}} \equiv \frac{1}{16\pi^2} G_F^2 M_W^2 (V_{tb}^* V_{ts})^2$.

In FG2HDM, new contributed Wilson coefficients to calculate the mass split $\Delta M_s = 2|M_{12}| \equiv 2|\langle B^0 | \mathcal{H}_{\text{eff}}^{B\bar{B}} | \bar{B}^0 \rangle|$, generated by integrating out heavy particles, are calculated to be

$$\begin{aligned}
\Delta C_{\text{VLL}} &= \frac{g^2}{4} \left(\frac{\sin^2 \theta'_2}{m_Z^2} + \frac{\cos^2 \theta'_2}{m_{Z'}^2} \right) (\Delta \mathcal{Q})^2 (V_{ib}^* V_{ts})^2 / N_{B\bar{B}}, \\
\Delta C_{\text{SLL}}^1 &= \Delta C_{\text{SRR}}^1 = -\frac{1}{v^2} \left(\frac{\cos^2(\alpha - \beta)}{m_h^2} + \frac{\sin^2(\alpha - \beta)}{m_H^2} - \frac{[U_\gamma^{21}]^2}{m_{H_A}^2} \right) \left(2 + \frac{1}{\tan^2 \beta} + \tan^2 \beta \right) \times m_s^2 (V_{ib}^* V_{ts})^2 / N_{B\bar{B}}, \\
\Delta C_{\text{LR}}^2 &= -\frac{2}{v^2} \left(\frac{\cos^2(\alpha - \beta)}{m_h^2} + \frac{\sin^2(\alpha - \beta)}{m_H^2} + \frac{[U_\gamma^{21}]^2}{m_{H_A}^2} \right) \left(2 + \frac{1}{\tan^2 \beta} + \tan^2 \beta \right) m_s^2 (V_{ib}^* V_{ts})^2 / N_{B\bar{B}}, \quad (44)
\end{aligned}$$

with $\Delta \mathcal{Q} \equiv (\mathcal{Q}_{i_R} - \mathcal{Q}_{u_R})$. The nonperturbative matrix elements can be parametrized by bag parameters, which are detailed in Appendix C. In Table II, a summary of numerical values of bag parameters at $\mu_b = m_b$ have been presented.

For completeness, here, we also exhibit the SM matrix element

$$\begin{aligned}
\langle B^0 | H_{\text{eff}}^{B\bar{B}} | \bar{B}^0 \rangle_{\text{SM}} &= \frac{4}{3} N_{B\bar{B}} [\alpha_s^{(5)}(\mu_b)]^{-\frac{6}{23}} \left[1 + \frac{\alpha_s^{(5)}(\mu_b)}{4\pi} J_5 \right] \\
&\times m_{B_s} \eta_B f_{B_s}^2 B_1 S_0(x_t), \quad (45)
\end{aligned}$$

where the QCD factor is $\eta_B = 0.5510$, $J_5 = 1.627$ in the naive dimensional regularization (NDR) scheme [98], and loop function $S_0(x_t)$ is extracted from [99]. The evolution of Wilson coefficients, running from Λ to μ_b , can be depicted by the following evolution matrix [98]:

$$U(\mu_b, \Lambda) = \begin{pmatrix} [\eta(\mu_b, \Lambda)]_{\text{VLL}} & 0 & 0 \\ 0 & [\eta(\mu_b, \Lambda)]_{\text{LR}} & 0 \\ 0 & 0 & [\eta(\mu_b, \Lambda)]_{\text{SLL}} \end{pmatrix}. \quad (46)$$

The VRR and SRR sectors share the common values as the above VLL and SLL ones. Numerical values of related parameters have been summarized in Table III.

D. Electroweak and Higgs physics

Various high-precision measurements around the Z pole ($\sqrt{s} \approx m_Z$) provide an excellent testing ground for NP. Most of these measurements were conducted by LEP1 and SLC [100–105] with remarkable precision, and they are summarized in Table V. Rather than elaborating on their extensive history, we refer interested readers to the cited references and instead focus on their connection to new physics through

$$\begin{aligned}
g_v^f &= I_3^f - Q_f \sin^2 \theta_W, & g_a^f &= I_3^f, \\
\delta g_v^f &= \frac{g'}{2} \sin \theta_2' (Q_{fR} + Q_{fL}) / N_g, \\
\delta g_a^f &= \frac{g'}{2} \sin \theta_2' (Q_{fR} - Q_{fL}) / N_g, \quad (47)
\end{aligned}$$

$$\begin{aligned}
G_f &\equiv (g_v^f)^2 + (g_a^f)^2, & \delta G_f &\equiv 2(g_v^f \delta g_v^f + g_a^f \delta g_a^f), \\
\delta \mathcal{F}_f &= \sum_f N_c^f \delta G_f, & \delta \mathcal{F}_q &= \sum_q N_c^q \delta G_q, & \mathcal{F}_f &= \sum_f N_c^f G_f, & \mathcal{F}_q &= \sum_q N_c^q G_q, \quad (48)
\end{aligned}$$

where the prefactor is $N_g = e \cos \theta_2' / (2 \sin \theta_W \cos \theta_W)$. The corresponding Z-pole formulas are given below [106]:

$$\delta \Gamma_{Z, \text{NP}} = \frac{\alpha m_Z}{12 c_W^2 s_W^2} \delta \mathcal{F}_f, \quad \delta \Gamma_{Z, \text{NP}}^f = \frac{\alpha m_Z}{12 c_W^2 s_W^2} N_c^f \delta G_f, \quad (49)$$

$$\delta \sigma_{\text{had, NP}}^0 = \frac{12\pi G_e \mathcal{F}_q}{m_Z^2 \mathcal{F}_f^2} \left(\frac{\delta G_e}{G_e} + \frac{\delta \mathcal{F}_q}{\mathcal{F}_q} - 2 \frac{\delta \mathcal{F}_f}{\mathcal{F}_f} \right), \quad (50)$$

$$\delta A_{f, \text{NP}} = -\frac{2[(g_v^f)^2 - (g_a^f)^2]}{G_f^2} (g_a^f \delta g_v^f - g_v^f \delta g_a^f), \quad (51)$$

$$\delta A_{\text{FB, NP}}^{0, f} = -\left[\frac{3g_v^f g_a^f [(g_v^e)^2 - (g_a^e)^2]}{G_f G_e^2} (g_a^e \delta g_v^e - g_v^e \delta g_a^e + e \leftrightarrow f) \right], \quad (52)$$

$$\begin{aligned}
\delta R_{\ell, \text{NP}}^0 &= N_c^\ell \left(\frac{\delta \mathcal{F}_q}{G_\ell} - \frac{\mathcal{F}_q \delta G_\ell}{G_\ell^2} \right), \\
\delta R_{c, \text{NP}}^0 &= N_c^q \left(\frac{\delta G_c}{\mathcal{F}_q} - \frac{G_c \delta \mathcal{F}_q}{\mathcal{F}_q^2} \right). \quad (53)
\end{aligned}$$

With these formulas in hand, we can systematically construct the corresponding χ_{EWPOs}^2 . Additionally, the formulas for SM predictions and the effective weak mixing

angles $\sin\theta_{\text{eff}}^{b,\ell}$ and $\sin\theta_{\text{eff}}^{\text{Up,Down}}$ used in our numerical analysis can be found in [107,108]. These results have been adopted and rigorously benchmarked by the ZFIT-TER group and its successors [109,110].

Beyond the Z-pole observables, the hff vertices are also constrained by current flavor-conserving Higgs experiments [111,112]. These constraints are typically derived by performing a fit to the signal strengths, defined as

$$\mu_{if} \equiv \frac{\sigma_i \cdot \mathcal{B}_f}{\sigma_i^{\text{SM}} \cdot \mathcal{B}_f^{\text{SM}}}, \quad (54)$$

where i represents a given initial production mode,⁴ and $f = b\bar{b}, c\bar{c}, \tau\bar{\tau}$, and $\mu\bar{\mu}$ correspond to various decay channels. Under the Born approximation [113,114], the partial width of Higgs boson decays into fermion pairs can be simplified as

$$\frac{\Gamma_{hff}^{\text{NP}}}{\Gamma_{hff}^{\text{SM}}} = \left| \frac{g_{hff}^{\text{NP}}}{g_{hff}^{\text{SM}}} \right|^2 = \left| \sin(\alpha - \beta) - \cos(\alpha - \beta) \frac{N_f}{m_f} \right|^2. \quad (55)$$

We do not include constraints from light quark or lepton Yukawa couplings due to the lack of available measurements. However, we incorporate the correlation matrix from CMS [111] in the construction of χ_{Higgs}^2 .

IV. NUMERICAL ANALYSIS

In this section, we perform a numerical analysis of the FG2HDM. Our primary objective is to efficiently identify viable solutions after extending the particle spectrum and interactions beyond the SM. In the following analysis, we explicitly examine the parameter space for the two $U(1)'$ charges $\mathcal{Q}_{d_R}$ and $\mathcal{Q}_{\mu_R}$, along with g' , $\sin\theta_2$, $s_{\alpha\beta}$,⁵ $\tan\beta$, and the masses of four new particles: $m_{Z'}$, m_H , $m_{H^\pm}$, and m_{H_A} , with a focus on the aforementioned flavor observables.

A. Input parameters and experimental status

With more data accumulated on the decay related to $b \rightarrow c\ell\nu$ since the first report of $R_{D^{(*)}}$ anomaly in 2012,

tensions still exist up to now. For the $R_{D^{(*)}}$ problem, there still retains a discrepancy of 3.31σ after recent updates [12,115]. Two additional LFU observables, $R_{J/\Psi} \equiv \mathcal{B}(B_c \rightarrow J/\Psi\tau\bar{\nu})/\mathcal{B}(B_c \rightarrow J/\Psi\mu\bar{\nu})$ and $R_{\Lambda_c} \equiv \mathcal{B}(\Lambda_b \rightarrow \Lambda_c\tau\bar{\nu})/\mathcal{B}(B_c \rightarrow \Lambda_c\mu\bar{\nu})$, supplement the $b \rightarrow c\tau\bar{\nu}$ current [116–118]. For the τ longitudinal-polarization asymmetry $P_\tau^{D^*}$ [119] in D^* and the fraction of the D^* longitudinal mode $F_L^{D^*}$ [119], the latest observations are maintained by BELLE [120] and LHCb [15], respectively. All the observations related to $b \rightarrow c\ell\nu$, except $R_{D^{(*)}}$, are within 2σ errors of the SM predictions. Besides, we do not include P_τ^D and $R_{\Upsilon(3S)}$ in the numerical analysis, due to the lack of experimental data for P_τ^D and the absence of scalar Wilson coefficients for $R_{\Upsilon(3S)}$, as reported in Ref. [27]. The latest experimental status of aforementioned observables is summarized in Table I.

The $R_{K^{(*)}}$ released in December 2022 [1] were finally found to be compatible with SM prediction, indicating that the long expected LFU anomalies in $b \rightarrow s\ell^+\ell^-$ have faded away. However, the NP opportunity has not been excluded in the binned branching fractions and particular components of angular distributions. All the binned global fit works [18–23] in recent years exhibit the common behaviors that, allowing a deviation of C_9 , can accommodate data. Recently, the unbinned amplitude analysis with improved Breit-Wigner function [3–5] also showed a deviation of C_9 (without a discrimination of lepton flavor species) from SM of about 2σ . Incorporating around 200 observables related to $b \rightarrow s\ell^+\ell^-$, including the latest $R_{K^{(*)}}$ values, the recent model-independent global fit of NP parameters [20] indicates that

$$\begin{aligned} \Delta C_9^\mu &= -0.872 \pm 0.215, & \Delta C_9^e &= -1.511^{+0.561}_{-0.533}, \\ \Delta C_{10}^\mu &= 0.171^{+0.157}_{-0.175}, & \Delta C_{10}^e &= 0.383^{+0.840}_{-0.424}, \end{aligned} \quad (56)$$

and

$$\begin{aligned} C_S^\mu &= 0.009^{+0.858}_{-0.845}, & C_S^e &= -0.806^{+1.900}_{-1.238}, & C_S'^\mu &= 0.012^{+0.858}_{-0.862}, & C_S'^e &= -0.803^{+1.861}_{-1.194}, \\ C_P^\mu &= 0.124^{+0.902}_{-0.910}, & C_P^e &= -1.837^{+1.376}_{-0.930}, & C_P'^\mu &= 0.038^{+0.894}_{-0.913}, & C_P'^e &= -1.652^{+1.200}_{-0.979}, \end{aligned} \quad (57)$$

in its S-IV scenario involving as general as 20 Wilson coefficients. The around 4σ and 3σ deviations in C_9^μ and C_9^e , respectively, as well as other Wilson coefficients will be taken as inputs of numerical analysis in this work.

For the consideration of NP in the $B_s^0 - \bar{B}_s^0$ system, we take ΔM_s as the typical observable in this analysis. Combing the recent updated PDG value [123] with an inclusion of the latest value measured by LHCb [124],

⁴Here, we follow the CMS assumption $\sigma_i = \sigma_i^{\text{SM}}$ when extracting μ_f [111].

⁵Here, $s_{\alpha\beta}$ denotes $\sin(\alpha - \beta)$.

TABLE I. Summary of current experimental values and SM predictions related to the $b \rightarrow c\ell\nu$ process. The correlation between R_D and R_{D^*} is -0.39 after Moriond 2024.

	R_D	R_{D^*}	$P_\tau^{D^*}$	$F_L^{D^*}$	$R_{J/\psi}$	R_{Λ_c}
SM	0.302(8) [90]	0.257(5) [90]	$-0.518(6)$ [90]	0.430(7) [90]	0.258(4) [90,121]	0.332(10) [90]
Exp.	0.342(26) [14]	0.287(12) [14]	$-0.38(^{+53}_{-55})$ [120]	0.49(5) [27]	0.61(18) [27]	0.271(72) [27,122]

$$\Delta M_s^{\text{PDG}} = 17.765 \pm 0.005 \text{ ps}^{-1}, \quad (58)$$

as well as theoretical prediction in SM $\Delta M_s^{\text{SM}} = 18.23 \pm 0.63 \text{ ps}^{-1}$ [125], the NP effect can be discriminated from the SM and parametrized [126,127] as

$$\left| 1 + \frac{M_{12}^{\text{NP}}}{M_{12}^{\text{SM}}} \right| \sim \frac{\Delta M_s^{\text{exp}}}{\Delta M_s^{\text{SM}}} = 0.974 \pm 0.034, \quad (59)$$

leading to

$$-0.094 \leq \frac{M_{12}^{\text{NP}}}{M_{12}^{\text{SM}}} \leq 0.042 \quad (60)$$

within 2σ standard deviations. The range in Eq. (60), together with Tables II and III, serve as experimental and theoretical respective inputs in the following numerical calculations of the B_s mixing system.

Much more has been learned about the properties of the Higgs boson in the decade since its discovery. In this work, we focus on its fermionic decay modes, which are of particular interest in the following numerical analysis. The latest results, based on partial Run-2 datasets from CMS [111,128] and ATLAS [112], are summarized in Table IV.

TABLE II. The averaged bag parameters $f_{B_s}^2 B_i$ (in the unit of GeV^2) adopted in the calculation of $B_s^0 - \bar{B}_s^0$ mixing, at $\mu_b = \bar{m}_b$, and in the $\overline{\text{MS}}$ -NDR scheme [126].

i	1	2	3	4	5
$f_{B_s}^2 B_i$	0.0452(14)	0.0441(17)	0.0454(27)	0.0544(19)	0.0507(17)

TABLE III. The inputs for evaluating evolution matrix $U(\mu_b, \Lambda)$ in Eq. (46), with $\Lambda = 100 \text{ GeV}$ at μ_b scale, and α_s is taken as $\alpha_s^{(5)} = 0.1179$ [98].

$[\eta]_{\text{VLL}}$	$[\eta]_{1\text{LR}}$	$[\eta]_{12\text{LR}}$	$[\eta]_{21\text{LR}}$	$[\eta]_{22\text{LR}}$	$[\eta]_{11\text{SLL}}$	$[\eta]_{12\text{SLL}}$	$[\eta]_{21\text{SLL}}$	$[\eta]_{22\text{SLL}}$
0.855	0.926	-0.037	-0.784	2.109	1.588	1.820	-0.006	0.577

TABLE IV. The measurements on signal strength extracted from various decay channels with the assumption $\sigma_i = \sigma_i^{\text{SM}}$, in which Gaussian error approximation has been adopted [111,128].

f	$b\bar{b}$	$c\bar{c}$	$\tau\bar{\tau}$	$\mu\bar{\mu}$
μ_f	1.05(22)	9.4(203)	0.85(10)	1.21(45)

Extensive measurements of EW observables have been conducted over decades, providing a robust foundation for precision tests of the SM and searches for NP. In this section, we present a systematic compilation of key observables near the Z pole, including their experimental values, SM theoretical predictions, and corresponding NP corrections in the working model of the current paper. These quantities are organized in Table V and serve as inputs in the subsequent analysis.

In addition to the essential physical observables mentioned above and their associated parameters, Table VI provides a summary of other basic input parameters required for the subsequent theoretical calculations.

TABLE V. Principal Z-pole observables, their SM predictions, and NP formulas. Measurements not separated by horizontal lines are correlated through correlation matrices [101]. The first set of A_e and A_τ measurements are extracted from the combination of leptonic polarization and left-right asymmetry at SLD, while the second set follows from the LEP-1 experiments.

Observables	Measurements	SM predictions	NP corrections
m_Z [GeV]	91.1876(21) [123]	91.1884	
Γ_Z [GeV]	2.4955(23) [100,101]	2.4946	Eq. (49)
σ_{had}^0 [nb]	41.4802(325) [100,101]	41.4922	Eq. (50)
R_e	20.804(50) [101]	20.733	
R_μ	20.785(33) [101]	20.733	Eq. (53)
R_τ	20.764(45) [101]	20.780	
$A_{\text{FB}}^{0,e}$	0.0145(25) [101]	0.0163	
$A_{\text{FB}}^{0,\mu}$	0.0169(13) [101]	0.0163	Eq. (52)
$A_{\text{FB}}^{0,\tau}$	0.0188(17) [101]	0.0163	
R_b	0.21629(66) [101]	0.21584	
R_c	0.1721(30) [101]	0.17222	Eq. (53)
A_{FB}^b	0.0996(16) [101]	0.1035	
A_{FB}^c	0.0707(35) [101]	0.0739	Eq. (52)
A_b	0.923(20) [101]	0.935	
A_c	0.670(27) [101]	0.668	Eq. (51)
A_e	0.1516(21) [101]	0.1476	
A_μ	0.142(15) [101]	0.1476	Eq. (51)
A_τ	0.136(15) [101]	0.1476	
A_e	0.1498(49) [101]	0.1476	
A_τ	0.1439(43) [101]	0.1476	Eq. (51)
A_s	0.895(91) [101]	0.936	
$A_{\text{FB}}^{0,s}$	0.0976(114) [101]	0.1036	Eq. (52)
R_{uc}	0.166(9) [123]	0.1722	Eq. (53)

TABLE VI. Basic input parameters in the numerical analysis.

Parameters	Values	Parameters	Values
m_b	$4.18^{(+3)}_{(-2)} \text{ GeV}$ [123]	m_t	$173.50(30) \text{ GeV}$ [123]
m_c	$1.27(2) \text{ GeV}$ [123]	m_s	$93^{(+11)}_{(-5)} \text{ MeV}$ [123]
m_d	$4.67^{(+48)}_{(-17)} \text{ MeV}$ [123]	m_u	$2.16^{(+49)}_{(-26)} \text{ MeV}$ [123]
m_e	$0.5109989461(31) \text{ MeV}$ [123]	m_μ	$105.6583745(24) \text{ MeV}$ [123]
m_τ	$1776.86(12) \text{ MeV}$ [123]	m_W	$80.377(12) \text{ GeV}$ [123]
m_Z	$91.1876(21) \text{ GeV}$ [123]	m_h	$125.12(17) \text{ GeV}$ [123]
$\overline{m}_t(\overline{m}_t)$	$163.53(83) \text{ GeV}$ [123]	m_{ν_i}	$< 10^{-11} \text{ GeV}$
m_{B_u}	$5279.34(12) \text{ MeV}$ [123]	τ_{B_u}	$1.638(4) \text{ ps}$ [123]
f_{B_u}	$190.0(13) \text{ MeV}$ [123]	m_{B_s}	$5366.92(10) \text{ MeV}$ [123]
G_F	$1.1663787(6) \text{ GeV}^{-2}$ [123]		
$\alpha_s(m_Z)$	$0.1179(9)$ [123]	$\alpha_e(m_Z)$	$1/127.944(14)$ [123]
$\sin^2 \theta_W$	$0.23121(4)$ [123]	$S_0(x_t)$	$2.322(18)$ [99]
λ	$0.22500(67)$ [123]	A	$0.826^{(+18)}_{(-15)}$ [123]
$\bar{\rho}$	$0.159^{(+10)}_{(-10)}$ [123]	$\bar{\eta}$	$0.348^{(+10)}_{(-10)}$ [123]

B. Numerical results

The decays $b \rightarrow c\ell\nu$ and $b \rightarrow s\ell^+\ell^-$ are, respectively, attributed by charged current and neutral current interactions. Hence, separated analyses are performed as the first step. Although it appears that the two processes are less correlated, they can be connected in certain specific UV-completed NP models. Such a correlation within the framework of the FG2HDM will be explored below.

We first take $b \rightarrow c\ell\nu$ alone to put constraints on NP parameters in terms of both model-independent and dependent ways, as presented in Fig. 1. In the left panel, best fits of model-independent coefficients $C_{S_L}^{\tau^3}$ and $C_{S_R}^{\tau^3}$ at the confidence levels (CL) of 68% and 95% are exhibited on the top left, incorporating data in Table I. The best-fit point $(C_{S_L}^{\tau^3}, C_{S_R}^{\tau^3}) = (-0.35, 0.42)$ indicates that a negative $C_{S_L}^{\tau^3}$ and positive $C_{S_R}^{\tau^3}$ are preferred by data at the 95% CL. This shows clearly why 2HDM-II is not preferred since the signs

of the two corresponding coefficients in 2HDM-II are identical. Though not measured yet, a future measurement of decay $B_c \rightarrow \tau\nu$ will further discriminate the parameter space. By conjecturing that $\mathcal{B}(B_c \rightarrow \tau\nu) < 15\%(5\%, 1\%)$, the solution region (dot-dashed contours) moves down to the right entirely, with a possibility to flip the sign of $C_{S_L}^{\tau^3}$ at the 2σ level. The magnitude of the coefficients also decreases, leading to even larger charged Higgs mass and relative smaller $\tan\beta$, as shown with the dashed contours in the left panel as well. This change can be well interpreted from the allowed region of $\mathcal{B}(B_c \rightarrow \tau\nu)$ marked in light green in the same plot. It is noticed that the measured branching ratio $\mathcal{B}(B_u \rightarrow \tau\nu)_{\text{exp}} = (1.09 \pm 0.24) \times 10^{-4}$ [123] is indeed smaller than that of the $B_c \rightarrow \tau\nu$ process, as assumed above. The right panel of Fig. 1 illustrates that the decay process $B_u \rightarrow \tau\nu$ can offer complementary information within the framework of the

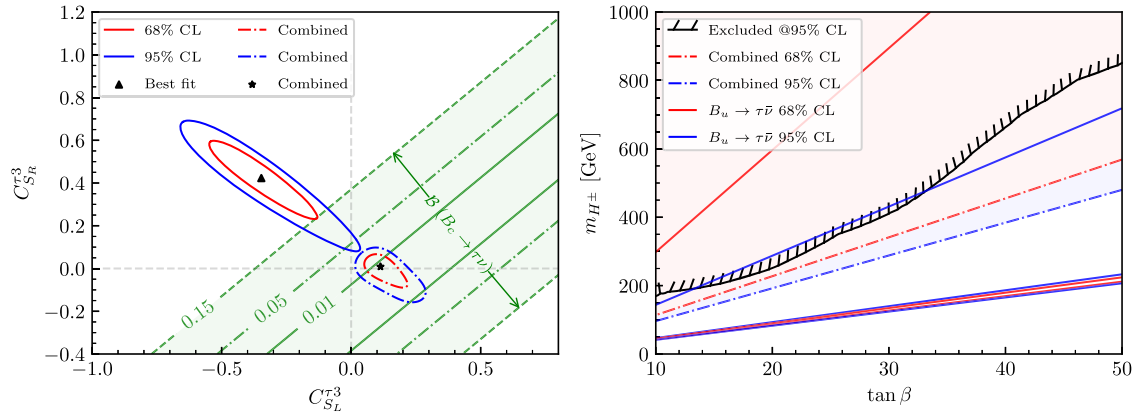


FIG. 1. The allowed parameter space by the $b \rightarrow c\ell\nu$ process: left: model-independent Wilson coefficients $C_{S_L}^{\tau^3}$ and $C_{S_R}^{\tau^3}$, and right: model-dependent allowed regions for $\tan\beta$ as well as $m_{H^\pm}$, where the excluded areas are extracted from ATLAS [129] for reference.

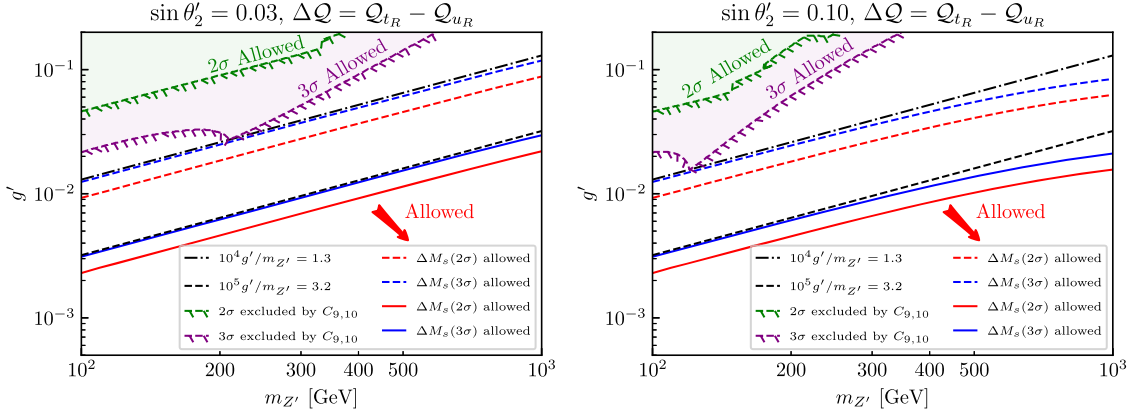


FIG. 2. Solution space constrained by the new Z' -boson-mediated $b \rightarrow s\ell\ell$ and $B_s - \bar{B}_s$ mixing; the region with green (purple) shade represents the $2(3)\sigma$ survival area provided by $C_{9,10}$. The boundary with red (blue) and its filling side represents the $2(3)\sigma$ requirement of ΔM_s at fixed ΔQ . The solid (dashed) boundary represents $\Delta Q = 4(1)$. The value of $\sin \theta'_2$ holds 0.03 (0.10) in the left (right) panel.

FG2HDM. Especially, when compared to the combined constraints from the $b \rightarrow c\ell\nu$ transitions, it provides even stronger bounds on the parameter space of $(\tan\beta, m_{H^\pm})$. We also put a direct search bound provided by ATLAS [129] in the right panel, and the corresponding recasting will be combined in a separate work.

The $b \rightarrow s$ observables receive contributions from both vector and scalar operators generated from neutral gauge bosons and neutral scalars. Here, we firstly focus on the Z' sector by combing the restrictions on $\Delta C_{9,10}$ and ΔC_{VLL} . As shown in Eqs. (40) and (44), the interaction strength is determined by the mixing angle $\sin \theta'_2$ and two $U(1)'$ charges Q_{d_R} and Q_{μ_R} . Two conjectures on mixing, SM-like (with $\sin \theta'_2 = 0.03$) and SM-deviated (with $\sin \theta'_2 = 0.10$), are made to carry out further numerical analysis. For the consideration of naturalness, the two free $U(1)'$ charges are set to be within the range $Q_{d_R}, Q_{\mu_R} \in [-1, 1]$, leading to $\Delta Q \in [-4, 4]$. In practical calculation, we choose $\Delta Q = 4$ and 1 as two typic test options. The remaining task is then to capture the allowed regions in the $(g', m_{Z'})$ plane. The survival space allowed by $b \rightarrow s\ell^+\ell^-$ is obtained from the $\chi^2(C_9, C_{10})$ constructed by a combination of $C_{9,10}^{e,\mu}$, while the allowed region for B_s^0 mixing originates from Eq. (60). Numerical results for the two conjectures with specific charge assignments are presented in Fig. 2. Taking the SM-like conjecture (left panel of Fig. 2) as an example, the permitted areas for $b \rightarrow s\ell^+\ell^-$ and ΔM_s occupy the upper left and lower right portions, respectively. The overlapping space of these areas represents the desired survival region. To provide complementary information by a comparison with charge-mass ratio $g_1/m_Z = e/m_W = 3.8 \times 10^{-3} \text{ GeV}^{-1}$ in the SM, we also plot two lines, $g'/m_{Z'} = 1.3(0.32) \times 10^{-4} \text{ GeV}^{-1}$, in black in each panel of Fig. 2. Apparently, data favor the small ΔQ case since no overlapping region exists for the $\Delta Q = 4$ scenario in the left panel. Therefore, with the condition of $\Delta Q \leq 1$ and $g'/m_{Z'} > 1.3 \times 10^{-4}$, the solution space for $(g', m_{Z'})$ will

be allowed. In the conjecture of the SM-deviated case, as shown in the right panel, the solution space could exist but with smaller Z' mass, which will receive more strict examination from collider physics.

The parameters $(m_H, m_{H_A}, \tan\beta, s_{\alpha\beta}, \text{ and } U_\gamma^{21})$ are related to scalar contribution to $b \rightarrow s$ processes from Eqs. (41) and (44). Assuming a heavy scalar singlet, as already adopted in this work, the relevant U_γ entry is close to unity, $U_\gamma^{21} \approx 1$. The yet-to-be measured rotation between the Higgs basis is conjectured as $s_{\alpha\beta} = 0.99$ and 0.95 , respectively. Hence, a contribution from the observed neutral scalar h is negligible. Without loss of generality, we fix the pseudoscalar mass as $m_{H_A} = 500 \text{ GeV}$ to explore the behavior of the remaining heavy neutral Higgs. After setting the above parameters, we present numerical relations between $\tan\beta$ and m_H in Fig. 3. As illustrated in the left panel of Fig. 3, although large $\tan\beta$ is supported by $\Delta C_{S,P}^{(\ell)}$ constrained from $b \rightarrow s\ell^+\ell^-$, ΔM_s with a less constrained neutral scalar mass prefers smaller $\tan\beta$. Hence, the overlapping area, mainly originated from ΔM_s , suggests a strong preference for $\tan\beta < 28$, while the range of heavy quark mass m_H remains broad, extending from hundreds of GeV to TeV as long as the $\tan\beta$ is sufficiently small. For the SM-deviated ($s_{\alpha\beta} = 0.95$) conjecture in the right panel of Fig. 3, the constraint from $C_{S,P}^{(\ell)}$ becomes more strict, while $B_s - \bar{B}_s$ mixing further suppresses the selections of value of $\tan\beta$, similar to the situation in the SM-like one. Nevertheless, the solution space for SM-deviated conjecture can accommodate all the related data as well.

A comparison of the phenomenological results from flavor physics in Fig. 2 with the phenomenological results from electroweak physics in Fig. 4 is both insightful and interesting. As we discuss in the narrative of Fig. 2, B_s^0 mixing provides us with the basic requirements $\Delta Q < 1$ and $g'/m_{Z'} > 1.3 \times 10^{-4}$. When combined with the

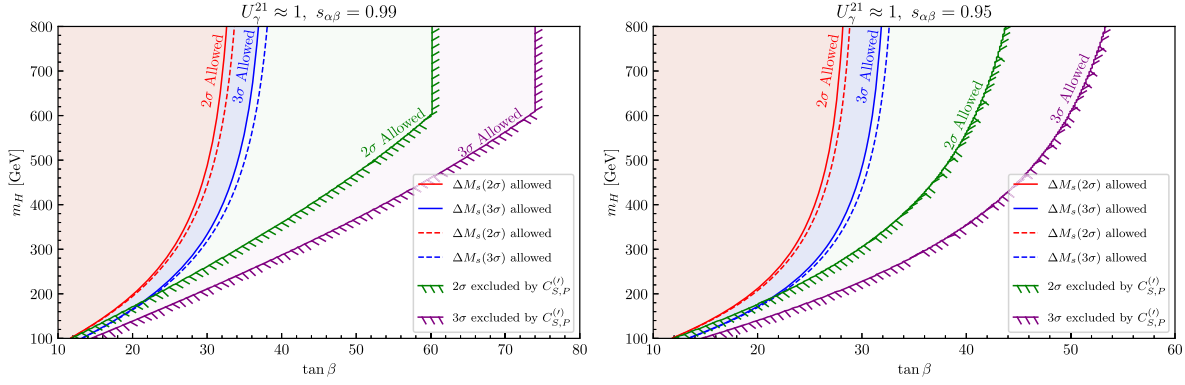


FIG. 3. Solution space constrained by scalar-boson-mediated $b \rightarrow s\ell\ell$ and $B - \bar{B}$ mixing: the region with green (purple) shade represents the $2(3)\sigma$ allowed area provided by $C_{S,P}^{(\ell)}$ at fixed value $m_{H_A} = 500$ GeV. The solid red (blue) boundary and its filling shade denotes the constraint from ΔM_s mixing at fixed value $m_{H_A} = 500$ GeV. The dashed ones show the changes when m_{H_A} is fixed at 520 GeV.

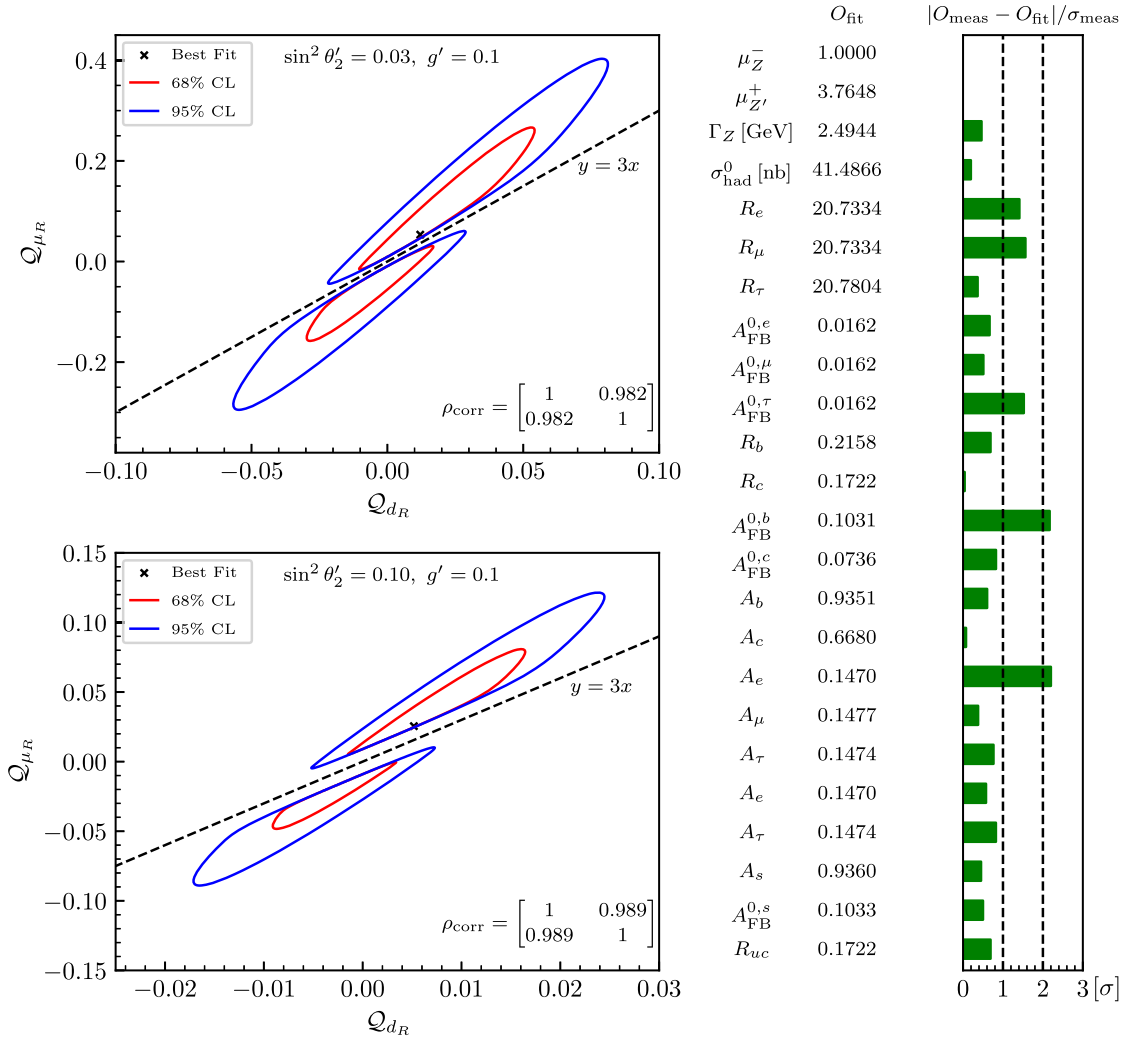


FIG. 4. Left panel: solution space constrained by electroweak precision observables (EWPOs) at Z-pole. The red (blue) contour represents the boundary at the 68% (95%) confidence level. Right panel: fit results and their deviations from experimental measurements. The values are obtained under the parameter settings: $v_S = 4 \times 10^5$ GeV and $\tan \beta = 15$, and two typical combinations of g' and $\sin \theta'_2$.

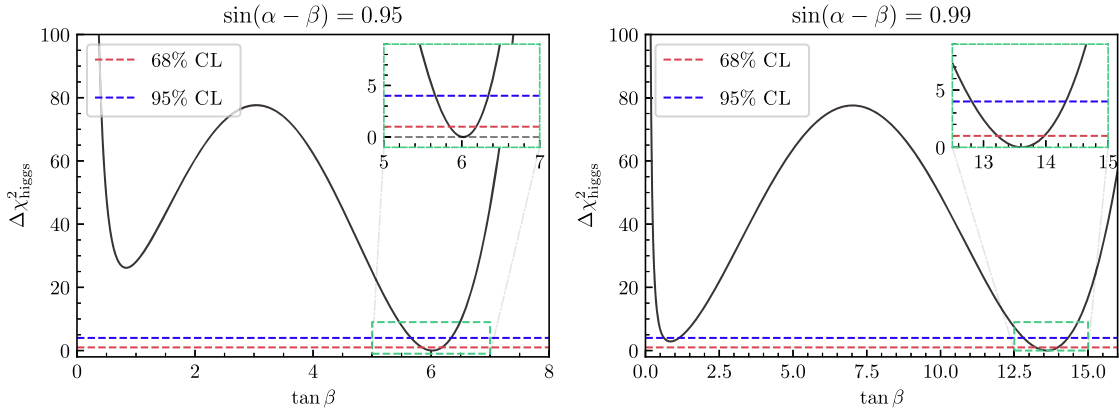


FIG. 5. Profiles of $\Delta\chi^2 = \chi^2_{\text{Higgs}} - \min(\chi^2_{\text{Higgs}})$ in different scenarios. The left (right) panel corresponds to the SM-deviated (SM-like) conjecture. The dashed red (blue) line denotes the boundary at the 68% (95%) confidence level. The insets in each subplot provide zoomed-in views around the respective minima.

$g' = 0.1$ setting in Fig. 4, the value of $m_{Z'}$ is restricted to be less than 800 GeV. Furthermore, when combined with the limitations of C_9 and C_{10} , the range below 500 GeV is deemed to be more secure. Figure 4 demonstrates refined $U(1)'$ charge assignments, and the key findings are as follows: (1) the results of the Z-pole measurements suggest that viable solutions for constrained $U(1)'$ charges exist in both SM-like and SM-deviated scenarios, albeit within a narrow parameter space. Within these ellipse areas, the corresponding values of ΔQ vary from -0.2 to 0.2 , which are accommodated with the results derived from the flavor constraints in Fig. 2. (2) The elements of the correlation matrix and the auxiliary relation ($Q\mu_R = 3Qd_R$) indicate that the BGL symmetry remains compatible with current electroweak precision data. (3) The right panel of Fig. 4 demonstrates that the best-fit point in a SM-like situation is consistent with the existing constraints from the Z-pole measurements. The trial solution $\mu_{Z'}^+ = 3.76$ indicates that solutions that do not exceed the limits of flavor physics $m_{Z'} < 500$ GeV can also exist.

The constraints on the observed Higgs properties, derived from Fig. 5, reveal two key features. First, the value of $\tan\beta$ in the vicinity of the relatively large global minimum decreases as $s_{\alpha\beta}$ deviates further from unity. Second, extrapolating the SM limit $s_{\alpha\beta} \rightarrow 1$ suggests the existence of a local minimum with $\tan\beta < 1$, which could arise in regions of the parameter space where Higgs couplings deviate significantly from SM predictions. These trends highlight the interplay between $\tan\beta$ and the Higgs mixing angle $\alpha - \beta$ in shaping the phenomenological implications of the observed scalar sector.

In the previous individual analyses (presented in Figs. 1–3), correlations among parameters and observables have not yet been taken into account. Given that the FG2HDM parameter $\tan\beta$ influences both charged and neutral Higgs interactions, and that $b \rightarrow s$ observables

receive contributions from both neutral Higgs and neutral gauge bosons, correlation effects may be significant. Therefore, we perform a combined analysis that incorporates measurements of $b \rightarrow c\ell\nu$, $b \rightarrow s\ell^+\ell^-$, and $B_s^0 - \bar{B}_s^0$ mixing, with all necessary FG2HDM parameters involved.

In the SM-like conjecture with $\sin\theta_2' = 0.03$, $s_{\alpha\beta} = 0.99$, and $m_{H_A} = 500$ GeV, the higher-dimensional global fit is carried out based on the χ^2 function, incorporating all previous measurements. With the best-fit point located at $\chi^2_{\min} = 26.69$, the 2σ and 3σ allowed regions for $(m_{Z'}, m_H, \tan\beta, \text{ and } g')$ are plotted in red and blue, as presented in Fig. 6. We indeed find a proper solution space within FG2HDM to accommodate flavor data studied in this work. Compared with constraints given by single observables, parameter spaces have shifted due to the correlation effect. As displayed in the left panel of Fig. 2, the mass of Z' could be around 220 GeV, while in the combined analysis this value is extended to 450 GeV, as shown in Fig. 6(a). On the other hand, the range for some parameters, like $\tan\beta$, has shrunk significantly in the combined analysis displayed in Fig. 6(e). This further implies that contributions to $b \rightarrow s$, from the gauge and scalar sectors, are in direct competition with one another. Now, we may safely conclude that typical model parameters g' , $m_{Z'}$, and $\tan\beta$ have been found to have their explicit boundaries

$$m_{Z'} < 450 \text{ GeV}, \quad \tan\beta < 28, \quad g' > 10^{-2} \quad (61)$$

at the 3σ level, while the masses of scalars are still less constrained.

The ranges here obtained indirectly from flavor physics are expected to be further improved by a direct search of new particles in colliders, which will be studied separately.

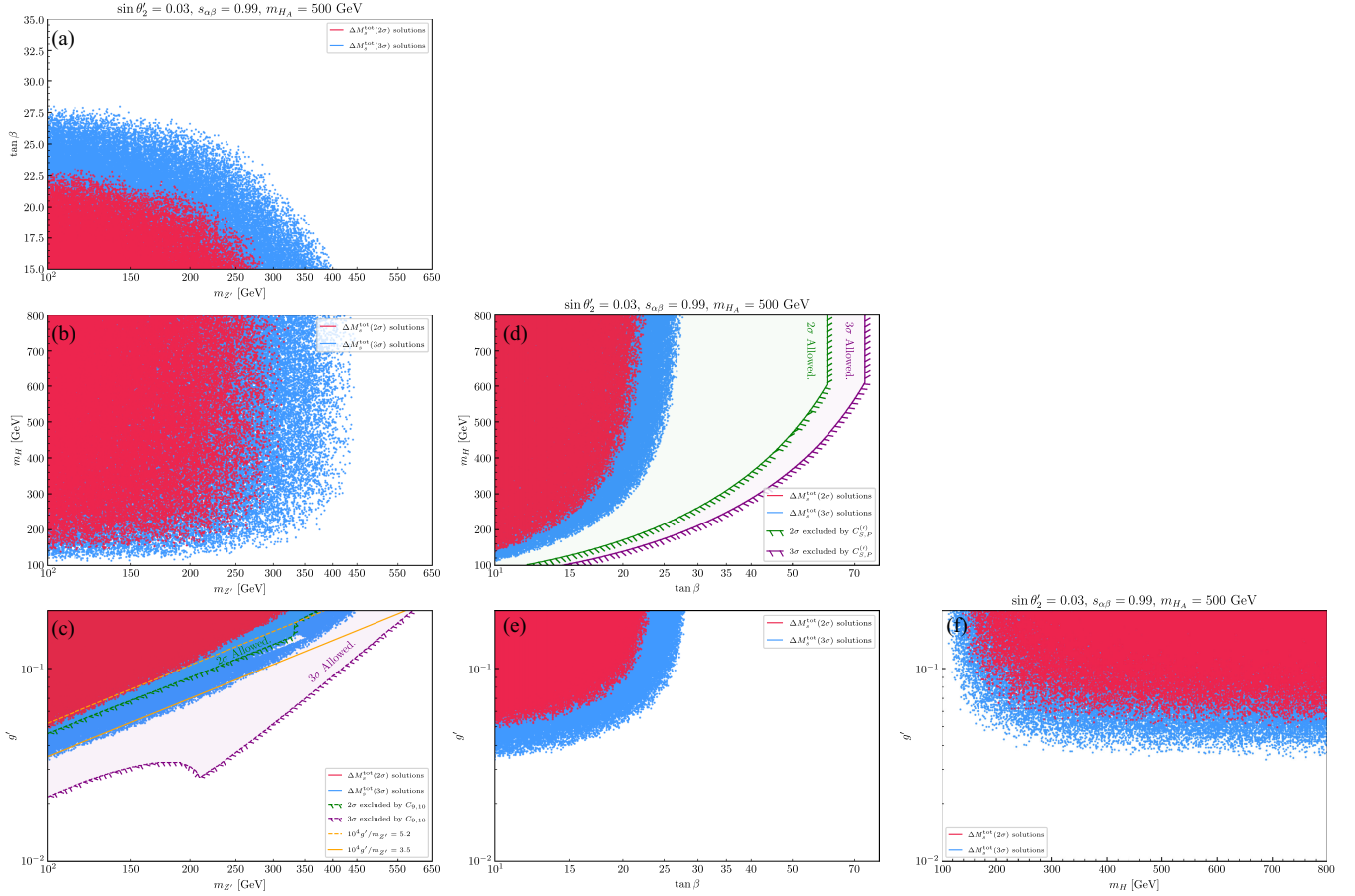


FIG. 6. Solution space constrained by both the scalar sector and gauge sector together. Subplots (a)–(f) present two-dimensional slices of the high-dimensional parameter space, each plotted with respect to a different pair of variables. For example, subplot (a) shows the slice defined by $\tan\beta$ and $m_{Z'}$ within the full parameter space. The regions with green (purple) shade have been explained in Fig. 2 as well as in Fig. 3. The region formed by red (blue) data points indicates the projection of solution space into the corresponding plane at the confidence level of $2(3)\sigma$. These solutions are obtained under the assumption of fixed $\sin\theta_2 = 0.03$, $s_{\alpha\beta} = 0.99$, and $m_{H_A} = 500$ GeV.

V. CONCLUDING REMARKS

In this work, we systematically studied a unique UV-completed two-Higgs-doublet model extended by a scalar singlet. Imposed with a specific $U(1)$ flavor gauge symmetry, the Yukawa matrices were designed with special structures so that tree-level FCNC interactions, controlled by the CKM matrix, occurred uniquely in the down-type quark sector. In the decoupling limit, the charged and neutral scalars, except the superheavy one from the singlet scalar, contributed to processes of interest. The additional neutral gauge boson Z' arising from the gauged $U(1)$ flavor symmetry also induced tree-level FCNC processes in the down-type quark sector.

We investigated the processes $b \rightarrow c\ell\nu$ and $b \rightarrow s\ell^+\ell^-$, where some anomalies still exist, in conjunction with the related and well-measured observable in $B_s^0 - \bar{B}_s^0$ mixing, EWPOs, and Higgs fermionic decays, to further explore the FG2HDM parameter space. We found that:

- (i) The FG2HDM had distinct features compared to the general 2HDM. For example, the identical signs of

$C_{S_L}^{\tau 3}$ and $C_{S_R}^{\tau 3}$ generated in 2HDM-II caused tension between 2HDM-II and $R_{D^{(*)}}$. This tension can be alleviated by flipping one of the signs, which was realized in the FG2HDM by integrating the charged Higgs, as shown in Fig. 1. A combination of the measured branching fraction, $\mathcal{B}(B_u \rightarrow \tau\nu)$, and the conjecture regarding the $B_c \rightarrow \tau\nu$ process indicates that the charged Higgs mass must be heavy.

- (ii) The tree-level down-type quark FCNC interactions can be induced by both the heavy neutral scalar and the neutral gauge boson, with competition between each other. Constraints on individual sectors can be obtained from the data, as presented in Figs. 2 and 3.
- (iii) As illustrated in Figs. 4 and 5, the current constraints imposed by the EW Z -pole observables and the Higgs physics can be simultaneously accommodated by setting a new tiny $U(1)'$ charge and establishing the value of $\tan\beta$ to be approximately $\mathcal{O}(10)$.

- (iv) Correlations established via common model parameters and physical observables shifted the survival parameter space of a single observable. In the comprehensive combined analysis, we obtained allowed parameters in the SM-like conjecture. The upper bound of the Z' mass was found to be 450 GeV, while the upper bound for $\tan\beta$ was 28. On the other hand, the lower bound of the coupling constant of the new $U(1)$ interaction was 10^{-2} , leading to a lower bound on the charge-mass ratio $g'/m_{Z'}$ of around 3.5×10^{-4} .

By simultaneously accommodating the flavor anomalies existing in both $b \rightarrow c\ell\nu$ and $b \rightarrow s\ell^+\ell^-$ processes, the survival parameter space of the FG2HDM was found to be large. Preliminary explorations of the model, focusing on electroweak physics, as well as Higgs boson fermionic decay modes, have been conducted. Meanwhile, the obtained mass bound of Z' requires a further examination from collider physics, which will be discussed separately in further work.

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APPENDIX A: QUANTUM NUMBERS FOR FLAVOR GAUGE SYMMETRY

The structure of Yukawa matrices in Eqs. (28) and (30) is ensured by the subtle symmetry introduced by an additional $U(1)$ transformation. Under this extended $U(1)$ group, the transformation behaviors of all relevant fields are given by

$$\phi \rightarrow \phi' = e^{i\theta X_\phi} \phi, \quad (\text{A1})$$

where the $U(1)$ charges required to maintain the desired Yukawa structures are chosen as follows:

$$\begin{aligned} X_{Q_L} &= \frac{1}{2} \begin{pmatrix} Q_{u_R} + Q_{d_R} & & \\ & Q_{u_R} + Q_{d_R} & \\ & & Q_{t_R} + Q_{d_R} \end{pmatrix}, & X_{u_R} &= \begin{pmatrix} Q_{u_R} & & \\ & Q_{u_R} & \\ & & Q_{t_R} \end{pmatrix}, \\ X_{d_R} &= \begin{pmatrix} Q_{d_R} & & \\ & Q_{d_R} & \\ & & Q_{d_R} \end{pmatrix}, & X_\Phi &= \frac{1}{2} \begin{pmatrix} Q_{u_R} - Q_{d_R} & & \\ & Q_{t_R} - Q_{d_R} & \\ & & \end{pmatrix}, & X_S &= \frac{1}{2} (X_{\Phi_1} - X_{\Phi_2}), \\ X_{L_L} &= \begin{pmatrix} Q_{e_L} & & \\ & Q_{\mu_L} & \\ & & Q_{\tau_L} \end{pmatrix}, & X_{\ell_R} &= \begin{pmatrix} Q_{e_R} & & \\ & Q_{\mu_R} & \\ & & Q_{\tau_R} \end{pmatrix}, & X_{\nu_R} &= 0. \end{aligned} \quad (\text{A2})$$

The $U(1)'$ charge matrices for different types of fermions are given by

$$\begin{aligned} Q_{uL} &= \frac{1}{2} \begin{pmatrix} Q_{u_R} + Q_{d_R} & & \\ & Q_{u_R} + Q_{d_R} & \\ & & Q_{t_R} + Q_{d_R} \end{pmatrix}, \\ Q_{dL} &= \frac{1}{2} (Q_{u_R} + Q_{d_R}) \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} + \frac{1}{2} (Q_{t_R} - Q_{u_R}) \begin{pmatrix} |c_1|^2 & c_1^* c_2 & c_1^* c_3 \\ c_2^* c_1 & |c_2|^2 & c_2^* c_3 \\ c_3^* c_1 & c_3^* c_2 & |c_3|^2 \end{pmatrix}, & Q_{uR} &= \begin{pmatrix} Q_{u_R} & & \\ & Q_{u_R} & \\ & & Q_{t_R} \end{pmatrix}, \\ Q_{dR} &= Q_{d_R} \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}, & Q_{\ell L} &= \begin{pmatrix} Q_{e_L} & & \\ & Q_{\mu_L} & \\ & & Q_{\tau_L} \end{pmatrix}, & Q_{\ell R} &= \begin{pmatrix} Q_{e_R} & & \\ & Q_{\mu_R} & \\ & & Q_{\tau_R} \end{pmatrix}, \\ (Q_{\nu L})_{ij} &= Q_{e_L} V_{ei}^* V_{ej} + Q_{\mu_L} V_{\mu i}^* V_{\mu j} + Q_{\tau_L} V_{\tau i}^* V_{\tau j}, & i, j &= 1, 2, 3, \\ Q_{\nu R} &= 0, \end{aligned} \quad (\text{A3})$$

where $c_i \equiv V_{ti}$, $i = d, s, b$, and $d \equiv U_{uR}|_{33}$. To obtain the above charges coupling to Z' , we have made use of the structures of U_{uL} and U_{uR} ,

$$U_{uL} = \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad U_{uR} = \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (\text{A4})$$

and

$$U_{\ell L} = U_{\ell R} = 1, \quad (\text{A5})$$

which are required by the form of mass matrices. Apparently, FCNC occurs only in the down-type quark sector with left-handed chirality.

The exact charges of specific fermions are constrained by anomaly cancellation conditions, which yield the following relations:

$$\begin{aligned} \mathcal{Q}_{u_R} &= -\mathcal{Q}_{d_R} - \frac{1}{3} \mathcal{Q}_{\mu_R}, & \mathcal{Q}_{t_R} &= -4\mathcal{Q}_{d_R} + \frac{2}{3} \mathcal{Q}_{\mu_R}, \\ \mathcal{Q}_{\tau_L} &= \mathcal{Q}_{d_R} + \frac{1}{6} \mathcal{Q}_{\mu_R}, & \mathcal{Q}_{\mu_L} &= -\mathcal{Q}_{d_R} + \frac{5}{6} \mathcal{Q}_{\mu_R}, \\ \mathcal{Q}_{e_L} &= \frac{9}{2} \mathcal{Q}_{d_R} - \mathcal{Q}_{\mu_R}, & \mathcal{Q}_{\tau_R} &= 2\mathcal{Q}_{d_R} + \frac{1}{3} \mathcal{Q}_{\mu_R}, \\ \mathcal{Q}_{e_R} &= 7\mathcal{Q}_{d_R} - \frac{4}{3} \mathcal{Q}_{\mu_R}. \end{aligned} \quad (\text{A6})$$

The above relations leave only two degrees of freedom,

denoted as $\mathcal{Q}_{d_R}$ and $\mathcal{Q}_{\mu_R}$. The results presented here are consistent with those found in [78] by interchanging e and τ .

APPENDIX B: SUMMARY OF ROTATION MATRIX

The detailed rotation matrices between the mass basis and the interaction basis in Higgs potential are summarized as

$$\begin{aligned} U_1 &\equiv \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}, & U_2 &\equiv \begin{pmatrix} 1 & 0 \\ 0 & U_\gamma \end{pmatrix} \begin{pmatrix} U_1 & 0 \\ 0 & 1 \end{pmatrix}, \\ U_3 &\equiv \begin{pmatrix} -\cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & -\cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}, \end{aligned} \quad (\text{B1})$$

where the 2×2 matrix U_γ and $\tan 2\alpha$ have the explicit forms

$$U_\gamma = \begin{pmatrix} \frac{2v_1 v_2}{\sqrt{4v_1^2 v_2^2 + v^2 v_S^2}} & \frac{-v v_S}{\sqrt{4v_1^2 v_2^2 + v^2 v_S^2}} \\ \frac{v v_S}{\sqrt{4v_1^2 v_2^2 + v^2 v_S^2}} & \frac{2v_1 v_2}{\sqrt{4v_1^2 v_2^2 + v^2 v_S^2}} \end{pmatrix}, \quad \tan 2\alpha = \frac{2l}{m-n}. \quad (\text{B2})$$

The rotation matrices defined in Eq. (15) for explaining the mass diagonalization procedure are

$$U_{AZ} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W & 0 \\ \sin\theta_W & \cos\theta_W & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad U_{Z\hat{Z}'} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta'_2 & -\sin\theta'_2 \\ 0 & \sin\theta'_2 & \cos\theta'_2 \end{pmatrix}, \quad \text{and} \quad \tan 2\theta'_2 = \frac{4|\Delta|}{4\delta - 1}. \quad (\text{B3})$$

APPENDIX C: FURTHER DETAILS ON PHENOMENOLOGY

The coefficients for the $bsZ^{(\prime)}$ and $Z^{(\prime)}\ell\ell$ vertices, extracted from Eq. (18) and utilized in Eq. (40), are summarized as follows:

$$\begin{aligned} \mathcal{A}_L^{bsZ} &= g' \sin\theta'_2 \mathcal{Q}_{dL}, \\ \mathcal{A}_L^{bsZ'} &= g' \cos\theta'_2 \mathcal{Q}_{dL}, \\ \mathcal{B}^{Z\ell\ell} &= \frac{1}{2} \left[\frac{g_2 \cos\theta'_2}{\cos\theta_W} \left(-\frac{1}{2} + 2\sin^2\theta_W \right) + g' \sin\theta'_2 (\mathcal{Q}_{\ell L} + \mathcal{Q}_{\ell R}) \right], \\ \mathcal{B}_5^{Z\ell\ell} &= \frac{1}{2} \left[\frac{g_2 \cos\theta'_2}{\cos\theta_W} \left(-\frac{1}{2} \right) + g' \sin\theta'_2 (\mathcal{Q}_{\ell L} - \mathcal{Q}_{\ell R}) \right], \\ \mathcal{B}^{Z'\ell\ell} &= \frac{1}{2} \left[\frac{-g_2 \sin\theta'_2}{\cos\theta_W} \left(-\frac{1}{2} + 2\sin^2\theta_W \right) + g' \cos\theta'_2 (\mathcal{Q}_{\ell L} + \mathcal{Q}_{\ell R}) \right], \\ \mathcal{B}_5^{Z'\ell\ell} &= \frac{1}{2} \left[\frac{-g_2 \sin\theta'_2}{\cos\theta_W} \left(-\frac{1}{2} \right) + g' \cos\theta'_2 (\mathcal{Q}_{\ell L} - \mathcal{Q}_{\ell R}) \right]. \end{aligned} \quad (\text{C1})$$

Here, it is noteworthy that the FCNC part $\mathcal{A}_L^{bsZ}$ from the SM bsZ vertex is modified by the $U(1)'$ down-type charges due to mixing effects with the bsZ' vertex.

Regarding the bsZ/Z' vertex in $B_s - \bar{B}_s$ oscillations, in the SM only the single operator $\mathcal{O}_{\text{VLL}}$ is present at the loop level. In the FG2HDM, from discussions involving $C_{9,10}$, it is inferred that no right-handed $\mathcal{O}_{\text{VRR}}$ or $\mathcal{O}_{\text{LR}}^1$ operators are generated due to the diagonal $\mathcal{Q}_{dR}$. These matrix elements in ΔM_s can typically be parametrized as bag parameters [126],

$$\begin{aligned}\langle B_s^0 | \mathcal{O}_{\text{VRR}} | \bar{B}_s^0 \rangle &= \langle B_s^0 | \mathcal{O}_{\text{VLL}} | \bar{B}_s^0 \rangle = \frac{1}{3} m_{B_s} f_{B_s}^2 B_1, \\ \langle B_s^0 | \mathcal{O}_{\text{LR1}} | \bar{B}_s^0 \rangle &= -\frac{1}{6} \mathcal{R}_{B_s} f_{B_s}^2 B_5, \\ \langle B_s^0 | \mathcal{O}_{\text{LR2}} | \bar{B}_s^0 \rangle &= \frac{1}{4} \mathcal{R}_{B_s} f_{B_s}^2 B_4, \\ \langle B_s^0 | \mathcal{O}_{\text{SRR1}} | \bar{B}_s^0 \rangle &= \langle B_s^0 | \mathcal{O}_{\text{SLL1}} | \bar{B}_s^0 \rangle = -\frac{5}{24} \mathcal{R}_{B_s} f_{B_s}^2 B_2, \\ \langle B_s^0 | \mathcal{O}_{\text{SRR2}} | \bar{B}_s^0 \rangle &= \langle B_s^0 | \mathcal{O}_{\text{SLL2}} | \bar{B}_s^0 \rangle = -\frac{5}{6} \mathcal{R}_{B_s} f_{B_s}^2 B_2 + \frac{1}{3} \mathcal{R}_{B_s} f_{B_s}^2 B_3,\end{aligned}\tag{C2}$$

where $\mathcal{R}_{B_s} = \frac{m_{B_s}^3}{(\bar{m}_b + \bar{m}_s)^2}$ represents the generic factorized evolution of density-density operators.

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