

Testing the nuclear TMD gluon densities with heavy flavor production in proton-lead collisions at the LHC

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We employ a simple model for nuclear modification of ordinary parton densities in a proton to evaluate the transverse momentum dependent gluon and quark distributions in nuclei (nTMDs) within the popular Kimber-Martin-Ryskin/Watt-Martin-Ryskin approach. The model is based on a global analysis of available deep inelastic scattering data for different nuclear targets within the rescaling model, incorporating Fermi motion effects. The derived nTMDs are tested with the latest CMS data on inclusive b -jet and B^+ meson production in proton-lead collisions collected at $\sqrt{s} = 5.02$ and 8.16 TeV using the high energy factorization framework. We predict the corresponding nuclear medium modification factors to be about of 0.8–1.2 in the probed kinematical region, which is consistent with other estimations. Specially we highlight a possibility to investigate the nuclear modification of parton densities by applying different cuts on the final states in such processes.

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I. INTRODUCTION

It is known that, at present, any theoretical investigation of QCD processes usually relies on a factorization of the effects of short and long distances at some scale. A key ingredient of such consideration is the parton (gluon and quark) distribution functions in a proton (PDFs), $f_a(x, \mu^2)$ with $a = g$ or q and x being the longitudinal momentum fraction of proton carried by parton a , which describe the parton content of a proton at the scale μ^2 . In the conventional QCD factorization, the PDFs obey the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations [1]. At high energies, an alternative description can be achieved within the high energy factorization [2], or k_T -factorization approach [3], which is based on the Balitsky-Fadin-Kuraev-Lipatov [4] or Catani-Ciafaloni-Fiorani-Marchesini [5] evolution equations for transverse momentum dependent (TMD, or unintegrated) gluon densities in a proton, $f_g(x, \mathbf{k}_T^2, \mu^2)$. Such an approach has certain technical advantages in the ease of including higher-order radiative corrections corresponding to the gluon emissions in the initial state (see, for example, reviews [6,7] for more information).

When it comes to nuclei, a number of complications arise. As is known, the nucleus is far from a simple picture

of quasifree nucleons [8–11]. Instead one finds so-called nuclear shadowing, antishadowing, European Muon Collaboration (EMC) effect, and Fermi motion dominance regions at $x \leq 0.1$, $0.1 \leq x \leq 0.3$, $0.3 \leq x \leq 0.7$, and $x \geq 0.7$, respectively. Defining the nuclear medium modification factor as a ratio of per-nucleon deep inelastic structure functions in nuclei A and deuteron $R_{AD} = F_2^A(x, Q^2)/F_2^D(x, Q^2)$, one then can refer to the shadowing and antishadowing effects as having $R_{AD} < 1$ and $R_{AD} > 1$, whereas EMC effect and Fermi motion correspond to a drop of R_{AD} in the valence-dominant region and an increasing of R_{AD} at larger x . So, corresponding perturbative QCD (pQCD) calculations demand nuclear PDFs and/or TMDs (nPDFs or nTMDs), which essentially differ from the usual parton distributions in a proton. Of course, their detailed knowledge—in particular, knowledge of nuclear gluon densities—is necessary for any theoretical description and proper interpretation of pA processes studied at modern (LHC, the relativistic heavy ion collider) and future colliders (Future Circular Collider (FCC-he) [12], Electron-Ion Collider (EIC) [13], Electron-Ion Collider in China (EIC) [14], Nuclotron based Ion Collider Facility (NICA) [15,16]). In turn, pA collisions is a natural reference for more complex nucleus-nucleus (AA) interactions, where nuclear matter can reach extremely high energy densities and temperatures, transforming into its hot and dense deconfined phase, the quark-gluon plasma (QGP) [17,18].

Despite significant theoretical efforts made in recent years, nPDFs and especially nTMDs are still poorly known at present and have large uncertainties due to shortage of experimental data and/or their limited kinematic coverage

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(see [19–21] and references therein). Several approaches could be used to evaluate these objects. In fact, they could be extracted from a global fit to nuclear data and then their scale dependence can be determined as a solution of QCD evolution equations (see, for example, [22–25] for nPDFs and [26] for nTMDs). This technique is in close analogy with the standard derivation of the PDFs and/or TMDs. The second strategy is based on special models [27–32], in particular, the rescaling model [29,30] and approaches where the geometrical scaling [31,32] is applied [33,34]. So, in the rescaling model it is assumed that the effective size of gluon and quark confinement in the nucleus is greater than in the free nucleon, thus giving a shift in the hard scale μ^2 (see also review [35] and Appendix below).

In this note, we concentrate mainly on the nTMD gluon densities and derive them from nPDFs using the popular Kimber-Martin-Ryskin (KMR) [36] or Watt-Martin-Ryskin (WMR) [37] approach. The WMR prescription is an extension and further development of KMR formalism and explored currently at the next-to-leading order [38]. In the KMR/WMR method, the DGLAP strong ordering condition is relaxed at the last step of the parton evolution cascade, so the transverse momentum is no longer negligible compared to the evolution scale μ^2 , thus giving rise to the transverse momentum dependence of the parton densities. Such prescription, where ordinary PDFs are employed as an input for the KMR/WMR procedure, is widely used in phenomenological applications (see, for example, [39–44] and references therein). The KMR/WMR method to generate nTMDs has been used already [45,46]. So, the analytical leading order (LO) expressions [47–49] (see also [50,51]) for PDFs obtained at small x in the generalized double asymptotic scaling approximation [52–54] have been extended to nPDFs using the rescaling model adopted for low x [49] and then applied as an input for the KMR/WMR formulas [46].

Recently, an early derivation [47–49] was significantly improved by taking into account the exact asymptotics at low and large x and by incorporating subasymptotic terms fixed by the momentum conservation and/or Gross-Llewellyn-Smith and Gottfried sum rules [55,56]. Moreover, a simple model for nuclear modification of these improved PDFs has been proposed very recently [57]. The approach [57] is based on the global analysis of available deep inelastic scattering data for different nuclear targets within the low- x enhanced rescaling model combined with the effects of Fermi motion. Here we employ the KMR/WMR formalism to calculate nTMDs within the proposed model [57]. Then, we will test the derived TMDs with inclusive beauty production in pA collisions at the LHC. Such processes are known to be sensitive to the gluon content of the nucleus and provide us with a possibility to reconstruct the full map of the latter (see, for example, [58]). Moreover, they provide a unique opportunity to study the QGP [59,60]. In fact, by studying how the nuclear medium

affects heavy quarks, the properties of the medium can be determined. Using the Monte-Carlo event generator PEGASUS [61], which implements now the TMD gluon dynamics in nuclei, we calculate the transverse momentum and rapidity distributions of b -jets and B^+ mesons and compare our predictions with the latest experimental data [62–64] collected by the CMS Collaboration at $\sqrt{s} = 5.02$ and 8.16 TeV. Of course, these data are of a great importance to test the derived predictions for nTMDs. Then we investigate corresponding effects of nuclear medium modifications in different kinematical regions. The consideration below extends and continues the line of our previous studies [46,49,57].

II. THEORETICAL FRAMEWORK

We start from a very brief description of our approach. In the fixed-flavor-number-scheme (FFNS) with $N_f = 4$, where b and t quarks are separated out, the nonsinglet (NS) and valence (V) parts of quark distribution functions at the LO can be represented in the following form [56]:

$$f_i(x, \mu^2) = \left[A_i(s) x^{\lambda_i} (1-x) + \frac{B_i(s)x}{\Gamma(1+\nu_i(s))} + D_i(s)x(1-x) \right] \times (1-x)^{\nu_i(s)}, \quad (1)$$

where $i = \text{NS or V}$, $s = \ln[\alpha_s(\mu_0^2)/\alpha_s(\mu^2)]$ and

$$\begin{aligned} A_i(s) &= A_i(0)e^{-d(n_i)s}, & B_i(s) &= B_i(0)e^{-ps}, \\ \nu_i(s) &= \nu_i(0) + rs, & r &= \frac{16}{3\beta_0}, & p &= r(\gamma_E + \hat{c}), \\ \hat{c} &= -\frac{3}{4}, & d(n) &= \frac{\gamma_{\text{NS}}(n)}{2\beta_0}, & n_i &= 1 - \lambda_i. \end{aligned} \quad (2)$$

Here $\Gamma(z)$ is the Riemann's Γ function, $\gamma_E \simeq 0.5772$ is the Euler's constant, $\beta_0 = 11 - 2N_f/3$ is the LO QCD β function, $\lambda_{\text{NS}} = \lambda_V = 0.5$, $\gamma_{\text{NS}}(n)$ is the LO nonsinglet anomalous dimension, and $A_i(0)$, $B_i(0)$, and $\nu_i(0)$ are the free parameters. The term proportional to $D_i(s)$ is a subasymptotics one and its scale dependence is fixed by the Gross-Llewellyn-Smith and Gottfried sum rules [56],

$$D_i(s) = (2 + \nu_i(s)) \left[N_i - A_i(s) \frac{\Gamma(\lambda_i)\Gamma(2 + \nu_i(s))}{\Gamma(\lambda_i + 2 + \nu_i(s))} - \frac{B_i(s)}{\Gamma(2 + \nu_i(s))} \right], \quad (3)$$

where $N_V = 3$ and $N_{\text{NS}} \equiv I_G(\mu^2) = 0.705 \pm 0.078$ (see [56] and references therein for more information). The singlet (SI) part of quark densities and gluon distribution in a proton can be represented as combinations of “ $\pm$ ” terms (see also [47–49]),

$$f_i(x, \mu^2) = f_i^+(x, \mu^2) + f_i^-(x, \mu^2), \quad (4)$$

where $i = SI$ or g and

$$f_{SI}^+(x, \mu^2) = \left[\frac{N_f}{9} \left(A_g + \frac{4}{9} A_q \right) \rho I_1(\sigma) e^{-\bar{d}^+ s} (1-x)^{m_q^+} + D^+(s) \sqrt{x} (1-x)^{n^+} - \frac{K^+}{\Gamma(2 + \nu^+(s))} \times \frac{B^+(s)x}{\hat{c} - \ln(1-x) + \Psi(2 + \nu^+(s))} \right] (1-x)^{\nu^+(s)+1}, \quad (5)$$

$$f_{SI}^-(x, \mu^2) = \left[A_q e^{-d^- s} (1-x)^{m_q^-} + \frac{B^-(s)x}{\Gamma(1 + \nu^-(s))} + D^-(s) \sqrt{x} (1-x)^{n^-} \right] (1-x)^{\nu^-(s)}, \quad (6)$$

$$f_g^+(x, \mu^2) = \left[\left(A_g + \frac{4}{9} A_q \right) I_0(\sigma) e^{-\bar{d}^+ s} (1-x)^{m_g^+} + \frac{B^+(s)x}{\Gamma(1 + \nu^+(s))} \right] (1-x)^{\nu^+(s)}, \quad (7)$$

$$f_g^-(x, \mu^2) = \left[-\frac{4}{9} A_q e^{-d^- s} (1-x)^{m_g^-} + \frac{K^-}{\Gamma(2 + \nu^-(s))} \times \frac{B^-(s)x}{\hat{c} - \ln(1-x) + \Psi(2 + \nu^-(s))} \right] (1-x)^{\nu^-(s)+1}. \quad (8)$$

Here $\Psi(z)$ is the Riemann's Ψ function, $I_0(z)$ and $I_1(z)$ are the modified Bessel functions and

$$\begin{aligned} \nu^\pm(s) &= \nu^\pm(0) + r^\pm s, & B^\pm(s) &= B^\pm(0) e^{-p^\pm s}, \\ p^\pm &= r^\pm (\gamma_E + \hat{c}^\pm), & r^+ &= \frac{12}{\beta_0}, & r^- &= \frac{16}{3\beta_0}, \\ \hat{c}^+ &= -\frac{\beta_0}{12}, & \hat{c}^- &= -\frac{3}{4}, & K^+ &= \frac{3N_f}{10}, \\ K^- &= \frac{2}{5}, & \rho &= \frac{\sigma}{2\ln(1/x)}, & \sigma &= 2\sqrt{|\hat{d}^+| s \ln \frac{1}{x}}, \\ \hat{d}^+ &= -\frac{12}{\beta_0}, & \bar{d}^+ &= 1 + \frac{20N_f}{27\beta_0}, & d^- &= \frac{16N_f}{27\beta_0} \end{aligned} \quad (9)$$

with A_g , A_q , $B^\pm(0)$, $\nu^\pm(0)$, $m_q^\pm$, $m_g^\pm$, and $n^\pm$ being free parameters. The expressions for subasymptotic terms $D^\pm(s)$ were derived from the momentum conservation law and could be found elsewhere [56]. All the parameters in the formulas above have been determined from a rigorous fit to precision BCDMS, H1, and ZEUS experimental data on the proton structure function $F_2(x, Q^2)$ in a wide kinematical region, $2 \times 10^{-5} \leq x \leq 0.75$ and $1.2 \leq Q^2 \leq 30000 \text{ GeV}^2$ [56]. The fit was comprising a total of 933 points from five datasets. A rather good value of $\chi^2/\text{d.o.f.} = 1.408$ (d.o.f. means degrees of freedom) has been achieved. For the reader's convenience, we collect the

results of the fit in Table 1 (see [56] for more detailed information).

Our next step is connected with nuclear modifications of derived PDFs. The combination of the rescaling model extended to low x (or shadowing region) and Fermi motion was applied [57]. So, for a nucleus A , the valence and nonsinglet parts are modified as,

$$f_i^A(x, \mu^2) = f_i(x, \mu_{A,i}^2), \quad (10)$$

where $i = V$ or NS and scale $\mu_{A,i}^2$ is related to μ^2 by,

$$s_i^A \equiv \ln \left(\frac{\ln(\mu_{A,i}^2/\Lambda_{\text{QCD}}^2)}{\ln(\mu_0^2/\Lambda_{\text{QCD}}^2)} \right) = s + \ln(1 + \delta_i^A), \quad (11)$$

with δ_i^A being the scale independent free parameters (see [49] and Appendix below) and analytical expressions for $f_i(x, \mu^2)$ are given by (1)–(3). For singlet quark and gluon parts, which have two (“+” and “−”) independent components, one has two additional rescaling parameters $\delta_\pm^A$ (see [49]). So,

$$\begin{aligned} f_i^A(x, \mu^2) &= f_i^{A,+}(x, \mu^2) + f_i^{A,-}(x, \mu^2), \\ f_i^{A,\pm}(x, \mu^2) &= f_i^\pm(x, \mu_{A,\pm}^2), \end{aligned} \quad (12)$$

where $i = SI$ or g and expressions for $f_i^\pm(x, \mu^2)$ are given by (4)–(9). The definition of $\mu_{A,\pm}^2$ is the same as above and

TABLE I. Fitted values of various phenomenological parameters involved in our analytical expressions for PDFs and TMD parton densities in a proton.

A_q	A_g	$A_V(0)$	$B_V(0)$	$A_{NS}(0)$	$B_{NS}(0)$
0.75 ± 0.02	1.13 ± 0.01	2.82 ± 0.01	$(1.06 \pm 0.05) \times 10^3$	0.279 ± 0.006	$(1.19 \pm 0.04) \times 10^2$
$B^+(0)$	$B^-(0)$	$\nu_V(0)$	$\nu_{NS}(0)$	$\nu^-(0)$	n^-
$(6.2 \pm 0.5) \times 10^1$	$(2.27 \pm 0.10) \times 10^2$	4.05 ± 0.02	3.23 ± 0.01	3.54 ± 0.02	4.66 ± 0.05

corresponding values of $s_{\pm}^A$ turned out to be [49]

$$s_{\pm}^A \equiv \ln \left(\frac{\ln(\mu_{A,\pm}^2/\Lambda_{\text{QCD}}^2)}{\ln(\mu_0^2/\Lambda_{\text{QCD}}^2)} \right) = s + \ln(1 + \delta_{\pm}^A), \quad (13)$$

where the free parameters $\delta_{\pm}^A$ are scale independent and have to be negative (see Appendix below).

The full model [57] takes into account the effects of Fermi motion of nucleons inside the nuclear target. These effects deform the nuclear structure functions mainly at large $x > 0.7$. Such deformation could be described by the convolution [65,66],

$$f_i^{A(F)}(x, \mu^2) = \frac{1}{R_i} \int \frac{dy}{y} [y f_N(y)] f_i^A(x/y, \mu^2), \quad (14)$$

where $i = V, NS, SI$ or g . The expressions for $f_N(y)$ and R_i can be found elsewhere [57]. The rescaling parameters δ_V^A , δ_{NS}^A , and $\delta_{\pm}^A$ have been extracted [57] from the global fit on experimental data on structure function ratios $F_2^A(x, Q^2)/F_2^{A'}(x, Q^2)$ for various nuclear targets A and A' (namely, ${}^4\text{He}$, ${}^6\text{Li}$, ${}^9\text{Be}$, ${}^{12}\text{C}$, ${}^{27}\text{Al}$, ${}^{40}\text{Ca}$, ${}^{56}\text{Fe}$, ${}^{64}\text{Cu}$, ${}^{118}\text{Sn}$, and ${}^{208}\text{Pb}$, see [57] for more detailed information) in the deep inelastic region. Moreover, their nuclear dependence was investigated and parametrized in several ways, which are often assumed in the literature (see, for example, [24,67–73] and references therein): $\delta_i^A \sim A^{1/3}$ (Fit A), $\delta_i^A \sim \ln A$ (Fit B), and $\delta_i^A \sim A^{1/3} + A^{-1/3}$ (Fit C). Thus, within the model [57], the nuclear medium modification factor R can be calculated for any nucleus, even for unmeasured ones.

Now we are able to calculate the nTMDs using the approach [57] and LO KMR/WMR formalism. There are known differential and integral definitions of the latter (see discussions [74–77]). Here we employ the integral definition, which is more preferable¹ in the phenomenological applications [74]. In this way, the nTMDs could be written as

$$f_a^A(x, \mathbf{k}_T^2, \mu^2) = T_a(\mathbf{k}_T^2, \mu^2) \frac{\alpha_s(\mathbf{k}_T^2)}{2\pi\mathbf{k}_T^2} \sum_b \int_x^{1-\Delta} dz P_{ab}(z) \times f_b^{A(F)}\left(\frac{x}{z}, \mathbf{k}_T^2\right), \quad (15)$$

where $a = q$ or g , $P_{ab}(z)$ are the usual unregulated LO DGLAP splitting functions and Δ is some cutoff parameter. Usually it has one of two physically motivated forms, $\Delta = \Delta_1 = |\mathbf{k}_T|/\mu$ and $\Delta = \Delta_2 = |\mathbf{k}_T|/(|\mathbf{k}_T| + \mu)$, which reflects the strong ordering or angular ordering (AO) conditions for parton emissions at the last evolution step. The Sudakov form factors $T_a(\mathbf{k}_T^2, \mu^2)$ give the probability

of evolving from a scale $\mathbf{k}_T^2$ to a scale μ^2 without parton emission,

$$\ln T_a(\mathbf{k}_T^2, \mu^2) = - \int_{\mathbf{k}_T^2}^{\mu^2} \frac{d\mathbf{p}_T^2}{\mathbf{p}_T^2} \frac{\alpha_s(\mathbf{p}_T^2)}{2\pi} \sum_b \int_0^{1-\Delta} dz z P_{ba}(z). \quad (16)$$

The analytical expressions for $T_a(\mathbf{k}_T^2, \mu^2)$ can be found in [78]. It is important to note that the KMR/WMR prescription is only correct at $\mathbf{k}_T^2 > \mu_0^2$, where $\mu_0^2 \sim 1 \text{ GeV}^2$ is the minimum scale where the perturbative QCD is still applicable. At low $\mathbf{k}_T^2$ special model assumptions, usually related with the overall normalization of the TMDs/nTMDs, are necessary (see [36,37,74,77]). However, in our consideration, these parton densities are well defined in the whole $\mathbf{k}_T^2$ range since “frozen” treatment of the QCD coupling in the infrared region is applied, where $\alpha_s(\mu^2) \rightarrow \alpha_s(\mu^2 + M_\rho^2)$ with $M_\rho \sim 1 \text{ GeV}$. Such treatment was used in all the fits [56,57] and results in a good description of the data on structure function $F_2(x, Q^2)$ and $F_2^A(x, Q^2)$, or rather their ratios.

In Fig. 1 we show the nTMD gluon distributions in ${}^{197}\text{Au}$ and ${}^{208}\text{Pb}$ calculated as functions of $\mathbf{k}_T^2$ for several values of x and scale μ^2 using mentioned above scenarios for nuclear dependence, Fits A, B, and C (see also [57]). These results were obtained with the angular and strong ordering conditions. As it was expected, the nTMDs extend into the $\mathbf{k}_T^2 > \mu^2$ region in the case of angular ordering and vanish at large $\mathbf{k}_T^2$ in the case of strong ordering condition. We find that different assumptions on the A dependence of rescaling parameters have only weak impact on predicted nTMDs. In fact, results of calculations based on Fits A, B, and C are very close to each other in a wide kinematical range.

III. NUMERICAL RESULTS

As has already been noted above, here we will test the obtained nTMDs with the data on beauty production in proton-lead collisions at the LHC. We strictly follow the previous evaluations [46,79] done in the high energy factorization [2], or k_T -factorization [3] approach and employ the FFNS scheme with $N_f = 4$. In such a way, the main contribution to beauty production comes from the off-shell gluon-gluon fusion subprocess $g^* + g^* \rightarrow b + \bar{b}$, which is implemented into the Monte-Carlo event generator PEGASUS [61]. The contribution from the quark-induced processed is of almost no importance because of comparatively low quark densities. The cross section is given by the convolution of the TMD gluon densities in a proton and/or nuclei and off-shell (dependent on the transverse momenta of incoming partons) hard scattering amplitudes (see [6,7] for more information). Note that Monte Carlo generator PEGASUS was used, in particular, by the ATLAS [80,81] and ALICE [82,83] Collaborations in

¹The analytical expressions for TMDs were obtained [56] in both differential and integral definitions of the KMR/WMR approach.

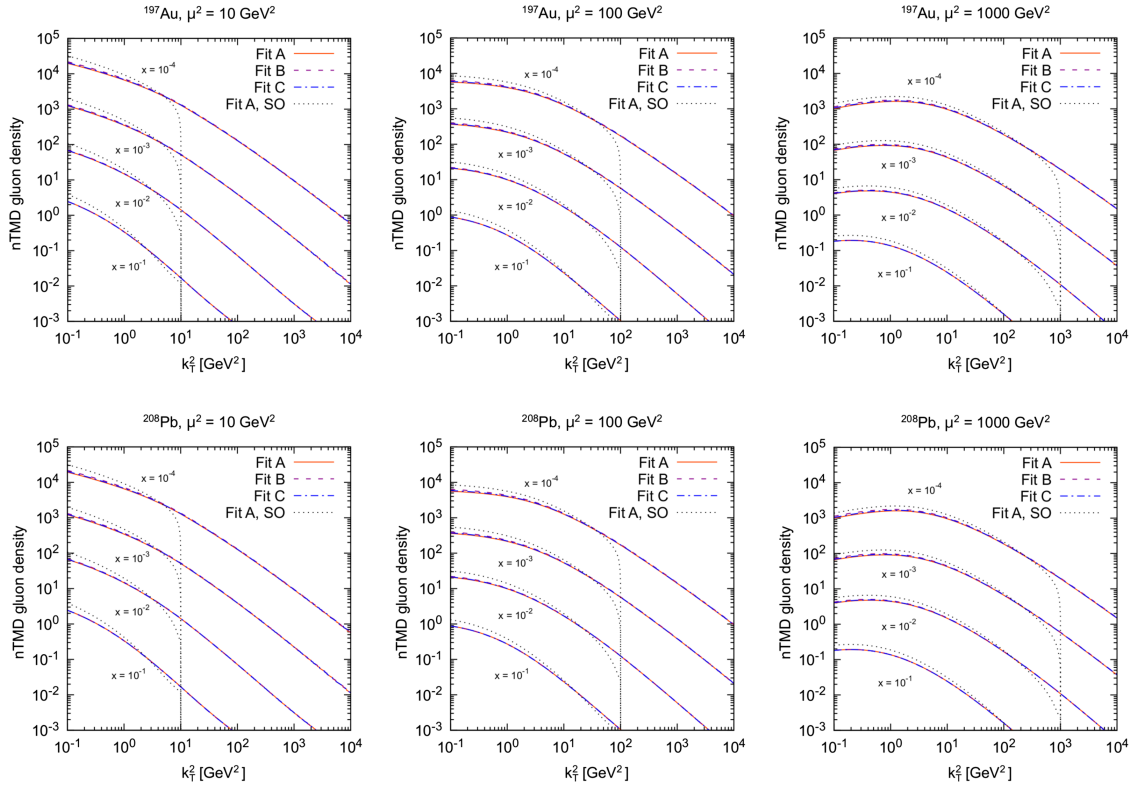


FIG. 1. nTMD gluon densities in ^{197}Au and ^{208}Pb calculated as functions of k_T^2 for several x and μ^2 using the angular ordering and strong ordering conditions. Different scenarios for their nuclear dependence, namely, Fits A, B, and C, described in the text (see also [57]), are applied.

the analyses of their data on heavy quark bound states production at the LHC.

We consider the latest measurements of inclusive b -jet and B^+ meson production performed by the CMS Collaborations at $\sqrt{s} = 5.02$ and 8.16 TeV [62–64]. The CMS data on b -jet transverse momentum spectra [62] refer to the transverse momentum range $55 < p_T < 400$ GeV and were obtained at $\sqrt{s} = 5.02$ TeV in four different proton-lead center-of-mass pseudorapidity subdivisions, namely, $-2.5 < \eta_{\text{c.m.}} < -1.5$, $-1.5 < \eta_{\text{c.m.}} < -0.5$, $-0.5 < \eta_{\text{c.m.}} < 0.5$, and $0.5 < \eta_{\text{c.m.}} < 1.5$. Note that the positive rapidities refer to the beam orientation where the proton is moving toward the positive z direction. The transverse momentum and rapidity distributions of B^+ mesons have been measured at $10 < p_T < 60$ GeV, $|y_{\text{lab}}| < 2.4$ and $\sqrt{s} = 5.02$ TeV [63]. Very recently, first data on B^+ meson production at $\sqrt{s} = 8.16$ TeV have been reported in the kinematical range $3 < p_T < 50$ GeV and $|y_{\text{lab}}| < 1.8$ [64]. In the present calculations, we identify the produced b quark with the beauty jet. To convert b quarks to B^+ mesons, we use the standard approximation where direction of b -hadrons coincides with direction of b quarks, so that the QCD evolution of corresponding fragmentation function can be described by the DGLAP equations. Numerically, we apply standard Peterson fragmentation function with shape parameter $\epsilon_b = 0.003$, which is often

used in the pQCD calculations. We set $m(b) = 4.183$ GeV, $m(B^+) = 5.279$ GeV and branching ratios $B(b \rightarrow B^+) = 0.408$ [84]. Following to fits [56,57], the calculations were made with the one-loop formula for QCD coupling α_s with $\Lambda_{\text{QCD}}^{(4)} = 118$ MeV.

Our predictions are shown in Figs. 2 and 3. In Fig. 2 we plot the transverse momentum spectra of inclusive b -jet production compared to the CMS data [62], normalized double differential cross sections as functions of longitudinal momentum fractions x_p and x_A (carried by the gluons originated from proton and nuclear target, respectively) and corresponding nuclear modification factors $R_{\text{pPb}}^{b\text{-jet}} = 1/A(d\sigma_{\text{pPb}}^{b\text{-jet}}/dp_T)/(d\sigma_{pp}^{b\text{-jet}}/dp_T)$, where $A = 208$ is the number of nucleons in the Pb nucleus. The results of calculations for B^+ meson production are shown in Fig. 3. The AO condition is applied everywhere and calculations are done in the each of pseudorapidity subdivisions considered in the CMS analysis. Solid, dashed and dash-dotted histograms corresponds to the central results for different scenarios of nuclear dependence, where we fixed both renormalization and factorization scales at their default values, $\mu_R^2 = \mu_F^2 = m_T^2$ with m_T being the transverse mass of leading b -jet. The shaded bands correspond to scale uncertainties of the calculations, shown for Fit A scenario. These uncertainties have been estimated in a

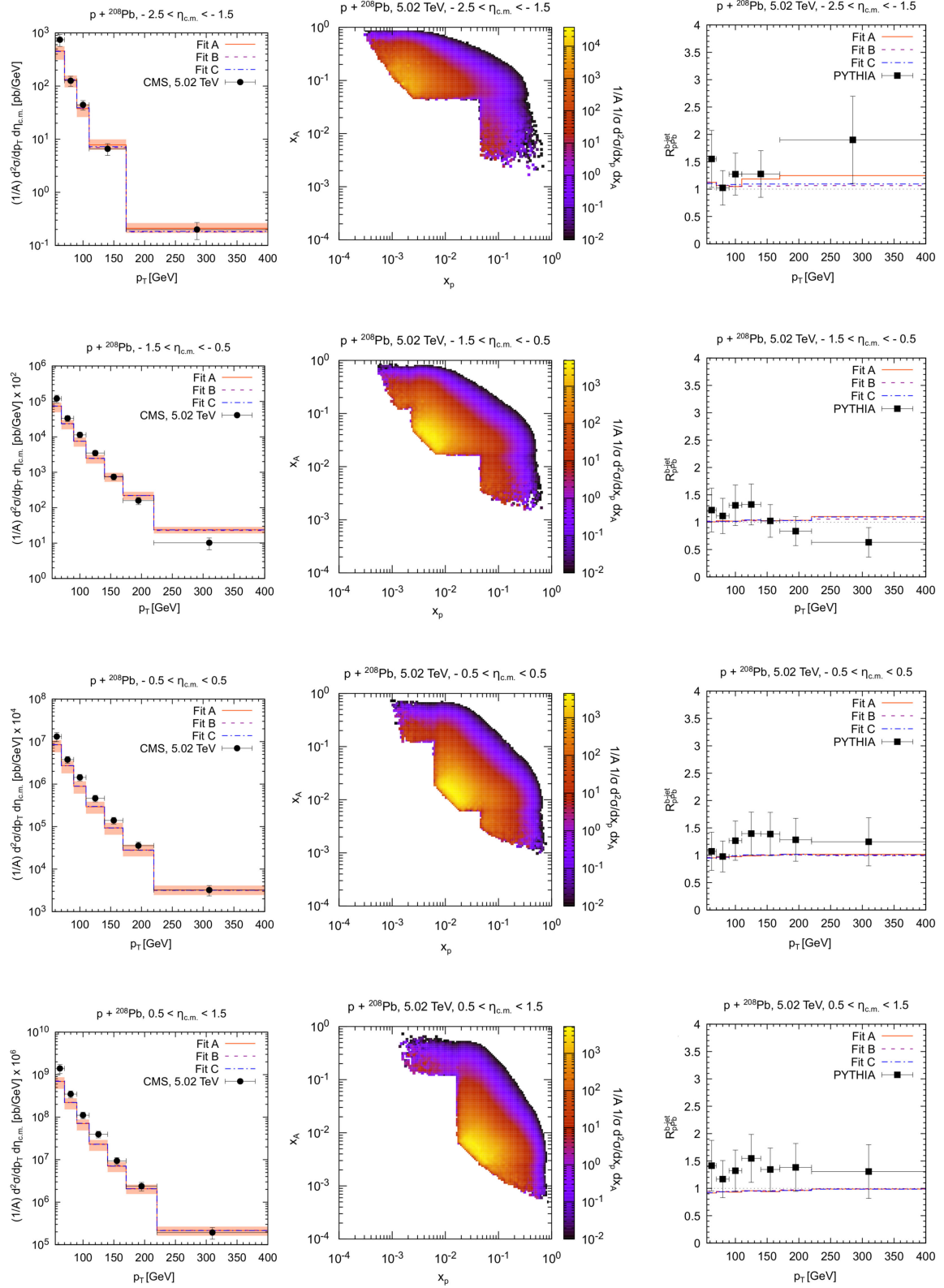


FIG. 2. Transverse momentum spectra of inclusive b -jet production compared to CMS data [62] (left panels), normalized double cross sections as functions of longitudinal momentum fractions of proton, x_p , and nucleus, x_A (center panels), and nuclear modification factors $R_{pPb}^{b\text{-jet}}$ (right panels). Different scenarios for their nuclear dependence, namely, Fits A, B, and C, described in the text (see also [57]), with AO condition are applied. The uncertainty band is shown for Fit A results only.

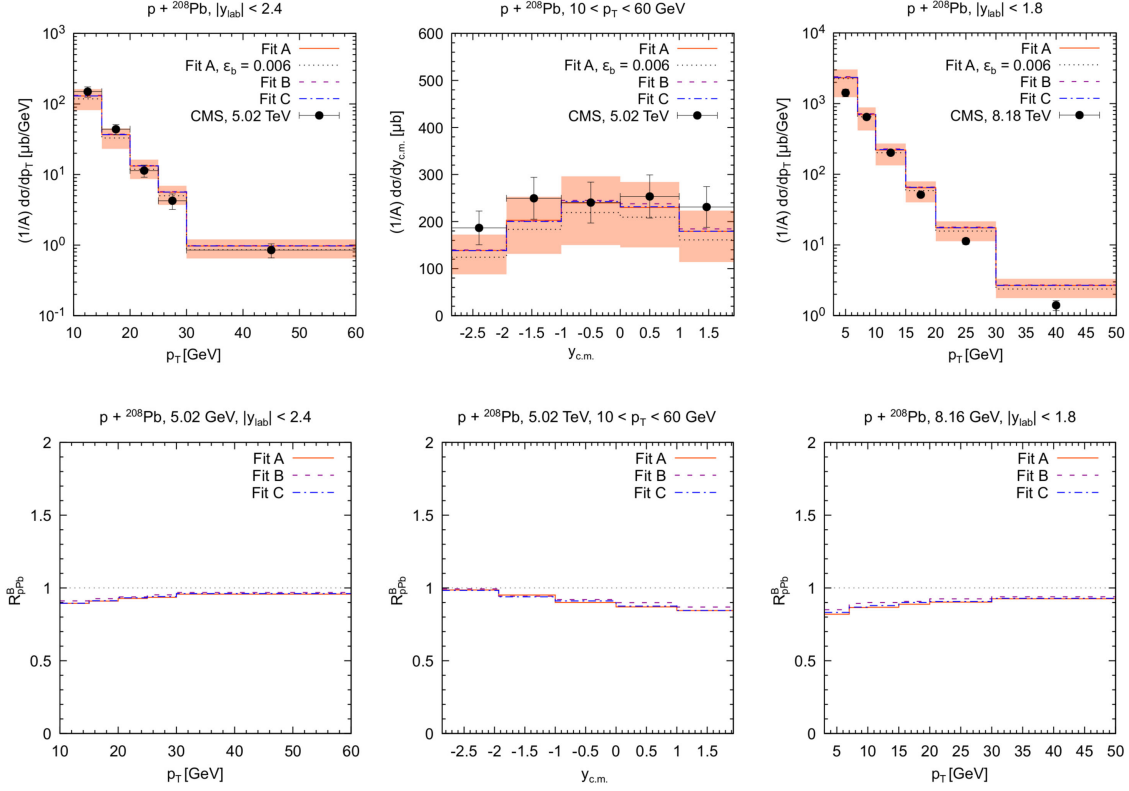


FIG. 3. Transverse momentum and rapidity spectra of inclusive B^+ -jet production compared to CMS data [63,64] (upper panels) and corresponding nuclear modification factors $R_{pPb}^{B^+}$ (lower panels). Different scenarios for their nuclear dependence, namely, Fits A, B and C, described in the text (see also [57]), with AO condition are applied. The uncertainty band is shown for Fit A results only.

standard way, by varying the scales μ_R and μ_F by a factor of 2 around their default values. We find that our predictions based on Fit A, Fit B and Fit C parametrizations of nuclear dependence are rather close to each other and consistent with the data within the theoretical and experimental uncertainties. However, overall agreement achieved in backward kinematical region (negative $\eta_{\text{c.m.}}$) is somewhat better than in the forward one. In fact, our results tend to slightly underestimate the CMS data at forward rapidities. It is clear that the main contribution in this region comes from events where essentially a small- x_A region of nuclear target A is probed (see Fig. 2). Thus, the observed difference may indicate that the nuclear shadowing effects, which manifest themselves mainly at low x_A , could be overestimated in our analysis. Another point is related with effects of parton showers and/or hadronization in final state. However, it is difficult to give a definite conclusion due to relatively large theoretical uncertainties of our calculations and shortage of the nuclear experimental data used in the determination of rescaling parameters (see also [57]). Moreover, additional higher-order pQCD contributions (not covered by the noncollinear pQCD evolution encoded in the TMDs/nTMDs) also could play a role (see, for example, [85,86] and references therein). An accurate addressing of all of these issues needs a dedicated study and is out of the present consideration.

It was already pointed out that special kinematical cuts on the final state provide possibilities to achieve the wanted region of x_A [58]. So, with increasing $\eta_{\text{c.m.}}$ the peak of the normalized cross section $1/\sigma d^2\sigma/dx_p dx_A$ shifts gradually to lower x_A values, which immediately reflects in the predicted nuclear medium modification factor $R_{pPb}^{b\text{-jet}}$ (see Fig. 2). In particular, our calculations show sizeable anti-shadowing and shadowing effects with $R_{pPb}^{b\text{-jet}} \sim 1.1\text{--}1.2$ and $R_{pPb}^{b\text{-jet}} \sim 0.9\text{--}1.0$ in the backward and forward kinematical regions, respectively. Similar values are obtained for B^+ mesons (see Fig. 3). Thus, we demonstrate that one can investigate the nuclear medium effects by applying the different cuts on the final states produced in the high energy pA collisions. This is important for a precise determination of the partonic content of nuclei, where there are a lot of differences among predictions from different groups.²

Finally, to investigate the dependence of our predictions on the $b \rightarrow B^+$ fragmentation function, we repeated the calculations with the shifted value of the Peterson shape

²The pseudorapidity-integrated value predicted by the PYTHIA event generator [87] is $R_{pPb}^{b\text{-jet}} = 1.22 \pm 0.15$ (stat. + syst. exp.) ± 0.27 (syst. PYTHIA), see [62]. To compare with our results, in Fig. 2 we show the PYTHIA predictions (taken from [62]) separately for each rapidity subdivision.

parameter, $\epsilon_b = 0.006$. The results of our calculations are shown in Fig. 3 by dotted histograms. We find that the obtained cross sections are somewhat smaller for larger ϵ_b values. However, these predictions lie within the estimated theoretical uncertainties.

IV. CONCLUSION

We have evaluated nTMDs at the leading order in QCD coupling using the popular Kimber-Martin-Ryskin/Watt-Martin-Ryskin approach. The calculations have been based on a rescaling model taking into account Fermi motion effects and global analysis of available deep inelastic scattering data for different nuclear targets. The obtained nTMDs have been successfully tested with the latest CMS data on beauty production in proton-lead collisions collected at $\sqrt{s} = 5.02$ and 8.16 TeV. We predict the corresponding nuclear medium modification factors to be about 0.8–1.2. Specially we have highlighted the possibility to investigate the nuclear modification of parton densities by applying different cuts on the final states produced in the pA collisions. It can be important for forthcoming studies of lepton-nucleus, proton-nucleus and nucleus-nucleus interactions at modern and future colliders, where nuclear parton dynamics could be examined directly.

Note that the consideration can be improved by taking into account effects of parton showers and/or higher-order pQCD corrections. We plan to investigate these issues in ensuing studies.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [57–59].

APPENDIX: RESCALING MODEL

Here we present the basic elements of the rescaling model [29,30]. We are following the recent preprints [35,49]. Consider first the solution for the Mellin moments of the nonsinglet (NS) parton density at LO³

$$\begin{aligned} M_n^{\text{NS}}(Q^2) &= M_n^{\text{NS}}(Q_0^2) \left[\frac{a_s(Q_0^2)}{a_s(Q^2)} \right]^{d_n^{\text{NS}}} \\ &= M_n^{\text{NS}}(Q_0^2) \left[\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)} \right]^{d_n^{\text{NS}}}, \\ d_n^{\text{NS}} &= \frac{\gamma_n^{\text{NS}}}{2\beta_0}, \end{aligned} \quad (\text{A1})$$

where γ_n^{NS} is the LO NS anomalous dimension.

The function $\ln(M_n^{\text{NS}}(Q^2))$ depends on $\ln[a_s(Q_0^2)/a_s(Q^2)] \equiv s$ in the form of a straight line with the slope d_n^{NS} . Experimental data confirm the behavior for each nuclei A and, moreover, $M_n^A(Q^2) < M_n^{A'}(Q^2)$ for $A' > A$. Changing $Q^2 \rightarrow Q_{A'}^2(n) \equiv \xi_n^{AA'}(Q^2)Q^2$, we can obtain

$$M_n^{A'}(Q^2) < M_n^A(\xi_n^{AA'}(Q^2)Q^2) \quad (\text{A2})$$

with $\xi_n^{AA'}(Q^2) > 1$ for $A' > A$ in the NS case. Then one could easily derive

$$\begin{aligned} M_n^A(\xi_n^{AA'}(Q^2)Q^2) \\ = M_n^A(\xi_n^{AA'}(Q_0^2)Q_0^2) \left[\frac{a_s(\xi_n^{AA'}(Q_0^2)Q_0^2)}{a_s(\xi_n^{AA'}(Q^2)Q^2)} \right]^{d_n^{\text{NS}}}, \end{aligned} \quad (\text{A3})$$

which is equal to

$$M_n^{A'}(Q^2) = M_n^{A'}(Q_0^2) \left[\frac{a_s(Q_0^2)}{a_s(Q^2)} \right]^{d_n^{\text{NS}}}. \quad (\text{A4})$$

Comparing last two equations, we obtain

$$\frac{a_s(\xi_n^{AA'}(Q^2)Q^2)}{a_s(\xi_n^{AA'}(Q_0^2)Q_0^2)} = \frac{a_s(Q^2)}{a_s(Q_0^2)}. \quad (\text{A5})$$

Since $a_s(\xi Q^2) = a_s(Q^2)/(1 + \beta_0 a_s(Q^2) \ln \xi)$ at LO, we have

$$\frac{1 + \beta_0 a_s(Q_0^2) \ln \xi_n^{AA'}(Q_0^2)}{1 + \beta_0 a_s(Q^2) \ln \xi_n^{AA'}(Q^2)} = 1, \quad (\text{A6})$$

i.e., $a_s(Q_0^2) \ln \xi_n^{AA'}(Q_0^2) = a_s(Q^2) \ln \xi_n^{AA'}(Q^2)$, or

$$\xi_n^{AA'}(Q^2) = \xi_n^{AA'}(Q_0^2)^{a_s(Q_0^2)/a_s(Q^2)}. \quad (\text{A7})$$

A very important observation was made [30] (see also [29]): $\xi_n^{AA'}(Q^2)$ is independent of n , i.e., $\xi_n^{AA'}(Q^2) = \xi^{AA'}(Q^2)$. Thus, it is the same for all Mellin moments and even for the parton density itself, i.e., we have in the NS case

$$f_A^{\text{NS}}(x, Q^2) = f_A^{\text{NS}}(x, \xi^{AA'}(Q^2)Q^2), \quad (\text{A8})$$

³The results for the valence and NS parts are very similar. Therefore, from now on, we will consider only the NS part.

that is, the parton densities in the nucleus and the nucleon are related in a very simple way within the framework of the scaling model.

Note that the property (A7) is not so convenient for applications. It can be simplified (see [49]). Indeed, let us consider $\ln[(\xi^{AA'}(Q^2)Q^2)/\Lambda^2] = \ln[(\xi^{AA'}(Q^2))] + \ln[Q^2/\Lambda^2]$, where

$$\begin{aligned} \ln[(\xi^{AA'}(Q^2))] &= \frac{a_s(Q_0^2)}{a_s(Q^2)} \ln[(\xi^{AA'}(Q_0^2))] \\ &= \frac{\ln[Q^2/\Lambda^2]}{\ln[Q_0^2/\Lambda^2]} \ln[(\xi^{AA'}(Q_0^2))]. \end{aligned} \quad (\text{A9})$$

So, we have

$$\begin{aligned} \ln[(\xi^{AA'}(Q^2)Q^2)/\Lambda^2] &= \ln[Q^2/\Lambda^2] \left[1 + \frac{\ln[(\xi_n^{AA'}(Q_0^2))]}{\ln[Q_0^2/\Lambda^2]} \right] \\ &= \ln[Q^2/\Lambda^2] (1 + \delta^{AA'}), \end{aligned} \quad (\text{A10})$$

where

$$\delta^{AA'} = \frac{\ln[(\xi_n^{AA'}(Q_0^2))]}{\ln[Q_0^2/\Lambda^2]} = \beta_0 a_s(Q_0^2) \ln[(\xi_n^{AA'}(Q_0^2))] \quad (\text{A11})$$

is Q^2 independent.

The coefficient $\delta^{AA'}$ combines the normalization of the strong coupling $a_s(Q_0^2)$ and shift $\xi_n^{AA'}(Q_0^2)$ of the normalization argument when we move from A kernels to A' kernels. In a way, this is very close to introducing the QCD Λ parameter:

$$\begin{aligned} a_s(Q^2) &= \frac{a_s(Q_0^2)}{1 + \beta_0 a_s(Q_0^2) \ln(Q^2/Q_0^2)} \\ &= \frac{1}{\beta_0 a_s(Q_0^2) \ln(Q^2/\Lambda^2)}, \end{aligned} \quad (\text{A12})$$

which takes together the normalization of the strong coupling $a_s(Q_0^2)$ and normalization argument Q_0^2 .

The variable $s^{A'} = \ln[(\ln[\xi_n^{AA'}(Q^2)Q^2/\Lambda^2])]/(\ln[Q_0^2/\Lambda^2]) = s + \ln(1 + \delta^{AA'})$. When A is a nucleon (and

$A' \rightarrow A$), we have $s^A = s + \ln(1 + \delta^{NA})$ and so we reproduce (11) with $\delta^{NA} = \delta^A$.

In the NS case under consideration, we have $\xi_n^{NA}(Q_0^2) > 1$ and $\delta_{\text{NS}}^{NA} \equiv \delta_{\text{NS}}^A > 0$. Thus, δ_{NS}^A is a macroscopic characteristic of the evolution of the NS parton density in nuclei, which transforms it into nucleon parton density at higher values of Q^2 .

Now we consider the singlet quark and gluon densities, which obey a system of DGLAP equations. After diagonalization, the corresponding diagonal $\pm$ moments have the following form:

$$M_n^\pm(Q^2) = M_n^\pm(Q_0^2) \left[\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)} \right]^{d_n^\pm}, \quad d_n^\pm = \frac{\gamma_n^\pm}{2\beta_0}, \quad (\text{A13})$$

where diagonal anomalous dimensions $\gamma_n^\pm$ are

$$\gamma_n^\pm = \gamma_n^{gg} - \gamma_n^{qq} \pm \sqrt{(\gamma_n^{gg} - \gamma_n^{qq})^2 + 4\gamma_n^{gq}\gamma_n^{qg}} \quad (\text{A14})$$

and γ_n^{ab} ($a, b = g, q$) are the LO parton anomalous dimensions. So, the result (A14) is very close to the one (A1). By repeating the previous studies for the NS part, we can see that the values of $\delta_{\pm}^{AA'}$ are independent on Q^2 .

The “+” component contains singular terms at $n \rightarrow 1$ and thus it gives a dominant contribution in the low x range. So, the shadowing effect can be interpreted within the framework of the rescaling model as a change in scale with $\xi_n^{AA'}(Q^2) < 1$ and, thus, $\delta_+^A < 0$. The “−” component, along with the “+” and NS components, is important at $x \sim 0.1 \div 0.2$, and thus all these components are responsible for the possible antishadowing effect. In the pure rescaling model, antishadowing should occur due to momentum conservation, because $R^A < 1$ at $x \geq 0.3$ and $x \leq 0.1$ (here $R^A = F_2^A(x, Q^2)/F_2^A(x, Q^2)$). In the case under consideration, where we also took into account the Fermi motion, $R^A > 1$ for $x \geq 0.7$. Thus, the antishadowing effect may or may not manifest itself. Thus, there are no strict restrictions for the value of δ_-^A and its value should be obtained based on a comparison of experimental data. Recent studies [57] lead to negative values of δ_-^A .

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