

Statistical repulsion on hyperons in two-color dense QCD

Masato Nagatsuka^{1,*} and Toru Kojo^{2,3,†}

¹*Department of Physics, Tohoku University, Sendai 980-8578, Japan*

²*Theory Center, IPNS, High Energy Accelerator Research Organization (KEK),
1-1 Oho, Tsukuba, Ibaraki, 305-0801, Japan*

³*Graduate Institute for Advanced Studies, SOKENDAI, 1-1 Oho, Tsukuba, Ibaraki, 305-0801, Japan*



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We investigate the onset of hyperons in baryonic (diquark) matter in two-color QCD (QC₂D) by introducing heavy quark doublets that emulate strange quarks. An even number of flavors is chosen to construct a theory amenable to sign-problem-free lattice Monte Carlo simulations. To explore QC₂D matter containing both light and heavy quarks, we construct a model in which quarks interact with light-light, light-heavy (hyperonic), and heavy-heavy diquarks via Yukawa couplings. As the quark chemical potential increases, the light diquarks condense first and form baryonic matter, and this onset density can be understood in hadronic terms. In contrast, the onset density of hyperons is substantially higher than that estimated from the hadronic sector of the model. This shift reflects an effective repulsion among baryons induced by the preoccupied light quarks. The Pauli blocking of light quarks suppresses the attractive diquark correlations responsible, in vacuum, for making hyperons lighter than the sum of the constituent light and heavy quark masses. Implications for three-color QCD are also briefly discussed.

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I. INTRODUCTION

Baryonic matter in QCD consists of composite particles and, as the baryon density (n_B) increases, is expected to transform into quark matter. In the dilute regime, hadrons serve as natural effective degrees of freedom, allowing for reliable computations of the properties of dense matter [1–5]. *Ab initio* nuclear calculations and statistical analyses of neutron star observations indicate that purely nucleonic computations remain valid only up to baryon densities of roughly $1.5\text{--}2n_0$ ($n_0 \simeq 0.16\text{ fm}^{-3}$, the nuclear saturation density) [4–6]. Beyond this regime, various medium effects—such as the enhancement of many-body forces, the appearance of hadrons beyond nucleons, and structural modifications of hadrons—are expected to play an important role [7–12].

One of the important problems in neutron star physics is the so-called *hyperon softening problem*, or *hyperon puzzle* [13–22]. Typical hadronic calculations predict that hyperons appear at baryon densities around $n_B = 2\text{--}3n_0$, but their emergence softens the equation of state (EOS) too

much, rendering it incompatible with the well-established two-solar-mass constraints on neutron stars [23–26]. One possible resolution is to introduce short-range repulsive interactions among hyperons and nucleons, such as YN and YNN forces, which delay the appearance of hyperons. Two-body YN repulsion alone is typically insufficient, necessitating the inclusion of three-body YNN repulsion [13]. Many-body calculations incorporating these interactions, with strengths constrained by YN scattering data [27] and hypernuclear spectroscopy [28–34] (for a recent review, see Ref. [35]), suggest that the resulting EOS can be compatible with the two-solar-mass constraints [36–39].

However, scenarios based on many-body repulsion raise questions regarding the convergence of the many-body expansion. When two- and three-body repulsive forces are comparable, it is natural to expect that four-, five-, and higher-body forces may also become important. Moreover, within the hadronic picture, it is not guaranteed that many-body effects always manifest as repulsion. We also note that extrapolating N -body ($N > 2$) repulsive contributions to the energy density, $\varepsilon_{N\text{-body}} \sim n_B^N$, to high densities eventually leads to a squared sound speed $c_s^2 \sim N - 1$, which violates the causality constraint $c_s^2 \leq 1$ [40]. To study the global behavior of many-body forces at high density, analyses based on more fundamental degrees of freedom are essential.

Recently, there has been growing interest in quark-level descriptions of baryons in dense matter [41–57]. Since

*Contact author: masato.nagatsuka.r4@dc.tohoku.ac.jp

†Contact author: torukojo@post.kek.jp

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baryons are composed of quarks, the quark states as fermions must gradually become saturated as the baryon density increases. This phenomenon, referred to as *quark saturation* [48], is expected to occur first at low momenta. As the density rises, the saturated region extends to higher momenta, eventually forming a quark Fermi sea. Recent studies suggest that quark saturation can occur at $n_B \sim 1-3n_0$ [58–60], even before baryons spatially overlap, thereby imposing nontrivial constraints on baryon dynamics. In the context of the hyperon puzzle, quark saturation induces statistical repulsion among nucleons and hyperons, since the preoccupied light quarks disfavor the formation of hyperons containing up and down quarks [61].

It is important to test the idea of statistical repulsion induced by quark degrees of freedom, as it provides a more fundamental mechanism than descriptions based on many-body forces. Direct tests from first principles are unfortunately not feasible, mainly because lattice QCD simulations of dense systems suffer from the notorious sign problem. However, simulations in two-color QCD (QC₂D) [62,63,63–76] and in isospin QCD (QCD_I) [77–80] at finite chemical potentials are feasible. Indeed, by comparing schematic model calculations with lattice results, both macroscopic and microscopic insights have been obtained. In this paper, we perform a model study on the properties of hyperons in dense QC₂D and discuss how this framework can be exploited to explore the concept of statistical repulsion.

Introducing strange quarks into QC₂D is formally straightforward at least within an effective model. However, considering our aim to test our conceptions in lattice simulations, we should consider theories with an even number of flavors; adding only a single flavor reintroduces the sign problem in lattice simulations, since the resulting Dirac determinant is no longer positive definite. To avoid this issue, we consider a heavy quark doublet, Q_u and Q_d , with equal masses M_Q [81]. A hyperon analogous to $\Lambda(uds)$, $\Sigma_0(uds)$, and similar states can then be constructed as a bound state of a light-heavy diquark. For simplicity, we focus on the isosinglet diquarks $u-Q_d$ and $d-Q_u$, with masses denoted by m_Y . In QC₂D, light diquarks are degenerate with the pion due to the Pauli-Gürsey symmetry. For hyperons, we expect m_Y to be of the order of the kaon mass or roughly the sum of the constituent light and heavy quark masses.

In this model setup, the physics at low quark chemical potential μ is identical to that of two-flavor QC₂D. At $\mu = m_\pi/2$, the light diquarks condense, initiating the formation of baryonic matter. Near this threshold, the system can be well described as a Bose-Einstein condensate (BEC) of light diquarks. As μ increases, the BEC regime gradually evolves into the Bardeen-Cooper-Schrieffer (BCS) regime, where diquark pairs coexist on top of the quark Fermi sea. (For reviews on the BEC-BCS crossover, see, e.g., Refs. [82–84].)

Our central question concerns the behavior of hyperons when the baryon chemical potential, $\mu_B = 2\mu$, exceeds the

hyperon mass m_Y . Neglecting interactions or constraints from the quark substructure, hyperons would appear at $\mu_B = m_Y$. In three-color QCD, such an onset softens the EOS, giving rise to the hyperon problem. We argue that including light-quark loops in the effective potential generates an effective repulsion for hyperons, thereby substantially increasing the critical chemical potential and mitigating the hyperon-induced softening. This finding is consistent with results from the IdylliQ model [49] for quarkyonic matter [85]. In this paper, we compute the critical chemical potential for hyperon emergence by examining the sign of the quadratic terms in the hyperon fields. The EOS after hyperon onset is computed elsewhere.

This paper is structured as follows. In Sec. II, we define our effective model for QC₂D with light and heavy flavors. In Sec. III, we construct the renormalized effective potential. In Sec. IV, we delineate the onset of hyperons. Section V is devoted to the conclusions.

II. MODEL

A. Lagrangian

The Lagrangian for the light quark sector is [86–91]

$$\begin{aligned} \mathcal{L}_q = & \bar{q}[\mathbf{i}\not{\partial} + \mu\gamma^0]q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\pi^a)^2 \\ & + \frac{1}{2}(\partial_\mu - 2i\mu\delta_{\mu 0})D \cdot (\partial^\mu + 2i\mu\delta^{\mu 0})D^* \\ & - g\bar{q}[\sigma + i\gamma_5\pi^a\tau^a]q \\ & - \frac{g}{2}[D^*(q^T C i\gamma_5\lambda_2\tau_2 q) + D(\bar{q} C i\gamma_5\lambda_2\tau_2 \bar{q}^T)] \\ & - \frac{m_\phi^2}{2}(\phi_a^2 + |D|^2) - \frac{\lambda}{24}(\phi_a^2 + |D|^2)^2 \\ & + h_\sigma\sigma, \end{aligned} \quad (1)$$

where $q = (u, d)^T$ is the isodoublet and bosonic fields $\phi_a = (\sigma, \vec{\pi})$. Here, D and D^* are an isosinglet diquark and antidiquark. The mass ($m_\phi^2 < 0$) and Yukawa coupling (g) for mesons and diquarks are related by the Pauli-Gürsey symmetry. The term $h_\sigma\sigma$ explicitly breaks the chiral symmetry.

To mimic the strangeness degrees of freedom, we introduce heavy quarks with the constituent masses of ~ 500 MeV as counterparts of strange quarks. But introducing only a single flavor causes the sign problem in lattice simulations. To employ the lattice as a simulator free from the sign problem, we introduce these heavy quarks as (iso)doublets so that the fermion determinant becomes positive definite. The Lagrangian is¹

¹A four-flavor linear sigma model with the $SU(8)$ Pauli-Gürsey symmetry has been studied recently in Ref. [81]. In this work, we do not use such symmetry to constrain the model parameters.

$$\begin{aligned}\mathcal{L}_Q = & \bar{Q}(i\not{\partial} - M_Q + \mu\gamma^0)Q \\ & + \frac{1}{2}(\partial_\mu - 2i\mu\delta_{\mu 0})D_Q \cdot (\partial^\mu + 2i\mu\delta^{\mu 0})D_Q^* \\ & - \frac{g_Q}{2}[D_Q^*(Q^T C i\gamma_5 \lambda_2 \tau_2 Q) + D_Q(\bar{Q} C i\gamma_5 \lambda_2 \tau_2 \bar{Q}^T)] \\ & - \frac{m_{D_Q}^2}{2}D_Q D_Q^* - \frac{\lambda_Q}{24}|D_Q D_Q^*|^2,\end{aligned}\quad (2)$$

where $Q = (Q_u, Q_d)^T$ is an isodoublet and D_Q is a color- and isosinglet $Q_u Q_d$ -diquark.²

Finally, we consider couplings between the light and heavy quark sectors. A diquark made of a light and a heavy quark is regarded as a hyperon. Writing the isosinglet light-heavy diquark as $Y_u \sim u Q_d$ and $Y_d \sim d Q_u$, the Lagrangian is $\mathcal{L}_Y = \sum_{i=u,d} \mathcal{L}_Y^i$, where

$$\begin{aligned}\mathcal{L}_Y^i = & \frac{1}{2}(\partial_\mu - 2i\mu\delta_{\mu 0})Y_i \cdot (\partial^\mu + 2i\mu\delta^{\mu 0})Y_i^* \\ & - \frac{g_Y}{2}[Y^*(f^T C i\gamma_5 \lambda_2 \tau_2 f) + Y(\bar{f} C i\gamma_5 \lambda_2 \tau_2 \bar{f}^T)]_i \\ & - \frac{m_Y^2}{2}Y_i Y_i^* - \frac{\lambda_Y}{24}(Y_i Y_i^*)^2 \\ & - \kappa_{\phi Y}|\phi|Y_i Y_i^* - \lambda_{\phi Y}(\phi_a^2 + D^2)Y_i Y_i^*,\end{aligned}\quad (3)$$

where $f_u = (u, Q_d)^T$ and $f_d = (d, Q_u)^T$ and $|\phi| = \sqrt{\phi_a^2}$ is the chiral scalar field.

In practice, it is useful to redefine fermion fields as $\psi = q$, $\psi_Q = Q$, and $\psi_f = f$ with the relations

$$(\psi^C, \psi_Q^C, \psi_f^C) = \lambda_2 \tau_2 C(\bar{q}^T, \bar{Q}^T, \bar{f}^T). \quad (4)$$

For example, $\psi_{uR}^C = C\bar{d}_G^T$ and $u_R^T C \lambda_2 \tau_2 = \bar{\psi}_{dG}^C$. In particular,

$$\begin{aligned}(q^T C i\gamma_5 \lambda_2 \tau_2 q) &= \bar{\psi}^C i\gamma_5 \psi, \\ (\bar{q} C i\gamma_5 \lambda_2 \tau_2 \bar{q}^T) &= \bar{\psi} i\gamma_5 \psi^C,\end{aligned}\quad (5)$$

where the color and flavor are diagonal for ψ -bilinear as in usual mesonic operators. The same applies for the other doublets. Using these expressions, we do not have to deal with color and flavor matrices, and the relationship with isospin QCD can be made explicit.

In the Namub-Gor'kov bases, the Lagrangian takes the form³

²The coupling g_Q is required to describe physics beyond the onset of heavy diquarks, such as the equation of state at higher densities.

³To describe the diquark condensate with zero momentum, $\Delta \sim q(-p)q(p) \sim \bar{\psi}_C(-p)\psi(p)$, it is convenient to take the Fourier transform of $\Psi(x)$ to be $\Psi(p) = (\psi(p), \psi_C(-p))$. Then, the Fourier transform of the Ψ bilinear becomes $\bar{\Psi}(p)[\not{\not{p}} - \Sigma + \mu\gamma^0 \not{\not{p}} - \Sigma + \mu\gamma^0] \Psi(p)$. This form is identical to the quark bilinear in QCD_I. Hence, the analytic results of QC₂D can be transferred to QCD_I by simply modifying the factor $\bar{N}_c$ and N_f .

$$\begin{aligned}\mathcal{L}_q = & \bar{\Psi} \begin{bmatrix} i\not{\partial} - \Sigma + \mu\gamma^0 & \Delta i\gamma_5 \\ \Delta^* i\gamma_5 & -i\not{\partial} - \Sigma - \mu\gamma^0 \end{bmatrix} \Psi \\ & + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \pi^a)^2 \\ & + \frac{1}{2}(\partial_\mu - 2i\mu\delta_{\mu 0})D \cdot (\partial^\mu + 2i\mu\delta^{\mu 0})D^* \\ & - \frac{m_\phi^2}{2}(\phi_a^2 + |D|^2) - \frac{\lambda}{24}(\phi_a^2 + |D|^2)^2 \\ & + h_\sigma \sigma + h_D(D + D^*),\end{aligned}\quad (6)$$

with $\Psi = (\psi, \psi^C)^T/\sqrt{2}$, $\Sigma = g(\sigma + i\gamma_5 \pi^a \tau^a)$, and $\Delta = gD$. The heavy quark part is

$$\begin{aligned}\mathcal{L}_Q = & \bar{\Psi}_Q \begin{bmatrix} i\not{\partial} - M_Q + \mu\gamma^0 & \Delta_Q i\gamma_5 \\ \Delta_Q^* i\gamma_5 & -i\not{\partial} - M_Q - \mu\gamma^0 \end{bmatrix} \Psi_Q \\ & + \frac{1}{2}(\partial_\mu - 2i\mu\delta_{\mu 0})D_Q \cdot (\partial^\mu + 2i\mu\delta^{\mu 0})D_Q^* \\ & - \frac{m_{D_Q}^2}{2}D_Q D_Q^* - \frac{\lambda_Q}{24}|D_Q D_Q^*|^2,\end{aligned}\quad (7)$$

with $\Psi_Q = (\psi_Q, \psi_Q^C)^T/\sqrt{2}$ and $\Delta_Q = g_Q D_Q$. Finally, for later convenience, we decompose the light-heavy sector as $\mathcal{L}_Y = \mathcal{L}_Y^0 + \mathcal{L}_Y^{\text{mix}}$, where

$$\begin{aligned}\mathcal{L}_Y^0 = & \frac{1}{2}(\partial_\mu - 2i\mu\delta_{\mu 0})Y_i \cdot (\partial^\mu + 2i\mu\delta^{\mu 0})Y_i^* \\ & - \frac{m_Y^2}{2}Y_i Y_i^* - \frac{\lambda_Y}{24}(Y_i Y_i^*)^2,\end{aligned}\quad (8)$$

for the kinetic terms and self-couplings among Y_u and Y_d , and, for the coupling between fermion pairs and bosonic fields,

$$\begin{aligned}\mathcal{L}_Y^{\text{mix}} = & -g_Y[Y_i^*(\bar{\psi}_Q^C i\gamma_5 \psi_i) + Y_i(\bar{\psi}_i i\gamma_5 \psi_Q^C)] \\ & - \kappa_{\phi Y}|\phi|Y_i Y_i^* - \lambda_{\phi Y}(\phi_a^2 + D^2)Y_i Y_i^*,\end{aligned}\quad (9)$$

where we used

$$\bar{\psi}_{Q_i}^C i\gamma_5 \psi_i = \bar{\psi}_i^C i\gamma_5 \psi_{Q_i}, \quad \bar{\psi}_{Q_i} i\gamma_5 \psi_i^C = \bar{\psi}_i i\gamma_5 \psi_{Q_i}^C. \quad (10)$$

B. Counterterms

We compute the effective potential up to single fermion loop for which the UV divergences must be handled. For the single fermion loop, we have only to renormalize the boson self-energies and vertices. In practice, it is convenient to use the counterterm formalism. First, we attach indices B to the fields, masses, and couplings, e.g., $\psi \rightarrow \psi_B$, $m \rightarrow m_B$, $g \rightarrow g_B$, and so on, and then decompose them into the renormalized and counterterms. Schematically, $\mathcal{L}(\Phi_B, g_B) = \mathcal{L}(\Phi, g) + \mathcal{L}^{c.t.}(\Phi, \delta g, \delta Z_\Phi)$,

where (Φ_B, g_B) are general bare fields and couplings, (Φ, g) are renormalized fields and couplings, and $(\delta g, \delta Z_\Phi)$ are counterterms for couplings and field normalization. Explicitly, the counterterm Lagrangian is [88,89]

$$\begin{aligned} \mathcal{L}_q^{c.t.} = & \frac{\delta Z_\phi}{2} (\partial_\mu \sigma)^2 + \frac{\delta Z_\phi}{2} (\partial_\mu \pi^a)^2 \\ & + \frac{\delta Z_\phi}{2} (\partial_\mu - 2i\mu\delta_{\mu 0}) D \cdot (\partial^\mu + 2i\mu\delta^{\mu 0}) D^* \\ & - \frac{\delta m_\phi^2}{2} (\phi_a^2 + |D|^2) - \frac{\delta \lambda}{24} (\phi_a^2 + |D|^2)^2, \end{aligned} \quad (11)$$

for the light quark sector, and

$$\begin{aligned} \mathcal{L}_Q^{c.t.} = & \frac{\delta Z_{D_Q}}{2} (\partial_\mu - 2i\mu\delta_{\mu 0}) D_Q \cdot (\partial^\mu + 2i\mu\delta^{\mu 0}) D_Q^* \\ & - \frac{\delta m_{D_Q}^2}{2} D_Q D_Q^* - \frac{\delta \lambda_Q}{24} |D_Q|^4, \end{aligned} \quad (12)$$

for the heavy quark sector, and

$$\begin{aligned} (\mathcal{L}_Y^{u+d})^{c.t.} = & \frac{\delta Z_Y}{2} (\partial_\mu - 2i\mu\delta_{\mu 0}) Y_i \cdot (\partial^\mu + 2i\mu\delta^{\mu 0}) Y_i^* \\ & - \frac{\delta m_Y^2}{2} Y_i Y_i^* - \frac{\delta \lambda_Y}{24} (Y_i Y_i^*)^2 \\ & - \delta \kappa_{\phi Y} |\phi| Y_i Y_i^* - \delta \lambda_{\phi Y} (\phi_a^2 + D^2) Y_i Y_i^*. \end{aligned} \quad (13)$$

These counterterms are to be determined in vacuum and define our model.

Computing fermion loops inevitably yields the quartic terms. Using the counterterms, It is possible to define our model so that the renormalized quartic couplings λ_Q and λ_Y vanish in vacuum at a specifically chosen renormalization scale. With such setup, we simplify our notation and then the nonzero quartic couplings solely arise from medium effects.

C. Mean-field propagators

In this work, we consider the density interval where σ , D , and D^* condense but the other bosonic fields π , D_Q , D_Q^* , and $Y_{u,d}$ are vanishing. With these Ansätze, we use our model to just examine the onset of “hyperons” $Y_{u,d}$. Below, we assume $\pi = 0$ and use the effective mass and gap $M_q = g\langle\sigma\rangle$ and $\Delta = g\langle D\rangle$ to describe the mean-field propagators for light quarks.

The mean-field propagators for light quarks in such bosonic condensates are given by

$$\mathcal{S}_q(p) = \begin{bmatrix} S_{11}(p) & S_{12}(p) \\ S_{21}(p) & S_{22}(p) \end{bmatrix} = \begin{bmatrix} \langle \psi \bar{\psi} \rangle & \langle \psi \bar{\psi}^C \rangle \\ \langle \psi^C \bar{\psi} \rangle & \langle \psi^C \bar{\psi}^C \rangle \end{bmatrix}, \quad (14)$$

where the normal (S_{11}, S_{22}) and the anomalous (S_{12}, S_{21}) propagators are

$$\begin{aligned} S_{11}(p) &= \sum_{\xi=p,a} S_{11}^\xi(p) \Lambda_\xi \gamma^0, \\ S_{12}(p) &= \sum_{\xi=p,a} S_{12}^\xi(p) \Lambda_\xi \gamma^5, \\ S_{21}(p) &= \sum_{\xi=p,a} S_{21}^\xi(p) \Lambda_\xi^C \gamma^5, \\ S_{22}(p) &= \sum_{\xi=p,a} S_{22}^\xi(p) \Lambda_\xi^C \gamma^0, \end{aligned} \quad (15)$$

with

$$\begin{aligned} S_{11}^\xi(p) &= i \left(\frac{|u_\xi(\mathbf{p})|^2}{p_0 - \eta_q^\xi \epsilon_q^\xi(\mathbf{p})} + \frac{|v_\xi(\mathbf{p})|^2}{p_0 + \eta_\xi^\xi \epsilon_q^\xi(\mathbf{p})} \right), \\ S_{12}^\xi(p) &= -i \left(\frac{u_\xi^*(\mathbf{p}) v_\xi^*(\mathbf{p})}{p_0 - \epsilon_q^\xi(\mathbf{p})} - \frac{u_\xi^*(\mathbf{p}) v_\xi^*(\mathbf{p})}{p_0 + \epsilon_q^\xi(\mathbf{p})} \right), \\ S_{21}^\xi(p) &= i \left(\frac{u_\xi(\mathbf{p}) v_\xi(\mathbf{p})}{p_0 - \epsilon_q^\xi(\mathbf{p})} - \frac{u_\xi(\mathbf{p}) v_\xi(\mathbf{p})}{p_0 + \epsilon_q^\xi(\mathbf{p})} \right), \\ S_{22}^\xi(p) &= i \left(\frac{|v_\xi(\mathbf{p})|^2}{p_0 - \eta_\xi^\xi \epsilon_q^\xi(\mathbf{p})} + \frac{|u_\xi(\mathbf{p})|^2}{p_0 + \eta_q^\xi \epsilon_q^\xi(\mathbf{p})} \right), \end{aligned} \quad (16)$$

where we introduced $\eta_p = +1$ and $\eta_a = -1$. In these expressions, we have defined the positive-energy and negative-energy projection operators Λ_p and Λ_a by

$$\Lambda_\xi(\mathbf{p}) = \gamma^0 \frac{E_q(\mathbf{p}) \gamma^0 + \eta_\xi (M_q + \boldsymbol{\gamma} \cdot \mathbf{p})}{2E_q(\mathbf{p})} \quad (17)$$

with $E_q(\mathbf{p}) = \sqrt{\mathbf{p}^2 + M_q^2}$, and $\Lambda_{p(a)}^C = \Lambda_{a(p)}$, and

$$\epsilon_q^\xi(\mathbf{p}) = \sqrt{(E_q - \eta_\xi \mu)^2 + |\Delta|^2}, \quad (18)$$

are the dispersion relations for quasiparticles. The factors $u_p(\mathbf{p})$, $v_p(\mathbf{p})$, $u_a(\mathbf{p})$, and $v_a(\mathbf{p})$ satisfy relations

$$\begin{aligned} |u_\xi(\mathbf{p})|^2 &= \frac{1}{2} \left(1 + \frac{E_q - \eta_\xi \mu}{\epsilon_q^\xi(\mathbf{p})} \right), \\ |v_\xi(\mathbf{p})|^2 &= \frac{1}{2} \left(1 - \frac{E_q - \eta_\xi \mu}{\epsilon_q^\xi(\mathbf{p})} \right). \end{aligned} \quad (19)$$

Finally, the heavy quark propagator is

$$\mathcal{S}_Q(p) = \begin{bmatrix} S_Q^{11}(p) & 0 \\ 0 & S_Q^{22}(p) \end{bmatrix}, \quad (20)$$

where

$$\begin{aligned} S_Q^{11}(p) &= (S_Q^{11})^p \Lambda_p^Q \gamma^0 + (S_Q^{11})^a \Lambda_a^Q \gamma^0, \\ S_Q^{22}(p) &= (S_Q^{22})^p (\Lambda_p^Q)^C \gamma^0 + (S_Q^{22})^a (\Lambda_a^Q)^C \gamma^0, \end{aligned} \quad (21)$$

with $(\Lambda_{p/a}^Q)^C = \Lambda_{a/p}^Q$, and the projectors are defined in the same way as those for light quarks. The coefficients are

$$\begin{aligned} (S_Q^{11})^\xi &= i \left(\frac{\Theta(E_Q - \eta_\xi \mu)}{p_0 - \eta_Q^\xi \epsilon_\xi(\mathbf{p})} + \frac{\Theta(\eta_\xi \mu - E_Q)}{p_0 + \eta_Q^\xi \epsilon_\xi(\mathbf{p})} \right), \\ (S_Q^{22})^\xi &= i \left(\frac{\Theta(\eta_\xi \mu - E_Q)}{p_0 - \eta_Q^\xi \epsilon_\xi(\mathbf{p})} + \frac{\Theta(E_Q - \eta_\xi \mu)}{p_0 + \eta_Q^\xi \epsilon_\xi(\mathbf{p})} \right), \end{aligned} \quad (22)$$

where $\epsilon_Q^\xi = |E_Q - \eta_\xi \mu|$. Especially, for $M_Q > \mu$, the expression is simplified as

$$\begin{aligned} (S_Q^{11})^\xi(p) &= \frac{i}{p_0 + \mu - \eta_\xi E_Q}, \\ (S_Q^{22})^\xi(p) &= \frac{i}{p_0 - \mu + \eta_\xi E_Q}. \end{aligned} \quad (23)$$

In the above expressions, we omit the usual small imaginary part $i\delta$ to save the space, but in actual computations, we make the replacement $p_0 \rightarrow p_0 + ip_0\delta$ to describe the correct boundary condition.

III. EFFECTIVE POTENTIAL

We compute the effective potential for possible condensates $\Phi = (\sigma, \pi, D, D_Q)$ and $Y = (Y_u, Y_d)$. In the following, we treat all fields as real, without loss of generality. The effective action can be computed by shifting bosonic fields into the condensed and fluctuation parts, $\Phi \rightarrow \Phi + \hat{\Phi}$, $Y \rightarrow Y + \hat{Y}$, and by computing the one-particle irreducible (1PI) graphs;

$$e^{i\Gamma(\Phi, Y)} = \int \mathcal{D}F \mathcal{D}\hat{\Phi} \mathcal{D}\hat{Y} e^{i \int_x \mathcal{L}(F; \Phi + \hat{\Phi}, Y + \hat{Y})}, \quad (24)$$

where F collectively denotes fermion fields ψ and ψ_Q and their conjugates.

In the mean-field effective action, the fluctuations $\hat{\Phi}$ and $\hat{Y}$ are neglected,

$$e^{i\Gamma_{\text{MF}}(\Phi, Y)} = \int \mathcal{D}F e^{i \int_x \mathcal{L}(F; \Phi, Y)}. \quad (25)$$

We treat the coupling between hyperon fields Y and light-heavy quark fields as perturbation,

$$\begin{aligned} e^{i\Gamma_{\text{MF}}(\Phi, Y)} &\simeq \int \mathcal{D}F e^{i \int_x [\mathcal{L}_q + \mathcal{L}_Q + \mathcal{L}_Y^0]} \\ &\times \left(1 + i \int_x \mathcal{L}_Y^{\text{mix}} + \dots \right). \end{aligned} \quad (26)$$

The first nonzero contributions arise from the second-order perturbation. This generates the self-energy terms for the Y fields.

We note that the ψ_Q fields do not yield the anomalous propagators, i.e., in the density interval of $D_Q = 0$. The self-energy term comes from the product of the normal propagators,

$$-i \frac{\Pi_Y}{2} |Y_i|^2 \equiv (-ig_Y)^2 (-1) \text{Tr}[S_q^{11} i\gamma_5 S_Q^{22} i\gamma_5] |Y_i|^2. \quad (27)$$

where the trace runs over momenta and color and Dirac indices, but not over flavors. We delineate this term in the next section.

The effective potential $V_{\text{MF}} = -\Gamma_{\text{MF}}$ is

$$V_{\text{MF}} = V_{\text{tree}} + V_{ct} + V_{\text{loop}} + V_{\text{pert}}^Y. \quad (28)$$

Assuming $\pi = D_Q = 0$, writing $M_q = g\sigma$, $\Delta = gD$, and $\Delta_{Y_i} = g_Y Y_i$, the tree-level potential is

$$\begin{aligned} V_{\text{tree}} &= \frac{m_\phi^2}{2g^2} M_q^2 + \frac{m_\phi^2 - 4\mu^2}{2g^2} \Delta^2 + \frac{\lambda}{24g^4} (M_q^2 + \Delta^2)^2 \\ &\quad - \frac{h_\sigma}{g} M_q - \frac{2h_D}{g} \Delta \\ &\quad + \frac{m_Y^2 - 4\mu^2}{2g_Y^2} \Delta_{Y_i}^2 + \frac{\lambda_Y}{24} (\Delta_{Y_i}^2)^2 \\ &\quad + \frac{\kappa_{\phi Y}}{gg_Y^2} M_q \Delta_{Y_i}^2 + \frac{\lambda_{\phi Y}}{g^2 g_Y^2} (M_q^2 + \Delta^2) \Delta_{Y_i}^2. \end{aligned} \quad (29)$$

The counterterms are

$$\begin{aligned} V_{ct} &= \frac{\delta m_\phi^2}{2g^2} M_q^2 + \frac{\delta m_\phi^2 - 4\mu^2 \delta Z_\phi}{2g^2} \Delta^2 + \frac{\delta \lambda}{24g^4} (M_q^2 + \Delta^2)^2 \\ &\quad + \frac{\delta m_Y^2 - 4\mu^2 \delta Z_Y}{2g_Y^2} \Delta_{Y_i}^2 + \frac{\delta \lambda_Y}{24g_Y^4} (\Delta_{Y_i}^2)^2 \\ &\quad + \frac{\delta \kappa_{\phi Y}}{gg_Y^2} M_q \Delta_{Y_i}^2 + \frac{\delta \lambda_{\phi Y}}{g^2 g_Y^2} (M_q^2 + \Delta^2) \Delta_{Y_i}^2. \end{aligned} \quad (30)$$

The fermion loop yields ($N_c = N_f = 2$),

$$\begin{aligned} V_{\text{loop}} &= -N_f N_c \int_p [\epsilon_q^p(\mathbf{p}) + \epsilon_q^a(\mathbf{p})] \\ &\quad - N_f N_c \int_p [|E_Q(\mathbf{p}) - \mu| + |E_Q(\mathbf{p}) + \mu|], \end{aligned} \quad (31)$$

with $E_Q(\mathbf{p}) = \sqrt{\mathbf{p}^2 + M_Q^2}$. For $M_Q > \mu$, the μ -dependence of the heavy quark contributions vanish, $|E_Q - \mu| + |E_Q + \mu| = E_Q - \mu + E_Q + \mu = 2E_Q$. We discuss the isolation of the UV divergence shortly.

Finally, the perturbative contribution is

$$V_{\text{pert}}^Y = \frac{\Pi_Y}{2g_Y^2} \Delta_{Y_i}^2. \quad (32)$$

Below, we first fix the counterterms and then move to computations of Π_Y . Meanwhile, the determination of the renormalized parameters will be done in separate computations.

A. Light quark sector

We evaluate

$$\begin{aligned} V_{\text{loop}}^q(\mu) &= -N_c N_f \int_p [\epsilon_q^p(\mathbf{p}) + \epsilon_q^a(\mathbf{p})] \\ &= V_q^R(\mu) + V_q^{(1)}(M_q, \Delta) + \mu^2 V_q^{(2)}(M_q, \Delta), \end{aligned} \quad (33)$$

where the last two terms are divergent,

$$\begin{aligned} V_q^{(1)} &= -2N_c N_f \int_p \sqrt{\mathbf{p}^2 + M_q^2 + \Delta^2}, \\ V_q^{(2)} &= -N_c N_f \int_p \frac{\Delta^2}{(\mathbf{p}^2 + M_q^2 + \Delta^2)^{3/2}}. \end{aligned} \quad (34)$$

Subtracting these terms from V_q , the twice subtracted potential V_q^R , defined by $V_q - V_q^{(1)} - V_q^{(2)}$, is UV finite.

The divergent $V_q^{(1)}$ and $V_q^{(2)}$ can be computed analytically in the dimensional regularization [88,89],

$$\begin{aligned} V_q^{(0)} &= \frac{N_c N_f}{(4\pi)^2} \left(\frac{1}{\epsilon} + \frac{3}{2} - \ln \frac{M_q^2 + \Delta^2}{\Lambda^2} \right) (M_q^2 + \Delta^2)^2, \\ V_q^{(2)} &= \frac{2N_c N_f}{(4\pi)^2} \left(\frac{1}{\epsilon} - \ln \frac{M_q^2 + \Delta^2}{\Lambda^2} \right) (-2\Delta^2), \end{aligned} \quad (35)$$

where Λ is a renormalization scale at which the renormalized parameters in our model are defined.

We use the $\overline{\text{MS}}$ scheme to remove the UV divergences. The counterterms are

$$\begin{aligned} \delta m_\phi^2 &= 0, \quad \delta Z_\phi = -\frac{2N_c N_f g^2}{(4\pi)^2} \frac{1}{\epsilon}, \\ \delta \lambda &= -\frac{24N_c N_f g^4}{(4\pi)^2} \frac{1}{\epsilon}. \end{aligned} \quad (36)$$

Then, the effective potential for the light quark sector is

$$\begin{aligned} V_{\text{MF}}^q &= (V_{\text{tree}} + V_{ct} + V_{\text{loop}})^q \\ &= V_q^R(\mu) - \frac{h_\sigma}{g} M_q - \frac{2h_D}{g} \Delta + \frac{m_\phi^2}{2g^2} (M_q^2 + \Delta^2) - \frac{2\mu^2 \Delta^2}{g^2} \left[1 - g^2 \frac{2N_c N_f}{(4\pi)^2} \ln \frac{M_q^2 + \Delta^2}{\Lambda^2} \right] \\ &\quad + \frac{1}{24g^4} \left[\lambda + g^4 \frac{24N_c N_f}{(4\pi)^2} \left(\frac{3}{2} - \ln \frac{M_q^2 + \Delta^2}{\Lambda^2} \right) \right] (M_q^2 + \Delta^2)^2. \end{aligned} \quad (37)$$

Later, we express the physical pion mass m_π in terms of m_ϕ and the other parameters and show the onset of diquark condensed phase to be $\mu^{\text{onset}} = m_\pi/2$.

B. Heavy quark sector

Similarly, the heavy quark loop for $M_Q > \mu$ is

$$V_Q = \frac{N_c N_f}{(4\pi)^2} \left(\frac{1}{\epsilon} + \frac{3}{2} - \ln \frac{M_Q^2}{\Lambda^2} \right) M_Q^4. \quad (38)$$

Since M_Q is not dynamical, this constant can be simply removed by the vacuum subtraction or can be neglected. Hence, we may write

$$V_{\text{MF}}^Q = 0, \quad (39)$$

at sufficiently low densities with $M_Q > \mu$ and $\Delta_Q = 0$.

C. Light-heavy quark sector

1. Structure of the potential

The light-heavy potential is

$$V_{\text{MF}}^Y = \frac{C_2^Y}{2g_Y^2} \Delta_{Y_i}^2 + \frac{\lambda_Y + \delta \lambda_Y}{24g_Y^4} (\Delta_{Y_i}^2)^2, \quad (40)$$

where the coefficient of the quadratic Δ_{Y_i} term is

$$\begin{aligned} C_2^Y &= m_Y^2 - 4\mu^2 + \frac{2\kappa_{\phi Y}}{g} M_q + \frac{2\lambda_{\phi Y}}{g^2} (M_q^2 + \Delta^2) \\ &\quad + \delta m_Y^2 - 4\mu^2 \delta Z_Y + \frac{2\delta \kappa_{\phi Y}}{g} M_q + \frac{2\delta \lambda_{\phi Y}}{g^2} (M_q^2 + \Delta^2) \\ &\quad + \Pi_Y(\mu). \end{aligned} \quad (41)$$

Below, we evaluate the self-energy at zero momentum,

$$\begin{aligned}\Pi_Y(\mu) &= 2ig_Y^2 \text{Tr}[S_q^{11}(p)i\gamma_5 S_Q^{22}(p)i\gamma_5] \\ &= 2ig_Y^2 N_c \sum_{\xi, \xi' = p, a} \int_p \text{tr}[\Lambda_\xi \gamma_0 i\gamma_5 (\Lambda_{\xi'}^Q)^C \gamma_0 i\gamma_5] S_{11}^\xi (S_{22}^Q)^{\xi'},\end{aligned}\quad (42)$$

where $\Lambda_\xi = \Lambda_\xi(\mathbf{p})$, $\Lambda_{\xi'}^Q = \Lambda_{\xi'}^Q(\mathbf{p})$, $S_{11}^\xi = S_{11}^\xi(p)$, and $(S_{22}^Q)^{\xi'} = (S_{22}^Q)^{\xi'}(p)$. This function is UV divergent. But its twice-subtracted function Π_Y^R can be made UV finite. Then, Π_Y can be written as (the prime means μ^2 -derivative)

$$\Pi_Y(\mu) = \Pi_Y^R(\mu) + \Pi_Y^{(1)}(M_q, \Delta) + \mu^2 \Pi_Y^{(2)}(M_q, \Delta), \quad (43)$$

which defines Π_Y^R with the divergent functions $\Pi_Y^{(1)}$ and $\Pi_Y^{(2)}$ to be determined shortly in Eq. (59). The divergences in $\Pi_Y^{(1)}$ and $\Pi_Y^{(2)}$ are canceled by the counterterms.

First, we carry out the p_0 integration,

$$i\mathcal{M}^{\xi\xi'}(\mathbf{p}; \mu) = \int_{p_0} S_{11}^\xi (S_{22}^Q)^{\xi'}. \quad (44)$$

For $M_Q > \mu$, it yields ($\epsilon_Q^\xi \equiv |E_Q - \eta_\xi \mu|$)

$$\mathcal{M}^{\xi\xi} = \frac{|v_\xi(\mathbf{p})|^2}{\epsilon_q^\xi(\mathbf{p}) + \epsilon_Q^\xi(\mathbf{p})} \quad (\xi = \xi'), \quad (45)$$

for $\xi = \xi'$ corresponding to the particle-particle and anti-particle-antiparticle contributions, and

$$\mathcal{M}^{\xi\xi'} = \frac{|v_\xi(\mathbf{p})|^2}{\epsilon_q^\xi(\mathbf{p}) + \epsilon_Q^{\xi'}(\mathbf{p})} \quad (\xi \neq \xi'), \quad (46)$$

for $\xi \neq \xi'$ representing the particle-antiparticle contributions.

Next, we compute the trace of γ -matrices,

$$\begin{aligned}\mathcal{K}^{\xi\xi'} &= \text{tr}[\Lambda_\xi \gamma_0 i\gamma_5 (\Lambda_{\xi'}^Q)^C \gamma_0 i\gamma_5] \\ &= 1 + \eta_\xi \eta_{\xi'} \frac{\mathbf{p}^2 + M_q M_Q}{E_q E_Q},\end{aligned}\quad (47)$$

which is μ independent.

Now, the self-energy is

$$\Pi_Y(\mu) = -2g_Y^2 N_c \sum_{\xi, \xi'} \int_p \mathcal{K}^{\xi\xi'}(\mathbf{p}) \mathcal{M}^{\xi\xi'}(\mathbf{p}; \mu). \quad (48)$$

Before its renormalization, this (divergent) function is negative definite, since both $\mathcal{K}^{\xi\xi'}$ and $\mathcal{M}^{\xi\xi'}$ are positive definite. As we see later, however, the vacuum subtraction changes the sign.

2. Sign of the self-energy

Before proceeding to the renormalization of the self-energy for general Δ , it is useful to examine the sign of the self-energy holding (M_q, Δ) fixed to the vacuum values $(M_0, 0)$. Fixing the values of dynamical gaps makes the structure of the self-energy very transparent, as we see shortly. The self-energy contains the integrands

$$\begin{aligned}\mathcal{K}^{\text{pp}} \mathcal{M}^{\text{pp}} + \mathcal{K}^{\text{aa}} \mathcal{M}^{\text{aa}} &= \mathcal{K}^{\text{pp}} \left[\frac{\Theta(E_q - \mu)}{E_q + E_Q - 2\mu} + \frac{1}{E_q + E_Q + 2\mu} \right] \\ &= \mathcal{K}^{\text{pp}} \left[-\frac{\Theta(\mu - E_q)}{E_q + E_Q - 2\mu} \right. \\ &\quad \left. + \frac{2}{E_q + E_Q} + \frac{8\mu^2}{(E_q + E_Q)^3} + \dots \right],\end{aligned}\quad (49)$$

and (reminder: $E_Q > \mu$)

$$\mathcal{K}^{\text{pa}} \mathcal{M}^{\text{pa}} + \mathcal{K}^{\text{ap}} \mathcal{M}^{\text{ap}} = \mathcal{K}^{\text{pa}} \frac{\Theta(\mu - E_q)}{E_Q - E_q}. \quad (50)$$

The terms without the step function depend on μ only through M_q and Δ ; these terms can be completely eliminated by the counterterms δm_Y^2 and δZ_Y , as we are holding M_q and Δ fixed to the vacuum value. The resultant self-energy contributions are positive,

$$\begin{aligned}\Pi_Y(\mu) + \delta m_Y^2 - 4\mu^2 \delta Z_Y &= 2g_Y^2 N_c \int_p \Theta(\mu - E_q) \left[\frac{\mathcal{K}^{\text{pp}}}{E_q + E_Q - 2\mu} - \frac{\mathcal{K}^{\text{pa}}}{E_Q - E_q} \right] \\ &> 2g_Y^2 N_c \int_p \Theta(\mu - E_q) \mathcal{K}^{\text{pp}} \left[\frac{1}{E_q + E_Q - 2\mu} - \frac{1}{E_Q - E_q} \right] \\ &= 2g_Y^2 N_c \int_p \Theta(\mu - E_q) \mathcal{K}^{\text{pp}} \frac{2(\mu - E_q)}{(E_q + E_Q - 2\mu)(E_Q - E_q)} \\ &> 0,\end{aligned}\quad (51)$$

where $\mathcal{K}^{\text{pp}} > \mathcal{K}^{\text{pa}}$ is used. Thus, the effective energy of the Y -diquarks increases.

This expression can be analytically estimated in the case with $M_Q \gg \mu (> M_q)$ as

$$\Pi_Y + \delta m_Y^2 - 4\mu^2 \delta Z_Y \simeq \frac{4g_Y^2 N_c}{M_Q} \int_p \Theta(\mu - E_q) = \frac{4g_Y^2 N_c n_B}{M_Q}. \quad (52)$$

We note that $\mathcal{K}^{\text{pp}} - \mathcal{K}^{\text{pa}} \simeq 2$ is justified when $|\mathbf{p}| \ll M_q$. The effective energy of Y grows with n_B ; this represents an effective repulsion between Y and light quarks. If we retain the momentum dependence in the denominator, the growth

of the repulsion becomes milder, with powers of n_B less than the linear.

Below, we generalize these considerations by including the in-medium modifications of M_q and Δ .

3. Structure of the UV divergence

To isolate the UV divergence and apply the dimensional regularization, we express the integral in (inverse) powers of $\bar{E} \equiv \sqrt{\mathbf{p}^2 + M_q^2 + \Delta^2}$. Expanding in this way, the effective potential can be expressed manifestly in powers of $M_q^2 + \Delta^2$, which respects the Pauli-Gürsey symmetry in the original Lagrangian (in the chiral limit). It is useful to note $E_q^2 = \bar{E}^2 - \Delta^2$ and $E_Q^2 = \bar{E}^2 - \Delta^2 + (M_Q^2 - M_q^2)$.

To specify the UV-divergent terms, we first examine the asymptotic behavior of $\mathcal{K}^{\xi\xi'}$. It is

$$\mathcal{K}^{\xi\xi'} = 1 + \eta_\xi \eta_{\xi'} - \frac{\eta_\xi \eta_{\xi'} (M_Q - M_q)^2}{2\bar{E}^2} + O(\bar{E}^{-4}). \quad (53)$$

For $\xi = \xi'$, this factor approaches constant at large energy, and hence the divergence is severe; we need to compute the other factor $\mathcal{M}^{\xi\xi'}$ to $O(1/\bar{E}^4)$. For $\xi \neq \xi'$, the leading behavior is $\sim 1/E_q^2$ so that we have only to compute $\mathcal{M}^{\xi\xi'}$ to $O(1/\bar{E}^2)$.

Next, we examine the asymptotic behavior of $\mathcal{M}^{\xi\xi'}$. We first note $\mathcal{K}^{\text{pp}} = \mathcal{K}^{\text{aa}}$ and $\mathcal{K}^{\text{pa}} = \mathcal{K}^{\text{ap}}$ so that we encounter the combination of $\mathcal{M}^{\text{pp+aa}} = \mathcal{M}^{\text{pp}} + \mathcal{M}^{\text{aa}}$ and $\mathcal{M}^{\text{pa+ap}} = \mathcal{M}^{\text{pa}} + \mathcal{M}^{\text{ap}}$. Then, it is manifest that each sum is symmetric for $\mu \leftrightarrow -\mu$ so that Π_Y is a function of μ^2 . We expand the energies

$$\begin{aligned} \epsilon_q^\xi &\simeq \bar{E} - \eta_\xi \mu + \eta_\xi \frac{\mu^2}{2\bar{E}^2} + O(\bar{E}^{-3}), \\ \epsilon_Q^\xi &\simeq \bar{E} - \eta_\xi \mu + \frac{M_Q^2 - M_q^2 - \Delta^2}{2\bar{E}} + O(\bar{E}^{-3}). \end{aligned} \quad (54)$$

We also expand the spinors as

$$\begin{aligned} |u_\xi|^2 &\simeq 1 - \frac{\Delta^2}{4\bar{E}^2} + O(\bar{E}^{-3}), \\ |v_\xi|^2 &\simeq \frac{\Delta^2}{4\bar{E}^2} + O(\bar{E}^{-3}). \end{aligned} \quad (55)$$

Combining all these, we find

$$\begin{aligned} \mathcal{M}^{\text{pp+aa}} &\simeq \frac{1}{\bar{E}} - \frac{M_Q^2 - M_q^2}{4\bar{E}^3} + \frac{\mu^2}{\bar{E}^3} + O(\bar{E}^{-5}), \\ \mathcal{M}^{\text{pa+ap}} &\simeq \frac{\Delta^2}{4\bar{E}^3} + O(\bar{E}^{-5}). \end{aligned} \quad (56)$$

Then, we can identify the UV divergences from the particle-particle and antiparticle-antiparticle contributions for the product $\mathcal{KM}$ as

$$\mathcal{K}^{\text{pp}} \mathcal{M}^{\text{pp+aa}} \simeq \frac{2}{\bar{E}} - \frac{M_Q(M_Q - M_q)}{\bar{E}^3} + \frac{2\mu^2}{\bar{E}^3} + O(\bar{E}^{-5}), \quad (57)$$

while it turns out that the particle-antiparticle contributions do not yield the UV divergent term in Π_Y ;

$$\mathcal{K}^{\text{pa}} \mathcal{M}^{\text{pa+ap}} \simeq O(\bar{E}^{-5}). \quad (58)$$

The divergent functions $\Pi_Y^{(1)}$ and $\Pi_Y^{(2)}$, which are to be used in Eq. (43), are

$$\begin{aligned} \Pi_Y^{(1)} &= 4g_Y^2 N_c \int_p \left[-\frac{1}{\bar{E}} + \frac{M_Q(M_Q - M_q)}{2\bar{E}^3} \right], \\ \Pi_Y^{(2)} &= 4g_Y^2 N_c \int_p \frac{-1}{\bar{E}^3}. \end{aligned} \quad (59)$$

Each term can be analytically evaluated by the dimensional regularization. We find

$$\begin{aligned} \int_p \frac{1}{\bar{E}} &= \frac{M_q^2 + \Delta^2}{8\pi^2} \left[-\frac{1}{\epsilon} - 1 + \ln \frac{M_q^2 + \Delta^2}{\Lambda^2} \right], \\ \int_p \frac{1}{\bar{E}^3} &= \frac{1}{4\pi^2} \left[\frac{1}{\epsilon} - \ln \frac{M_q^2 + \Delta^2}{\Lambda^2} \right]. \end{aligned} \quad (60)$$

Hence, the divergent functions are

$$\begin{aligned} \Pi_Y^{(1)} &= \frac{1}{\epsilon} \frac{g_Y^2 N_c}{2\pi^2} [\Delta^2 + M_q^2 - M_Q M_q + M_Q^2] \\ &\quad + \frac{g_Y^2 N_c}{2\pi^2} (\Delta^2 + M_q^2) \\ &\quad - \frac{g_Y^2 N_c}{2\pi^2} (\Delta^2 + M_q^2 - M_Q M_q + M_Q^2) \ln \frac{\Delta^2 + M_q^2}{\Lambda^2}, \\ \Pi_Y^{(2)} &= \frac{g_Y^2 N_c}{\pi^2} \left[-\frac{1}{\epsilon} + \ln \frac{M_q^2 + \Delta^2}{\Lambda^2} \right]. \end{aligned} \quad (61)$$

Now, the light-heavy effective potential can be renormalized. In the $\overline{\text{MS}}$ scheme, we set the counterterms as

$$\begin{aligned} \delta Z_Y &= -\frac{g_Y^2 N_c}{4\pi^2} \frac{1}{\epsilon}, & \delta m_Y^2 &= -\frac{g_Y^2 N_c M_Q^2}{2\pi^2} \frac{1}{\epsilon}, \\ \delta \kappa_{\phi Y} &= \frac{gg_Y^2 N_c M_Q}{4\pi^2} \frac{1}{\epsilon}, & \delta \lambda_{\phi Y} &= -\frac{g^2 g_Y^2 N_c}{4\pi^2} \frac{1}{\epsilon}. \end{aligned} \quad (62)$$

Canceling the divergences by counterterms, the renormalized light-heavy potential (41) now reads

$$V_{\text{MF}}^Y = \frac{(C_2^Y)_R}{2g_Y^2} \Delta_{Y_i}^2 + \frac{\lambda_Y + \delta \lambda_Y}{24g_Y^4} (\Delta_{Y_i}^2)^2 \quad (63)$$

with the renormalized quadratic coefficient

$$\begin{aligned}
(C_2^Y)_R = & m_Y^2 - 4\mu^2 + \frac{2\kappa_{\phi Y}}{g}M_q + \frac{2\lambda_{\phi Y}}{g^2}(M_q^2 + \Delta^2) \\
& + \frac{g_Y^2 N_c}{2\pi^2}(M_q^2 + \Delta^2) \\
& - \frac{g_Y^2 N_c}{2\pi^2}(\Delta^2 + M_q^2 - M_Q M_q + M_Q^2) \ln \frac{M_q^2 + \Delta^2}{\Lambda^2} \\
& + \mu^2 \frac{g_Y^2 N_c}{2\pi^2} \ln \frac{M_q^2 + \Delta^2}{\Lambda^2} \\
& + \Pi_Y^R(\mu).
\end{aligned} \tag{64}$$

To determine $\delta\lambda_Y$, we also need to compute a four-point function of fields Y . We skip such computations, since the coefficients of the Δ_Y^2 terms determine the onset of $Y_{u,d}$, provided that the appearance of $Y_{u,d}$ does not accompany the first order phase transition (Δ_Y smoothly increases from zero).

IV. ONSET OF HYPERONS

From now on, we read off the coefficients of Δ_Y^2 terms in Eq. (64). We determine the critical chemical potential where the coefficients of Δ_Y^2 terms vanish.

Below, we avoid details of hadronic interactions by setting $\kappa_{\phi Y}$ and $\lambda_{\phi Y}$ to zero. The effects of finite $\kappa_{\phi Y}$ and $\lambda_{\phi Y}$ are discussed in the Appendix. We also take our renormalization scale Λ to be M_0 , the vacuum effective quark mass, and all couplings should be regarded as those defined at $\Lambda = M_0$. Then, the renormalized vacuum mass of Y is given by

$$(m_Y^{\text{vac}})^2 = m_Y^2 + \Pi_Y^R(\mu = 0) + \frac{g_Y^2 N_c M_0^2}{2\pi^2}. \tag{65}$$

Eliminating the tree-level mass m_Y in favor of the renormalized vacuum mass m_Y^{vac} , the in-medium quadratic coefficient of Y can be written as

$$\begin{aligned}
(C_2^Y)_R = & (m_Y^{\text{vac}})^2 - 4\mu^2 + [\Pi_Y^R(\mu) - \Pi_Y^R(0)] \\
& + \frac{g_Y^2 N_c}{2\pi^2}(\Delta^2 + M_q^2 - M_0^2) \\
& - \frac{g_Y^2 N_c}{2\pi^2}(\Delta^2 + M_q^2 - M_Q M_q + M_Q^2) \ln \frac{M_q^2 + \Delta^2}{M_0^2} \\
& + \mu^2 \frac{g_Y^2 N_c}{2\pi^2} \ln \frac{M_q^2 + \Delta^2}{M_0^2}.
\end{aligned} \tag{66}$$

Assuming a transition to the phase of condensed Y is the second-order type, the Y -diquarks begin to condense at the chemical potential where the coefficient of the quadratic term vanishes.

A. Model parameters

In our analyses, we use the set of parameters

$$\begin{aligned}
g & \simeq 4.3, & \sqrt{-m_\phi^2} & \simeq 510 \text{ MeV}, \\
\lambda & \simeq 137, & h_\sigma^{1/3} & \simeq 112 \text{ MeV},
\end{aligned} \tag{67}$$

with which we obtain

$$\begin{aligned}
M_0 & \simeq 300 \text{ MeV}, & m_\sigma & \simeq 545 \text{ MeV}, \\
m_\pi & \simeq 140 \text{ MeV}, & f_\pi & \simeq 85 \text{ MeV}.
\end{aligned} \tag{68}$$

For parameters involving heavy quarks, we use

$$M_Q = 500 \text{ MeV}, \quad \kappa_{\phi Y} = \lambda_{\phi Y} = 0. \tag{69}$$

We take $g_Y/g = 1.0$ and $m_Y^{\text{vac}} = 500 \text{ MeV}$ as our reference values but vary them to delineate the effects of the statistical repulsion.

B. Baryonic matter of light quarks

First, we describe the evolution of a baryonic matter made of light quarks. Here, we switch off the hyperon and heavy quark sectors. The equation of state of matter is given by

$$P(\mu) = -V_{\text{MF}}^q(\mu; M_q^*, \Delta^*), \tag{70}$$

where M_q^* and Δ^* are the solutions of the gap equations. The quark number (n) and baryon number (n_B) densities are given by $n = N_c n_B = \partial P / \partial \mu$. The energy density is $\varepsilon = \mu n - P$. The general trends have been studied in the previous works, so here we review only a few important points; see Refs. [90,92] for more details.

We compare the results of purely hadronic model, i.e., tree-level and chiral perturbation theory (ChPT) results, to the one-loop results with the quark-diquark coupling. The low-energy theorem leads to the universal behaviors in the low density limit; the deviations of these model results at higher densities reflect the sensitivity to the hadron-hadron interactions and the importance of the quark substructure of hadrons.

Shown in Fig. 1 are the evolution of gaps (M_q, Δ) as functions of μ , at the one-loop and tree levels. The light diquarks begin to condense at $\mu = m_\pi/2$, and the chiral effective quark mass begins to decrease accordingly. The Δ grows rapidly just beyond the threshold. At higher density, the behaviors are very different in the purely hadronic model and model with the quark-diquark coupling. At tree level,⁴ Δ grows as $\Delta \sim \mu$, since, within the hadronic part, the quadratic term $\sim -\mu^2 \Delta^2$ must be balanced by the

⁴At tree level, we have to retune model parameters as they do not contain the quark loop effects. We use the tree-level relations $m_{\phi, \text{tree}}^2 = -\frac{m_\sigma^2 - 3m_\pi^2}{2}$, $\lambda_{\text{tree}} = \frac{2(m_\sigma^2 - m_\pi^2)}{\langle \sigma \rangle_{\text{tree}}^2}$, $\langle \sigma \rangle_{\text{tree}} \simeq 0.81 f_\pi$, with which the tree-level and one-loop results coincide for $\mu \leq m_\pi/2$.

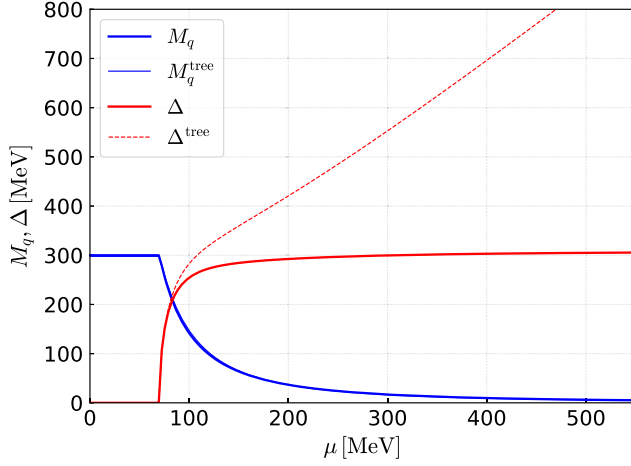


FIG. 1. The evolution of (M_q, Δ) as functions of μ . The onset of diquark condensate is $\mu = m_\pi/2$. The Δ at tree level grows as $\Delta \sim \mu$, while inclusion of the quark coupling tempers the growth, resulting $\Delta \sim M_0$. M_q and M_q^{tree} largely overlap, and the difference is not visible.

quartic (repulsive) term $\sim \lambda \Delta^4$. In contrast, with the quark-diquark couplings, the quark loop generates a μ^2 term, which can balance with the $\sim -\mu^2 \Delta^2 \ln \Delta^2$ term; hence, the strong μ^2 dependence is factored out, and the resulting Δ can be insensitive to μ . The asymptotic behavior of the pressure is $P \sim C_0 \mu^4 + C_1 \mu^2 \Delta^2 + \dots$, where C_0 and C_1 depend on the hadronic parameters only weakly; the details of hadronic parameters affect the value of Δ but not the coefficients. The bulk part is fixed by the quark descriptions. See Refs. [90,92] for more details.

Next, we examine the baryon density n_B as a function of μ (Fig. 2). We take $n_0 = 0.16 \text{ fm}^{-3}$ for our unit as this unit is often used for three-color QCD. We expect that the sizes of a baryon for two-color and three-color QCD do not differ significantly, $\sim 0.5\text{--}0.8 \text{ fm}$; then, the overlap of diquarks in

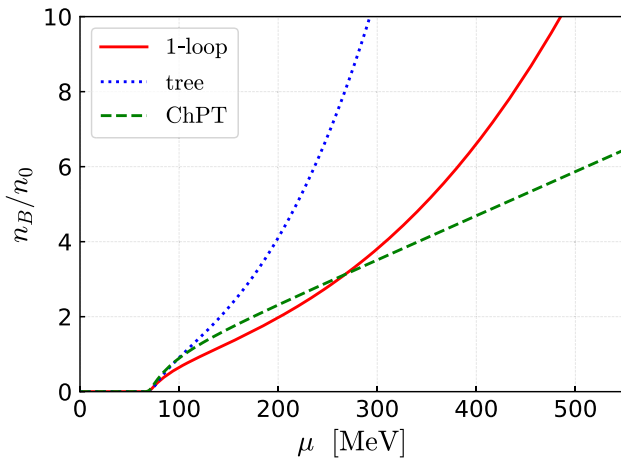


FIG. 2. The baryon density n_B as functions of μ for the one-loop, tree, and ChPT cases. We take $n_0 = 0.16 \text{ fm}^{-3}$ as a unit.

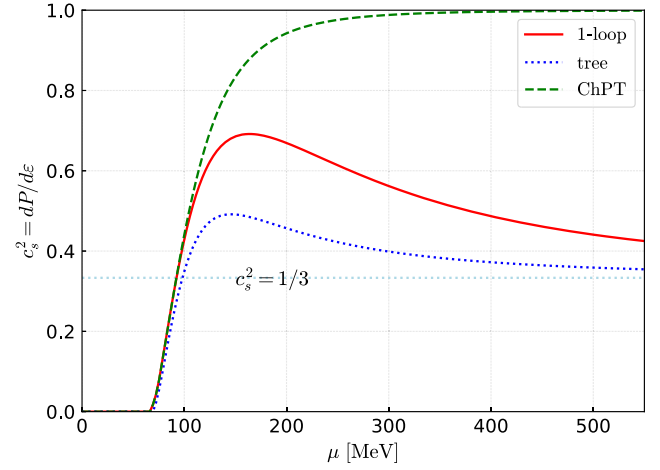


FIG. 3. The squared sound speed $c_s^2 = dP/d\varepsilon$ as functions of μ for the one-loop, tree, and ChPT cases. The conformal limit $1/3$ is also plotted for an eye guide.

QC₂D should occur around $\sim 5n_0$, which corresponds to $\mu \simeq 350 \text{ MeV}$. The results for different models coincide near the threshold $\mu = m_\pi/2$, but they soon begin to deviate at $\mu \sim 90\text{--}100 \text{ MeV}$.

In Fig. 3, we plot the squared sound speed $c_s^2 = dP/d\varepsilon$. All models show the similar growth of c_s^2 just above the threshold. Beyond $\mu \simeq 90\text{--}100 \text{ MeV}$, the ChPT approaches $c_s^2 = 1$, which comes from the behavior $P \sim f_\pi^2 \mu^2$, which should be regarded as an artifact at very large μ . The other two models yield the sound speed peaks around $\mu \sim 150\text{--}200 \text{ MeV}$ or $n_B \sim 1.5\text{--}2n_0$ but with the different magnitude, and then both relax to the conformal value $c_s^2 = 1/3$ after μ^4 -term dominates the pressure.

C. Onset of hyperons

Using the above-mentioned density-dependent gaps, we numerically evaluate the evolution of C_2^Y .

Shown in Fig. 4 are C_2^Y for various strength of the coupling g_Y and $(m_Y^{\text{vac}}, M_Q) = (500, 500) \text{ MeV}$. At tree level including the hadronic part only, $\sqrt{C_2^Y}$ drops from the vacuum value m_Y^{vac} to zero at $\mu = m_Y^{\text{vac}}/2$ or $\mu_B = m_Y^{\text{vac}}$. We recall that we are switching off the interactions among light hadrons (σ, π, D) and hyperons Y , i.e., setting $\kappa_{\phi Y} = \lambda_{\phi Y} = 0$, so the onset is determined independently from the light hadron sector.

Including quark loops, the hyperons Y are influenced by a matter formed by light quarks. The quark substructure effects in Y become stronger for a larger g_Y ; explicitly, g_Y characterizes the strength of the transition

$$Y \leftrightarrow qQ, \quad (71)$$

so that with a greater g_Y one has more chances to observe the composite structure through the coupling of qQ to the chemical potential. The $g_Y = 0$ corresponds to the tight-

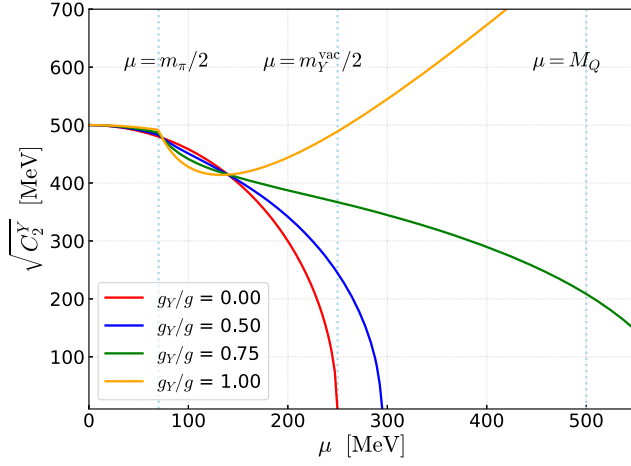


FIG. 4. The density evolution of the coefficient C_2^Y for the quadratic term of $Y_{u,d}$. When C_2^Y becomes negative, the hyperon fields condense. At tree level, the hyperon condensates emerge at $\mu = m_Y^{\text{vac}}/2$ or $\mu_B = m_Y^{\text{vac}}$. After including the coupling g_Y between quark and heavy-light diquark, the onset chemical potential is shifted to a higher value for a greater g_Y .

binding limit, and we only observe the elementary particle aspect of Y . For more realistic considerations, it is natural to expect its strength to be comparable to the Yukawa coupling between a light quark and a light-light diquark. In particular, if we assume $g_Y = g$, hyperons do not condense at least before the quark chemical potential reaches the heavy quark mass, $\mu = M_Q$. The region $\mu \geq M_Q$ is beyond the scope of our paper, since one needs to include heavy quarks as dynamical degrees of freedom.

Whether the statistical repulsion becomes important or not depends on how early light quark states are occupied. This can be seen in Figs. 5 for $g_Y/g = 0.75$ and 6 for

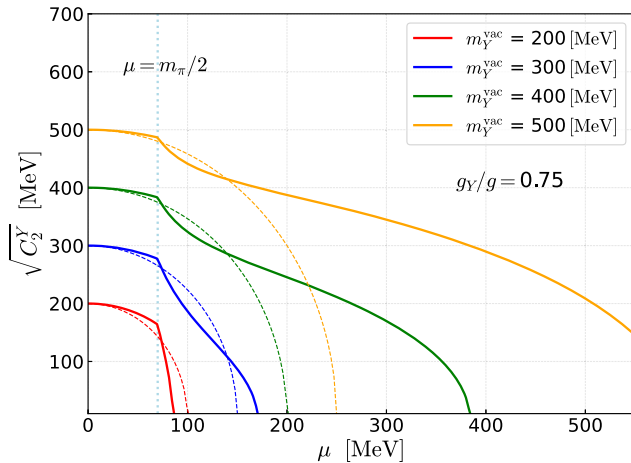


FIG. 5. The density evolution of C_2^Y for the vacuum hyperon mass, $m_Y^{\text{vac}} = 200, 300, 400$, and 500 MeV, with $g_Y/g = 0.75$. The dashed lines are the results for $g_Y = 0$. For a larger m_Y^{vac} , more quark states are occupied so that the statistical repulsion sets in before Y condenses.

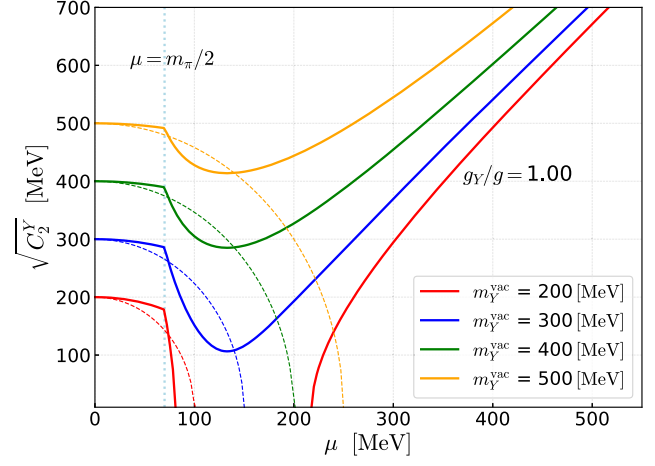


FIG. 6. The same plot as Fig. 5 except $g_Y/g = 1.0$.

$g_Y/g = 1.0$, where m_Y^{vac} is varied from 200 to 500 MeV. If m_Y^{vac} is very close to m_π , the critical chemical potential $\mu = m_Y^{\text{vac}}/2$ is reached before the light quarks substantially occupy the low momentum states. In this case, the shift of the critical chemical potential is small.

The case with $m_Y^{\text{vac}} = 200$ MeV and $g_Y/g = 1.0$ deserves some comment. Here, Y condenses at $\mu < m_Y^{\text{vac}}/2$, but the condensates melt around $\mu \simeq 220$ MeV. The condensation below the tree-level estimate $\mu < \mu^{\text{tree}} = m_Y^{\text{vac}}/2$ is not regarded as generic; just above $\mu = m_\pi/2$, the details of the chiral restoration and diquark formation are as important as the statistical repulsion. At higher μ , the trend of M_q and Δ becomes stable, and the self-energy of Y is mainly determined by the statistical repulsion. In particular, for $g_Y/g = 1.0$ and $m_Y^{\text{vac}} = 200$ MeV, the repulsive effects which grow with density can destroy the condensates of Y .

V. CONCLUSIONS

We study the onset of “hyperons” in dense QC₂D by introducing heavy quarks that mimic strange quarks. The introduction of the heavy doublet is motivated to avoid the sign problem in lattice Monte Carlo simulations.

QCD and QC₂D differ because baryons are fermions in the former but bosons in the latter. Meanwhile, one can still study the statistical constraints due to quarks in the same way for both cases. In this respect, it should be useful to delineate the properties of matter with hyperons in QC₂D.

In the dilute regime, hyperons behave effectively like elementary particles in response to an increasing chemical potential. Once light diquarks condense, however, light quarks in hyperons are subject to the statistical constraint and affect the hyperon self-energies. If we drop off quark contributions by hand (tree-level approximation), hyperons begin to condense at $\mu_B (= 2\mu) = m_Y^{\text{vac}}$. This trend changes by allowing hyperons to transform into quark intermediate states which are subject to the statistical constraints. A stronger coupling shifts the onset chemical potential for

hyperons to a larger value. For $g_Y = g$, we found that hyperons do not condense except m_Y very close to m_π , and the strangeness enters the system not through composite hadrons but through elementary strange quarks which appear for $\mu > M_Q$.

In this work, we study hyperons up to the chemical potential just before the onset of Y or heavy quarks Q , i.e., $\mu_B < m_Y^{\text{vac}}$ or $\mu_B < 2M_Q$. With this restriction, we use light quark propagators in the presence of M_q and Δ , while treating heavy quarks as impurities. This allows us simple computations. Meanwhile, it is also important to study how hyperons behave beyond their onset and how the EOS softens. We expect that the statistical repulsion caused by preoccupied light quarks not only delays the appearance of hyperons but also tempers the softening associated with their emergence. The EOS after the emergence of hyperons can be computed by using light and heavy propagators in the background of M_q , Δ , and Y . This will be discussed elsewhere.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX: HADRONIC INTERACTION TERMS

While the interactions among light hadrons and hyperons are not considered in the main text, we roughly

examine the effect of the coefficients $\lambda_{\phi Y}$ and $\kappa_{\phi Y}$. Based on Eq. (64), the in-medium quadratic coefficient of Y with finite $\lambda_{\phi Y}$ and $\kappa_{\phi Y}$ can be written as

$$(\tilde{C}_2^Y)_R = (C_2^Y)_R + \frac{2\kappa_{\phi Y}}{g}(M_q - M_0) + \frac{2\lambda_{\phi Y}}{g^2}(\Delta^2 + M_q^2 - M_0^2). \quad (\text{A1})$$

The coefficient $(C_2^Y)_R$ is defined in Eq (66) and contains the renormalized vacuum mass of Y , which should be modified as

$$(m_Y^{\text{vac}})^2 = m_Y^2 + \Pi_Y^R(\mu = 0) + \frac{g_Y^2 N_c M_0^2}{2\pi^2} + \frac{2\kappa_{\phi Y}}{g}M_0 + \frac{2\lambda_{\phi Y}}{g^2}M_0^2. \quad (\text{A2})$$

First, we consider the term with $\lambda_{\phi Y}$ in $(\tilde{C}_2^Y)_R$. From the observation in Fig. 1, Δ and M_q almost satisfy $M_q^2 + \Delta^2 \simeq M_0^2$, which can fluctuate just above the onset of the diquark condensate at $\mu = m_\pi/2$. Therefore, this term only gives rise to a slight fluctuation regardless of the sign of $\lambda_{\phi Y}$.

On the other hand, the term with $\kappa_{\phi Y}$ could have a sizable effect at $\mu \geq m_\pi/2$. For negative values of $\kappa_{\phi Y}$, this term shifts the onset of hyperons to higher densities, whereas for positive values, it can cancel the onset shift induced by statistical repulsion. For example, in the case with $m_Y^{\text{vac}} = 500$ MeV and $g_Y/g = 1.0$, we roughly estimate that $\kappa_{\phi Y} \simeq 1600$ MeV, which is unrealistically large, is required to cancel the shift of the onset induced by the statistical repulsion. For smaller hyperon masses or weaker coupling such as $g_Y/g = 0.5$, hyperons can emerge before the statistical repulsion sufficiently develops, and as a consequence, the critical value of $\kappa_{\phi Y}$ drops to ~ 100 MeV; in the dilute regime, the EOS is sensitive to the hadronic coupling, as expected.

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