

Scalar-induced gravitational waves as a probe of primordial parity-violation

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Parity violation leaves telltale trails in many cosmological observables. We illustrate parity-odd primordial scalar trispectra, that despite being of modest strength, impart detectable chirality to scalar-induced gravitational waves (SIGWs). This allows us to impose strong bounds on the parity-odd part of trispectrum. Over certain scales, we find SIGWs directly quantify parity violation in primordial non-Gaussianity, unobscured by the Gaussian contribution. Our results call for treatment of SIGWs and parity-odd trispectrum as complementary predictions of parity-violating theories.

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I. INTRODUCTION

The phenomenon of scalar perturbations inducing tensor perturbations at the second order of perturbation theory has been long examined in the literature [1–10] (see [11] for a review). Recently, these scalar-induced gravitational waves (SIGWs) have been considered as a potential candidate to explain the signature of stochastic gravitational waves (GWs) claimed by the Pulsar Timing Array Collaboration [12–24]. There have also been efforts in the literature to examine the generation and direct interferometric detection of chirality in stochastic GWs [25–35,35–45,45–60].

Violation of the symmetry of parity in the Universe is a compelling phenomenon with several cosmological implications, such as cosmic birefringence [61–66]. The presence of chirality in cosmological GW shall be a signature of parity violation in the Universe. Another signature shall be in the N -point correlation functions of the perturbations, with the lowest order (for scalar perturbations) sensitive to parity being the irreducible four-point correlation, the trispectrum $\mathcal{T}$. There have been recent attempts searching for the parity-odd component of the trispectrum in the dataset of the Baryon Oscillation Spectroscopic Survey [67–70].

In the presence of a non-negligible trispectrum $\mathcal{T}$, it is logical to expect its signatures in sourcing SIGWs. Moreover, if there is a parity-odd component to the trispectrum $\mathcal{T}_{\text{odd}}$, the signature of parity violation can be inherited by the SIGWs, thereby acquiring a nonvanishing chirality [44]. In this analysis, we examine such an acquisition of chirality by SIGWs from $\mathcal{T}_{\text{odd}}$. We quantify the chirality by computing the ratio of spectral densities of the degree of circular polarization V to the intensity I of the scalar-induced tensor perturbations using certain generic templates of the trispectrum. The templates of trispectrum we employ are fair representatives of the models of inflation violating parity and enable us to relate the induced chirality to the behavior of $\mathcal{T}_{\text{odd}}$. We arrive at theoretical bound on the amplitude of $\mathcal{T}_{\text{odd}}$, utilizing the inequality relating the Stokes parameters V and intensity I of the GWs. We also identify interesting regimes of wave numbers, in cases with peaked forms of scalar power and trispectra, where the chirality is determined by the ratio of parity-odd to parity-even parts of $\mathcal{T}$.

This generic mechanism shall have interesting implications for different classes of parity-violating scenarios. For instance, models with gravitational Chern-Simons coupling predict a highly suppressed chirality for primary tensor perturbations (for example, [71–78]), but could lead to substantial amplitude of $\mathcal{T}_{\text{odd}}$ [76–78]. Hence SIGWs shall play a dominant role in determining the chirality of their total system of GWs. On the other hand, models with coupling between axion and gauge fields predict their primary GW to be maximally chiral (for instance, [27,29,79–81]). In such models, the chirality of the total GWs shall acquire new contributions from SIGWs.

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Regarding notations, we denote the conformal time as η , the reduced Planck mass as $M_{\text{Pl}} = (8\pi G)^{-1/2}$, and set $\hbar = c = 1$ throughout the manuscript.

II. ROLE OF TRISPECTRUM IN SIGWS

The two-point correlation of the scalar-induced tensor perturbations in Fourier space h_k^λ is given by [10,82–84]

$$\begin{aligned} \langle h_{k_1}^{\lambda_1} h_{k_2}^{\lambda_2} \rangle &= \frac{8}{k_1 k_2 \eta^2} \int \int \frac{d^3 \mathbf{q}_1 d^3 \mathbf{q}_2}{(2\pi)^6} Q^{\lambda_1}(\mathbf{k}_1, \mathbf{q}_1) Q^{\lambda_2}(\mathbf{k}_2, \mathbf{q}_2) \\ &\times \tilde{I}_2 \left(\frac{q_1}{k_1}, \frac{|\mathbf{k} - \mathbf{q}_1|}{k_1}, \frac{q_2}{k_2}, \frac{|\mathbf{k} - \mathbf{q}_2|}{k_2} \right) \\ &\times \langle \mathcal{R}_{\mathbf{q}_1} \mathcal{R}_{\mathbf{k}_1 - \mathbf{q}_1} \mathcal{R}_{\mathbf{q}_2} \mathcal{R}_{\mathbf{k}_2 - \mathbf{q}_2} \rangle. \end{aligned} \quad (1)$$

The function $\tilde{I}_2$ captures the factors arising from the transfer functions relating the Bardeen potential to the primordial scalar perturbation $\mathcal{R}_k$ along with the Green's function associated with the inhomogeneous solution of h_k^λ . The factor of Q^λ for a given polarization λ arises from the contraction of the polarization tensor with the momenta of integration (cf. the Appendix). The scalar four-point function sourcing the two-point function of h_k^λ shall have a component reducible to the scalar power spectrum $\mathcal{P}_S$ and an irreducible component, the trispectrum $\mathcal{T}$, which is defined by the relation

$$\begin{aligned} \langle \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \mathcal{R}_{\mathbf{k}_3} \mathcal{R}_{\mathbf{k}_4} \rangle_c &= (2\pi)^3 \mathcal{T}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) \\ &\times \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4). \end{aligned} \quad (2)$$

The dimensionless power spectrum associated with h_k^λ , $\mathcal{P}_h^{\lambda_1 \lambda_2}$ can then be computed as

$$\langle h_{\mathbf{k}}^{\lambda_1} h_{\mathbf{k}'}^{\lambda_2} \rangle = (2\pi)^3 \frac{2\pi^2}{k^3} \mathcal{P}_h^{\lambda_1 \lambda_2}(k, \eta) \delta^{(3)}(\mathbf{k} + \mathbf{k}'), \quad (3)$$

which shall be related to $\mathcal{P}_S$ and $\mathcal{T}$ as

$$\begin{aligned} \mathcal{P}_h^{\lambda_1 \lambda_2}(k, \eta) &= \frac{4}{\pi} \frac{k^3}{(k\eta)^2} \left[\int d^3 \mathbf{q} Q^{\lambda_1}(\mathbf{k}, \mathbf{q}) Q^{\lambda_2}(-\mathbf{k}, -\mathbf{q}) \right. \\ &\times \tilde{I}_1 \left(\frac{q}{k}, \frac{|\mathbf{k} - \mathbf{q}|}{k} \right) \frac{\mathcal{P}_S(q) \mathcal{P}_S(|\mathbf{k} - \mathbf{q}|)}{q^3 |\mathbf{k} - \mathbf{q}|^3} \\ &+ \frac{1}{\pi} \int \int \frac{d^3 \mathbf{q}_1 d^3 \mathbf{q}_2}{(2\pi)^6} Q^{\lambda_1}(\mathbf{k}, \mathbf{q}_1) Q^{\lambda_2}(-\mathbf{k}, -\mathbf{q}_2) \\ &\times \tilde{I}_2 \left(\frac{q_1}{k}, \frac{|\mathbf{k} - \mathbf{q}_1|}{k}, \frac{q_2}{k}, \frac{|\mathbf{k} - \mathbf{q}_2|}{k} \right) \\ &\left. \times \mathcal{T}(\mathbf{q}_1, \mathbf{k} - \mathbf{q}_1, -\mathbf{q}_2, \mathbf{q}_2 - \mathbf{k}) \right]. \end{aligned} \quad (4)$$

We have retained the arbitrary combination of λ_1 and λ_2 in defining $\mathcal{P}_h$. If we inspect the possible combinations of λ_1 ,

λ_2 in the above expression, we shall find that only terms with $\lambda_1 = \lambda_2$ survive the integration, essentially due to the statistical homogeneity and isotropy of the perturbations.

We may identify $\mathcal{P}_h$ associated with the intensity I of the correlation as

$$\mathcal{P}_h^I(k, \eta) = \sum_{\lambda_1, \lambda_2} \mathcal{P}_h^{\lambda_1 \lambda_2}(k, \eta) \delta^{\lambda_1 \lambda_2} = \sum_{\lambda} \mathcal{P}_h^{\lambda \lambda}(k, \eta). \quad (5)$$

Written explicitly in helical basis $\lambda = [L, R]$, $\mathcal{P}_h^I = \mathcal{P}_h^{LL} + \mathcal{P}_h^{RR}$. We may note here that only the even part of trispectrum $\mathcal{T}_{\text{even}}$ contributes to $\mathcal{P}_h^I$. Note that the energy density of SIGW ρ_{GW} is related to $\mathcal{P}_h^I$ (using the sub-Hubble behavior of h_k^λ) as

$$\frac{d\langle \rho_{\text{GW}} \rangle}{d \ln k} = \frac{M_{\text{Pl}}^2}{16} \left(\frac{k^2}{a^2} \right) \mathcal{P}_h^I(k, \eta), \quad (6)$$

with a being the scale factor.

On the other hand, we may compute $\mathcal{P}_h$ associated with the degree of circular polarization V as

$$\begin{aligned} \mathcal{P}_h^V(k, \eta) &= \frac{4k^3}{\pi^2 (k\eta)^2} \int \int \frac{d^3 \mathbf{q}_1 d^3 \mathbf{q}_2}{(2\pi)^6} [Q^L(\mathbf{k}, \mathbf{q}_1) Q^L(-\mathbf{k}, -\mathbf{q}_2) \\ &- Q^R(\mathbf{k}, \mathbf{q}_1) Q^R(-\mathbf{k}, -\mathbf{q}_2)] \\ &\times \tilde{I}_2 \left(\frac{q_1}{k}, \frac{|\mathbf{k} - \mathbf{q}_1|}{k}, \frac{q_2}{k}, \frac{|\mathbf{k} - \mathbf{q}_2|}{k} \right) \\ &\times \mathcal{T}_{\text{odd}}(\mathbf{q}_1, \mathbf{k} - \mathbf{q}_1, -\mathbf{q}_2, \mathbf{q}_2 - \mathbf{k}). \end{aligned} \quad (7)$$

By construction, $\mathcal{P}_h^V$ is the difference between the power spectra of left and right helical modes $\mathcal{P}_h^{LL} - \mathcal{P}_h^{RR}$. The factor $[Q^L(\mathbf{k}, \mathbf{q}_1) Q^L(-\mathbf{k}, \mathbf{q}_2) - Q^R(\mathbf{k}, \mathbf{q}_1) Q^R(-\mathbf{k}, \mathbf{q}_2)]$ in $\mathcal{P}_h^V$ is imaginary and parity odd, such that it extracts the contribution from the parity-odd component of the trispectrum $\mathcal{T}_{\text{odd}}$. Notice that if we consider individual helicities, $\mathcal{T}_{\text{odd}}$ leads to an imbalance between the power in left and right helical modes, by diminishing one while amplifying the other by the same amount. Apart from $\mathcal{P}_S$ and $\mathcal{T}$, $\mathcal{P}_h^I$ may receive contribution from the scalar bispectrum [17,82–89], but $\mathcal{P}_h^V$ shall remain insensitive to it as the bispectrum preserves parity. As to the other Stokes parameters of this system, it can be shown that $Q = U = 0$.

Similar to $\langle \rho_{\text{GW}} \rangle$ we may relate the degree of circular polarization in the SIGW V_{GW} , to $\mathcal{P}_h^V$ as

$$\frac{d\langle V_{\text{GW}} \rangle}{d \ln k} = \frac{M_{\text{Pl}}^2}{16} \left(\frac{k^2}{a^2} \right) \mathcal{P}_h^V(k, \eta). \quad (8)$$

The degree of chirality in k space is then quantified by the ratio

$$\Pi_k \equiv \left(\frac{d\langle V_{\text{GW}} \rangle}{d \ln k} \right) / \left(\frac{d\langle \rho_{\text{GW}} \rangle}{d \ln k} \right) \quad (9)$$

$$= \frac{\mathcal{P}_h^V(k, \eta)}{\mathcal{P}_h^I(k, \eta)} = \frac{\mathcal{P}_h^{LL}(k, \eta) - \mathcal{P}_h^{RR}(k, \eta)}{\mathcal{P}_h^{LL}(k, \eta) + \mathcal{P}_h^{RR}(k, \eta)}. \quad (10)$$

Since the time dependences are identical in both $\mathcal{P}_h^I(k, \eta)$ and $\mathcal{P}_h^V(k, \eta)$, the ratio is time independent. Additionally, these time dependences are also canceled by the prefactor $(k/a)^2$ while relating them to ρ_{GW} and V_{GW} [cf. Eqs. (6) and (8)]. So, for convenience in further usage, we define the combination $\mathcal{P}_h^{I,V}(k) = (k\eta)^2 \mathcal{P}_h^{I,V}(k, \eta)$.

Note that the Stokes parameters are constrained by the relation $I^2 \geq Q^2 + U^2 + V^2$. With $Q = U = 0$, the relation implies $|V/I| \leq 1$.

III. CHIRALITY INDUCED

For computing the chirality induced in SIGWs, we use parametric forms of the trispectrum $\mathcal{T}$ that capture the salient features of it when generated by parity-violating inflationary models [76,77,90–98]. The parity-odd part of the trispectrum shall be purely imaginary and contain vector triple products of the wave vectors involved to capture the change of signature under inversion of coordinates. With these considerations, we examine two parametrizations of $\mathcal{T}$.

Template 1: We construct the odd and even parts of the first template of $\mathcal{T}$ as

$$\begin{aligned} \mathcal{T}_{\text{odd}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) &= i\tilde{\tau}_{\text{NL}}\beta(\widehat{\mathbf{k}_1 + \mathbf{k}_2}, \hat{\mathbf{k}}_1, \hat{\mathbf{k}}_3)P(k_1)P(k_3) \\ &\quad \times P(|\mathbf{k}_1 + \mathbf{k}_2|) + 23 \text{ permutations}, \end{aligned} \quad (11a)$$

$$\begin{aligned} \mathcal{T}_{\text{even}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) &= 2\tau_{\text{NL}}P(k_1)P(k_3)P(|\mathbf{k}_1 + \mathbf{k}_2|) \\ &\quad + 11 \text{ permutations}. \end{aligned} \quad (11b)$$

The overall amplitudes of $\mathcal{T}_{\text{odd}}$ and $\mathcal{T}_{\text{even}}$ are set by the dimensionless real parameters $\tilde{\tau}_{\text{NL}}$ and τ_{NL} , respectively. The parity-violating role in $\mathcal{T}_{\text{odd}}$ is played by the function $\beta(\hat{\mathbf{k}}_1, \hat{\mathbf{k}}_2, \hat{\mathbf{k}}_3) = \hat{\mathbf{k}}_1 \cdot (\hat{\mathbf{k}}_2 \times \hat{\mathbf{k}}_3)$, which under inversion ($\mathbf{k}_i \rightarrow -\mathbf{k}_i$) acquires a negative sign. The quantity $P(k)$ is the dimensionful scalar power spectrum, related to the dimensionless $\mathcal{P}_S(k)$ as $P(k) = 2\pi^2 \mathcal{P}_S(k)/k^3$. Having the structure of $\mathcal{T}$ expressible in terms of $\mathcal{P}_S$, the corresponding $\mathcal{P}_h^V$ and $\mathcal{P}_h^I$ can be computed, once the form of $\mathcal{P}_S$ is specified. We shall work with two cases of $\mathcal{P}_S$: a scale-invariant form of $\mathcal{P}_S(k) = A_S$ and $\mathcal{P}_S(k)$ containing a log-normal peak.

We shall begin with the case of $\mathcal{P}_S(k) = A_S$. Rewriting $\mathcal{T}_{\text{odd}}$ and $\mathcal{T}_{\text{even}}$ in terms of A_S , τ_{NL} , and $\tilde{\tau}_{\text{NL}}$, we compute $\mathcal{P}_h^V$ and $\mathcal{P}_h^I$ numerically to obtain the measure of chirality as

$$\Pi_k = \frac{\tilde{\tau}_{\text{NL}} A_S^3 \mathcal{I}_V^{(2)}}{A_S^2 \mathcal{I}_I^{(1)} + \tau_{\text{NL}} A_S^3 \mathcal{I}_I^{(2)}}, \quad (12)$$

where $\mathcal{I}_I^{(1)}$, $\mathcal{I}_I^{(2)}$, and $\mathcal{I}_V^{(2)}$ are the numerical factors arising from the integrals over $\mathbf{q}_1$ and $\mathbf{q}_2$ in Eqs. (4) and (7). The factor $\mathcal{I}_I^{(1)}$ arises from performing the three-dimensional integral containing product of $\mathcal{P}_S$, while $\mathcal{I}_I^{(2)}$ arises from the six-dimensional integral containing $\mathcal{T}_{\text{even}}$ in Eq. (4), while computing $\mathcal{P}_h^I = \sum_{\lambda} \mathcal{P}_h^{\lambda\lambda}$. The factor $\mathcal{I}_V^{(2)}$ arises from performing the six-dimensional integral containing $\mathcal{T}_{\text{odd}}$ in Eq. (7). We should note that, if we are to include contributions from the bispectrum at the same order in A_S , parametrized by f_{NL} with a template of choice (say local), then the expression of Π_k shall extend to be

$$\Pi_k = \frac{\tilde{\tau}_{\text{NL}} A_S^3 \mathcal{I}_V^{(2)}}{A_S^2 \mathcal{I}_I^{(1)} + A_S^3 \left(f_{\text{NL}}^2 \mathcal{I}_I^{(3)} + \tau_{\text{NL}} \mathcal{I}_I^{(2)} \right)}, \quad (13)$$

where the additional factor of $\mathcal{I}_I^{(3)}$ shall arise from the six-dimensional integral capturing the contribution from the bispectrum [82–85]. However, since we are interested in the imprints due to the trispectrum, we shall not delve into explicit computation related to bispectrum in our analysis.

Motivated by perturbativity, in this case we assume that the contribution from the bispectrum and $\mathcal{T}_{\text{even}}$ to $\mathcal{P}_h^I$ is subdominant to the contribution from $\mathcal{P}_S^2$, i.e., $(f_{\text{NL}}^2 \mathcal{I}_I^{(3)}, \tau_{\text{NL}} \mathcal{I}_I^{(2)}) A_S < \mathcal{I}_I^{(1)}$ [99]. Hence,

$$\Pi_k \simeq \frac{\tilde{\tau}_{\text{NL}} A_S^3 \mathcal{I}_V^{(2)}}{A_S^2 \mathcal{I}_I^{(1)}} \simeq -\frac{7}{2\pi} \tilde{\tau}_{\text{NL}} A_S, \quad (14)$$

with the prefactor determined from numerical computation of the integrals (see Appendix for details of numerical integration). Setting $A_S = 2 \times 10^{-9}$ consistent with the normalization of $\mathcal{P}_S$ over cosmic microwave background (CMB) scales [100] and $\tilde{\tau}_{\text{NL}} = 1$, we obtain $\Pi_k \simeq -2 \times 10^{-9}$. Since, we have assumed $\mathcal{P}_S(k) = A_S$ and constructed $\mathcal{T}_{\text{odd}}$ in terms of it, we obtain scale-invariant $\mathcal{P}_h^V$ and $\mathcal{P}_h^I$. The Stokes inequality $|V/I| \leq 1$ in this case becomes $|\Pi_k| \leq 1$, which translates to model parameters as

$$|\tilde{\tau}_{\text{NL}}| A_S \lesssim 1. \quad (15)$$

This limit mimics the bound due to perturbativity on τ_{NL} , although it is on $\tilde{\tau}_{\text{NL}}$ and stems from Stokes inequality. It sets a bound of $|\tilde{\tau}_{\text{NL}}| \lesssim 5 \times 10^8$. Note that this is a purely theoretical constraint and it does not rely on the exact amplitude of tensor power spectra over CMB scales. For comparison with observational bound on $\tilde{\tau}_{\text{NL}}$, the closest available constraint is on a template of trispectrum arising from Chern-Simons coupling during inflation, which resembles Eq. (11), barring factors involving scalar products of wave vectors [68,90]. The constraint on parameters

equivalent to $\tilde{\tau}_{\text{NL}}$ in this case is $\tilde{\tau}_{\text{NL}} = 570 \pm 780$ from galaxy surveys [68] and $\tilde{\tau}_{\text{NL}} = (-1.4 \pm 1.6) \times 10^4$ from CMB [101]. Our constraint on $\tilde{\tau}_{\text{NL}}$ is weaker by about 3–4 orders of magnitude. However, we can apply the above arguments to scales smaller than CMB. There, if A_S reaches higher amplitudes and stays scale invariant over a range of scales, the bound of $\tilde{\tau}_{\text{NL}} \lesssim A_S^{-1}$ shall become proportionately stronger.

We repeat the exercise for the case where $\mathcal{P}_S$ has a log-normal peak and is enhanced over small scales. Such a case is relevant for generating SIGWs of amplitudes detectable by current or upcoming observational missions and also widely considered in the context of production of primordial black holes (PBHs) [102–108]. The form of $\mathcal{P}_S$ in this case is parametrized as

$$\mathcal{P}_S(k) = A_S \left(\frac{k}{k_*} \right)^{n_S-1} + \frac{A_p}{\sqrt{2\pi\sigma_p^2}} \exp \left[-\frac{1}{2\sigma_p^2} \ln^2 \left(\frac{k}{k_p} \right) \right]. \quad (16)$$

We set $A_S = 2 \times 10^{-9}$ as before and $n_S = 0.96$ to be consistent with CMB over large scales. We choose $A_p = 10^{-2}$ so as to mimic models that generate a significant amount of PBHs and SIGWs. Such $\mathcal{P}_S$, when plugged in to Eqs. (11), in turn, leads to a peaked trispectrum. In this case, we account for the contribution from $\mathcal{T}_{\text{even}}$ to $\mathcal{P}_h$ along with the reducible term. We set $\tau_{\text{NL}} = \tilde{\tau}_{\text{NL}} = 1$ to numerically compute $\mathcal{P}_h^I$ and $\mathcal{P}_h^V$ and examine the chirality induced.

We present the behavior of $\mathcal{P}_h^V(k)$ and $\mathcal{P}_h^I(k)$ in this case along with their ratio Π_k in Fig. 1. We vary the width of the peak σ_p over a range of values to vary the feature from a sharp spike to a smooth curve and observe several interesting features. First, chirality alters sign around the peak, i.e., Π_k is negative (right circular) away from the peak

over $k \ll k_p$ and positive (left circular) just after the peak $k \gtrsim k_p$. Second, $\tilde{\tau}_{\text{NL}} = 1$ easily induces chirality of at least $\Pi_k \simeq 10^{-3}$ around the peak. Importantly, at $k \simeq 3k_p$, Π_k reaches as high as $\Pi_k \simeq 0.1$ –1, depending on the width of the peak.

Focusing specifically over this region, we find that the behavior of $\mathcal{P}_h^I(k)$ around $k \simeq 3k_p$ is dominated by contribution from $\mathcal{T}_{\text{even}}$ (and bispectrum, refer to [84], for instance). Hence, the degree of chirality can be approximated as

$$\Pi_{k \simeq 3k_p} \simeq \frac{\tilde{\tau}_{\text{NL}} \mathcal{I}_V^{(2)}}{\tau_{\text{NL}} \mathcal{I}_I^{(2)}}, \quad (17)$$

where $\mathcal{I}_{I,V}^{(2)}$ are numerical factors arising from the integrations [cf. Eqs. (4) and (7)], as mentioned before. For the trispectrum with similar shapes for parity-odd and parity-even components (barring β), $\mathcal{I}_I^{(2)} \simeq \mathcal{I}_V^{(2)}$, implying degree of chirality to be $\Pi_{k \simeq 3k_p} \simeq \tilde{\tau}_{\text{NL}}/\tau_{\text{NL}}$, the ratio of amplitudes of the parity-odd to parity-even parts of the trispectrum. Examining Fig. 1 in this range, Π_k reaches substantial values informing us of the above-mentioned ratio. However, as noted earlier, the above estimate does not include the contribution from bispectrum $A_S^3 f_{\text{NL}}^2 \mathcal{I}_I^{(3)}$ to $\mathcal{P}_h^I$ [cf. Eq. (13)]. If we compute this contribution, the ratio shall get modified as

$$\Pi_{k \simeq 3k_p} \simeq \frac{\tilde{\tau}_{\text{NL}} \mathcal{I}_V^{(2)}}{f_{\text{NL}}^2 \mathcal{I}_I^{(3)} + \tau_{\text{NL}} \mathcal{I}_I^{(2)}}, \quad (18)$$

with factors of $\mathcal{I}_I^{(3)} \simeq \mathcal{I}_I^{(2)}$ arising from similar integrals. For values of $f_{\text{NL}} < \tau_{\text{NL}}$, $\Pi_{k \simeq 3k_p}$ reduces from Eq. (17) with

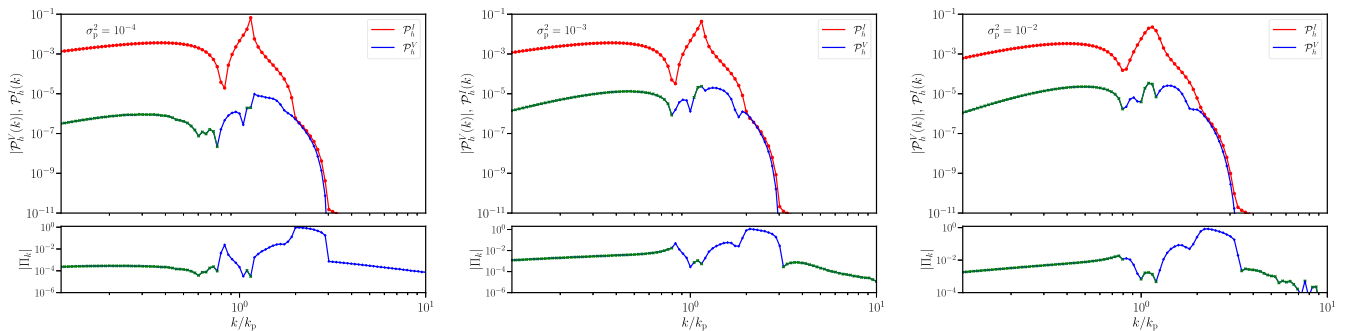


FIG. 1. We illustrate the spectral densities $\mathcal{P}_h^V(k)$ and $\mathcal{P}_h^I(k)$ computed for template 1 of $\mathcal{T}$ [Eq. (11)], for the case of $\mathcal{P}_S(k)$ with a log-normal peak [Eq. (16)], focusing over a range of scales close to the peak. We present $\mathcal{P}_h^I(k)$ and $\mathcal{P}_h^V(k)$ in red and blue curves, respectively, and indicate negative values of $\mathcal{P}_h^V(k)$ in green. We have set $A_p = 10^{-2}$, $\tau_{\text{NL}} = \tilde{\tau}_{\text{NL}} = 1$, and vary the width of the peak as $\sigma_p^2 = [10^{-4}, 10^{-3}, 10^{-2}]$ (as marked in each panel), inspecting the peak shift from a very sharp feature to a rather smooth feature in the spectrum. We also present the degree of chirality Π_k at the bottom of each panel (positive values in blue and negative in green). Note that $\mathcal{P}_h^I(k)$ is computed including contributions from both $\mathcal{P}_S$ and $\mathcal{T}_{\text{even}}$. We observe the following behaviors from this exercise: Π_k is highly scale dependent, changing signature around the peak ($k \simeq k_p$), and of opposite sign compared to the region away from the peak (say, $k \ll k_p$). Π_k is considerably large over the wave numbers past the peak at $k \simeq 3k_p$, reaching as high as $\Pi_k \simeq 0.1$ –1.

corrections of order $f_{\text{NL}}^2/\tau_{\text{NL}}$. When $f_{\text{NL}}^2 = \tau_{\text{NL}}$, we see that $\Pi_{k \simeq 3k_p}$ is diminished roughly by a factor of 2. For $f_{\text{NL}}^2 > \tau_{\text{NL}}$, $\Pi_{k \simeq 3k_p} \simeq \tilde{\tau}_{\text{NL}}/f_{\text{NL}}^2$. Therefore, the chirality of SIGWs, over a range of scales adjacent to the peak, provides a direct window to observe the relative amplitude of parity violation present in primordial scalar non-Gaussianity.

Template 2: We consider another template of $\mathcal{T}$ to inspect the scenario where the trispectrum has an intrinsic scale dependence, apart from the behavior of $\mathcal{P}_S$. Such a template is motivated by models of inflation that generate peaks in the spectra, often leading to non-Gaussianity parameters such as f_{NL} , having nontrivial scale dependences [83,94,109–114]. We model the odd and even parts of $\mathcal{T}$ as containing intrinsic peaks as

$$\begin{aligned} \mathcal{T}_{\text{odd}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = & i\tilde{\tau}_{\text{NL}} \left\{ 1 + \frac{\alpha}{\sqrt{2\pi\sigma_t^2}} \exp \left[\frac{-1}{2\sigma_t^2} \ln^2 \left(\frac{k_1 + k_2 + k_3 + k_4}{4k_t} \right) \right] \right\} [\beta(\widehat{\mathbf{k}_1 + \mathbf{k}_2}, \hat{\mathbf{k}}_1, \hat{\mathbf{k}}_3) P(k_1) P(k_3) \\ & \times P(|\mathbf{k}_1 + \mathbf{k}_2|) + 23 \text{ permutations}], \end{aligned} \quad (19a)$$

$$\begin{aligned} \mathcal{T}_{\text{even}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = & 2\tau_{\text{NL}} \left\{ 1 + \frac{\alpha}{\sqrt{2\pi\sigma_t^2}} \exp \left[\frac{-1}{2\sigma_t^2} \ln^2 \left(\frac{k_1 + k_2 + k_3 + k_4}{4k_t} \right) \right] \right\} [P(k_1) P(k_3) P(|\mathbf{k}_1 + \mathbf{k}_2|) + 11 \text{ permutations}]. \end{aligned} \quad (19b)$$

The parameter α quantifies the relative enhancement of the overall amplitude at the peak k_t , compared to the value away from the peak. We use $\mathcal{P}_S(k)$ of the form of Eq. (16) in this template and set $k_p = k_t$, as is often observed in inflationary models. We vary the width of the peak in $\mathcal{T}$, σ_t , while fixing the value of σ_p to inspect the interplay of scale dependences between power and trispectra in the resultant behaviors of $\mathcal{P}_h^V$ and $\mathcal{P}_h^I$.

We repeat the numerical analysis for this template and present the behaviors of $\mathcal{P}_h^V$, $\mathcal{P}_h^I$, and Π_k around the peak in Fig. 2. We observe that, for $\sigma_t^2 \ll \sigma_p^2$, the contribution from $\mathcal{T}$ is highly suppressed and hence $\Pi_k \leq 10^{-2}$ around the peak because of the contribution due to $\mathcal{T}$ being very much restricted over the ranges of the integrations involved. When $\sigma_t^2 = \sigma_p^2$, we find that Π_k is enhanced beyond 0.1 around $k \simeq 3k_p$. Curiously, $\mathcal{P}_h^I$ is affected by features from the trispectrum and hence incurs a corrugated shape in this regime.¹ With $\sigma_t^2 \gg \sigma_p^2$, we find Π_k recovering a behavior close to template 1 as in Fig. 1. This can be understood as the peak in $\mathcal{T}$ appearing wide enough for the peak in $\mathcal{P}_S$, to seem like a constant rescaling of τ_{NL} and $\tilde{\tau}_{\text{NL}}$. Further, inclusion of the contribution containing $f_{\text{NL}}^2 A_p^3$ shall increase $\mathcal{P}_h^I$, thereby reducing Π_k as remarked in case of template 1. In addition to the exact effect due to the peak in $\mathcal{T}$, this exercise underlines the necessity to appropriately account for intrinsic scale dependence in $\mathcal{T}$, while computing the spectra and chirality of SIGWs, for it may lead to

nontrivial features. Hence, a careful estimation of a scale-dependent Π_k with GW surveys will lend tighter constraint on $\tilde{\tau}_{\text{NL}}$, along with τ_{NL} and A_S over corresponding scales. Such bounds over small scales shall complement the bounds on the parameters over large scales from CMB and galaxy surveys, thereby informing us of the extended scale dependence of primordial correlations.

IV. CONCLUSION

In this work, we demonstrate that the parity-odd trispectrum leads to non-negligible chirality in SIGWs. We illustrate this phenomenon using two templates of $\mathcal{T}$, with two cases of $\mathcal{P}_S$ and numerically computing the resulting $\mathcal{P}_h^V$ and $\mathcal{P}_h^I$. $\mathcal{P}_h^V$ receives leading contribution from $\mathcal{T}_{\text{odd}}$ and so is directly proportional to $\tilde{\tau}_{\text{NL}}$. The simple case of $\mathcal{P}_S(k) = A_S$ leads to a limit of $\tilde{\tau}_{\text{NL}} \lesssim A_S^{-1}$, sheerly due to the Stokes inequality $|V| \leq I$. It translates to a conservative bound of $|\tilde{\tau}_{\text{NL}}| \lesssim 5 \times 10^8$ over large scales, consistent with the current observations [68,69,101,115]. Our bound is purely theoretical and independent of the observability of GWs over the scales of interest. Conversely, if $\tilde{\tau}_{\text{NL}}$ is detected with sufficient confidence using CMB or galaxy surveys, our analysis provides a lower bound of Π_k , motivating the search for chirality in stochastic GWs over corresponding frequencies.

For the case of $\mathcal{P}_S(k)$ with a log-normal peak, we find that for $\tilde{\tau}_{\text{NL}} = \tau_{\text{NL}} = 1$ chirality can be as large as $\Pi_k \geq 10\%$ over scales adjacent to the peak. Moreover, it alters in sign between scales close to and away from the peak. We should emphasize that such values are well within the sensitivities of future observational missions. For instance, the parameter choices of $A_p \simeq 10^{-2}$ leads to $\Omega_{\text{GW}} \simeq 10^{-10}$ at

¹The value of $\mathcal{P}_h^I$ turning negative when dominated by $\mathcal{T}_{\text{even}}$ indicates the necessity to account for other non-Gaussian contributions, such as the positive-definite term proportional to $f_{\text{NL}}^2 A_p^3$.

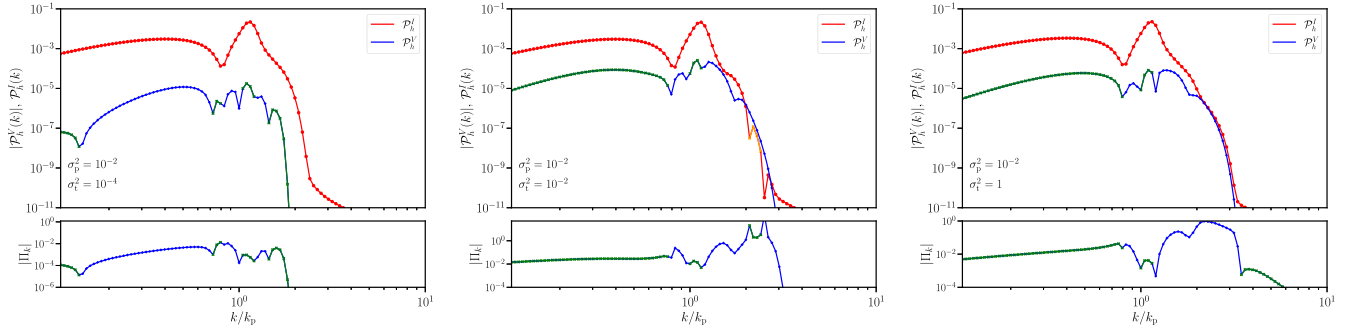


FIG. 2. We present $\mathcal{P}_h^V(k)$ and $\mathcal{P}_h^I(k)$, along with Π_k (with same color choices as in Fig. 1) computed for template 2 with $\mathcal{P}_S(k)$ and $\mathcal{T}$ containing log-normal peaks [cf. Eqs. (16) and (19)]. We also indicate the regimes where $\mathcal{P}_h^V(k)$ and $\mathcal{P}_h^I(k)$ turn negative (in green and orange, respectively). We set the parameters to be $\alpha = 10$, $k_t = k_p$, $\sigma_p^2 = 10^{-2}$, and others as mentioned in the text. We vary $\sigma_t^2 = [10^{-4}, 10^{-2}, 1]$ to inspect the effect of such an intrinsic scale dependence of $\mathcal{T}$ on $\mathcal{P}_h^V(k)$ and $\mathcal{P}_h^I(k)$. We learn that, for $\sigma_t^2 = 10^{-4}$, the contribution from $\mathcal{T}$ to $\mathcal{P}_h^I$ and $\mathcal{P}_h^V$ is lesser and hence $\Pi_k \leq 10^{-2}$. For $\sigma_t^2 = 10^{-2} = \sigma_p^2$, the contributions from $\mathcal{T}$ and $\mathcal{P}_S$ to $\mathcal{P}_h^I$ compete closely around k_p , leading to corrugated features in it. Further, $\mathcal{P}_h^V$ becomes comparable and even dominant over $\mathcal{P}_h^I$ in this brief window. When $\sigma_t^2 = 1$, the peak just rescales τ_{NL} and $\tilde{\tau}_{\text{NL}}$ and we recover behaviors of spectra similar to the case of $\sigma_p^2 = 10^{-2}$ in template 1 (cf. Fig. 1).

the peak [10,11]. If the peak occurs at small scales of $k_p = \mathcal{O}(10^{15}) \text{ Mpc}^{-1}$, it corresponds to frequencies of $f = \mathcal{O}(1) \text{ Hz}$. Such Ω_{GW} along with the V mode $\Omega_{\text{GW}}^V = \Pi_k \Omega_{\text{GW}}$, can be probed using upcoming GW interferometers [51,55,57–59]. Note that $\Pi_k \geq 10\%$ over scales adjacent to the peak for nominal values of $\tilde{\tau}_{\text{NL}} = \tau_{\text{NL}} = 1$. When contributions in terms of f_{NL} are included, the resultant Ω_{GW} and the associated Ω_{GW}^V should be able to probe parameters f_{NL} , $\tilde{\tau}_{\text{NL}}$, τ_{NL} of values $\mathcal{O}(10)$. Hence, the bounds that we may obtain on $\tilde{\tau}_{\text{NL}}$ over small scales through potential measurement of Π_k , with distinct scale-dependent signatures, can be comparable to or even stronger than the limits obtained from CMB or galaxy surveys. Such a window to directly assess the magnitude and signature of $\mathcal{T}_{\text{odd}}$ is observationally rare and possibly unique over small scales.

With template 2, we illustrate the importance of scale dependence of $\mathcal{T}$ for the behavior of $\mathcal{P}_h^V$ and $\mathcal{P}_h^I$. Crucially, using peaked spectra in both templates, we find a window of scales where Π_k turns out to be the ratio of parity-odd to parity-even parts of non-Gaussian contributions to SIGWs. Hence, Π_k therein quantifies the parity violation in the higher-order correlations of primordial scalar perturbations.

On the theoretical front, our results call for careful analyses of SIGWs from parity-violating inflationary models. As mentioned earlier, in models with suppressed chirality of primary GWs, $\mathcal{T}_{\text{odd}}$ through SIGWs shall determine the overall behavior of Π_k . In models with maximally chiral primary GWs, the total Π_k shall be attenuated by the contribution from SIGWs. Hence, in general, $\mathcal{T}_{\text{odd}}$ shall strongly impact the magnitude, sign, and scale dependence of Π_k . In essence, the predictions of $\mathcal{T}_{\text{odd}}$ and Π_k arising from parity-violating models may be treated as twin predictions, informing about each other and alleviating observational degeneracies.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX: DETAILS OF INTEGRATION REQUIRED FOR $\mathcal{P}_h^I$ AND $\mathcal{P}_h^V$

The factor Q^λ in Eq. (1) is a dimensionless function that arises from the contraction of the polarization tensor (for a given polarization λ) defined with respect to the propagation direction of $\hat{\mathbf{k}}$ and the wave vector of integration $\mathbf{q}$ in

the following fashion:

$$\mathcal{Q}^\lambda(\mathbf{k}, \mathbf{q}) = e_{ij}^\lambda(\mathbf{k}) \frac{q_i q_j}{k^2}. \quad (\text{A1})$$

Here $e_{ij}^\lambda(\mathbf{k})$ is the polarization tensor that can be written in terms of the polarization vectors orthogonal to $\hat{\mathbf{k}}$, in any basis, say $\lambda = [+,\times]$ or $\lambda = [R,L]$ (refer to [11] and references therein for explicit constructions).

The function $\tilde{I}_2$ that appears in Eq. (1) is a dimensionless function arising from the evolution of the inhomogeneous solution of h_k^λ [which is denoted as $\tilde{I}(kv, ku, k\eta)$ in [82,84]]. It results from averaging over fast oscillations of the solution and the overall time dependence is extracted out as $1/(2k^2\eta^2)$. The explicit expression of the function $\tilde{I}_2$ is given by [82]

$$\begin{aligned} \tilde{I}_2(u_1, v_1, u_2, v_2) = & I_A(u_1, v_1) I_A(u_2, v_2) \left[I_B(u_1, v_1) I_B(u_2, v_2) \right. \\ & \left. + \pi^2 I_C(u_1, v_1) I_C(u_2, v_2) \right], \end{aligned} \quad (\text{A2})$$

where

$$I_A(u, v) = \frac{3(u^2 + v^2 - 3)}{4u^3 v^3}, \quad (\text{A3})$$

$$I_B(u, v) = -4uv + (u^2 + v^2 - 3) \ln \left| \frac{3 - (u+v)^2}{3 - (u-v)^2} \right|, \quad (\text{A4})$$

$$I_C(u, v) = (u^2 + v^2 - 3) \Theta(u + v - 3). \quad (\text{A5})$$

From the expression above, we may obtain $\tilde{I}_1$ that appears in Eq. (4) as

$$\tilde{I}_1(u, v) = \tilde{I}_2(u, v, u, v), \quad (\text{A6})$$

$$= I_A^2(u, v) \left[I_B^2(u, v) + \pi^2 I_C^2(u, v) \right]. \quad (\text{A7})$$

For numerical implementation, we perform the integrations in Eqs. (4) and (7) to compute $\mathcal{P}_h^I$ and $\mathcal{P}_h^V$ using the Monte Carlo method of importance sampling with 10^9 samples for each k value. Specifically, for the two- and five-dimensional integrals (with one angular integration always performed trivially due to homogeneity and isotropy) that

correspond to terms with $\mathcal{P}_S^2$ and $\mathcal{T}$ in the integrand, we perform uniform sampling over the ranges of polar angles as $\cos \theta_{1,2} \in [-1, 1]$ and the combination of azimuthal angles $(\phi_1 - \phi_2) \in [0, 2\pi]$. As to the ranges of $q_{1,2}$, we perform importance sampling with a Gaussian sampler in the logarithmic range. In case of the spectra without peaks, we center the sampler at $q_{1,2} = k$, i.e., $\log_{10}(q_{1,2}/k) = 0$ with its standard deviation set to 5, i.e., five decades of wave numbers around the center. In case of peaks in $\mathcal{P}_S$ (and in $\mathcal{T}$ in template 2), we center the sampler at the location of the peak as $q_{1,2} = k_p$, i.e., $\log_{10}(q_{1,2}/k) = \log_{10}(k_p/k)$, with the standard deviation set to $5\sigma_p$. The center and standard deviation of the sampler are chosen such that the sampling covers the prominent volume of the integrand in each case with sufficient convergence. The convergence for a given k is checked by computing the sampling variance and ensuring the ratio of square root of this variance to the integral to be $\lesssim 10\%$.

Regarding the behavior of the integrand in case of scale-invariant spectra, there is a logarithmic dependence of the integrand in the form of $[\log(q_1/k)]^2$, specifically when $q_1/k \rightarrow \infty$ with $q_2 = k$ and vice versa ($q_1 \leftrightarrow q_2$). On the other hand, certain terms in the trispectrum contain $|\mathbf{q}_1 - \mathbf{q}_2|^{-3}$ and diverge when $\mathbf{q}_1 \rightarrow \mathbf{q}_2$. Through change of variables, it can be shown that it is the same as the divergence of $q_{1,2}^{-3}$ when $q_{1,2} \rightarrow 0$. For numerical implementation, we have chosen a finite minimum value of integrated wave numbers as $q_{1,2}/k = 10^{-3}$ and have checked that the results do not strongly depend on this choice. We also note that these divergences are highly suppressed in the case of peaked power and trispectra.

We should note that $\mathcal{P}_h^V$ is sensitive to the three-dimensional nature of the trispectrum, as $\mathcal{T}_{\text{odd}}$ vanishes if any of the constituting wave vectors are taken to zero or, equivalently, any two of them are set to be (anti)parallel. Additionally, not all configurations of $\mathcal{T}_{\text{odd(even)}}$ contribute to $\mathcal{P}_h^{V(I)}$. Terms of $\mathcal{T}$ that are insensitive to angles between wave vectors of integration $\mathbf{q}_1$ and $\mathbf{q}_2$ vanish upon integration over azimuthal angles. Also, of the different permutations of terms in $\mathcal{T}$ [cf. Eqs. (11) and (19)], when substituted in the integrals of Eq. (4), some of them can be equated to others through change of variables, thereby reducing the unique types of terms that have to be integrated.

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