

Erratum: About electroweak domain walls in Majoron models **[Phys. Rev. D 113, 043515 (2026)]**

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The operator in the Lagrangian $\mathcal{L}_{B+L}$ in Eq. (5) of this article does not actually pick up a coupling to the Majoron when performing the field redefinitions of only L and Q with the transformation parameters α_L and α_B indicated between Eqs. (4) and (5). One can show that under these redefinitions, $\mathcal{L}_{B+L} \rightarrow e^{-i(\alpha_B + \alpha_L)} \mathcal{L}_{B+L}$ in terms of $\alpha_B + \alpha_L = -\theta_{EW}$. The problem is that the operator in Eq. (5) accidentally conserves $B - L$, whereas the Majoron coupling arises from $\alpha_B - \alpha_L = j/v_L - \theta_{EW} - 2\text{Arg}[Y_L]$. This situation can be rectified either by invoking a different operator, breaking both $B - L$ and $B + L$, or by changing the anomaly structure of the theory by allowing for chiral $B - L$ charges (compared to the vector-like charge assignment of $B - L$ for the SM fermions): We can add, e.g., a pair of fermion doublets $\Psi_{L,R} \sim (\mathbf{1}, \mathbf{2}, \mp 1/2)$, which are vector-like under the SM gauge symmetries, but chiral under lepton number, and obtain a mass $M_\Psi \equiv Y_\Psi v_L / \sqrt{2}$ from the spontaneous breaking of lepton number via the operator

$$Y_\Psi \sigma \Psi_L \Psi_R. \quad (1)$$

Before continuing with the impact on the Majoron, let us comment on the phenomenological viability of the additional doublet fermions: In order to avoid a relic abundance of stable Ψ , we can either suppress their thermal production or make them unstable. The first option requires $T_{RH} < M_\Psi$ for sufficient Boltzmann suppression, where T_{RH} is the postinflationary reheating temperature in the instantaneous decay approximation. Since the Majoron domain wall discussion in the main text assumes postinflationary $B - L$ breaking, we have to impose $v_L < T_{RH} < M_\Psi$, which amounts to a narrow range for the Yukawa coupling close to its perturbative maximum: $1 < Y_\Psi / \sqrt{2} < \sqrt{2\pi} \simeq 2.5$. On the other hand, we can also add a decay mode for Ψ . For instance, with the normalization $Q_{B-L}[\sigma] = -2$, we can choose the charge assignment and $Q_{B-L}[\Psi_L] = -1$, $Q_{B-L}[\Psi_R] = 3$, which allows for the operators $\Psi_L H N$ and $\Psi_L \tilde{H} e$, where e is a charged lepton singlet and $\tilde{H}$ is the $SU(2)_W$ conjugate of H . Here, all gauge and generation indices are suppressed. The first operator is only present in the Type I and Type III seesaw models, and the second operator works for all kinds of seesaw models. If the Yukawa coupling of this operator is not suppressed, one can have fast decays $\Psi \rightarrow H e^\dagger$, so we do not expect any Ψ relics to be present at late times.

We can rotate the Majoron into the topological term of $SU(2)_L$ via the transformations $\Psi_{L,R} \rightarrow e^{-i\alpha_{\Psi_{L,R}}} \Psi_{L,R}$ with $\alpha_{\Psi_L} + \alpha_{\Psi_R} = j/v_L$, which induces the shift of the topological angle of $\theta_{EW} \rightarrow \theta_{EW} + j/v_L$. The rotation of the lepton doublet in terms of α_L is left as in this article, but now we can remove the Majoron from the topological term via a transformation of Q with $\alpha_B = -\theta_{EW} - \text{Arg}[Y_L] - j/(2v_L)$. This leads to $\alpha_B + \alpha_L = -\theta_{EW} - j/v_L$, so the Majoron now appears in the operator in Eq. (5). In the presence of the vector-like pair of doublets, additional fermionic zero modes arise that contribute to the 't Hooft vertex in Fig. 1 of this article, which can be tied up via the Y_Ψ Yukawa coupling. Furthermore, we have to include the change in the running of the $SU(2)_L$ due to these new states. Using the methods of Ref. [1], the estimate for the Majoron mass in Eq. (6) of this article changes as

$$m_j^2 \rightarrow m_j^2 \times \left(\frac{M_\Psi}{M_{UV}} \right) \times \left(\frac{M_{UV}}{M_\Psi} \right)^{\frac{N_S+2}{3}}, \quad (2)$$

where the first term comes from the additional zero modes, and the second one from the change of the β function. Here, N_S denotes the number of additional pairs of scalar doublets with the same mass M_Ψ as the fermions. For $N_S = 0$, the additional fermions slightly reduce the estimate for the Majoron mass, and for $N_S = 1$, the suppression from the fermionic zero modes

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is exactly canceled by the enhancement from the change in the running due to both fermions and scalars, as first demonstrated in Refs. [2,3]. The same subtlety might also be relevant for the case of the QCD axion discussed in Ref. [1].

This analysis illustrates that the results and conclusions of this article that pertain to the estimate of the Majoron mass do not change. If anything, the preceding discussion demonstrates that it is more difficult than naively expected to generate a Majoron mass from electroweak instanton effects, which aligns well with the central message of this article.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

- [1] C. Csáki, R. T. D’Agnolo, E. Kuflik, and M. Ruhdorfer, Instanton NDA and applications to axion models, *J. High Energy Phys.* **04** (2024) 074.
- [2] Y. Nomura, T. Watari, and T. Yanagida, Quintessence axion potential induced by electroweak instanton effects, *Phys. Lett. B* **484**, 103 (2000).
- [3] M. Ibe, M. Yamazaki, and T. T. Yanagida, Quintessence axion revisited in light of Swampland conjectures, *Classical Quantum Gravity* **36**, 235020 (2019).
- [4] A. Herrero-Brocal, Non-anomalous axions: Lessons from the Majoron, [arXiv:2602.22311](https://arxiv.org/abs/2602.22311).