

# 3D stress tensor for gravity in 4D flat spacetime

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Three-dimensional (3D) Carrollian conformal field theories (CFTs) are potential codimension-one holographic duals of 4D asymptotically flat spacetimes that live on the whole of the null boundary. In this paper, we show that the *local stress tensor* of the 3D Carrollian conformal theory (without any additional sources) constructed in terms of the geometric structure at asymptotic null infinity naturally encodes *both* the leading and subleading soft graviton theorems. We relate the 3D Carroll stress tensor to the 2D Celestial one and show how the 3D version naturally localizes the nonlocal 2D Celestial stress tensor. We also comment on the relation with stress tensors in relativistic 3D CFTs and connections to the flat limit of AdS/CFT.

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## I. INTRODUCTION

The very rich structure of infrared physics in asymptotically flat spacetimes (AFS) has become the cynosure of recent interest and a rather remarkable story has emerged relating asymptotic symmetries, gravitational memory effects and soft theorems in an intricate web of relations [1–4]. The asymptotic symmetries algebra of four-dimensional (4D) AFS at its null boundary ( $\mathcal{I}^\pm$ ) is given by the Bondi-van der Burg-Metzner-Sachs (BMS<sub>4</sub>) algebra [5–8],

$$[L_n^\pm, L_m^\pm] = (n - m)L_{m+n}^\pm, \quad [M_{r,s}, M_{p,q}] = 0, \quad (1a)$$

$$[L_n^+, M_{r,s}] = \frac{1}{2}(n - 1 - 2r)M_{n+r,s}, \quad (1b)$$

$$[L_n^-, M_{r,s}] = \frac{1}{2}(n - 1 - 2s)M_{r,n+s}. \quad (1c)$$

Soft theorems are universal statements about the IR physics of scattering amplitudes involving massless particles [9–13]. It turns out that the soft theorems can be understood as BMS Ward identities [14,15]. The leading Weinberg soft theorem is a consequence of supertranslation

invariance [15] and the super-rotation Ward identity leads to the subleading soft graviton theorem [16].

Hand in hand with the developments in the IR physics in flat spacetimes, there are renewed attempts to understand holography in AFS. Recent developments have proceeded in two principle directions called Celestial [2–4] and Carrollian holography [17–25]. The more popular approach, Celestial holography, is a codimension two approach where the holographic dual to 4D AFS is proposed to be a 2D relativistic conformal field theory (CFT) that lives on the celestial sphere at null infinity [2]. Carrollian holography, on the other hand, purports a codimension one dual for AFS in terms of a non-Lorentzian CFT. A body of recent work has emerged connecting 3D Carrollian CFTs and bulk physics in 4D AFS [26–42] and in particular, the connection between 3D Carrollian correlation functions and 4D S-matrix elements has been established via the so-called *modified Mellin* transformation [26,43].

In AdS/CFT, the stress tensor of the boundary theory is dual to metric fluctuations in the bulk [44,45] and its correlation functions are related to scattering of gravitons [46–49]. In the flat space limit, the anti-de Sitter (AdS) scattering amplitudes go over to flatspace amplitudes [48,49]. The large radius limit that sends AdS to flatspace is a Carroll limit on the boundary CFT. Thus, it is very tempting to presume that a properly defined Carrollian stress tensor [24,32,50–57] would capture the details of scattering of gravitons in flat spacetime. The stress tensor in 2D Celestial CFT captures the subleading soft graviton theorem while one has to postulate the existence of a spin-one current in the theory to account for the leading Weinberg soft theorem. This is somewhat unsatisfactory,

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since if (some aspects of) flat holography were to arise from a consistent limit of AdS/CFT, presumably both the leading and subleading soft theorems should be related to stress tensor Ward identities in the dual theory. In this paper, we show that by considering a 3D Carrollian CFT, instead of a 2D Celestial CFT, we are able to fulfil this expectation. Both soft graviton theorems indeed are packaged into the 3D Carroll stress tensor Ward identities. The 2D Celestial stress tensor is also known to have issues with locality. We comment on how the 3D Carrollian stress tensor helps alleviate such concerns.

## II. CARROLLIAN BASIS FOR SCATTERING AMPLITUDES

The graviton S-matrix admits a universal leading and subleading soft factorization [11,13,58]. The leading soft graviton theorem is expressed as

$$\lim_{\omega \rightarrow 0} \omega \mathcal{S}_{n+1}(k^\mu, p_i^\mu) = \sum_{i=1}^n \frac{\epsilon_{\mu\nu} p_i^\mu p_i^\nu}{p_i \cdot \hat{k}} \mathcal{S}_n(p_i^\mu). \quad (2)$$

Here  $\mathcal{S}_n$  denotes scattering amplitudes of  $n$  massless particles,  $k^\mu$  is the graviton 4-momentum with energy  $\omega$  and unit vector  $\hat{k}^\mu$ , i.e.,  $k^\mu = \omega \hat{k}^\mu$ .  $p_i^\mu$  are momenta of other hard particles and  $\epsilon_{\mu\nu}$  denotes the polarization tensor. Similarly the subleading piece is

$$(1 + \omega \partial_\omega) \mathcal{S}_{n+1}(k^\mu, p_i^\mu) = \sum_{i=1}^n \frac{\epsilon_{\mu\nu} p_i^\mu \hat{J}^{\nu\lambda} k_\lambda}{p_i \cdot \hat{k}} \mathcal{S}_n(p_i^\mu), \quad (3)$$

where  $\hat{J}^{\nu\lambda}$  is the angular momentum of the graviton.

We wish to express these soft theorems in a Carrollian basis. We recall that 4D S-matrix elements are connected correlation functions in 3D Carroll CFT by a modified Mellin transformation [26,43,61] of the plane wave basis,

$$\begin{aligned} & \langle \Phi_{h_i, \bar{h}_1}^{\epsilon_1}(u_1, z_1, \bar{z}_1) \dots \Phi_{h_n, \bar{h}_n}^{\epsilon_n}(u_n, z_n, \bar{z}_n) \rangle \\ &= \int \prod_{i=1}^n d\omega_i \omega_i^{\Delta_i-1} e^{i\epsilon_i \omega_i u_i} \mathcal{S}_n(\omega_i, z_i, \bar{z}_i, \sigma_i). \end{aligned} \quad (4)$$

Here  $\Phi_{h_i, \bar{h}_i}$  are 3D Carroll primary fields with weights  $(h, \bar{h})$  and  $\mathcal{S}_n$  are  $n$ -point scattering amplitudes of massless particles with helicity  $\sigma_i$  and momenta,

$$p^\mu = \omega q^\mu, \quad q^\mu = (1 + z\bar{z}, z + \bar{z}, i(z - \bar{z}), 1 - z\bar{z}), \quad (5)$$

where  $\omega$  is the energy of the particle, and  $z, \bar{z}$  are the stereographic coordinates on the celestial sphere. A local Carrollian primary field on  $\mathcal{S}^\pm$  can be defined using bulk momentum space operators [26],

$$\Phi_{h, \bar{h}}^\epsilon(u, z, \bar{z}) = \int_0^\infty d\omega e^{i\omega \epsilon u} \omega^{\Delta-1} a(\epsilon \omega, z, \bar{z}, \sigma), \quad (6)$$

with  $h + \bar{h} = \Delta$  and  $h - \bar{h} = \sigma$ .  $\epsilon = \pm 1$  denotes a creation/annihilation operator.

Let us now briefly review the asymptotic expansion near  $\mathcal{I}^+$  [16], which would allow us to relate the boundary Carrollian operators in terms of fields that appear in this expansion later on. In Bondi coordinates, asymptotically flat metrics admit the following expansion near  $\mathcal{I}^+$ :

$$\begin{aligned} ds^2 &= -du^2 - 2dudr + 2r^2 \gamma_{z\bar{z}} dz d\bar{z} \\ &+ \frac{2m_B}{r} du^2 + r C_{z\bar{z}} dz^2 + r C_{\bar{z}\bar{z}} d\bar{z}^2 \\ &+ D^z C_{z\bar{z}} dudz + D^{\bar{z}} C_{\bar{z}\bar{z}} d\bar{u} d\bar{z} + \dots \end{aligned} \quad (7)$$

Here  $\gamma_{z\bar{z}}$  denotes the metric on the celestial sphere, and  $D_{(z, \bar{z})}$  are the associated covariant derivatives. The information about the gravitational radiation is contained in the asymptotic shear function  $C_{AB}$ . In flat Minkowski coordinates, the mode expansion for the outgoing graviton is given by

$$\begin{aligned} h_{\mu\nu}^{\text{out}}(x^0, \vec{x}) &= \sum_{\alpha=\pm} \int \frac{d^3 q}{(2\pi)^3} \frac{1}{2\omega_{\vec{q}}} \left[ \epsilon_{\mu\nu}^{(\alpha)}(\vec{q}) a_\alpha^{\text{out}}(\vec{q}) e^{iq \cdot x} \right. \\ &\quad \left. + \epsilon_{\mu\nu}^{(\alpha)}(\vec{q}) a_\alpha^{\text{out}\dagger}(\vec{q}) e^{-iq \cdot x} \right]. \end{aligned} \quad (8)$$

Here  $\alpha$  designates the helicities and  $q^\mu$  is 4-null momentum. The ladder operators satisfy canonical commutation relations. In Bondi coordinates, the outgoing graviton  $h_{\mu\nu}(u, r, z, \bar{z})$  is related to the shear via

$$C_{z\bar{z}}(u, z, \bar{z}) = \mathcal{D} \lim_{r \rightarrow \infty} \frac{1}{r} \partial_z x^\mu \partial_{\bar{z}} x^\nu h_{\mu\nu}^{\text{out}}(u + r, r\Omega), \quad (9)$$

where  $\mathcal{D}$  is the normalization constant that sets the Canonical normalization for the graviton field and  $\Omega$  denotes the angular coordinates on the sphere. The integral near  $\mathcal{I}^+$  in the saddle point approximation yields

$$\begin{aligned} C_{z\bar{z}}(u, z, \bar{z}) &= -\frac{i\mathcal{D}}{8\pi^2} \hat{e}_{z\bar{z}} \int_0^\infty d\omega_q [a_-^{\text{out}}(\omega_q \Omega) e^{-i\omega_q u} \\ &\quad - a_+^{\text{out}}(\omega_q \Omega)^\dagger e^{i\omega_q u}]. \end{aligned} \quad (10)$$

The polarization vector is given by

$$\hat{e}_{z\bar{z}} = \frac{1}{r^2} \partial_z x^\mu \partial_{\bar{z}} x^\nu \epsilon_{\mu\nu}^{(+)}(\omega_q \Omega) = \frac{2}{(1 + z\bar{z})^2}. \quad (11)$$

Since the news tensor is defined as

$$N_{AB} \equiv \partial_u C_{AB}, \quad (12)$$

we can use integrated versions of this operator to construct soft insertions.

For instance, the soft graviton theorems in (2) and (3), could be expressed as insertion of leading and subleading soft modes in the scattering amplitudes. These modes could be extracted from the News tensor by

$$\begin{aligned} N_{\bar{z}\bar{z}}^{(0)} &\equiv \mathcal{A} \int du N_{zz} = -\hat{e}_{\bar{z}\bar{z}} \lim_{\omega \rightarrow 0} \omega [a_+(\omega\Omega) + a_-(\omega\Omega)^\dagger], \\ N_{\bar{z}\bar{z}}^{(1)} &\equiv \mathcal{A} \int duu N_{zz} = -\hat{e}_{\bar{z}\bar{z}} \lim_{\omega \rightarrow 0} (1 + \omega\partial_\omega) \\ &\quad \times [a_+(\omega\Omega) - a_-(\omega\Omega)^\dagger]. \end{aligned} \quad (13)$$

We have included an overall normalization constant  $\mathcal{A}$  that takes care of extraneous numerical factors. Here  $N_{zz}^{(0)}$  and  $N_{zz}^{(1)}$  are subsequently leading and subleading soft graviton operators. Using the parametrization of the null momenta in (5), the leading soft graviton theorem becomes

$$\begin{aligned} \lim_{\omega \rightarrow 0} \omega \mathcal{M}_{n+1}(\omega, z, \bar{z}, \omega_i, z_i, \bar{z}_i) \\ = - \left( \sum_i \frac{\epsilon_i \omega_i (\bar{z} - \bar{z}_i)}{z - z_i} \right) \mathcal{M}_n(\omega_i, z_i, \bar{z}_i, \sigma_i). \end{aligned} \quad (14)$$

Alternatively, we can write

$$\begin{aligned} \left\langle N_{\bar{z}\bar{z}}^{(0)}(z, \bar{z}) \prod_{i=1}^n a_i(\epsilon_i \omega_i, z_i, \bar{z}_i, \sigma_i) \right\rangle \\ = - \left( \sum_i \frac{\epsilon_i \omega_i (z - z_i)}{\bar{z} - \bar{z}_i} \right) \left\langle \prod_{i=1}^n a_i(\epsilon_i \omega_i, z_i, \bar{z}_i, \sigma_i) \right\rangle. \end{aligned} \quad (15)$$

Furthermore, the modified Mellin transformation (42) on the hard particles yields [62,63]

$$\begin{aligned} \left\langle N_{\bar{z}\bar{z}}^{(0)}(z, \bar{z}) \prod_{i=1}^n \Phi_{h_i, \bar{h}_i}^{\epsilon_i}(u_i, z_i, \bar{z}_i, \sigma_i) \right\rangle \\ = - \sum_i \epsilon_i \frac{z - z_i}{\bar{z} - \bar{z}_i} \partial_{u_i} \left\langle \prod_{i=1}^n \Phi_{h_i, \bar{h}_i}^{\epsilon_i}(u_i, z_i, \bar{z}_i, \sigma_i) \right\rangle. \end{aligned} \quad (16)$$

Using similar arguments, we can express the subleading soft graviton theorem in modified Mellin space as [62]

$$\begin{aligned} \left\langle N_{\bar{z}\bar{z}}^{(1)}(z, \bar{z}) \prod_{i=1}^n \Phi_{h_i, \bar{h}_i}^{\epsilon_i}(u_i, z_i, \bar{z}_i) \right\rangle \\ = - \left( \sum_i \epsilon_i \frac{(z_i - z)^2 + 2(h_i + u_i \partial_{u_i})(z_i - z) \partial_{z_i}}{\bar{z} - \bar{z}_i} \right) \end{aligned} \quad (17)$$

$$\left\langle \prod_{i=1}^n \Phi_{h_i, \bar{h}_i}^{\epsilon_i}(u_i, z_i, \bar{z}_i) \right\rangle. \quad (18)$$

We shall comeback to these universal soft graviton theorems in modified Mellin space, i.e., (16) and (17) in a later section, where we identify these relations as the Carrollian stress tensor Ward identities.

### III. BOUNDARY CARROLLIAN STRESS TENSOR

We now turn our attention to the intrinsic construction of boundary Carroll stress tensors. Carrollian manifolds [64] are endowed with a covariant degenerate 2-tensor field  $h_{\gamma\beta}$  and a nowhere vanishing contravariant vector field  $\tau^\alpha$  orthogonal to it. The null boundary of 4D AFS is a Carroll manifold with an induced metric,

$$ds^2 = 0 \cdot du^2 + l_{ij} dx^i dx^j, \quad (19)$$

where  $u$  is retarded time,  $x^i$  are the coordinates of the celestial sphere, and  $l_{ij}$  is the round 2-sphere metric. We identify

$$\tau^\alpha = (1, 0^i), \quad h_{\gamma\beta} = \text{diag}(0, l_{ij}). \quad (20)$$

The asymptotic symmetries of AFS induces a conformal structure on the intrinsic Carrollian geometry on null infinity leading to [19]

$$\mathcal{L}_\xi \tau^\alpha = \lambda(u, x^i) \tau^\alpha, \quad \mathcal{L}_\xi h_{\beta\gamma} = -2\lambda(u, x^i) h_{\beta\gamma}. \quad (21)$$

(21) has a solution given by

$$\xi = \left( \beta(x^i) + \frac{u}{2} \partial_i g^i(x^i) \right) \partial_u + g^i(x^i) \partial_i. \quad (22)$$

$\beta(x^i)$  is an arbitrary function of  $x^i$  and parametrizes supertranslations.  $g^z, g^{\bar{z}}$  are holomorphic/antiholomorphic;  $g^z \equiv g^z(z)$  and  $g^{\bar{z}} \equiv g^{\bar{z}}(\bar{z})$  and parametrize superrotations.

These symmetries allow us define a Carrollian CFT and its associated stress tensor on null infinity. We define the Carroll stress tensor as variation of the boundary action  $\mathcal{S}$  with respect to the Carrollian metric (written in vielbeins [65]),

$$\delta \mathcal{S} = \int du d^2 x e [\tau^\nu \delta \tau_\mu + e_a^\nu \delta e_\mu^a] T_\nu^\mu. \quad (23)$$

Local Carroll symmetries fix [66]  $T_u^i = 0, T_j^i = T_i^j$ , where the vanishing of  $T_u^i$  is a telltale sign of Carroll symmetry. We further decompose  $T_j^i$  into a trace and symmetric traceless part,

$$T_j^i = t(u, x^i) \delta_j^i + \theta_j^i(u, x^i). \quad (24)$$

Weyl symmetries force the stress tensor to be traceless:  $t(u, x^i) = -\frac{1}{2} T_u^u(t, x^i)$ . Conservation equations lead to

$$\partial_u T_u^u = 0, \quad \partial_u T_i^u - \partial_j \left( \frac{1}{2} T_u^u \delta_i^j - \theta_i^j \right) = 0. \quad (25)$$

Integration (25) with respect to  $u$  yields

$$T_i^u - \frac{u}{2} \partial_i T_u^u = C_i(z, \bar{z}) - \int du \partial_j \theta_i^j(u, z, \bar{z}). \quad (26)$$

Here  $C_i(z, \bar{z})$  is an integration constant. We thus can separately define conserved components,

$$T_u(z, \bar{z}) \equiv T_u^u, \quad T_i(z, \bar{z}) \equiv T_i^u - \frac{u}{2} \partial_i T_u^u. \quad (27)$$

Currents associated with BMS symmetries are obtained by contracting the stress tensor with the associated Killing vectors  $J^\mu = T_\nu^{\mu} \xi^\nu$ . The corresponding charges are [29],

$$Q_\xi = \int d^2 z [T_u \beta(z, \bar{z}) + T_z g^z(z) + T_{\bar{z}} g^{\bar{z}}(\bar{z})]. \quad (28)$$

These can be decomposed as

$$L_n^+ = Q_\xi^{[\beta=0, g^z=z^{n+1}, g^{\bar{z}}=0]} = \int d^2 z \sqrt{q} T_z \cdot z^{n+1}, \quad (29a)$$

$$L_n^- = Q_\xi^{[\beta=0, g^z=0, g^{\bar{z}}=\bar{z}^{n+1}]} = \int d^2 z \sqrt{q} T_{\bar{z}} \cdot \bar{z}^{n+1}, \quad (29b)$$

$$M_{r,s} = Q_\xi^{[\beta=z^r \bar{z}^s, g^z=0, g^{\bar{z}}=0]} = \int d^2 z \sqrt{q} T_u \cdot z^r \bar{z}^s. \quad (29c)$$

$L_n^\pm, M_{r,s}$  denote superrotation and supertranslation charges respectively and close to form the BMS<sub>4</sub> algebra (1).

#### IV. CARROLL OPES AND WARD IDENTITIES

A Carrollian conformal primary field  $\Phi(u, z, \bar{z})$  is labeled as

$$[L_0^+, \Phi(0)] = h\Phi(0), \quad [L_0^-, \Phi(0)] = \bar{h}\Phi(0), \quad (30)$$

and the primary conditions [26]

$$[L_n^\pm, \Phi(0)] = 0, \quad [M_{r,s}, \Phi(0)] = 0 \quad \forall n, r, s > 0, \quad (31)$$

lead to transformation rules for primary fields

$$\begin{aligned} [L_n^+, \Phi(\vec{x})] &= z^{n+1} \partial_z \Phi(\vec{x}) + \left( h + \frac{u}{2} \partial_u \right) \Phi(\vec{x}) \partial_z (z^{n+1}), \\ [L_n^-, \Phi(\vec{x})] &= \bar{z}^{n+1} \partial_{\bar{z}} \Phi(\vec{x}) + \left( \bar{h} + \frac{u}{2} \partial_u \right) \Phi(\vec{x}) \partial_{\bar{z}} (\bar{z}^{n+1}), \\ [M_{r,s}, \Phi(\vec{x})] &= z^r \bar{z}^s \partial_u \Phi(\vec{x}), \end{aligned} \quad (32)$$

at an arbitrary point  $\vec{x} \equiv (u, z, \bar{z})$  on the Carroll manifold. The transformation rules of global Poincaré subgroup

(i.e.,  $n = 0, \pm 1, r, s = 0, 1$ ) coincide with that of (6) defined using bulk momentum space operators. These transformation rules can alternatively be encoded in terms of the operator product expansion (OPE) of the stress tensor and concerned operators. Using (29) it is possible to rewrite (32) as

$$:T_u(z, \bar{z}) \Phi(u, \omega, \bar{\omega}): \sim \partial_u \Phi(u, \omega, \bar{\omega}) \delta^2(z - \omega), \quad (33a)$$

$$:T_z(z, \bar{z}) \Phi(u, \omega, \bar{\omega}): \sim \left( h + \frac{u}{2} \partial_u \right) \Phi(u, \omega, \bar{\omega}) \partial_\omega \delta^2(z - \omega) + \delta^2(z - \omega) \partial_\omega \Phi(u, \omega, \bar{\omega}). \quad (33b)$$

A similar expression exists for the antiholomorphic component. These OPEs are generic operator statements, which when inserted inside correlation functions lead to Ward identities. Stress tensor Ward identities associated with a generic 3D Carrollian CFT are

$$\left\langle T_u(z, \bar{z}) \prod_{i=1}^n \Phi_i(\vec{x}_i) \right\rangle = - \sum_i \hat{\delta}_i^2 \partial_{u_i} \left\langle \prod_{i=1}^n \Phi_i(\vec{x}_i) \right\rangle, \quad (34a)$$

$$\begin{aligned} \left\langle T_z(z, \bar{z}) \prod_{i=1}^n \Phi_i(\vec{x}_i) \right\rangle &= \sum_i \left( \hat{\delta}_i^2 \partial_{z_i} + \left( h_i + \frac{u_i}{2} \partial_{u_i} \right) \partial_{z_i} \hat{\delta}_i^2 \right) \\ &\times \left\langle \prod_{i=1}^n \Phi_i(\vec{x}_i) \right\rangle. \end{aligned} \quad (34b)$$

Above, to reduce clutter, we have defined  $\hat{\delta}_i^2 = \delta^2(z - z_i)$ .

#### V. SOFT THEOREMS AS STRESS TENSOR WARD IDENTITIES

Now we establish the primary result of our paper; Ward identities of the boundary Carrollian stress tensor are manifestations of the bulk soft graviton theorems. We recall (16) which is the leading soft-graviton theorem with negative helicity. Acting twice with  $\partial_z$  on either side we get

$$\left\langle \partial_z^2 N_{\bar{z}\bar{z}}^{(0)} \prod_{i=1}^n \Phi_i(\vec{x}_i) \right\rangle = - \sum_i \hat{\delta}_i^2 \epsilon_i \partial_{u_i} \left\langle \prod_{i=1}^n \Phi_i(\vec{x}_i) \right\rangle. \quad (35)$$

The above relation can be recognized as Ward identity of  $T_u(z, \bar{z})$  (34a) if we identify

$$T_u = \partial_z^2 N_{\bar{z}\bar{z}}^{(0)}. \quad (36)$$

Similarly, for the positive helicity soft graviton, we would have the identification  $T_u = \partial_{\bar{z}}^2 S_0^+(z, \bar{z})$ . So we have a relation between the positive/negative helicity soft graviton and the  $(uu)$  component of the 3D Carroll stress tensor,

$$T_u(z, \bar{z}) = \partial_z^2 N_{\bar{z}\bar{z}}^{(0)} = \partial_{\bar{z}}^2 S_{zz}^{(0)}. \quad (37)$$

Now recall the subleading soft graviton theorem in modified Mellin space given by (17) [62]. This time we act with  $\partial_z^3$  on both sides,

$$\begin{aligned} & \left\langle \partial_z^3 N_{\bar{z}\bar{z}}^{(1)}(z, \bar{z}) \prod_{i=1}^n \Phi_i(\vec{x}_i) \right\rangle \\ &= - \left( \sum_i \hat{\delta}_i^2 \epsilon_i \partial_{z_i} + \epsilon_i \left( h_i + \frac{u_i}{2} \partial_{u_i} \right) \partial_{z_i} \hat{\delta}_i^2 \right) \left\langle \prod_{i=1}^n \Phi_i(\vec{x}_i) \right\rangle. \end{aligned} \quad (38)$$

This is identical to (34b) if we have

$$T_z(z, \bar{z}) = \partial_z^3 N_{\bar{z}\bar{z}}^{(1)}(z, \bar{z}). \quad (39)$$

Similarly using the positive helicity subleading soft graviton theorem, one can verify that  $T_{\bar{z}}(z, \bar{z}) = \partial_{\bar{z}}^3 S_1^+(z, \bar{z})$ . Hence  $\{T_u, T_z, T_{\bar{z}}\}$  built out of the 3D Carroll stress tensor are connected to the leading and subleading soft graviton operators in 4D AFS through the identifications,

$$\begin{aligned} T_u(z, \bar{z}) &= \partial_z^2 N_{\bar{z}\bar{z}}^{(0)}(z, \bar{z}) = \partial_{\bar{z}}^2 N_{zz}^{(0)}(z, \bar{z}), \\ T_z(z, \bar{z}) &= \partial_z^3 N_{\bar{z}\bar{z}}^{(1)}(z, \bar{z}), \quad T_{\bar{z}}(z, \bar{z}) = \partial_{\bar{z}}^3 N_{zz}^{(1)}(z, \bar{z}). \end{aligned}$$

Carroll stress tensor Ward identities can be recast as soft theorems as a result of these identifications.

## VI. CELESTIAL AND CARROLLIAN SOFT OPERATORS

In the Celestial framework, one uses the *conformal primary basis* to relate scattering amplitudes in 4D AFS with 2D conformal correlators on the celestial sphere,

$$\langle \mathcal{O}_1(z_1) \dots \mathcal{O}_n(z_n) \rangle = \int_0^\infty \prod_{i=1}^n \omega_i^{\Delta_i-1} \mathcal{S}_n(\omega_i, z_i, \sigma_i), \quad (40)$$

where we have suppressed the  $\bar{z}$  dependences. Helicity  $\sigma$  thus maps to spin of the boundary operator, whereas energy  $\omega$  is traded with scaling dimension  $\Delta$ . Hence, poles in the soft expansion are mapped to poles in  $\Delta$  of the associated primary operators. Leading and subleading soft contributions are extracted from a graviton conformal primary operator  $G_\Delta(z, \bar{z})$  as [67]

$$\begin{aligned} N_{zz}^{(0)}(z, \bar{z}) &= \text{Res}_{\Delta=1} G_{\Delta, \sigma=2}(z, \bar{z}), \\ N_{zz}^{(1)}(z, \bar{z}) &= \text{Res}_{\Delta=0} G_{\Delta, \sigma=2}(z, \bar{z}). \end{aligned} \quad (41)$$

Using these graviton conformal primaries, one can define operators of the dual 2D CFT [1,14,15,67,68],

$$P_{(2d)}(z, \bar{z}) = \partial_z N_{zz}^{(0)}(z, \bar{z}), \quad (42a)$$

$$\bar{T}_{(2d)}(\bar{z}) = \int d^2 w \frac{1}{(\bar{z} - \bar{w})} \partial_{\bar{w}}^3 N_{zz}^{(1)}(w, \bar{w}). \quad (42b)$$

TABLE I. Weights of Carroll stress-tensor components.

| Weights   | $T_u$ | $T_z$ | $T_{\bar{z}}$ |
|-----------|-------|-------|---------------|
| $\Delta$  | 3     | 3     | 3             |
| $h$       | 3/2   | 2     | 1             |
| $\bar{h}$ | 3/2   | 1     | 2             |

The “shadow” integral transformation [67] in the second equation of (42) maps a primary operator of holomorphic weight  $(h, \bar{h})$  to another one of  $(1-h, 1-\bar{h})$ . As  $S_1^+(z, \bar{z})$  has weights  $(1, -1)$ , its shadow is a primary of weight  $(0, 2)$  and this is identified as the antiholomorphic stress tensor of the 2D Celestial CFT. In addition, another Kac-Moody operator  $P_{(2d)}(z, \bar{z})$  of weight  $(\frac{3}{2}, \frac{1}{2})$  is required accounting for the leading soft graviton. With these identifications, the soft theorems in this basis takes the form of standard 2D CFT Ward identities.

We have seen that in Carrollian framework the universal soft sector is entirely contained in the stress tensors and the associated Ward identities coincide with the soft theorems. Stress tensor transformation rules in a CFT is dictated by the algebra of charges. In a 3D Carrollian CFT, this is the BMS<sub>4</sub> charge algebra (1) using which one can readily show that  $T_u, T_z, T_{\bar{z}}$  transform like primaries with the conformal weights  $(\frac{3}{2}, \frac{3}{2})$ ,  $(2, 1)$ , and  $(1, 2)$  respectively [29] (summarized in Table I). From this, it is immediately suggestive that celestial and Carrollian soft operators related via the following equations:

$$T_u(z, \bar{z}) = \partial_z P_{(2d)}(z, \bar{z}), \quad (43a)$$

$$T_z(z, \bar{z}) = \partial_z T_{(2d)}(z), T_{\bar{z}}(z, \bar{z}) = \partial_z \bar{T}_{(2d)}(\bar{z}). \quad (43b)$$

As a consequence of the integral transformation in (42), the celestial stress tensor  $\bar{T}_{(2d)}(\bar{z})$  is a nonlocal object. Hence, the action of  $\partial_z$  on  $\bar{T}_{(2d)}(\bar{z})$  does not vanish. On the contrary, it localizes the shadow integral giving rise to another local primary of weight  $(1, 2)$ , i.e.,

$$\begin{aligned} T_{\bar{z}}(z, \bar{z}) &= \partial_z \bar{T}_{(2d)}(\bar{z}) = \partial_z \int d^2 w \frac{1}{\bar{z} - \bar{w}} \partial_{\bar{w}}^3 N_{zz}^{(1)}(w, \bar{w}) \\ &= \int d^2 w \delta^2(z - w) \partial_{\bar{w}}^3 N_{zz}^{(1)}(w, \bar{w}) = \partial_{\bar{z}}^3 N_{zz}^{(1)}(z, \bar{z}). \end{aligned} \quad (44)$$

This primary operator naturally arises in the stress tensor sector of a generic 3D Carroll CFT. A similar result holds for the holomorphic  $T_z(z, \bar{z})$  which also localizes the nonlocal 2D holomorphic stress tensor.

## VII. DISCUSSIONS

We conclude our article with some preliminary remarks about the limit from AdS/CFT and the possible emergence of the hard sector.

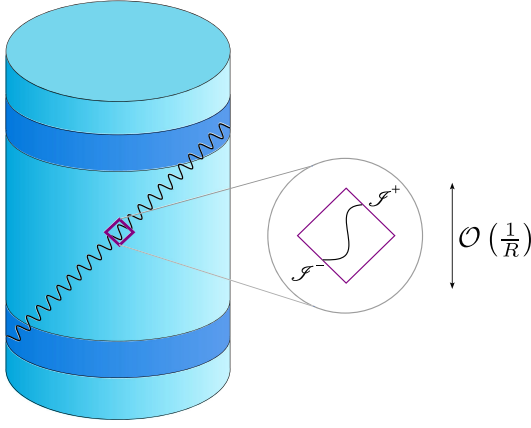


FIG. 1. Wavelengths of bulk scattering processes in AdS get stretched out as we zoom into the Minkowski diamond at the center of AdS.

### A. The limit from AdS/CFT

Let us offer some explanations as to why the Carroll stress tensor should holographically encode the soft sector in AFS as a limit of AdS/CFT.

Consider the traceless Noetherian stress tensor for a massless relativistic scalar  $\phi$  in 3D flat spacetime;  $T_\nu^\mu = \partial^\mu \phi \partial_\nu \phi - \frac{1}{2} \delta_\nu^\mu (\partial_\rho \phi \partial^\rho \phi) - \frac{1}{8} (-\delta_\nu^\mu \square + \partial^\mu \partial_\nu) \phi^2$ . We now implement the Carroll limit  $(t, x^i) \rightarrow (\epsilon t, x^i)$ ,  $\epsilon \rightarrow 0$  and find,

$$\begin{aligned} T_u^\mu &= \epsilon^2 T_t^\mu = -\frac{1}{2} (\partial_u \phi)^2, \\ T_i^\mu &= \epsilon T_t^\mu = -\frac{3}{4} \partial_u \phi \partial_i \phi + \frac{1}{4} \phi \partial_u \partial_i \phi. \end{aligned} \quad (45)$$

So  $T_u^\mu$  is the leading piece and  $T_i^\mu$  is next-to-leading piece in the Carroll expansion.

In standard AdS/CFT, the fluctuations of the bulk metric or gravitons are captured by the correlations of the boundary stress tensor. The flat limit of AdS/CFT can be viewed as a very high-energy limit where one focuses on a tiny Minkowski diamond at the center of AdS. Thus, if we do not scale the energy of the bulk scattering processes with the AdS radius, we end up with particles going soft in the flat limit. This is roughly due to the wavelength being stretched by an order of the AdS radius as depicted in Fig. 1. A Carrollian CFT can be thought of as a very high-energy limit of a usual relativistic CFT [69]. So a flat space limit in the bulk, which is Carroll limit on the boundary focuses only on the very high-energy modes of the original theory. Any finite energy solution in this limit would thus naturally scale to zero energy and become soft.

The mode of the graviton corresponding to the stress tensor component  $T_u^\mu$ , which generates supertranslations [see (28)] goes soft fastest because this is the leading term in the  $c \rightarrow 0$  limit.  $T_i^\mu$  is subleading in powers of  $c$  as compared to  $T_u^\mu$ . This combination also generates superrotations and corresponds to the subleading soft graviton

theorem. The components of the Carroll stress tensor thus naturally capture the soft sector of gravitational radiation in the bulk 4D AFS. From the point of view of a limit of AdS/CFT, this is because the finite energy gravitons are stretched out in the infinite radius limit which corresponds to the zero lightspeed limit in the dual CFT.

### B. Hard sector?

An immediate question is the precise relation between the scattering of hard gravitons and our Carroll stress tensor. Consider  $\theta_j^i$  of (24) which has the properties  $\theta_i^i = 0, \theta_j^i = \theta_i^j$ . These two conditions leave us with two independent components, which in  $z, \bar{z}$  coordinates can be expressed as  $\theta_{zz}$  and  $\theta_{\bar{z}\bar{z}}$ .

$T_z(z, \bar{z})$  transform as a Virasoro primary of weight  $(2, 1)$ . Also the coordinates  $(u, z, \bar{z})$  scale with weights  $(-\frac{1}{2}, -\frac{1}{2})$ ,  $(-1, 0)$ , and  $(0, -1)$ , respectively. Thus,  $\theta_{zz}, \theta_{\bar{z}\bar{z}}$  must have scaling dimensions  $(\frac{3}{2}, \frac{1}{2})$  and  $(\frac{1}{2}, \frac{5}{2})$ . So  $h - \bar{h} = \pm 2$  for these components and hence  $\theta_j^i$  captures the pure spin-2 sector of the Carrollian stress tensor.

The modified Mellin transformation relates spin of the boundary operators with the helicity of the massless particles involved in bulk scattering. So it is tempting to postulate that the correlation functions of these  $\theta_j^i$ 's would capture the hard-scattering amplitudes from boundary perspective. However, the precise relation to the spin-2 primary fields we constructed from the flat space limit in [33] remains to be worked out. It is instructive to understand the set of boundary conditions in AdS [71–73] and a suitable variational principle that reduces to our stress tensor in the flat space limit. This would firmly establish an important result in the holographic dictionary; the relation between the bulk metric degrees of freedom and the boundary stress tensor similar to AdS holography [74].

In conclusion, we have been able to find that the intrinsically defined local stress tensor of a 3D Carroll CFT naturally encodes both the leading and subleading soft graviton theorems, making this an improvement on the 2D Celestial stress tensor, which accounted for only the subleading one. We also showed relation between the Carroll and Celestial soft sectors and how the nonlocal Celestial stress tensor is localized by virtue of this relation. We finally offered some explanation of our findings as a limit of AdS/CFT.

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