

Z_{cs}^+ production in B^+ decaysZhuo Yu,^{1,*} Qi Wu^{2,†} and Dian-Yong Chen^{1,3,‡}¹*School of Physics, Southeast University, Nanjing 210094, China*²*Institute of Particle and Nuclear Physics, Henan Normal University, Xinxiang 453007, China*³*Lanzhou Center for Theoretical Physics, Lanzhou University, Lanzhou 730000, China*

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In the present work, we study the Z_{cs}^+ exotic state production in the B^+ meson decays through the meson loop mechanism, where the B^+ meson decays into an anticharmed meson and a charmed-strange meson pair, and consequently couples to Z_{cs} and a light meson ($\phi, \rho^0, \omega, \eta, \eta'$, and π^0) via exchanging a proper charmed or charmed-strange meson. Using the effective Lagrangian approach, we estimate the branching fractions of different channels, finding them to be of the order of 10^{-4} , and their ratios are almost independent of the model parameter. The fit fractions of Z_{cs}^+ in different B^+ decay processes are also estimated, and we propose searching for Z_{cs}^+ in the $B^+ \rightarrow J/\psi K^+ \eta'$ process, which should be accessible to both Belle II and LHCb Collaborations.

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I. INTRODUCTION

The quark model categorizes the simplest hadrons as mesons, composed of a quark and an antiquark, and baryons, which consist of three quarks. However, since the first observation of $X(372)$ by Belle Collaboration in 2003 [1], an increasing number of exotic candidates have been reported by LHCb, Belle, BABAR, and BESIII Collaborations (see Refs. [2–14] for recent reviews). Among these states, the charmoniumlike states consisting of a $c\bar{c}$ pair have formed a large family. Unlike the neutral $X(372)$, the first charged charmoniumlike state, $Z_c^-(4430)$, was observed in the $\pi^-\psi(2S)$ mass spectrum of the $B \rightarrow K\pi^-\psi(2S)$ process by Belle Collaboration in 2007 [15–17], and later confirmed by LHCb Collaboration in the same process [18]. Following $Z_c^-(4430)$, several more charged charmoniumlike states have been observed, including $Z_c^-(4240)$, observed in the $\pi^-\psi'$ invariant mass spectrum of the $B^0 \rightarrow \psi'\pi^-K^+$ decay [18], $Z_c^+(4050)$ and $Z_c^+(4250)$, seen in the $\pi^+\chi_{c1}$ invariant mass spectrum of the $\bar{B}^0 \rightarrow K^-\pi^+\chi_{c0}$ decay [19], $Z_c^\pm(3900)$, reported in the $\pi^\pm J/\psi$ invariant mass spectrum of the process $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ [20,21], $Z_c^+(4200)$, found in the $J/\psi\pi^+$ invariant mass spectrum of the

$\bar{B}^0 \rightarrow J/\psi K^-\pi^+$ decay [22], $Z_c^-(4100)$, observed in the $\eta_c\pi^-$ invariant mass spectrum of the $B^0 \rightarrow \eta_c(1S)K^+\pi^-$ decay [23], and $Z_c^\pm(4020)$, reported in the $\pi^\pm h_c$ invariant mass spectrum of the process $e^+e^- \rightarrow \pi^+\pi^-h_c$ [24].

Among these Z_c states, $Z_c^\pm(3900)$ and $Z_c^\pm(4020)$ stand out as two extensively investigated particles, both theoretically and experimentally, with their existence confirmed by multiple collaborations across various decay channels. The $Z_c^\pm(3900)$ was the first confirmed charged charmoniumlike state, initially observed by BESIII [20] and Belle [21] Collaborations in 2013 in the $J/\psi\pi^\pm$ invariant mass distributions of the process $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ at $\sqrt{s} = 4.26$ GeV. This state was subsequently confirmed by the authors of Ref. [25] in the same process but at $\sqrt{s} = 4.17$ GeV using the data collected by the CLEO-c detector. As the heavy quark spin symmetry partner of $Z_c^\pm(3900)$, $Z_c^\pm(4020)$ was observed in the $\pi^\pm h_c$ mass spectrum via the $e^+e^- \rightarrow \pi^+\pi^-h_c$ process [24] by BESIII Collaboration in the same year. In addition to the hidden charm process, BESIII Collaboration also observed both states in the $(D^*\bar{D}^*)^\pm$ invariant mass spectrum of the $e^+e^- \rightarrow \pi D^*\bar{D}^*$ processes [26,27].

The observed masses of $Z_c(3900)$ and $Z_c(4020)$ lie close to the threshold of $D^*\bar{D}$ and $D^*\bar{D}^*$, respectively. This particular property has led to the proposition of the molecular interpretations with $I = 1$. In the framework of the one-boson-exchange model, $Z_c(3900)$ and $Z_c(4020)$ were accommodated as $D^*\bar{D}$ and $D^*\bar{D}^*$ molecular states with $I = 1$ [28,29]. In Ref. [30], the authors applied the QCD sum rule in the $D^*\bar{D}^*$ molecular scenario to reproduce the mass of $Z_c(4020)$. The decay behaviors of $Z_c(3900)/Z_c(4020)$ were also investigated within the

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molecular framework in Refs. [31–37]. Since these two charmoniumlike states were observed in the hidden charm final states, the most possible quark components of charged charmoniumlike states are $c\bar{c}q\bar{q}$, which indicate that they could be regarded as tetraquark states [38–44]. In addition to QCD exotic states' interpretations, $Z_c(3900)$ and $Z_c(4020)$ could also be well reproduced by the initial single pion emission mechanism [45–48].

According to SU(3) flavor symmetry, the strange partners of Z_c states with quark content $c\bar{c}s\bar{q}$ should also exist and have in fact been predicted by several theoretical models [49–53]. In 2021, BESIII Collaboration observed the strangeness flavor partner of the $Z_c(3900)$, named $Z_{cs}(3985)$, in the K^+ recoil mass spectrum of the process $e^+e^- \rightarrow K^+(D_s^-D^{*0} + D_s^{*-}D^0)$ [54]. Subsequently, a structure, $Z_{cs}(4000)$, with $J^P = 1^+$ was observed by LHCb Collaboration in the $B^+ \rightarrow J/\psi\phi K^+$ decay [55]. The resonance parameters of these two states are

$$\begin{aligned} Z_{cs}(3985): & \quad M = 3982.5 \pm 2.1_{-2.6}^{+1.8} \text{ MeV}, \\ & \quad \Gamma = 12.8_{-4.4}^{+5.3} \pm 3.0 \text{ MeV}, \\ Z_{cs}(4000): & \quad M = 4003.0 \pm 6.0_{-14.0}^{+4.0} \text{ MeV}, \\ & \quad \Gamma = 131.0 \pm 15 \pm 26 \text{ MeV}. \end{aligned} \quad (1)$$

One can find that the masses of these two states are pretty close, while their widths differ significantly. Because of this property, whether $Z_{cs}(3985)$ and $Z_{cs}(4000)$ are the same state remains an open question. Following Refs. [56–58], we assume them as the same state and use Z_{cs} to refer to them in the present work.

The theoretical interpretations of Z_{cs} are in line with those of $Z_c^\pm(3900)$ and $Z_c^\pm(4020)$, which range from molecular states [56,59–66], to compact tetraquark states [60,67–71], to kinematic effects [72,73]. Given that the mass of Z_{cs} lies near the threshold of $D_s\bar{D}^*/D_s^*\bar{D}$, it is natural to regard it as a $D_s^{(*)}\bar{D}^{(*)}$ molecular state [56,59–66]. However, studies employing the one-boson-exchange model do not support Z_{cs} as a hadronic molecular state or a pure molecular state [74]. In the tetraquark scenario, analyses based on QCD sum rules [60,67,68] and quark models [69–71] suggest that Z_{cs} may instead be a compact tetraquark state. Besides the molecular and tetraquark states, the reflection mechanism [72] and threshold effect [73] were also proposed to reveal the nature of the Z_{cs} state.

The inner structure of the Z_{cs} remains elusive. In order to better understand its nature, one might look for more production modes. One of the most important production processes of exotic states is the B^+ decay process, where LHCb Collaboration has observed Z_{cs} in $B^+ \rightarrow J/\psi\phi K^+$ decays, reporting a fit fraction [55],

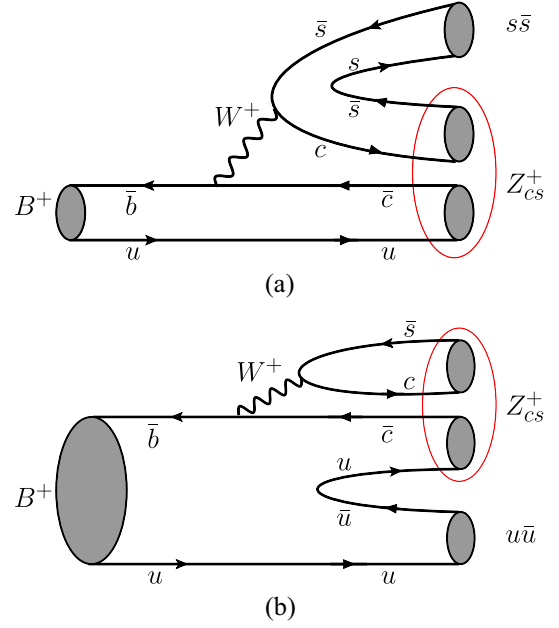


FIG. 1. Diagrams contributing to $B^+ \rightarrow Z_{cs}^+ s\bar{s}$ (diagram a) and $B^+ \rightarrow Z_{cs}^+ u\bar{u}$ (diagram b) at quark level.

$$\frac{\mathcal{B}[B^+ \rightarrow Z_{cs}^+ \phi \rightarrow J/\psi\phi K^+]}{\mathcal{B}[B^+ \rightarrow J/\psi\phi K^+]} = (9.4 \pm 2.1 \pm 3.4)\%. \quad (2)$$

Considering the Particle Data Group (PDG) average of the branching fraction of $B^+ \rightarrow J/\psi\phi K^+$ to be $(5.0 \pm 0.4) \times 10^{-5}$ [75], one can conclude that the branching fraction of the cascade process is

$$\mathcal{B}[B^+ \rightarrow Z_{cs}^+ \phi \rightarrow J/\psi K^+ \phi] = (4.6 \pm 2.0) \times 10^{-6}. \quad (3)$$

Such a large branching fraction in B decay processes is helpful for us to investigate the property of Z_{cs} . In our previous works, we have studied the hidden charm decays [76] and the production of Z_{cs} in B/B_s decays [58]. The branching fraction of the cascade process $B^+ \rightarrow Z_{cs}^+ \phi \rightarrow J/\psi K^+ \phi$ is estimated to be $(1.86_{-1.37}^{+2.12}) \times 10^{-6}$ by considering the triangle loop with $D_s^{(*)}\bar{D}^{(*)}D_s^{(*)}$ [58], which is comparable with the experimental measurement from LHCb Collaboration. According to the Review of Particle Physics [75], the branching fractions of $B^+ \rightarrow \bar{D}^0 D_{s1}^+(2460)$ and $B^+ \rightarrow \bar{D}^{*0} D_{s1}^+(2460)$ are $(3.1_{-0.9}^{+1.0}) \times 10^{-3}$ and $(12.0 \pm 3.0) \times 10^{-3}$, respectively, which are in the same order of $B^+ \rightarrow \bar{D}^{(*)0} D_s^{*+}$.¹ Moreover, the $D_s^{(*)+} D_s^{(*)+} \phi$ vertices have a P -wave coupling, while the D_{s1}^+ couples to the $D_s^{(*)+} \phi$ in S -waves. Thus, the triangle

¹In this work, $D_{s1}^+(2460)^+$ is treated as a P -wave charmed-strange meson with $J^P = 1^+$ and it will be abbreviated as D_{s1}^+ in the following.

TABLE I. The considered processes in the present work and their branching fractions in the PDG average [75].

Process	Branching fraction
$B^+ \rightarrow J/\psi K^+ \phi$	$(5.0 \pm 0.4) \times 10^{-5}$
$B^+ \rightarrow J/\psi K^+ \omega$	$(3.2^{+0.60}_{-0.32}) \times 10^{-4}$
$B^+ \rightarrow J/\psi K^+ \eta$	$(1.24 \pm 0.14) \times 10^{-4}$
$B^+ \rightarrow J/\psi K^+ \eta'$	$(3.1 \pm 0.4) \times 10^{-5}$
$B^+ \rightarrow J/\psi K^+ \pi^0$	$(1.14 \pm 0.1) \times 10^{-3}$
$B^+ \rightarrow J/\psi K^+ \rho^0$	—

loop with $D_{s1}^{*+} \bar{D}^{(*)} D_s^{(*)}$ could also contribute to the process $B^+ \rightarrow \phi Z_{cs}^+$, and we will revisit the process $B^+ \rightarrow \phi Z_{cs}^+$ by including the triangle loop with $D_{s1}^{*+} \bar{D}^{(*)} D_s^{(*)}$ in this work.

In the present work, we aim to systematically investigate the production of Z_{cs} in B decay with a light unflavored meson. Given that the quark components of Z_{cs}^+ are the molecular state composed of $D_s^+ \bar{D}^0$, there are two different production mechanisms at the quark level, as shown in Fig. 1. In Fig. 1(a), the antibottom quark transitions into an anticharmed quark by emitting a W^+ boson, which consequently decays into a charmed quark and an antistrange quark. The charmed quarks hadronize as D_s^+ with the antistrange quark created from the vacuum, while the rest of the antistrange quarks from W^+ decay and strange quarks created from the vacuum form a light unflavored hadron containing $s\bar{s}$ quark components, such as ϕ , η , and η' . The anticharmed quark coming from the antibottom quark and the observed up quark form a $\bar{D}^0$, then the $\bar{D}^0$ and D_s^+ are bounded as the molecular state Z_{cs}^+ . A similar mechanism can be found in Fig. 1(b). Different from the mechanism in Fig. 1(a), the quark pair created from the vacuum is $u\bar{u}$, and the $u\bar{u}$ in the final states could hadronize as π^0 , η , η' , ω , and ρ^0 mesons.

In the present work, we evaluate the possibility of searching for Z_{cs}^+ in the $J/\psi K^+$ invariant mass distributions of the decay processes $B^+ \rightarrow J/\psi K^+ \mathbb{M}$, where $\mathbb{M}$ can be $\phi/\omega/\rho^0/\pi^0/\eta/\eta'$. Fortunately, all the above processes except for $B^+ \rightarrow J/\psi K^+ \rho^0$ have been measured experimentally, and the branching fractions range from 10^{-5} to 10^{-3} , which are listed in Table I. Using the branching fraction of $Z_{cs}^+ \rightarrow J/\psi K^+$ estimated in Ref. [76], we will revisit the fit fraction of Z_{cs} in $B^+ \rightarrow J/\psi K^+ \phi$ and predict the fit fractions of Z_{cs} in $B^+ \rightarrow J/\psi K^+ \omega/\eta/\eta'/\pi^0$. The branching fraction of $B^+ \rightarrow J/\psi K^+ \rho^0$ remains unclear up till now, but we still consider it in the present work.

The rest of this work is organized as follows. In Sec. II, we present the model used in the estimation of Z_{cs} productions. The numerical results and the relevant discussions are given in Secs. III and IV is devoted to a brief summary.

II. THEORETICAL FRAMEWORK

In the present work, we estimate the production processes at the hadron level, where the molecular state

TABLE II. All the loops considered in the present work contribute to $B^+ \rightarrow Z_{cs}^+ \phi/\omega/\rho^0/\eta/\eta'/\pi^0$.

Process	Loops
$B^+ \rightarrow Z_{cs}^+ \phi$	$D_s^+ \bar{D}^0 D_s^{*+}, D_s^{*+} \bar{D}^{*0} D_s^+, D_s^{*+} \bar{D}^0 D_s^{*+},$ $D_s^{*+} \bar{D}^{*0} D_s^+, D_{s1}^{*+} \bar{D}^0 D_s^{*+}, D_{s1}^{*+} \bar{D}^{*0} D_s^+,$
$B^+ \rightarrow Z_{cs}^+ \omega/\rho^0$	$\bar{D}^0 D_s^+ \bar{D}^{*0}, \bar{D}^{*0} D_s^+ \bar{D}^{*0},$ $\bar{D}^0 D_s^{*+} \bar{D}^0, \bar{D}^{*0} D_s^{*+} \bar{D}^0,$
$B^+ \rightarrow Z_{cs}^+ \eta/\eta'$	$D_s^+ \bar{D}^0 D_s^{*+}, D_s^{*+} \bar{D}^0 D_s^{*+}, D_s^{*+} \bar{D}^{*0} D_s^+, D_{s1}^{*+} \bar{D}^0 D_s^{*+},$ $\bar{D}^0 D_s^+ \bar{D}^{*0}, \bar{D}^{*0} D_s^+ \bar{D}^{*0}, \bar{D}^{*0} D_s^{*+} \bar{D}^0$
$B^+ \rightarrow Z_{cs}^+ \pi^0$	$\bar{D}^0 D_s^+ \bar{D}^{*0}, \bar{D}^{*0} D_s^+ \bar{D}^{*0}, \bar{D}^{*0} D_s^{*+} \bar{D}^0$

Z_{cs}^+ could be produced in B^+ meson decay through the triangle loop mechanism. In our model, the B^+ meson first decays into a charm-strange meson and an anticharmed meson through weak interaction, and they transition into Z_{cs} and a light unflavor meson by exchanging a proper charm-strange or charmed meson. In Table II, we collect the possible meson loops contributing to the discussed processes, where $\mathbb{M}_1 \mathbb{M}_2 \mathbb{M}_3$ refers to the meson loop constructed by $\mathbb{M}_1$, $\mathbb{M}_2$, and $\mathbb{M}_3$. Here, we take $B^+ \rightarrow Z_{cs}^+ \phi$ as an example, and the relevant diagrams are presented in Fig. 2. We employ the effective Lagrangian approach to calculate the triangle loop diagrams. In the following subsections, we present the relevant effective Lagrangian and decay amplitude for calculating these production processes.

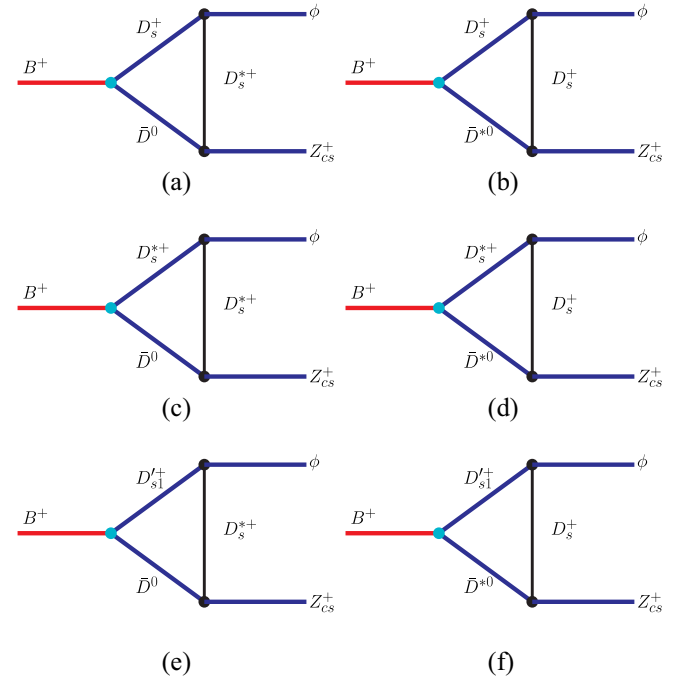


FIG. 2. Diagrams contributing to $B^+ \rightarrow Z_{cs}^+ \phi$. (a)–(f) correspond to different meson loop contributions as labeled.

A. Effective Lagrangian

In the present work, Z_{cs}^+ is assumed to be an S -wave bound state of $D_s^+ \bar{D}^{*0} + D_s^{*+} \bar{D}^0$ with $I(J^P) = \frac{1}{2}(1^+)$, which is

$$Z_{cs}^+ = \frac{1}{\sqrt{2}}(|D_s^{*+} \bar{D}^0\rangle + |D_s^+ \bar{D}^{*0}\rangle), \quad (4)$$

and the effective Lagrangian depicting the interaction of Z_{cs} with its components can be

$$\mathcal{L}_{Z_{cs}} = \frac{g_{Z_{cs}}}{\sqrt{2}} Z_{cs}^\mu (D_{s\mu}^* \bar{D} + D_s \bar{D}_\mu^*), \quad (5)$$

where $g_{Z_{cs}}$ is the effective coupling constant.

As for the B^+ weak decay vertex, we utilized the parametrized hadronic matrix elements obtained by the effective Hamiltonian at the quark level, which are [77,78]

$$\begin{aligned} \langle 0 | J_\mu | P(p_1) \rangle &= -if_P p_{1\mu}, \\ \langle 0 | J_\mu | V(p_1, \epsilon) \rangle &= f_V \epsilon_\mu m_V, \\ \langle P(p_2) | J_\mu | B(p) \rangle &= \left[P_\mu - \frac{m_B^2 - m_P^2}{Q^2} Q_\mu \right] F_1(Q^2) + \frac{m_B^2 - m_P^2}{Q^2} Q_\mu F_0(Q^2), \\ \langle V(p_2, \epsilon) | J_\mu | B(p) \rangle &= \frac{i\epsilon^\nu}{m_B + m_V} \left\{ i\epsilon_{\mu\nu\alpha\beta} P^\alpha Q^\beta V(Q^2) - (m_B + m_V)^2 g_{\mu\nu} A_1(Q^2) + P_\mu P_\nu A_2(Q^2) \right. \\ &\quad \left. + 2m_V(m_B + m_V) \frac{P_\nu Q_\mu}{Q^2} [A_3(Q^2) - A_0(Q^2)] \right\}, \end{aligned} \quad (6)$$

with $J_\mu = \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2$, $P_\mu = p_\mu + p_{2\mu}$, and $Q_\mu = p_\mu - p_{2\mu}$. $V, A_{0,1,2}(Q^2)$ and $F_{0,1}(Q^2)$ are the weak transition form factors, while $A_3(Q^2)$ is the linear combination of form factors $A_1(Q^2)$ and $A_2(Q^2)$, which is [77]

$$A_3(Q^2) = \frac{m_B + m_V}{2m_V} A_1(Q^2) - \frac{m_B - m_V}{2m_V} A_2(Q^2). \quad (7)$$

More details of these form factors will be discussed in the following section.

The amplitudes of the B^+ weak transition could be constructed by the products of two hadronic matrix elements. Here, we take the vertex of $B^+ \rightarrow D_{s1}^+ \bar{D}^*$ as an example, and the amplitude reads

$$\begin{aligned} \mathcal{A}(B^+(p_0) \rightarrow D_{s1}^+(p_1) \bar{D}^*(p_2)) &= \mathcal{A}_{\mu\nu}^{B^+ \rightarrow D_{s1}^+ \bar{D}^*}(p_1, p_2) \epsilon_{D_{s1}}^\mu \epsilon_{\bar{D}^*}^\nu \\ &= \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cs} a_1 \langle D_{s1}^+(p_1, \epsilon_{D_{s1}}) | J_\mu^\dagger | 0 \rangle \langle \bar{D}^*(p_2, \epsilon_{\bar{D}^*}) | J_\mu | B^+(p_0) \rangle \epsilon_{D_{s1}}^\mu \epsilon_{\bar{D}^*}^\nu, \end{aligned} \quad (8)$$

where G_F is the Fermi constant, V_{cb}^* and V_{cs} are the Cabibbo–Kobayashi–Maskawa matrix elements, and $a_1 = c_1^{\text{eff}} + c_2^{\text{eff}}/N_c$ with $c_{1,2}^{\text{eff}}$ are taken to be effective Wilson coefficients obtained by the factorization approach [79]. In the present estimations, we adopt $G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$, $V_{cb} = 0.041$, $V_{cs} = 0.987$, and $a_1 = 1.05$, as in Refs. [80–82].

The interaction of light unflavored mesons and heavy-light mesons can be described by effective Lagrangians constructed based on the heavy quark limit and chiral symmetry. The relevant effective Lagrangians are given by

$$\begin{aligned} \mathcal{L}_{\mathcal{D}^* \mathcal{D}^{(*)} \mathcal{P}} &= -ig_{\mathcal{D}^* \mathcal{D} \mathcal{P}} (\mathcal{D}_i^\dagger \partial_\mu \mathcal{P}_{ij} \mathcal{D}_j^{*\mu} - \mathcal{D}_i^{*\mu\dagger} \partial_\mu \mathcal{P}_{ij} \mathcal{D}_j) + \frac{1}{2} g_{\mathcal{D}^* \mathcal{D}^* \mathcal{P}} \epsilon_{\mu\nu\alpha\beta} \mathcal{D}_i^{*\mu\dagger} \partial^\nu \mathcal{P}_{ij} \overleftrightarrow{\partial}^\alpha \mathcal{D}_j^{*\beta} + \text{H.c.}, \\ \mathcal{L}_{\mathcal{D}^{(*)} \mathcal{D}^{(*)} \mathcal{V}} &= -ig_{\mathcal{D} \mathcal{D} \mathcal{V}} \mathcal{D}_i^\dagger \overleftrightarrow{\partial}_\mu \mathcal{D}_j (\mathcal{V}^\mu)_j^i - 2f_{\mathcal{D}^* \mathcal{D} \mathcal{V}} \epsilon_{\mu\nu\alpha\beta} (\partial^\mu \mathcal{V}^\nu)_j^i (\mathcal{D}_i^\dagger \overleftrightarrow{\partial}^\alpha \mathcal{D}_j^{*\beta} - \mathcal{D}_i^{*\beta\dagger} \overleftrightarrow{\partial}^\alpha \mathcal{D}_j) \\ &\quad + ig_{\mathcal{D}^* \mathcal{D}^* \mathcal{V}} \mathcal{D}_i^{*\nu\dagger} \overleftrightarrow{\partial}_\mu \mathcal{D}_j^{*\mu} (\mathcal{V}^\mu)_j^i + 4if_{\mathcal{D}^* \mathcal{D}^* \mathcal{V}} \mathcal{D}_{i\mu}^{*\dagger} (\partial^\mu \mathcal{V}^\nu - \partial^\nu \mathcal{V}^\mu)_j^i \mathcal{D}_\nu^{*j} + \text{H.c.}, \\ \mathcal{L}_{\mathcal{D}'_1 \mathcal{D}^* \mathcal{P}} &= g_{\mathcal{D}'_1 \mathcal{D}^* \mathcal{P}} [\mathcal{D}'_{1b\mu} \overleftrightarrow{\partial}^\mu \mathcal{D}_{a\nu}^* \partial^\nu \mathcal{P}_{ba} - \mathcal{D}'_{1b\mu} \overleftrightarrow{\partial}^\nu \mathcal{D}_a^{*\mu\dagger} \partial_\nu \mathcal{P}_{ba} + \mathcal{D}'_{1b\mu} \overleftrightarrow{\partial}^\nu \mathcal{D}_\nu^{*\dagger} \mathcal{P}_{ba}], \\ \mathcal{L}_{\mathcal{D}'_1 \mathcal{D}^{(*)} \mathcal{V}} &= g_{\mathcal{D}'_1 \mathcal{D} \mathcal{V}} \mathcal{D}_b \mathcal{D}_{1a}^{*\dagger} (\mathcal{V}^\mu)_{ba} + ig_{\mathcal{D}'_1 \mathcal{D}^* \mathcal{V}} \epsilon_{\mu\nu\alpha\beta} \mathcal{D}^{*\mu} \overleftrightarrow{\partial}^\beta \mathcal{D}'_1^{*\alpha\dagger} (\mathcal{V}^\nu)_{ba}, \end{aligned} \quad (9)$$

where $\mathcal{D}'_1 = (D'_1(2430)^0, D'_1(2430)^+, D'_{s1}(2460)^+)$, $\mathcal{D}^{(*)\dagger} = (D^{(*)0}, D^{(*)+}, D_s^{(*)+})$, and $A \overset{\leftrightarrow}{\partial}_\mu B = A \partial_\mu B - B \partial_\mu A$. $\mathcal{P}$ and $\mathcal{V}$ are 3×3 matrices representing pseudoscalar and vector mesons, with their concrete forms being

$$\mathcal{P} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \alpha\eta + \beta\eta' & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \alpha\eta + \beta\eta' & K^0 \\ K^- & \bar{K}^0 & \gamma\eta + \delta\eta' \end{pmatrix}, \quad (10)$$

$$\mathcal{V} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}, \quad (11)$$

where the parameters α, β, γ , and δ in Eq. (10) related to the mixing angle θ are defined as

$$\alpha = \frac{\cos\theta - \sqrt{2}\sin\theta}{\sqrt{6}}, \quad \beta = \frac{\sin\theta + \sqrt{2}\cos\theta}{\sqrt{6}}, \\ \gamma = \frac{-2\cos\theta - \sqrt{2}\sin\theta}{\sqrt{6}}, \quad \delta = \frac{-2\sin\theta + \sqrt{2}\cos\theta}{\sqrt{6}}, \quad (12)$$

with the mixing angle $\theta = -14.1^\circ$ [83,84].

B. Decay amplitude

With the effective Lagrangians discussed in the previous section, we can now obtain the amplitudes corresponding to the processes shown in Fig. 2, which read

$$\begin{aligned} \mathcal{M}_a &= i^3 \int \frac{d^4 q}{(2\pi)^4} [\mathcal{A}^{B^+ \rightarrow D_s^+ \bar{D}^0}(p_1, p_2)] [2f_{D^*} \mathcal{D} \mathcal{V} \varepsilon_{\mu\delta\alpha\beta} (ip_3^\mu) (p_1^\alpha + q^\alpha)] [ig_{Z_{cs} D_s^* D} g_{\delta\phi}] \left[\frac{1}{p_1^2 - m_1^2} \right] \left[\frac{1}{p_2^2 - m_2^2} \right] \\ &\quad \times \left[\frac{-g^{\beta\phi} + q^\beta q^\phi / m_q^2}{q^2 - m_q^2} \right] \mathcal{F}^2(q^2, m_q^2), \\ \mathcal{M}_b &= i^3 \int \frac{d^4 q}{(2\pi)^4} [\mathcal{A}^{B^+ \rightarrow D_s^+ \bar{D}^{*0}}(p_1, p_2)] [-g_{D^*} \mathcal{D} \mathcal{V} (p_{1\delta} + q_\delta)] [ig_{Z_{cs} D_s^* D}] \left[\frac{1}{p_1^2 - m_1^2} \right] \left[\frac{-g^{\nu\rho} + p_2^\nu p_2^\rho / m_2^2}{p_2^2 - m_2^2} \right] \left[\frac{1}{q^2 - m_q^2} \right] \mathcal{F}^2(q^2, m_q^2), \\ \mathcal{M}_c &= i^3 \int \frac{d^4 q}{(2\pi)^4} [\mathcal{A}^{B^+ \rightarrow D_s^{*+} \bar{D}^0}(p_1, p_2)] [ig_{D^*} \mathcal{D}^* \mathcal{V} (p_1 + q)_\delta g_{\alpha\beta} + 4if_{D^*} \mathcal{D}^* \mathcal{V} ((ip_{3\beta}) g_{\alpha\delta} - (ip_{3\alpha}) g_{\delta\beta})] [ig_{Z_{cs} D_s^* D} g_{\delta\phi}] \\ &\quad \times \left[\frac{-g^{\mu\alpha} + p_1^\mu p_1^\alpha / m_1^2}{p_1^2 - m_1^2} \right] \left[\frac{1}{p_2^2 - m_2^2} \right] \left[\frac{-g^{\beta\phi} + q^\beta q^\phi / m_q^2}{q^2 - m_q^2} \right] \mathcal{F}^2(q^2, m_q^2), \\ \mathcal{M}_d &= i^3 \int \frac{d^4 q}{(2\pi)^4} [\mathcal{A}^{B^+ \rightarrow D_s^{*+} \bar{D}^{*0}}(p_1, p_2)] [-2f_{D^*} \mathcal{D} \mathcal{V} \varepsilon_{\mu\delta\alpha\kappa} (ip_3^\mu) (p_1^\kappa + q^\kappa)] [ig_{Z_{cs} D_s^* D} g_{\delta\rho}] \left[\frac{-g^{\nu\alpha} + p_1^\nu p_1^\alpha / m_1^2}{p_1^2 - m_1^2} \right] \\ &\quad \times \left[\frac{-g^{\rho\rho} + p_2^\rho p_2^\rho / m_2^2}{p_2^2 - m_2^2} \right] \left[\frac{1}{q^2 - m_q^2} \right] \mathcal{F}^2(q^2, m_q^2), \\ \mathcal{M}_e &= i^3 \int \frac{d^4 q}{(2\pi)^4} [\mathcal{A}^{B^+ \rightarrow D_{s1}^{*+} \bar{D}^0}(p_1, p_2)] [ig_{D_{s1}^*} \mathcal{D}^* \mathcal{V} \varepsilon_{\beta\delta\alpha\tau} (p_1^\tau + q^\tau)] [ig_{Z_{cs} D_s^* D} g_{\delta\phi}] \left[\frac{-g^{\mu\alpha} + p_1^\mu p_1^\alpha / m_1^2}{p_1^2 - m_1^2} \right] \\ &\quad \times \left[\frac{1}{p_2^2 - m_2^2} \right] \left[\frac{-g^{\beta\phi} + q^\beta q^\phi / m_q^2}{q^2 - m_q^2} \right] \mathcal{F}^2(q^2, m_q^2), \\ \mathcal{M}_f &= i^3 \int \frac{d^4 q}{(2\pi)^4} [\mathcal{A}^{B^+ \rightarrow D_{s1}^{*+} \bar{D}^{*0}}(p_1, p_2)] [ig_{D_{s1}^*} \mathcal{D} \mathcal{V} g_{\delta\alpha}] [ig_{Z_{cs} D_s^* D} g_{\delta\rho}] \left[\frac{-g^{\mu\alpha} + p_1^\mu p_1^\alpha / m_1^2}{p_1^2 - m_1^2} \right] \\ &\quad \times \left[\frac{-g^{\rho\rho} + p_2^\rho p_2^\rho / m_2^2}{p_2^2 - m_2^2} \right] \left[\frac{1}{q^2 - m_q^2} \right] \mathcal{F}^2(q^2, m_q^2). \end{aligned} \quad (13)$$

In the above amplitudes, we introduce a form factor $\mathcal{F}(q^2, m^2)$ to depict the internal structures of the involved hadrons. It also plays the role of making the amplitudes finite in the ultraviolet region. In the present work, we adopt the form factor in the dipole form, and its concrete form is

$$\mathcal{F}(q^2, m^2) = \frac{m^2 - \Lambda^2}{q^2 - \Lambda^2}, \quad (14)$$

where the parameter Λ can be reparametrized as $\Lambda = m + \alpha\Lambda_{\text{QCD}}$, where $\Lambda_{\text{QCD}} = 220$ MeV. α is the model parameter [85], which should be of order unity.

Then, the amplitude of $B^+ \rightarrow Z_{cs}^+\phi$ reads

$$\mathcal{M}_{B^+ \rightarrow Z_{cs}^+\phi} = \mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c + \mathcal{M}_d + \mathcal{M}_e + \mathcal{M}_f, \quad (15)$$

and the partial width of $B^+ \rightarrow Z_{cs}^+\phi$ can be estimated by

$$\Gamma_{B^+ \rightarrow Z_{cs}^+\phi} = \frac{1}{8\pi} \frac{|\vec{p}|}{m_{B^+}^2} |\overline{\mathcal{M}_{B^+ \rightarrow Z_{cs}^+\phi}}|^2, \quad (16)$$

where $\vec{p}$ is the momentum of final states in the rest frame of B^+ , and the overline above the amplitude indicates the sum over the spins of final states.

III. NUMERICAL RESULTS AND DISCUSSIONS

A. Coupling constants

Before presenting the numerical results, some relevant constants should be clarified. The decay constants in Eq. (6) are taken as $f_{D_s} = 250$ MeV, $f_{D_s^*} = 272$ MeV, and $f_{D_{s1}^*} = 158$ MeV from Refs. [86,87]. In addition, the form factors in Eq. (6) are usually estimated in the quark model and are known only in the spacelike region. Some methods, like analytic continuation, could cover the time-like region where the physical decay processes occur. In Refs. [77,78], the form factors are parametrized in the form

$$F(Q^2) = \frac{F(0)}{1 - a\zeta + b\zeta^2}, \quad (17)$$

with $\zeta = Q^2/m_B^2$. The relevant parameters $F(0)$, a , and b for each form factor are collected in Table III. Notably, the weak transition form factors are introduced due to the internal structures of the involved mesons, and similar to the form factor in Eq. (14), these form factors also serve to regularize the loop integral in the ultraviolet region. To evaluate the loop integrals via the Feynman parametrization method, we further parametrize these form factors in the form

TABLE III. Values of parameters $F(0)$, a , and b in form factors [77].

Parameter	$F(0)$	a	b
F_0	0.67	0.65	0.00
F_1	0.67	1.25	0.39
A_0	0.64	1.30	0.31
A_1	0.63	0.65	0.02
A_2	0.61	1.14	0.52
V	0.75	1.29	0.45

TABLE IV. Values of parameters Λ_1 and Λ_2 obtained by fitting Eq. (18) with Eq. (17).

Parameter	F_0	F_1	A_0	A_1	A_2	V
Λ_1	7.75	6.53	6.5	8.95	6.65	6.30
Λ_2	11.00	6.84	7.1	8.75	7.55	7.10

$$F(Q^2) = F(0) \frac{\Lambda_1^2}{Q^2 - \Lambda_1^2} \frac{\Lambda_2^2}{Q^2 - \Lambda_2^2}. \quad (18)$$

By fitting Eq. (17) with Eq. (18), we can obtain the values of Λ_1 and Λ_2 for each form factor, which are listed in Table IV.

Considering the chiral symmetry and heavy quark limit, the coupling constants of charmed mesons with pseudo-scalar and vector mesons can be related to gauge couplings by [88–90]

$$\begin{aligned} g_{D^*\mathcal{D}\mathcal{P}} &= \frac{2g}{f_\pi} \sqrt{m_{\mathcal{D}}m_{\mathcal{D}^*}}, & g_{D^*\mathcal{D}^*\mathcal{P}} &= \frac{g_{D^*\mathcal{D}\mathcal{P}}}{\sqrt{m_{\mathcal{D}^*}m_{\mathcal{D}}}}, \\ g_{\mathcal{D}\mathcal{D}\mathcal{V}} &= g_{D^*\mathcal{D}^*\mathcal{V}} = \frac{\beta g_V}{\sqrt{2}}, & f_{D^*\mathcal{D}\mathcal{V}} &= \frac{f_{D^*\mathcal{D}^*\mathcal{V}}}{m_{\mathcal{D}^*}} = \frac{\lambda g_V}{\sqrt{2}}, \\ g_{\mathcal{D}_1'\mathcal{D}^*\mathcal{P}} &= \frac{2h}{f_\pi}, & g_{\mathcal{D}_1'\mathcal{D}\mathcal{V}} &= -\sqrt{2}g_V\zeta\sqrt{m_{\mathcal{D}}m_{\mathcal{D}_1'}}, \\ g_{\mathcal{D}_1'\mathcal{D}^*\mathcal{V}} &= \sqrt{2}g_V\zeta, \end{aligned} \quad (19)$$

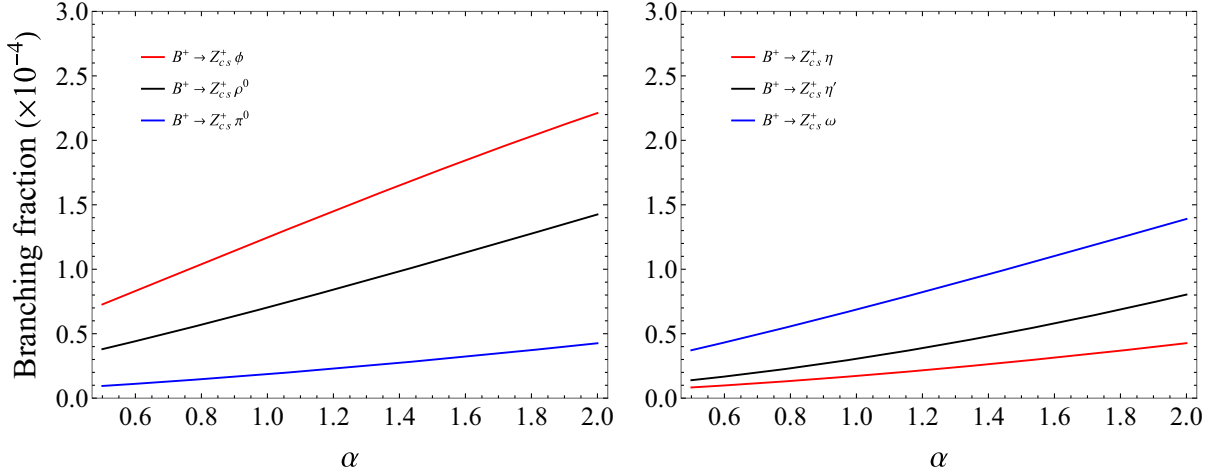
where the parameters $\beta = 0.9$, $\zeta = 0.1$, $g_V = m_\rho/f_\pi$ with $f_\pi = 0.132$ GeV, and $h = -0.56$ [90]. By matching the form factor obtained from the light cone sum rules and lattice QCD, one can obtain the parameters $\lambda = 0.56$ GeV⁻¹ and $g = 0.59$ [88].

In addition to the above coupling constants, the coupling constant $g_{Z_{cs}D_s^*D}$ was investigated in the decay properties of Z_{cs} [76], and the coupling constant $g_{Z_{cs}D_s^*D}$ was determined to be 6.0–6.7, which is weakly dependent on the model parameter. In the following, we take $g_{Z_{cs}D_s^*D} = 6.0$, as in Ref. [58], which investigated the production of Z_{cs} in B and B_s decays.

B. Branching fractions

Now, all the relevant coupling constants and the parameters in the weak transition form factors are prepared except the model parameter α in Eq. (14), since the model parameter α cannot be determined by first-principles methods and it should be of order unity, as clarified in Ref. [85]. In the present work, we vary the model parameter α from 0.5 to 2.0 to check the α dependence of the branching fractions of the considered processes.

As shown in Fig. 3, the α dependence of branching fractions for the considered processes is presented. On the whole, the branching fractions are estimated to around 10^{-4} and increase with the increase of α . All the estimated branching fractions are compiled in Table V, showing


 FIG. 3. The branching fractions of $B^+ \rightarrow Z_{cs}^+ \mathbb{M}$ depending on the model parameter α , where $\mathbb{M}$ can be $\phi/\omega/\rho^0/\pi^0/\eta/\eta'$.

$B^+ \rightarrow \phi Z_{cs}^+$ as the largest channel in the whole parameter range, naturally explaining why LHCb first observed Z_{cs} in $B^+ \rightarrow J/\psi K^+ \phi$. In Ref. [76], we investigated the hidden charm decays of the Z_{cs} state, and the branching fraction of $Z_{cs}^+ \rightarrow J/\psi K^+$ was estimated to be $(4.0^{+4.3}_{-2.7})\%$. Considering Z_{cs} is a narrow resonance, the branching fraction of the cascade process is

$$\begin{aligned} \mathcal{B}[B^+ \rightarrow Z_{cs}^+ \phi \rightarrow J/\psi K^+ \phi] \\ = \mathcal{B}[B^+ \rightarrow Z_{cs}^+ \phi] \times \mathcal{B}[Z_{cs}^+ \rightarrow J/\psi K^+] \\ = (0.95 \sim 18.34) \times 10^{-6}, \end{aligned} \quad (20)$$

which is comparable to the experimental measurement from LHCb Collaboration shown in Eq. (3).

As shown in Table I, most considered decay processes $B^+ \rightarrow J/\psi K \mathbb{M}$, $\mathbb{M} = (\rho^0, \omega, \eta^{(\prime)}, \pi^0)$ have been measured experimentally, thus, we can further consider the fit fractions of the Z_{cs}^+ in the $B^+ \rightarrow J/\psi K^+ \mathbb{M}$ decays, which is the more important criterion of searching for the Z_{cs}^+ state in these processes. The fit fraction reads

$$FF[B^+ \rightarrow Z_{cs}^+ \mathbb{M} \rightarrow J/\psi K^+ \mathbb{M}] = \frac{\mathcal{B}[B^+ \rightarrow Z_{cs}^+ \mathbb{M} \rightarrow J/\psi K^+ \mathbb{M}]}{\mathcal{B}[B^+ \rightarrow J/\psi K^+ \mathbb{M}]}. \quad (21)$$

 TABLE V. The estimated branching fractions for all processes (unit of 10^{-4}).

Process	Branching fraction ($\times 10^{-4}$)
$B^+ \rightarrow Z_{cs}^+ \phi$	[0.73 ~ 2.21]
$B^+ \rightarrow Z_{cs}^+ \omega$	[0.37 ~ 1.39]
$B^+ \rightarrow Z_{cs}^+ \rho^0$	[0.38 ~ 1.42]
$B^+ \rightarrow Z_{cs}^+ \eta$	[0.08 ~ 0.43]
$B^+ \rightarrow Z_{cs}^+ \eta'$	[0.14 ~ 0.80]
$B^+ \rightarrow Z_{cs}^+ \pi^0$	[0.09 ~ 0.43]

Then, the ratio of the fit fractions of Z_{cs}^+ in the processes $B^+ \rightarrow Z_{cs}^+ \mathbb{M} \rightarrow J/\psi K^+ \mathbb{M}$ and $B^+ \rightarrow Z_{cs}^+ \phi \rightarrow J/\psi K^+ \phi$ could be simplified as

$$\frac{FF[B^+ \rightarrow Z_{cs}^+ \mathbb{M} \rightarrow J/\psi K^+ \mathbb{M}]}{FF[B^+ \rightarrow Z_{cs}^+ \phi \rightarrow J/\psi K^+ \phi]} = \frac{\mathcal{B}[B^+ \rightarrow J/\psi K^+ \phi]}{\mathcal{B}[B^+ \rightarrow J/\psi K^+ \mathbb{M}]} \mathcal{R}_{Z_{cs}^+ \mathbb{M}}, \quad (22)$$

where the ratio $\mathcal{R}_{Z_{cs}^+ \mathbb{M}}$ reads

$$\mathcal{R}_{Z_{cs}^+ \mathbb{M}} = \frac{\mathcal{B}[B^+ \rightarrow Z_{cs}^+ \mathbb{M}]}{\mathcal{B}[B^+ \rightarrow Z_{cs}^+ \phi]}. \quad (23)$$

In Fig. 4, the α dependence of the $\mathcal{R}_{Z_{cs}^+ \mathbb{M}}$ ratios is presented, and one can find that these ratios are very weakly dependent on the model parameter α even in a very large parameter range. The concrete values of these ratios are collected in Table VI. From the table, one can find that the uncertainties of these ratios are very small, and these

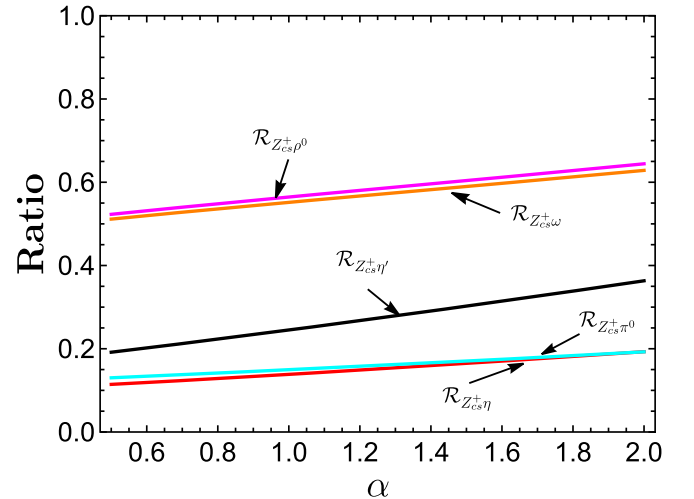

 FIG. 4. The branching fraction ratios depending on the model parameter α .

TABLE VI. The estimated ratios and fit fractions for the relevant processes, in units of %. The fit fractions of Z_{cs}^+ in the $B^+ \rightarrow J/\psi K^+ \phi$ are the measured data from LHCb Collaboration [55].

Process	Ratio	FF
$B^+ \rightarrow J/\psi K^+ \phi$	...	$(9.4 \pm 2.1 \pm 3.4)$
$B^+ \rightarrow J/\psi K^+ \omega$	0.51–0.63	0.58–1.11
$B^+ \rightarrow J/\psi K^+ \rho^0$	0.52–0.64	...
$B^+ \rightarrow J/\psi K^+ \eta$	0.11–0.19	0.34–0.88
$B^+ \rightarrow J/\psi K^+ \eta'$	0.19–0.36	2.35–6.77
$B^+ \rightarrow J/\psi K^+ \pi^0$	0.13–0.19	0.05–0.09

almost model-independent predictions can be tested by further experiment measurements by LHCb and Belle II Collaborations and serve as an important test to the present estimations.

It is worth mentioning that the coupling constants and the parameters in the weak transition form factors also lead to the uncertainty of the branching fraction of $B^+ \rightarrow Z_{cs}^+ \mathbb{M}$. However, these variables appear in both numerators and denominators of the branching fraction ratios $R_{Z_{cs}^+ \mathbb{M}}$, thus, their associated uncertainties are also expected to largely cancel.

With the ratio $R_{Z_{cs}^+ \mathbb{M}}$ estimated in the present work and the measured fit fraction of Z_{cs}^+ in the process $B^+ \rightarrow J/\psi K^+ \phi$, we can estimate the fit fractions of Z_{cs}^+ in various processes, except for $B^+ \rightarrow J/\psi K^+ \rho^0$ by Eq. (22), which are also collected in Table VI. From the table, one can find that the fit fraction of $B^+ \rightarrow Z_{cs}^+ \phi \rightarrow J/\psi K^+ \phi$ is the largest one, which further explains the results from LHCb Collaboration on the observation of Z_{cs} in the $B^+ \rightarrow J/\psi K^+ \phi$ process. Moreover, the fit fraction of Z_{cs} in the $B^+ \rightarrow J/\psi K^+ \eta'$ process is estimated to be $(2.35 \sim 6.77)\%$, which is comparable to the reported fit fraction of Z_{cs} in $B^+ \rightarrow J/\psi K^+ \phi$ by LHCb Collaboration, indicating that it is accessible for future experimental measurements.

IV. SUMMARY

In 2021, BESIII and LHCb Collaborations reported their new observations in the hidden charm and open strange

meson analysis. Two states, named $Z_{cs}(3985)$ and $Z_{cs}(4000)$, were seen as the same state since their masses were very close. The mass of the observed states was close to the threshold of $D_s^* D$, leading to a large number of works investigating them as a molecular state composed of $D_s \bar{D}^* + D_s^* \bar{D}$. The large branching ratio of the cascade process $B^+ \rightarrow Z_{cs}^+ \phi \rightarrow J/\psi K^+ \phi$, of the order of 10^{-6} , motivated us to investigate its properties in the B^+ decay process. In the present work, we estimated the production of Z_{cs}^+ in B^+ decay via the triangle loop mechanism. The branching fraction of $B^+ \rightarrow Z_{cs}^+ \phi \rightarrow J/\psi K^+ \phi$ was estimated to be $(0.95 \sim 18.34) \times 10^{-6}$, which is comparable with the experimental measurement by LHCb Collaboration.

Besides $B^+ \rightarrow Z_{cs}^+ \phi$, we also estimated some other final states, such as $Z_{cs}^+ \omega / \rho^0 / \eta^{(\prime)} / \pi^0$, in the same mechanism, which are of the order of $10^{-5} - 10^{-4}$. Furthermore, with the fit fraction of Z_{cs}^+ in $B^+ \rightarrow Z_{cs}^+ \phi \rightarrow J/\psi K^+ \phi$ reported by LHCb Collaboration, the fit fractions of other processes were also predicted. Our results indicated that the branching fraction of $B^+ \rightarrow Z_{cs}^+ \phi$ and the fit fraction of Z_{cs} in $B^+ \rightarrow J/\psi K^+ \phi$ were the largest ones in the considered processes, which provides a natural explanation for why LHCb Collaboration first observed Z_{cs}^+ in $B^+ \rightarrow J/\psi K^+ \phi$. The fit fraction of Z_{cs}^+ in $B^+ \rightarrow Z_{cs}^+ \eta' \rightarrow J/\psi K^+ \eta'$ was estimated to be $(2.35 \sim 6.77)\%$, which is comparable with the reported fit fraction of Z_{cs} in $B^+ \rightarrow J/\psi K^+ \phi$ by LHCb Collaboration. Thus, we suggest searching for Z_{cs}^+ in $B^+ \rightarrow J/\psi K^+ \eta'$ in future experiments.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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