

# Revealing the nature of double-strangeness pentaquark states via femtoscopic correlation functions

Zheng-Ting Lai<sup>1</sup>, Jun-Xu Lu<sup>1,\*</sup>, Zhi-Wei Liu<sup>2,3,†</sup>, A. Martínez Torres<sup>4,‡</sup>,  
K. P. Khemchandani<sup>4,§</sup> and Li-Sheng Geng<sup>1,5,6,||</sup>

<sup>1</sup>*School of Physics, Beihang University, Beijing 102206, People's Republic of China*

<sup>2</sup>*Institute for Advanced Study in Nuclear Energy and Safety, College of Physics and Optoelectronic Engineering, Shenzhen University, Shenzhen 518060, People's Republic of China*

<sup>3</sup>*Shenzhen Key Laboratory of Nuclear and Radiation Safety, Shenzhen 518060, People's Republic of China*

<sup>4</sup>*Universidade Federal de São Paulo, C. P. 01302-907, São Paulo, Brazil*

<sup>5</sup>*Peng Huanwu Collaborative Center for Research and Education, Beihang University, Beijing 100191, People's Republic of China*

<sup>6</sup>*Southern Center for Nuclear-Science Theory (SCNT), Institute of Modern Physics, Chinese Academy of Sciences, Huizhou 516000, People's Republic of China*



(Received 21 July 2026; accepted 24 September 2026; published 13 October 2026)

Recent discoveries of exotic hadrons, which cannot be classified within the conventional quark model of  $q\bar{q}$  mesons and  $qqq$  baryons, strongly imply the existence of dynamically generated hadronic molecules. Some of these hadron-hadron interactions are accompanied by coupled-channel effects, which remain challenging to quantitatively determine. In this work, we demonstrate that femtoscopy provides a sensitive probe of such coupled-channel dynamics. We calculate correlation functions for the double-strangeness pentaquark candidates  $P_{css}(4493)$  ( $J^P = 1/2^-$ ) and  $P_{css}(4633)$  ( $J^P = 1/2^-$  or  $3/2^-$ ), revealing clear signatures of the attractive interactions that can form bound states. The results are markedly different from those obtained in scenarios that neglect off-diagonal transitions, highlighting the importance of coupled-channel effects for understanding the structure of these hadrons.

DOI: [10.1103/PhysRevD.114.074031](https://doi.org/10.1103/PhysRevD.114.074031)

## I. INTRODUCTION

Quantum chromodynamics, the fundamental theory governing the strong interaction, dictates that quarks are confined within color-singlet states, i.e., hadrons, in the low-energy region. Consequently, understanding hadron-hadron interactions is a core goal of modern theoretical particle and nuclear physics, as it can deepen our understanding of the nonperturbative behavior of QCD [1,2] and is essential for studies of beyond-the-Standard-Model physics [3–6].

Since 2003, numerous exotic hadrons with structures beyond the conventional  $q\bar{q}$  mesons and  $qqq$  baryons have been observed [7–21]. Various interpretations have been

proposed, including compact multiquark states, hadronic molecules, glueballs, and quark-gluon hybrid states. Some of these observed exotic structures may alternatively arise from kinematic effects, such as triangle singularities [22–25]. Among these interpretations, hadronic molecules are the most favored because most of these hadrons lie near the two-hadron thresholds of conventional hadron pairs [13,26]. This molecular picture is analogous to the deuteron, a bound state of a proton and a neutron [27]. It is worth mentioning that research on exotic three-hadron molecular states has also advanced significantly [28–30].

The interpretation of hadronic molecules relies heavily on our understanding of hadron-hadron interactions. However, accurate characterization of these hadron-hadron interactions is severely hampered by the scarcity of experimental data, with only a few exceptions, such as nucleon-nucleon and  $\pi N$  interactions. The primary reason is that these hadrons, typically hyperons or heavy-flavored hadrons, have extremely short lifetimes. It is impossible to produce them in sufficient amounts to form fixed targets or particle beams, thereby making conventional scattering experiments infeasible. Therefore, hadron-hadron interactions associated with hadronic-molecule candidates are mainly constrained by invariant-mass distributions. Recently, three approaches have been reviewed in Ref. [26] to help reveal the nature of

\*Contact author: [ljxwohool@buaa.edu.cn](mailto:ljxwohool@buaa.edu.cn)

†Contact author: [liuzhw@szu.edu.cn](mailto:liuzhw@szu.edu.cn)

‡Contact author: [amartine@if.usp.br](mailto:amartine@if.usp.br)

§Contact author: [kanchan.khemchandani@unifesp.br](mailto:kanchan.khemchandani@unifesp.br)

||Contact author: [lisheng.geng@buaa.edu.cn](mailto:lisheng.geng@buaa.edu.cn)

hadronic molecules, including studying multiplets of hadronic molecules based on heavy-quark spin/flavor symmetry or SU(3) symmetry, studying three-body hadronic molecules that can help verify the molecular nature of related two-body systems, and employing the femtoscopic technique with which one can extract insights into hadron-hadron interactions directly from  $pp$ ,  $pA$ , and  $AA$  collisions. Notably, the femtoscopic technique can reveal the internal structure of many such exotic hadrons [31–36].

In the present work, we focus on a key issue in hadron-hadron interactions: coupled-channel effects, i.e., off-diagonal transitions. Quantum mechanics allows the mixing of different reaction channels with identical quantum numbers, a manifestation of underlying symmetries. Thus, in addition to elastic channels, scattering processes naturally involve inelastic channels. Coupled-channel effects modify the scattering amplitudes and consequently alter the pole structure of the scattering matrix, ultimately resulting in observable signatures in invariant-mass distributions, including threshold effects and line shapes. The well-known two-pole structure of the  $\Lambda(1405)$  [37–40] is a typical manifestation of coupled-channel effects. In scattering processes featuring identical initial and final hadron pairs, coupled channels merely emerge as intermediate states, whose contributions are frequently treated as minor effects, though this assumption is not always justified.

Inspired by the capability of femtoscopia to extract interaction information and reveal the intrinsic nature of hadronic states, we further demonstrate in this work that the technique is sensitive to coupled-channel dynamics and can uncover the essential properties of related hadronic molecules. We take the double-strangeness pentaquark states  $P_{css}$ , dynamically generated via charmed meson-baryon interactions, as typical examples in which coupled-channel effects dominate.

The existence of double-strangeness hidden-charm pentaquark states has been expected since the discovery of the pentaquark states  $P_c(4380)$ ,  $P_c(4312)$ ,  $P_c(4440)$ ,  $P_c(4457)$ , and  $P_c(4337)$  [41–43], and strange hidden-charm pentaquark states  $P_{cs}(4459)$  [44] and  $P_{cs}(4338)$  [45], though no solid experimental evidence has been found so far. At present, the pentaquark states are widely interpreted as meson-baryon molecular states, though several ambiguities remain, such as the spins of  $P_c(4440)$  and  $P_c(4457)$ . See Refs. [10–14,17,19,26] for recent reviews. Double-strangeness hidden-charm pentaquark states were first proposed in Ref. [46] within the light-meson (no pion) exchange model, but this approach requires an unphysical regularization cutoff. Later in Ref. [47], via the vector meson exchange model, two  $P_{css}$  states of molecular nature located at  $(M_R, \Gamma_R) = (4493.35, 73.67)$  MeV due to pseudoscalar meson-baryon (PB) interactions, denoted as  $P_{css}(4493)$  with  $J^P = 1/2^-$ , and  $(4633.38, 79.58)$  MeV due to vector meson-baryon (VB) interactions, denoted as  $P_{css}(4633)$  with  $J^P = 1/2^-$  or  $3/2^-$ , are predicted.

The two bound states are generated predominantly through coupled-channel dynamics. The dominant attraction arises from the strong off-diagonal interactions between the two heaviest meson-baryon channels, whereas the diagonal interactions are significantly suppressed or even repulsive. A similar model was applied in Ref. [48], using an alternative approach to evaluate the *baryon-baryon-vector* (BBV) vertices leading to the prediction of four  $P_{css}$  states.

In the present work, by taking the  $P_{css}(4493)$  and  $P_{css}(4633)$  states dominated by coupled-channel effects as representative examples, we investigate how coupled-channel dynamics are reflected in femtoscopic momentum correlation functions. To further illustrate the generality of the mechanism, we also consider the  $\Omega(2012)$  state as a more extreme example in which the diagonal interactions vanish at leading order in the  $I = 0$  sector, together with a simplified two-channel model that isolates the role of the off-diagonal transition.

The manuscript is organized as follows: in Sec. II, we briefly revisit the mechanisms underlying the dynamical generation of the  $P_{css}$  states and introduce the Koonin-Pratt (KP) formalism for correlation functions. In Sec. III, we present the numerical results and corresponding discussions. Finally, a summary is given in Sec. IV.

## II. FORMALISM

In this section, we briefly review the mechanisms underlying the dynamical generation of the two  $P_{css}$  states in the unitarized coupled-channel approach based on vector-meson exchange interactions. More details are available in Ref. [47]. The thresholds for the relevant channels are listed in Table I. Notably, these channels are not subject to the Coulomb interaction, which allows the contributions from the strong interaction to be isolated.

### A. Dynamical generation of the $P_{css}(4493)$ and $P_{css}(4633)$

Following Ref. [47], when considering the interactions between the pseudoscalar mesons and baryons, the coupled channels in the particle basis are  $\eta_c \Xi^-$ ,  $D_s^- \Xi_c^0$ ,  $D_s^- \Xi_c'^0$ , and  $D^- \Omega_c^0$ . Note that such a choice of charge states ensures that the interaction is free of Coulomb effects. The  $t$ -channel vector-meson exchange diagrams reduce to contact

TABLE I. Thresholds of the coupled channels considered in the PB and VB systems.

| PB system        |                 | VB system           |                 |
|------------------|-----------------|---------------------|-----------------|
| Channel          | Threshold (MeV) | Channel             | Threshold (MeV) |
| $\eta_c \Xi^-$   | 4298            | $J/\psi \Xi^-$      | 4415            |
| $D_s^- \Xi_c^0$  | 4437            | $D_s^{*-} \Xi_c^0$  | 4581            |
| $D_s^- \Xi_c'^0$ | 4545            | $D_s^{*-} \Xi_c'^0$ | 4689            |
| $D^- \Omega_c^0$ | 4564            | $D^{*-} \Omega_c^0$ | 4706            |

TABLE II.  $C_{ij}$  coefficients of the PB interaction in the  $(I, S) = (1/2, -2)$  sector, where  $\kappa_c \simeq 1/4$  and  $\kappa_{cc} \simeq 1/9$ . We note that there is a typo in Table I of Ref. [47] as pointed out in Ref. [48].

|                  | $\eta_c \Xi^-$ | $D_s^- \Xi_c^0$               | $D_s^- \Xi_c'^0$              | $D^- \Omega_c^0$ |
|------------------|----------------|-------------------------------|-------------------------------|------------------|
| $\eta_c \Xi^-$   | 0              | $\sqrt{\frac{3}{2}} \kappa_c$ | $\frac{1}{\sqrt{2}} \kappa_c$ | $-\kappa_c$      |
| $D_s^- \Xi_c^0$  |                | $-1 + \kappa_{cc}$            | 0                             | 0                |
| $D_s^- \Xi_c'^0$ |                |                               | $-1 + \kappa_{cc}$            | $-\sqrt{2}$      |
| $D^- \Omega_c^0$ |                |                               |                               | $\kappa_{cc}$    |

interactions up to  $\mathcal{O}(p^2)$ , where  $p$  denotes a generic small momentum. Upon projection onto the  $S$ -wave, the potential becomes

$$V_{ij}(\sqrt{s}) = -C_{ij} \frac{1}{4f^2} (2\sqrt{s} - M_i - M_j) N_i N_j \quad (1)$$

where  $N_i$  and  $N_j$  are the normalization factors for baryon spinors,  $N_i = \sqrt{(E_i + M_i)/2M_i}$ , and  $M_i(E_i)$  and  $M_j(E_j)$  are the masses (energies) of the baryons in the incoming and outgoing channels, respectively. The  $C_{ij}$  coefficients of the PB interaction are listed in Table II.

We employ the Bethe-Salpeter equation in coupled channels to account for the nonperturbative effects,

$$T_{ij} = V_{ij} + V_{il} G_l T_{lj}, \quad (2)$$

where  $G_l$  denotes the meson and baryon propagator labeled by the  $l$ th intermediate channel. Under the on-shell approximation, the above integral equation reduces to a simple algebraic one:  $T = (1 - VG)^{-1}V$ , where  $G$  is a diagonal matrix whose elements are meson-baryon loop functions defined as

$$G_l(P) = i \int \frac{d^4 q}{(2\pi)^4} \frac{2M_l}{(P-q)^2 - M_l^2 + i\epsilon} \frac{1}{q^2 - m_l^2 + i\epsilon}, \quad (3)$$

in which  $M_l$  and  $m_l$  correspond to the masses of baryon and meson inside the loop. In Eq. (3),  $P = p + k = (\sqrt{s}, \mathbf{0})$  is the total four-momentum in the c.m. frame, and  $q$  is the internal meson four-momentum.

We closely follow the treatment of Ref. [47] and employ the *cutoff method* with  $\Lambda = 800$  MeV to regularize the loop function  $G_l$ . A double-strangeness pentaquark state with a mass of  $M_R = 4493$  MeV and a width of  $\Gamma_R = 74$  MeV is dynamically generated and strongly couples to  $D^- \Omega_c^0$ . As mentioned in Ref. [47], the attractive interaction of the  $D^- \Omega_c^0$  channel alone is insufficient to generate the state, and the off-diagonal transition amplitude  $D^- \Omega_c^0 \rightarrow D_s^- \Xi_c'^0$  provides the additional attraction required. A similar picture is also found in Ref. [48], where the *vector-vector-baryon* (VVB) vertices are evaluated by directly using the flavor and spin wave functions after appropriate

symmetrization [49,50], instead of employing the SU(4) symmetry adopted in Ref. [47].

On the other hand, in Ref. [51], an off-shell coupled-channel formalism based on the one-boson-exchange model was applied, and a double-strangeness pentaquark at  $(M_R, \Gamma_R) = (4504, 0.4)$  MeV was found. The three dominant coupled channels are  $D_s^- \Xi_c^0$ ,  $D_s^- \Xi_c'^0$ , and  $D^- \Omega_c^0$ . However, the  $P_{css}$  state was shown to persist even without the  $\bar{D}_s \Xi_c'$  contribution [51], indicating a distinct binding mechanism compared with the one in Ref. [47].

The  $P_{css}(4633)$  state in the vector-meson-baryon sector is generated analogously. In the VB sector, four coupled channels in the particle basis— $J/\psi \Xi^-$ ,  $D_s^{*-} \Xi_c^0$ ,  $D_s^{*-} \Xi_c'^0$ , and  $D^{*-} \Omega_c^0$ —are taken into account, in which the  $P_{css}(4633)$  state is dominated by the transition amplitude between the last two heavier channels. Note that the VB interaction has the same structure and coupling strength as the PB interaction, except for the scalar product between the polarization vectors of the incoming and outgoing vector mesons.

## B. Femtoscopic correlation functions

Femtoscopic correlation functions are calculated via the KP formula [52,53],

$$C(\mathbf{p}_1, \mathbf{p}_2) \simeq \int d\mathbf{r} S_{12}(r) |\Psi(\mathbf{r}, \mathbf{k})|^2, \quad (4)$$

where we adopt a Gaussian source function

$$S_{12}(r) = \frac{1}{(2\sqrt{\pi}R)^3} e^{-r^2/(4R^2)}. \quad (5)$$

In Eq. (4), the  $\Psi(\mathbf{r}, \mathbf{k})$  is the relative wave function of the two-body outgoing state, which, taking the on-shell approximation, can be written as [35]

$$\Psi(\mathbf{r}, \mathbf{k}) = e^{i\mathbf{k}\cdot\mathbf{r}} + T(k, k) \cdot \theta(q_{\max} - k) \cdot \tilde{G}(r, \sqrt{s}), \quad (6)$$

where  $T(k, k)$  is the scattering amplitude in Eq. (2) and  $\tilde{G}(r, \sqrt{s})$  is given by

$$\tilde{G}(r, \sqrt{s}) = \int_0^{q_{\max}} \frac{d^3 \mathbf{k}'}{(2\pi)^3} \frac{M_B}{E_B(\mathbf{k}') E_M(\mathbf{k}')} \cdot \frac{j_0(k' r)}{\sqrt{s} - (E_B(\mathbf{k}') + E_M(\mathbf{k}')) + i\epsilon}, \quad (7)$$

where  $E_B(\mathbf{k}') = \sqrt{M_B^2 + \mathbf{k}'^2}$  and  $E_M(\mathbf{k}') = \sqrt{M_M^2 + \mathbf{k}'^2}$  are the energies of the corresponding baryon and meson. The cutoff  $q_{\max}$  is set to 800 MeV in the calculation of  $G_l$ .

Substituting the wave function into the KP formula and considering coupled channels, one obtains

$$\begin{aligned}
C_i(k) = & 1 + \theta(q_{\max} - k) \int_0^\infty dr 4\pi r^2 S_{12}(r) \\
& \times \left[ |j_0(kr) + T_{ii}(\sqrt{s}) \cdot \tilde{G}_i(r, \sqrt{s})|^2 - |j_0(kr)|^2 \right. \\
& \left. + \sum_{i \neq j} \omega_j |T_{ji}(\sqrt{s}) \cdot \tilde{G}_j(r, \sqrt{s})|^2 \right], \quad (8)
\end{aligned}$$

where  $\omega_j$  is the weight for each component of the multichannel wave function, and the sum runs over all possible coupled channels. For simplicity, we assume the weights are all equal to 1 in this study. The subscripts  $i, j$  label the corresponding coupled channels and  $\sqrt{s} = E_B(k) + E_M(k)$ .

### III. RESULTS AND DISCUSSIONS

#### A. Coupled-channel effects in the correlation functions

Coupled-channel effects enter momentum correlation functions through two distinct mechanisms. The first

mechanism originates from the nonperturbative structure of the scattering amplitude, as illustrated in Eq. (8), which includes all intermediate channels. The second mechanism arises from the off-diagonal elements of the scattering matrix,  $T_{ji}\tilde{G}_j$  with  $i \neq j$ , which correspond to transitions between different reaction channels. Physically, this represents the contributions to the  $i$ th channel scattering wave function from the process in which particles in the initial state of channel  $j$  undergo rescattering and convert into particles in channel  $i$ .

To explore the influence of coupled-channel dynamics on hadron-hadron interactions, we calculate the momentum correlation functions for the PB and VB systems associated with the  $P_{css}(4493)$  and  $P_{css}(4633)$  states. By comparing results obtained with and without off-diagonal interactions, we aim to elucidate how coupled-channel effects modify the correlation functions and encode the underlying attractive or repulsive interactions.

In Fig. 1, we present the correlation functions for  $\eta_c \Xi^-$ ,  $D_s^- \Xi_c^0$ ,  $D_s^- \Xi_c'^0$ , and  $D^- \Omega_c^0$  for four typical representative

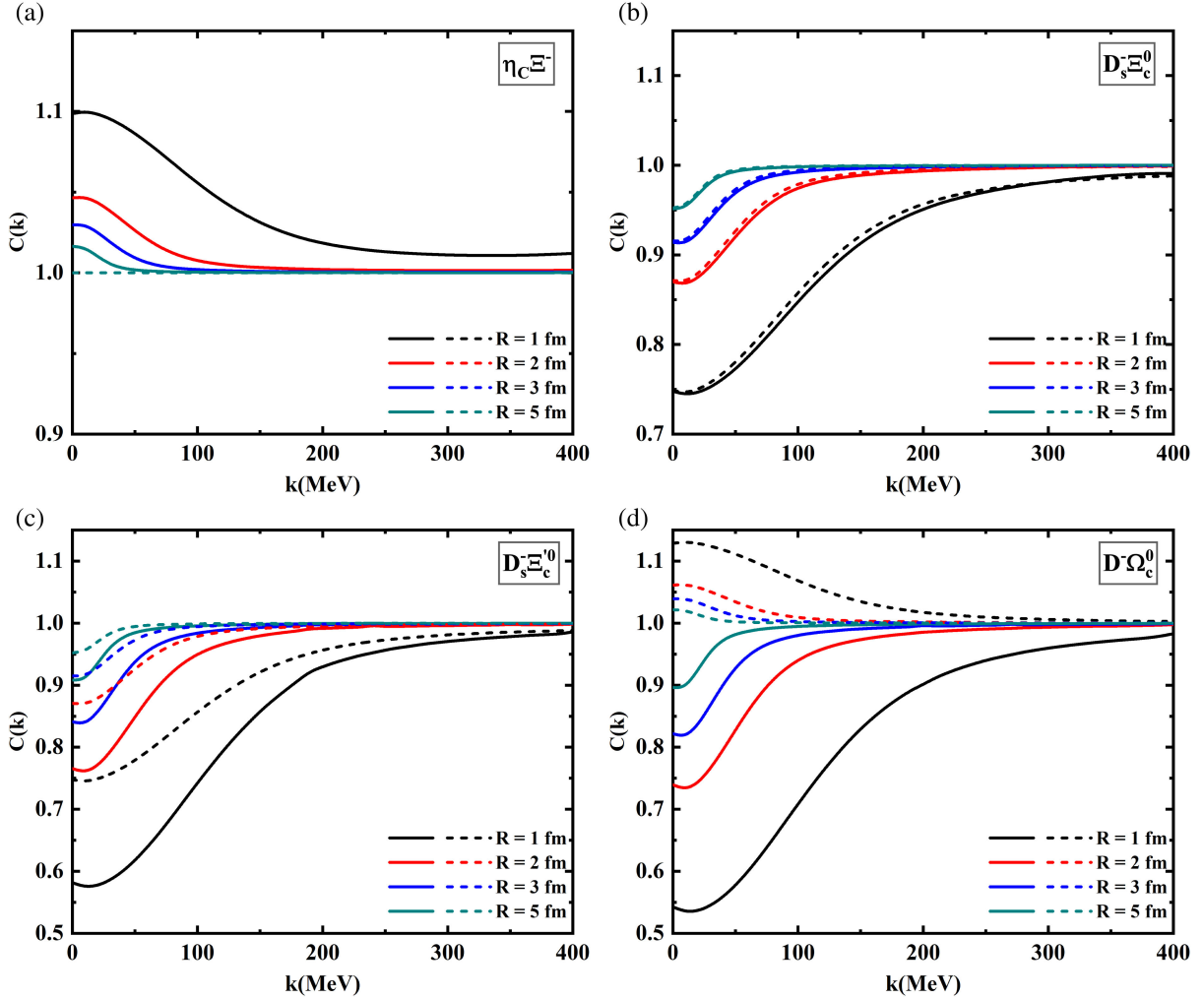


FIG. 1.  $\eta_c \Xi^-$  (a),  $D_s^- \Xi_c^0$  (b),  $D_s^- \Xi_c'^0$  (c), and  $D^- \Omega_c^0$  (d) correlation functions as a function of the relative momentum  $k$  for different source sizes  $R = 1, 2, 3$ , and  $5$  fm. The solid lines represent the results obtained in the coupled-channel case, while the short dashed lines represent the results obtained in the single-channel scattering process.

source sizes  $R = 1, 2, 3$ , and  $5$  fm. For comparison, we also show the results obtained in the uncoupled-channel case, in which all off-diagonal elements in Table II are set to zero, denoted by dashed lines. According to the general analysis in Ref. [54], the correlation functions of  $D_s^- \Xi_c^0$  and  $D^- \Omega_c^0$  channels exhibit clear signatures of strong attractive interactions when taking into account full coupled-channel effects, which is consistent with the existence of the  $P_{css}(4493)$  below the thresholds of both channels.

To further clarify the role of the coupled-channel dynamics, we have also examined the pole structure by switching off all the off-diagonal interactions. In the uncoupled-channel calculation, the poles associated with  $P_{css}(4493)$  and  $P_{css}(4633)$  in the full coupled-channel calculation no longer survive in the relevant energy region. This confirms that the  $P_{css}(4493)$  and  $P_{css}(4633)$  states cannot be generated by the diagonal interactions alone.

This observation can be understood from the diagonal interactions themselves. The  $D^- \Omega_c^0 \rightarrow D^- \Omega_c^0$  interaction is strongly suppressed, since it proceeds only via heavy  $J/\psi$  exchange. The  $D_s^- \Xi_c^0 \rightarrow D_s^- \Xi_c^0$  interaction is even repulsive due to the exchange of the  $\phi$  meson. This correspondence can be seen more explicitly in Ref. [48], where the relevant diagonal coefficients are expressed in terms of the masses of the exchanged vector mesons, as follows:

$$C_{\bar{D}_s \Xi_c^0 \rightarrow \bar{D}_s \Xi_c^0} \propto -\frac{1}{m_\phi^2} + \frac{1}{m_{J/\psi}^2}, \quad C_{\bar{D} \Omega_c^0 \rightarrow \bar{D} \Omega_c^0} \propto \frac{1}{m_{J/\psi}^2}.$$

Thus, without considering coupled channels, one expects the correlation function of  $D^- \Omega_c^0$  to exhibit a totally different line shape, as shown in Fig. 1 with a dashed line. The  $D^- \Omega_c^0$  correlation function in the uncoupled-channel calculation is enhanced at low momenta, exhibiting a weakly attractive behavior. For the  $D_s^- \Xi_c^0$  channel, on the other hand, the correlation function is suppressed below unity at low momenta, especially for sources with small sizes, as shown by the short dashed lines, indicating a repulsive interaction kernel. However, the line shapes of correlation functions for deeply bound states and repulsive interactions are similar [54]. Consequently, the line-shape variations in the correlation function of  $D_s^- \Xi_c^0$  are unlikely to be as pronounced as that of  $D^- \Omega_c^0$ , regardless of whether coupled-channel effects are included. As for the  $\eta_c \Xi^-$  channel, the deviation of the correlation function from unity originates entirely from coupled-channel effects, since the interaction vanishes in the uncoupled case. This contrast indicates that the scattering amplitudes receive substantial contributions from coupled-channel transitions, which dominate their attractive behavior.

It is worth noting that although the  $P_{css}(4493)$  state emerges as a resonance in the  $D_s^- \Xi_c^0$  channel, the correlation function of this channel still exhibits a repulsive pattern [54], which is suppressed in the low-momentum region and rises as momentum increases, rather than the

typical resonance behavior observed in Ref. [32], which is enhanced in the low-momentum region and decreases with increasing momentum. This is mainly because  $D_s^- \Xi_c^0$  cannot directly couple to the two dominant  $D_s^- \Xi_c^0$  and  $D^- \Omega_c^0$  channels, as shown in Table II, but only indirectly via  $\eta_c \Xi^-$ . Such a mechanism naturally leads to a rather weak coupling of  $D_s^- \Xi_c^0$  to  $P_{css}$ , and the coupled-channel effects have only a minor impact on the  $T_{22}$  matrix element. The small differences between the correlation functions calculated with full coupled-channel effects and those without any coupled-channel effects confirm this picture, as shown in Fig. 1. This suggests that the repulsive nature of the  $D_s^- \Xi_c^0$  channel does not substantially weaken the attractive interactions in the  $D_s^- \Xi_c^0$  and  $D^- \Omega_c^0$  channels. To further explore the role of this channel, we increase the coupling strength between  $D_s^- \Xi_c^0$  and  $D_s^- \Xi_c^0$  to approximately 75% of that between the  $D_s^- \Xi_c^0$  and  $D^- \Omega_c^0$  channels. Under this assumption, the correlation function of  $D_s^- \Xi_c^0$  exhibits typical behavior in which a resonance emerges, consistent with the findings of Ref. [32]. This tells us that  $D_s^- \Xi_c^0$  is not an optimal channel to study the  $P_{css}(4493)$  state. A similar pattern is also observed in the VB systems. Finally, we note that the source size  $R$  is selected arbitrarily for illustration. The coupled-channel effects discussed above are valid for any reasonable  $R$  and are not sensitive to the specific choice of the source size  $R$ .

In Fig. 2, we show the correlation functions of the vector-meson-baryon counterparts,  $J/\psi \Xi^-$ ,  $D_s^{*-} \Xi_c^0$ ,  $D_s^{*-} \Xi_c^0$ , and  $D^{*-} \Omega_c^0$  pairs related to the  $P_{css}(4633)$  state. Given that the spin dependence is not resolved in the standard measurements of correlation functions, we have to take the spin average for the VB interactions with  $J^P = 1/2^-$  and  $3/2^-$ . However, in the present calculation, because the potentials and the momentum cutoff are identical, the resulting correlation functions for  $J^P = 1/2^-$  and  $3/2^-$  are identical. The solid curves obtained from the coupled-channel calculation indicate strong attractive interactions in the  $D_s^{*-} \Xi_c^0$  and  $D^{*-} \Omega_c^0$  channels. In contrast, in the  $D_s^{*-} \Xi_c^0$  channel the dashed curve, which reflects the behavior of the diagonal interactions alone, exhibits a suppression below unity, indicating a repulsive interaction, while in the  $D^{*-} \Omega_c^0$  channel, the dashed curve exhibits only a mild enhancement, corresponding to a weakly attractive interaction [54]. These features are very similar to those in the PB system and support the interpretation that the strong enhancement represented by the solid curves does not originate from the diagonal terms, but rather originates predominantly from the coupled-channel dynamics encoded in  $V_1^{\text{eff}}$  of Eq. (13).

In the above discussion, we have compared the calculation with no coupled-channel effects at all (the short dashed lines), where the off-diagonal elements of the interaction  $V$  are set to zero, against the full calculation that includes the coupled-channel effects both from  $T_{ii} \tilde{G}_i$  and the transition terms  $T_{ji} \tilde{G}_j$  with  $j \neq i$  (the solid lines).

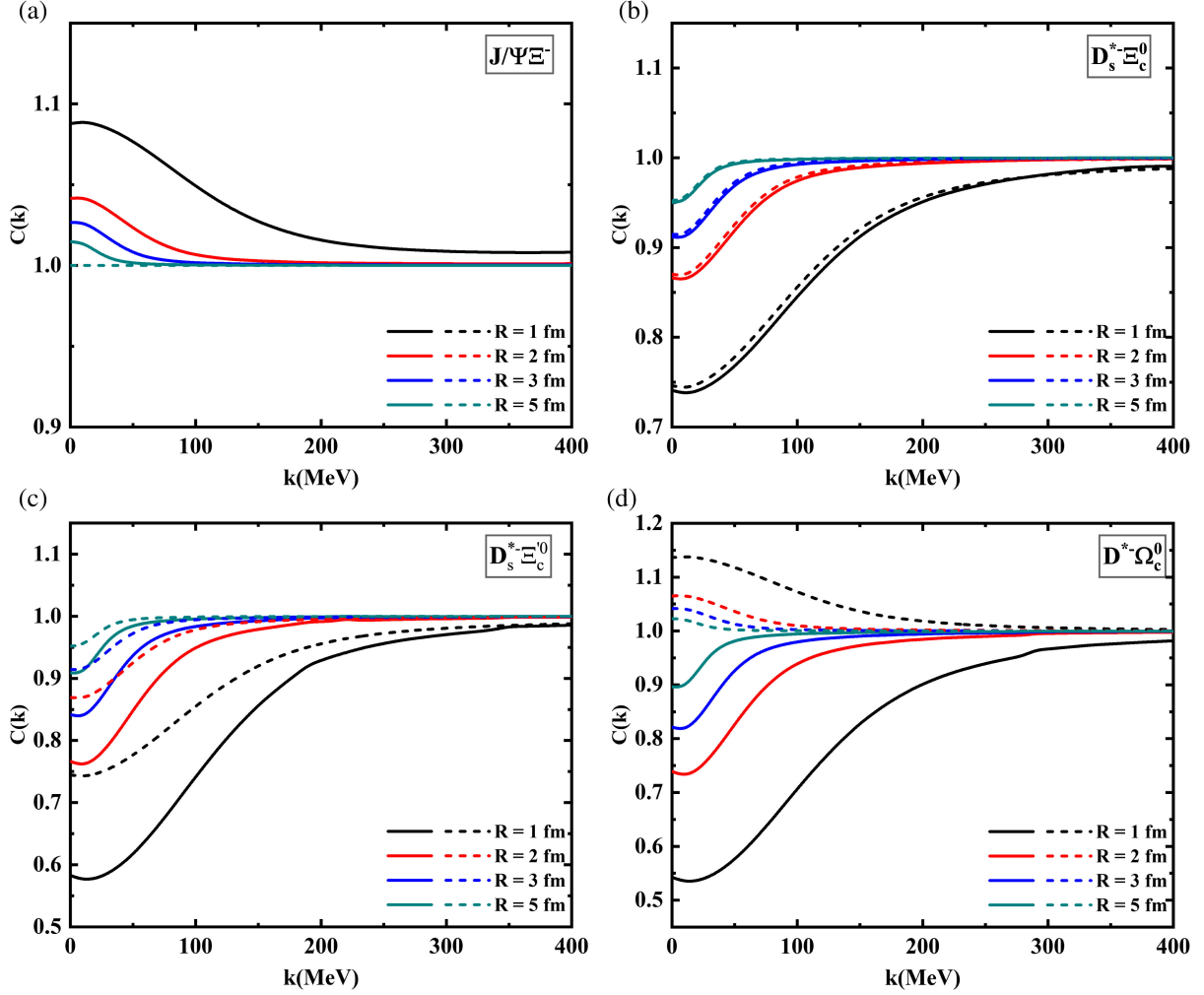


FIG. 2. Same as Fig. 1 but for the  $J/\psi\Xi^-$  (a),  $D_s^{*-}\Xi_c^0$  (b),  $D_s^{*-}\Xi_c^0$  (c), and  $D^{*-}\Omega_c^0$  (d) correlation functions.

In the following, we further distinguish their relative magnitudes and show how these coupled-channel effects are manifested in the correlation function. In the PB and VB interactions considered here, we find that the  $T_{ji}\tilde{G}_j$  terms are significantly smaller than  $T_{ii}\tilde{G}_i$  when computing the correlation function for channel  $i$ . Especially, given a relatively small relative momentum  $k$  of channel  $i$  within our energy range of interest, e.g.,  $k \in [0, 400]$  MeV, the ratio of the loop function of channel  $j$  (which has a higher threshold) to that of channel  $i$ , i.e.,  $\tilde{G}_j(r, \sqrt{s})/\tilde{G}_i(r, \sqrt{s})$  is much less than unity, due to the spherical Bessel function  $j_0(kr)$ . Moreover, the ratio drops sharply as the threshold difference between channels  $j$  and  $i$  increases. Meanwhile, for channels with lower thresholds than channel  $i$ , the off-diagonal element of the  $T$  matrix is smaller than the diagonal element  $T_{ii}$  in the present system. As a consequence, the contribution from the off-diagonal element,  $T_{ji}\tilde{G}_j$ , to the correlation function is negligible compared with the diagonal elements. Therefore, the coupled-channel dynamics affect the correlation function mainly through the modification of  $T_{ii}$ , rather than through the explicit  $T_{ji}\tilde{G}_j$

terms in Eq. (8). We confirm this conclusion in Fig. 3 by calculating the contributions from the diagonal and off-diagonal  $T$ -matrix elements.

In the above study, we have set the weight factors  $\omega_j = 1$ . It is important to examine whether different weights could affect the resulting correlation functions. To quantify the impact of  $\omega_j$ , i.e., the relative importance of explicit coupled-channel terms  $T_{ji}\tilde{G}_j$ , we define the following ratio:

$$R_{ji}(k) = \frac{I[|T_{ji}(\sqrt{s}) \cdot \tilde{G}_j(r, \sqrt{s})|^2]}{I[|j_0(kr) + T_{ii}(\sqrt{s}) \cdot \tilde{G}_i(r, \sqrt{s})|^2 - |j_0(kr)|^2]}, \quad (9)$$

for  $i \neq j$ , where

$$I[f] \equiv \theta(q_{\max} - k) \int_0^\infty dr 4\pi r^2 S_{12}(r) f(r, k). \quad (10)$$

The values of  $R_{ji}$  are summarized in Table III. One observes that all off-diagonal contributions satisfy  $R_{ji} \ll 1$ , indicating that the explicit coupled-channel terms are much

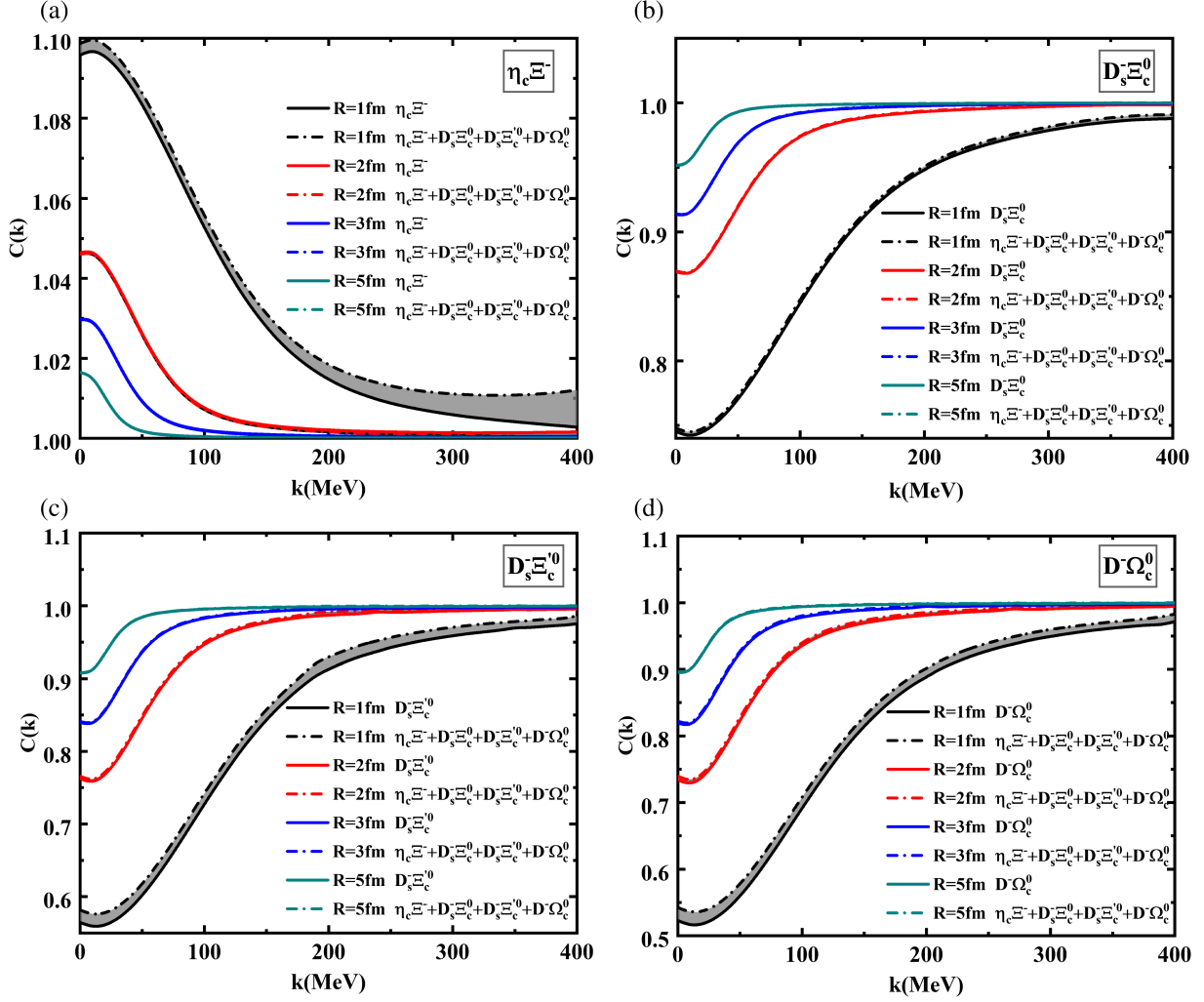


FIG. 3.  $\eta_c \Xi^-$  (a),  $D_s^- \Xi_c^0$  (b),  $D_s^- \Xi_c^0$  (c), and  $D^- \Omega_c^0$  (d) correlation functions as a function of the relative momentum  $k$  for source sizes  $R = 1, 2, 3$ , and  $5$  fm. The short dash-dotted lines represent the full coupled-channel results, while the solid lines are obtained by retaining only the elastic-channel term involving the full coupled-channel amplitude  $T_{ii}$ , namely  $T_{ii} \tilde{G}_i$ , and omitting the explicit coupled-channel terms  $T_{ji} \tilde{G}_j$  with  $j \neq i$ . The gray-shaded area represents the overlap between the full coupled-channel calculation and the calculation including only the diagonal term  $T_{ii} \tilde{G}_i$ , indicating that the explicit  $T_{ji} \tilde{G}_j$  terms provide only a small additional contribution to the correlation function.

TABLE III. Relative contributions of coupled channels to the PB correlation functions for a source size  $R = 1$  fm. The tabulated values are arithmetic averages over uniformly sampled momentum points  $k \in [0, 400]$  MeV. Note that  $R_{ji}(k)$  depends on  $k$  and varies within this range, so the listed numbers should be understood as representative averages rather than fixed constants.

| $i \setminus j$  | $\eta_c \Xi^-$ | $D_s^- \Xi_c^0$ | $D_s^- \Xi_c^0$ | $D^- \Omega_c^0$ |
|------------------|----------------|-----------------|-----------------|------------------|
| $\eta_c \Xi^-$   | ...            | 0.029           | 0.037           | 0.074            |
| $D_s^- \Xi_c^0$  | -0.013         | ...             | -0.006          | -0.011           |
| $D_s^- \Xi_c^0$  | -0.013         | -0.003          | ...             | -0.070           |
| $D^- \Omega_c^0$ | -0.013         | -0.004          | -0.058          | ...              |

smaller than the corresponding diagonal contributions. Consequently, even if weight factors  $\omega_j$  other than 1 are assigned to different channels, their influence on the correlation functions remains negligible unless extremely large values of  $\omega_j$  are adopted [55]. Once experimental data become available, the corresponding production weights can be determined following the methods proposed in Refs. [56–58], thereby enabling a more quantitative understanding of the coupled-channel effects.

In this situation, we are safe to omit the last term in Eq. (8), and the correlation function for channel  $i$  can be simplified as

$$C_i(k) = 1 + \int_0^\infty dr 4\pi r^2 S_{12}(r) [ |j_0(kr) + T_{ii}(\sqrt{s}) \cdot \tilde{G}_i(r, \sqrt{s})|^2 - |j_0(kr)|^2 ]. \quad (11)$$

It is even more interesting that such a conclusion is not only valid for the present PB or VB interactions. This conclusion, though maybe not universal, has also been verified in studies of the correlation function for various systems, including the  $DK - D_s\eta$  system associated with  $D_{s0}^*(2317)$  [54] and the  $\bar{K}\Xi(1530) - \eta\Omega - \bar{K}\Xi$  system associated with  $\Omega(2012)$  [55]. Note that the contributions from coupled channels belonging to isospin multiplets cannot be neglected, since their thresholds are compatible.

To examine whether the above observation is specific to the  $P_{css}$  systems, it is useful to consider another interesting case in which the role of the off-diagonal interaction is particularly transparent. The  $\Omega(2012)$  state, in the molecular picture involving  $\bar{K}\Xi(1530)$  and  $\eta\Omega$  within the chiral perturbation theory, provides such an example of a resonance dynamically generated by coupled-channel effects [58–74]. We note that, in the charge basis, the  $\bar{K}^0\Xi^-(1530)$  or  $K^-\Xi^0(1530)$  channel contains both  $I = 0$  and  $I = 1$  components, and their diagonal interactions do not vanish. In contrast, the  $\eta\Omega$  channel contains only an  $I = 0$  component, and its leading-order diagonal interaction  $\eta\Omega^- \rightarrow \eta\Omega^-$  vanishes. For this reason, the  $\eta\Omega^-$  correlation function provides a particularly clean reference for illustrating the coupled-channel mechanism. More importantly, unlike the  $\eta_c\Xi^-$  channel in the  $P_{css}$  system, which is not among the dominant channels responsible for the generation of the  $P_{css}$  state, the  $\eta\Omega^-$  channel participates directly in the coupled-channel mechanism responsible for the dynamical generation of the  $\Omega(2012)$  state. The correlation functions of the  $\eta\Omega^-$ ,  $\Xi^{*0}K^-$  and  $\Xi^{*-}\bar{K}^0$  channels relevant to the  $\Omega(2012)$  have been studied in Ref. [55]. The  $\eta\Omega^-$  correlation function is exactly unity in the uncoupled-channel calculation, whereas the correlation functions that account for off-diagonal effects again exhibit the typical features of a strongly attractive interaction. This feature clearly indicates that the deviation of the  $\eta\Omega^-$  correlation function from unity arises solely from off-diagonal coupled-channel interactions, similar to the correlation functions of the  $\eta_c\Xi^-$  channel in Fig. 1 and the  $J/\psi\Xi^-$  channel in Fig. 2.

## B. Two-channel illustration of coupled-channel dynamics

One notices that the correlation functions for the  $D_s^-\Xi_c^0$  and  $D^-\Omega_c^0$  channels in the PB sector, as well as  $D_s^{*-}\Xi_c^0$  and  $D^{*-}\Omega_c^0$  channels in the VB sector are comparable in magnitude when coupled-channel effects are included in the calculations.

To gain further insight into how coupled-channel dynamics manifests itself in momentum correlation functions, it is

useful to consider the simplest two-channel system as an illustration. The simplified two-channel model represents an extreme case designed to isolate the mechanism by which off-diagonal transitions generate an effective interaction in the elastic channel. We assume that the diagonal interactions vanish, while only the off-diagonal transition potential survives, i.e.,

$$V = \begin{pmatrix} 0 & V' \\ V' & 0 \end{pmatrix}, \quad G = \begin{pmatrix} G_1 & 0 \\ 0 & G_2 \end{pmatrix}, \quad (12)$$

with  $V'$  the off-diagonal elements of the potential.

From the perspective of channel 1, the effect of channel 2 can be absorbed into an effective interaction. Physically, the particles first make a transition from channel 1 to channel 2 through the off-diagonal potential  $V'$ , propagate in channel 2, and then return to channel 1 through another transition. This process generates the effective interaction

$$V_1^{\text{eff}} = V' G_2 V', \quad (13)$$

where  $G_2$  is the loop function of channel 2. Correspondingly, the scattering amplitude reads

$$T_{11} = \frac{V_1^{\text{eff}}}{1 - V_1^{\text{eff}} G_1}. \quad (14)$$

Remarkably, a bound state or resonance may emerge in the absence of any direct interaction, given the condition

$$1 - G_1 V_1^{\text{eff}} = 0,$$

which is entirely governed by the off-diagonal transitions.

Now the correlation function for channel 1 can be further simplified as

$$C_1(k) = 1 + \frac{e^{-4k^2 R^2} - 1}{4k^2 R^2} + \int_0^\infty dr 4\pi r^2 S_{12}(r) \left| j_0(kr) - \mathcal{F}(r, k) + \mathcal{F}(r, k) \frac{1}{1 - G_1(k) V_1^{\text{eff}}(k)} \right|^2, \quad (15)$$

where

$$\mathcal{F}(r, k) = \tilde{G}_1(r, k)/G_1(k). \quad (16)$$

The correlation function of channel 2 can be obtained similarly. Notably, in this simplest two-channel situation, the only difference between the correlation functions of channels 1 and 2 lies in the particle masses in  $\mathcal{F}(r, k)$  of Eq. (15). Particularly, under the nonrelativistic expansion of the loop functions, which is equivalent to solving the Lippmann-Schwinger equation instead of the Bethe-Salpeter equation, the ratio  $\mathcal{F}(r, k)$  is independent of particle

masses. In the present situation involving heavy mesons and baryons, such a difference is relatively small, and it is anticipated that the correlation functions of these two channels will be very similar not only in line shapes but also in magnitude.

Although the  $P_{css}$  systems contain nonvanishing diagonal interactions, the qualitative similarity of the two correlation functions is consistent with the mechanism illustrated by Eq. (15), namely that the common off-diagonal transition largely governs their dynamics. Such a similarity further implies that the off-diagonal elements dominate the dynamics, while the diagonal elements play only a minor role, given the markedly different behavior in the uncoupled-channel calculations. Thus, if future experimental measurements confirm that both channels exhibit strong attraction, this would support the physical picture in which the  $P_{css}(4493)$  and  $P_{css}(4633)$  states are molecular states dynamically generated by the attractive interaction arising from coupled-channel effects. Specifically, the magnitude of the difference between the correlation functions of these two channels directly reveals the strength of the coupled-channel dynamics. This is challenging for  $D_s^{(*)-}\Xi_c^0$ , because the correlation functions corresponding to a bound state and to repulsive interactions behave similarly.

#### IV. SUMMARY

In this work, we explored the capability of femtoscopy to probe coupled-channel effects in hadron-hadron interactions and unravel the intrinsic nature of relevant hadronic molecules. To this end, we revisited interactions of heavy pseudoscalar mesons and baryons, including  $\eta_c\Xi^-$ ,  $D_s^-\Xi_c^0$ ,  $D_s^-\Xi_c^0$ , and  $D^-\Omega_c^0$ , associated with the double strangeness pentaquark state  $P_{css}(4493)$ , which is governed by vector meson exchanges. Using the Koonin-Pratt formula with a Gaussian source and a unitarized coupled-channel approach, we calculated the correlation functions for all coupled channels. We further extended our study to heavy vector meson-baryon interactions and the associated  $P_{css}(4633)$  state. To reveal the coupled-channel effects, we also computed the correlation functions in the uncoupled-channel case, in which all off-diagonal elements are set to zero.

In the heavy pseudoscalar meson-baryon interactions, our calculations revealed that the correlation functions of the  $D_s^-\Xi_c^0$  and  $D^-\Omega_c^0$  channels display typical features of strong attractive interactions, even though their diagonal potentials are either suppressed or even repulsive. This indicates that the nature of the state is driven predominantly by the coupled-channel mechanism. Meanwhile, the correlation function of  $\eta_c\Xi^-$  manifests weak attractive behavior,

consistent with the nature of an above-threshold resonance [32]. Notably, the elastic interaction of  $\eta_c\Xi^-$  vanishes entirely, so such behavior arises purely from coupled-channel effects. In contrast, since the  $D_s^-\Xi_c^0$  channel is very weakly coupled to the two dominant  $D_s^-\Xi_c^0$  and  $D^-\Omega_c^0$  channels, its correlation function is almost unaffected whether coupled-channel effects are incorporated or not. A similar scenario is also found in the heavy-flavor vector meson-baryon interactions relevant to the  $P_{css}(4633)$  state.

We notice that a similar physical picture exists in the interactions between decuplet baryons and pseudoscalar mesons, i.e.,  $\bar{K}\Xi(1530)$  and  $\eta\Omega^-$ , related to the  $\Omega(2012)$ , in the  $I = 0$  sector, where only off-diagonal elements survive at leading order. The  $\eta\Omega^-$  correlation function exhibits the characteristic behavior of a strong attraction, even though the leading-order diagonal interaction vanishes.

These results demonstrate that future femtosopic measurements of these correlation functions will serve as a sensitive probe of coupled-channel dynamics, providing direct insight into the mechanisms underlying the generation of these states.

#### ACKNOWLEDGMENTS

L. S. G. thanks Angel Ramos for useful communications regarding the details of Ref. [47]. This work is partly supported by the National Key R&D Program of China under Grant No. 2023YFA16067003 and the National Natural Science Foundation of China under Grants No. W2543006 and No. 12435007. Z.-W. L. acknowledges support from the National Natural Science Foundation of China under Grants No. 12522505 and No. 12405133, and the Shenzhen Science and Technology Program under Grant No. ZDSYS20230626091501002. K. P. K. and A. M. T. acknowledge support from the Brazilian funding agency CNPq (through Grants No. 407437/2023-1, No. 306461/2023-4, and No. 304510/2023-8). Z.-T. L. acknowledges support from Beijing Natural Science Foundation under Grant No. QY26113. J.-X. L. acknowledges support from the National Natural Science Foundation of China under Grants No. 12522505 and the Fundamental Research Funds for the Central Universities.

#### DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

- [1] ALICE Collaboration, *Nature (London)* **588**, 232 (2020); **590**, 13(E) (2021).
- [2] L. Fabbietti, V. Mantovani Sarti, and O. Vazquez Doce, *Annu. Rev. Nucl. Part. Sci.* **71**, 377 (2021).
- [3] N. Brambilla *et al.*, *Eur. Phys. J. C* **74**, 2981 (2014).
- [4] A. Belley *et al.*, *Phys. Rev. Lett.* **132**, 182502 (2024).
- [5] R. Aliberti *et al.*, *Phys. Rep.* **1143**, 1 (2025).
- [6] A. Boccaletti *et al.*, *Nature (London)* **653**, 373 (2026).
- [7] E. Oset *et al.*, *Int. J. Mod. Phys. E* **25**, 1630001 (2016).
- [8] J.-M. Richard, *Few Body Syst.* **57**, 1185 (2016).
- [9] A. Hosaka, T. Hyodo, K. Sudoh, Y. Yamaguchi, and S. Yasui, *Prog. Part. Nucl. Phys.* **96**, 88 (2017).
- [10] H.-X. Chen, W. Chen, X. Liu, and S.-L. Zhu, *Phys. Rep.* **639**, 1 (2016).
- [11] A. Esposito, A. Pilloni, and A. D. Polosa, *Phys. Rep.* **668**, 1 (2017).
- [12] R. F. Lebed, R. E. Mitchell, and E. S. Swanson, *Prog. Part. Nucl. Phys.* **93**, 143 (2017).
- [13] F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, and B.-S. Zou, *Rev. Mod. Phys.* **90**, 015004 (2018); **94**, 029901(E) (2022).
- [14] A. Ali, J. S. Lange, and S. Stone, *Prog. Part. Nucl. Phys.* **97**, 123 (2017).
- [15] S. L. Olsen, T. Skwarnicki, and D. Zieminska, *Rev. Mod. Phys.* **90**, 015003 (2018).
- [16] M. Karliner, J. L. Rosner, and T. Skwarnicki, *Annu. Rev. Nucl. Part. Sci.* **68**, 17 (2018).
- [17] Y.-R. Liu, H.-X. Chen, W. Chen, X. Liu, and S.-L. Zhu, *Prog. Part. Nucl. Phys.* **107**, 237 (2019).
- [18] E. van Beveren and G. Rupp, *Prog. Part. Nucl. Phys.* **117**, 103845 (2021).
- [19] H.-X. Chen, W. Chen, X. Liu, Y.-R. Liu, and S.-L. Zhu, *Rep. Prog. Phys.* **86**, 026201 (2023).
- [20] M. Mai, U.-G. Meißner, and C. Urbach, *Phys. Rep.* **1001**, 1 (2023).
- [21] Z.-Y. Bai, D.-Y. Chen, Qi-Huang, X. Liu, S.-Q. Luo, and J.-Z. Wang, *Phys. Rept.* **1204**, 1 (2027).
- [22] X.-H. Liu, Q. Wang, and Q. Zhao, *Phys. Lett. B* **757**, 231 (2016).
- [23] J.-J. Xie, L.-S. Geng, and E. Oset, *Phys. Rev. D* **95**, 034004 (2017).
- [24] F.-K. Guo, X.-H. Liu, and S. Sakai, *Prog. Part. Nucl. Phys.* **112**, 103757 (2020).
- [25] G. D. Alexeev *et al.* (COMPASS Collaboration), *Phys. Rev. Lett.* **127**, 082501 (2021).
- [26] M.-Z. Liu, Y.-W. Pan, Z.-W. Liu, T.-W. Wu, J.-X. Lu, and L.-S. Geng, *Phys. Rep.* **1108**, 1 (2025).
- [27] S. Weinberg, *Phys. Rev.* **137**, B672 (1965).
- [28] X.-L. Ren, B. B. Malabarba, L.-S. Geng, K. P. Khemchandani, and A. Martínez Torres, *Phys. Lett. B* **785**, 112 (2018).
- [29] A. Martínez Torres, K. P. Khemchandani, and L.-S. Geng, *Phys. Rev. D* **99**, 076017 (2019).
- [30] T. W. Wu, Y. W. Pan, M. Z. Liu, and L. S. Geng, *Sci. Bull.* **67**, 1735 (2022).
- [31] Z.-W. Liu, J.-X. Lu, M.-Z. Liu, and L.-S. Geng, *Phys. Rev. D* **108**, L031503 (2023).
- [32] Z.-W. Liu, J.-X. Lu, M.-Z. Liu, and L.-S. Geng, *Sci. Bull.* **70**, 3515 (2025).
- [33] Z.-W. Liu, J.-M. Xie, J.-X. Lu, and L.-S. Geng, *arXiv:2512.10459*.
- [34] H.-N. Liu, Z.-W. Liu, L. Abreu, and L.-S. Geng, *arXiv:2511.19098*.
- [35] I. Vidana, A. Feijoo, M. Albaladejo, J. Nieves, and E. Oset, *Phys. Lett. B* **846**, 138201 (2023).
- [36] L. M. Abreu, P. Gubler, K. P. Khemchandani, A. Martínez Torres, and A. Hosaka, *Phys. Lett. B* **860**, 139175 (2025).
- [37] J. A. Oller and U. G. Meissner, *Phys. Lett. B* **500**, 263 (2001).
- [38] D. Jido, J. A. Oller, E. Oset, A. Ramos, and U. G. Meissner, *Nucl. Phys. A* **725**, 181 (2003).
- [39] U.-G. Meißner, *Symmetry* **12**, 981 (2020).
- [40] J.-M. Xie, J.-X. Lu, L.-S. Geng, and B.-S. Zou, *Phys. Rev. D* **108**, L111502 (2023).
- [41] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **115**, 072001 (2015).
- [42] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **122**, 222001 (2019).
- [43] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **128**, 062001 (2022).
- [44] R. Aaij *et al.* (LHCb Collaboration), *Sci. Bull.* **66**, 1278 (2021).
- [45] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **131**, 031901 (2023).
- [46] F.-L. Wang, R. Chen, and X. Liu, *Phys. Rev. D* **103**, 034014 (2021).
- [47] J. A. Marsé-Valera, V. K. Magas, and A. Ramos, *Phys. Rev. Lett.* **130**, 091903 (2023).
- [48] L. Roca, J. Song, and E. Oset, *Phys. Rev. D* **109**, 094005 (2024).
- [49] V. R. Debastiani, J. M. Dias, W. H. Liang, and E. Oset, *Phys. Rev. D* **97**, 094035 (2018).
- [50] W.-F. Wang, A. Feijoo, J. Song, and E. Oset, *Phys. Rev. D* **106**, 116004 (2022).
- [51] S. Clymton, H.-C. Kim, and T. Mart, *Phys. Rev. D* **112**, 034015 (2025).
- [52] S. E. Koonin, *Phys. Lett.* **70B**, 43 (1977).
- [53] S. Pratt, T. Csorgo, and J. Zimanyi, *Phys. Rev. C* **42**, 2646 (1990).
- [54] Z.-W. Liu, J.-X. Lu, and L.-S. Geng, *Phys. Rev. D* **107**, 074019 (2023).
- [55] J.-X. Lin, P. Encarnación, A. Feijoo, and M. Albaladejo, *arXiv:2603.18610*.
- [56] V. Vovchenko and H. Stoecker, *Comput. Phys. Commun.* **244**, 295 (2019).
- [57] P. Encarnación, A. Feijoo, V. M. Sarti, and A. Ramos, *Phys. Rev. D* **111**, 114013 (2025); **113**, 099901(E) (2026).
- [58] Y.-H. Lin and B.-S. Zou, *Phys. Rev. D* **98**, 056013 (2018).
- [59] M. P. Valderrama, *Phys. Rev. D* **98**, 054009 (2018).
- [60] Y. Huang, M.-Z. Liu, J.-X. Lu, J.-J. Xie, and L.-S. Geng, *Phys. Rev. D* **98**, 076012 (2018).
- [61] R. Pavao and E. Oset, *Eur. Phys. J. C* **78**, 857 (2018).
- [62] M. V. Polyakov, H.-D. Son, B.-D. Sun, and A. Tandogan, *Phys. Lett. B* **792**, 315 (2019).
- [63] Y. Lin, Y.-H. Lin, F. Wang, B. Zou, and B.-S. Zou, *Phys. Rev. D* **102**, 074025 (2020).
- [64] N. Ikeno, G. Toledo, and E. Oset, *Phys. Rev. D* **101**, 094016 (2020).
- [65] X. Hu and J. Ping, *Phys. Rev. D* **106**, 054028 (2022).
- [66] J.-X. Lu, C.-H. Zeng, E. Wang, J.-J. Xie, and L.-S. Geng, *Eur. Phys. J. C* **80**, 361 (2020).

- [67] N. Ikeno, W.-H. Liang, G. Toledo, and E. Oset, [Phys. Rev. D \*\*106\*\*, 034022 \(2022\)](#).
- [68] N. Ikeno, G. Toledo, W.-H. Liang, and E. Oset, [Few Body Syst. \*\*64\*\*, 55 \(2023\)](#).
- [69] Q.-F. Lü, H. Nagahiro, and A. Hosaka, [Phys. Rev. D \*\*107\*\*, 014025 \(2023\)](#).
- [70] C.-H. Zeng, J.-X. Lu, E. Wang, J.-J. Xie, and L.-S. Geng, [Phys. Rev. D \*\*102\*\*, 076009 \(2020\)](#).
- [71] J. Song, W.-H. Liang, C.-W. Xiao, J. M. Dias, and E. Oset, [Eur. Phys. J. C \*\*84\*\*, 1311 \(2024\)](#).
- [72] Q.-H. Shen, J.-X. Lu, L.-S. Geng, X. Liu, and J.-J. Xie, [arXiv:2510.13623](#).
- [73] J.-J. Xie and L.-S. Geng, [Chin. Phys. Lett. \*\*41\*\*, 081402 \(2024\)](#).
- [74] S.-W. Liu, W.-T. Lyu, and J.-J. Xie, [arXiv:2604.25630](#).