Erratum: Kalb-Ramond field and gravitational parity violation [Phys. Rev. D 110, 044067 (2024)]

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We here look to describe the changes to our manuscript that are necessary due to a calculation mistake, which led to a minor but meaningful revision of the paper's conclusions. The equations of motion (30) and (31) were originally given by

$$h_{\mathrm{L}}'' + 2\mathcal{H}h_{\mathrm{L}}' - \partial_z^2 h_{\mathrm{L}} - \frac{\xi}{m_p^2}\{2\mathcal{V}_m\mathcal{V}_e h_{\mathrm{L}}(3\mathcal{H}^2 - a''/a) - i\mathcal{H}((5\mathcal{V}_m^2 + 3\mathcal{V}_e^2)\partial_z h_{\mathrm{L}} + 2i\mathcal{V}_m\mathcal{V}_e h_{\mathrm{L}}') + i(\mathcal{V}_m^2 + \mathcal{V}_e^2)\partial_z h_{\mathrm{L}}'\} = 0, \quad (1)$$

$$h_{\mathrm{R}}'' + 2\mathcal{H}h_{\mathrm{R}}' - \partial_z^2 h_{\mathrm{R}} - \frac{\xi}{m_p^2}\{2\mathcal{V}_m\mathcal{V}_e h_{\mathrm{R}}(3\mathcal{H}^2 - a''/a) + i\mathcal{H}((5\mathcal{V}_m^2 + 3\mathcal{V}_e^2)\partial_z h_{\mathrm{R}} - 2i\mathcal{V}_m\mathcal{V}_e h_{\mathrm{R}}') - i(\mathcal{V}_m^2 + \mathcal{V}_e^2)\partial_z h_{\mathrm{R}}'\} = 0. \quad (2)$$

However, accounting for a missed term results in a noteworthy simplification. In momentum space, the correct result is

$$h_{\mathrm{R,L}}'' + 2\mathcal{H}h_{\mathrm{R,L}}' + k^2\left(1 - \lambda_{\mathrm{R,L}}\frac{\mathcal{H}}{k}\frac{\xi\tilde{v}^2}{1 - 3\xi\tilde{\mathcal{V}}_e\tilde{\mathcal{V}}_m}\right)h_{\mathrm{R,L}} = 0, \quad (3)$$

where $\lambda_{\mathrm{R,L}} = \pm 1$, $\tilde{\mathcal{V}}_e = \mathcal{V}_e/m_p$, $\tilde{\mathcal{V}}_m = \mathcal{V}_m/m_p$, and $\tilde{v} = v/m_p = \tilde{\mathcal{V}}_m^2 - \tilde{\mathcal{V}}_e^2$. This result changes a number of equations that follow. In full summary, the following equations are modified:

$$(30) \to h_{\mathrm{L}}'' + 2\mathcal{H}h_{\mathrm{L}}' - \partial_z^2 h_{\mathrm{L}} + \frac{\xi\tilde{v}^2}{1 - 3\tilde{\mathcal{V}}_e\tilde{\mathcal{V}}_m} i\partial_z h_{\mathrm{L}},$$

$$(31) \to h_{\mathrm{R}}'' + 2\mathcal{H}h_{\mathrm{R}}' - \partial_z^2 h_{\mathrm{R}} - \frac{\xi\tilde{v}^2}{1 - 3\tilde{\mathcal{V}}_e\tilde{\mathcal{V}}_m} i\partial_z h_{\mathrm{R}},$$

$$(34) \to A = B = 0, \qquad C = -\frac{\xi\tilde{v}^2}{1 - 3\tilde{\mathcal{V}}_e\tilde{\mathcal{V}}_m}$$

$$(35) \to \tilde{A} = \tilde{B} = \tilde{D} = 0, \qquad C = \tilde{C},$$

$$(36) \to \beta_1 = 0, \qquad \gamma_0 = C = -\frac{\xi\tilde{v}^2}{1 - 3\tilde{\mathcal{V}}_e\tilde{\mathcal{V}}_m},$$

$$(38) \to \bar{B} = 0, \qquad \bar{C} = \gamma_0 \simeq -\xi\tilde{v}^2,$$

$$(43) \to h_{\mathrm{R,L}} = h_{\mathrm{R,L}}^{\mathrm{GR}} \exp\left(\mp \frac{i}{2}\xi\tilde{v}^2 z_0\right),$$

$$(45) \to \tilde{\lambda}_{\mathrm{R,L}} = -\lambda_{\mathrm{R,L}}\xi\tilde{v}^2\mathcal{H},$$

$$(50) \to v^2\left(\frac{\mathcal{H}^2}{k^2 m_p^4}\right) < 6 \times 10^{-15},$$

$$(51) \to v\sqrt{\frac{H_0}{ka}} \lesssim 10^{-4} m_p.$$

Parity violation manifests solely as velocity birefringence, not as amplitude birefringence. This is due to the quantity $\bar{B}$ as originally defined in Eq. (38) in the manuscript, being zero.

Within the treatment of our constraints, we then derive the following. Using the SH0ES result [1] for the Hubble constant $H_0 \approx 73.04$ km/s/Mpc, we can write our final constraint as a function of redshift z and the gravitational wave frequency f as

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$$\frac{v}{m_p} \lesssim 4.4 \times 10^5 \left(\frac{f/\mathrm{Hz}}{1+z}\right)^{1/2}. \tag{4}$$

This illustrates that observations of lower frequency gravitational waves at high redshift would give the most meaningful constraint[1] on v. This is particularly challenging, however, since the lowest frequency gravitational waves observed in pulsar timing arrays are from the stochastic GW background [2]. Since the source of the background is not yet known, redshift determination is not feasible. If the GW background is determined to be dominantly caused by the physics of inflation, (4) will be useful in constraining the energy scale of the KR fields.

[1] A. G. Riess *et al.*, A comprehensive measurement of the local value of the Hubble constant with 1 km s^{-1} Mpc^{-1} uncertainty from the Hubble space telescope and the SH0ES team, Astrophys. J. Lett. **934**, L7 (2022).
[2] G. Agazie *et al.* (NANOGrav Collaboration), The NANOGrav 15 yr data set: Evidence for a gravitational-wave background, Astrophys. J. Lett. **951**, L8 (2023).

[1]We note that in an upcoming paper, we find a very similar model involving an interaction between the Kalb-Ramond fields and the Riemann tensor, instead of its dual, produces an analogous constraint that is a few orders of magnitude weaker than our initial estimate.