

Twist-3 gluon contribution to the single transverse-spin asymmetry in J/ψ production in pp collisions

Longjie Chen^{1,*} and Shinsuke Yoshida^{2,3,†}

¹*Institute of Nuclear Physics Polish Academy of Sciences,
ul. Radzikowskiego 152, PL-31-342 Kraków, Poland*

²*State Key Laboratory of Nuclear Physics and Technology, Institute of Quantum Matter,
South China Normal University, Guangzhou 510006, China*

³*Guangdong Basic Research Center of Excellence for Structure and Fundamental Interactions of Matter,
Guangdong Provincial Key Laboratory of Nuclear Science, Guangzhou 510006, China*



(Received 24 May 2026; accepted 9 July 2026; published 29 July 2026)

We present our results for the twist-3 gluon contribution to the single transverse-spin asymmetry (SSA) in J/ψ production in proton-proton collisions. Although the data were reported by the RHIC experiment more than a decade ago, a theoretical calculation based on rigorous collinear factorization has remained unavailable for the dominant gluonic contribution. Our results show that only the C -even type twist-3 gluon distribution, which has a direct relationship with the gluon transverse-momentum-dependent distribution function, contributes to the J/ψ SSA. Therefore, this observable serves as a key probe to understand the three-dimensional motion of gluons inside the proton. We also perform numerical simulations of the J/ψ SSA at RHIC and LHC energies. Our simulations indicate that a sizable SSA could be generated through a mechanism different from that responsible for the SSA in light hadron and D -meson production.

DOI: [10.1103/fcl3-kr8l](https://doi.org/10.1103/fcl3-kr8l)

I. INTRODUCTION

The understanding of the nucleon's internal structure has made dramatic progress over the past half century through constant interactions between theory and experiment. At present, those efforts continue to be directed toward the next-generation electron-ion collider (EIC) [1]. The central aim of this experiment is to reveal the role of gluons in various aspects of the nucleon as its main constituents. Gluons show their unique characteristics in several fundamental problems, the origin of the mass (trace anomaly), the nucleon spin puzzle (gluon spin and the less-known orbital angular momentum), and the small- x structure (dense gluonic system). One of the significant advantages of the EIC is the availability of polarized electron and polarized proton beams with a high-energy collider environment, thereby providing a distinct experimental capability compared with existing facilities such as Jefferson Lab. Comprehensive studies of various polarized processes will provide a new perspective on our understanding of the

nucleon spin structure. The measurement of the gluon spin inside the proton is one of the major achievements made at the Relativistic Heavy Ion Collider (RHIC) [2,3]. The EIC will focus on the further understanding of the origin of the proton spin, including the gluon spin contribution from the small- x regimes and the orbital motion of the gluons. The single transverse-spin asymmetry has drawn attention recently as a key probe to investigate such orbital motion of the partons inside the proton. When large asymmetries were first observed in the late 1970s [4,5], they were regarded as mysteries in high energy hadron physics since the conventional parton model calculation could not reproduce them at all [6]. The transverse-momentum-dependent (TMD) factorization [7], which additionally takes into account the transverse momenta of partons in a scattering process, has been well developed over the past couple of decades as a possible solution to the large SSAs. Reasonable descriptions of the existing SSA data within the TMD framework have made us recognize the importance of the orbital motion of the partons in high energy hadron reactions. It can now be said that SSA has transformed from merely a mystery in QCD into an ideal tool to investigate the orbital motion of the partons. Another successful framework for describing the large SSAs is the twist-3 mechanism in collinear factorization. This framework describes incoherent multiparton scatterings in a hard process and can be applied to single-scale processes such as $pp \rightarrow \pi X$ at the RHIC. The two

*Contact author: chenlongjieusc@163.com

†Contact author: shinyoshida85@gmail.com

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](https://creativecommons.org/licenses/by/4.0/). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

frameworks mentioned above have different applicable regions with respect to the transverse momentum of the particle produced in the final state. Although they seemingly describe different mechanisms as the origin of the SSA, it was found that there exists an intermediate region where the two frameworks give consistent cross section formulas for the SSAs in certain processes [8–14]. This equivalence supports the idea that the two frameworks describe a common origin of the large SSAs. From a phenomenological perspective, global analyses incorporating the relations between TMD functions and collinear twist-3 functions have succeeded in providing a unified description of experimental data across their applicable domains [15]. Heavy flavored hadron production serves as an important tool to investigate the gluon structure in the proton because the heavy quark that fragments into the final state heavy hadron is mainly produced through the gluon fusion process [16]. The orbital motion of gluons can be accessed through measurements of the SSAs in heavy meson production. The SSAs in heavy meson production have been extensively studied in recent years within the TMD framework for both D -meson production [17–21] and J/ψ production [19,21–30]. On the other hand, the twist-3 gluon contribution to the SSA was calculated for D -meson production about 15 years ago [31–35] and the first calculation for the J/ψ SSA in semi-inclusive deep inelastic scattering was done very recently [36]. The calculation of the SSA in $pp^\uparrow \rightarrow J/\psi X$ has been done only for quark contribution [37] although the corresponding data were reported in 2010 by the RHIC experiment [38].

In this paper, we present the first result for the twist-3 gluon contribution to the SSA in this process. We also perform numerical simulations of the SSA at RHIC and LHC energies with a simple model for the twist-3 gluon distribution functions by taking into account the data points reported in [38].

The remainder of this paper is organized as follows: In Sec. II, we introduce the definitions of the twist-3 gluon distribution functions relevant to our study and show some required relations for the calculation. In Sec. III, we show our result of the unpolarized and the polarized cross section formulas within the collinear factorization. In Sec. IV, we show numerical simulations for the J/ψ SSA at RHIC and LHC energies. Section V is devoted to a summary of our study. All the materials required for the twist-3 calculation are summarized in the Appendix.

II. DEFINITIONS OF TWIST-3 GLUON DISTRIBUTION FUNCTIONS

We introduce the definitions of the twist-3 gluon distribution functions. In general, the twist-3 cross section can be expressed in terms of the intrinsic, kinematical and dynamical twist-3 functions. The contribution from the intrinsic function is canceled in the case of the Sivers effect because of PT invariance, and, therefore, only the other two types are relevant to our study. The kinematical functions for a transversely polarized proton are defined as follows [39]:

$$\begin{aligned} \Phi_\delta^{[\pm]\alpha\beta\gamma}(x) &= i \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle pS | F^{\beta n}(0) [0, \lambda n] D^\gamma(\lambda n) F^{\alpha n}(\lambda n) | pS \rangle \\ &\quad - \int \frac{d\lambda}{2\pi} e^{i\lambda x} \int_\lambda^{\mp\infty} d\mu \langle pS | F^{\beta n}(0) [0, \mu n] g F^{\gamma n}(\mu n) [\mu n, \lambda n] F^{\alpha n}(\lambda n) | pS \rangle \\ &= \frac{M_N}{2} g_\perp^{\alpha\beta} \epsilon^{pnS_\perp\gamma} G_T^{[\pm](1)}(x) + i \frac{M_N}{2} \epsilon^{pn\alpha\beta} S_\perp^\gamma \Delta G_T^{[\pm](1)}(x) + \frac{M_N}{8} (\epsilon^{pnS_\perp\{\alpha} g_\perp^{\beta\}\gamma} + \epsilon^{pn\gamma\{\alpha} S_\perp^{\beta\}}) \Delta H_T^{[\pm](1)}(x) + \dots, \end{aligned} \quad (1)$$

where the index γ is restricted to the transverse components and $p, S_\perp$ and M_N respectively denote the proton's momentum, spin, and mass. We use the simplified notation $\epsilon^{pnS_\perp\gamma} = \epsilon^{\mu\nu\rho\gamma} p_\mu n_\nu S_{\perp\rho}$. $[0, \lambda n]$ is a shorthand notation of the gauge-link operator in the adjoint representation,

$$[0, \lambda n] \equiv \text{Pexp} \left(ig \int_\lambda^0 dt A^n(tn) \right), \quad (2)$$

which guarantees the gauge invariance of the matrix elements. The vector n is lightlike and satisfies $p \cdot n = 1, n^2 = 0$. Naively T -odd observables such as the SSA receive contributions from $G_T^{(1)}(x)$ and $\Delta H_T^{(1)}(x)$.

The dynamical gluon distribution functions are defined by matrix elements composed of three gluon field strength tensors [33,40]. The dynamical functions are categorized into two types, C -even function $N(x_1, x_2)$ and C -odd function $O(x_1, x_2)$, reflecting the fact that there are two structure constants, f^{abc} and d^{abc} in $SU(N_c)$ group,

$$\begin{aligned}
N^{\alpha\beta\gamma}(x_1, x_2) &= i \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle pS_\perp | i f^{bca} F_b^{\beta n}(0) g F_c^{\gamma n}(\mu n) F_a^{\alpha n}(\lambda n) | pS_\perp \rangle \\
&= 2iM_N [g_\perp^{\alpha\beta} \epsilon^{\gamma pn S_\perp} N(x_1, x_2) - g_\perp^{\beta\gamma} \epsilon^{\alpha pn S_\perp} N(x_2, x_2 - x_1) - g_\perp^{\alpha\gamma} \epsilon^{\beta pn S_\perp} N(x_1, x_1 - x_2)] + \dots, \quad (3)
\end{aligned}$$

$$\begin{aligned}
O^{\alpha\beta\gamma}(x_1, x_2) &= i \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle pS_\perp | d^{bca} F_b^{\beta n}(0) g F_c^{\gamma n}(\mu n) F_a^{\alpha n}(\lambda n) | pS_\perp \rangle \\
&= 2iM_N [g_\perp^{\alpha\beta} \epsilon^{\gamma pn S_\perp} O(x_1, x_2) + g_\perp^{\beta\gamma} \epsilon^{\alpha pn S_\perp} O(x_2, x_2 - x_1) + g_\perp^{\alpha\gamma} \epsilon^{\beta pn S_\perp} O(x_1, x_1 - x_2)] + \dots, \quad (4)
\end{aligned}$$

where we omitted gauge links for simplicity. These dynamical functions satisfy the following symmetries.

$$\begin{aligned}
O(x_1, x_2) &= O(x_2, x_1), \quad O(x_1, x_2) = O(-x_1, -x_2), \\
N(x_1, x_2) &= N(x_2, x_1), \quad N(x_1, x_2) = -N(-x_1, -x_2). \quad (5)
\end{aligned}$$

The following relations were derived in [39]:

$$\begin{aligned}
G_T^{[\pm](1)}(x) &= \pm 4\pi(N(x, x) - N(x, 0)), \\
\Delta H_T^{[\pm](1)}(x) &= \mp 8\pi N(x, 0). \quad (6)
\end{aligned}$$

Using these relations, the polarized cross section can be written solely in terms of the dynamical functions as shown in the next section. The superscript “(1)” of a kinematical function denotes the first $\mathbf{k}_T^2$ moment of the corresponding gluon TMD function [41]. The determination of $N(x, x)$ and $N(x, 0)$ could provide indirect information on the orbital motion of partons.

III. CALCULATION OF THE SSA IN J/ψ PRODUCTION IN pp COLLISIONS

A. Nonrelativistic QCD(NRQCD) framework and the unpolarized cross section for J/ψ production

We consider J/ψ production in proton-proton collisions,

$$p^\uparrow(p, S_\perp) + p(p') \rightarrow J/\psi(P_J) + X. \quad (7)$$

We adopt the NRQCD framework [42,43] for the hadronization of the charm quark pair into J/ψ . The above scattering is illustrated within this framework as

$$p^\uparrow(p, S_\perp) + p(p') \rightarrow \sum_n c\bar{c}[n](P_J) + X, \quad (8)$$

where n denotes all possible Fock states of the charm quark pair. As a first step, we consider the color-singlet contribution $n = {}^3S_1^{[1]}$ in which the hadronization of J/ψ is represented as Fig. 1 and gives the following mathematical structure in the diagram calculation:

$$\mathcal{N} \langle O^{J/\psi} [{}^3S_1^{(1)}] \rangle [\epsilon(P_J + m_J)]_{ij} \delta_{i'j'}, \quad (9)$$

where ϵ^ρ is the polarization vector of J/ψ , $i, j(i', j')$ are the spinor(color) indices, $\mathcal{N}$ is the normalization factor, and $\langle O^{J/\psi} [{}^3S_1^{(1)}] \rangle$ is the long-distance-matrix-element which describes the nonperturbative transition from the color-singlet charm quark pair into J/ψ . The unpolarized J/ψ gives

$$\sum \epsilon^\rho \epsilon^\sigma = -g^{\rho\sigma} + \frac{P_J^\rho P_J^\sigma}{m_J^2}. \quad (10)$$

There are 12 nonvanishing diagrams shown in Fig. 2 at the level of the amplitude. The unpolarized cross section can be calculated by taking the square of the amplitude as

$$\begin{aligned}
P_J^0 \frac{d\sigma}{d^3P_J} &= \frac{4\pi\alpha_s^3}{S} (\mathcal{N} \langle O^{J/\psi} [{}^3S_1^{(1)}] \rangle) \int \frac{dx}{x} G(x) \\
&\times \int \frac{dx'}{x'} G(x') \hat{\sigma}^U \delta(\hat{s} + \hat{t} + \hat{u} - m_J^2) \\
&= \frac{4\pi\alpha_s^3}{S} (\mathcal{N} \langle O^{J/\psi} [{}^3S_1^{(1)}] \rangle) \frac{G(x)}{x} \\
&\times \int_{x'_{\min}}^1 \frac{dx'}{x'} G(x') \frac{1}{x'S + T - m_J^2} \hat{\sigma}^U \Big|_{x=\bar{x}}, \quad (11)
\end{aligned}$$

where α_s is the QCD coupling, S is the center-of-mass energy, $G(x)$ is the unpolarized gluon distribution function of the proton, $\bar{x}$ is given by

$$\bar{x} = -\frac{x'U + m_J^2(1-x')}{x'S + T - m_J^2}, \quad x'_{\min} = -\frac{T}{S + U - m_J^2}, \quad (12)$$

and Mandelstam variables are defined by

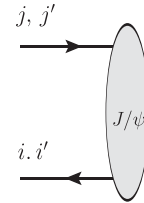


FIG. 1. Diagrammatic expression of the hadronization process in the color-singlet contribution.

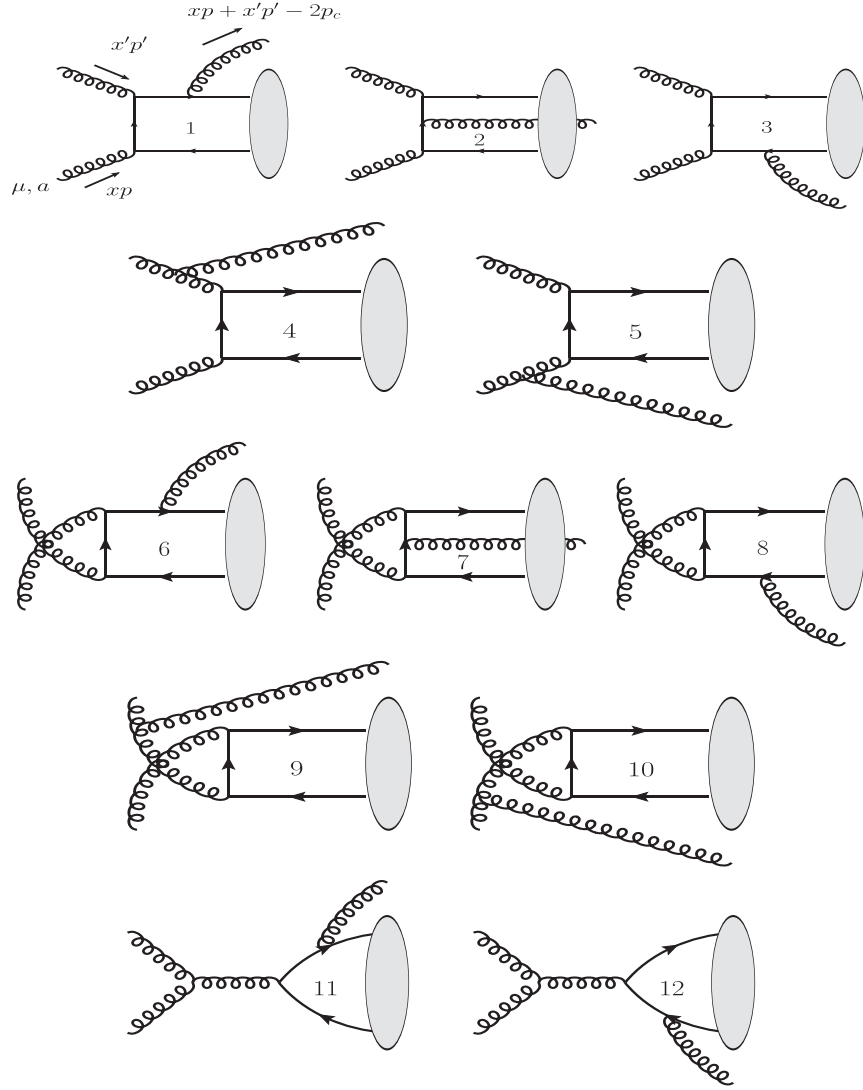


FIG. 2. Leading-order (LO) diagrams with $^3S_1^{(1)}$ charm quark pair in the final state. We express each numbered diagram as $C_i^a \mathcal{M}_{i\mu}(xp)$ ($i = 1, 2, \dots, 12$).

$$\begin{aligned}
 S &= (p + p')^2, \quad \hat{s} = (xp + x'p')^2 = xx'S, \\
 T &= (p - P_J)^2, \quad \hat{t} = (xp - P_J)^2 = xT + m_J^2(1 - x), \\
 U &= (p' - P_J)^2, \quad \hat{u} = (x'p' - P_J)^2 = x'U + m_J^2(1 - x'). \quad (13)
 \end{aligned}$$

The i -th diagram in Fig. 2 can be decomposed into the color part C_i^a and the tensor part $\mathcal{M}_{i\mu}$. The color factor of the

squared amplitude is obtained as

$$C_i^a C_j^{b*} = C_{ij} \delta^{ab}, \quad (i, j = 1, 2, \dots, 12). \quad (14)$$

Thus one can obtain the known result for the hard cross section $\hat{\sigma}^U$ [44] as

$$\begin{aligned}
 \hat{\sigma}^U &= \sum_{i,j=1}^{12} C_{ij} \left(-\frac{1}{2} g_{\perp}^{\mu\nu}(p) \right) \mathcal{M}_{i\mu}(xp) \mathcal{M}_{i\nu}^*(xp) \\
 &= \left(-\frac{1}{N_c^2 C_F} + \frac{1}{4 C_F} \right) \frac{128 m_J^2 ((\hat{t}^2 + \hat{t} \hat{u} + \hat{u}^2)^2 + m_J^4 (\hat{t}^2 + \hat{t} \hat{u} + \hat{u}^2) - m_J^2 (2 \hat{t}^3 + 3 \hat{t}^2 \hat{u} + 3 \hat{t} \hat{u}^2 + 2 \hat{u}^3))}{(\hat{s} - m_J^2)^2 (\hat{t} - m_J^2)^2 (\hat{u} - m_J^2)^2}, \quad (15)
 \end{aligned}$$

where $g_{\perp}^{\mu\nu}(p) = g^{\mu\nu} - p^\mu n^\nu - p^\nu n^\mu$.

B. Twist-3 polarized cross section for J/ψ production in pp collisions

We calculate the twist-3 gluon distribution contribution to the polarized cross section. The basic formulation for deriving the cross section formula was established based on the modern nonpole technique in [45] for ep collisions. Some extension is needed in pp collisions because of the fact that there are both the initial state interaction (ISI) and

the final state interaction(FSI). We present here the polarized cross section formula for pp collisions, and the derivation in detail will be discussed in a separate paper because it is quite lengthy and technical. When a process involves both ISI and FSI, we can write down the formula for the three-gluon distribution contribution to the polarized cross section as

$$\begin{aligned}
 P_J^0 \frac{d\Delta\sigma}{d^3P_J} = & \frac{1}{16\pi^2 S} (\mathcal{N} \langle O^{J/\psi} [^3S_1^{(1)}] \rangle) \omega_\alpha^\mu \omega_\beta^\nu \omega_\gamma^\lambda \int \frac{dx'}{x'} G(x') \\
 & \times \left[\int \frac{dx}{x^2} \sum_{i,j=1}^{12} (\Phi_\partial^{[-]\alpha\beta\gamma}(x) (C_{Lij}^{f(+)[-]} + C_{Lij}^{f(-)[-]} + \Phi_\partial^{[+]\alpha\beta\gamma}(x) C_{Lij}^{f(-)[+]} \right. \\
 & + 2\pi C_{Lij}^{d(-)[+]} O^{\alpha\beta\gamma}(x, x)) \frac{\partial}{\partial k^\lambda} [\mathcal{M}_{i\mu}(k) \mathcal{M}_{j\nu}^*(k)] \Big|_{k=xp} \\
 & - \frac{1}{2} \int dx_1 \int dx_2 \left[\frac{-if^{abc}}{N_c(N_c^2 - 1)} N^{\alpha\beta\gamma}(x_1, x_2) + \frac{N_c d^{abc}}{(N_c^2 - 4)(N_c^2 - 1)} O^{\alpha\beta\gamma}(x_1, x_2) \right] \\
 & \times \left(\frac{1}{x_2 x_1 \pm i\epsilon} \frac{1}{x_2 - x_1 \mp i\epsilon} \frac{1}{\mp i\epsilon} \mathcal{M}_{L\mu\lambda}^{(\pm)[\mp]ac}(x_1 p, (x_2 - x_1)p) \left[\sum_{i=1}^{12} C_i^{b*} \mathcal{M}_{iv}^*(x_2 p) \right] \right. \\
 & \left. + \frac{1}{x_1 x_2 \pm i\epsilon} \frac{1}{x_2 - x_1 \mp i\epsilon} \left[\sum_{i=1}^{12} C_i^a \mathcal{M}_{i\mu}(x_1 p) \right] \mathcal{M}_{R\nu\lambda}^{(\pm)[\mp]bc}(x_2 p, (x_2 - x_1)p) \right) \Big], \quad (16)
 \end{aligned}$$

where the color factors C_{Lij}^f and C_{Lij}^d are shown in the Appendix. $\mathcal{M}_{L\mu\lambda}^{(\pm)[\mp]ac}(x_1 p, (x_2 - x_1)p)$ is the sum of the diagrams including an additional gluon line with momentum $(x_2 - x_1)p$, and there are 86 diagrams as shown in Fig. 6. $\mathcal{M}_{R\nu\lambda}^{(\pm)[\mp]bc}(x_2 p, (x_2 - x_1)p)$ is its complex conjugate. The complicated pole structure composed of $1/(x_1 \pm i\epsilon)$, $1/(x_2 \pm i\epsilon)$, and $1/(x_2 - x_1 \pm i\epsilon)$ reflects the presence of ISI and FSI, and all contributions have to be combined. We show the explicit forms of $\mathcal{M}_{L\mu\lambda}^{(\pm)[\mp]ac}(x_1 p, (x_2 - x_1)p)$ in the Appendix. After evaluating all diagrams, the polarized cross section is given by

$$\begin{aligned}
 P_J^0 \frac{d\Delta\sigma}{d^3P_J} = & \frac{4\pi\alpha_s^3(-\pi M_N)}{S} (\mathcal{N} \langle O^{J/\psi} [^3S_1^{(1)}] \rangle) \int \frac{dx}{x'} G(x') \int \frac{dx}{x^2} \delta(\hat{s} + \hat{t} + \hat{u} - m_J^2) \\
 & \times (\hat{s} \epsilon^{P_J p n S_\perp} - (\hat{t} - m_J^2) x' \epsilon^{n p p' S_\perp}) \left(-\frac{1}{N_c^2 C_F} + \frac{1}{4C_F} \right) \left[x \frac{d}{dx} N(x, x) \left(-\frac{4}{\hat{s} \hat{u}} \hat{\sigma}^U \right) + x \frac{d}{dx} N(x, 0) \hat{\sigma}_D \right. \\
 & \left. + N(x, x) \hat{\sigma}_{ND1} + N(x, 0) \hat{\sigma}_{ND2} + N(x, Hx) \hat{\sigma}_{H1} + N(x - Hx, x) \hat{\sigma}_{H2} + N(Hx, Hx - x) \hat{\sigma}_{H3} \right], \quad (17)
 \end{aligned}$$

where $H = -(\hat{u} - m_J^2)/(\hat{s} - \hat{u})$. We find that the contribution from the C -odd function $O(x_1, x_2)$ is completely canceled as in the case of semi-inclusive deep inelastic scattering (SIDIS) [34]. Note that the result is independent of the arbitrary vector n ,

$$\hat{s} \epsilon^{P_J p n S_\perp} - (\hat{t} - m_J^2) x' \epsilon^{n p p' S_\perp} = 2x x' \epsilon^{P_J p p' S_\perp}, \quad (18)$$

which follows from the identity

$$g^{\alpha\beta} \epsilon^{\mu\nu\rho\sigma} = g^{\alpha\mu} \epsilon^{\beta\nu\rho\sigma} + g^{\alpha\nu} \epsilon^{\mu\beta\rho\sigma} + g^{\alpha\rho} \epsilon^{\mu\nu\beta\sigma} + g^{\alpha\sigma} \epsilon^{\mu\nu\rho\beta}. \quad (19)$$

The hard cross sections are given by

$$\begin{aligned}
\hat{\sigma}_D &= \frac{1}{\hat{s} \hat{u}} \frac{512m_J^2((\hat{t}^2 + \hat{t} \hat{u} + \hat{u}^2)^2 + m_J^4(\hat{t}^2 + \hat{u}^2) - 2m_J^2(\hat{t}^3 + \hat{t}^2 \hat{u} + \hat{t} \hat{u}^2 + \hat{u}^3))}{(\hat{s} - m_J^2)^2(\hat{t} - m_J^2)^2(\hat{u} - m_J^2)^2}, \\
\hat{\sigma}_{ND1} &= -\frac{1}{\hat{s} \hat{u}} \frac{256m_J^2}{(\hat{s} - m_J^2)^3(\hat{t} - m_J^2)^3(\hat{u} - m_J^2)^3} (-4\hat{t}^7 - 32\hat{t}^6 \hat{u} - 89\hat{t}^5 \hat{u}^2 - 142\hat{t}^4 \hat{u}^3 - 135\hat{t}^3 \hat{u}^4 - 76\hat{t}^2 \hat{u}^5 - 22\hat{t} \hat{u}^6 - 4\hat{u}^7 \\
&\quad + m_J^2(20\hat{t}^6 + 119\hat{t}^5 \hat{u} + 268\hat{t}^4 \hat{u}^2 + 330\hat{t}^3 \hat{u}^3 + 239\hat{t}^2 \hat{u}^4 + 96\hat{t} \hat{u}^5 + 22\hat{u}^6) - m_J^4(38\hat{t}^5 + 168\hat{t}^4 \hat{u} + 291\hat{t}^3 \hat{u}^2 \\
&\quad + 264\hat{t}^2 \hat{u}^3 + 135\hat{t} \hat{u}^4 + 40\hat{u}^5) + m_J^6(34\hat{t}^4 + 106\hat{t}^3 \hat{u} + 127\hat{t}^2 \hat{u}^2 + 78\hat{t} \hat{u}^3 + 31\hat{u}^4) \\
&\quad - 2m_J^8(7\hat{t}^3 + 12\hat{t}^2 \hat{u} + 8\hat{t} \hat{u}^2 + 5\hat{u}^3) + m_J^{10}(2\hat{t}^2 - \hat{t} \hat{u} + \hat{u}^2)), \\
\hat{\sigma}_{ND2} &= \frac{1}{\hat{s} \hat{u}} \frac{256m_J^2}{(\hat{s} - m_J^2)^3(\hat{t} - m_J^2)^3(\hat{u} - m_J^2)^3} (-4\hat{t}^7 - 32\hat{t}^6 \hat{u} - 91\hat{t}^5 \hat{u}^2 - 146\hat{t}^4 \hat{u}^3 - 125\hat{t}^3 \hat{u}^4 - 44\hat{t}^2 \hat{u}^5 + 6\hat{t} \hat{u}^6 + 4\hat{u}^7 \\
&\quad + m_J^2(20\hat{t}^6 + 121\hat{t}^5 \hat{u} + 264\hat{t}^4 \hat{u}^2 + 286\hat{t}^3 \hat{u}^3 + 141\hat{t}^2 \hat{u}^4 + 8\hat{t} \hat{u}^5 - 6\hat{u}^6) - m_J^4(42\hat{t}^5 + 184\hat{t}^4 \hat{u} + 277\hat{t}^3 \hat{u}^2 \\
&\quad + 168\hat{t}^2 \hat{u}^3 + 21\hat{t} \hat{u}^4) + m_J^6(46\hat{t}^4 + 134\hat{t}^3 \hat{u} + 113\hat{t}^2 \hat{u}^2 + 18\hat{t} \hat{u}^3 + 5\hat{u}^4) \\
&\quad - 2m_J^8(13\hat{t}^3 + 20\hat{t}^2 \hat{u} + 6\hat{t} \hat{u}^2 + 3\hat{u}^3) + m_J^{10}(6\hat{t}^2 + \hat{t} \hat{u} + 3\hat{u}^2)), \\
\hat{\sigma}_{H1} &= \frac{1}{\hat{s} \hat{u}} \frac{256m_J^2}{(\hat{s} - m_J^2)^3(\hat{t} - m_J^2)^3(\hat{u} - m_J^2)^3} (-2\hat{t}^7 - 12\hat{t}^6 \hat{u} - 33\hat{t}^5 \hat{u}^2 - 56\hat{t}^4 \hat{u}^3 - 61\hat{t}^3 \hat{u}^4 - 46\hat{t}^2 \hat{u}^5 - 22\hat{t} \hat{u}^6 - 8\hat{u}^7 \\
&\quad + m_J^2(10\hat{t}^6 + 47\hat{t}^5 \hat{u} + 98\hat{t}^4 \hat{u}^2 + 122\hat{t}^3 \hat{u}^3 + 99\hat{t}^2 \hat{u}^4 + 60\hat{t} \hat{u}^5 + 26\hat{u}^6) - m_J^4(20\hat{t}^5 + 68\hat{t}^4 \hat{u} + 97\hat{t}^3 \hat{u}^2 + 84\hat{t}^2 \hat{u}^3 \\
&\quad + 61\hat{t} \hat{u}^4 + 34\hat{u}^5) + m_J^6(20\hat{t}^4 + 42\hat{t}^3 \hat{u} + 33\hat{t}^2 \hat{u}^2 + 26\hat{t} \hat{u}^3 + 23\hat{u}^4) - 2m_J^8(5\hat{t}^3 + 4\hat{t}^2 \hat{u} + \hat{t} \hat{u}^2 + 4\hat{u}^3) \\
&\quad + m_J^{10}(2\hat{t}^2 - \hat{t} \hat{u} + \hat{u}^2)), \\
\hat{\sigma}_{H2} &= -\frac{1}{\hat{s} \hat{u}} \frac{256m_J^2}{(\hat{s} - m_J^2)^3(\hat{t} - m_J^2)^3(\hat{u} - m_J^2)^3} (-4\hat{t}^7 - 24\hat{t}^6 \hat{u} - 67\hat{t}^5 \hat{u}^2 - 112\hat{t}^4 \hat{u}^3 - 119\hat{t}^3 \hat{u}^4 - 82\hat{t}^2 \hat{u}^5 - 34\hat{t} \hat{u}^6 - 8\hat{u}^7 \\
&\quad + m_J^2(20\hat{t}^6 + 99\hat{t}^5 \hat{u} + 220\hat{t}^4 \hat{u}^2 + 282\hat{t}^3 \hat{u}^3 + 221\hat{t}^2 \hat{u}^4 + 108\hat{t} \hat{u}^5 + 30\hat{u}^6) - m_J^4(40\hat{t}^5 + 156\hat{t}^4 \hat{u} + 261\hat{t}^3 \hat{u}^2 \\
&\quad + 244\hat{t}^2 \hat{u}^3 + 139\hat{t} \hat{u}^4 + 46\hat{u}^5) + m_J^6(40\hat{t}^4 + 114\hat{t}^3 \hat{u} + 133\hat{t}^2 \hat{u}^2 + 90\hat{t} \hat{u}^3 + 37\hat{u}^4) - 4m_J^8(5\hat{t}^3 + 9\hat{t}^2 \hat{u} + 7\hat{t} \hat{u}^2 \\
&\quad + 4\hat{u}^3) + m_J^{10}(4\hat{t}^2 + 3\hat{t} \hat{u} + 3\hat{u}^2)), \\
\hat{\sigma}_{H3} &= -\frac{\hat{t}}{\hat{s} \hat{u}} \frac{512m_J^2}{(\hat{s} - m_J^2)^3(\hat{t} - m_J^2)^3(\hat{u} - m_J^2)^3} (-\hat{t}(\hat{t} + 2\hat{u})(\hat{t}^2 + 2\hat{t} \hat{u} + 2\hat{u}^2)^2 + m_J^2(5\hat{t}^5 + 23\hat{t}^4 \hat{u} + 43\hat{t}^3 \hat{u}^2 + 38\hat{t}^2 \hat{u}^3 \\
&\quad + 10\hat{t} \hat{u}^4 - 4\hat{u}^5) - m_J^4(10\hat{t}^4 + 32\hat{t}^3 \hat{u} + 33\hat{t}^2 \hat{u}^2 + 4\hat{t} \hat{u}^3 - 10\hat{u}^4) + m_J^6(10\hat{t}^3 + 18\hat{t}^2 \hat{u} + \hat{t} \hat{u}^2 - 10\hat{u}^3) \\
&\quad + m_J^8(-5\hat{t}^2 - 2\hat{t} \hat{u} + 5\hat{u}^2) + m_J^{10}(\hat{t} - \hat{u})).
\end{aligned} \tag{20}$$

IV. NUMERICAL SIMULATIONS OF THE SSA IN THE J/ψ PRODUCTION

We perform numerical simulations of the $J/\psi A_N$,

$$A_N = \frac{d\Delta\sigma}{d^3P_J} / \frac{d\sigma}{d^3P_J}, \tag{21}$$

at the RHIC energy, $\sqrt{S} = 200$ GeV [38], and the LHCspin energy, $\sqrt{S} = 115$ GeV [46]. The C -even function $N(x_1, x_2)$ has not yet been well constrained by experiments. Our simulations aim to clarify which hard scattering cross sections can be enhanced in these experiments by using the same model for the five types of the functions in (17),

$$\begin{aligned}
N(x, x) &= N(x, 0) = N(x, Hx) = N(x - Hx, x) \\
&= N(Hx, Hx - x) = 0.002xG(x).
\end{aligned} \tag{22}$$

We use GJR08 LO [47] for the unpolarized gluon distribution function $G(x)$. This model function reproduces the upper bound of existing experimental results [48–50]. The scale dependence of the C -even function requires knowledge of its two-variable dependence [51]. Although this two-variable dependence can be probed through naively T -even observables such as the double-spin asymmetry A_{LT} [52], no experimental data are currently available. In our numerical analysis, each C -even function is modeled using the unpolarized gluon distribution, and its scale dependence is therefore estimated using the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) evolution equation. Figure 3 shows our simulations for the x_F dependence

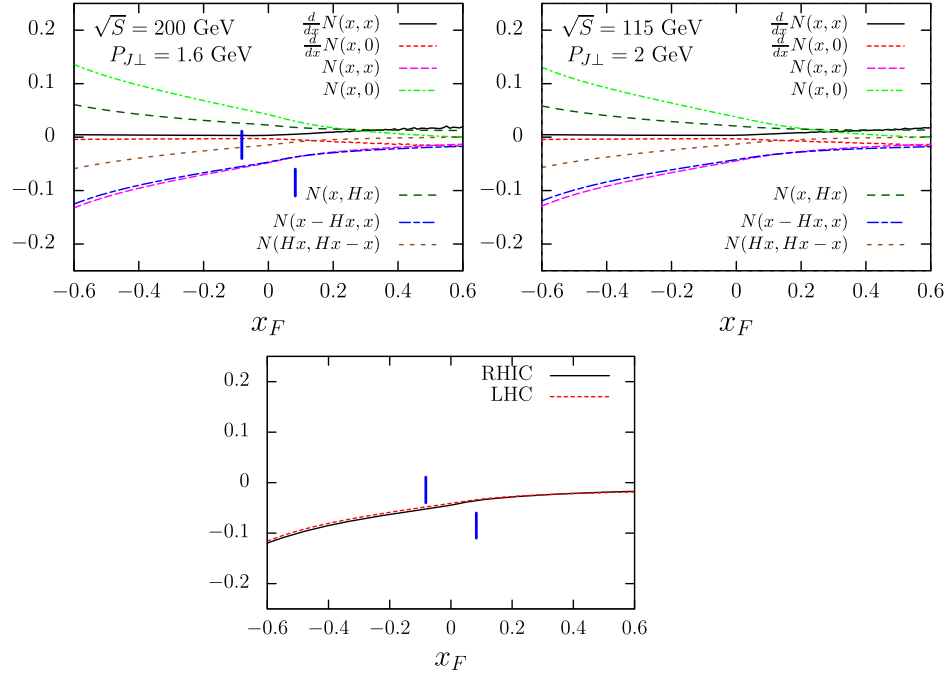


FIG. 3. The top left (right) figure shows the result of the $J/\psi A_N$ for the RHIC (LHC) energy. The contribution from each term in (17) is separately plotted. The thick error bars in the top left figure are taken from [38]. The bottom figure shows the total contribution obtained by combining the separate curves shown in the top RHIC and LHC figures.

of A_N . They show that the $J/\psi A_N$ does not exhibit the typical behavior observed in light hadron production, where A_N increases as x_F becomes larger in the positive region [53]. This behavior can be reproduced by the derivative term of the twist-3 distribution function in light

hadron production, and a similar behavior has also been observed in D -meson production [34]. However, in the present case, the derivative term is found to be small over the entire range of x_F . Although our calculation provides the first theoretical prediction for A_N in $pp \rightarrow J/\psi X$ within

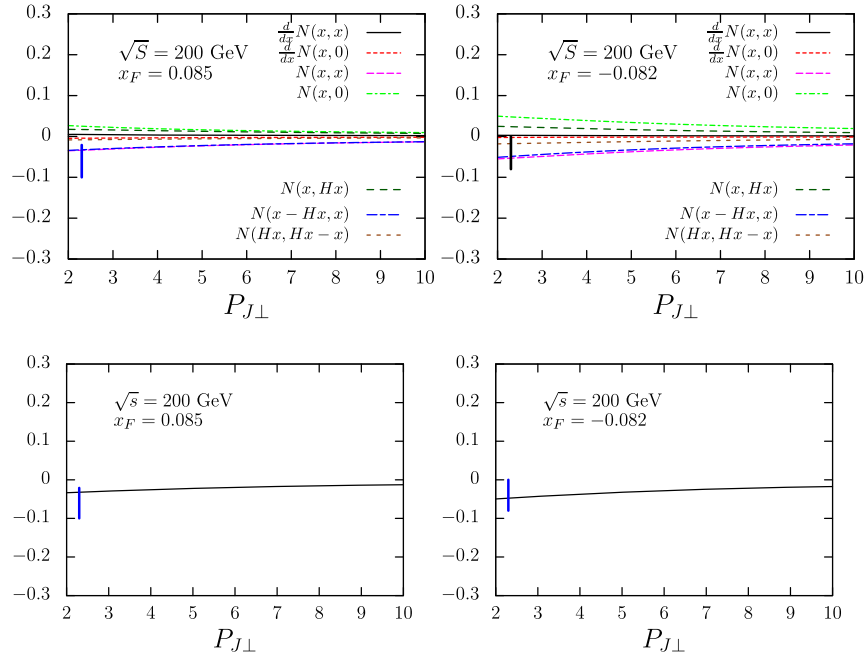


FIG. 4. The top left (right) figure shows the $P_{J\perp}$ dependence of the $J/\psi A_N$ in positive (negative) x_F regions. The thick error bars are taken from [38]. The bottom figures show the total contributions obtained by combining the separate curves shown in the top figures.

the collinear framework, predictions based on the color-gauge invariant generalized parton model (CGI-GPM) using TMD functions already exist [19,21,28,29]. Within this model framework, qualitatively different results are obtained for RHIC and LHC. In contrast, our collinear twist-3 calculation predicts no significant difference between the two experiments. A comparison of the results from these experiments would therefore provide valuable insight into the origin of the SSA. Figure 4 shows the $P_{J\perp}$ dependence of A_N at the RHIC energy in both the positive- and negative- x_F regions. The results for LHCspin are omitted because it yields nearly identical values similarly to the x_F dependence shown in Fig. 3. RHIC has measured only a single data point at $P_{J\perp} > 1$ GeV in both the positive- and negative- x_F regions, which is insufficient to determine the C -even function. In contrast, the LHCspin experiment has the potential to cover a much wider $P_{J\perp}$ range, $P_{J\perp} \leq 18$ GeV for charmonium production. Therefore, it is expected to provide sufficient data to constrain not only the magnitude but also the functional form of the C -even function. From the viewpoint of understanding TMD functions, we find that the contributions from $N(x, x)$ and $N(x, 0)$, which are directly related to TMD functions, are relatively significant in both the x_F dependence and the $P_{J\perp}$ dependence. This indicates that the $J/\psi A_N$ is an interesting observable for indirectly accessing three-dimensional partonic information through collinear twist-3 correlation functions. Our result will give a basis to further analyses in future experiments and lead to a phenomenological understanding of the poorly known gluon Sivers effect. In SIDIS, the contributions from $N(x, x)$ and $N(x, 0)$ are not particularly large, while functions whose arguments depend on the Mandelstam variables provide relatively large contributions [36]. Therefore, SIDIS is a suitable observable for constraining the functions in the region $x_1 \neq x_2$ and $x_{1,2} \neq 0$. As discussed above, in pp collisions the TMD-related functions $N(x, x)$ and $N(x, 0)$ provide the dominant contributions. Consequently, ep and pp reactions play complementary roles in constraining the three-gluon correlation functions.

V. SUMMARY

We have derived the first result of the twist-3 gluonic contribution to the J/ψ SSA in pp collisions in the color-singlet case. We find that the contribution from the C -odd type function $O(x_1, x_2)$ is completely canceled in the polarized cross section similar to the case of ep collision, and therefore, this is another ideal observable to pin down the C -even type function $N(x_1, x_2)$. The derivative terms $\frac{d}{dx} N(x, x)$ and $\frac{d}{dx} N(x, 0)$, which are dominant sources of the SSA in D -meson production, also appear in J/ψ production. We have performed numerical simulations at RHIC and LHC energies using the same model as that employed in D -meson production. Our results suggest that the contribution from the derivative terms is small even in the large x_F regions, which differs from the typical behavior

observed in light hadron and D -meson production. On the other hand, the nonderivative terms associated with $N(x, x)$ and $N(x, 0)$ provide relatively significant contributions. Therefore, the J/ψ SSA is an interesting observable for extracting three-dimensional information through their relations to TMD functions. The RHIC experiment confirmed nonzero A_N suggesting non-negligible orbital motion of gluons inside the proton. This finding motivates further investigations over a broader kinematic region to achieve a more comprehensive understanding of the underlying gluon dynamics. The LHCspin experiment has the potential to expand the accessible $P_{J\perp}$ regions and is essential for a more complete understanding of the poorly known gluon Sivers function. Our results provide a basis for future investigations of the twist-3 gluon distribution function which is responsible for describing the J/ψ SSA in high- $P_{J\perp}$ regions. Past studies of J/ψ production in unpolarized reactions show that the color-octet contribution could be as large as the color-singlet contribution. Extending the present analysis to the color-octet channel based on the formalism established in this work is one of our future research objectives.

ACKNOWLEDGMENTS

This work is supported by Polish National Science Center Grant No. UMO-2023/49/B/ST2/03665.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX: EXPLICIT FORMS OF THE HARD PARTS

Here we show the explicit forms of $\mathcal{M}_{L\mu\lambda}^{(\pm)[\mp].ac}$, which are needed for the calculation of the polarized cross section (17). We assign the color and Lorentz indices in each diagram as shown in Fig. 5. There are 86 diagrams, $\mathcal{M}_{L\mu\lambda}^{ac}(i = 1, 2, \dots, 86)$, where the color indices d and f are suppressed for simplicity.

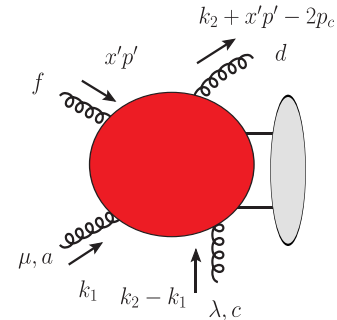


FIG. 5. Labels of external gluon lines in $\mathcal{M}_{L\mu\lambda}^{ac}(k_1, k_2 - k_1)$.

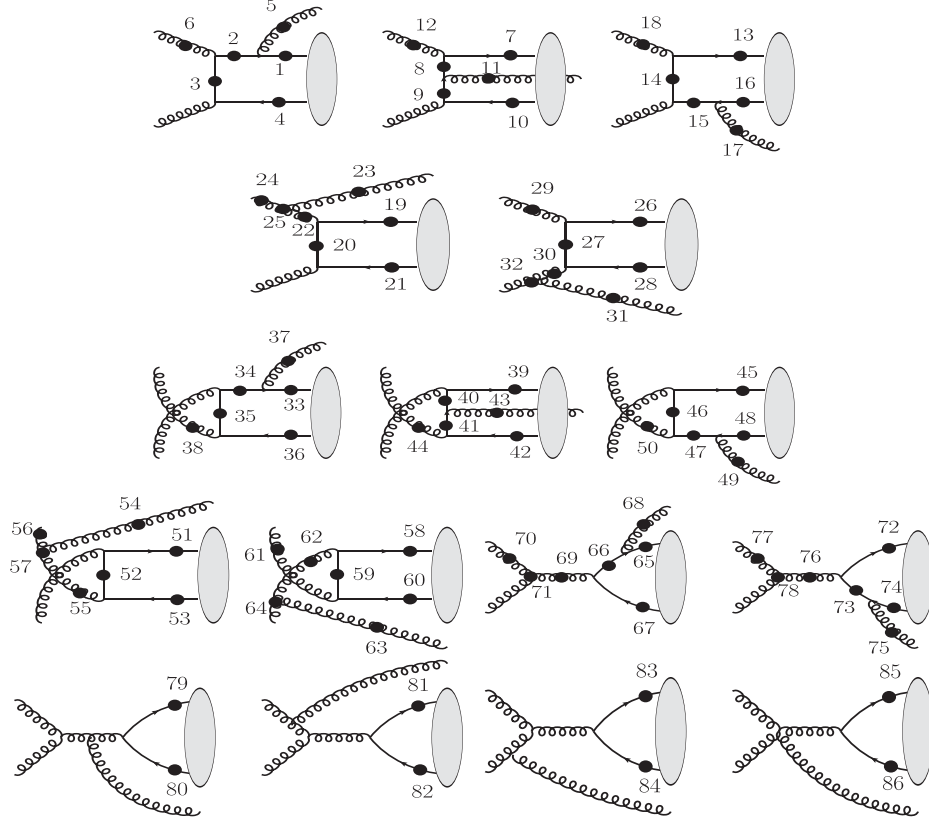


FIG. 6. The gluon with momentum $k_2 - k_1$ is connected to one of the black dots. The diagram with the $k_2 - k_1$ gluon line connected to the i -th black dot gives $\mathcal{M}_{Li\mu\lambda}^{ac}(k_1, k_2 - k_1)$.

The diagrams including a four-gluon vertex can be decomposed as follows:

$$\begin{aligned}
 \mathcal{M}_{L25\mu\lambda}^{ac} &= (-if^{ced})(-if^{feg})\text{Tr}[T^g T^a] \mathcal{M}_{25\mu\lambda}^1 + (-if^{cfe})(-if^{edg})\text{Tr}[T^g T^a] \mathcal{M}_{25\mu\lambda}^2, \\
 \mathcal{M}_{L32\mu\lambda}^{ac} &= (-if^{ced})(-if^{age})\text{Tr}[T^f T^g] \mathcal{M}_{32\mu\lambda}^1 + (-if^{cfe})(-if^{agd})\text{Tr}[T^e T^g] \mathcal{M}_{32\mu\lambda}^2, \\
 \mathcal{M}_{L57\mu\lambda}^{ac} &= (-if^{feg})(-if^{ced})\text{Tr}[T^a T^g] \mathcal{M}_{57\mu\lambda}^1 + (-if^{cfe})(-if^{edg})\text{Tr}[T^a T^g] \mathcal{M}_{57\mu\lambda}^2, \\
 \mathcal{M}_{L64\mu\lambda}^{ac} &= (-if^{age})(-if^{ced})\text{Tr}[T^g T^f] \mathcal{M}_{64\mu\lambda}^1 + (-if^{cfe})(-if^{agd})\text{Tr}[T^e T^g] \mathcal{M}_{64\mu\lambda}^2, \\
 \mathcal{M}_{L71\mu\lambda}^{ac} &= (-if^{afe})(-if^{cgd})\text{Tr}[T^g T^e] \mathcal{M}_{71\mu\lambda}^1 + (-if^{cfe})(-if^{aeg})\text{Tr}[T^d T^g] \mathcal{M}_{71\mu\lambda}^2, \\
 \mathcal{M}_{L78\mu\lambda}^{ac} &= (-if^{afe})(-if^{cgd})\text{Tr}[T^g T^e] \mathcal{M}_{78\mu\lambda}^1 + (-if^{cfe})(-if^{aeg})\text{Tr}[T^g T^d] \mathcal{M}_{78\mu\lambda}^2, \\
 \mathcal{M}_{L85\mu\lambda}^{ac} &= (-if^{afe})(-if^{deg})\text{Tr}[T^c T^g] \mathcal{M}_{85\mu\lambda}^1 + (-if^{aeg})(-if^{fde})\text{Tr}[T^c T^g] \mathcal{M}_{85\mu\lambda}^2, \\
 \mathcal{M}_{L86\mu\lambda}^{ac} &= (-if^{afe})(-if^{deg})\text{Tr}[T^c T^g] \mathcal{M}_{86\mu\lambda}^1 + (-if^{aeg})(-if^{fde})\text{Tr}[T^c T^g] \mathcal{M}_{86\mu\lambda}^2,
 \end{aligned} \tag{A1}$$

where T^a is a generator of the $SU(N_c)$ group. $\mathcal{M}_{L\mu\lambda}^{(\pm)[\mp]ac}$ in the polarized cross section (17) are given by

$$\mathcal{M}_{L\mu\lambda}^{(+)[+]ac}(k_1, k_2 - k_1) = 0, \tag{A2}$$

$$\begin{aligned}
\mathcal{M}_{L\mu\lambda}^{(-)[+]\text{ac}}(k_1, k_2 - k_1) = & (-if^{cfe})[\text{Tr}[T^d T^e T^a] \mathcal{M}_{3\mu\lambda} + \text{Tr}[T^d T^e T^a] \mathcal{M}_{6\mu\lambda} + \text{Tr}[T^e T^d T^a] \mathcal{M}_{8\mu\lambda} + \text{Tr}[T^e T^d T^a] \mathcal{M}_{9\mu\lambda} \\
& + \text{Tr}[T^a T^e T^d] \mathcal{M}_{12\mu\lambda} + \text{Tr}[T^e T^a T^d] \mathcal{M}_{14\mu\lambda} + \text{Tr}[T^a T^d T^e] \mathcal{M}_{18\mu\lambda} + (-if^{edg})\text{Tr}[T^g T^a] \mathcal{M}_{20\mu\lambda} \\
& + (-if^{edg})\text{Tr}[T^a T^g] \mathcal{M}_{22\mu\lambda} + (-if^{edg})\text{Tr}[T^a T^g] \mathcal{M}_{24\mu\lambda} + (-if^{edg})\text{Tr}[T^g T^a] \mathcal{M}_{25\mu\lambda}^2 \\
& + (-if^{agd})\text{Tr}[T^e T^g] \mathcal{M}_{27\mu\lambda} + (-if^{agd})\text{Tr}[T^e T^g] \mathcal{M}_{29\mu\lambda} - (-if^{agd})\text{Tr}[T^e T^g] \mathcal{M}_{30\mu\lambda} \\
& + (-if^{agd})\text{Tr}[T^e T^g] \mathcal{M}_{32\mu\lambda}^2 - \text{Tr}[T^d T^a T^e] \mathcal{M}_{35\mu\lambda} + \text{Tr}[T^d T^a T^e] (\mathcal{M}_{38\mu\lambda} - \text{Tr}[T^a T^d T^e] \mathcal{M}_{40\mu\lambda} \\
& - \text{Tr}[T^a T^d T^e] \mathcal{M}_{41\mu\lambda} + \text{Tr}[T^a T^d T^e] \mathcal{M}_{44\mu\lambda} - \text{Tr}[T^a T^e T^d] \mathcal{M}_{46\mu\lambda} + \text{Tr}[T^a T^e T^d] \mathcal{M}_{50\mu\lambda} \\
& - (-if^{edg})\text{Tr}[T^a T^g] \mathcal{M}_{52\mu\lambda} + (-if^{edg})\text{Tr}[T^a T^g] \mathcal{M}_{55\mu\lambda} + (-if^{edg})\text{Tr}[T^a T^g] \mathcal{M}_{56\mu\lambda} \\
& + (-if^{edg})\text{Tr}[T^g T^a] \mathcal{M}_{57\mu\lambda}^2 + (-if^{adg})\text{Tr}[T^g T^e] \mathcal{M}_{59\mu\lambda} - (-if^{adg})\text{Tr}[T^g T^e] \mathcal{M}_{61\mu\lambda} \\
& + (-if^{adg})\text{Tr}[T^e T^g] \mathcal{M}_{62\mu\lambda} + (-if^{adg})\text{Tr}[T^e T^g] \mathcal{M}_{64\mu\lambda}^2 + (-if^{aeg})\text{Tr}[T^d T^g] \mathcal{M}_{70\mu\lambda} \\
& - (-if^{aeg})\text{Tr}[T^d T^g] \mathcal{M}_{71\mu\lambda}^2 - (-if^{adg})\text{Tr}[T^e T^g] \mathcal{M}_{77\mu\lambda} - (-if^{adg})\text{Tr}[T^e T^g] \mathcal{M}_{78\mu\lambda}^2], \tag{A3}
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_{L\mu\lambda}^{(-)[+]\text{ac}}(k_1, k_2 - k_1) = & \text{Tr}[T^c T^d T^f T^a] \mathcal{M}_{1\mu\lambda} + \text{Tr}[T^d T^c T^f T^a] \mathcal{M}_{2\mu\lambda} + \text{Tr}[T^a T^d T^c T^f] \mathcal{M}_{3\mu\lambda} + \text{Tr}[T^d T^a T^f T^c] \mathcal{M}_{4\mu\lambda} \\
& + (-if^{ced})\text{Tr}[T^a T^e T^f] \mathcal{M}_{5\mu\lambda} + \text{Tr}[T^c T^f T^d T^a] \mathcal{M}_{7\mu\lambda} + \text{Tr}[T^c T^f T^d T^a] \mathcal{M}_{8\mu\lambda} \\
& + (\text{Tr}[T^c T^f T^d T^a] - (-if^{ced})\text{Tr}[T^f T^e T^a]) \mathcal{M}_{9\mu\lambda} + (\text{Tr}[T^a T^f T^d T^c] - (-if^{aed})\text{Tr}[T^f T^e T^c]) \mathcal{M}_{10\mu\lambda} \\
& + (-if^{ced})\text{Tr}[T^a T^f T^e] (\mathcal{M}_{11\mu\lambda} + \text{Tr}[T^c T^f T^a T^d] \mathcal{M}_{13\mu\lambda} + \text{Tr}[T^c T^f T^a T^d] \mathcal{M}_{14\mu\lambda} \\
& + \text{Tr}[T^a T^f T^c T^d] \mathcal{M}_{15\mu\lambda} + \text{Tr}[T^a T^f T^d T^c] \mathcal{M}_{16\mu\lambda} + (-if^{ced})\text{Tr}[T^a T^f T^e] \mathcal{M}_{17\mu\lambda} \\
& + (-if^{fde})\text{Tr}[T^c T^e T^a] \mathcal{M}_{19\mu\lambda} + ((-if^{fde})\text{Tr}[T^e T^c T^a] - (-if^{cfe})(-if^{edg})\text{Tr}[T^g T^a]) \mathcal{M}_{20\mu\lambda} \\
& + ((-if^{fde})\text{Tr}[T^a T^e T^c] + (-if^{aed})(-if^{fge})\text{Tr}[T^g T^c]) \mathcal{M}_{21\mu\lambda} - (-if^{ced})(-if^{feg})\text{Tr}[T^g T^a] \mathcal{M}_{22\mu\lambda} \\
& + (-if^{ced})(-if^{feg})\text{Tr}[T^g T^a] \mathcal{M}_{23\mu\lambda} + (-if^{ced})(-if^{feg})\text{Tr}[T^g T^a] \mathcal{M}_{25\mu\lambda}^1 + (-if^{aed})\text{Tr}[T^c T^f T^e] \mathcal{M}_{26\mu\lambda} \\
& + (-if^{aed})\text{Tr}[T^c T^f T^e] \mathcal{M}_{27\mu\lambda} + (-if^{aed})\text{Tr}[T^f T^e T^c] \mathcal{M}_{28\mu\lambda} + \text{Tr}[T^c T^d T^f T^a] \mathcal{M}_{33\mu\lambda} \\
& + \text{Tr}[T^d T^c T^f T^a] \mathcal{M}_{34\mu\lambda} + \text{Tr}[T^c T^d T^a T^f] \mathcal{M}_{35\mu\lambda} + \text{Tr}[T^d T^a T^f T^c] \mathcal{M}_{36\mu\lambda} + (-if^{ced})\text{Tr}[T^a T^e T^f] \mathcal{M}_{37\mu\lambda} \\
& + (\text{Tr}[T^c T^d T^f T^a] + (-if^{aed})\text{Tr}[T^c T^e T^f]) \mathcal{M}_{39\mu\lambda} + (\text{Tr}[T^a T^c T^d T^f] + (-if^{cfe})\text{Tr}[T^a T^d T^e]) \mathcal{M}_{40\mu\lambda} \\
& + \text{Tr}[T^c T^a T^d T^f] \mathcal{M}_{41\mu\lambda} + \text{Tr}[T^a T^d T^f T^c] \mathcal{M}_{42\mu\lambda} + (-if^{ced})\text{Tr}[T^a T^e T^f] \mathcal{M}_{43\mu\lambda} + \text{Tr}[T^c T^f T^a T^d] \mathcal{M}_{45\mu\lambda} \\
& + \text{Tr}[T^a T^f T^c T^d] \mathcal{M}_{46\mu\lambda} + \text{Tr}[T^a T^f T^c T^d] \mathcal{M}_{47\mu\lambda} + \text{Tr}[T^a T^f T^d T^c] \mathcal{M}_{48\mu\lambda} + (-if^{ced})\text{Tr}[T^a T^f T^e] \mathcal{M}_{49\mu\lambda} \\
& + ((-if^{fde})\text{Tr}[T^c T^e T^a] - (-if^{aed})(-if^{fge})\text{Tr}[T^c T^g]) \mathcal{M}_{51\mu\lambda} \\
& + ((-if^{fde})\text{Tr}[T^c T^a T^e] + (-if^{feg})(-if^{ced})\text{Tr}[T^a T^g]) \mathcal{M}_{52\mu\lambda} + (-if^{fde})\text{Tr}[T^a T^e T^c] \mathcal{M}_{53\mu\lambda} \\
& + (-if^{ced})(-if^{feg})\text{Tr}[T^g T^a] \mathcal{M}_{54\mu\lambda} - (-if^{ced})(-if^{feg})\text{Tr}[T^g T^a] \mathcal{M}_{55\mu\lambda} \\
& + (-if^{ced})(-if^{feg})\text{Tr}[T^g T^a] \mathcal{M}_{57\mu\lambda}^1 + (-if^{aed})\text{Tr}[T^c T^e T^f] \mathcal{M}_{58\mu\lambda} + (-if^{aed})\text{Tr}[T^c T^e T^f] \mathcal{M}_{59\mu\lambda} \\
& + (-if^{aed})\text{Tr}[T^e T^f T^c] \mathcal{M}_{60\mu\lambda} + (-if^{aed})(-if^{fge})\text{Tr}[T^c T^g] \mathcal{M}_{81\mu\lambda} + (-if^{aed})(-if^{fge})\text{Tr}[T^g T^c] \mathcal{M}_{82\mu\lambda} \\
& + (-if^{aed})(-if^{fge})\text{Tr}[T^c T^g] \mathcal{M}_{83\mu\lambda} + (-if^{aed})(-if^{fge})\text{Tr}[T^g T^c] \mathcal{M}_{84\mu\lambda} \\
& + (-if^{aed})(-if^{fge})\text{Tr}[T^g T^c] \mathcal{M}_{85\mu\lambda}^1 + (-if^{aed})(-if^{fge})\text{Tr}[T^c T^g] \mathcal{M}_{85\mu\lambda}^2, \tag{A4}
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_{L\mu\lambda}^{(+)[-]ac}(k_1, k_2 - k_1) = & (-if^{afe})[\text{Tr}[T^d T^e T^c] \mathcal{M}_{4\mu\lambda} + \text{Tr}[T^e T^d T^c] \mathcal{M}_{10\mu\lambda} + \text{Tr}[T^e T^c T^d] \mathcal{M}_{15\mu\lambda} + \text{Tr}[T^e T^d T^c] \mathcal{M}_{16\mu\lambda} \\
& + (-if^{cgd})\text{Tr}[T^e T^g] \mathcal{M}_{17\mu\lambda} - (-if^{\text{deg}})\text{Tr}[T^g T^c] \mathcal{M}_{21\mu\lambda} + (-if^{cgd})\text{Tr}[T^e T^g] \mathcal{M}_{31\mu\lambda} \\
& + (-if^{cgd})\text{Tr}[T^e T^g] \mathcal{M}_{32\mu\lambda}^1 - \text{Tr}[T^c T^d T^e] \mathcal{M}_{33\mu\lambda} - \text{Tr}[T^d T^c T^e] \mathcal{M}_{34\mu\lambda} - (-if^{cgd})\text{Tr}[T^g T^e] \mathcal{M}_{37\mu\lambda} \\
& - \text{Tr}[T^c T^d T^e] \mathcal{M}_{39\mu\lambda} - \text{Tr}[T^c T^e T^d] \mathcal{M}_{45\mu\lambda} + (-if^{\text{deg}})\text{Tr}[T^c T^g] \mathcal{M}_{51\mu\lambda} + (-if^{cgd})\text{Tr}[T^g T^e] \mathcal{M}_{63\mu\lambda} \\
& + (-if^{cgd})\text{Tr}[T^g T^e] \mathcal{M}_{64\mu\lambda}^1 + \text{Tr}[T^c T^d T^e] \mathcal{M}_{65\mu\lambda} + \text{Tr}[T^d T^c T^e] \mathcal{M}_{66\mu\lambda} + \text{Tr}[T^d T^e T^c] \mathcal{M}_{67\mu\lambda} \\
& + (-if^{cgd})\text{Tr}[T^g T^e] \mathcal{M}_{68\mu\lambda} + (-if^{ceg})\text{Tr}[T^d T^g] \mathcal{M}_{69\mu\lambda} + (-if^{ceg})\text{Tr}[T^d T^g] \mathcal{M}_{71\mu\lambda}^1 \\
& + \text{Tr}[T^c T^e T^d] \mathcal{M}_{72\mu\lambda} + \text{Tr}[T^e T^c T^d] \mathcal{M}_{73\mu\lambda} + \text{Tr}[T^e T^d T^c] \mathcal{M}_{74\mu\lambda} + (-if^{cgd})\text{Tr}[T^e T^g] \mathcal{M}_{75\mu\lambda} \\
& + (-if^{ceg})\text{Tr}[T^g T^d] \mathcal{M}_{76\mu\lambda} + (-if^{ceg})\text{Tr}[T^g T^d] \mathcal{M}_{78\mu\lambda}^1 + (-if^{\text{deg}})\text{Tr}[T^c T^g] \mathcal{M}_{79\mu\lambda} \\
& + (-if^{\text{deg}})\text{Tr}[T^c T^g] \mathcal{M}_{80\mu\lambda} - (-if^{\text{deg}})\text{Tr}[T^c T^g] \mathcal{M}_{81\mu\lambda} - (-if^{\text{deg}})\text{Tr}[T^c T^g] \mathcal{M}_{82\mu\lambda} \\
& + (-if^{\text{deg}})\text{Tr}[T^c T^g] \mathcal{M}_{85\mu\lambda}^1 + (-if^{\text{deg}})\text{Tr}[T^c T^g] \mathcal{M}_{86\mu\lambda}^1].
\end{aligned} \tag{A5}$$

They satisfy the Ward-Takahashi identity,

$$\begin{aligned}
(k_2 - k_1)^\lambda \mathcal{M}_{L\mu\lambda}^{(\pm)[\pm]ac}(k_1, k_2 - k_1) &= \sum C_{Liac}^{(\pm)[\pm]} \mathcal{M}_{i\mu}(k_2), \\
k_1^\mu \mathcal{M}_{L\mu\lambda}^{(\pm)[\pm]ac}(k_1, k_2 - k_1) &= - \sum C_{Liac}^{(\pm)[\pm]} \mathcal{M}_{i\lambda}(k_2),
\end{aligned} \tag{A6}$$

where the color factors are given by

$$\begin{aligned}
C_{L1ac}^{(-)[+]} &= (-if^{cfe})\text{Tr}[T^d T^e T^a], & C_{L2ac}^{(-)[+]} &= (-if^{cfe})\text{Tr}[T^e T^d T^a], & C_{L3ac}^{(-)[+]} &= (-if^{cfe})\text{Tr}[T^e T^a T^d], \\
C_{L4ac}^{(-)[+]} &= (-if^{cfe})(-if^{edg})\text{Tr}[T^g T^a], & C_{L5ac}^{(-)[+]} &= (-if^{cfe})(-if^{edg})\text{Tr}[T^g T^a], & C_{L6ac}^{(-)[+]} &= (-if^{cfe})\text{Tr}[T^d T^a T^e], \\
C_{L7ac}^{(-)[+]} &= (-if^{cfe})\text{Tr}[T^a T^d T^e], & C_{L8ac}^{(-)[+]} &= (-if^{cfe})\text{Tr}[T^a T^e T^d], & C_{L9ac}^{(-)[+]} &= (-if^{cfe})(-if^{edg})\text{Tr}[T^a T^g], \\
C_{L10ac}^{(-)[+]} &= (-if^{cfe})(-if^{agd})\text{Tr}[T^e T^g], & C_{L11ac}^{(-)[+]} &= C_{L12ac}^{(-)[+]} = (-if^{cfe})(-if^{aeg})\text{Tr}[T^d T^g],
\end{aligned} \tag{A7}$$

$$\begin{aligned}
C_{L1ac}^{(+)[-]} &= -(-if^{afe})\text{Tr}[T^d T^e T^c], & C_{L2ac}^{(+)[-]} &= -(-if^{afe})\text{Tr}[T^e T^d T^c], & C_{L3ac}^{(+)[-]} &= -(-if^{afe})\text{Tr}[T^e T^c T^d], \\
C_{L4ac}^{(+)[-]} &= (-if^{afe})(-if^{\text{deg}})\text{Tr}[T^g T^c], & C_{L5ac}^{(+)[-]} &= -(-if^{afe})(-if^{cgd})\text{Tr}[T^e T^g], \\
C_{L6ac}^{(+)[-]} &= -(-if^{afe})\text{Tr}[T^d T^c T^e], & C_{L7ac}^{(+)[-]} &= -(-if^{afe})\text{Tr}[T^c T^d T^e], \\
C_{L8ac}^{(+)[-]} &= -(-if^{afe})\text{Tr}[T^c T^e T^d], & C_{L9ac}^{(+)[-]} &= (-if^{afe})(-if^{\text{deg}})\text{Tr}[T^c T^g], \\
C_{L10ac}^{(+)[-]} &= -(-if^{afe})(-if^{cgd})\text{Tr}[T^g T^e], & C_{L11ac}^{(+)[-]} &= C_{L12ac}^{(+)[-]} = -(-if^{afe})(-if^{ceg})\text{Tr}[T^d T^g],
\end{aligned} \tag{A8}$$

$$\begin{aligned}
C_{L1ac}^{(-)[-]} &= C_{L2ac}^{(-)[-]} = C_{L3ac}^{(-)[-]} = C_{L6ac}^{(-)[-]} = C_{L7ac}^{(-)[-]} = C_{L8ac}^{(-)[-]} = -\text{Tr}[T^d T^a T^f T^c] + \text{Tr}[T^a T^d T^c T^f], \\
C_{L4ac}^{(-)[-]} &= C_{L5ac}^{(-)[-]} = C_{L9ac}^{(-)[-]} = C_{L10ac}^{(-)[-]} = C_{L11ac}^{(-)[-]} = C_{L12ac}^{(-)[-]} = 0.
\end{aligned} \tag{A9}$$

The color factors in the polarized cross section (17) are obtained as

$$C_{Liac}^{(\pm)[\pm]} C_j^{b*} = C_{Lij}^{f(\pm)[\pm]} i f^{abc} + C_{Lij}^{d(\pm)[\pm]} d^{abc}. \tag{A10}$$

- [1] A. Accardi, J. L. Albacete, M. Anselmino, N. Armesto, E. C. Aschenauer, A. Bacchetta, D. Boer, W. K. Brooks, T. Burton, N. B. Chang *et al.*, *Eur. Phys. J. A* **52**, 268 (2016).
- [2] A. Adare *et al.* (PHENIX Collaboration), *Phys. Rev. Lett.* **103**, 012003 (2009); *Phys. Rev. D* **79**, 012003 (2009); L. Adamczyk *et al.* (STAR Collaboration), *Phys. Rev. D* **86**, 032006 (2012); P. Djawotho (STAR Collaboration), *Nuovo Cimento Soc. Ital. Fis.* **036C**, 35 (2013); A. Adare *et al.* (PHENIX Collaboration), *Phys. Rev. D* **90**, 012007 (2014).
- [3] D. de Florian, R. Sassot, M. Stratmann, and W. Vogelsang, *Phys. Rev. Lett.* **113**, 012001 (2014).
- [4] R. D. Klem, J. E. Bowers, H. W. Courant, H. Kagan, M. L. Marshak, E. A. Peterson, K. Ruddick, W. H. Dragoset, and J. B. Roberts, *Phys. Rev. Lett.* **36**, 929 (1976).
- [5] G. Bunce *et al.*, *Phys. Rev. Lett.* **36**, 1113 (1976).
- [6] G. L. Kane, J. Pumplin, and W. Repko, *Phys. Rev. Lett.* **41**, 1689 (1978).
- [7] R. Boussarie, M. Burkardt, M. Constantinou, W. Detmold, M. Ebert, M. Engelhardt, S. Fleming, L. Gamberg, X. Ji, Z. B. Kang *et al.*, [arXiv:2304.03302](https://arxiv.org/abs/2304.03302).
- [8] X. Ji, J. W. Qiu, W. Vogelsang, and F. Yuan, *Phys. Lett. B* **638**, 178 (2006).
- [9] X. Ji, J. W. Qiu, W. Vogelsang, and F. Yuan, *Phys. Rev. D* **73**, 094017 (2006).
- [10] Y. Koike, W. Vogelsang, and F. Yuan, *Phys. Lett. B* **659**, 878 (2008).
- [11] A. Bacchetta, D. Boer, M. Diehl, and P. J. Mulders, *J. High Energy Phys.* **08** (2008) 023.
- [12] F. Yuan and J. Zhou, *Phys. Rev. Lett.* **103**, 052001 (2009).
- [13] J. Zhou, F. Yuan, and Z. T. Liang, *Phys. Rev. D* **81**, 054008 (2010).
- [14] R. Ikarashi, Y. Koike, and S. Yoshida, *Phys. Rev. D* **113**, 074033 (2026).
- [15] J. Cammarota, L. Gamberg, Z.-B. Kang, J. A. Miller, D. Pitonyak, A. Prokudin, T. C. Rogers, and N. Sato (Jefferson Lab Angular Momentum Collaboration), *Phys. Rev. D* **102**, 054002 (2020).
- [16] J. W. Qiu, X. P. Wang, and H. Xing, *Chin. Phys. Lett.* **38**, 041201 (2021).
- [17] M. Anselmino, M. Boglione, U. D'Alesio, E. Leader, and F. Murgia, *Phys. Rev. D* **70**, 074025 (2004).
- [18] R. M. Godbole, A. Kaushik, and A. Misra, *Phys. Rev. D* **94**, 114022 (2016).
- [19] U. D'Alesio, F. Murgia, C. Pisano, and P. Tael, *Phys. Rev. D* **96**, 036011 (2017).
- [20] R. M. Godbole, A. Kaushik, and A. Misra, *Phys. Rev. D* **97**, 076001 (2018).
- [21] U. D'Alesio, C. Flore, F. Murgia, C. Pisano, and P. Tael, *Phys. Rev. D* **99**, 036013 (2019).
- [22] R. M. Godbole, A. Kaushik, A. Misra, and V. S. Rawoat, *Phys. Rev. D* **91**, 014005 (2015).
- [23] A. Mukherjee and S. Rajesh, *Eur. Phys. J. C* **77**, 854 (2017).
- [24] S. Rajesh, R. Kishore, and A. Mukherjee, *Phys. Rev. D* **98**, 014007 (2018).
- [25] H. Sun, Tichouk, and X. Luo, *Phys. Rev. D* **100**, 014007 (2019).
- [26] U. D'Alesio, F. Murgia, C. Pisano, and P. Tael, *Phys. Rev. D* **100**, 094016 (2019).
- [27] R. Kishore, A. Mukherjee, and S. Rajesh, *Phys. Rev. D* **101**, 054003 (2020).
- [28] U. D'Alesio, F. Murgia, C. Pisano, and S. Rajesh, *Eur. Phys. J. C* **79**, 1029 (2019).
- [29] U. D'Alesio, L. Maxia, F. Murgia, C. Pisano, and S. Rajesh, *Phys. Rev. D* **102**, 094011 (2020).
- [30] N. Kato, L. Maxia, and C. Pisano, *Phys. Rev. D* **110**, 034038 (2024).
- [31] Z. B. Kang and J. W. Qiu, *Phys. Rev. D* **78**, 034005 (2008).
- [32] Z. B. Kang, J. W. Qiu, W. Vogelsang, and F. Yuan, *Phys. Rev. D* **78**, 114013 (2008).
- [33] H. Beppu, Y. Koike, K. Tanaka, and S. Yoshida, *Phys. Rev. D* **82**, 054005 (2010).
- [34] Y. Koike and S. Yoshida, *Phys. Rev. D* **84**, 014026 (2011).
- [35] H. Beppu, Y. Koike, K. Tanaka, and S. Yoshida, *Phys. Rev. D* **85**, 114026 (2012).
- [36] L. Chen, H. Xing, and S. Yoshida, *Phys. Rev. D* **108**, 054021 (2023).
- [37] A. Schafer and J. Zhou, *Phys. Rev. D* **88**, 014008 (2013).
- [38] A. Adare *et al.* (PHENIX Collaboration), *Phys. Rev. D* **82**, 112008 (2010); **86**, 099904(E) (2012).
- [39] Y. Koike, K. Yabe, and S. Yoshida, *Phys. Rev. D* **101**, 054017 (2020).
- [40] X. D. Ji, *Phys. Lett. B* **289**, 137 (1992).
- [41] P. J. Mulders and J. Rodrigues, *Phys. Rev. D* **63**, 094021 (2001).
- [42] W. E. Caswell and G. P. Lepage, *Phys. Lett.* **167B**, 437 (1986).
- [43] G. T. Bodwin, E. Braaten, and G. P. Lepage, *Phys. Rev. D* **51**, 1125 (1995); **55**, 5853(E) (1997).
- [44] R. Baier and R. Ruckl, *Phys. Lett.* **102B**, 364 (1981).
- [45] S. Yoshida and D. Zheng, *Phys. Rev. D* **106**, 034019 (2022).
- [46] C. A. Aidala, A. Bacchetta, M. Boglione, G. Bozzi, V. Carassiti, M. Chiosso, R. Cimino, G. Ciullo, M. Contalbrigo, U. D'Alesio *et al.*, [arXiv:1901.08002](https://arxiv.org/abs/1901.08002).
- [47] M. Gluck, P. Jimenez-Delgado, and E. Reya, *Eur. Phys. J. C* **53**, 355 (2008).
- [48] H. Liu (PHENIX Collaboration), *AIP Conf. Proc.* **1149**, 439 (2009).
- [49] U. A. Acharya *et al.* (PHENIX Collaboration), *Phys. Rev. Lett.* **127**, 162001 (2021).
- [50] N. J. Abdulameer *et al.* (PHENIX Collaboration), *Phys. Rev. D* **107**, 052012 (2023).
- [51] V. M. Braun, A. N. Manashov, and B. Pirnay, *Phys. Rev. D* **80**, 114002 (2009); **86**, 119902(E) (2012).
- [52] Y. Hatta, K. Kanazawa, and S. Yoshida, *Phys. Rev. D* **88**, 014037 (2013).
- [53] E. C. Aschenauer, C. Aidala, A. Bazilevsky, M. Diehl, R. Fatemi, C. Gagliardi, Z. Kang, Y. V. Kovchegov, J. Jalilian-Marian, J. Lajoie *et al.*, [arXiv:1602.03922](https://arxiv.org/abs/1602.03922).