

Global coupled-channel analysis of $e^+e^- \rightarrow c\bar{c}$ processes in $\sqrt{s} = 3.75$ to 4.7 GeV

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(Received 8 February 2024; accepted 12 August 2025; published 15 September 2025)

Recent high-precision $e^+e^- \rightarrow c\bar{c}$ data from BESIII and Belle are highly useful to understand vector charmonium (ψ) pole structures and puzzling line shapes due to the exotic hadron candidates Y . We, thus, perform a global coupled-channel analysis of most of the available data (ten two-body, nine three-body, and one four-body final states) in $\sqrt{s} = 3.75 - 4.7$ GeV. Not only cross sections, but also invariant-mass distributions of subsystems are fitted. The $e^+e^- \rightarrow \mu^+\mu^-$ cross sections are also predicted. Our model includes dozens of (quasi-)two-body states that nonperturbatively couple with each other through bare ψ excitations, particle-exchange, and short-range mechanisms; approximate three-body unitarity is considered. The amplitudes obtained from the fit are analytically continued to ψ and Z_c poles. We find ψ states similar to those in the Particle Data Group listing and $Y(4320)$. Moreover, several ψ states, including new ones, are found close to open-charm thresholds. Trajectories and compositeness of the near-threshold poles suggest dominant hadron-molecule contents in their internal structures. Two Z_c poles are found as virtual states ~ 40 MeV below the $D^*\bar{D}^{(*)}$ thresholds, consistent with lattice QCD results. This work presents the first global analysis to determine ψ and Z_c poles, thereby paving the way to extracting detailed properties of the prominent exotic hadron candidates from data.

DOI: 10.1103/fch8-xwb8

I. INTRODUCTION

The Y sector of XYZ exotic hadrons was opened with the discovery of $Y(4260)$ by the BABAR Collaboration [1] in $e^+e^- \rightarrow \gamma_{\text{ISR}}\pi^+\pi^-J/\psi$ (γ_{ISR} is initial state radiation γ) and subsequent confirmations by the CLEO and Belle Collaborations [2,3].¹ The $Y(4260)$ has been considered an exotic state. One reason is for its peculiar decay patterns. Usually, charmonium states above open-charm thresholds dominantly decay into the open-charm channels. However, $Y(4260)$ signals were not seen in $e^+e^- \rightarrow (\text{open-charm channels})$ data but were seen in hidden-charm channels. Second, $Y(4260)$ does not have a quark-model counterpart

[5]. The discovery of Y continued: $Y(4360)$ in $e^+e^- \rightarrow \gamma_{\text{ISR}}\pi^+\pi^-\psi'$ by BABAR [6] and Belle [7] and $Y(4660)$ by Belle [7]. The exotic hadrons are considered a key to deepening our understanding of QCD and, thus, invited lots of studies; see reviews [8–15].

BESIII pursued the precision frontier of the Y sector with direct Y productions without γ_{ISR} and found that Y widths appear differently in their different decay modes (Y -width problem) [16–24]; see Fig. 4 in [24]. BESIII also found that $Y(4260)$ in $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ consists of $Y(4220)$ and $Y(4320)$ [25] and that $Y(4320)$ does not appear in other final states. $Y(4360)$ and $Y(4660)$ were confirmed with higher precision [20,26].

Actually, the process-dependent Y line shapes can be caused by process-dependent interferences between various charmonia and by kinematical effects such as threshold cusp and triangle singularity. Thus, the Y -width problem indicates the limitation of determining the resonance parameters by collecting results of single-channel analyses; a single-channel analysis determines resonance parameters from fitting only one process. We should understand the process-dependent Y line shapes by analyzing the different final states simultaneously with a unified coupled-channel model; there is no need to resort to the process-dependent Y widths.

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¹While we basically follow the particle name convention of the Particle Data Group (PDG) [4], we sometimes use historical names such as Y . Often, $D_1(2420)$, $D_1(2430)$, $D_2^*(2460)$, $D_0^*(2300)$, and $D_{s1}(2536)$ are simplified as D_1 , D_1' , D_2 , D_0 , and D_{s1} , respectively. Either or all of $D\bar{D}$, $D^*\bar{D}$, and $D^*\bar{D}^*$ are collectively denoted by $D^{(*)}\bar{D}^{(*)}$.

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The $Z_c^+(3900)$ is an outstanding exotic $c\bar{c}u\bar{d}$ candidate and was discovered in the $J/\psi\pi^+$ invariant-mass distribution of $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ [27,28]. Then, $Z_c^+(4020)$ was discovered in a study of $e^+e^- \rightarrow \pi^+\pi^-h_c$ [29]. Furthermore, Z_c^+ signals were also observed in the invariant-mass distributions of two-body subsystems in $e^+e^- \rightarrow D^*\bar{D}\pi$ [30], $D^*\bar{D}^*\pi$ [31], $\psi'\pi\pi$ [26], and $\eta_c\rho\pi$ [32]. The properties of Z_c and Y should be correlated, since Z_c appear as $Y \rightarrow Z_c\pi$. Inevitably, the above coupled-channel analysis considers the Z_c signals in the data and addresses their nature.

Regarding previous coupled-channel studies, Cleven *et al.* [33] fitted $e^+e^- \rightarrow J/\psi\pi^+\pi^-$, $h_c\pi^+\pi^-$, and $D\bar{D}^*\pi$ cross sections and invariant-mass line shapes to study $Y(4260)$. This pioneering work showed that the asymmetric $Y(4260)$ line shapes can be naturally explained with a $D_1\bar{D}$ -molecule scenario of $Y(4260)$. However, the data available at that time were rather scarce, compared to what we have today. Recently, von Detten *et al.* [34] performed a similar study including updated data and more final states in the $\psi(4230)$ region. To explain the process-dependent line shapes, they considered interferences between $\psi(4160)$ and $\psi(4230)$. They concluded that the data are consistent with the $D_1\bar{D}$ -molecule interpretation of $\psi(4230)$. However, as they noted, their study was still exploratory. Their model did not account for the unitarity that should be important to describe overlapping resonances. Also, they did not include data such as $e^+e^- \rightarrow D^*\bar{D}^*\pi$ [35] and $\psi'\pi\pi$ [20] in their fit. It remains to be seen whether the $D_1\bar{D}$ -molecule scenario for $\psi(4230)$ can also explain the $e^+e^- \rightarrow D^*\bar{D}^*\pi$ and $\psi'\pi\pi$ data showing $\psi(4230)$ signals.

Chen *et al.* [36] performed a Breit-Wigner fit to $e^+e^- \rightarrow D^*\bar{D}\pi$, $J/\psi\pi\pi$, and $h_c\pi\pi$ cross-section data and concluded that $Y(4320)$ and $Y(4390)$ signals in the data can be explained with interferences between $\psi(4160)$, $\psi(4230)$, and $\psi(4415)$. However, their analysis did not include $e^+e^- \rightarrow \psi'\pi\pi$ data [26], available at that time, that clearly show a $Y(4360/4390)$ signal. Zhou *et al.* [37] fitted $e^+e^- \rightarrow D^{(*)}\bar{D}^{(*)}$ and $D\bar{D}\pi$ data with a two-body unitary coupled-channel model. Their interesting conclusion is that $\psi(4160)$ and $\psi(4230)$ are the same state. However, one of the bare charmonium states in their model has an unreasonably small mass, which could be an artifact of not including nonresonant mechanisms. Also, the $D\bar{D}\pi$ data were not reasonably fitted. Overall, the previous analyses used rather limited datasets, considering the recent data discussed below. Accordingly, several major coupled channels and unitarity were not considered, which would question the reliability of their conclusions.

In the past several years, BESIII has accumulated high-quality data for various $e^+e^- \rightarrow c\bar{c}$ cross sections and their invariant-mass distributions over a wide energy region [16–26,29–32,35,38–48]; see Figs. 3–9. It is timely to analyze these data simultaneously with a coupled-channel framework and extract vector charmonium properties such

as their poles (masses and widths) and residues (coupling strengths to decay channels). These resonance properties are a primary basis to study the nature of Y and a prerequisite to understand the Y line shapes. They are also new information for well-established charmonia [$\psi(4040)$, $\psi(4160)$, and $\psi(4415)$], since their properties have been mainly from analyzing the inclusive ($e^+e^- \rightarrow \text{hadrons}$) data [49]. In this work, we perform such a global coupled-channel analysis over $\sqrt{s} = 3.75 - 4.7$ GeV for the first time and present the fit results and the vector charmonium and Z_c pole positions.

Possible interpretations of the internal structures of the Y states have been proposed. $\psi(4230)$ as a $D_1\bar{D}$ hadron molecule has been proposed in Refs. [50–54]. Similarly, Refs. [52,53] interpreted $\psi(4360)$ as a $D_1\bar{D}^*$ hadron molecule. Possible other interpretations of $\psi(4230)$ are a $c\bar{c}$ -gluon hybrid state [55–57] and a hadrocharmonium [58].² Our analysis model is flexible enough to capture the open-charm hadron-molecule structures of the vector charmonium states. The internal structures of the extracted states will be explored by examining the pole trajectories and the compositeness [59–61].

We also cross-check our coupled-channel model by comparing the model's prediction of $e^+e^- \rightarrow \mu^+\mu^-$ cross sections with data [62]. Since our model includes all major channels involving $c\bar{c}$ quarks, it provides the $c\bar{c}$ contribution to the vacuum polarization (VP) that occurs in $e^+e^- \rightarrow \mu^+\mu^-$. The light-hadron contribution to the VP is obtainable through a dispersion relation applied to the inclusive ($e^+e^- \rightarrow \text{hadrons}$) cross-section data [49] subtracted by our model's ($e^+e^- \rightarrow c\bar{c}$ -hadrons) cross sections.

The organization of this paper is as follows. In Sec. II, we describe our coupled-channel amplitudes for $e^+e^- \rightarrow c\bar{c}$ processes and cross-section formulas. In Sec. III, our fit results are presented, and reaction mechanisms are discussed. We compare the inclusive R value from our model with data in Sec. IV and predict $e^+e^- \rightarrow \mu^+\mu^-$ cross sections in Sec. V. In Sec. VI, we present vector-charmonium poles extracted from the coupled-channel amplitudes and examine their pole trajectories and compositeness. The Z_c poles are also presented. A summary is given in Sec. VII. Appendixes discuss two-body scattering models that are main building blocks of our coupled-channel model, our pole-uncertainty estimation methods, and model parameter values.

II. MODEL

A. $e^+e^- \rightarrow c\bar{c}$ reaction amplitudes from coupled-channel model

Our coupled-channel model for the e^+e^- annihilation processes is primarily based on the manifestly three-body

²Recent BESIII data indicate that the Y states substantially decay into open-charm channels, disfavoring the hadrocharmonium scenario. See related discussions in Ref. [14].

unitary formulation presented in Refs. [63–66]. For three-body final states, $e^+e^- \rightarrow abc$, the full amplitude [Figs. 1(a) and 1(b)] is given by³

$$A_{abc,e^+e^-} = \sum_{abc}^{\text{cyclic}} \sum_{RR'\bar{s}_R} \Gamma_{ab,R}(\mathbf{p}_a^*) \tau_{R,R'}(p_c, E - E_c) \times \left[\sum_{ij} \bar{\Gamma}_{R'c,\psi_i}^{\mu}(\mathbf{p}_c, E) \bar{G}_{ij}(E) \bar{\Gamma}_{\psi_j,\gamma^*}(E) + \bar{\Gamma}_{R'c,\gamma^*}^{\mu}(\mathbf{p}_c, E) \right] \frac{1}{s} l_{\mu}, \quad (1)$$

where the first and second terms in the square bracket are resonant (ψ) and nonresonant (NR) parts, respectively. The symbol R is a two-meson resonance such as $D_1(2420)$; cyclic permutations (abc) , (cab) , and (bca) are indicated by $\sum_{abc}^{\text{cyclic}}$; ψ_i indicates the i th bare ψ state; $E(=\sqrt{s})$ denotes the abc invariant mass. The amplitude includes $R \rightarrow ab$ vertex $\Gamma_{ab,R}$, dressed R propagator $\tau_{R,R'}$ [Fig. 1(c)], dressed $\psi \rightarrow Rc$ vertex $\bar{\Gamma}_{Rc,\psi}^{\mu}$ [Fig. 1(f)], dressed NR Rc production mechanism $\bar{\Gamma}_{Rc,\gamma^*}^{\mu}$ [Fig. 1(g)], dressed ψ production mechanism $\bar{\Gamma}_{\psi,\gamma^*}$ [Fig. 1(h)], and dressed ψ propagator $\bar{G}_{ij}$ [Fig. 1(i)]. The virtual photon propagator is $1/s$, and the lepton current matrix element is $l_{\mu}(=e\bar{v}_{e^+}\gamma_{\mu}u_{e^-})$. Amplitudes for two-body final states (A_{ab,e^+e^-}) are obtained from Eq. (1) by removing $\Gamma_{ab,R}\tau_{R,R'}$ and identifying $R'c$ with ab .

We consider Rc channels summarized in Table I. These channels are understood to be negative C -parity ($C = -1$) states. Taking a convention of $D_J \xrightarrow{C} \bar{D}_J$ for all charmed mesons D_J , we use a $C = -1$ base for an open-charm channel as

$$\frac{1}{\sqrt{2}}(D_J \bar{D}_{J'} - \bar{D}_J D_{J'}), \quad (2)$$

where $D_J \neq D_{J'}$ and $m_{D_J} > m_{D_{J'}}$. We group the Rc channels into (A)–(C) in Table I. A (bare) R state is excited in a partial-wave two-body scattering as $ab \rightarrow R \rightarrow a'b'$. The partial wave is specified by $\{L, I\}$ or $IJ^{P(C)}$, where L , I , J , and P are the orbital angular momentum, total isospin, total angular momentum, and parity of the ab (or $a'b'$) system, respectively. Models for these $ab \rightarrow a'b'$ two-body scatterings are basic building blocks of the three-body $e^+e^- \rightarrow c\bar{c}$ reaction model described in this subsection and are discussed in detail in Appendix A. In particular, numerical values for $R \rightarrow ab$ couplings $g_{ab,R}^{LS}$, cutoffs

³We denote a particle x 's mass, momentum, energy, width, and spin state in the abc center-of-mass (c.m.) frame by $m_x, \mathbf{p}_x, E_x, \Gamma_x$, and s_x^z , respectively; $E_x = \sqrt{m_x^2 + p_x^2}$ with $p_x^2 = |\mathbf{p}_x|^2$. The mass and width values are from the PDG [4]. Our model is isospin symmetric, and the averaged mass is used for isospin partners.

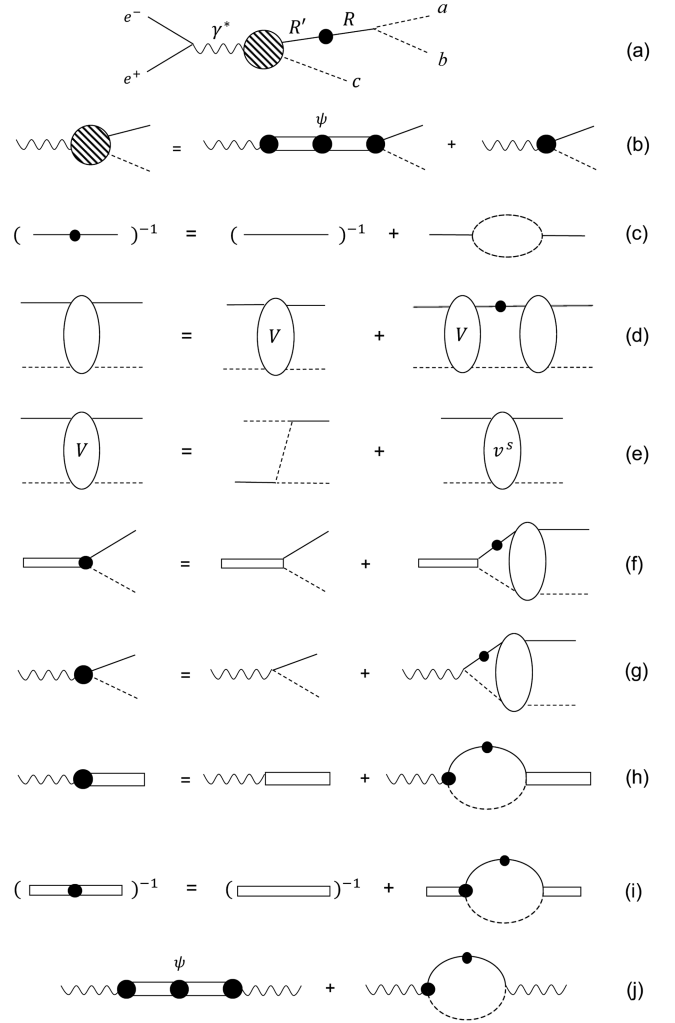


FIG. 1. (a) $e^+e^- \rightarrow abc$ mechanism in our model. The dashed lines represent stable particles, and abc are final three-body states shown in Figs. 3 and 4. The solid lines are bare [Breit-Wigner] resonance states R listed in Table I(C) [Table I(A) and I(B)]. (b) Resonant and nonresonant mechanisms. The double line represents bare charmonium (ψ) states. (c) Dressed R propagator: the first [second] diagram is a bare R propagator [self-energy]. (d) Lippmann-Schwinger-like equation for Rc scattering driven by V . The white oval is a $Rc \rightarrow R'c'$ scattering amplitude. (e) Rc interactions V from particle-exchange and short-range (v^s) mechanisms. (f) Dressed ψ decay vertex. (g) Dressed nonresonant Rc photoproduction vertex. (h) Dressed ψ photoproduction vertex. In (f)–(h), the first [second] diagram is a bare vertex [rescattering term]. (i) Dressed ψ propagator: the first [second] diagram is a bare ψ propagator [self-energy]. (j) Charm vacuum polarization (for $e^+e^- \rightarrow \mu^+\mu^-$, not for $e^+e^- \rightarrow abc$).

$c_{ab,R}$, and (bare) R masses m_R are determined from analyzing two-body data and given in Appendix A; $g_{ab,R}^{LS}$ and $c_{ab,R}$ will appear in Eq. (4) and m_R in Eqs. (5) and (8). For groups (A) and (B), the R propagations are described in a Breit-Wigner (BW) form. For group (B), we do not consider $R \rightarrow ab$ couplings. For group (C), $R = D_0^*(2300)$,

TABLE I. Quasi-two-body (Rc) coupled channels with $J^{PC} = 1^{--}$. See the text for grouping (A)–(C).

(A)	$D_1(2420)\bar{D}^{(*)}$, $D_1(2430)\bar{D}^{(*)}$, $D_2^*(2460)\bar{D}^{(*)}$, $D^{(*)}\bar{D}^{(*)}$, $D_{s1}(2536)\bar{D}_s$
(B)	$D_s^{(*)}\bar{D}_s^{(*)}$, $J/\psi\eta$, $J/\psi\eta'$, $\omega\chi_{c0}$, $\Lambda_c\bar{\Lambda}_c$
(C)	$D_0^*(2300)\bar{D}^*$, f_0J/ψ , f_2J/ψ , $f_0\psi'$, f_0h_c , $Z_c\pi$, $Z_{cs}\bar{K}$

$f_{0(2)}$, and $Z_{c(s)}$ indicate bare states that are dressed to form poles in unitary coupled-channel scattering amplitudes for $\{L, I\} = \{0, 1/2\}$ $D\pi$, $\{0(2), 0\}$ $\pi\pi - K\bar{K}$, and $IJ^{PC} = 11^{+-}$ $D^*\bar{D} - D^*\bar{D}^* - J/\psi\pi - \psi'\pi - h_c\pi - \eta_c\rho$ ($IJ^P = \frac{1}{2}1^+$ $D_s^*\bar{D} - D_s\bar{D}^* - J/\psi K$), respectively. We refer to the amplitudes as the $D_0^*(2300)$, $f_{0(2)}$, and $Z_{c(s)}$ amplitudes, respectively.

The $R \rightarrow ab$ vertex is given by

$$\Gamma_{ab,R}(\mathbf{p}_a^*) = (t_a t_b^z t_b^z | t_R t_R^z) \sum_{LL^z SS^z} (s_a s_a^z s_b s_b^z | SS^z) \times (LL^z SS^z | s_R s_R^z) Y_{LL^z}(\hat{\mathbf{p}}_a^*) \times \sqrt{\frac{E_a(p_a^*) E_b(p_a^*)}{E_a(p_a) E_b(p_b)}} f_{ab,R}^{LS}(p_a^*), \quad (3)$$

where $\mathbf{p}_a^*$ is the momentum of a particle a in the ab c.m. frame and $p_a^* = |\mathbf{p}_a^*|$. The spherical harmonics is denoted by $Y_{LL^z}(\hat{\mathbf{p}}_a^*)$ with $\hat{\mathbf{p}}_a^* = \mathbf{p}_a^*/|\mathbf{p}_a^*|$. The parentheses are Clebsch-Gordan (CG) coefficients, where t_x and t_x^z are the isospin of a particle x and its z component, respectively. The total spin of ab is denoted by S . We also used the vertex function

$$f_{ab,R}^{LS}(q) = \frac{g_{ab,R}^{LS}}{\sqrt{4E_a(q)E_b(q)}} \frac{q^L/m_\pi^{L-1}}{(1 + q^2/c_{ab,R}^2)^{2+\frac{L}{2}}}, \quad (4)$$

where $g_{ab,R}^{LS}$ and $c_{ab,R}$ are the coupling constant and cutoff, respectively, as already discussed; the factor m_π^{L-1} is introduced just for making the coupling constant dimensionless. The phase of $g_{\bar{a}\bar{b},\bar{R}}^{LS}$ (charge-conjugate partner of $g_{ab,R}^{LS}$) is fixed to give $\Gamma_{ab,R} = \Gamma_{\bar{a}\bar{b},\bar{R}}$ in Eq. (3); for example, $g_{D\pi,D^*}^{11} = -g_{\bar{D}\pi,\bar{D}^*}^{11}$. Also in Eq. (3), $\Gamma_{ab,R} = -\Gamma_{ba,R}$ can happen due to the CG coefficients. To avoid this, our rule of ordering ab in $\Gamma_{ab,R}$ is to satisfy $m_a > m_b$. For a $m_a = m_b$ case ($ab = K\bar{K}$), the ordering is particle-antiparticle.

The dressed R propagator [Fig. 1(c)], used as the dressed Rc Green function in Eq. (1), is given for R listed in Table I (C) by

$$[\tau^{-1}(p, E)]_{R,R'} = (W_R^2 - m_R^2) \delta_{R,R'} - \Sigma_{R,R'}(p, E), \quad (5)$$

with $W_R^2 = E^2 - p^2$ and m_R being a bare mass of R . We also introduced the R self-energy given as (see Appendix of Ref. [67] for derivation)

$$\Sigma_{R,R'}(p, E) = \sum_{ab,LS} \mathcal{B}_{ab}(t_a t_b^z t_b^z | t_R, t_a^z + t_b^z)^2 \int q^2 dq \times \frac{M_{ab}(q)}{\sqrt{M_{ab}^2(q) + p^2}} f_{R,ab}^{LS}(q) f_{ab,R'}^{LS}(q) \times \left\{ \frac{1}{E - \sqrt{M_{ab}^2(q) + p^2} + i\frac{\Gamma_a}{2} + i\frac{\Gamma_b}{2}} - \frac{1}{E + \sqrt{M_{ab}^2(q) + p^2}} \right\}, \quad (6)$$

with $M_{ab}(q) = E_a(q) + E_b(q)$; $s_R = s_{R'}$ is implied in Eq. (6). Because of the Bose symmetry, we have a factor $\mathcal{B}_{ab}$: $\mathcal{B}_{ab} = 1/2$ for identical particles a and b ; $\mathcal{B}_{ab} = 1$ otherwise. The width of a particle $a(b)$ is denoted by $\Gamma_{a(b)}$. In our model, $\Gamma_a = \Gamma_b = 0$ applies to most channels (only the $\eta_c\rho$ channel in the Z_c amplitude has $\Gamma_\rho = 150$ MeV), and, in such cases, the above formula reduces to

$$\Sigma_{R,R'}(W_R^2) = \sum_{ab,LS} \mathcal{B}_{ab}(t_a t_b^z t_b^z | t_R, t_a^z + t_b^z)^2 \int q^2 dq \times \frac{2M_{ab}(q) f_{R,ab}^{LS}(q) f_{ab,R'}^{LS}(q)}{W_R^2 - M_{ab}^2(q) + i\epsilon}. \quad (7)$$

Meanwhile, for R states listed in Table I(A) and (B), we use BW propagators to make the calculations more tractable:

$$\tau_{R,R}(p, E) = \frac{1}{2E_R(p)} \frac{1}{E - E_R(p) + i\frac{\Gamma_R}{2}}, \quad (8)$$

where m_R (hidden in E_R) and Γ_R are the BW mass and width for R , respectively. The use of the BW forms causes a partial violation of three-body unitarity.

The dressed $\psi_i \rightarrow Rc$ vertex [Fig. 1(f)] is given as

$$\bar{\Gamma}_{Rc,\psi_i}^\mu(\mathbf{p}_c, E) = x_{Rc}(t_R t_R^z t_c t_c^z | l t_R^z + t_c^z) \sum_{sll^z} (s_R s_R^z s_c s_c^z | ss_R^z + s_c^z) (ll^z ss_R^z + s_c^z | J\mu) \times Y_{l,l^z}(-\hat{\mathbf{p}}_c) \bar{F}_{(Rc)_{ls},\psi_i}(p_c, E), \quad (9)$$

where $l(s)$ is the relative orbital angular momentum (total spin) of Rc . For the present case, the total angular momentum is $J = 1$ and the total isospin $I = 0$, and, thus, IJ indices are suppressed in the notation $(Rc)_{ls}$. The Rc state in Eq. (9) decays into a final abc state, as indicated in Eq. (1). A factor x_{Rc} is introduced in Eq. (9). If either $D_J \bar{D}_{J'}$ or $\bar{D}_J D_{J'}$ (but not both) from Eq. (2) decays into abc , $x_{Rc} = +1/\sqrt{2}$ for $D_J \bar{D}_{J'}$ and $-1/\sqrt{2}$ for $\bar{D}_J D_{J'}$; $x_{Rc} = 1$ otherwise. Similarly, in Eq. (3) with $R = Z_c$, we should have included a factor of $+1/\sqrt{2}$ and $-1/\sqrt{2}$ for

$ab = D^*\bar{D}$ and $\bar{D}^*D$, respectively, since the Z_c amplitude includes the $(D^*\bar{D} - \bar{D}^*D)/\sqrt{2}$ channel.

The dressed $\psi_i \rightarrow (Rc)_{ls}$ form factor in Eq. (9) is

$$\begin{aligned} \bar{F}_{(Rc)_{ls},\psi_i}(p_c, E) &= F_{(Rc)_{ls},\psi_i}(p_c) + \sum_{c'R'R''l's'} \int q^2 dq \\ &\times X_{(Rc)_{ls},(R''c')_{l's'}}(p_c, q; E) \\ &\times \tau_{R'',R'}(q, E - E_{c'}) F_{(R'c')_{l's'},\psi_i}(q), \quad (10) \end{aligned}$$

where the first and second terms are direct decay and rescattering mechanisms, respectively. We use bare dipole form factors parametrized as

$$F_{(Rc)_{ls},\psi_i}(q) = \frac{C_{(Rc)_{ls}}^i}{\sqrt{4E_c(q)m_{\psi_i}}} \frac{q^l/m_\pi^{l-1}}{[1 + q^2/(\Lambda_{(Rc)_{ls}}^i)^2]^{2+\frac{l}{2}}}, \quad (11)$$

where $C_{(Rc)_{ls}}^i$, $\Lambda_{(Rc)_{ls}}^i$, and m_{ψ_i} are the coupling constant, cutoff, and bare ψ_i mass, respectively. We have also

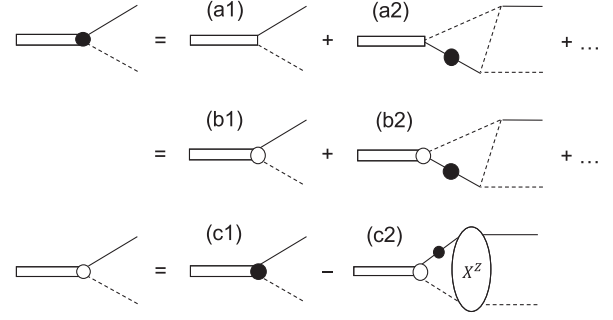


FIG. 2. Expansion of dressed ψ (γ^*) decay vertex [Figs. 1(f) and 1(g)]. (a1) Direct decay; (a2) single-triangle rescattering; (b1) partially dressed (PD) decay; (b2) PD single-triangle rescattering. (c1), (c2) Definition of the PD decay; X^Z refers to a scattering amplitude driven solely by Z diagrams.

introduced the $Rc \rightarrow R'c'$ partial wave ($IJ^{PC} = 01^{--}$) amplitude $X_{(Rc)_{ls},(R'c')_{l's'}}$ that is obtained by solving the scattering equation [Fig. 1(d)]:

$$\begin{aligned} X_{(R'c')_{l's'},(Rc)_{ls}}(p', p; E) &= V_{(R'c')_{l's'},(Rc)_{ls}}(p', p; E) + \sum_{c''R''R'''l''s''} \int q^2 dq V_{(R'c')_{l's'},(R''c'')_{l''s''}}(p', q; E) \\ &\times \tau_{R''',R''}(q, E - E_{c''}) X_{(R''c'')_{l''s''},(Rc)_{ls}}(q, p; E), \quad (12) \end{aligned}$$

with driving terms [Fig. 1(e)]

$$\begin{aligned} V_{(R'c')_{l's'},(Rc)_{ls}}(p', p; E) &= Z_{(R'c')_{l's'},(Rc)_{ls}}^{\bar{c}}(p', p; E) \\ &+ v_{(R'c')_{l's'},(Rc)_{ls}}^s(p', p). \quad (13) \end{aligned}$$

The so-called Z diagram $Z_{(R'c')_{l's'},(Rc)_{ls}}^{\bar{c}}$ is shown by the first term of the rhs in Fig. 1(e); $\bar{c}$ indicates a potentially on-shell exchanged particle. These long-range interactions couple the Rc channels listed in Table I(A) and (C) but not (B). The formulas and a list of the considered Z diagrams are given in Appendix B and Table XIV therein. We also consider short-range potentials v^s between open-charm channels; see Appendix A 3 for details. In Refs. [50,51], the ρ -, ω -, and σ -exchange mechanisms are considered to generate $\psi(4230)$ as a $D_1\bar{D}$ hadron-molecule state. The interactions v^s can simulate the sum of such meson-exchange mechanisms. Also, the resummation of v^s and the consequent pole formations significantly enhance threshold cusps, which will play an important role in fitting the data.

The dressed NR Rc production amplitude [Fig. 1(g)] is obtained from Eqs. (9) and (10) by replacing the labels “ ψ_i ” with “ γ^* ” and using the form factor below:

$$F_{(Rc)_{ls},\gamma^*}(q) = \frac{C_{(Rc)_{ls}}^{\gamma^*}}{\sqrt{2E_c(q)}} \frac{q^l/m_\pi^{l-1}}{[1 + q^2/(\Lambda_{(Rc)_{ls}}^{\gamma^*})^2]^{2+\frac{l}{2}}}. \quad (14)$$

For $Rc = D_0^*(2300)\bar{D}^*$, $Z_c\pi$, and $Z_{cs}\bar{K}$ in Table I(C), we assume that these channels are generated through rescatterings via the Z diagrams and set $C_{(Rc)_{ls}}^{i(\gamma^*)} = 0$. Also, we do not enforce SU(3) relations, for example, between $C_{(D^{(*)}\bar{D}^{(*)})_{ls}}^{i(\gamma^*)}$ and $C_{(D_s^{(*)}\bar{D}_s^{(*)})_{ls}}^{i(\gamma^*)}$.

For later purposes, we define several terminologies related to reaction mechanisms. We expand the above dressed vertices of Eqs. (9) and (10) [Figs. 1(f) and 1(g)] into “direct decay” [Fig. 2(a1)] and “single-triangle rescattering” [Fig. 2(a2)] terms. The direct decay mechanisms have been defined below Eq. (10). The single-triangle rescattering terms are obtained by expanding the rescattering terms of Eq. (10) using Eqs. (12) and (13) as

$$\begin{aligned} \sum_{c'R'R''l's'} \int q^2 dq Z_{(Rc)_{ls},(R''c')_{l's'}}^{\bar{c}}(p_c, q; E) \\ \times \tau_{R'',R'}(q, E - E_{c'}) F_{(R'c')_{l's'},\psi_i}(q). \quad (15) \end{aligned}$$

The “partially dressed (PD) decay” mechanisms [Fig. 2(b1)] are obtained from the dressed vertex of Eq. (10) by removing all terms in which the last interaction is a Z mechanism; see Figs. 2(c1) and 2(c2). This subset of the mechanisms encompasses the resummed short-range potentials v^s in Eq. (13). Finally, the “PD single-triangle rescattering”

mechanisms [Fig. 2(b2)] are obtained from Eq. (15) by replacing the bare form factor F with the above-defined PD decay amplitude.

The dressed ψ production amplitude [Fig. 1(h)] is given as

$$\bar{\Gamma}_{\psi_i, \gamma^*}(E) = \Gamma_{\psi_i, \gamma^*} + \sum_{cRR'ls} \int q^2 dq F_{(Rc)ls, \psi_i}(q) \times \tau_{R,R'}(q, E - E_c) \bar{F}_{(R'c)ls, \gamma^*}(q), \quad (16)$$

with a bare $\gamma^* \rightarrow \psi_i$ amplitude $\Gamma_{\psi_i, \gamma^*}$:

$$\Gamma_{\psi_i, \gamma^*} = \frac{1}{\sqrt{2m_{\psi_i}}} \frac{em_{\psi_i}^2}{g_{\psi_i}}, \quad (17)$$

and g_{ψ_i} is a coupling parameter. The dressed ψ propagator [Fig. 1(i)] is given by

$$[\bar{G}^{-1}(E)]_{ij} = (E - m_{\psi_i})\delta_{ij} - [\Sigma_{\psi}(E)]_{ij}, \quad (18)$$

where the ψ self-energy in the second term is given by

$$[\Sigma_{\psi}(E)]_{ij} = \sum_{cRR'ls} \int q^2 dq F_{(Rc)ls, \psi_i}(q) \tau_{R,R'}(q, E - E_c(q)) \times \bar{F}_{(R'c)ls, \psi_j}(q, E). \quad (19)$$

For fitting the $e^+e^- \rightarrow c\bar{c}$ data in $\sqrt{s} \leq 4.7$ GeV, we need to include resonances heavier than 4.6 GeV such as $\psi(4660)$ [20] and $\psi(4710)$ [18]. See the structure at $\sqrt{s} \sim 4.66$ GeV in Fig. 4(f), for example. However, the currently available data in the $\sqrt{s} > 4.6$ GeV region are insufficient to include these states in the coupled-channel framework. More data in $\sqrt{s} > 4.6$ GeV are necessary, including charm-strange final states such as $D^{(*)}\bar{D}_s^{(*)}K$. Thus, we consider $\psi(4660)$ - and $\psi(4710)$ -excitation amplitudes of the BW form. Specifically, in the square bracket in Eq. (1), we include additional terms of

$$\sum_{Y=\psi(4660), \psi(4710)} e^{i\phi_Y} \frac{\bar{\Gamma}_{R'c,Y}^{\mu}(\mathbf{p}_c) \Gamma_{Y, \gamma^*}}{E - m_Y + i\frac{\Gamma_Y}{2}}, \quad (20)$$

where m_Y and Γ_Y are the BW mass and width, respectively, and ϕ_Y adjusts the amplitude phase. The production and decay vertices (Γ_{Y, γ^*} and $\bar{\Gamma}_{R'c,Y}^{\mu}$) are given in Eqs. (17) and (9), respectively.

The formulas in this subsection are based on two-body interactions via bare R excitations, $ab \rightarrow R \rightarrow a'b'$. For a case where two-body interactions also include separable contact interactions, we can extend the above formulas in a straightforward manner, as detailed in Ref. [65]. In particular, the $Z_{c(s)}$ amplitude is solely from a set of contact interactions, without R excitations.

The vertex functions in Eqs. (3) and (9) can be related to matrix elements of a Hermitian interaction Hamiltonian by multiplying the imaginary unit i to Eq. (3) [Eq. (9)] when $s_a + s_b + L + s_R$ [$s_R + s_c + l + J$] is odd. Or, for our convenience, $-i$ is multiplied to Eq. (9) for $Rc = D_2^*(2460)\bar{D}$ and $D^*\bar{D}$. In addition, the factor of $1/\sqrt{2E_R}$ needs to be moved from $\tau_{R,R}$ in Eqs. (5) or (8) to the vertex functions. When considering the imaginary unit in the vertex functions as above, we must consistently multiply i [$-i$] to the rhs in Eq. (B2) when $s_R + s_c + l + J$ [$s_{R'} + s_{c'} + l' + J$] is odd; $i \leftrightarrow -i$ for $R^{(l)}c^{(l')} = D_2^*(2460)\bar{D}$ and $D^*\bar{D}$. Then, no change happens to our results including parameter values.

B. Cross-section formulas

The cross section for a three-meson (abc) production from an e^+e^- annihilation ($e^+e^- \rightarrow abc$) is given by

$$d\sigma_{e^+e^- \rightarrow abc} = \sum_{if} \mathcal{B} \frac{(2\pi)^4 \delta^{(4)}(p_i - p_f)}{4v_{\text{rel}} E_{e^+} E_{e^-}} |\mathcal{M}_{fi}|^2 (2m_e)^2 \times \frac{d^3 p_a}{(2\pi)^3 2E_a} \frac{d^3 p_b}{(2\pi)^3 2E_b} \frac{d^3 p_c}{(2\pi)^3 2E_c}, \quad (21)$$

with a Bose factor $\mathcal{B} = 1/3!$ for three identical particles abc , $\mathcal{B} = 1/2!$ for two identical particles among abc , and $\mathcal{B} = 1$ otherwise; $\sum_{if}$ indicates the average (sum) of initial (final) spin states. The invariant amplitude $\mathcal{M}_{fi}$ is related to Eq. (1) by

$$\mathcal{M}_{fi} = -(2\pi)^3 \sqrt{8E_a E_b E_c} A_{abc, e^+e^-} = \tilde{\mathcal{M}}_{abc}^{\mu} \frac{1}{s} l_{\mu}. \quad (22)$$

Then, the cross section in the total c.m. frame can be written as

$$d\sigma_{e^+e^- \rightarrow abc} = \sum_f \mathcal{B} \frac{\alpha}{512\pi^3 s^3} (|\tilde{\mathcal{M}}_{abc}^x|^2 + |\tilde{\mathcal{M}}_{abc}^y|^2) \times dm_{ab}^2 dm_{ac}^2 d\cos\theta_c d\bar{\phi}_a, \quad (23)$$

where α is the fine structure constant and the z axis is taken along the e^+e^- beam direction; m_{ab} and m_{ac} are the invariant masses of the ab and ac subsystems, respectively; θ_c is the polar angle of c in the total c.m. frame; $\bar{\phi}_a$ is the azimuthal angle of a in the ab c.m. frame, relative to the azimuthal angle of c .

Similarly, for a two-meson (ab) production, the invariant amplitude is

$$\begin{aligned}\mathcal{M}_{fi} &= -\sqrt{(2\pi)^3 4E_a E_b A_{ab,e^+e^-}} \\ &= \tilde{\mathcal{M}}_{ab}^\mu \frac{1}{s} l_\mu,\end{aligned}\quad (24)$$

and its cross section in the total c.m. frame is

$$\begin{aligned}d\sigma_{e^+e^- \rightarrow ab} &= \sum_{if} \mathcal{B} \frac{(2\pi)^4 \delta^{(4)}(p_i - p_f)}{4v_{\text{rel}} E_{e^+} E_{e^-}} |\mathcal{M}_{fi}|^2 (2m_e)^2 \\ &\times \frac{d^3 p_a}{(2\pi)^3 2E_a} \frac{d^3 p_b}{(2\pi)^3 2E_b} \\ &= \sum_f \frac{\mathcal{B} \alpha p_a}{8(\sqrt{s})^5} (|\tilde{\mathcal{M}}_{ab}^x|^2 + |\tilde{\mathcal{M}}_{ab}^y|^2) d\cos\theta_a.\end{aligned}\quad (25)$$

For $e^+e^- \rightarrow \Lambda_c \bar{\Lambda}_c$, the $\Lambda_c \bar{\Lambda}_c$ scattering due to the Coulomb force may significantly enhance the cross section near the threshold. We, thus, multiply Eq. (25) by the Sommerfeld factor [68,69]:

$$\frac{\pi\alpha}{\beta} \frac{\sqrt{1-\beta^2}}{1 - \exp(-\pi\alpha\sqrt{1-\beta^2}/\beta)}, \quad (26)$$

with $\beta = p_{\Lambda_c}/E_{\Lambda_c}$ in the c.m. frame. Because of this factor, the $e^+e^- \rightarrow \Lambda_c \bar{\Lambda}_c$ cross section is nonzero at the threshold.

For $e^+e^- \rightarrow \eta_c \rho \pi$ followed by $\rho \rightarrow \pi\pi$, the $e^+e^- \rightarrow \eta_c \rho \pi$ cross section of Eq. (23) is multiplied by the ρ propagator and ρ -decay vertex. See Eq. (10) in Ref. [70] for a similar formula.

Near thresholds, the available phase spaces in our isospin-symmetric model can be non-negligibly different from those in data. To correct this, for $e^+e^- \rightarrow D^{*+} D^{*-}$ as an example, the phase-space factor in Eq. (25) is calculated after modifying $\sqrt{s}$ as

$$\sqrt{s} \rightarrow \sqrt{s} + 2(m_{D_{\text{iso}}^*} - m_{D^{*+}}), \quad (27)$$

with $m_{D_{\text{iso}}^*} = (m_{D^{*+}} + m_{D^{*0}})/2$. The amplitude $\mathcal{M}_{fi}$ is calculated with the unmodified $\sqrt{s}$.

III. FIT RESULTS

A. Fitting parameters, fitting method, χ^2

Our coupled-channel model is fitted to $e^+e^- \rightarrow c\bar{c}$ cross-section data (20 final states) as shown in Figs. 3 and 4 and also to currently available invariant-mass and angular distribution data shown in Figs. 5–9. We include five bare ψ states that are minimally needed to obtain a reasonable fit. In addition, we also consider $\psi(4660)$ and $\psi(4710)$ as BW amplitudes.

We have 200 fitting parameters in total: m_{ψ_i} ; real coupling constants in Γ_{Rc,ψ_i}^μ , Γ_{Rc,γ^*}^μ , Γ_{ψ_i,γ^*} , $Z_{c(s)}$ amplitude, and $v_{R'c',Rc}^s$; BW masses, widths, and complex $\psi \rightarrow Rc$ couplings for $\psi(4660)$ and $\psi(4710)$. We adjust cutoffs in

Γ_{Rc,γ^*}^μ for $Rc = D_{(s)}^{(*)} \bar{D}_{(s)}^{(*)}$ and $\Lambda_c \bar{\Lambda}_c$ to control the energy dependences of the NR contributions (green dash-dotted curves) in Figs. 3(a)–3(f) and 4(i). Most (some) of the other cutoffs in $\Gamma_{Rc,\psi_i(\gamma^*)}^\mu$ are fixed to 1 (0.7) GeV.

The parameters are adjusted to minimize the χ^2 :

$$\chi^2 = \sum_{i=1}^{N_{\text{data}}} \chi_i^2 = \sum_{i=1}^{N_{\text{data}}} \frac{[O_i(\text{model}) - O_i(\text{data})]^2}{[\delta O_i(\text{data})]^2}, \quad (28)$$

where $O_i(\text{data})$ and $\delta O_i(\text{data})$ are the i th experimental data and its error (statistical and systematic errors are quadratically summed), respectively, and $O_i(\text{model})$ is the corresponding model calculation. The number of data points is $N_{\text{data}} = 1635$. To obtain a physically reasonable solution, it is necessary to apply weighting factors w_i to some of the data by $\chi_i^2 \rightarrow w_i \times \chi_i^2$ in the above χ^2 function. Otherwise, the fit will be poor for data with relatively large errors such as $e^+e^- \rightarrow \pi D \bar{D}$ [Fig. 3(g)]. Also, sharp structures, such as the peaks at $M_{J/\psi\pi^+} \sim 3.9$ GeV in Figs. 5(b) and 5(e), require weights, as the data points are much more scarce compared to the other smooth regions. No established weighting scheme exists for this type of problem. Thus, we apply the weighting in an *ad hoc* manner to obtain an overall reasonable fit. Furthermore, we set allowed ranges of the fitting parameters to avoid unreasonably large cancellations among different mechanisms. We obtained an overall reasonable fit with unweighted $\chi^2/\text{n.d.f.} = 2320/(1635 - 200) \simeq 1.6$ [default fit]. The parameter values for the default fit are given in Appendix D 1, Tables XI, XIII, and XVI–XVIII. Also, a discussion is given in Appendix D 2 on whether the parameter values could be consistent with the heavy quark spin symmetry (HQSS).

B. Comparison with data

The full calculations of the $e^+e^- \rightarrow c\bar{c}$ cross sections from the default-fit model are shown by the red circles connected by lines in Figs. 3 and 4 along with the data. The NR contributions [last term in Fig. 1(b)] are shown by the green dash-dotted curves. We also show contributions from various subsets of the ψ and γ^* decay mechanisms, defined around Eq. (15), such as the direct decay [Fig. 2(a1), blue dashed line], single-triangle rescattering [Fig. 2(a2), magenta dotted line], PD decay [Fig. 2(b1), cyan short-dashed line], and PD single-triangle rescattering [Fig. 2(b2), brown dash-two-dotted line].

Let us explain why some theoretical curves appear to be missing in Figs. 3 and 4. In Figs. 3(d)–3(f), 4(a)–4(c), and 4(i), (PD) single-triangle rescattering contributions do not exist, and the PD decay contributions are the same as the full calculations. This is because the final two-body channels for these processes belong to Table I(B) that do not *directly* couple with the Z diagrams. Furthermore, $J/\psi\eta^{(\prime)}$ and $\omega\chi_{c0}$ channels do not directly couple with the

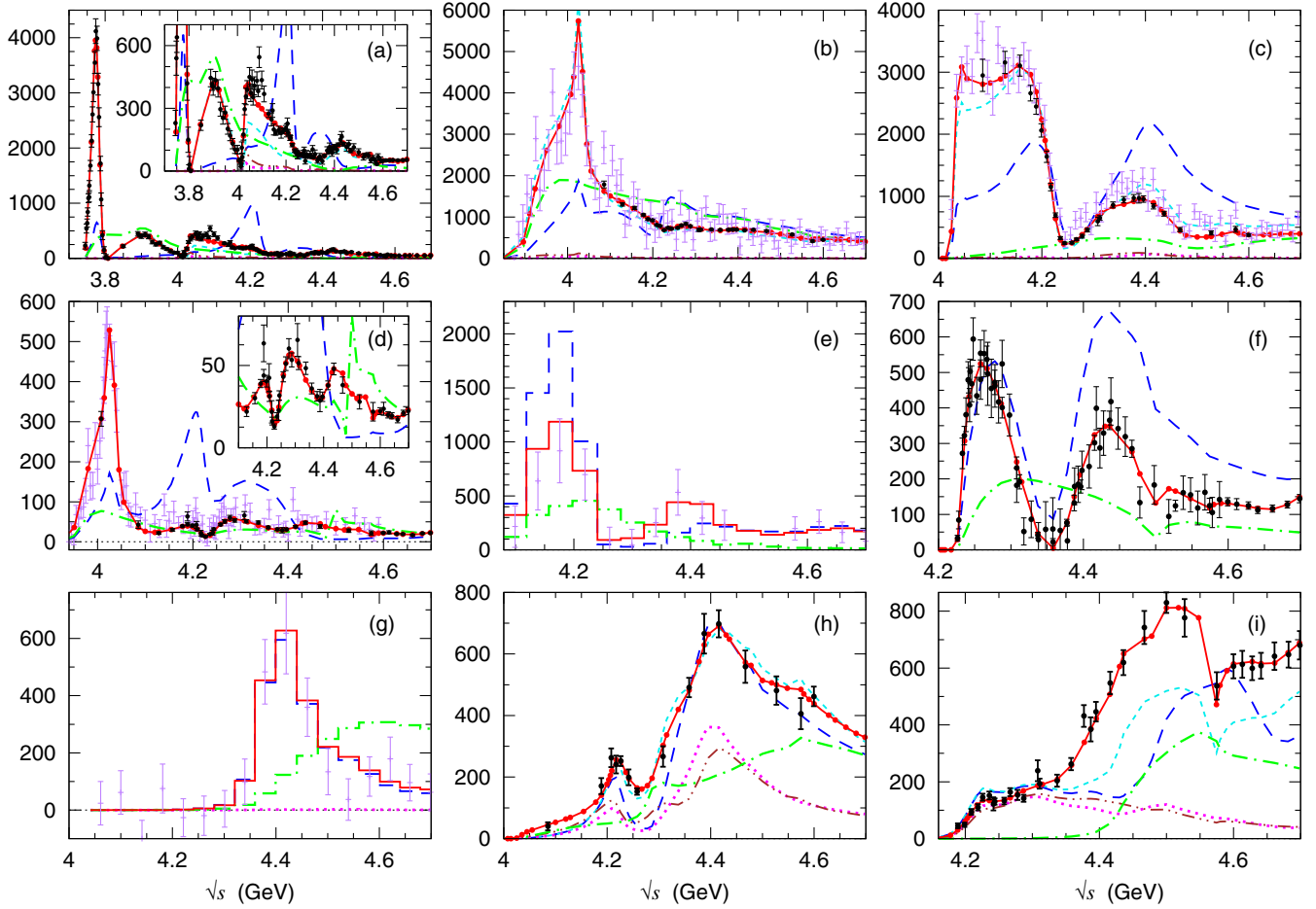


FIG. 3. Cross sections (unit:picobarn), as functions of the total energy $\sqrt{s}$, for e^+e^- annihilations into open-charm final states such as: (a) D^+D^- ; (b) $D^{*+}D^-$; (c) $D^{*+}D^{*-}$; (d) $D_s^+D_s^-$; (e) $D_s^{*+}D_s^-$; (f) $D_s^{*+}D_s^{*-}$; (g) $\pi^+D^0D^-$; (h) $\pi^+D^0D^{*-}$; (i) $\pi^+D^{*0}D^{*-}$; charge-conjugates included. The red points are from our coupled-channel calculation; the lines are just for guiding eyes. Various contributions are shown such as direct decays of Fig. 2(a1) (blue dashed line), single-triangle rescattering of Fig. 2(a2) (magenta dotted line), partially dressed decays of Fig. 2(b1) (cyan short-dashed line), partially dressed single-triangle rescattering of Fig. 2(b2) (brown dash-two-dotted line), and nonresonant mechanisms (green dash-dotted line). The BESIII (Belle ISR) data are shown by the black circles (purple bars) with error bars, and they are from Refs. [38,71] in (a); [39] (black) and [72] (purple) in (b) and (c); [40] (black and purple) in (d); [73] in (e); [41] in (f); [74] in (g); [23] in (h); and [35] in (i). The experimental uncertainties include statistical and systematic ones. In (e) and (g), the calculated cross sections have been averaged within each bin to compare with the ISR data.

short-range potentials v^s either, and, in this case, the direct-decay contributions are the same as the full calculations. In Figs. 3(g) and 4(d)–4(g), the direct-decay and the PD decay contributions are the same. This is because the final quasi-two-body channels⁴ do not directly couple with v^s . In Fig. 4(h), direct-decay mechanisms are absent.

We also show our fit results for invariant-mass and angular distributions in Figs. 5–8. The various contributions discussed above are also shown. The red histograms are obtained by averaging the differential cross sections in

each of the bins. In each panel of the figures, the histogram is normalized so that the total event number from the histogram equals that of the data (central value). The same normalization factor is also multiplied to the other curves in the same panel.

C. Remarks on reaction mechanisms

We begin with general remarks that are common to several processes.

- (i) The coherent sum of the PD decay and PD single-triangle rescattering contributions mostly saturates the full calculation. An exception is $e^+e^- \rightarrow \rho\eta_c\pi$ [Fig. 4(h)], for which direct decays are absent and double-triangle mechanisms [one-order higher than Fig. 2(b2)] give a sizable contribution.

⁴Final quasi-two-body channels directly decay to a final three-body state. For example, f_0J/ψ , f_2J/ψ , and $Z_c\pi$ are, among those listed in Table I, possible final quasi-two-body channels for $e^+e^- \rightarrow J/\psi\pi\pi$.

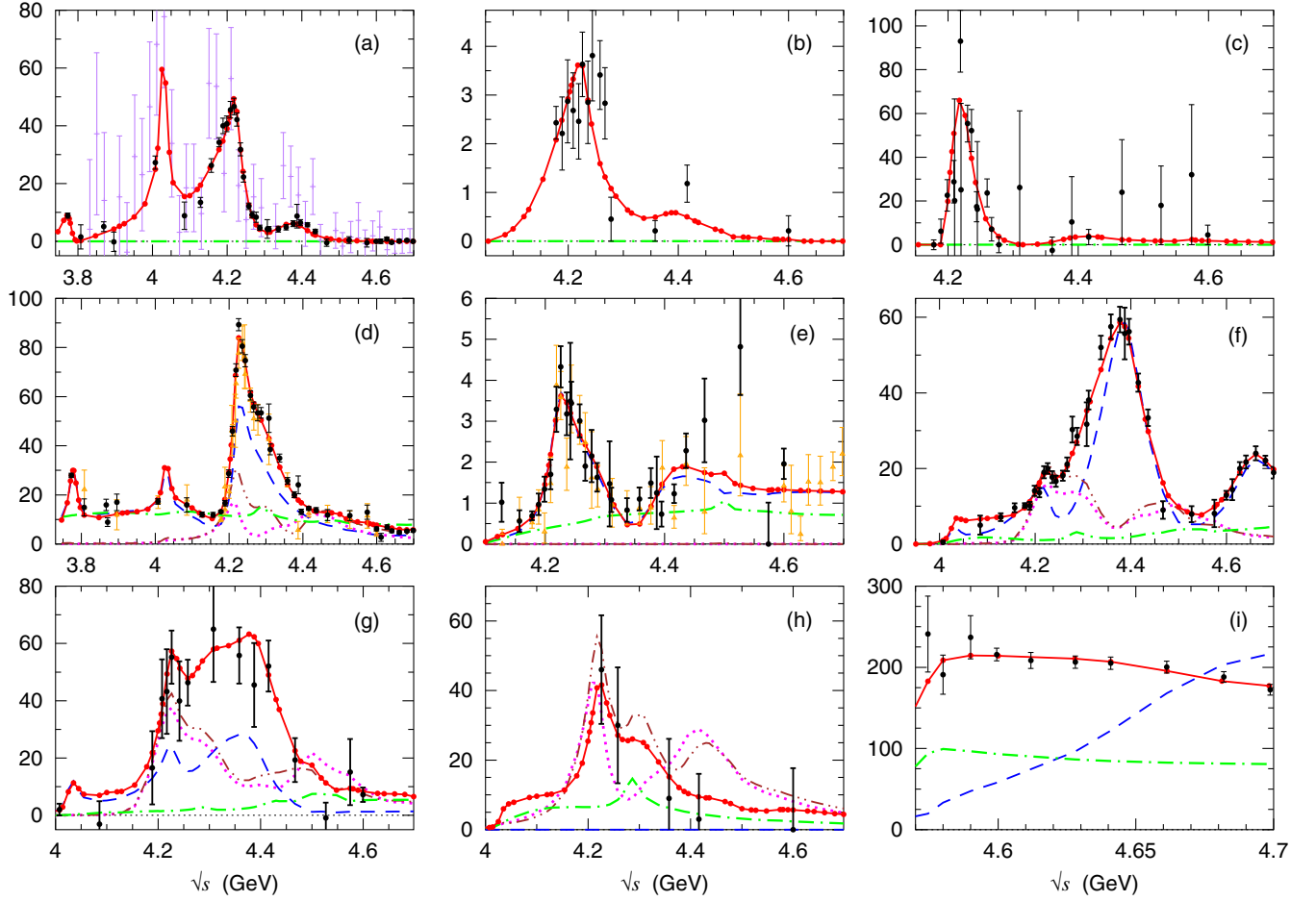


FIG. 4. Cross sections (unit:picobarn) for e^+e^- annihilations into hidden-charm and baryonic final states such as: (a) $J/\psi\eta$; (b) $J/\psi\eta'$; (c) $\omega\chi_{c0}$; (d) $J/\psi\pi^+\pi^-$, $J/\psi\pi^0\pi^0$; (e) $J/\psi K^+K^-$, $J/\psi K_S^0 K_S^0$; (f) $\psi'\pi^+\pi^-$; (g) $h_c\pi^+\pi^-$; (h) $\eta_c\rho^+\pi^-$; (i) $\Lambda_c^+\Lambda_c^-$. $\rho \rightarrow \pi\pi$ is considered in (h). The data are from Refs. [24,42] (black) and [75] (purple) in (a); [43] in (b); [22,44,45] in (c); [16] for $J/\psi\pi^+\pi^-$ (black) and [17] for $J/\psi\pi^0\pi^0$ (orange triangles, doubled) in (d); [19] for $J/\psi K^+K^-$ (black) and [18] for $J/\psi K_S^0 K_S^0$ (orange, doubled) in (e); [20] in (f); [21] in (g); [32] in (h); and [47,48] in (i). Other features are the same as those in Fig. 3.

- (ii) For two-body open-charm final states, $e^+e^- \rightarrow D_{(s)}^{(*)}\bar{D}_{(s)}^{(*)}, \Lambda_c\bar{\Lambda}_c$, the PD decay mechanisms dominate, and the PD single-triangle rescatterings contribute only slightly or not at all [Figs. 3(a)–3(f) and 4(i)]. The resummed v^s significantly contributes to the processes, as indicated by the differences between the PD decay (cyan short-dashed line) and the direct decay (blue dashed line) in Figs. 3(a)–3(c) and those between the full (red solid line) and the direct decay (blue dashed line) in Figs. 3(d)–3(f) and 4(i). Actually, the resummed v^s generates several hadron molecules near open-charm thresholds, as will be discussed later in Sec. VIB. Thus, the resummed v^s contributions can be viewed as effects from the hadron molecules, and the effects are larger near the thresholds.
- (iii) The resummed v^s significantly contributes to three-body open-charm final states $\pi D^*\bar{D}^{(*)}$ [Figs. 3(h)

and 3(i)]. The effect can also be seen in the invariant-mass distributions [Figs. 5(c) and 5(f)] by the difference between the PD decay (cyan short-dashed line) and the direct decay (blue dashed line) contributions.

- (iv) For three-body hidden-charm final states in Figs. 4(d)–4(h), the resummed v^s effects appear as the difference between the PD single-triangle rescattering (brown dash-two-dotted line) and the single-triangle rescattering (magenta dotted line) contributions. In particular, the effect is large for the $J/\psi\pi\pi$ final state and significantly enhancing the $Z_c(3900)$ peak in Figs. 5(b) and 5(e).
- (v) The data show several cusp structures at $\sqrt{s} = 4431$ MeV ($D_1\bar{D}^*$ threshold) in Fig. 3(a); 4289 MeV ($D_1\bar{D}$) in Fig. 3(b); 4503 MeV ($D_{s1}\bar{D}_s$) in Fig. 3(f); and 4573 MeV ($\Lambda_c\bar{\Lambda}_c$) in Fig. 3(i). Our calculation fits these structures with the threshold cusps

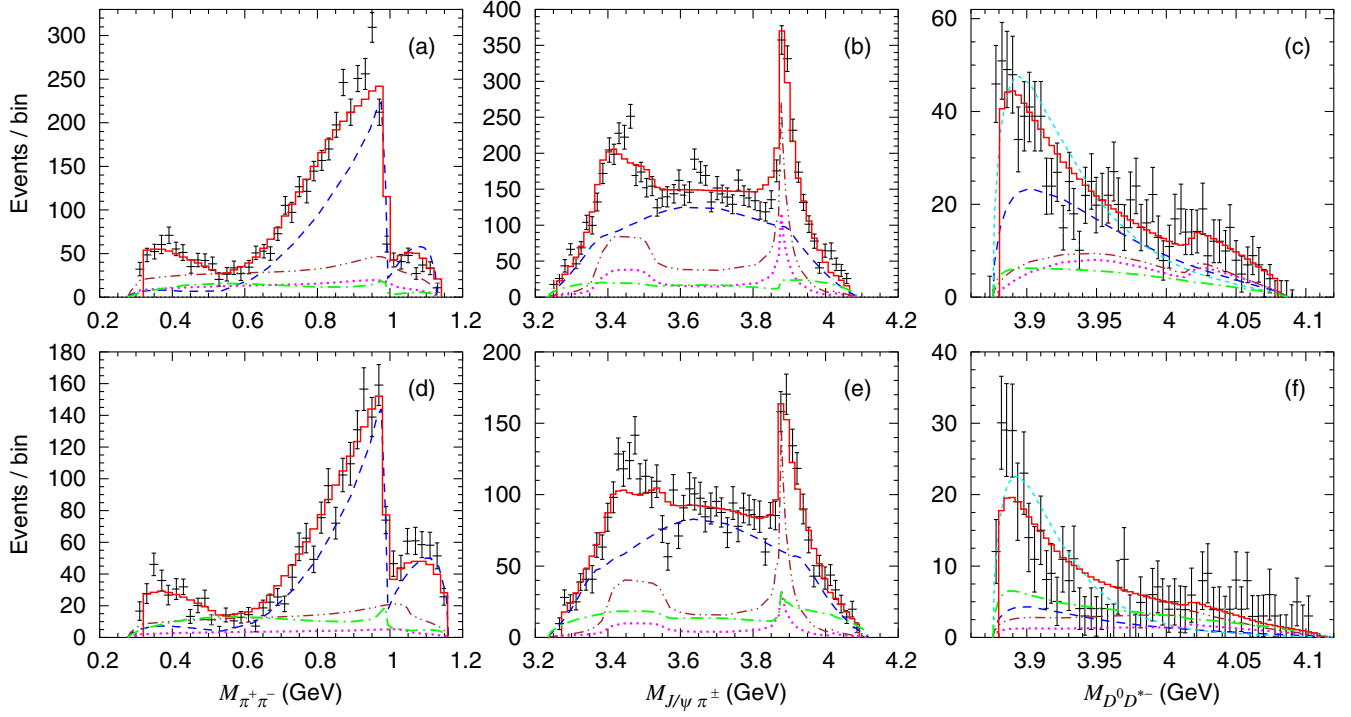


FIG. 5. (a), (d) $\pi^+\pi^-$ invariant-mass distributions in $e^+e^- \rightarrow J/\psi\pi^+\pi^-$. (b), (e) $J/\psi\pi^\pm$ invariant-mass distributions in $e^+e^- \rightarrow J/\psi\pi^+\pi^-$. (c), (f) D^0D^{*-} invariant-mass distributions in $e^+e^- \rightarrow \pi^+D^0D^{*-}$. $\sqrt{s} = 4.23$ GeV for (a)–(c) and $\sqrt{s} = 4.26$ GeV for (d)–(f). Data are from Ref. [46] for (a), (b), (d), and (e) and Ref. [30] for (c) and (f). For the other features, see the caption of Fig. 3.

caused and enhanced by the resummed v^s . Conversely speaking, the strengths of v^s and the associated molecule poles are constrained by fitting the cusps.

- (vi) As a consequence of the coupled-channel fit, our model creates common structures in different processes, even when not necessarily required by the data. For example, $\psi(4040)$ peaks appear in $D^*\bar{D}$ [Fig. 3(b)] and $D_s\bar{D}_s$ [Fig. 3(d)] to fit the data, and

- they also appear in other processes [Figs. 4(a), 4(d), 4(f), and 4(g)] for which data are lacking at the peak.
- (vii) For two-body open-charm final states, the fit quality is sensitive to the $\sqrt{s}$ dependence of the NR contributions, as seen in Figs. 3(a)–3(f) and 4(i). Therefore, we adjusted the cutoffs in Γ_{Rc,γ^*}^μ with $Rc = D_{(s)}^{(*)}\bar{D}_{(s)}^{(*)}, \Lambda_c\bar{\Lambda}_c$. Although not used, s -dependent form factors may be an alternative option for

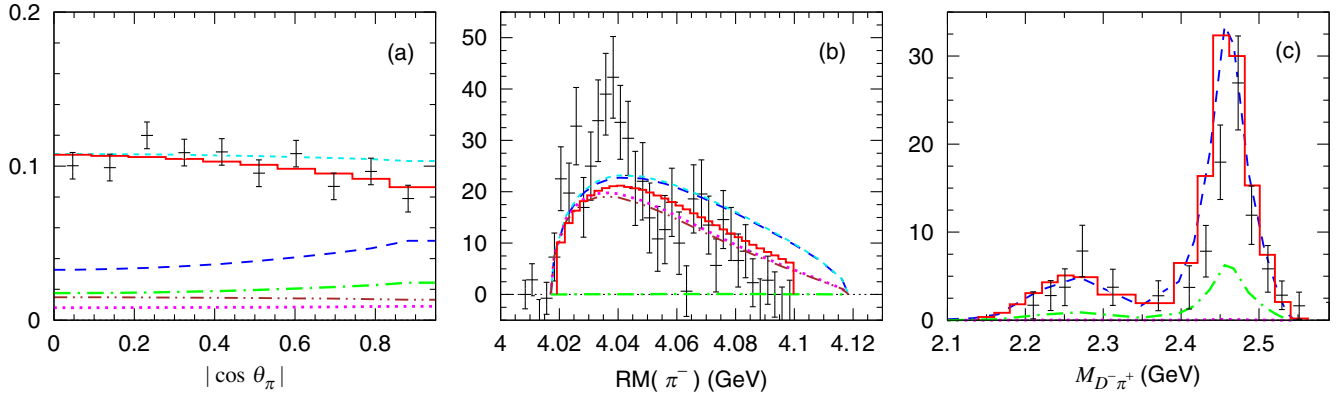


FIG. 6. (a) $\cos\theta_\pi$ distribution (fractional yield) in $e^+e^- \rightarrow \pi^+D^0D^{*-}$ at $\sqrt{s} = 4.23$ and 4.26 GeV; θ_π is the π angle from the beam direction in the CM frame. (b) π^- recoil mass spectrum (events per bin) in $e^+e^- \rightarrow \pi^-(D^*\bar{D}^*)^+$ at $\sqrt{s} = 4.26$ GeV. (c) $D\pi$ invariant-mass distribution (events per bin) in $e^+e^- \rightarrow \pi^+D^0D^-$ at $\sqrt{s} = 4.42$ GeV. The data, from which backgrounds have been subtracted, are from [30] in (a); [31] in (b); and [74] in (c).

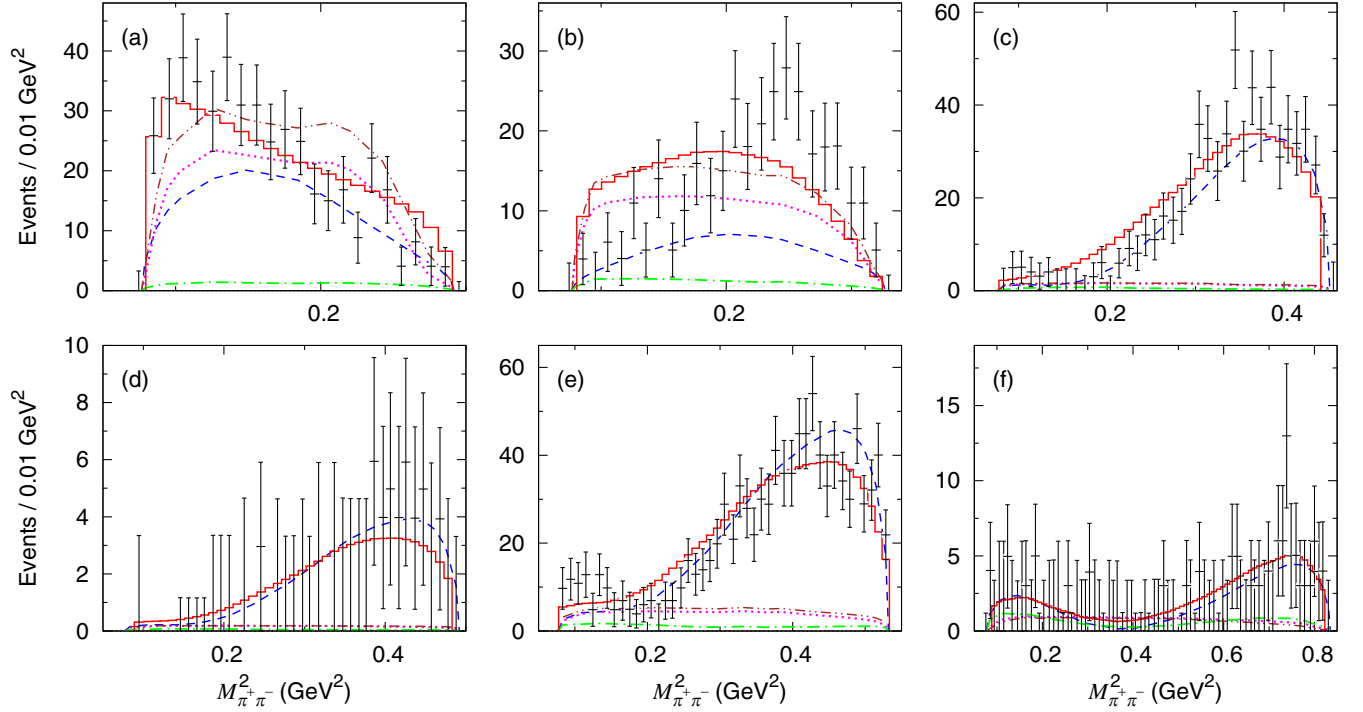


FIG. 7. $\pi^+\pi^-$ invariant-mass distributions in $e^+e^- \rightarrow \psi'\pi^+\pi^-$ at (a) $\sqrt{s} = 4226$ MeV, (b) 4258 MeV, (c) 4358 MeV, (d) 4387 MeV, (e) 4416 MeV, and (f) 4600 MeV. Only for panel (e), the vertical axis is “events/0.02 GeV^2 ”. Data are from Ref. [26]. See the caption of Fig. 3 for other features.

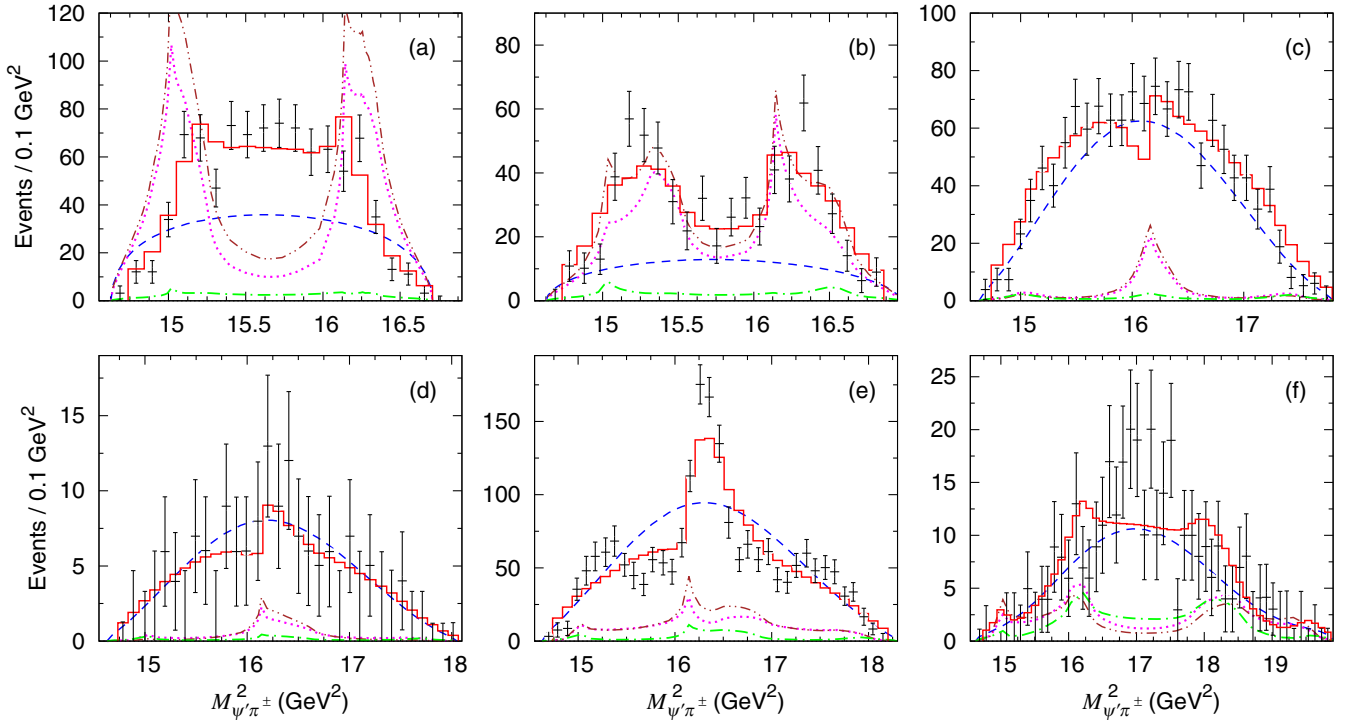


FIG. 8. $\psi'\pi^\pm$ invariant-mass distributions in $e^+e^- \rightarrow \psi'\pi^+\pi^-$ at (a) $\sqrt{s} = 4226$ MeV, (b) 4258 MeV, (c) 4358 MeV, (d) 4387 MeV, (e) 4416 MeV, and (f) 4600 MeV. Other features are the same as Fig. 7.

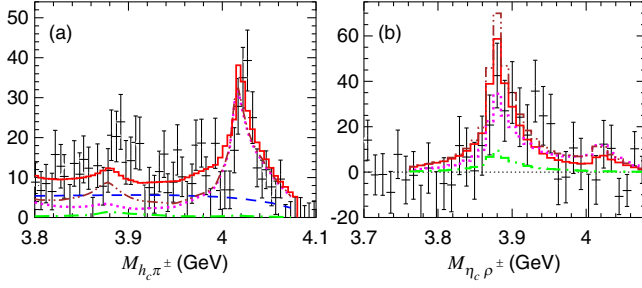


FIG. 9. (a) $h_c \pi^\pm$ invariant-mass distributions in $e^+ e^- \rightarrow h_c \pi^+ \pi^-$ at $\sqrt{s} = 4.23$ and 4.26 GeV. (b) $\eta_c \rho^\pm$ invariant-mass distributions in $e^+ e^- \rightarrow \eta_c \rho^+ \pi^-$ at $\sqrt{s} = 4.23$ GeV. The data (unit: events per bin), from which backgrounds have been subtracted, are from Ref. [29] in (a) and [32] in (b). See the caption of Fig. 3 for other features.

these NR photon couplings to the open-charm hadron pairs.

- (viii) For understanding line shapes of cross-section data and then correctly extracting vector charmonium properties, it is essential to consider the openings of the final quasi-two-body (Rc) channels at their quasithresholds.⁵ This is because the threshold effects can significantly alter line shapes caused by vector charmonium resonances. For $e^+ e^- \rightarrow \pi D \bar{D}$ as an example, the final d -wave $D_2 \bar{D}$ contribution equals the full calculation. This ψ (γ^*) decay sequence is supported by the $D\pi$ invariant-mass distribution data [Fig. 6(c)]. The cross section is suppressed below the $D_2 \bar{D}$ quasithreshold ($\sqrt{s} = 4.33$ GeV), as seen in Fig. 3(g). Furthermore, the suppression near the quasithreshold occurs due to the centrifugal barrier. We find that the line shape peak position (4.42 GeV) is shifted from the resonance mass position (4.39 GeV from Table III) by these threshold effects. On the other hand, in Ref. [74], the data in Fig. 3(g) were fitted without considering the centrifugal barrier effect, resulting in 4.411 ± 0.007 GeV for the $\psi(4415)$ mass. The threshold effects are also important to understand the $e^+ e^- \rightarrow \pi D^* \bar{D}^{(*)}$ line shapes in Figs. 3(h) and 3(i). This can be seen in Fig. 10, where the final $D_1 \bar{D}^{(*)}$ and $D_2 \bar{D}^{(*)}$ contributions rapidly grow around their quasithresholds but are strongly suppressed below them. In Ref. [36], the cross-section data are fitted with the squared charmonium (BW) propagators multiplied by the three-body phase space, missing the threshold effects. This prescription cannot be justified, since the line shapes in Figs. 5(c) and 5(f) are very different from the phase-

⁵For a given quasi-two-body channel, its quasithreshold is the sum of the channel-particle nominal masses. Because of finite widths, their exact thresholds do not exist.

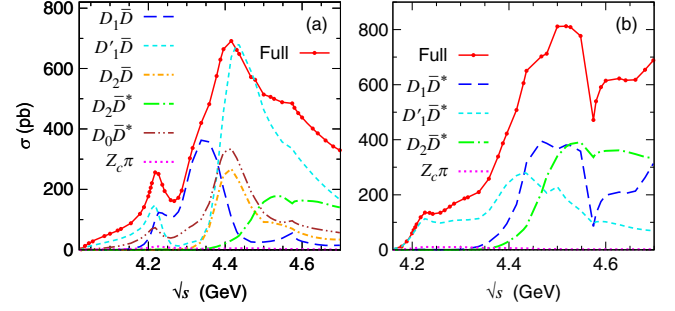


FIG. 10. Contributions from various final quasi two-body channels to (a) $e^+ e^- \rightarrow \pi^+ D^0 D^{*-}$ and (b) $e^+ e^- \rightarrow \pi^+ D^{*0} D^{*-}$. The quasi thresholds are at 4289 MeV for $D_1 \bar{D}$, 4328 MeV for $D_2 \bar{D}$, 4431 MeV for $D_1 \bar{D}^*$, and 4470 MeV for $D_2 \bar{D}^*$, with the widths of $\Gamma_{D_1} = 31$ MeV and $\Gamma_{D_2} = 47$ MeV.

space shape, indicating the dominant open-charm Rc contributions.

- (ix) Various final Rc contributions to $\pi D^* \bar{D}^{(*)}$ are shown in Fig. 10. Their coherent sums are well constrained by the $e^+ e^- \rightarrow \pi D^* \bar{D}^{(*)}$ cross-section data. To reliably control the individual final Rc contributions, we need detailed experimental information such as invariant-mass distributions or Dalitz plots at various $\sqrt{s}$. Currently, they are available only near the $\psi(4230)$ region, as shown in Figs. 5(c), 5(f), 6(a), and 6(b). As remarked above, understanding the individual final Rc contributions is important for correctly determining vector charmonium properties.
- (x) To examine resonance contributions and understand the process-dependent Y line shapes, we need to construct resonance amplitudes with poles and residues extracted from the coupled-channel amplitudes. This study will be done in the future.
- (xi) The NR contributions (green dash-dotted curves) to some processes exhibit resonant structures, e.g., near $D_1 \bar{D}$ threshold (4289 MeV) in Figs. 3(h), 4(d), and 4(f)–4(h). The structures are mainly caused by hadron-molecule poles generated by the interactions of Eq. (13) without coupling to bare ψ states; see Fig. 15. In a unitary model like what we use, these molecule poles in the NR amplitude [second term in the square bracket in Eq. (1)] are canceled in the full amplitude of Eq. (1). This point is well discussed in Ref. [76].

In the following, remarks are made about process-specific reaction mechanisms.

1. $e^+ e^- \rightarrow \pi D^* \bar{D}$

Various final Rc states contribute to this process as seen in Fig. 10(a). The final $D_1 \bar{D}$ contribution is enhanced at ~ 4230 MeV even below its quasithreshold and rapidly grows above its quasithreshold. This indicates that $D_1 \bar{D}$ is

an important decay channel of $\psi(4230)$. In the $\psi(4230)$ region, the contributions from the broad $D'_1\bar{D}$ and $D_0\bar{D}^*$ channels are comparable to the $D_1\bar{D}$ contribution, while the $Z_c\pi$ contribution is rather small. We note that the $D_0\bar{D}^*$ and $Z_c\pi$ contributions arise from triangle mechanisms of Fig. 2(b2), which is a consequence of the coupled-channel unitarity, and the model has no bare $\psi_i \rightarrow D_0\bar{D}^*, Z_c\pi$ couplings.

Examining the reaction mechanisms in Fig. 15 in Ref. [34], their “tree-level” and “triangle” contributions are similar to our counterparts ($D_1\bar{D}$ and $Z_c\pi$) in magnitude. Their model has large contributions from contact $\psi(4160, 4230) \rightarrow Z_c\pi$ mechanisms that they argue are from $\psi(4160, 4230) \rightarrow D'_1\bar{D} - \text{loop} \rightarrow Z_c\pi$. However, in our analysis, such mechanisms are included in the final $Z_c\pi$ contribution and, thus, small. On the other hand, they do not consider tree-level $\psi(4160, 4230) \rightarrow D'_1\bar{D} \rightarrow \pi D^*\bar{D}$ contributions, which is inconsistent with the above argument for the contact Z_c mechanisms. They also do not consider the $D_0\bar{D}^*$ contribution required by the unitarity. In addition, in Ref. [34], the interference between overlapping $\psi(4160)$ and $\psi(4230)$ is not constrained by the unitarity. Thus, the above comparison between the model in Ref. [34] and ours clarifies the crucial importance of unitarity in describing this coupled-channel system.

The enhanced line shapes near the D^0D^{*-} threshold in Figs. 5(c) and 5(f) are largely caused by the final $D_1\bar{D}$ contribution. However, in Ref. [30], the BESIII analysis found that the $D_1\bar{D}$ contribution is very small. More experimental information (Dalitz plots, amplitude analysis results) is necessary to further test our model and examine BESIII’s finding.

2. $e^+e^- \rightarrow \pi D^*\bar{D}^*$

In the $\psi(4230)$ region, the final $D'_1\bar{D}^*$ contribution dominates, as seen in Fig. 10(b). Since the D'_1 width is broad (314 MeV), no significant kinematical suppression happens even if the $D'_1\bar{D}^*$ quasithreshold is ~ 210 MeV above. On the other hand, the other open-charm channels $D_1\bar{D}^*$ and $D_2\bar{D}^*$ are kinematically much suppressed due to their narrow widths. The $Z_c\pi$ contribution is also small, but its interference with other mechanisms is not so small (at most $\sim 20\%$ of the cross section at $\sqrt{s} \sim 4.3$ GeV). As seen in Fig. 3(i), the final $D'_1\bar{D}^*$ contribution is comparably from the direct-decay [Fig. 2(a1)] and single-triangle-rescattering [Fig. 2(a2)] mechanisms. This is possibly because the $D'_1 \rightarrow D^*\pi$ coupling is large. The resummed v^s enhancement is small, because the $D'_1\bar{D}^*$ channel does not directly couple to v^s .

The recoil pion spectrum peak in Fig. 6(b) does not seem to be well fitted. This problem might be due to a significant detection efficiency effect not considered in our fit. Indeed, while the kinematical end point is ~ 4.12 GeV, the data extend only up to ~ 4.09 GeV, suggesting considerable loss

of low-momentum pion events. Efficiency-corrected data are highly desirable for analyzing this and also other data overall.

3. $e^+e^- \rightarrow J/\psi\pi^+\pi^-, J/\psi\pi^0\pi^0$

An issue is to understand the asymmetric line shape of the $\psi(4230)$ in the cross-section data. BESIII introduced $\psi(4320)$ in addition to $\psi(4230)$ [16]. However, $\psi(4320)$ is not necessary to fit other processes, which casts doubt about its existence. In Ref. [34], a $D_1\bar{D}D^*$ triangle diagram and $D_1\bar{D}D^*\bar{D}^{(*)}$ box diagrams get enhanced at the $D_1\bar{D}$ threshold (4289 MeV), causing the asymmetric line shape; $Y(4320)$ is unnecessary. Our coupled-channel model has a similar triangle mechanism in Fig. 2(a2). However, the box diagrams are absorbed by the bare $\psi_i \rightarrow f_{0(2)}J/\psi$ vertices, losing the ability to cause the threshold enhancement. Possibly due to this partial lack of the threshold enhancement, we find a $Y(4320)$ pole in our coupled-channel amplitude; see Sec. VI A. In the future, we will introduce direct couplings between the hidden-charm and open-charm channels via v^s to account for the threshold effects, as done in Ref. [34] via the box diagrams, and examine if $Y(4320)$ still exists. Also, this development could make one bare ψ state redundant. Yet, it is unclear whether the conclusion of Ref. [34] remains valid after they include a dataset as comprehensive as ours.

The $J/\psi\pi$ invariant-mass distributions are well fitted in Figs. 5(b) and 5(e). The figures show that the $Z_c(3900)$ peak is caused by the (PD) single-triangle rescattering [Fig. 2(b2)] that mainly includes $D_1^{(*)}\bar{D}D^*$ triangle loop $\rightarrow Z_c\pi$. The triangle loops causes the $D^*\bar{D}$ threshold cusp that is further enhanced by a pole in the Z_c amplitude. While the $D_1\bar{D}D^*$ triangle singularity also occurs near the $D_1\bar{D}$ quasithreshold, $\sqrt{s}$ for Figs. 5(b) and 5(e) are somewhat lower, and the enhancement due to the triangle singularity would not be drastic.

4. $e^+e^- \rightarrow \psi'\pi^+\pi^-$

The $\psi'\pi$ invariant-mass line shape in Fig. 8 sensitively depends on $\sqrt{s}$. Let us see how this happens. The $Z_c\pi$ contribution forms $D^*\bar{D}^{(*)}$ threshold cusps while the $\psi'f_0$ contribution resembles a phase-space shape. This can be seen in Fig. 8, because the PD single-triangle contribution (brown dash-two-dotted curves) equals the $Z_c\pi$ contribution, and the direct-decay contribution (blue dashed curves) equals the $\psi'f_0$ contribution. The relative magnitude of the $Z_c\pi$ and $\psi'f_0$ contributions varies significantly with $\sqrt{s}$. As a result, their interference pattern (line shape) sensitively depends on $\sqrt{s}$.

Let us see how the $Z_c\pi$ and $\psi'f_0$ contributions interfere with each other. In Fig. 8(a), for example, the $Z_c\pi$ contribution shows the $D^*\bar{D}$ and $D^*\bar{D}^*$ threshold cusps

at $M_{\psi'\pi}^2 \sim 15$ and 16.1 GeV^2 , respectively, and their reflections (the cusps from another $\psi'\pi$ pair) appear in $M_{\psi'\pi}^2 = 16.2\text{--}16.3$ and $15.1\text{--}15.2 \text{ GeV}^2$, respectively. In $M_{\psi'\pi}^2 = 15.4\text{--}16 \text{ GeV}^2$, the $Z_c\pi$ and $\psi'f_0$ contributions interfere constructively. In the cusp region, the phase of the $Z_c\pi$ amplitude changes rapidly, and the interference becomes destructive. As a result of these interferences, the cusp structures are mostly wiped out.

In Fig. 8(b), where $\sqrt{s}$ is $\sim 30 \text{ MeV}$ larger than that in Fig. 8(a), the $D^*\bar{D}$ and $D^*\bar{D}^*$ threshold cusps stay at the same $M_{\psi'\pi}^2$, but their reflections appear in $M_{\psi'\pi}^2 = 16.4\text{--}16.6$ and $15.3\text{--}15.5 \text{ GeV}^2$, respectively. Compared with Fig. 8(a), the $\psi'f_0$ contribution relative to the $Z_c\pi$ contribution is significantly smaller. This can be seen in Fig. 4(f), where, from $\sqrt{s} = 4.22$ to 4.26 GeV , the $\psi'f_0$ contribution (blue dashed curve) sharply drops, while the $Z_c\pi$ contribution (brown dash-two-dotted curve) decreases only slightly. As a result, both constructive and destructive interferences between the $Z_c\pi$ and $\psi'f_0$ contributions are significantly smaller compared to Fig. 8(a), and the cusp structures remain. This explains the rather rapid change of the line shape from Figs. 8(a) to 8(b). The fairly constant $Z_c\pi$ contribution in this region is related to the flat $D_1'\bar{D}^*$ contribution to $e^+e^- \rightarrow D^*\bar{D}^*\pi$ [Fig. 10(b)], because the $D_1'\bar{D}^*D^*$ triangle rescattering is a main $Z_c\pi$ production mechanism. Different processes are related in this way by the coupled-channel dynamics.

In Fig. 8(e), a prominent peak is formed. This is partly because the $D^*\bar{D}^*$ threshold cusp is enhanced by a $D_1'\bar{D}^*D^*$ triangle singularity, as can be inferred from the $D_1'\bar{D}^*$ contribution in Fig. 10(b). Still, the peak height seems lower than the data. This might indicate that the $D^*\bar{D}^*$ interaction in the Z_c amplitude needs to be more attractive. In our current Z_c amplitude, the $D^*\bar{D}$ and $D^*\bar{D}^*$ interaction strengths are the same following the HQSS. In the future, we relax this constraint to see if the fit improves. This might also improve the fit in Fig. 6(b).

The invariant-mass distribution data in Figs. 7 and 8 are fitted using $\sqrt{s}$ -dependent coupling parameters in Ref. [77]. The cross-section data in Fig. 4(f) were not fitted. Therefore, the reason for the $\sqrt{s}$ dependence of the $M_{\psi'\pi}^2$ line shape was not clarified. The Z_c propagations were treated with BW amplitudes.

In Ref. [78], both cross-section and invariant-mass distribution data were fitted—single-channel analysis. The $D^*\bar{D}^{(*)}$ threshold cusps (or Z_c structures) are generated with the initial single pion emission (ISPE) mechanisms. The ISPE mechanisms correspond to our single-triangle rescattering mechanisms shown in Fig. 2(a2), but unstable charm mesons (solid line with a solid circle in the figure) shrunk to a point. Thus, kinematical effects such as $D_1'\bar{D}^{(*)}$ threshold cusps and $D_1'\bar{D}^{(*)}D^*$ triangle singularities are lost

in the ISPE mechanisms. Also, the ISPE mechanisms in Ref. [78] do not cause Z_c poles.

5. $e^+e^- \rightarrow h_c\pi^+\pi^-$

For the $M_{h_c\pi}$ line shape in Fig. 9(a), the final $Z_c\pi$ contribution (brown dash-two-dotted curve) causes the $D^*\bar{D}$ and $D^*\bar{D}^*$ threshold cusps. In particular, the $D^*\bar{D}^*$ cusp is prominent and has been interpreted as $Z_c(4020)$. The final h_cf_0 contribution (blue dashed curve) is relatively small, compared to the final $J/\psi f_0$ and $\psi'f_0$ contributions in $e^+e^- \rightarrow J/\psi\pi\pi$ and $\psi'\pi\pi$, respectively.

As discussed in Ref. [34], this process is a HQSS-violating process; however, its cross section is comparable to that of $e^+e^- \rightarrow J/\psi\pi\pi$, which is HQSS conserving. For a similar case in the b -quark sector, it is argued in Ref. [79] that comparable cross sections of HQSS-conserving $\Upsilon(10860) \rightarrow \pi Z_b^{(\prime)} \rightarrow \Upsilon\pi\pi$ and HQSS-violating $\Upsilon(10860) \rightarrow \pi Z_b^{(\prime)} \rightarrow h_b\pi\pi$ can be understood with two parameters $m_{Z_b} - m_{Z_b'}$ and $\Gamma_{Z_b^{(\prime)}}$, and the HQSS restores at $m_{Z_b} = m_{Z_b'}$. An assumption in Ref. [79] is that Z_b and Z_b' are almost equally excited in both processes; data support this. For the c -quark sector, this assumption is not valid. As seen in Figs. 5(b), 5(e), and 9(a), $Z_c(3900)$ and $Z_c(4020)$ are strongly excited in $e^+e^- \rightarrow J/\psi\pi\pi$ and $h_c\pi\pi$, respectively, but not vice versa. Thus, the argument of Ref. [79] cannot be directly applied to the c -quark sector.

6. $e^+e^- \rightarrow J/\psi K^+K^-, J/\psi K_S K_S$

The $J/\psi K^+K^-$ data [Fig. 4(e)] show an enhancement suggesting $Y(4500)$ [19]. However, our model does not fit it, since the data are rather fluctuating in this region, and the $J/\psi K_S K_S$ data do not indicate the same enhancement.

For $\sqrt{s}$ slightly above the $D_{s1}\bar{D}_s$ threshold (4503 MeV), the $D_{s1}\bar{D}_s D^*$ triangle diagram [Fig. 2(b2)] causes a triangle singularity, and attractive $D_{s1}\bar{D}_s$ and $\bar{D}_s D^*$ (Z_{cs} amplitude) interactions further enhance the triangle amplitude. However, this contribution alone is rather small, as indicated by the brown dash-two-dotted curve in Fig. 4(e), and

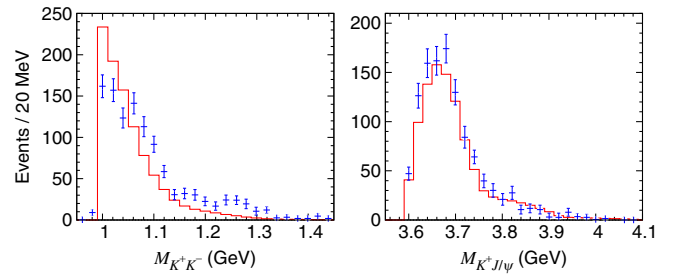


FIG. 11. Predictions for K^+K^- (left) and $J/\psi K^+$ (right) invariant-mass distributions of $e^+e^- \rightarrow J/\psi K^+K^-$ from $\sqrt{s} = 4.1$ to 4.6 GeV . Data are from Ref. [19].

causes a modest change in the line shape. This triangle contribution sensitively depends on bare $\psi_i \rightarrow D_{s1}\bar{D}_s$ coupling strengths. We constrained the couplings using $e^+e^- \rightarrow D_{s1}\bar{D}_s$ cross-section data near the threshold [80]. In Ref. [54], a $D_{s1}\bar{D}_s$ bound state is predicted and assigned to $Y(4500)$. However, considering the smallness of the above triangle contribution, it seems difficult to fit the $Y(4500)$ fluctuation in the $J/\psi K^+K^-$ data with this $D_{s1}\bar{D}_s$ molecule's contribution under the constraint from the $e^+e^- \rightarrow D_{s1}\bar{D}_s$ data [80].

We also present in Fig. 11 our model's prediction for $J/\psi K^+$ and K^+K^- invariant-mass distributions of $e^+e^- \rightarrow J/\psi K^+K^-$ over $\sqrt{s} = 4.1\text{--}4.6$ GeV. The agreement with data is fair.

IV. INCLUSIVE CROSS SECTIONS

Let us see if our coupled-channel model reasonably gives the inclusive $e^+e^- \rightarrow c\bar{c}$ cross sections or the conventional R value defined by

$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma_{e^+e^- \rightarrow \mu^+\mu^-}^{\text{tree}}(s)} = R_c(s) + R_{uds}(s), \quad (29)$$

where we have separated the R value into the contribution from the open- and hidden-charm channels (R_c) and that from light-hadron channels (R_{uds}), assuming that the couplings between the separated channels are small. We have also introduced the tree-level $e^+e^- \rightarrow \mu^+\mu^-$ cross sections⁶

$$\sigma_{e^+e^- \rightarrow \mu^+\mu^-}^{\text{tree}}(s) = \frac{4\pi\alpha^2}{3s}. \quad (30)$$

With our coupled-channel model, $R_c \times \sigma_{e^+e^- \rightarrow \mu^+\mu^-}^{\text{tree}}$ is obtained by summing all the calculated cross sections in Figs. 3 and 4 and their isospin partners. The obtained R_c is shown by the magenta dotted curve in Fig. 12 along with the experimental R values [49]. We assume that the difference between them is from light-hadron contributions, R_{uds} . We express R_{uds} by

$$R_{uds}(s) = (\sqrt{s} - E_1)(\sqrt{s} - E_2)(\sqrt{s} - E_3) \times (c_1s + c_2\sqrt{s} + c_3) + R_{uds}^0, \quad (31)$$

for $E_1 < \sqrt{s} < E_3$, and

$$R_{uds}(s) = R_{uds}^0, \quad (32)$$

otherwise, and then adjust the parameters as in Table II to reproduce the experimental R . The R_{uds} (blue dashed line) as

⁶We neglect the lepton mass, which is a good approximation for $\sqrt{s} > 3.7$ GeV.

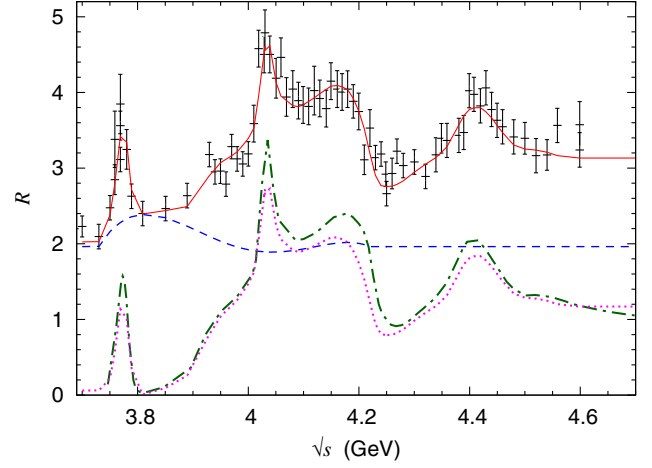


FIG. 12. The R value. The sum of $\sigma(e^+e^- \rightarrow c\bar{c} \text{ hadrons})$ from our coupled-channel model gives the magenta dotted curve. The light-hadron contribution is shown by the blue dashed curve. Their sum is shown by the red solid curve. The green dash-dotted curve is calculated with an optical theorem of Eq. (33). Data are from Ref. [49].

well as $R_c + R_{uds}$ (red solid line) obtained from the fit are shown in Fig. 12. The resonant structures in the data are well reproduced by our coupled-channel model.

If our coupled-channel model were three-body unitary, R_c can also be calculated using an optical theorem

$$R_c(s) = -\frac{3}{\alpha} \text{Im}[\Pi_c(s)], \quad (33)$$

where we have introduced a charm VP by

$$\Pi_c(s) = \frac{1}{s} \left(\Sigma_{\gamma^*} + \sum_{ij} \bar{\Gamma}_{\gamma^*, \psi_i} \bar{G}_{ij} \bar{\Gamma}_{\psi_j, \gamma^*} \right), \quad (34)$$

where the NR contribution Σ_{γ^*} is obtained from Eq. (19) by replacing labels “ $\psi_{i(j)}$ ” with “ γ^* .” The resonant part consists of $\bar{\Gamma}_{\psi_i, \gamma^*} (= \bar{\Gamma}_{\gamma^*, \psi_i})$ and $\bar{G}_{ij}$ that have been defined in Eqs. (16) and (18), respectively. A diagrammatic representation for Π_c is Fig. 1(j). The R_c calculated with Eq. (33) is shown in Fig. 12 (green dash-dotted curve). The difference between the magenta dotted and green dash-dotted curves is a measure of the three-body unitarity violation in our model. The violation is modest, which should allow our model to predict $e^+e^- \rightarrow \mu^+\mu^-$ cross sections in the next subsection.

TABLE II. Numerical parameter values in Eq. (31).

E_1 (GeV)	3.73	c_1 (GeV ⁻⁵)	-30.1	R_{uds}^0	1.96
E_2 (GeV)	3.98	c_2 (GeV ⁻⁴)	-7.93		
E_3 (GeV)	4.22	c_3 (GeV ⁻³)	545		

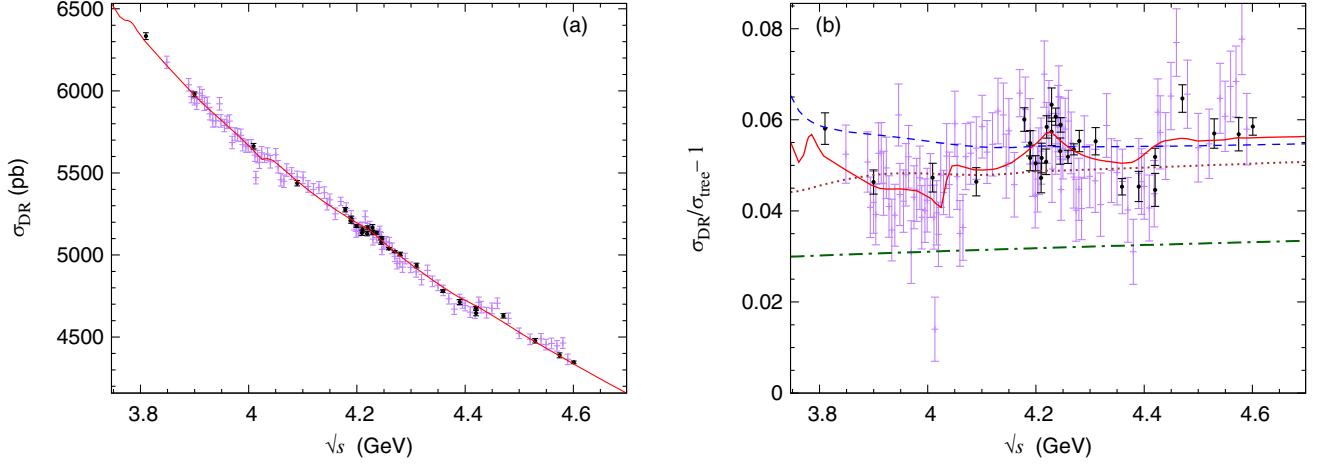


FIG. 13. (a) Dressed $e^+e^- \rightarrow \mu^+\mu^-$ cross sections. The red solid curve is our prediction from Eq. (35) including the pure leptonic tree and VP contributions. The black circles (purple bars) are higher (lower) luminosity data from Ref. [62]. The data have been slightly shifted as Eq. (39). (b) Ratios of the DR $e^+e^- \rightarrow \mu^+\mu^-$ cross sections shown in the left panel to the tree-level $e^+e^- \rightarrow \mu^+\mu^-$ cross sections (red solid curve and data). The contributions from Π_{lepton} (green dash-dotted line), $\Pi_{\text{lepton}} + \Pi_{uds}$ (brown dotted line), and $\Pi_{\text{lepton}} + \Pi_{uds} + \Pi_{J/\psi} + \Pi_{\psi'}$ (blue dashed line) are also shown. See Eq. (36) for the notations.

V. $e^+e^- \rightarrow \mu^+\mu^-$ CROSS SECTIONS

BESIII measured the $e^+e^- \rightarrow \mu^+\mu^-$ cross sections for $\sqrt{s} = 3.8\text{--}4.6$ GeV and subtracted initial state radiation effects to obtain the dressed (DR) cross sections [62]. Theoretically, the DR $e^+e^- \rightarrow \mu^+\mu^-$ cross section up to and including $\mathcal{O}(\alpha^3)$ is given by

$$\sigma_{e^+e^- \rightarrow \mu^+\mu^-}^{\text{DR}}(s) = \sigma_{e^+e^- \rightarrow \mu^+\mu^-}^{\text{tree}}(1 + 2\text{Re}[\Pi(s)]), \quad (35)$$

where $\sigma_{e^+e^- \rightarrow \mu^+\mu^-}^{\text{tree}}$ has been given in Eq. (30) and Π is VP including various intermediate states:

$$\Pi(s) = \Pi_{\text{lepton}} + \Pi_{uds} + \Pi_{J/\psi} + \Pi_{\psi'} + \Pi_c, \quad (36)$$

where Π_{lepton} includes the e^+e^- , $\mu^+\mu^-$, $\tau^+\tau^-$ one-loop contribution; we use Eq. (214) in Ref. [81] to calculate Π_{lepton} . The light hadron contributions Π_{uds} are calculated with the once-subtracted dispersion relation:

$$\text{Re}[\Pi_{uds}(s)] = \frac{s - s_0}{\pi} P \int_{4m_\pi^2}^{\infty} ds' \frac{(-\alpha/3)R_{uds}(s')}{(s' - s_0)(s' - s)}, \quad (37)$$

where R_{uds} has been introduced in Eqs. (31) and (32); P indicates the principal value integral. We take the subtraction point $s_0 = 0$ and set the subtraction constant to zero.⁷ Regarding the J/ψ and ψ' contributions Π_ψ ($\psi = J/\psi$ or ψ'), we use

$$\Pi_\psi(s) = \frac{3s \Gamma_{e^+e^-}^\psi}{\alpha m_\psi} \frac{1}{s - m_\psi^2 + im_\psi \Gamma_\psi}, \quad (38)$$

where Γ_ψ and $\Gamma_{e^+e^-}^\psi$ are the ψ total width and partial width to e^+e^- , respectively, and their values are from Ref. [4]. The other charm contributions Π_c is from our coupled-channel model as has been given in Eq. (34).

Our prediction from Eq. (35) is compared with the corresponding data [62] in Fig. 13(a). For this comparison, we followed Ref. [82] and shifted the data as

$$\sigma_i^{\text{DR,exp}} \rightarrow (f_0 + f_1 \sqrt{s_i}) \sigma_i^{\text{DR,exp}}, \quad (39)$$

where $\sigma_i^{\text{DR,exp}}$ is the i th data point at $s = s_i$ and f_0 and f_1 are fitting parameters. We find $f_0 = 0.917$ and $f_1 = 0.0259 \text{ GeV}^{-1}$ for the best fit. The shift amounts to 1.59% at $\sqrt{s} = 3.81$ GeV and 3.64% at $\sqrt{s} = 4.6$ GeV, which is fairly comparable to the systematic uncertainty of 2.91%.

For more detailed comparisons, we divide the cross sections in Fig. 13(a) by $\sigma_{e^+e^- \rightarrow \mu^+\mu^-}^{\text{tree}}$ and show the ratios subtracted by one in Fig. 13(b). Contributions from various VP terms in Eq. (36) are also shown. Our full result (red solid curve) does not have a sharp dip at $\sqrt{s} \sim 4.2$ GeV that the data seem to indicate. Also, the structure at $\sqrt{s} \sim 4.45$ GeV from our model is smaller than the data. Similar results were also obtained in previous theoretical studies [82] ([83]) where the full R (R_c) values of Eq. (29) were converted to the hadronic VP (Π_c).⁸ Farrar *et al.* [82]

⁷A possibly nonzero subtraction constant could be absorbed by the parameter f_0 in Eq. (39), as far as the difference between the calculation and the shifted data is concerned.

⁸See Fig. 1(bottom) in Ref. [83]. The notation h_c in Ref. [83] and our notation Π_c are related by $h_c = -\frac{\pi}{\alpha} \Pi_c$.

suggested the possibility of undetected hadronic final states and statistical fluctuations in the data.

On the other hand, the model of von Detten *et al.* [34] reproduces the dip at $\sqrt{s} \sim 4.2$ GeV. It is noted, however, that their model is not unitary and there is no clear relation with the inclusive R values. Furthermore, a fitting parameter, c_{mix} in Eq. (44) in [34], is introduced exclusively for $e^+e^- \rightarrow \mu^+\mu^-$, and it plays an important role in fitting the data, as seen in Fig. 18 in [34] (“mixing” contribution). However, the unitarity dictates that such mixing occurs not only in $e^+e^- \rightarrow \mu^+\mu^-$, but also in all the other $e^+e^- \rightarrow c\bar{c}$ processes. As discussed in the previous paragraph, our model and the dispersive approaches [82,83] do not have any freedom to fit the dip, once they begin with the R values. Thus, it is fair to say that the structures in the $e^+e^- \rightarrow \mu^+\mu^-$ cross-section data are not well understood. On the experimental side, more precise data would be highly desirable to verify the structures.

VI. POLE EXTRACTION AND RESONANCE PROPERTIES

A. Vector-charmonium states

We analytically continue the coupled-channel amplitude fitted to the dataset to the complex energy plane, using the method for three-body unitary models discussed in Refs. [84,85]. We can then find a pole location $E = E_\psi$ where $\det[\bar{G}^{-1}(E_\psi)] = 0$ for Eq. (18). We search for vector charmonium poles on the relevant Riemann sheets⁹: unphysical sheets of the open channels and physical sheets of the closed channels or sheets slightly deviating from this condition; $3.75 < M < 4.7$ GeV ($M \equiv \text{Re}[E_\psi]$), and $\Gamma \equiv -2 \times \text{Im}[E_\psi] < 0.2$ GeV.

Pole uncertainty estimates are generally difficult in global coupled-channel analyses, and simplified methods have been used [86,87]. For statistical uncertainty estimate, we introduce complex parameters δm_{ψ_i} as $m_{\psi_i} \rightarrow m_{\psi_i} + \delta m_{\psi_i}$ in Eq. (18). We also select parameters to which pole locations are sensitive such as bare couplings for “ $\psi, \gamma^* \rightarrow$ open-charm channels” that mainly dress bare ψ states, thereby shifting their masses and generating widths; diagonal and/or large couplings of v^s in Eq. (13) that generate hadron-molecule states. We then vary these parameters, 85 in total, around the default fit for the uncertainty estimate. This time, we neither weight the data nor limit the parameter ranges as we did to obtain the default fit. See Appendix C 1 for more details.

Regarding systematic uncertainty (model dependence) of the poles, there are certainly many possible sources such as the choice of form factors (cutoffs), the number of bare states, whether parameters are constrained by the HQSS or SU(3), etc. For a substantial model variation, which is crucial for the systematic uncertainty estimate, finding a

TABLE III. Vector charmonium poles and $\psi(4660)$ BW parameters. See the text for the notations.

This work		PDG(ψ) [4], BESIII [16,38,88]		
M (MeV)	Γ (MeV)	M (MeV)	Γ (MeV)	
3764.2 ± 2.0	47.3 ± 2.6	3751.9 ± 3.8	32.8 ± 5.8	$^1D\bar{D}$
3780.2 ± 1.2	29.9 ± 2.3	3778.1 ± 0.7	27.5 ± 0.9	$\psi(3770)$
3898.4 ± 0.9	127.5 ± 6.7	3872.5 ± 14.2	179.7 ± 14.1	$^1D^*\bar{D}$
3956.1 ± 1.0	96.8 ± 10.4	...	...	$^1D_s\bar{D}_s$
4029.2 ± 0.4	26.3 ± 1.0	4039 ± 1	80 ± 10	$\psi(4040)$
4052.4 ± 0.4	49.0 ± 0.3	...	...	$^3D_s^*\bar{D}_s$
4192.2 ± 2.2	129.3 ± 4.2	4191 ± 5	70 ± 10	$\psi(4160)$
4216.2 ± 0.5	40.3 ± 1.0	...	...	$^3D_s^*\bar{D}_s^*$
4229.9 ± 0.9	46.4 ± 2.6	4222.5 ± 2.4	48 ± 8	$\psi(4230)$
4308.1 ± 2.2	138.2 ± 4.4	4298 ± 12	127 ± 17	$Y(4320)$
4346.2 ± 3.8	122.8 ± 6.7	4374 ± 7	118 ± 12	$\psi(4360)$
4390.1 ± 2.0	106.5 ± 4.1	4421 ± 4	62 ± 20	$\psi(4415)$
4496.3 ± 3.1	16.4 ± 2.1	...	...	$^bD_{s1}\bar{D}_s$
4579.6 ± 1.7	-5.2 ± 7.6	...	...	$^b\Lambda_c\bar{\Lambda}_c$
4655.9 ± 3.0	134.9 ± 5.9	4630 ± 6	72^{+14}_{-12}	$\psi(4660)$

solution comparable to the default fit requires considerable effort that warrants an independent paper. We, thus, do not go into this task here. This important issue can be addressed when updating the model in the future by including more data and theoretical inputs.

We find 14 states, as listed in Table III, where experimental analysis results are also shown for comparison. A graphical presentation of this table is given in Fig. 14. Overall, the pole uncertainties from our coupled-channel analysis are smaller than those from the experimental single-channel analyses. This can be expected, since, in the former, the data of the various processes constrain the pole locations, and some data are very precise. Our analysis finds states that can be identified with all of the vector charmonia ($M > 3.75$ GeV) listed in the PDG [4]. However, there are sizable differences with the PDG average such as the $\psi(4040)$ width and the $\psi(4415)$ mass and width. One possible cause of the differences is threshold effects, as discussed in Sec. III C, that can shift a line shape peak position from a resonance mass. The $\psi(4415)$ as well as $\psi(4040)$ and $\psi(4160)$ resonance parameters in the PDG are basically from the BW fit to the R values [49] without considering any thresholds and coupled channels. The previous simple analyses might have introduced artifacts in the extracted resonance parameters.

Moreover, several states are found close to quasithresholds of open-charm ($H\bar{H}'$) channels. We denote these states as $^xH\bar{H}'$ with $x = b, r, v$ (bound, resonant, virtual), based on the pole locations, regardless of their internal structures. Given the branch point of

⁹See Sec. 50 in Ref. [4] for the definition of the (un)physical sheet.

$$E_{H\bar{H}'}^{\text{BP}} = (m_H + m_{\bar{H}'}) - \frac{i}{2}(\Gamma_H + \Gamma_{\bar{H}'}), \quad (40)$$

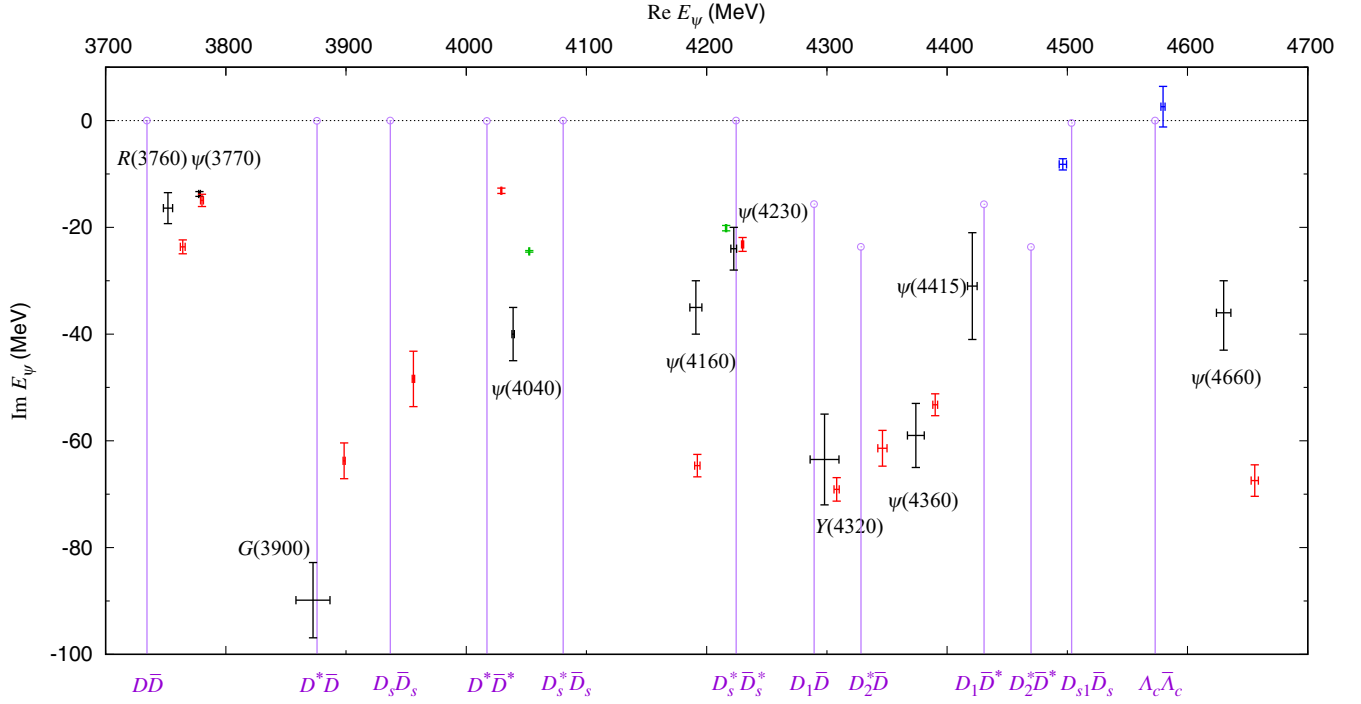


FIG. 14. Vector charmonium poles (E_ψ) with uncertainties. Red points indicate resonance poles [located on unphysical (physical) sheets of open (closed) channels], while blue and green points indicate bound and virtual poles, respectively, of the nearest-threshold channels. Black points are ψ states listed in PDG [4], $\mathcal{R}(3760)$ [88], $G(3900)$ [38], and $Y(4320)$ [16]. Open circles and accompanying vertical lines are branch points [Eq. (40)] and cuts, respectively, for open-charm channels indicated at the bottom.

$^b H\bar{H}'$ with $\text{Im}[E_\psi] \leq \text{Im}[E_{H\bar{H}'}^{\text{BP}}]$ ($\text{Im}[E_\psi] > \text{Im}[E_{H\bar{H}'}^{\text{BP}}]$ and $\text{Re}[E_\psi] > \text{Re}[E_{H\bar{H}'}^{\text{BP}}]$) is located on the physical (unphysical) sheet of the $H\bar{H}'$ channel. $^{r(v)} H\bar{H}'$ is located on the unphysical sheet of the $H\bar{H}'$ channel, and $\text{Re}[E_\psi] \geq \text{Re}[E_{H\bar{H}'}^{\text{BP}}]$ ($\text{Re}[E_\psi] < \text{Re}[E_{H\bar{H}'}^{\text{BP}}]$).

The $^b D\bar{D}$ state from our analysis is similar to $\mathcal{R}(3760)$ claimed by BESIII's analyses of $e^+e^- \rightarrow \text{hadrons}$ [88] and $e^+e^- \rightarrow \text{non-open-charm hadrons}$ [89]. The BESIII also found $\mathcal{R}(3810)$ with $M \sim 3805$ MeV and $\Gamma \sim 10$ MeV in Refs. [88,89]. Our analysis does not find $\mathcal{R}(3810)$, because our dataset does not show any structure associated with it. We find a $^b D^* \bar{D}$ state similar to $G(3900)$ from the BESIII analysis on $e^+e^- \rightarrow D\bar{D}$ [38]. Coupled-channel K -matrix analyses were done for the $e^+e^- \rightarrow D^{(*)} \bar{D}^{(*)}$ cross-section data [Figs. 3(a)–3(c)] and inclusive data for $\sqrt{s} < 4.2$ GeV [71,90]. A $G(3900)$ pole was found at $(3869.2 \pm 6.7) - (29.0 \pm 5.2)i$ MeV in Ref. [90] but not in Ref. [71]. A similar analysis of older data was done in Ref. [91] using a HQSS-based coupled-channel model, and $G(3900)$ was claimed at $3879 - 35i$ MeV. The $G(3900)$ widths from Refs. [90,91] are significantly narrower than our result. These theoretical analyses [71,90,91] did not find $\mathcal{R}(3760)$ and $\mathcal{R}(3810)$. A $\Lambda_c \bar{\Lambda}_c$ bound state was claimed at ~ 38 MeV below the threshold from a single-channel analysis of the $e^+e^- \rightarrow \Lambda_c \bar{\Lambda}_c$ data [92]. Our analysis found a similar pole but located above the threshold due to a coupled-channel effect. A $D_{s1} \bar{D}_s$ bound state predicted with a HQSS and SU

(3) symmetric model in Ref. [54] is similar to our $^b D_{s1} \bar{D}_s$. The authors of Ref. [54] interpreted this state as $Y(4500)$ with $\Gamma \sim 111$ MeV appearing in $e^+e^- \rightarrow J/\psi K^+ K^-$. In our analysis, $^b D_{s1} \bar{D}_s$ is not $Y(4500)$ but a much narrower state that causes a dip in the $e^+e^- \rightarrow D_s^* \bar{D}_s^*$ cross section [Fig. 3(f)]. The other $^x H\bar{H}'$ states in Table III are found for the first time in the present analysis.

Two reasons why our model can accommodate more poles than the five bare states are that (i) the short-range interactions among the open-charm channels can generate hadron-molecule states; and (ii) a bare state may cause more than one resonance by coupling with hadronic continuum states (Table I); see a demonstration in Ref. [93].

There exist four poles with $M \sim 4.23$ GeV and two poles with $M \sim 4.38$ GeV. It is speculated that these overlapping resonances interfere with each other differently in various processes, resulting in the process-dependent Y line shapes. Indeed, the previous analyses considered the interference of $\psi(4160)$ and $\psi(4230)$ to explain the seemingly process-dependent $\psi(4230)$ width for $e^+e^- \rightarrow J/\psi \eta$ [94] and for $e^+e^- \rightarrow J/\psi \pi \pi, \pi D^* \bar{D}, J/\psi \eta$ and more in Ref. [34]. We will address this issue in the future, taking into account the unitarity that was not considered in the previous analyses.

B. Pole trajectories

In our coupled-channel model, vector charmonium states can be formed from bare ψ states dressed by quasi-two-body continuum states in Table I, as can be seen in Eq. (18).

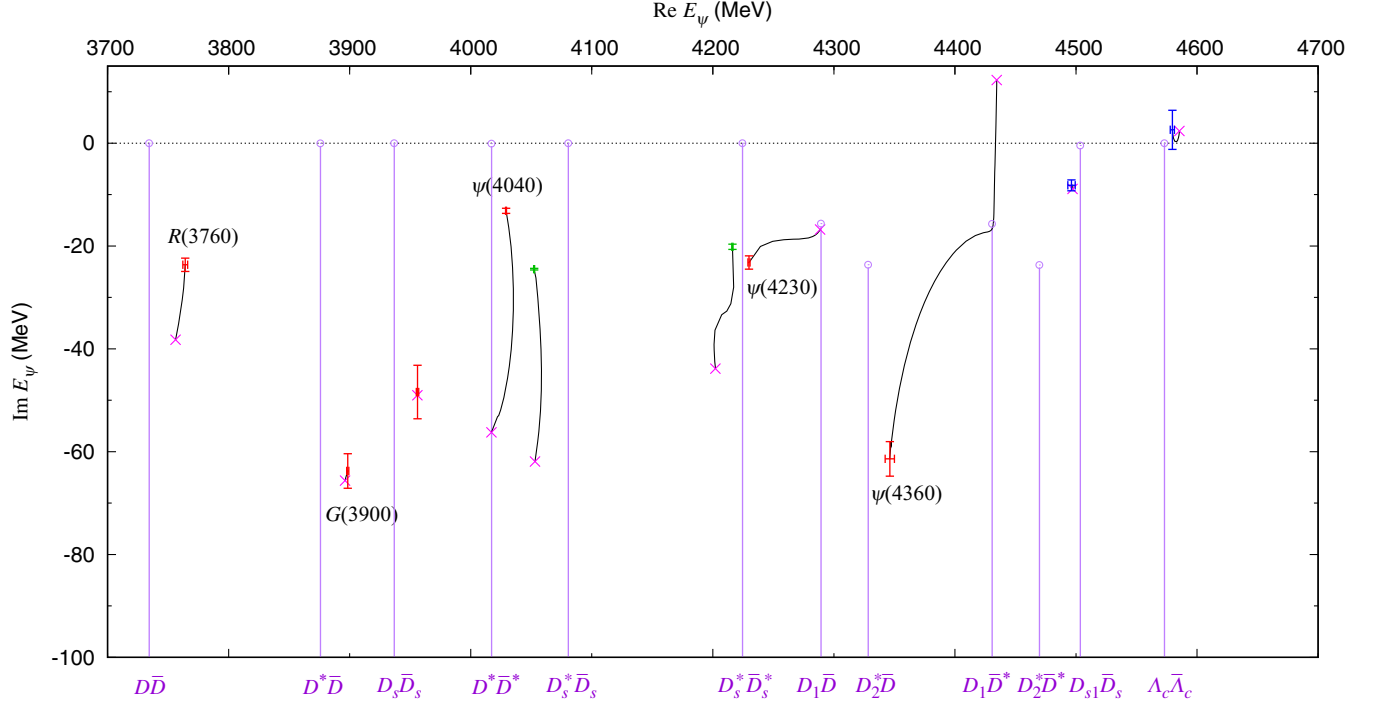


FIG. 15. Pole trajectories of hadron-molecule-dominated states. The magenta crosses are the locations of poles generated by the hadron-hadron interactions of Eq. (13), without couplings to bare ψ states. The poles move along the black curves (trajectories) as bare $\psi_i \rightarrow Rc$ couplings are gradually turned on. The other ends of the trajectories are the pole locations, listed in Table III, from the default model.

Another pole formation is from hadron-hadron interactions of Eq. (13). In this case, the $Rc \rightarrow R'c'$ partial wave amplitude $[X_{(Rc)_{ls},(R'c')_{l's'}}]$ in Eq. (12) include hadron-molecule poles, and, as a consequence, so does the NR amplitude in Eq. (1). We find such hadron-molecule poles near most of the open-charm thresholds, as shown by the magenta crosses in Fig. 15; v^s terms in Eq. (13) play a dominant role for these hadron-molecule formations.

The hadron-molecule poles further couple with bare ψ states via continuum states, yielding some of the poles in Table III. This development can be visualized in Fig. 15 as pole trajectories (black curves). To draw the trajectories, we multiply a common parameter λ to all of bare $\psi_i \rightarrow Rc$ couplings $[C_{(Rc)_{ls}}^i]$ in Eq. (11) of the default model. As we vary λ from 0 to 1, the pole locations move from the magenta crosses to the other ends (pole locations from the default model) following the black curves. An exception is the trajectory connecting to $\psi(4360)$, because we do not find a practical integral path for the analytic continuation to the pole location for $\lambda = 0$. Thus, we introduced one more parameter λ' by which the Z terms in Eq. (13) are multiplied. The trajectory is drawn by varying the parameters as $\lambda = 0 \rightarrow 1$ with $\lambda' = 0$ and then $\lambda' = 0 \rightarrow 1$ with $\lambda = 1$. We note that the trajectories shown in Fig. 15 are not unique but dependent on how the bare $\psi_i \rightarrow Rc$ couplings are turned on; even the end points of the trajectories may change.

Previous theoretical studies [50–54] speculated that $\psi(4230)$ is a $D_1\bar{D}$ bound state formed by short-range

interactions without coupling to charmonium states. References [52,53] also assigned a $D_1\bar{D}^*$ bound state to $\psi(4360)$. A HQSS model of Ref. [54] predicted a $D_1\bar{D}^*$ virtual state, which is more consistent with our result. Our analysis found not only such molecular poles, but also their significant shifts due to couplings with (bare) charmonium states. The $G(3900)$ pole was described as a p -wave $D^*\bar{D}$ molecule from a one-boson-exchange model in Ref. [90]. Our analysis found a similar $D^*\bar{D}$ molecular state that is slightly shifted by couplings with (bare) charmonium states. We also found a p -wave $D^*\bar{D}^*$ molecular state that is shifted to become $\psi(4040)$, one of the well-established vector charmonium states.

C. Compositeness

The pole trajectories shown in Fig. 15 imply that some of the vector charmonium states from the default model may have substantial hadron-molecule contents. To explore the internal structure, the compositeness [59–61] might serve as a qualitative measure. When a resonance state (Gamow state) $|\psi\rangle$ is normalized as $\langle\psi|\psi\rangle = 1$, a compositeness X_{Rc} is the contribution from a continuum (quasi-two-body) Rc channel to this normalization. With an elementariness Z_a from a bare state a , the sum rule $\sum_{Rc} X_{Rc} + \sum_a Z_a = 1$ is satisfied.

Here, we calculate the compositeness in a manner similar to Ref. [60]. The formulas in Ref. [60] are for models including bare states and two-body continuum states. Since our coupled-channel model includes three-body channels

that enter the calculations in the forms of the Z diagrams in Eq. (13) and the R self-energies and widths in Eqs. (5)–(7), we extend the formulas in Ref. [60].

The compositeness of a state ψ at $E = E_\psi$ can be expressed with its residue. The residue for an $Rc \rightarrow R'c'$ transition is given by

$$\begin{aligned} r_{(R'c')l's',(Rc)ls}^\psi(p_{c'}, p_c) &= \lim_{E \rightarrow E_\psi} (E - E_\psi) \sum_{ij} \bar{F}_{(R'c')l's',\psi_i}(p_{c'}, E) \\ &\quad \times \bar{G}_{ij}(E) \bar{F}_{(Rc)ls,\psi_j}(p_c, E) \\ &= \sum_{ij} \bar{F}_{(R'c')l's',\psi_i}(p_{c'}, E_\psi) [\bar{G}^{-1}(E_\psi)]_{ij} [\Delta'(E_\psi)]^{-1} \\ &\quad \times \bar{F}_{(Rc)ls,\psi_j}(p_c, E_\psi), \end{aligned} \quad (41)$$

where $\bar{F}_{(Rc)ls,\psi_j}$ and $\bar{G}_{ij}$ have been defined in Eqs. (10) and (18), respectively; $[\bar{G}^{-1}]_{ij}$ is the adjugate matrix of $[\bar{G}]_{ij}$, $\Delta(E)$ is the determinant of the matrix $[\bar{G}^{-1}(E)]_{ij}$, and $\Delta'(E_\psi) = d\Delta(E)/dE|_{E=E_\psi}$. The compositeness is given with the residue as

$$X_{Rc} = \int \frac{q^2 dq}{4E_R^2(q)} \frac{\sum_{ls} r_{(Rc)ls,(Rc)ls}^\psi(q, q)}{[E_\psi - E_R(q) - E_c(q) + i\frac{\Gamma_R}{2}]^2}. \quad (42)$$

The above compositeness formula can be reduced to Eq. (93) in Ref. [60] by turning off Z diagrams and R self-energies and widths, by nonrelativistic reductions, and by using the same normalizations of the form factors.

Caveats are in order regarding the compositeness calculated with Eq. (42). The compositeness is generally complex, and it is difficult to interpret its imaginary part. Furthermore, interpreting X_{Rc} is difficult for cases with $\text{Re}[X_{Rc}] < 0$ or $\text{Re}[X_{Rc}] > 1$. Therefore, only when the

imaginary part is significantly smaller than the real part and $0 \leq \text{Re}[X_{Rc}] \leq 1$ may we interpret X_{Rc} as an approximate probability of finding Rc continuum states (or a Rc molecule) in a resonance state. Also, Eqs. (41) and (42) indicate that the compositeness depends on the form factors, and this dependence would be more pronounced for $l > 0$. Thus, the compositeness of p -wave states from $D_{(s)}^{(*)}\bar{D}_{(s)}^{(*)}$ scattering should be viewed with more caution. The compositeness is given model independently for s -wave states in the weak binding limit only [59–61].

Tables IV and V present the compositeness for the vector charmonium states listed in Table III. The result confirms what the trajectory analysis suggested: The states shown in Fig. 15 have large compositeness. A noteworthy case is $\psi(4040)$ with $X_{D^*\bar{D}^*} \sim 0.86$. This well-established state has been assumed to be the $\psi(3S)$ state in quark models, and its experimentally determined mass has been used as an input in determining the quark-model parameters [5]. However, our comprehensive analysis might suggest reconsidering this conventional assumption.

As mentioned earlier, $\psi(4230)$ and $\psi(4360)$ have often been speculated to be $D_1\bar{D}$ and $D_1\bar{D}^*$ molecules, respectively [50–54]. However, our compositeness analysis suggests more complex structures than these expectations. Hadron dynamics cause large mixings among nearby molecular states [$D_1\bar{D}$, $D_1\bar{D}^*$, and $D_s^*\bar{D}_s^*$] and also bare ψ states to form $\psi(4230)$, $\psi(4360)$, and ${}^vD_s^*\bar{D}_s^*$ states. Because of these mixings, it is not straightforward to relate quark-model states to the vector charmonium states in this region. A possible idea is to introduce quark-model states as bare ψ states in our coupled-channel model; a related work can be found in Ref. [95].

D. Z_c poles

Finally, Table VI presents the Z_c poles in $D^*\bar{D} - D^*\bar{D}^* - J/\psi\pi - \psi'\pi - h_c\pi - \eta_c\rho$ coupled-channel scattering

TABLE IV. Compositeness X_{Rc} of vector charmonium states ψ listed in Table III. Contributions from open-charm channels are shown. Dots indicate $|X_{Rc}| < 0.01$.

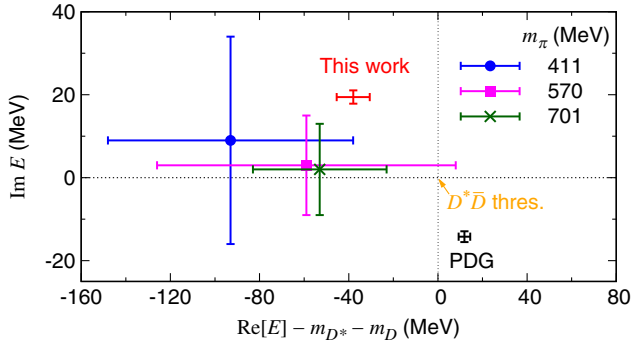
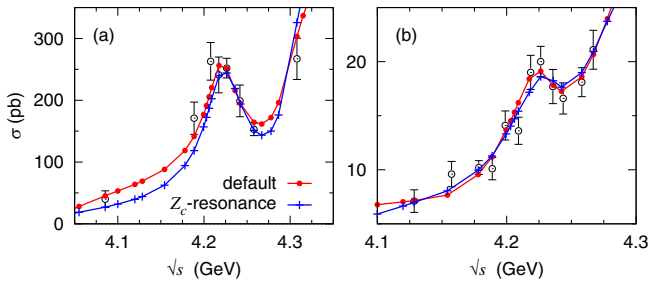
$Rc \setminus \psi$	${}^rD\bar{D}$	$\psi(3770)$	${}^rD^*\bar{D}$	${}^rD_s\bar{D}_s$	$\psi(4040)$	${}^vD_s^*\bar{D}_s^*$	$\psi(4160)$
$D\bar{D}$	$1.66 + 0.42i$	$-0.66 - 0.41i$	$-0.03 - 0.01i$	...	...	...	...
$D^*\bar{D}$	$0.01 - 0.01i$	...	$1.10 + 0.04i$	...	$-0.01 + 0.01i$	...	...
$D^*\bar{D}^*$	...	...	...	...	$0.86 + 0.22i$	$0.06 + 0.12i$	$0.26 - 0.03i$
$D_1\bar{D}$	...	...	...	...	...	...	$0.01 - 0.02i$
$D_1\bar{D}^*$	$0.01 - 0.01i$	...	...	...	...	...	$-0.00 + 0.01i$
$D_2^*\bar{D}^*$	...	...	...	...	...	...	$-0.01 - 0.02i$
$D_s\bar{D}_s$	...	...	...	$1.01 + 0.01i$	...	...	...
$D_s^*\bar{D}_s$	...	...	...	...	$0.01 - 0.03i$	$0.91 - 0.09i$	$-0.03 - 0.25i$
$D_s^*\bar{D}_s^*$	...	...	...	...	...	...	$-0.10 - 0.04i$
$D_{s1}\bar{D}_s$	...	...	...	...	...	...	...
$\Lambda_c\bar{\Lambda}_c$	...	...	...	...	...	...	...
Sum	$1.68 + 0.38i$	$-0.68 - 0.40i$	$1.05 + 0.02i$	$1.00 + 0.00i$	$0.86 + 0.19i$	$0.97 + 0.03i$	$0.14 - 0.36i$

TABLE V. Continued from Table IV.

$Rc \backslash \psi$	$^v D_s^* \bar{D}_s^*$	$\psi(4230)$	$Y(4320)$	$\psi(4360)$	$\psi(4415)$	$^b D_{s1} \bar{D}_s$	$^b \Lambda_c \bar{\Lambda}_c$
$D\bar{D}$	...	...	...	...	...	...	...
$D^* \bar{D}$	...	...	...	...	...	...	...
$D^* \bar{D}^*$	$0.01 + 0.02i$	...	$0.02 - 0.15i$	...	...	...	...
$D_1 \bar{D}$	$0.23 - 0.12i$	$0.18 + 0.13i$	...	$0.31 - 0.06i$	$-0.03 + 0.04i$	...	$0.03 - 0.00i$
$D_1 \bar{D}^*$	$0.04 - 0.06i$	$0.09 + 0.05i$	$0.01 - 0.01i$	$0.29 - 0.16i$	$0.08 + 0.13i$	...	$0.01 - 0.01i$
$D_2^* \bar{D}^*$	$0.02 - 0.01i$	...	$-0.01 - 0.00i$	...	$0.06 - 0.03i$	...	...
$D_s \bar{D}_s$	...	...	...	...	...	$0.02 - 0.03i$	...
$D_s^* \bar{D}_s$	$0.04 + 0.05i$	$0.02 - 0.05i$	$0.05 + 0.06i$	...	$0.01 - 0.02i$	...	...
$D_s^* \bar{D}_s^*$	$0.50 + 0.25i$	$0.35 - 0.27i$	$-0.11 - 0.03i$	...	$-0.03 - 0.00i$	...	...
$D_{s1} \bar{D}_s$	...	...	...	...	...	$0.99 + 0.00i$	...
$\Lambda_c \bar{\Lambda}_c$	...	...	...	$0.02 - 0.04i$	$0.01 + 0.01i$	...	$0.97 + 0.01i$
Sum	$0.83 + 0.12i$	$0.65 - 0.13i$	$-0.06 - 0.14i$	$0.61 - 0.27i$	$0.10 + 0.13i$	$1.01 - 0.01i$	$1.00 - 0.02i$

TABLE VI. $IJ^{PC} = 11^{+-}$ $D^* \bar{D} - D^* \bar{D}^* - J/\psi \pi - \psi' \pi - h_c \pi - \eta_c \rho$ coupled-channel scattering amplitude poles (unit: MeV). $Z_c(3900)$ and $Z_c(4020)$ are $D^* \bar{D}$ and $D^* \bar{D}^*$ virtual (resonance) poles in this work (PDG [4]).

$E_{Z_c}^{\text{This work}}$	$M_{Z_c}^{\text{PDG}}$	$\Gamma_{Z_c}^{\text{PDG}}$	
$(3837.7 \pm 7.4) + (19.4 \pm 1.6)i$	3887.1 ± 2.6	28.4 ± 2.6	$Z_c(3900)$
$(3989.9 \pm 5.6) + (26.1 \pm 4.3)i$	4024.1 ± 1.9	13 ± 5	$Z_c(4020)$

FIG. 16. $Z_c(3900)$ poles on the $D^* \bar{D}$ unphysical sheet from this work, PDG [4], and lattice QCD [105].FIG. 17. Comparison of the default model (Z_c -virtual-state solution) and Z_c -resonance solution on fitting (a) $e^+e^- \rightarrow \pi^+ D^0 D^{*-}$ and (b) $e^+e^- \rightarrow \psi' \pi^+ \pi^-$ cross-section data. Data are from Refs. [20,23].

amplitude ($IJ^{PC} = 11^{+-}$) implemented in our three-body scattering model.¹⁰ See Appendix C 2 for the uncertainty estimation method. One pole (the other) is a $D^* \bar{D}$ ($D^* \bar{D}^*$) virtual state, located at ~ 40 MeV below the threshold. The previous analyses fitted the $M_{\pi J/\psi}$ and $M_{D^* \bar{D}}$ line shape data [Figs. 5(b), 5(c), 5(e), and 5(f)] where the $Z_c(3900)$ signals are clearest but not fitting the cross-section data that can test Z_c production mechanisms and Z_c -pole residues. While some analyses [90,97–102] obtained virtual poles, the others [98,99,103,104] and the experimental ones [28,30,46] found resonance poles near the PDG value. Lattice QCD results [105–110] favor the virtual-state solution, providing $Z_c(3900)$ virtual poles¹¹ [105] as shown in Fig. 16, where our result compares fairly well. We also searched a Z_c resonance solution. The Z_c amplitude in the default-fit model is replaced by that including energy-dependent $D^* \bar{D}^{(*)}$ interactions [98]. Then we refitted the full dataset with 201 parameters, under a constraint that the Z_c amplitude has a resonance pole above the $D^* \bar{D}^{(*)}$ threshold. We could achieve $\chi^2 \sim 2510$ while $\chi^2 = 2320$ for the default model. Compared to the default model, the invariant-mass distributions are fitted equally well, but some of the cross-section data are not fitted comparably. Some cases are shown in Fig. 17. This may suggest the importance of fitting the cross-section data to discriminate between the Z_c pole locations.

VII. SUMMARY AND OUTLOOK

We performed a global coupled-channel analysis of most of the available $e^+e^- \rightarrow c\bar{c}$ data (20 channels) in $\sqrt{s} = 3.75 - 4.7$ GeV for the first time, considering

¹⁰Since we did not analyze data showing a $Z_{cs}(3985)$ structure [96], we do not discuss a pole in our $D_s^* \bar{D} - \bar{D}^* D_s - J/\psi K$ coupled-channel amplitude.

¹¹The poles listed in Ref. [105] have their conjugates that are physically more relevant, as pointed out in Ref. [111]. Figure 16 shows the conjugates.

three-body unitarity approximately and all relevant coupled channels. Channel couplings are caused by bare ψ excitations, long-range particle exchanges, and short-range contact interactions. We obtained overall reasonable fits to both cross-section and invariant-mass distribution data with $\chi^2/\text{n.d.f.} \simeq 1.6$. We clarified mechanisms that generate various structures in the data, paying special attention to open-charm threshold cusps enhanced by nearby poles.

We predicted the $e^+e^- \rightarrow \mu^+\mu^-$ cross sections using the vacuum polarization due to light hadrons, leptons, and charm contributions. The charm contribution was calculated with the tails of the J/ψ and ψ' , as well as with our coupled-channel model. While our prediction is consistent with previous calculations based on dispersion relations, it does not reproduce the fine structure in the data at $\sqrt{s} \sim 4.2$ GeV.

We analytically continued the coupled-channel amplitude to extract vector-charmonium poles. We obtained not only familiar vector charmonia, but also those near the open-charm thresholds; some of the states were found for the first time. We examined pole trajectories and compositeness of the poles to explore the internal structures of the vector-charmonium states. This study suggested that open-charm hadron-molecular structures dominated in many states. The $\psi(4230)$ and $\psi(4360)$ states are not simple $D_1\bar{D}$ and $D_1\bar{D}^*$ molecules, respectively, as proposed in the literature but rather mixtures of them plus 30%–40% elementariness; $\psi(4230)$ further includes substantial $D_s^*\bar{D}_s^*$ compositeness. Also, our analysis suggested a large $D^*\bar{D}^*$ compositeness in $\psi(4040)$, although $\psi(4040)$ has been assumed to be the $\psi(3S)$ quark-model state. While we estimated statistical uncertainties of the pole locations, we did not address systematic uncertainties (see Sec. VI A), leaving this task to future work.

We found Z_c poles as $D^*\bar{D}^{(*)}$ virtual states; similar conclusions are also from lattice QCD analyses. We suggested the importance of analyzing cross-section data in addition to the invariant-mass distribution data to discriminate whether Z_c is a resonance or a virtual state.

In the future, we will examine how the vector-charmonium states contribute to each of the $e^+e^- \rightarrow c\bar{c}$ processes through interferences with one another, causing the process-dependent Y line shapes. BESIII will provide more data covering higher energy and more channels. We will update the current analysis by including such data and address the properties of vector-charmonium states heavier than 4.6 GeV and those of Z_{cs} . More detailed data such as Dalitz plots are also expected to be available from BESIII. It is important to verify the existence and properties of the presented vector-charmonium states by analyzing such detailed experimental information while also incorporating and/or varying theoretical inputs.

ACKNOWLEDGMENTS

We acknowledge T. Doi, F.-K. Guo, C. Hanhart, T. Sato, Q. Wang, J.-J. Wu, W. Yamada, M.-J. Yan, and Z.-Y.

Zhou for useful discussions. This work is in part supported by National Natural Science Foundation of China (NSFC) under Contracts No. U2032103 and No. 11625523 and also by National Key Research and Development Program of China under Contract No. 2020YFA0406400.

APPENDIX A: TWO-MESON SCATTERING MODELS

Basic components [Eqs. (3)–(8) and (13)] in our three-body coupled-channel model are two-meson scattering models for $ab \rightarrow a'b'$ via (bare) R excitations [see Fig. 1(a) for the notation] and also via contact interactions. The (bare) R states are categorized into groups (A)–(C) in Table I; no $R \rightarrow ab$ couplings are considered for group (B) in our model. We also consider short-range $Rc \rightarrow R'c'$ interactions [v^s in Eq. (13)] between open-charm channels. These two-meson scattering models and parameters therein are described below.

TABLE VII. Parameter values for R listed in Table I(A): the BW mass m_R and width Γ_R in Eq. (8); couplings $g_{ab,R}^{LS}$ in Eq. (4). For the cutoff in Eq. (4), $c_{ab,R} = 1$ GeV is commonly used for all R in this table.

$m_{D_1(2420)}$ (MeV)	2422
$\Gamma_{D_1(2420)}$ (MeV)	31
$g_{D^*\pi,D_1(2420)}^{01}$	−0.860
$g_{D^*\pi,D_1(2420)}^{21}$	1.25
$m_{D_1(2430)}$ (MeV)	2412
$\Gamma_{D_1(2430)}$ (MeV)	314
$g_{D^*\pi,D_1(2430)}^{01}$	23.5
$g_{D^*\pi,D_1(2430)}^{21}$	0
$m_{D_2^*(2460)}$ (MeV)	2461
$\Gamma_{D_2^*(2460)}$ (MeV)	47
$g_{D^*\pi,D_2^*(2460)}^{21}$	0.840
$g_{D\pi,D_2^*(2460)}^{20}$	0.569
m_D (MeV)	1867
Γ_D (MeV)	0
$g_{D^*\pi,D}^{11}$	4.62
m_{D^*} (MeV)	2009
Γ_{D^*} (MeV)	0.069
$g_{D\pi,D^*}^{10}$	2.67
$g_{D^*\pi,D^*}^{11}$	3.92
$m_{D_{s1}(2536)}$ (MeV)	2535
$\Gamma_{D_1(2420)}$ (MeV)	0.92
$g_{D_{s1}(2536)}^{01}$	0
$g_{D_{s1}(2536)}^{21}$	1.14

1. Groups (A) and (B) in Table I

The R propagations are described by the BW form of Eq. (8), and the BW mass and width values are taken from the PDG [4] except that $\Gamma_{J/\psi} = \Gamma_{D_s^*} = 0$. The $R \rightarrow ab$ decay vertices ($\Gamma_{ab,R}$) are Eqs. (3) and (4). The coupling constants ($g_{ab,R}^{LS}$) in $\Gamma_{ab,R}$, whose numerical values are listed in Table VII, are determined, assuming that $D_1(2420) \rightarrow D^*\pi$ (mainly d -wave), $D_1(2430) \rightarrow D^*\pi$ (s -wave), $D_2^*(2460) \rightarrow D^*\pi + D\pi$ [$\Gamma(D\pi)/\Gamma(D^*\pi) \sim 1.5$ [4]], $D^* \rightarrow D\pi$, and $D_{s1}(2536) \rightarrow D^*K$ (d -wave) saturate their widths. The $D^* \rightarrow D\pi$ and $D^* \rightarrow D^*\pi$ coupling constants are related by the HQSS [112]; $g_{D^*\pi,D^*}^{11}/g_{D\pi,D^*}^{10} = \sqrt{2m_{D^*}/m_D}$ within our definition. A small s -wave decay of $D_1(2420)$ is also included to reproduce the helicity angle distribution [113]. The $D_1(2420)$ d -wave decay partial width is 98.7% of the total decay width. This is rather different from 48% in Ref. [34], where their d -wave $D_1(2420) \rightarrow D^*\pi$ coupling constant was determined using a HQSS relation with $D_2^*(2460) \rightarrow D^{(*)}\pi$ couplings; the $D_2^*(2460)$ couplings were fitted to experimental $\Gamma_{D_2^*(2460)}$.

2. Group (C) in Table I

Bare R [$D_0^*(2300)$, f_0 , f_2 , or $Z_{c(s)}$] states and/or contact interactions are implemented in a two-body unitary coupled-channel model and are dressed to form pole(s) in the scattering amplitude. These two-body models are described below.

We consider an $ab \rightarrow a'b'$ partial-wave scattering with the total energy E , the orbital angular momentum L , and the total isospin I . We denote the incoming and outgoing momenta by q and q' , respectively. First, we introduce a contact interaction for the scattering:

$$v_{a'b',ab}^{LI}(q', q) = w_{a'b'}^{LI}(q') h_{a'b',ab}^{LI} w_{ab}^{LI}(q), \quad (\text{A1})$$

where $h_{a'b',ab}^{LI}$ is a coupling constant. A vertex function $w_{ab}^{LI}(q)$ is also introduced in the form of

$$w_{ab}^{LI}(q) = \frac{1}{\sqrt{4E_a(q)E_b(q)}} \frac{(q/m_\pi)^L}{[1 + (q/b_{ab}^{LI})^2]^{2+\frac{L}{2}}}, \quad (\text{A2})$$

with a cutoff b_{ab}^{LI} . The corresponding partial wave amplitude is

$$t_{a'b',ab}^{LI}(q', q; E) = \sum_{a''b''} w_{a'b'}^{LI}(q') \tau_{a'b',a''b''}^{LI}(E) \times h_{a''b'',ab}^{LI} w_{ab}^{LI}(q), \quad (\text{A3})$$

with

$$[(\tau^{LI}(E))^{-1}]_{a'b',ab} = \delta_{a'b',ab} - \sigma_{a'b',ab}^{LI}(E), \quad (\text{A4})$$

$$\sigma_{a'b',ab}^{LI}(E) = \mathcal{B}_{ab} h_{a'b',ab}^{LI} \int dq q^2 [w_{ab}^{LI}(q)]^2 \times \left(\frac{1}{E - E_a(q) - E_b(q) + i\frac{\Gamma_a}{2} + i\frac{\Gamma_b}{2}} - \frac{1}{E + E_a(q) + E_b(q) + i\epsilon} \right), \quad (\text{A5})$$

where the Bose factor $\mathcal{B}_{ab}$ is $\mathcal{B}_{ab} = 1/2$ for identical particles a and b and $\mathcal{B}_{ab} = 1$ otherwise. For cases with $\Gamma_a = \Gamma_b = 0$, the above formula reduces to

$$\sigma_{a'b',ab}^{LI}(E) = \mathcal{B}_{ab} h_{a'b',ab}^{LI} \int dq q^2 \frac{2M_{ab}(q)[w_{ab}^{LI}(q)]^2}{E^2 - M_{ab}^2(q) + i\epsilon}. \quad (\text{A6})$$

Second, we include bare R -excitation mechanisms:

$$V_{a'b',ab}^{LI}(q', q; E) = \sum_R f_{a'b',R}^{LI}(q') \frac{1}{E^2 - m_R^2} f_{R,ab}^{LI}(q) + v_{a'b',ab}^{LI}(q', q). \quad (\text{A7})$$

The bare $R \rightarrow ab$ vertex function $f_{ab,R}^{LI}(q)$ [$f_{R,ab}^{LI}(q) = f_{ab,R}^{LI}(q)$] is defined in Eq. (4) and, thus, is plugged into Eq. (3). In this section, we use the superscript LI rather than LS in Eq. (4). The resulting scattering amplitude from the interaction of Eq. (A7) is

$$T_{a'b',ab}^{LI}(q', q; E) = \sum_{R',R} \bar{f}_{a'b',R'}^{LI}(q'; E) \tau_{R',R}^{LI}(0, E) \bar{f}_{R,ab}^{LI}(q; E) + t_{a'b',ab}^{LI}(q', q; E). \quad (\text{A8})$$

The second term has already been given in Eq. (A3). The dressed $R \rightarrow ab$ vertex $\bar{f}_{ab,R}$ is

$$\begin{aligned} \bar{f}_{ab,R}^{LI}(q; E) &= f_{ab,R}^{LI}(q) + \sum_{a'b'} \mathcal{B}_{a'b'} \int dq' q'^2 2M_{a'b'}(q') \\ &\times \frac{t_{ab,a'b'}^{LI}(q, q'; E) f_{a'b',R}^{LI}(q')}{E^2 - M_{a'b'}^2(q') + i\epsilon}, \\ \bar{f}_{R,ab}^{LI}(q; E) &= f_{R,ab}^{LI}(q) + \sum_{a'b'} \mathcal{B}_{a'b'} \int dq' q'^2 2M_{a'b'}(q') \\ &\times \frac{f_{R,a'b'}^{LI}(q') t_{a'b',ab}^{LI}(q', q; E)}{E^2 - M_{a'b'}^2(q') + i\epsilon}, \end{aligned} \quad (\text{A9})$$

where we assumed $\Gamma_{a'} = \Gamma_{b'} = 0$. The dressed R Green function in Eq. (A8), $\tau_{R',R}^{LI}(p, E)$, is obtained from Eqs. (5)–(7) by setting $p = 0$ and replacing only one of $f_{ab,R'}^{LS}$ or $f_{R,ab}^{LS}$ with the dressed one of Eq. (A9).

The S matrix is related to the partial wave amplitude in Eq. (A8) by

TABLE VIII. Description of two-meson scattering models. Partial waves are specified by the orbital angular momentum L and the isospin I ; $J^{P(C)}$ is used for $R = Z_c$ and Z_{cs} .

R	$\{L, I\}$	Coupled channels (ab)	No. of bare R states	Contact interactions	No. of poles
f_0	$\{0, 0\}$	$\pi\pi, K\bar{K}$	2	Included	3
f_2	$\{2, 0\}$	$\pi\pi, K\bar{K}$	1	Not included	1
$D_0^*(2300)$	$\{0, 1/2\}$	$D\pi$	1	Included	1

R	$I, J^{P(C)}$	Coupled channels (ab)	No. of bare R states	Contact interaction	No. of poles
Z_c	$1, 1^{+-}$	$\frac{D^*\bar{D}-\bar{D}^*D}{\sqrt{2}}, D^*\bar{D}^*, J/\psi\pi, \psi'\pi, h_c\pi, \eta_c\rho$	0	Included	2
Z_{cs}	$\frac{1}{2}, 1^+$	$\frac{D_s^*\bar{D}-\bar{D}^*D_s}{\sqrt{2}}, J/\psi K$	0	Included	1

$$s_{ab,ab}^{LI}(E) = \eta_{LI} e^{2i\delta_{LI}} = 1 - 2\pi i \rho_{ab} \mathcal{B}_{ab} T_{ab,ab}^{LI}(q_o, q_o; E), \quad (\text{A10})$$

where the phase shift and inelasticity are denoted by δ_{LI} and η_{LI} , respectively, and q_o is the on-shell momentum [$E = E_a(q_o) + E_b(q_o)$]; $\rho_{ab} = q_o E_a(q_o) E_b(q_o) / E$ is the phase-space factor. The above formalism is used to calculate the $f_0, f_2, D_0^*(2300), Z_c$, and Z_{cs} amplitudes for which Table VIII specifies details.

We make several remarks on the $Z_{c(s)}$ amplitudes. We consider $L = 0$ in Eq. (A1), except for the $h_c\pi$ channel for which $L = 1$. By considering the heavy quark spin and SU(3) symmetries [98], we set the coupling strengths in Eq. (A1) as $h_{[D^*\bar{D}], [D^*\bar{D}]} = h_{D^*\bar{D}^*, D^*\bar{D}^*} = h_{[D_s^*\bar{D}], [D_s^*\bar{D}]}$, where $[D^*\bar{D}] \equiv \frac{D^*\bar{D}-\bar{D}^*D}{\sqrt{2}}$ and $[D_s^*\bar{D}] \equiv \frac{D_s^*\bar{D}-\bar{D}^*D_s}{\sqrt{2}}$. Also, we set

$h_{[D_s^*\bar{D}], J/\psi K} \sim h_{[D^*\bar{D}], J/\psi\pi}$ from the SU(3). We assume no interactions between the hidden-charm channels such as $J/\psi\pi, \psi'\pi, h_c\pi$, and $\eta_c\rho$. For $\eta_c\rho$ loops, we consider the ρ width (Γ_ρ). Thus, we use Eq. (A5) with $\Gamma_a = \Gamma_\rho = 150$ MeV and $\Gamma_b = 0$ rather than Eq. (A6).

The coupling parameters in the two-meson scattering models are determined as follows. Our f_0 and f_2 amplitudes are fitted to the empirical $\pi\pi$ s - and d -wave phase shifts and inelasticities [114]. The $D\pi$ s -wave (D_0^*) amplitude is fitted to an amplitude based on the lattice QCD (LQCD) spectrum [115]. Our fits are shown in Fig. 18. The Z_c amplitude is determined in the global fit. Numerical values of the fitting parameters are given in Tables IX–XI.

After the fits, resonance poles are extracted from the amplitudes and presented in Table XII. The Z_c poles are presented in Table VI. Z_{cs} pole is not shown, because our

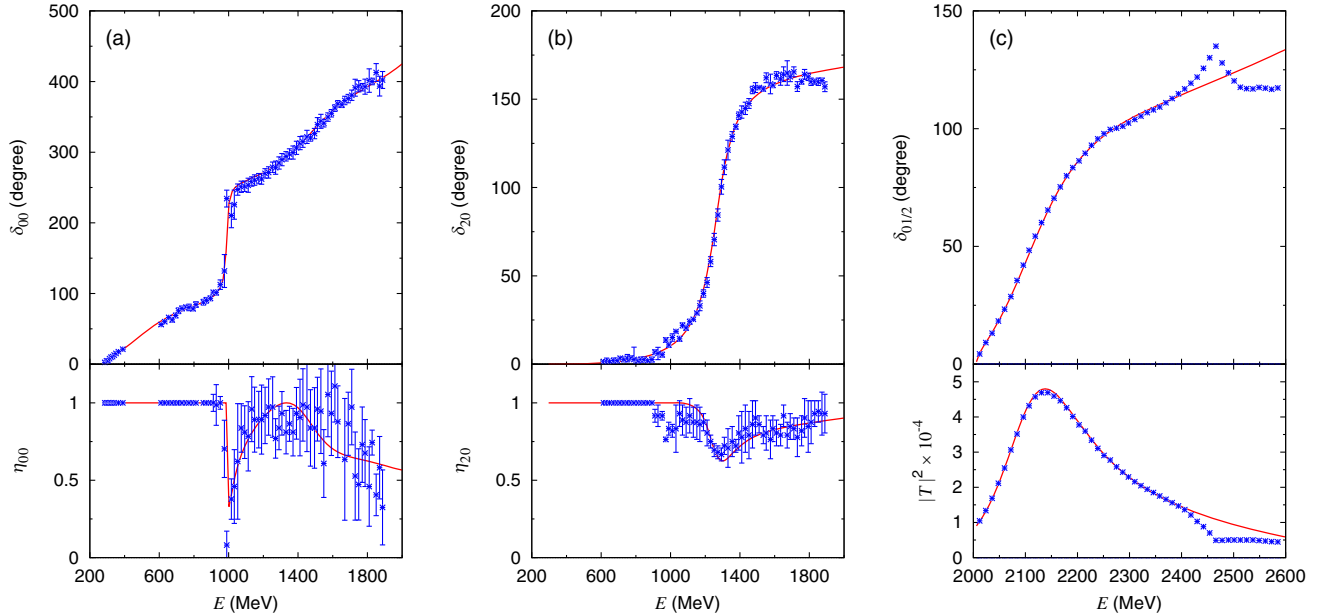


FIG. 18. (a),(b) The $\pi\pi$ scattering amplitudes. Phase shifts and inelasticities are shown in the upper and lower panels, respectively. Data are from Ref. [114]. (a) $\{L, I\} = \{0, 0\}$. (b) $\{L, I\} = \{2, 0\}$. (c) The $D\pi$ scattering amplitude for $\{L, I\} = \{0, 1/2\}$. The phase shifts and modulus of the amplitudes are shown in the upper and lower panels, respectively. The blue points are from Ref. [115] (errors are not shown), and the red curve is our fit.

TABLE IX. Parameter values for the $\pi\pi$ partial wave scattering model. The i th bare R states (R_i) has a mass of m_{R_i} , and it decays into h_1 and h_2 particles with a coupling ($g_{h_1 h_2, R_i}$) and a cutoff ($c_{h_1 h_2, R_i}$). Couplings and cutoffs for contact interactions are denoted by $h_{h_1 h_2, h_1 h_2}$ and $b_{h_1 h_2}$, respectively. For simplicity, we suppress the superscripts LI of the parameters. The mass and cutoff values are given in the unit of MeV, and the couplings are dimensionless.

$R\{L, I\}$	$f_0 \{0, 0\}$	$f_2 \{2, 0\}$
m_{R_1}	1055	1633
$g_{\pi\pi, R_1}$	22.0	-0.689
$c_{\pi\pi, R_1}$	1195	1448
$g_{K\bar{K}, R_1}$	-6.29	0.487
$c_{K\bar{K}, R_1}$	1173	1628
m_{R_2}	1769	...
$g_{\pi\pi, R_2}$	-27.7	...
$c_{\pi\pi, R_2}$	1195	...
$g_{K\bar{K}, R_2}$	17.2	...
$c_{K\bar{K}, R_2}$	1173	...
$h_{\pi\pi, \pi\pi}$	13.7	...
$h_{\pi\pi, K\bar{K}}$	-2.76	...
$h_{K\bar{K}, K\bar{K}}$	-3.68	...
$b_{\pi\pi}$	1195	...
$b_{K\bar{K}}$	1173	...

dataset hardly constrains it. The f_0 and f_2 pole locations are consistent with the PDG averages [4]. The LQCD-based amplitude has a D_0^* pole at $2105^{+6}_{-8} - i102^{+10}_{-12}$ MeV [115], and our D_0^* pole in Table XII is consistent.¹² D_0^* poles from previous LQCD analyses (at physical mass) are not yet well determined; for example, see Fig. 2 in Ref. [117]. Thus, our choice of the $D\pi$ s -wave amplitude in Fig. 18(c) should be regarded as an assumption in our coupled-channel model. Because the present global analysis does not include data of $D\pi$ invariant-mass distribution in $e^+e^- \rightarrow \pi D\bar{D}^*$, the fit quality would not sensitively depend on the D_0^* pole location.

3. Contact interactions between open-charm channels

For the interactions v^s in Eq. (13), we consider contact interactions between 11 open-charm channels ($IJ^{PC} = 01^{--}$): $D^{(*)}\bar{D}^{(*)}$, $D_s^{(*)}\bar{D}_s^{(*)}$, $D_1(2420)\bar{D}^{(*)}$, $D_2^*(2460)\bar{D}^*$, $D_{s1}(2536)\bar{D}_s$, and $\Lambda_c\bar{\Lambda}_c$. Labeling the channels with α (L -wave, total spin S) and β (L' , S'), our interaction potential for an $\alpha \rightarrow \beta$ process is given by

$$v_{\beta, \alpha}^s(p', p) = f_{\beta}^{L'}(p') C_{\beta\alpha} f_{\alpha}^L(p), \quad (\text{A11})$$

¹²The analyses of the LQCD spectra in Refs. [115, 116] also identified a higher pole slightly below the $D_s\bar{K}$ threshold.

TABLE X. Parameter values for the $D\pi$ partial wave scattering model. See Table IX for the description.

$R\{L, I\}$	$D_0^* \{0, 1/2\}$
m_{R_1}	2294
$g_{D\pi, R_1}$	24.9
$c_{D\pi, R_1}$	1000
$h_{D\pi, D\pi}$	6.26
$b_{D\pi}$	1000

TABLE XI. Parameter values for the Z_c and Z_{cs} amplitude models. The cutoff is $b_x = 1000$ MeV for all channels x . See Table IX for the description and the text for the notations.

$RIJ^{P(C)}$	$Z_c 11^{+-}, Z_{cs} \frac{1}{2} 1^+$
$h_{[D^* \bar{D}], [D^* \bar{D}]} = h_{D^* \bar{D}^*, D^* \bar{D}^*} = h_{[D_s^* \bar{D}], [D_s^* \bar{D}]}$	-4.00
$h_{[D^* \bar{D}], D^* \bar{D}^*}$	-3.59
$h_{[D^* \bar{D}], J/\psi\pi}$	-4.23
$h_{[D^* \bar{D}], \psi'\pi}$	2.77
$h_{[D^* \bar{D}], h_c\pi}$	0.805
$h_{[D^* \bar{D}], \eta_c\rho}$	-5.95
$h_{D^* \bar{D}^*, J/\psi\pi}$	-0.714
$h_{D^* \bar{D}^*, \psi'\pi}$	4.02
$h_{D^* \bar{D}^*, h_c\pi}$	2.25
$h_{D^* \bar{D}^*, \eta_c\rho}$	1.53
$h_{[D_s^* \bar{D}], J/\psi K}$	-4 (fixed)

where $C_{\beta\alpha}$ is a coupling constant and $C_{\beta\alpha} = C_{\alpha\beta}$. The dipole form factor f_{α}^L is given by

$$f_{\alpha}^L(p) = \frac{1}{\sqrt{4E_{1\alpha}E_{2\alpha}}} \left(\frac{\Lambda^2}{\Lambda^2 + p^2} \right)^{2+L/2} \left(\frac{p}{m_{\pi}} \right)^L, \quad (\text{A12})$$

where $E_{i\alpha}$ is the energy of an i th particle in the channel α ; $\Lambda = 1$ GeV is used. We consider $\{L, S\} = \{0, 1\}$ for $D_1(2420)\bar{D}^{(*)}$, $D_2^*(2460)\bar{D}^*$, $D_{s1}(2536)\bar{D}_s$, and $\Lambda_c\bar{\Lambda}_c$ and $\{L, S\} = \{1, 0\}$, $\{1, 1\}$, $\{1, 0\}$ for $D_{(s)}\bar{D}_{(s)}$, $D_{(s)}^*\bar{D}_{(s)}$, and $D_{(s)}^*\bar{D}_{(s)}^*$, respectively; $\{L, S\} = \{1, 2\}$ is not

TABLE XII. Pole positions (M_{pole}) in our $\pi\pi$ and $D\pi$ scattering amplitudes. The Riemann sheets (RS) of the pole positions are specified by $(s_{\pi\pi}, s_{K\bar{K}})$ for $\pi\pi$ and $(s_{D\pi})$ for $D\pi$; $s_x = p(u)$ indicates that a pole is on the physical (unphysical) sheet of the channel x .

$\{L, I\}$	M_{pole} (MeV)	RS	Name
$\{0, 0\}$	461 - 252 <i>i</i>	(<i>up</i>)	$f_0(500)$
	994 - 11 <i>i</i>	(<i>up</i>)	$f_0(980)$
	1426 - 204 <i>i</i>	(<i>uu</i>)	$f_0(1370)$
$\{2, 0\}$	1245 - 100 <i>i</i>	(<i>uu</i>)	$f_2(1270)$
$\{0, 1/2\}$	2104 - 100 <i>i</i>	(<i>u</i>)	$D_0^*(2300)$

TABLE XIII. Parameter values for $C_{\beta\alpha}$ in Eq. (A11). Dots indicate unused parameters.

$\beta \backslash \alpha$	$D\bar{D}$	$D^*\bar{D}$	$D^*\bar{D}^*$	$D_1\bar{D}$	$D_1\bar{D}^*$	$D_2^*\bar{D}^*$	$D_s\bar{D}_s$	$D_s^*\bar{D}_s$	$D_s^*\bar{D}_s^*$	$D_{s1}\bar{D}_s$	$\Lambda_c\bar{\Lambda}_c$
$D\bar{D}$	-1.02	0.475	0.385	...	6.43	-1.51	...	0.316	...	...	...
$D^*\bar{D}$		-1.02	0.135	-1.85	1.28	...	...	...	...	...	-1.27
$D^*\bar{D}^*$			-0.460	...	...	5.05	...	0.468	...	...	1.27
$D_1\bar{D}$				-20.9	...	...	...	...	...	...	11.3
$D_1\bar{D}^*$					-10.2	-2.02	-1.50	...	...	...	18.1
$D_2^*\bar{D}^*$						...	-0.887	...	...	...	-3.39
$D_s\bar{D}_s$							-1.01	-0.0724	0.508	-4.08	...
$D_s^*\bar{D}_s$								...	-1.15	...	...
$D_s^*\bar{D}_s^*$									-0.181	-2.60	-1.00
$D_{s1}\bar{D}_s$										-30.5	...
$\Lambda_c\bar{\Lambda}_c$											-20.1

considered for $D_{(s)}^*\bar{D}_{(s)}^*$. $C_{\alpha\beta}$ values from the global fit are listed in Table XIII.

We have introduced two types of contact interactions in Eqs. (A1) and (A11). The contact interactions in Eq. (A1) [Eq. (A11)] work on ab [Rc] pair in Fig. 1(a).

APPENDIX B: PARTICLE-EXCHANGE MECHANISMS (Z DIAGRAMS)

As introduced in Eq. (13), we consider particle-exchange mechanisms (Z diagrams) depicted in Fig. 19. These mechanisms are essential to satisfy the three-body coupled-channel unitarity. A particle($\bar{c}$)-exchange

$Rc \rightarrow R'c'$ mechanism is given by

$$Z_{R'c',Rc}^{\bar{c}}(\mathbf{p}_{c'}, \mathbf{p}_c; E) = \frac{[\Gamma_{c\bar{c},R'}(\mathbf{p}_c^*)]^* \Gamma_{c'\bar{c},R}(\mathbf{p}_{c'})}{E - E_c - E_{c'} - E_{\bar{c}} + ie}, \quad (\text{B1})$$

with $\mathbf{p}_{\bar{c}} = -\mathbf{p}_c - \mathbf{p}_{c'}$. The $R \rightarrow c'\bar{c}$ vertices $\Gamma_{c'\bar{c},R}(\mathbf{p}_a^*)$ have been defined in Eq. (3). For a case where $c'\bar{c}$ interact via a contact interaction of Eq. (A1), Eq. (B1) is modified to include $h_{c'\bar{c}',ab}^{LI}$ and $w_{c'\bar{c}}^{LI}$ in Eq. (A1), as detailed in Ref. [65]. Equation (B1) is projected onto a partial wave with the total angular momentum J , total isospin I , and $C = -1$:

$$Z_{(R'c')_{I'J'},(Rc)_{Is}}^{\bar{c}IJ^{PC}}(\mathbf{p}_{c'}, \mathbf{p}_c; E) = d_{R'c',Rc} \frac{\sqrt{4\pi(2I+1)}}{(2I+1)(2J+1)} \sum_{(\text{iso})\text{spins}} (t_R t_R^z t_c t_c^z | II^z) (t_{R'} t_{R'}^z t_{c'} t_{c'}^z | II^z) (s_R s_R^z s_c s_c^z | s s^z) \\ \times (s_{R'} s_{R'}^z s_{c'} s_{c'}^z | s' s'^z) (I 0 s s^z | JM) (I' m' s' s'^z | JM) \int d\Omega_{\hat{p}} Y_{I'm'}^*(\hat{p}) Z_{R'c',Rc}^{\bar{c}}(\mathbf{p}_{c'}, \mathbf{p}_c \mathbf{e}_z; E), \quad (\text{B2})$$

where $\mathbf{p}_c$ is taken along the z axis and $\hat{p} = -\hat{p}_{c'}$; $\sum_{(\text{iso})\text{spins}}$ indicates the summation of all z components in the CG coefficients. The use of $C = -1$ base states is accompanied by $d_{R'c',Rc}$ in Eq. (B2), and their values along with the exchanged particles $\bar{c}$ are listed in Tables XIV and XV. Since the partial wave form of Eq. (B2) with $IJ^{PC} = 01^{--}$ is used in the present analysis, the label IJ^{PC} is suppressed in Eq. (13).

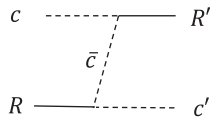


FIG. 19. Particle($\bar{c}$)-exchange $Rc \rightarrow R'c'$ mechanism (Z diagram).

APPENDIX C: POLE UNCERTAINTY ESTIMATIONS

1. Vector charmonium poles

In a standard procedure of estimating uncertainties of (resonance) pole values, all fitting parameters are varied around the minimum χ^2 to generate an error matrix of the parameters. The matrix is then used to propagate the parameter errors to the pole-value errors. In our global coupled-channel analysis, however, this procedure is practically impossible. This is because the cross-section calculation is rather time consuming for the four-dimensional phase-space integral in Eq. (23), and we have too many 200 fitting parameters (Tables XI, XIII, and XVI–XVIII) in our model to reach a convergence in the χ^2 minimization. This problem is common in global coupled-channel analyses for

TABLE XIV. Values of $d_{R'c', Rc}$ in Eq. (B2) for the $Rc \rightarrow R'c' Z$ diagrams. Exchanged particles ($\bar{c}$) or their charge conjugates are either indicated in the parentheses or $\bar{c} = \pi$ for other entries with nonzero $d_{R'c', Rc}$.

$Rc \backslash R'c'$	$D\bar{D}$	$D^*\bar{D}$	$D^*\bar{D}^*$	$D_0^*\bar{D}^*$	$D_1\bar{D}$	$D_1'\bar{D}$	$D_2^*\bar{D}$	$D_1\bar{D}^*$	$D_1'\bar{D}^*$	$D_2^*\bar{D}^*$	f_0J/ψ	f_2J/ψ	$f_0\psi'$	f_0h_c	$Z_c\pi$
$D\bar{D}$	...	...	-2	$-\sqrt{2}$	...	...	...	...	...	$-\sqrt{2}$	...	...	...	...	$\sqrt{2}(D^*)$
$D^*\bar{D}$		-1	$-\sqrt{2}$	-1	...	...	-1	-1	-1	-1	...	...	...	...	$1(D^*)$
$D^*\bar{D}^*$			-2	...	$-\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{2}$	...	...	...	...	$-\sqrt{2}(D), -2(D^*)$
$D_0^*\bar{D}^*$				...	-1	-1	-1	...	...	...	...	...	...	...	$-1(D)$
$D_1\bar{D}$					...	...	...	...	...	-1	...	...	...	...	$1(D^*)$
$D_1'\bar{D}$						...	...	...	...	-1	...	...	...	...	$1(D^*)$
$D_2^*\bar{D}$							-1	...	...	-1	...	...	...	...	$1(D^*)$
$D_1\bar{D}^*$								-1	-1	-1	...	...	...	...	$-\sqrt{2}(D^*)$
$D_1'\bar{D}^*$									-1	-1	...	...	...	...	$-\sqrt{2}(D^*)$
$D_2^*\bar{D}^*$										-1	...	...	...	...	$-1(D), -\sqrt{2}(D^*)$
f_0J/ψ											...	...	...	...	1
f_2J/ψ												...	...	...	1
$f_0\psi'$													...	...	1
f_0h_c														...	1
$Z_c\pi$															$1(J/\psi), 1(\psi')$

nucleon resonances [86,87]. We, thus, use a practical uncertainty estimation method hinted by Ref. [86].

As discussed in Sec. III A, we first adjust 200 fitting parameters and obtain a default fit to the data shown in Figs. 3–9. Then, we introduce complex parameters δm_{ψ_i} into Eq. (18) as

$$m_{\psi_i} \rightarrow m_{\psi_i} + \delta m_{\psi_i}. \quad (\text{C1})$$

To obtain an error matrix, we select parameters that would relatively largely influence the pole values and refit them to the data. We vary 85 parameters in total: δm_{ψ_i} ; $\psi(4660)$ BW mass and width in Eq. (20); real scaling factors multiplied to the $\psi(4660)$ and $\psi(4710)$ amplitudes in Eq. (20) and to the NR amplitude $\bar{\Gamma}_{R'c, \gamma^*}^\mu$ in Eq. (1); bare $\gamma^* \rightarrow \psi_i$ couplings g_{ψ_i} in Eq. (17); coupling constants $C_{(Rc)_{1s}}^i$ in Eq. (11) and $C_{(Rc)_{1s}}^{\gamma^*}$ in Eq. (14) for bare $\psi_i, \gamma^* \rightarrow$ open-charm channels; coupling constants $C_{\beta\alpha}$ in Eq. (A11) that are diagonal ($\beta = \alpha$) or whose absolute values are larger than 10, 0.5, and $\sqrt{10} \times 0.5$ for s -wave, p -wave, and s -wave $\leftrightarrow$ p -wave interactions, respectively. Effects of the other fixed parameters to pole uncertainty are simulated by $\text{Im}[\delta m_{\psi_i}]$ in Eq. (C1). The obtained error matrix is used to estimate the vector charmonium pole uncertainties in Table III through the standard error propagation. We note that the pole values from the default fit and those (central values) obtained from the refit are slightly different for two reasons: (i) The imaginary parts of δm_{ψ_i} are degrees of freedom not existing in the default model. (ii) Some data points are weighted in the default fit so that the fit is overall reasonable. The above error analysis is done with unweighted data. The purpose of this error analysis is to estimate how much the pole values can fluctuate due to the

experimental errors. Thus, in Table III, the pole central values are from the default fit and their uncertainties are from the above uncertainty analysis.

2. Z_c poles

We estimate uncertainties of the Z_c poles as below. We select parameters that are presumably most relevant to the Z_c poles and vary them around the default values to refit the data and obtain an error matrix; the other parameters are fixed at their default-fit values. Then the error matrix is used to estimate Z_c -pole uncertainties through the standard error propagation. The 63 fitting parameters in the refit are the Z_c amplitude parameters in Table XI; δm_{ψ_i} in Eq. (C1); $\psi(4660)$ BW mass and width in Eq. (20); real scaling factors multiplied to the $\psi(4660)$ and $\psi(4710)$ amplitudes in Eq. (20) and to the NR amplitude; bare $\gamma^* \rightarrow \psi_i$ couplings g_{ψ_i} in Eq. (17); coupling constants $C_{(Rc)_{1s}}^i$ in Eq. (11) and $C_{(Rc)_{1s}}^{\gamma^*}$ in Eq. (14) for bare $\psi_i, \gamma^* \rightarrow (D_2\bar{D})_{22}, (h_c f_0^1)_{11}$, and $(Rc)_{01}$ with $Rc = D_1^{(\prime)}\bar{D}^{(*)}, D_2\bar{D}^*, J/\psi f_0^1$, and $\psi' f_0^1$; and coupling constants $C_{\beta\alpha}$ in Eq. (A11) for which $\alpha = \beta = D_1\bar{D}^{(*)}$. The Z_c pole uncertainties are listed in Table VI. The pole central values in Table VI are from the default fit for the reasons discussed in Appendix C 1.

TABLE XV. Continued from Table XIV.

$Rc \backslash R'c'$	$D_{s1}\bar{D}_s$	f_0J/ψ	f_2J/ψ	$Z_{cs}\bar{K}$
$D_{s1}\bar{D}_s$	...	...	...	$-1/\sqrt{2}(D^*)$
f_0J/ψ		...	...	$\sqrt{2}(K)$
f_2J/ψ			...	$\sqrt{2}(K)$
$Z_{cs}\bar{K}$				$1(J/\psi)$

TABLE XVI. Parameter values for i th bare ψ states ($i = 1, \dots, 5$). Bare ψ_i masses [m_{ψ_i} in Eq. (18)], bare $\psi_i \rightarrow Rc$ coupling constants [$C_{(Rc)ls}^i$ in Eq. (11)], and bare ψ_i photoproduction couplings [g_{ψ_i} in Eq. (17)] are listed. In the last column ($i = \gamma^*$), nonresonant $\gamma^* \rightarrow Rc$ coupling constants [$C_{(Rc)ls}^{\gamma^*}$ in Eq. (14)] are listed. Dots indicate parameters either unused (thus, 0) or nonexistent.

	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = \gamma^*$
m_{ψ_i} (MeV)	3785	4199	4354	4518	4522	...
$C_{(D_1\bar{D})_{01}}^i$	...	-19.1	...	18.7	29.2	2.29
$C_{(D_1\bar{D})_{21}}^i$	...	...	...	...	...	...
$C_{(D_1'\bar{D})_{01}}^i$	...	...	...	31.3	6.39	3.88
$C_{(D_1'\bar{D})_{21}}^i$	...	...	...	...	...	...
$C_{(D_2^*\bar{D})_{22}}^i$	...	...	...	2.54	...	-0.307
$C_{(D_1\bar{D}^*)_{01}}^i$	...	...	...	-26.1	-56.7	-1.70
$C_{(D_1\bar{D}^*)_{21}}^i$	...	...	...	...	...	...
$C_{(D_1'\bar{D}^*)_{01}}^i$	...	14.5	35.9	...	...	-0.760
$C_{(D_1'\bar{D}^*)_{21}}^i$	...	...	...	...	...	...
$C_{(D_2^*\bar{D}^*)_{01}}^i$	...	...	...	...	-39.3	3.19
$C_{(D_2^*\bar{D}^*)_{21}}^i$	...	...	...	...	...	...
$C_{(D\bar{D})_{10}}^i$	1.87	-0.378	-1.63	-1.15	-5.37	-0.197
$C_{(D^*\bar{D})_{11}}^i$	...	-1.07	0.446	-1.00	-1.94	0.980
$C_{(D^*\bar{D}^*)_{10}}^i$	...	6.87	-5.43	1.66	-2.34	0.418
$C_{(D_{s1}\bar{D}_s)_{01}}^i$	...	...	...	...	...	-0.0520
$C_{(D_s\bar{D}_s)_{10}}^i$	...	-0.246	1.18	0.0533	1.58	0.234
$C_{(D_s^*\bar{D}_s)_{11}}^i$	...	1.31	5.00	3.41	-3.15	0.557
$C_{(D_s^*\bar{D}_s^*)_{10}}^i$	...	-3.85	-3.42	0.0924	-2.87	1.04
$C_{(f_0^1 J/\psi)_{01}}^i$	...	-1.76	4.68	...	-3.46	0.582
$C_{(f_0^1 J/\psi)_{21}}^i$	...	0.209	-0.556	...	0.486	-0.0157
$C_{(f_0^2 J/\psi)_{01}}^i$	...	...	-8.63	...	...	...
$C_{(f_0^2 J/\psi)_{21}}^i$	...	0.214	-0.413	...	...	-0.0437
$C_{((\pi\pi)_{00} J/\psi)_{01}}^i$	...	-0.532	2.78	...	-0.606	...
$C_{((\pi\pi)_{00} J/\psi)_{21}}^i$	-0.0298	0.0356	-0.160	...	0.111	...
$C_{((K\bar{K})_{00} J/\psi)_{01}}^i$	...	...	-0.592	-0.174	-0.232	0.0347
$C_{((K\bar{K})_{00} J/\psi)_{21}}^i$	...	...	-0.0296	...	...	-0.0729
$C_{(f_2 J/\psi)_{01}}^i$	...	...	...	-0.311	-1.91	...
$C_{(f_2 J/\psi)_{21}}^i$	...	...	...	...	...	...
$C_{(f_0^1 \psi')_{01}}^i$	...	-6.09	7.27	11.4	-8.78	-0.858
$C_{(f_0^1 \psi')_{21}}^i$	...	...	1.24	-1.31	-0.329	...
$C_{(f_0^2 \psi')_{01}}^i$	...	6.37	-10.5	-12.2	16.2	-0.541
$C_{(f_0^2 \psi')_{21}}^i$	...	-2.75	-1.01	-1.96	-3.05	-0.286
$C_{((\pi\pi)_{00} \psi')_{01}}^i$	...	-1.15	1.68	2.26	-1.89	-0.236
$C_{((\pi\pi)_{00} \psi')_{21}}^i$	...	0.300	0.216	0.0751	0.399	0.0414
$C_{(f_0^1 h_c)_{11}}^i$	...	...	-2.14	1.31	1.74	...
$C_{(f_0^2 h_c)_{11}}^i$	...	...	...	...	...	...
$C_{((\pi\pi)_{00} h_c)_{11}}^i$	...	-0.190	...	0.584	...	...
$C_{(J/\psi\eta)_{11}}^i$	0.101	0.456	0.127	-0.347	-0.414	...
$C_{(J/\psi\eta')_{11}}^i$	...	-0.0659	-0.229	-0.0989	0.130	...
$C_{(\omega\chi_{c0})_{01}}^i$	...	-1.20	0.556	-2.53	-3.28	...
$C_{(\Lambda_c \bar{\Lambda}_c)_{01}}^i$	...	...	...	...	...	1.22
g_{ψ_i}	25.1	22.1	-109	115	-47.3	...

TABLE XVII. Numerical values of cutoffs (unit: MeV) for bare $\psi_i \rightarrow Rc$ vertices [$\Lambda_{(Rc)ls}^i$ in Eq. (11)] and nonresonant bare $\gamma^* \rightarrow Rc$ vertices [$\Lambda_{(Rc)ls}^{\gamma^*}$ in Eq. (14)]. We use cutoffs not dependent on the label “ i ” in ψ_i . Cutoffs not shown are fixed to 1000 MeV.

$\Lambda_{(D\bar{D})10}^{\gamma^*}$	1373
$\Lambda_{(D^*\bar{D})11}^{\gamma^*}$	1395
$\Lambda_{(D^*\bar{D}^*)10}^{\gamma^*}$	1999
$\Lambda_{(D_s\bar{D}_s)10}^{\gamma^*}$	1052
$\Lambda_{(D_s^*\bar{D}_s)11}^{\gamma^*}$	700
$\Lambda_{(D_s^*\bar{D}_s^*)10}^{\gamma^*}$	903
$\Lambda_{(\Lambda_c\bar{\Lambda}_c)01}^{\gamma^*}$	1838
$\Lambda_{(f_0^1\psi')01}^i$	700 (fixed)
$\Lambda_{(f_0^2\psi')21}^i$	700 (fixed)
$\Lambda_{(f_0^2\psi')01}^i$	700 (fixed)
$\Lambda_{(f_0^2\psi')21}^{\gamma^*}$	700 (fixed)
$\Lambda_{((\pi\pi)_{00}\psi')21}^{\gamma^*}$	700 (fixed)

TABLE XVIII. Numerical values for parameter in $\psi(4660)$ and $\psi(4710)$ BW amplitudes of Eq. (20); $\psi_6 = \psi(4660)$ and $\psi_7 = \psi(4710)$.

	$i = 6$	$i = 7$
m_{ψ_i} (MeV)	4655	4710 (fixed)
Γ_{ψ_i} (MeV)	134	152
ϕ_{ψ_i}	2.02	-2.87
$C_{(D_1\bar{D}^*)01}^i/g_{\psi_i}$	0.523	-0.559
$C_{(D_{s1}\bar{D}_s)01}^i/g_{\psi_i}$	0.0178	...
$C_{(D_s\bar{D}_s)10}^i/g_{\psi_i}$	0.00	-0.0157
$C_{(D_s^*\bar{D}_s)11}^i/g_{\psi_i}$	0.00663	-0.0221
$C_{(D_s^*\bar{D}_s^*)10}^i/g_{\psi_i}$	0.0365	0.0588
$C_{(\Lambda_c\bar{\Lambda}_c)01}^i/g_{\psi_i}$	-0.279	...
$C_{(f_0^1\psi')01}^i/g_{\psi_i}$	-0.253	...
$C_{((\pi\pi)_{00}\psi')01}^i/g_{\psi_i}$	-0.0473	...

APPENDIX D: PARAMETER VALUES FROM THE GLOBAL FIT AND HEAVY-QUARK SPIN SYMMETRY

1. Parameters from the global fit

Parameter values determined from the global fit to the $e^+e^- \rightarrow c\bar{c}$ data are listed in Tables XI, XIII, XVI–XVIII.

Parameters $C_{((ab)_{LI}c)ls}^i$ in Table XVI are defined as follows. For a two-meson scattering model including contact interactions, we consider a direct bare $\psi \rightarrow abc$ decay where the ab meson pair has an orbital angular momentum L and a total isospin I . This bare vertex function is given by [cf. Eq. (11)]

TABLE XIX. Short-range interaction parameters, $C_{a\beta}$ in Eq. (A11), from our global fit and those based on HQSS. The p -wave channels are labeled by $\alpha(\beta) = 1: D\bar{D}$; 2: $D^*\bar{D}$; 3: $D^*\bar{D}^*(S=0)$; 4: $D^*\bar{D}^*(S=2)$. See the text for details.

$C_{a\beta}$	Global fit	HQSS ($C_1 = C_3$)	HQSS ($C_1 \neq C_3$)
C_{11}	-1.023	-0.748	-0.987
C_{12}	0.475	0.160	0.298
C_{13}	0.384	0.171	0.426
C_{14}	...	-0.431	0.558
C_{22}	-1.022	-1.205	-1.022
C_{23}	0.135	-0.092	-0.172
C_{24}	...	0.749	-0.645
C_{33}	-0.459	-0.550	-0.495
C_{34}	...	0.249	-0.322
C_{44}	...	-1.476	-1.175
C_1	...	-0.451	-0.249
C_2	...	-2.279	-0.387
C_3	...	-0.451	-2.252
C_4	...	-0.799	-0.791

$$F_{((ab)_{LI}c)ls,\psi_i}(q) = C_{((ab)_{LI}c)ls}^i \left(\frac{q}{m_\pi}\right)^l \times \frac{[1 + q^2/(\Lambda_{((ab)_{LI}c)ls}^i)^2]^{-2-\frac{l}{2}}}{\sqrt{4E_c(q)m_{\psi_i}}}, \quad (\text{D1})$$

where $C_{((ab)_{LI}c)ls}^i$ and $\Lambda_{((ab)_{LI}c)ls}^i$ are coupling and cutoff parameters, respectively. The dressed vertices [Eq. (10)], dressed ψ productions [Eq. (16)], and self-energies [Eq. (19)] include the bare vertices $F_{(Rc)ls,\psi_i}$ in Eq. (11) and also $F_{((ab)_{LI}c)ls,\psi_i}$ in Eq. (D1) in a similar manner.

A remark on the cutoffs in Table XVII is in order. As discussed in the main text, we fix most cutoffs [$\Lambda_{(Rc)ls}^i$ in Eq. (11) for bare $\psi_i \rightarrow Rc$ vertices ($i = 1, \dots, 5$) and $\Lambda_{(Rc)ls}^{\gamma^*}$ in Eq. (14) for nonresonant $\gamma^* \rightarrow Rc$ vertices] to 1 GeV and some of them to 0.7 GeV. However, cutoffs in the nonresonant $\gamma^* \rightarrow D_{(s)}^{(*)}\bar{D}_{(s)}^{(*)}, \Lambda_c\bar{\Lambda}_c$ vertices need to be adjusted to fit the $e^+e^- \rightarrow D_{(s)}^{(*)}\bar{D}_{(s)}^{(*)}, \Lambda_c\bar{\Lambda}_c$ data.

2. HQSS

It is interesting to examine the consistency between the parameters determined from the global fit and those following the HQSS. The HQSS relation among p -wave $D\bar{D} - D^*\bar{D} - D^*\bar{D}^*$ coupled-channel interactions is given in Ref. [91] and that for s -wave $D_1\bar{D} - D_1\bar{D}^* - D_2^*\bar{D}^*$ interactions in Ref. [13].¹³ We find HQSS-based short-range interactions close to those in Table XIII by adjusting independent interaction parameters such as C_i in Eqs. (9) and (10) in Ref. [91] and $F_{Ij_e}^{d(c)}$ in Eq. (82) in Ref. [13].

¹³See Table VI and Eq. (85) in the arXiv version of [13].

TABLE XX. Continued from Table XIX. The s -wave channels are labeled by $\alpha(\beta) = 5: D_1\bar{D}$; 6: $D_1\bar{D}^*$; 7: $D_2^*\bar{D}^*$.

$C_{\alpha\beta}$	Global fit	HQSS
C_{55}	-20.9	-18.0
C_{56}	0	8.96
C_{57}	0	3.78
C_{66}	-10.2	-11.5
C_{67}	-2.02	-3.08
C_{77}	0	-6.70
F_{01}^d	...	-5.32
F_{02}^d	...	-12.8
F_{01}^c	...	0.365
F_{02}^c	...	12.8

TABLE XXI. HQSS-based relative branchings for S and D -wave charmonium decays to open-charm channels α , denoted by $\mathcal{B}_{\psi(S)\rightarrow\alpha}^{\text{HQSS}}$ and $\mathcal{B}_{\psi(D)\rightarrow\alpha}^{\text{HQSS}}$, respectively.

$\alpha(l=1)$	$D\bar{D}$	$D^*\bar{D}$	$D^*\bar{D}^*(S=0)$	$D^*\bar{D}^*(S=2)$
$\mathcal{B}_{\psi(S)\rightarrow\alpha}^{\text{HQSS}}$	$\frac{1}{12}$	$\frac{1}{3}$	$\frac{1}{36}$	$\frac{5}{9}$
$\mathcal{B}_{\psi(D)\rightarrow\alpha}^{\text{HQSS}}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{5}{36}$	$\frac{1}{36}$

$\alpha(l=0)$	$D_1\bar{D}$	$D_1\bar{D}^*$	$D_2^*\bar{D}^*$
$\mathcal{B}_{\psi(S)\rightarrow\alpha}^{\text{HQSS}}$	0	0	0
$\mathcal{B}_{\psi(D)\rightarrow\alpha}^{\text{HQSS}}$	$\frac{5}{8}$	$\frac{5}{16}$	$\frac{1}{16}$

The result is shown in¹⁴ Tables XIX and XX. In Table XIX, we show two cases where the HQSS-violating effect is either not included ($C_1 = C_3$) or included ($C_1 \neq C_3$). Our parameter set is fairly consistent with “HQSS($C_1 \neq C_3$)” in Table XIX and “HQSS” in Table XX.

Regarding the bare $\psi_i \rightarrow (Rc)_{I_S}$ branchings, the HQSS also provides relations as given by $\mathcal{B}_{\psi(S)\rightarrow\alpha}^{\text{HQSS}}$ and $\mathcal{B}_{\psi(D)\rightarrow\alpha}^{\text{HQSS}}$ in Table XXI; $\alpha \equiv (Rc)_{I_S}$. The relation between $D\bar{D} - D^*\bar{D} - D^*\bar{D}^*$ channels and that between $D_1\bar{D} - D_1\bar{D}^* - D_2^*\bar{D}^*$

¹⁴We can relate C_i in Ref. [91] (GeV^{-2}) and those in Table XIX (dimensionless) by dividing the latter by m_π^2 .

TABLE XXII. Ratios $R_{S,\alpha}^i$ and $R_{D,\alpha}^i$ defined in Eq. (D2).

$\alpha(l=1)$	$D\bar{D}$	$D^*\bar{D}$	$D^*\bar{D}^*(S=0)$
$R_{S,\alpha}^1$	42	...	...
$R_{D,\alpha}^1$	8	...	...
$R_{S,\alpha}^2$	2	3	1701
$R_{D,\alpha}^2$	0.3	3	340
$R_{S,\alpha}^3$	32	0.6	1060
$R_{D,\alpha}^3$	6	0.5	212
$R_{S,\alpha}^4$	16	3	99
$R_{D,\alpha}^4$	3	2	20
$R_{S,\alpha}^5$	346	11	198
$R_{D,\alpha}^5$	69	9	40

$\alpha(l=0)$	$D_1\bar{D}$	$D_1\bar{D}^*$	$D_2^*\bar{D}^*$
$R_{D,\alpha}^1$	...	...	...
$R_{D,\alpha}^2$	581	...	...
$R_{D,\alpha}^3$	...	...	...
$R_{D,\alpha}^4$	558	2185	...
$R_{D,\alpha}^5$	1364	10280	24662

channels are independent. We examine to what extent the relations hold in our model by listing in Table XXII

$$R_{S,\alpha}^i \equiv \frac{|C_\alpha^i|^2}{\mathcal{B}_{\psi(S)\rightarrow\alpha}^{\text{HQSS}}}, \quad R_{D,\alpha}^i \equiv \frac{|C_\alpha^i|^2}{\mathcal{B}_{\psi(D)\rightarrow\alpha}^{\text{HQSS}}}, \quad (\text{D2})$$

where C_α^i is the bare $\psi_i \rightarrow \alpha$ coupling constant defined in Eq. (11) and given in Table XVI. The HQSS indicates that, for a given i , either of $R_{S,\alpha}^i$ or $R_{D,\alpha}^i$ to be the same among $\alpha = D\bar{D}$, $D^*\bar{D}$, and $D^*\bar{D}^*$ and among $\alpha = D_1\bar{D}$, $D_1\bar{D}^*$, $D_2^*\bar{D}^*$, respectively. However, we do not find such a tendency in Table XXII, showing large HQSS violations in our model. We can find arguments that the HQSS can be often broken badly above the open-charm thresholds in Ref. [118]. Yet, an interesting future work would be to perform a global fit that implements the HQSS constraints to some extent.

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