

Kerr-Schild double copy of the Randall-Sundrum black string

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We construct the Kerr-Schild classical double copy of the black string in the Randall-Sundrum II model, deriving the single and zeroth copies and verifying the associated field equations. The single copy gauge field is independent of the holographic coordinate and satisfies a sourceless Maxwell equation on the curved background, in direct analogy with the Coulomb field of the Schwarzschild double copy. The zeroth copy scalar obeys a modified Klein-Gordon equation with a first-order derivative term along the extra dimension; a field redefinition yields a standard Klein-Gordon equation with effective mass $m^2 = 12/l^2$, induced by the warp factor. We further show that an alternative Kerr-Schild splitting, gravitationally equivalent to the canonical one, produces a physically inequivalent double copy: the gauge field is supported by a conserved but delocalized bulk current, and the zeroth copy satisfies a massless equation that carries no imprint of the warped extra dimension.

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I. INTRODUCTION

The relationship between gauge theories and gravity has long been a central theme in theoretical physics, appearing in contexts ranging from the AdS/CFT correspondence to the more recent double copy paradigm. The latter emerged from the Bern-Carrasco-Johansson color-kinematics duality in scattering amplitudes [1,2], which establishes that gravitational amplitudes can be systematically constructed from gauge-theory building blocks, schematically expressed as

$$\text{gravity} = (\text{gauge theory}) \times (\text{gauge theory}).$$

Although originally formulated for perturbative scattering amplitudes, this correspondence has been successfully extended to exact classical solutions [3], where spacetimes admitting a Kerr-Schild structure [4] possess a natural gauge counterpart.

In the Kerr-Schild double copy framework, the gravitational metric is written as

$$g_{MN} = \bar{g}_{MN} + \phi k_M k_N, \quad (1)$$

where $\bar{g}_{MN}$ is a background metric (typically flat or maximally symmetric), ϕ is a scalar function, and k^M is

a null vector field that is geodesic with respect to both the background and the full metric. A key feature of this ansatz is the linearization of the Einstein equations, enabling a direct identification of gauge-theory solutions. The single copy corresponds to the gauge field $A_M = \phi k_M$, satisfying Maxwell equations, while the zeroth copy is the scalar $\Phi = \phi$, obeying the corresponding scalar equation. The Kerr-Schild double copy has been extensively explored for a wide class of stationary, cosmological, and extended solutions [5–19].

An important subtlety of the Kerr-Schild ansatz is that the decomposition (1) is not unique. The rescaling

$$k_M \rightarrow a(x)k_M, \quad \phi \rightarrow \frac{\phi}{a(x)^2}, \quad (2)$$

leaves the metric invariant while producing different gauge-theory fields. As emphasized in [7], this ambiguity must be resolved by imposing physical criteria on the single copy, such as the absence of delocalized sources. Different splittings that are gravitationally equivalent can thus yield physically inequivalent gauge-theory descriptions, making the resolution of this ambiguity a nontrivial and physically meaningful step in the double copy construction.

A complementary line of research in gravitational physics concerns the possibility of extra spatial dimensions. This idea received renewed attention at the turn of the century with the Randall-Sundrum models [20,21], in which our four-dimensional universe is realized as a hypersurface (brane) embedded in a five-dimensional anti-de Sitter (AdS₅) bulk. Of particular relevance to the present work is the Randall-Sundrum II (RSII) model [21],

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where gravity is effectively localized on the brane even though the extra dimension is infinite and noncompact, provided the bulk geometry is appropriately warped [22]. In this setup, the metric takes the form

$$ds^2 = e^{-2|y|/l} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (3)$$

where l is the AdS_5 curvature radius and $y \in (-\infty, +\infty)$ is the extra-dimensional coordinate with a $\mathbb{Z}_2$ orbifold symmetry. The exponential warp factor renders the four-dimensional graviton zero mode normalizable, reproducing Newtonian gravity at large distances together with calculable short-distance corrections.

Black hole solutions incorporating the effects of the warped extra dimension have also been extensively studied [23–29], exhibiting genuine extra-dimensional features absent in standard four-dimensional general relativity. In particular, Ref. [23] showed that the Schwarzschild solution can be extended into the bulk as a “black string,” obtained by uniformly extending the four-dimensional geometry along the extra dimension.

Despite these developments, the classical double copy in the presence of warped extra dimensions remains, to the best of our knowledge, unexplored. The Randall-Sundrum framework provides a natural setting to address this question, combining AdS geometry, gravity localization, and exact black hole solutions that admit a Kerr-Schild representation. Understanding how characteristic brane-world features are encoded in gauge-theory data may offer new insights into both the double copy correspondence and extra-dimensional gravity.

In this work, we initiate the study of the classical double copy for black holes in Randall-Sundrum braneworlds by focusing on the RSII black string [23]. We derive the corresponding single and zeroth copies for the canonical Kerr-Schild splitting of [7] and verify the associated field equations, showing that the RSII warp factor leaves a direct imprint on the gauge-theory description. A central result of this work is that the ambiguity inherent in the Kerr-Schild decomposition is physically consequential in the warped setting: we construct an alternative splitting that is gravitationally equivalent to the canonical one but yields a physically inequivalent double copy, with a gauge field supported by a delocalized bulk current and a zeroth copy that carries no imprint of the warped extra dimension.

II. SETUP AND THEORETICAL FRAMEWORK

In this section we collect the technical ingredients underlying our analysis. We first review the Kerr-Schild double copy in curved backgrounds, focusing on the linearized field equations in the presence of a cosmological constant, and then present the RSII model and the black string solution that will serve as the gravitational starting point. We denote bulk indices by $M, N, \dots = 0, 1, \dots, D-1$, while

four-dimensional brane indices are denoted $\mu, \nu, \dots = 0, 1, 2, 3$. For the RSII model $D = 5$.

A. Classical double copy in curved backgrounds

The Kerr-Schild ansatz and the rescaling ambiguity were introduced in Eqs. (1) and (2). We collect here the additional properties needed in subsequent sections.

The nullity of k^M ensures that the inverse metric $g^{MN} = \bar{g}^{MN} - \phi k^M k^N$ remains linear in ϕ and that the determinant satisfies $\det(g) = \det(\bar{g})$. As a consequence, the Einstein equations linearize exactly in the perturbation $h_{MN} = \phi k_M k_N$. For a background $\bar{g}_{MN}$ solving Einstein’s equations with cosmological constant Λ_D , the linearized vacuum equations for the Kerr-Schild perturbation take the form [6,7]

$$\bar{\nabla}_P [2\bar{\nabla}_{(M} h_{N)}^P - \bar{\nabla}^P h_{MN}] = \frac{4\Lambda_D}{D-2} h_{MN}. \quad (4)$$

The gauge-theory copies are extracted by contracting this tensorial equation with Killing vectors of both the exact geometry and the background. Given a Killing vector ξ^M of both g_{MN} and $\bar{g}_{MN}$:

Single copy. A single contraction of (4) with ξ^M , combined with the Killing equation and the null geodesic properties of k_M , yields a Maxwell-type equation for the gauge field

$$A_M = \phi k_M. \quad (5)$$

The explicit form of this equation depends on the background geometry and the choice of Killing vector.

Zeroth copy. A further contraction with the same Killing vector produces a scalar equation for

$$\Phi = \phi. \quad (6)$$

In flat backgrounds ($\Lambda_D = 0$, $\bar{g}_{MN} = \eta_{MN}$), these reduce to the sourceless Maxwell and Klein-Gordon equations, respectively [3]. In curved backgrounds, the cosmological constant and the nontrivial connection generally modify both equations, as we shall see in detail for the RSII geometry. Moreover, the rescaling freedom (2) leaves the gravitational solution invariant but modifies the gauge-theory fields. We will exploit this freedom to construct both a canonical and an alternative realization of the double copy and show that they lead to physically distinct gauge-theory descriptions.

B. The RSII black string

The RSII model describes a single positive-tension three-brane embedded in an AdS_5 bulk, with the metric given by (3). Here $\eta_{\mu\nu}$ is the four-dimensional Minkowski metric, $y \in (-\infty, +\infty)$ is the extra-dimensional coordinate, and l is the AdS_5 curvature radius, related to the bulk cosmological

constant by $\Lambda_5 = -6/l^2$. The brane is located at $y = 0$ and a $\mathbb{Z}_2$ reflection symmetry $y \leftrightarrow -y$ is imposed so that the geometry on both sides of the brane is identical. The brane tension σ is related to l and the five-dimensional Newton constant G_5 through the Israel junction conditions [30,31] by $\sigma = 3/(4\pi G_5 l)$. A key property of this background is the exponential warp factor $e^{-2|y|/l}$, which localizes the four-dimensional graviton zero mode near the brane [21].

It will be convenient to introduce the conformal coordinate $z = le^{|y|/l}$, in terms of which the metric (3) becomes

$$ds^2 = \frac{l^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2). \quad (7)$$

Since $|y| \geq 0$, the conformal coordinate satisfies $z \geq l$, with the brane at $z = l$ and the Poincaré horizon at $z \rightarrow \infty$. The $\mathbb{Z}_2$ orbifold identifies the two sides of the brane, and we work on the fundamental domain $z \in [l, \infty)$. When needed for the Kerr-Schild construction, we extend the background metric (7) to the full Poincaré patch $z \in (0, \infty)$, which includes the AdS_5 boundary at $z \rightarrow 0$.

A central property underlying the construction of black hole solutions in this background is that the metric (7) is a conformal rescaling of flat space, so the five-dimensional Einstein equations with $\Lambda_5 = -6/l^2$ are automatically satisfied whenever the seed metric replacing $\eta_{\mu\nu}$ is Ricci flat [32]. Replacing $\eta_{\mu\nu}$ with the four-dimensional Schwarzschild metric therefore yields an exact solution known as the black string [23]

$$ds^2 = \frac{l^2}{z^2} (-f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_2^2 + dz^2), \quad (8)$$

where $f(r) = 1 - \frac{2M}{r}$, and $d\Omega_2^2 = d\theta^2 + \sin^2 \theta d\varphi^2$ is the metric on the unit two-sphere. The induced metric on the brane at $z = l$ is the four-dimensional Schwarzschild solution, consistent with the spherical symmetry of the seed. The solution describes a horizon at $r = 2M$ that extends uniformly along the extra dimension, forming a stringlike object in the five-dimensional bulk.

Although this is an Einstein metric, so that the Ricci scalar and the square of the Ricci tensor remain finite everywhere, the Kretschmann scalar

$$R_{MNPQ}R^{MNPQ} = \frac{1}{l^4} \left(40 + \frac{48M^2 z^4}{r^6} \right) \quad (9)$$

diverges both at $r = 0$ (the black string singularity) and as $z \rightarrow \infty$ (the Poincaré horizon), signaling a curvature singularity at the AdS_5 horizon. Moreover, the black string is subject to the Gregory-Laflamme instability [33] for sufficiently small masses. The end point of this instability is expected to be a stable “black cigar” that coincides with the black string in the vicinity of the brane but whose horizon closes off before reaching $z \rightarrow \infty$ [23].

Despite these limitations, the black string (8) provides an exact and analytically tractable solution that is well suited for studying the Kerr-Schild double copy in braneworld scenarios. In particular, its warped structure will play a central role in what follows.

III. CLASSICAL DOUBLE COPY OF THE RSII BLACK STRING

In this section we construct the classical double copy of the RSII black string (8). We recast the metric in Kerr-Schild form over the AdS_5 background, derive the associated single and zeroth copies following the prescription of [7], and verify the corresponding field equations on the curved background. We then consider an alternative Kerr-Schild splitting and show that it leads to a physically inequivalent double copy.

A. Kerr-Schild decomposition

To bring the RSII black string (8) into Kerr-Schild form over the AdS_5 background (7), we introduce ingoing Eddington-Finkelstein coordinates

$$dv = dt + \frac{dr}{f(r)}, \quad (10)$$

in terms of which the metric becomes

$$ds^2 = \frac{l^2}{z^2} \left(-dv^2 + 2dvdr + r^2 d\Omega_2^2 + dz^2 + \frac{2M}{r} dv^2 \right). \quad (11)$$

The first four terms are the AdS_5 background in Eddington-Finkelstein form, while the last is a rank-one perturbation. The metric therefore admits the Kerr-Schild decomposition (1) with

$$\phi = \frac{2M}{r} \frac{z^2}{l^2}, \quad k_M dx^M = \frac{l^2}{z^2} dv. \quad (12)$$

We now verify the required properties of k_M . The vector has a single nonvanishing component k_v . Writing the background as $\bar{g}_{MN} = (l^2/z^2)\hat{g}_{MN}$, where $\hat{g}_{MN}$ denotes the flat five-dimensional metric in Eddington-Finkelstein coordinates, the contravariant components are

$$k^M = \bar{g}^{MN} k_N = \hat{g}^{Mv} = \delta_r^M, \quad (13)$$

where we used $\hat{g}^{vv} = 0$ and $\hat{g}^{rv} = 1$. Nullity follows immediately:

$$\bar{g}_{MN} k^M k^N = \bar{g}_{rr} = \frac{l^2}{z^2} \hat{g}_{rr} = 0. \quad (14)$$

For the geodesic property, we compute

$$k^N \bar{\nabla}_N k_M = \partial_r \left(\frac{l^2}{z^2} \delta_M^v \right) - \bar{\Gamma}_{rM}^v \frac{l^2}{z^2}. \quad (15)$$

The first term vanishes as the only nonzero component of k_M is independent of r . For the second term, it can be shown that $\bar{\Gamma}_{rM}^v = 0$ for all M . Therefore k^M is an affinely parametrized null geodesic of the AdS_5 background.

B. Single copy

Having obtained the Kerr-Schild decomposition of the RSII black string, the single copy gauge field is defined through (5) and the identifications (12) as

$$A_M = \phi k_M = \frac{2M}{r} \delta_M^v, \quad (16)$$

so that the only nonvanishing component is

$$A_v = \frac{2M}{r}. \quad (17)$$

The field strength tensor, $F_{MN} = \partial_M A_N - \partial_N A_M$, has a single independent nonvanishing component,

$$F_{vr} = -\partial_r A_v = \frac{2M}{r^2}, \quad (18)$$

with all other components vanishing. In particular, the gauge field and its field strength are independent of the bulk coordinate z , reflecting the fact that the warp factor has been fully absorbed into the Kerr-Schild decomposition.

The single copy equation of motion is obtained by contracting the linearized Einstein equation (4) with a Killing vector ξ^M of the background. We choose $\xi = \partial_v$, which satisfies $\xi^M k_M = l^2/z^2$, naturally aligned with the direction along which k_M is supported. Specializing to RSII ($D = 5$, $\Lambda_5 = -6/l^2$), the contraction gives

$$\xi^N \bar{\nabla}_P [2\bar{\nabla}_{(M} h_{N)}^P - \bar{\nabla}^P h_{MN}] = -\frac{8}{z^2} \phi k_M. \quad (19)$$

Defining $T_{MNP} = 2\bar{\nabla}_{(M} h_{N)P} - \bar{\nabla}_P h_{MN}$, this can be written as

$$\bar{g}^{PQ} \bar{\nabla}_P (\xi^N T_{MNQ}) - \bar{g}^{PQ} (\bar{\nabla}_P \xi^N) T_{MNQ} = -\frac{8}{z^2} A_M, \quad (20)$$

where we identified $\phi k_M = A_M$ and moved ξ^N inside the covariant derivative, thus introducing a connection term.

Using adapted coordinates in which $\xi^M = \delta_v^M$ is a coordinate basis vector and evaluating the Christoffel symbols for the AdS_5 background, Eq. (20) reduces to

$$\bar{g}^{PQ} \bar{\nabla}_P \left[\frac{l^2}{z^2} F_{MQ} + \frac{2l^2}{z^3} \delta_Q^z A_M \right] = -\frac{10}{z^2} A_M. \quad (21)$$

The second term inside the brackets can be evaluated directly

$$\bar{g}^{PQ} \bar{\nabla}_P \left[\frac{2l^2}{z^3} \delta_Q^z A_M \right] = -\frac{10}{z^2} A_M, \quad (22)$$

which exactly cancels the right-hand side, leaving

$$\bar{\nabla}_N \left(\frac{l^2}{z^2} F_M^N \right) = 0. \quad (23)$$

Expanding the covariant divergence using $\bar{\nabla}_N (l^2/z^2) = -(2l^2/z^3) \delta_N^z$, this can be equivalently written as

$$\bar{\nabla}_N F_M^N - \frac{2}{z} F_M^z = 0, \quad (24)$$

making explicit the effect of the AdS_5 warp factor on the gauge dynamics through the coupling to F_M^z . Since F_{MN} has support only along the (v, r) directions, raising indices with the background metric does not generate a z component,

$$F_M^z = \bar{g}^{zP} F_{MP} = \frac{z^2}{l^2} F_{Mz} = 0, \quad (25)$$

and thus the second term in (24) vanishes identically. The equation of motion reduces then to

$$\bar{\nabla}_N F_M^N = 0, \quad (26)$$

away from the singularity at $r = 0$.

This equation must be understood in a distributional sense. The gauge field (17) is singular at $r = 0$, and therefore the divergence $\bar{\nabla}_N F_M^N$ vanishes everywhere except at the origin, where it produces a localized δ function source. In this sense, the solution describes a pointlike field generated by a localized charge at $r = 0$, in direct analogy with the Coulomb field arising from the double copy of the Schwarzschild geometry [3].

It is worth emphasizing that the z independence of A_v follows directly from the Kerr-Schild decomposition and should not be interpreted as a dynamical localization of the gauge field on the brane: gauge fields cannot be gravitationally trapped in the RSII scenario [34,35], and their confinement requires additional nongravitational mechanisms [36,37]. From the braneworld perspective, the restriction to the brane at $z = l$ reproduces the standard four-dimensional single copy, consistent with the role of the brane as the hypersurface where four-dimensional physics is recovered [22].

C. Zeroth copy

The final step in the classical double copy chain is to construct the zeroth copy, corresponding to a scalar field theory. This scalar is identified with the Kerr-Schild scalar function (12)

$$\Phi = \phi = \frac{2M}{r} \frac{z^2}{l^2}. \quad (27)$$

The equation of motion is obtained by contracting the linearized Einstein equation (4) twice with the same Killing

vector $\xi = \partial_v$ used in the single copy construction. Using the tensor T_{MNP} defined in Sec. III B, the double contraction gives

$$\bar{g}^{PQ}\bar{\nabla}_P(\xi^M\xi^NT_{MNQ}) - \bar{g}^{PQ}\bar{\nabla}_P(\xi^M\xi^N)T_{MNQ} = -\frac{8l^2}{z^4}\Phi. \quad (28)$$

Evaluating both terms on the left-hand side using adapted coordinates and the Christoffel symbols of the AdS_5 background, as in the single copy derivation, one arrives at

$$\square\Phi - \frac{8z}{l^2}\partial_z\Phi + \frac{20}{l^2}\Phi = 0, \quad (29)$$

away from the singularity at $r = 0$, where a localized δ function source appears, as in the single copy case.

Equation (29) differs from the standard Klein-Gordon equation by the presence of a first-order derivative term along the holographic direction z . This term has no counterpart in the flat-space or pure (A)dS zeroth copy equations of [3,7] and reflects the explicit z dependence of the zeroth copy scalar $\Phi \sim z^2$: unlike the single copy gauge field, the coupling to the conformal factor of the RSII spacetime is no longer trivial.

It is natural to ask whether a field redefinition can bring Eq. (29) into standard Klein-Gordon form. Defining

$$\Theta \equiv \frac{l^4}{z^4}\Phi, \quad (30)$$

a direct computation shows that the first-derivative coupling is removed and (29) transforms into

$$\left(\square - \frac{12}{l^2}\right)\Theta = 0. \quad (31)$$

The redefined field Θ therefore satisfies a Klein-Gordon equation with effective mass $m^2 = 12/l^2$, induced by the AdS_5 warp factor.

The two scalar fields have complementary bulk profiles and physical roles. The canonical zeroth copy $\Phi \sim z^2$ grows toward the Poincaré horizon, signaling that it is not localized near the brane and should be understood only as the geometrically natural variable directly encoding the Kerr-Schild structure. In contrast, $\Theta \sim z^{-2}$ decays along the holographic direction and is normalizable with respect to the natural bulk norm,¹ in line with the expected behavior of normalizable bulk modes in the RSII setup [21,22]. The fact that Θ satisfies a Klein-Gordon equation with a constant mass further supports its interpretation as the

physically propagating scalar degree of freedom in the bulk.

D. Alternative Kerr-Schild splitting

Motivated by the ambiguity in the Kerr-Schild decomposition discussed in the Introduction, we consider an alternative splitting of the RSII black string,

$$\phi = \frac{2M}{r}, \quad k_M dx^M = \frac{l}{z} dv. \quad (32)$$

This choice is related to the canonical splitting (12) through the rescaling (2) with $a(x) = z/l$, which leaves the metric (11) invariant. The vector k_M is null and geodesic with respect to the AdS_5 background, and the determinant satisfies $\det(g) = \det(\bar{g})$, confirming that (32) provides an equally valid Kerr-Schild representation.

Despite this equivalence at the gravitational level, the redistribution of the warp factor between ϕ and k_M leads to a physically distinct realization of the classical double copy, as we now show.

1. Single copy

The single copy gauge field now reads

$$\tilde{A}_v = \frac{2Ml}{rz}, \quad (33)$$

with all other components vanishing. The field strength has two independent nonvanishing components,

$$\tilde{F}_{vr} = \frac{2Ml}{r^2z}, \quad \tilde{F}_{vz} = \frac{2Ml}{rz^2}, \quad (34)$$

in contrast with the canonical single copy, where the gauge field is z independent and F_{MN} is purely four dimensional. Repeating the procedure of Sec. III B, i.e., contracting the linearized Einstein equation with $\xi = \partial_v$ (which now satisfies $\xi^M k_M = l/z$) and evaluating the Christoffel symbols of the AdS_5 background, one finds

$$\bar{\nabla}_N \tilde{F}_M{}^N = \tilde{J}_M, \quad (35)$$

with source current

$$\tilde{J}_M = -\frac{2M}{lr} \left(\frac{5}{z} \delta_M^v + \frac{1}{r} \delta_M^z \right). \quad (36)$$

Away from $r = 0$, this source is nonvanishing and extends along the holographic direction, in sharp contrast with the sourceless equation satisfied by the canonical single copy. The current is conserved, $\bar{\nabla}^M \tilde{J}_M = 0$, so the alternative splitting is internally consistent as a gauge theory, but the delocalized nature of the source signals that it does not admit a brane-localized interpretation.

¹For a scalar field in AdS_5 with metric (7), the natural norm is $\int_1^\infty dz \sqrt{-\bar{g}} |\Theta|^2 \sim \int_1^\infty dz (l/z)^5 |\Theta|^2$. For $\Theta \sim z^{-2}$ this integral converges, while for $\Phi \sim z^2$ it diverges.

The two single copies,

$$A_v = \frac{2M}{r}, \quad \tilde{A}_v = \frac{2Ml}{rz}, \quad (37)$$

are not related by a gauge transformation. Indeed, their difference

$$A_v - \tilde{A}_v = \frac{2M}{r} \left(1 - \frac{l}{z}\right) \quad (38)$$

would need to equal $\partial_v \Lambda$ for some scalar Λ , implying

$$\Lambda = \frac{2M}{r} \left(1 - \frac{l}{z}\right) v, \quad (39)$$

but then $\partial_r \Lambda \neq 0$, contradicting the fact that both fields have $A_r = 0$. Moreover, they satisfy different equations of motion: the canonical field is sourceless (away from $r = 0$), while the alternative one is supported by a delocalized bulk current.

2. Zeroth copy

The scalar associated with the alternative splitting is

$$\tilde{\Phi} = \frac{2M}{r}, \quad (40)$$

which is independent of the holographic coordinate. Repeating the double contraction of Sec. III C, one finds

$$\left[\square - \frac{12}{l^2}\right] \left(\frac{l^2}{z^2} \tilde{\Phi}\right) = 0. \quad (41)$$

In contrast with the canonical zeroth copy equation (29), Eq. (41) contains no first-derivative term along z , reflecting the z independence of $\tilde{\Phi}$. The equation of motion for the scalar field reduces to the massless Klein-Gordon equation

$$\square \tilde{\Phi} = 0. \quad (42)$$

The physical content of the two zeroth copies is markedly different. The canonical scalar $\Phi \sim z^2$ encodes the warp factor and, after the rescaling $\Theta \sim z^{-2}$, yields a normalizable mode localized near the brane satisfying a massive Klein-Gordon equation with $m^2 = 12/l^2$. The alternative scalar $\tilde{\Phi}$ is z independent and therefore not normalizable with respect to the bulk norm²: although it satisfies the massless equation $\square \tilde{\Phi} = 0$, the z dependence

²The norm $\int_{l^\infty} dz (l/z)^5 |\tilde{\Phi}|^2 \sim \int_{l^\infty} dz z^{-5}$ converges due to the AdS_5 measure and not to a nontrivial decay of the field profile, which carries no information about the warped geometry. In contrast, the canonical $\Phi \sim z^2$ does encode the warp factor, and its rescaled version $\Theta \sim z^{-2}$ is normalizable.

has been entirely stripped from the Kerr-Schild scalar, so that the zeroth copy retains no imprint of the extra dimension. The massive Klein-Gordon equation satisfied by $(l^2/z^2)\tilde{\Phi}$ is not a property of the alternative splitting itself, but rather a consequence of the same rescaling that defines Θ in the canonical construction.

Taken together, these results confirm that the physicality criterion of [7], requiring the absence of delocalized sources in the single copy and the retention of the warped bulk structure in the zeroth copy, selects the canonical splitting (12) as the physically preferred one. Different Kerr-Schild decompositions that are gravitationally equivalent encode different aspects of the warped geometry, and only the canonical choice leads to a double copy with a sourceless gauge field and a zeroth copy that carries a nontrivial imprint of the extra dimension.

IV. CONCLUSIONS

In this work we have constructed the Kerr-Schild classical double copy of the RSII black string in five dimensions, using the Poincaré patch of AdS_5 as the background. The canonical splitting of [7] yields a single copy gauge field that is independent of the holographic coordinate and satisfies a sourceless Maxwell equation on the curved background, in direct analogy with the Coulomb solution of the Schwarzschild double copy [3]. The zeroth copy scalar obeys a modified Klein-Gordon equation with a first-order derivative coupling along the extra dimension; a field redefinition removes this coupling and produces a standard Klein-Gordon equation with effective mass $m^2 = 12/l^2$, with the redefined field $\Theta \sim z^{-2}$ normalizable and localized near the brane. An alternative Kerr-Schild splitting, gravitationally equivalent to the canonical one, leads to a physically inequivalent double copy: the gauge field is supported by a conserved but delocalized bulk current, and the zeroth copy carries no imprint of the warped geometry. These results demonstrate that the Kerr-Schild double copy is sensitive to the warped bulk structure of braneworld models, and that the physicality criterion of [7]—requiring the absence of delocalized sources—plays an essential role in selecting the physically meaningful decomposition.

Our construction complements the recent analysis of [18], which studies the double copy of black strings in a four-dimensional cylindrical AdS background. While both works find gauge fields satisfying Maxwell-type equations on curved backgrounds, the five-dimensional RSII setting introduces qualitatively new features: the exponential localization of gravity along the extra dimension leaves a direct imprint on the gauge-theory description through the modified Klein-Gordon equation and the normalizability structure of the zeroth copy, which have no counterpart in that work.

Several directions remain open. A natural extension is to consider rotating and charged braneworld black

holes [27,29]; in particular, the tidal charge inherited from the bulk Weyl tensor in [27] offers a concrete probe of how extra-dimensional effects are encoded in the single copy. It would also be interesting to investigate whether the double copy can track the Gregory-Laflamme instability [33] of the RSII black string by comparing the gauge-theory data of the unstable solution with that of its expected stable end point and whether the construction extends to the stabilized black strings of [38]. From a holographic perspective, the effective mass $m^2 = 12/l^2$ corresponds via the standard AdS/CFT relation [39–41] to a dual operator of conformal dimension $\Delta = 6$; understanding whether the single and zeroth copies admit a natural boundary conformal field theory interpretation would connect the double copy program to the holographic description of braneworld gravity [42]. Finally, the Petrov type D character of the black string makes it a natural candidate for the Weyl double copy [43]. In particular, recent progress has extended the Weyl double copy construction beyond four dimensions, including five-dimensional type N and type D

spacetimes [44,45]. It would therefore be interesting to investigate whether the warped bulk geometry considered here admits a corresponding Weyl double copy description or whether the Randall-Sundrum warping introduces genuinely new features. We leave these questions for future work.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the author.

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